Discount Pricing*

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Abstract

This paper investigates discount pricing, the common marketing practice whereby a price is listed as a discount from an earlier, or regular, price. We discuss two reasons why a discounted price—as opposed to a merely low price—can make a rational consumer more willing to purchase the item. First, the information that the product was initially sold at a high price can indicate the product is high quality. Second, a discounted price can signal that the product is an unusual bargain, and there is little point searching for lower prices. We also discuss a behavioral model in which consumers have an intrinsic preference for paying a below-average price. Here, a seller has an incentive to offer different prices to identical consumers, so that a proportion of its consumers enjoy a bargain. We discuss in each framework when a seller has an incentive to offer false discounts, in which the reference price is exaggerated.

Keywords: Reference dependence, price discounts, sales tactics, false advertising.

1 Introduction

In his account of sales practices, Cialdini (2001, page 12) writes about

the Drubeck brothers, Sid and Harry, who owned a men’s tailor shop [...] in the 1930s. Whenever Sid had a new customer trying on suits in front of the shop’s three-sided mirror, he would admit to a hearing problem and repeatedly request that the man speak more loudly to him. Once the customer had found a suit he liked and asked for the price, Sid would call to his brother, the head tailor, at the back of the room, ‘Harry, how much for this suit?’ Looking up from his work—and greatly exaggerating the suit’s true price—Harry would call back, ‘For that beautiful, all wool suit, forty-two dollars.’ Pretending not

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to have heard and cupping his hand to his ear, Sid would ask again. Once more Harry would reply, ‘Forty-two dollars.’ At this point, Sid would turn to the customer and report, ‘He says twenty-two dollars.’ Many a man would hurry to buy the suit and scramble out of the shop with his [...] bargain before poor Sid discovered the ‘mistake’.

As this anecdote suggests, consumers are more likely to buy an item if they perceive it to be a bargain. This is easily understood when the consumer is given an accidental discount, as occurs for instance if she sees that the product she wants has been given the wrong price tag. If the product’s genuine price—which reflects its cost, quality and/or competitive environment—is $42, but by chance the consumer can get the product for $22, this represents genuine value-for-money and will make the consumer more inclined to purchase. This rational response to an accidental discount is exploited by the Drubecks’ fraudulent sales tactics.

What is more of a challenge is to explain why consumers might care about receiving a deliberate discount from a seller, as opposed simply to obtaining a low price. For instance, a consumer may be more likely to buy a jacket priced at $100 accompanied by a sign which reads “50% of its previous price” than he/she would be if the price were merely stated as $100. Alternatively, a retailer might claim its price was $100 even though the “manufacturer’s recommended price” was $200. Despite its prevalence, this pricing practice—which we term discount pricing—has apparently received little economic analysis. In the literature on sales (for instance, Lazear 1986), consumers care only about the price level, and whether a low price is framed as a discount off a higher price plays no role. In this paper, we explore the economics of discount pricing, focusing on the potential information content of a discount and its strategic implications. Our analysis is developed in two models that suggest different reasons why rational consumers care about discounts, as well as in a third model with behavioural consumers.

First, in section 2, uninformed consumers rationally take a monopoly seller’s initial price as a signal of its choice of quality, and so are willing to pay more for the product when they observe the initial price was high. The firm sells its product to two groups of consumers, one of which can accurately determine the product’s quality while the other group, the casual buyers, cannot. The monopolist can price discriminate between the two consumer groups using inter-temporal pricing, and the second group can use the price offered to the first group, when they observe it, as an indicator of quality. In this framework, it is more likely that the firm has an incentive to supply a high-quality product when casual buyers
can observe its initial price. Thus, the firm’s ability to write “was $200, now $100”, if credible, may induce it to provide a high-quality item.

In the second model, presented in section 3, the knowledge that a product is offered at a discounted price induces consumers to buy immediately rather than investigate a rival’s price. Two firms compete to sell to consumers, and either firm offers one of two prices: a full price or a sale price. (Price variation is generated by exogenous demand variation.) When a product is offered on sale, a consumer buys immediately even if that price is relatively high, and so a consumer cares about whether a discount is offered rather than the level of the actual price. If a consumer is given no credible information about whether the current price is discounted or not, she must judge how likely it is that the next price will be higher, given the current price, and buy accordingly. This inability to fine-tune her search process can cause welfare losses.

In our third model, in section 4, consumers intrinsically care about getting a bargain. Studies in behavioral economics (discussed shortly) have focused on how “reference prices”, which can sometimes be manipulated by a seller’s marketing activities, affect purchase decisions. In our model, a bargain is a price below the firm’s average offered price. If consumers observe the firm’s prices to all consumers, the firm responds to the “demand for bargains” by offering distinct prices to otherwise identical consumers. If the demand curve is concave, the firm follows a simple “high-low” pricing strategy with just two prices, a full price and a sale price. If instead consumers see only their own price, but hold equilibrium beliefs about the average price, the firm again has an incentive to pursue a high-low policy, but one with lower prices relative to when consumers see the prices offered to all consumers. When its prices are secret, the firm has a greater incentive to undercut its anticipated average price to some consumers, since others do not see this price cut and cannot react to it.

If, for whatever reason, consumers care about getting a discount, a seller may have an incentive to exploit this by making false claims about its previous or regular price. The outcome when these deceptive marketing tactics are used depends on the “savviness” of consumers. If consumers are aware that sellers are able to misrepresent their reference price without penalty, they will simply regard such sale signs as puffery and pay them no attention. The result is that a potentially useful channel of information is absent. However, if instead consumers are more gullible and believe a firm’s false claims (when such claims are plausible), the outcome is worse, as these consumers may be induced to pay more for the product than they would otherwise.
The media regularly features stories in which a seller’s claimed discounts are alleged to be fictitious. For instance, a supermarket’s heavily advertised 15% average price reduction may have been preceded by an unadvertised gradual price rise cancelling out the reduction. In Britain, a legal case involved the “Officers Club” chain of clothing stores, where it was alleged that only a tiny share of sales were made at the regular price and the great majority of items were sold at “70% off” this supposed regular price.¹ Several jurisdictions have rules in place to combat false discounting.² In the United States, the Federal Trade Commission’s Guides Against Deceptive Pricing (para. 233.1) distinguishes between genuine and fictitious discounts. For instance, “where an artificial, inflated price was established for the purpose of enabling the subsequent offer of a large reduction - the ‘bargain’ being advertised is a false one; the purchaser is not receiving the unusual value he expects. In such a case, the ‘reduced’ price is, in reality, probably just the seller’s regular price.”³

There are a number of earlier contributions which discuss issues related to our models. Our first model, where an initial price of a product signals its quality, builds on a large literature which studies how (current) price can signal quality. For instance, Bagwell and Riordan (1991) present a model where a firm has private information about the exogenous quality of its product. They find that high and declining prices signal high product quality: the firm distorts its price above the full-information level in order to signal high quality, and, as more consumers become informed, there is less price distortion in later periods. While their motivation is different from ours and their insights are derived mainly in a setting where the firm’s current price signals quality, they also consider an extension where consumers can observe the firm’s past price. In this case, the firm’s prices may be more distorted in period 1 but less distorted in period 2, compared to when past price is not observed, and they find that the high-quality firm has an incentive to reveal past price information to uninformed consumers. Thus, when a firm makes sequential sales of a product, the exogenous quality of which is the firm’s private information, a policy that

¹The England and Wales High Court (Chancery Division) found that the seller engaged in “misleading advertising”. See details in the judgement of 26 May 2005 of Justice Etherton of the case between the Office of Fair Trading and The Officers Club Ltd., www.bailii.org/ew/cases/EWHC/Ch/2005/1080.html. For instance, in paragraph 16 of this judgement, it states that between 1 September 2002 and 28 June 2003 only 0.15% of the total number of items sold in the chain of stores were at the “full price”. The judgement also discusses similar cases in other countries, such as Colorado vs. May Department Stores in the United States (para. 59), and Commissioner of Competition vs. Sears Canada Inc. in Canada (para. 63).

²Some jurisdictions also have policies to prevent permanent sales by requiring all sales to occur on stipulated dates. Thus the winter sales in Paris in 2012 had to take place between 11 January and 14 February.

³This document can be downloaded from www.ftc.gov/bcp/guides/deceptprc.htm.
bans false discounts would boost profit.

Muris (1991, section IIIC) and Rubin (2008, section III) discuss how the FTC has ceased fighting fictitious pricing cases in recent years, in part because it was often rival sellers—not consumers—who used the FTC’s Guides to prevent a firm’s heavy discounting, and in part because of a perception that any focus on price was potentially pro-competitive. However, our second model in section 3 suggests that complaints by rivals about a firm’s false sales can have a procompetitive motive: false discounts discourage consumers from investigating rival offers and deprive the rivals of opportunity to compete effectively. In these settings, preventing false discounting can lead to more effective competition.

Models and experiments from psychology and behavioral economics offer a number of insights on the use of discount pricing.\(^4\) Thaler (1985) proposes a model of consumer behaviour in which the context of a transaction matters to a consumer as well as the transaction itself. One implication of this theory is that firms can profit from a high “suggested retail price”, which serves as a reference price, and a lower selling price may then provide consumers with a “transaction utility”.

Bordalo, Gennaioli and Shleifer (2012) develop a model of salience in consumer decision making, which they use to explain a number of perplexing phenomena. Their analysis suggests that, by raising consumers’ valuation of quality through salience, firms can benefit from “misleading sales”—artificially inflating the regular price and simultaneously offering a generous discount. Jahedi (2011) experimentally investigates a kind of “bargain” which we do not study in this paper, where a seller offers two units of its product for little more than the price of one unit. He shows how consumers are less likely to buy two units when faced with the choice from \{buy nothing, buy two units for $1\} than they are when faced with the larger choice set \{buy nothing, buy one unit for $0.97, buy two units for $1\}. Jahedi designs the experiments so that subjects know that prices have no signaling role (such as the signaling roles we analyze in our first two models), and deduces that his subjects have an intrinsic “taste for bargains”.

Our third model is a model with consumer reference dependence, where consumers also have a taste for bargains. Spiegler (2011a, section 9.4.2) briefly outlines a related model, although his construction perhaps uses implausibly high prices (higher than any consumer’s raw valuation for the product). Most existing models of consumer reference dependence

\(^4\)Experimental evidence that consumers are influenced by false sales is discussed by Urbany, Bearden and Weibaker (1988). They also found more generally that an advertised reference price—plausible or exaggerated—raised consumers’ estimates of the firm’s regular price and the perceived offer value, and reduced consumer search for other sellers.
focus instead on loss-aversion, where a consumer’s propensity to buy falls when offered a price above her reference point. See Heidhues and Kőszegi (2005), Spiegler (2011b), Puppe and Rosenkranz (2011) and Zhou (2011) for models involving consumer loss aversion. Much of this literature finds that loss aversion makes a firm’s prices more rigid, for instance in response to cost variation, than would be the case in a “standard” model. By contrast, when consumers are bargain-loving, we show that a firm is more inclined to vary its prices than otherwise.

2 Initial Price as Signal of Product Quality

In this section we modify a standard static model of quality choice so that the firm sells over time. Specifically, a monopolist supplies a product over two periods, with its price in period \( t = 1, 2 \) denoted \( p_t \), and chooses its quality \textit{ex ante} which is then fixed for the two periods. The firm can choose one of two quality levels, \( L \) and \( H \), and it has constant unit cost \( c_i \) if it chooses quality \( i = L, H \). All consumers have unit demand. For simplicity, suppose the firm aims to maximize the sum of profits in the two periods.

A fraction \( \sigma \) of consumers are keen and particularly interested in the product: they can discern the product’s quality, and they are impatient and wish to buy only in period 1. Their valuation is \( v_i \) for the product when its quality is \( i = L, H \). The remaining \( 1 - \sigma \) consumers are casual buyers: they cannot directly observe quality and buy for simplicity only in period 2. (Little of substance in the analysis would be affected if some casual buyers also purchased in the first period.) Their valuation for the product is \( \theta v_i \) when quality is \( i = L, H \), where the parameter \( 0 < \theta \leq 1 \) reflects the plausible situation where casual buyers have a lower willingness-to-pay for the item. To avoid discussing sub-cases involving non-supply, we assume that

\[
\theta v_L > c_H \tag{1}
\]

so that the high-quality product can profitably be sold even to casual buyers who think quality is low. We also assume that providing the high-quality product is socially efficient, so that

\[
[\sigma + (1 - \sigma)\theta] \Delta_v > \Delta_c , \tag{2}
\]

where \( \Delta_v \equiv v_H - v_L \) and \( \Delta_c \equiv c_H - c_L \).

\(^5\)This static model is taken from Tirole (1988, section 2.3.1.1), which itself incorporates elements from a number of earlier contributions.
We study market equilibrium under alternative information assumptions. A consumer buys the item if the price is no higher than her willingness-to-pay, which depends on observed (if the consumer is keen) or anticipated (if casual) product quality. The firm’s strategy consists of its choice of quality and its two prices. In equilibrium the firm’s strategy is optimal given consumer buying behaviour, while the expectations of product quality by casual buyers, which may depend on observed prices, are consistent with the firm’s strategy.

Consider first the case where the casual buyers do not observe the firm’s initial price. A casual buyer’s anticipated quality might depend on the period-2 price. However, all that matters for the firm is the maximum price, say $P$, which induces a casual buyer to buy the product. (If the firm is going to sell to casual buyers it should set the highest possible price, regardless of its chosen quality.) Clearly, we have $\theta v_L \leq P \leq \theta v_H$, since the value of the item to the casual buyers is known to lie between these extremes. From (1), it is profitable to sell to these casual buyers, regardless of their beliefs about quality. Thus, given $P$, the firm’s profit if it chooses to supply the high-quality product is

$$\sigma (v_H - c_H) + (1 - \sigma)(P - c_H),$$

while its profit if it supplies the low-quality product is

$$\sigma (v_L - c_L) + (1 - \sigma)(P - c_L).$$

Comparing these two profits, we see that if

$$\sigma > \frac{\Delta_c}{\Delta_v},$$

the unique equilibrium is for the firm to provide a high-quality product, and the firm’s prices fully extract consumer surplus so that $p_1 = v_H$ and $p_2 = \theta v_H$. Thus, if the fraction of informed buyers is large enough, the firm makes more profit by serving these buyers with their preferred product than by supplying a low-cost product to all consumers. By contrast, if $\sigma < \Delta_c/\Delta_v$ the unique equilibrium is to provide a low-quality product, and prices are $p_1 = v_L$ and $p_2 = \theta v_L$. We summarize this discussion as:

**Lemma 1** Suppose that casual buyers cannot observe the firm’s initial price. If the fraction of keen buyers is large enough that (3) is satisfied, the unique rational expectations equilibrium is for the firm to supply a high-quality product, and to choose prices which fully extract consumer surplus (i.e., $p_1 = v_H, p_2 = \theta v_H$). If the fraction of keen buyers is small enough that (3) is strictly violated, the unique rational expectations equilibrium is for the
firm to supply a low-quality product, and to choose prices which fully extract consumer surplus (i.e., \( p_1 = v_L, p_2 = \theta v_L \)).

Consider next the case where casual buyers do observe the initial price. For instance, they see a price label which truthfully states “was $200, now $100”. A similar argument to that used for Lemma 1 establishes that when (3) holds, providing high quality is the unique equilibrium. But now, even if (3) fails, high quality can be supported in equilibrium. Specifically, suppose the firm chooses a particular initial price \( p_1 \) such that \( v_L < p_1 \leq v_H \). Suppose given \( p_1 \) that the maximum price which induces the casual buyers to buy is \( P \), where as before \( P \) lies in the range \( \theta v_L \leq P \leq \theta v_H \). Then the firm’s profit if it supplies a high-quality product is

\[
\sigma(p_1 - c_H) + (1 - \sigma)(P - c_H) ,
\]

while its profit if it provides a low-quality product is

\[
(1 - \sigma)(P - c_L) .
\]

(For this last expression, note that the firm does not sell to the informed buyers since \( v_L < p_1 \).) Thus, supplying a high-quality product is more profitable if

\[
\sigma(p_1 - c_L) > \Delta_c . \tag{4}
\]

In particular, we see that a higher initial price makes it more likely that offering a high-quality product is profitable, and in this sense a high initial price acts as a signal to casual buyers that quality is high. The reason is that a high initial price makes deviating to low quality more costly for the firm: if it deviates to low quality, it must forego serving the keen (informed) buyers and serving these buyers is more profitable with a higher initial price. Setting \( p_1 = v_H \) in (4) implies that first-best profit—where the firm supplies a high-quality product and chooses prices \( p_1 = v_H \) and \( p_2 = \theta v_H \)—is feasible if \( \sigma(v_H - c_L) > \Delta_c \), i.e., if

\[
\sigma > \frac{\Delta_c}{\Delta_c + [v_L - c_L]} . \tag{5}
\]

If this condition does not hold, there is no initial price which could convince casual buyers that quality is high. In this case the firm supplies a low-quality item and fully extracts the resulting consumer surplus.

Since condition (5) is less stringent than (3), we deduce that efficient quality provision is easier to achieve when the initial price is observed by casual buyers. When (5) holds but (3) does not, there is another equilibrium with low quality. As is usual in signaling games,
this multiplicity of equilibrium is due to the arbitrariness of beliefs off the equilibrium path. For clear-cut statements in the rest of this section, we assume that beliefs off the equilibrium path satisfy the “forward induction” refinement: when seeing a price off the equilibrium path, casual buyers reason what quality the firm could have rationally chosen given this price; if it is always optimal for the firm to choose $q$, then their belief is that quality is $q$. Then, when (5) holds, high quality is the unique equilibrium.\(^6\)

We summarize this discussion as:

**Lemma 2** Suppose that casual buyers can observe the firm’s initial price. If the fraction of keen buyers is large enough that (5) is satisfied, the unique rational expectations equilibrium is for the firm to supply a high-quality product, and to choose prices which fully extract consumer surplus (i.e., $p_1 = v_H, p_2 = \theta v_H$). If the fraction of keen buyers is small enough that (5) is strictly violated, the unique rational expectations equilibrium is for the firm to supply a low-quality product, and to choose prices which fully extract consumer surplus (i.e., $p_1 = v_L, p_2 = \theta v_L$).

If the firm can credibly reveal its initial price to casual buyers, then when the fraction of keen buyers lies in the range

$$\frac{\Delta_c}{\Delta_v + [v_L - c_L]} < \sigma < \frac{\Delta_c}{\Delta_v} \quad (6)$$

the firm will wish to do so. (When the fraction lies outside this range, communicating its initial price to casual buyers has no impact, as anticipated quality cannot be affected by the firm’s initial choice of price.) Welfare—which equals profit in this setting with full extraction of consumer surplus—also rises in this case.

We summarize the discussion as:

**Proposition 1** Relative to a setting where casual buyers cannot observe the initial price, if the firm can credibly communicate the initial price to casual buyers, this weakly (strictly if condition (6) holds) increases product quality, profit and welfare.

Now consider the scenario in which the firm is able to make any claim—true or false—about its initial price. If casual buyers are aware that the firm can make false claims about its discount without penalty, they will “discount the discount” and behave just as if they

\(^6\)At the potential low-quality equilibrium (with $p_1 = v_L$ and $p_2 = \theta v_L$), consider a deviation to high quality with $p_1 = v_H$ and $p_2 = \theta v_H$. Since with $p_1 = v_H$ it is always optimal for the firm to choose quality $H$, regardless of what $P$ is, the forward induction refinement implies that the casual buyers must believe that the firm has chosen $H$ upon seeing $p_1 = v_H$. This eliminates the low-quality equilibrium.
do not observe the initial price. When the fraction of keen buyers lies in the range (6), a policy which prevents firms making false claims about discounts will induce the firm to switch from offering a low-quality to a high-quality product, which will boost profit and welfare. The policy opens up a useful channel of information to otherwise uninformed buyers. In particular, if the casual buyers are savvy in this manner, the firm will welcome a policy which forbids it from making fictitious discount claims.

However, casual buyers might instead be “gullible” and believe the firm’s claims about its initial price when such claims are plausible. For instance, they might mistakenly think that effective consumer policy is already in place to prevent misleading price claims. If the fraction of keen consumers lies in the range (6), then faced with these more gullible casual buyers the firm would not switch to offering a high-quality product. Instead, the firm would produce a low-quality product, actually offer the initial price \( p_1 = v_L \) to the keen buyers, but claim to casual buyers that its initial price was \( p_1 = v_H \), who can therefore be charged price \( p_2 = \theta v_H \). The outcome is poor for casual buyers, who suffer negative consumer surplus. Thus, in the case with gullible consumers a policy which prevents misleading claims about initial prices not only ensures efficient quality choice (as was the case with savvy consumers), but now improves consumer welfare and reduces profit.

The idea that consumers care about a seller’s initial price because it signals product quality can be applied to other settings. Consider, for instance, the following variant of Lazear’s (1986) model of clearance sales. Suppose that the firm has only one unit of a product to sell, and that the quality of its product, denoted \( v \), is exogenous, uncertain, and initially unobserved even by the firm itself. In the first period, a keen consumer who observes \( v \) considers buying the product, and will buy if the initial price \( p_1 \) is below \( v \). If he chooses not to buy the product, a casual consumer in the second period considers whether to buy. The casual buyer does not directly observe \( v \), and bases her purchase decision on the expected value of \( v \), conditional on the item not having sold in the first period. In this setting, total supply is limited, and when the casual buyer sees the item on sale in period 2, she knows that demand from the keen buyer was low. This causes her to lower her estimate of quality. But the information content of the event that the item ends up on sale is less when the initial price was high, as fewer informed consumers would have been willing to buy at a higher price. That is, expected quality, conditional on the item remaining unsold,

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7 In an environment where casual buyers do not observe initial prices, there is no difference between “savvy” and “gullible” consumers, and both make rational inferences about a firm’s choice of quality.

8 In the fashion context, for example, \( v \) might represent whether or not the product’s colour or cut is fashionable that season, which is not something the firm knows in advance.
is higher when the initial price was higher.\textsuperscript{9} Hence, initial price again acts as a signal of quality, albeit for a reason very different to that in our endogenous quality model. It can be shown, however, that in this setting firm profit is lower, and consumer surplus is higher, when the initial price is \textit{not} observed. (We will obtain a similar result in our model presented in section 4.)

3 Discounts as a Signal to Buy Immediately

A second reason why consumers like a discounted price is because this may signal the price is unusually low, and they would do well to take advantage of it. This signal could potentially operate in two dimensions. In a monopoly context where the firm sets different prices over time, a discounted rather than full price might indicate the price is likely to go up, and the consumer should buy immediately rather than wait for a lower price. Alternatively, in a static oligopoly search context, a discounted price from one seller could indicate that rival prices are likely to be no lower, and there is little reason to investigate other sellers when search is costly. In this section we explore the latter possibility. (The dynamic monopoly model can be analyzed in a very similar manner.)

Before describing the analysis in detail, we point out that to investigate the question at hand we need a framework which is more complicated than standard models of search. As usual, we require a framework with price dispersion so that consumers sometimes have an incentive to search for a lower price. However, in order to discuss the impact of discounted prices, as opposed to merely low prices, we need the pattern of price dispersion itself to be uncertain from the consumer’s point of view. For instance, if the consumer knew the potential prices were \( p_L \) and \( p_H \), then if she first encounters \( p_L \) she knows the other price is either \( p_L \) or \( p_H \) and so does not benefit from additional information about whether the price is discounted.

In more detail, suppose two firms compete to sell a homogeneous product to consumers. The two firms sell repeatedly over time, although all consumers are short-lived and can buy only in their own period. A firm’s price is either \( p_L \) or \( p_H > p_L \) in each period with the probability of the latter being \( \alpha \), and price is independently realized in each period and across firms. We refer to \( p_L \) as the “sale” (or discounted) price and \( p_H \) as the “regular” (or full) price. The market parameters \((p_L, p_H, \alpha)\) are unchanging over time. Thus the regular and the sale prices are the same for both firms, although with probability \( 2\alpha(1 - \alpha) \) one

\textsuperscript{9}In Lazear’s model, the second consumer is also well informed about \( v \), and so does not care about the initial price.
firm runs a sale while its rival does not. For now we take the process of price determination to be exogenous. (The model will be “closed” in a particular way shortly.)

Suppose there are a number of “searchers” who are imperfectly informed about market prices. Specifically, they can travel to their local firm for free and see its price and, if desired, buy immediately from that firm (with an equal proportion of consumers local to each firm), but they need to incur a cost $s_1$ to travel to the second, to them more remote, firm and discover its price. Suppose a consumer can return to buy from her local firm after investigating the remote firm by incurring the further search cost $s_2$. Suppose prices are such that these searchers will always wish to buy the product from one firm or the other.

The ideal search rule for such a consumer, given known tariff parameters $(p_L, p_H, \alpha)$, is simple. If the consumer knows the local price is the sale price she will buy immediately, as the rival’s price cannot be lower. If the consumer knows the local price is the full price, she may decide to investigate the rival’s price in case it turns out to be discounted. If her local firm offers $p_H$, the risk-neutral consumer has an incentive to investigate the remote firm whenever $p_H \geq s_1 + \alpha p_H + (1 - \alpha)p_L$, i.e., when the expected sale discount $(1 - \alpha)(p_H - p_L)$ satisfies

$$ (1 - \alpha)(p_H - p_L) \geq s_1 . \quad (7) $$

If the local price is the full price, the consumer will nevertheless buy locally if (7) does not hold, as it is not worth incurring the search cost to obtain the small expected discount at the rival. A consumer will never return to buy from her local firm after travelling to the remote firm. This ideal stopping rule depends on whether the local product is offered on sale, and on the size and frequency of the sale discount, but not on price levels.

Now suppose a consumer is initially offered price $p$ from her local firm, without any credible information about whether this price is discounted. She must then decide whether to buy immediately purely on the basis of the price level. Moreover, a consumer might sometimes return to buy locally after travelling to the remote firm, thus incurring a double search cost $s_1 + s_2$. Suppose that tariff parameters $(p_L, p_H, \alpha)$ are uncertain from the viewpoint of the consumer. A consumer conditions the distribution of rival’s price $\tilde{p}$ on the local firm’s price $p$, and a consumer who sees local price $p$ will buy immediately if and only if

$$ p \leq s_1 + \mathbb{E}[\min\{\tilde{p}, p + s_2\} \mid p] . \quad (8) $$

Here, the right-hand side is the expected expense involved if the consumer travels to the remote firm: the search cost $s_1$ is sunk, but then the consumer has the ability to buy from whichever supplier is cheaper (after taking the cost of returning to the local seller into
account). The search rule in (8) will in general be inefficient compared to the search rule when the consumer knows when the local price is the discounted price, and so we expect that credible information about discounts will benefit consumers.

To investigate in more detail, we specialize and close the model in the following manner. Here, the firms’ price variation is generated by local demand shifts. Specifically, suppose in each period there are also a number of “inert” consumers can buy only from their local firm (to which they can travel costlessly). These consumers have unit demand, and their valuation for the unit can take one of three values: $V_L, V_M$ or $V_H$, where $0 < V_L < V_M < V_H$. The market operates in one of two states. In the first state, the possible valuations are $\{V_L, V_M\}$, and in any period and for either firm these two demand realizations are equally likely. In the other state, the possible valuations are $\{V_M, V_H\}$ and again these two demand realizations are equally likely. Each market state $\{V_L, V_M\}$ or $\{V_M, V_H\}$ is realized ex ante with equal probability. A firm knows which market state is realized, but in a given period does not observe its rival’s local demand realization. Suppose each firm’s production is costless.

The searchers are willing to pay up to $V_H$ for a single unit and their search costs are $s_1 = s_2 = s$, where $0 < s < V_H - V_M$. (The condition $s < V_H - V_M$ will ensure that the consumer will return to buy locally if she discovers the remote price is higher.) The key feature of this set-up is that when a searcher knows that local demand is $V_M$, she does not know if the market state is $\{V_L, V_M\}$ or $\{V_M, V_H\}$. We will derive an equilibrium in which each firm sets its price to fully extract surplus from their inert consumers, i.e., a firm chooses $p = V_i$ when its realized local demand is $V_i$. Intuitively, this pricing behaviour is an equilibrium whenever the proportion of searchers is small enough, as then a firm’s incentive to extract surplus from the inert consumers dominates the incentive to keep the searchers from investigating the rival firm.

For now, take as given this pricing rule by firms. What is the optimal search rule for the searchers? From (8), and given $s < V_H - V_M$, a consumer has an incentive to travel to the remote firm when the local price is $p = V_M$ (and then to travel back to the local firm if the remote price turns out to be $p = V_H$) if and only if

$$s \leq \frac{1}{s}(V_M - V_L).$$

A consumer has an incentive to travel to the remote firm when the local price is $p = V_H$ if

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10There are other ways to close the model. For instance, we might have inert consumers with a constant downward-sloping demand curve, and each firm has idiosyncratic shocks to its unit cost.
and only if
\[ s \leq \frac{1}{2}(V_H - V_M) \, . \] (10)
(Of course, a consumer will buy immediately if she is offered the lowest price \( V_L \).) A complicating factor is that this search rule may not be monotonic; that is, a consumer might search on when she sees the intermediate price \( p = V_M \) but not if she sees the highest price \( p = V_H \). The reason is that in the latter case, the consumer knows that the low price \( p = V_L \) is not a possibility, and it might be that this chance of the low price is what drives search incentives when \( p = V_M \). This possibility is ruled out if condition (9) implies condition (10), i.e., if \( \frac{1}{2}(V_H - V_M) \geq \frac{1}{5}(V_M - V_L) \). In particular, if the search cost is small enough that
\[ s < \frac{1}{5}(V_M - V_L) \leq \frac{1}{2}(V_H - V_M) \, , \] (11)
the optimal search rule is to buy immediately if the local price is \( p = V_L \) and otherwise to travel to the remote firm. (If the local price is \( p = V_M \) and the remote price is \( p = V_H \), the consumer will then return to buy locally.) Of course, this search rule is inefficient, as when the market state is \( \{V_M, V_H\} \) and the consumer is first offered price \( p = V_M \), she travels to the remote firm even though the price cannot be lower there. Nevertheless, the consumer always buys the product at the cheapest price available.

The following result describes market equilibrium when consumers do not know whether their local price is discounted or not:

**Lemma 3** Suppose parameters satisfy (11). Provided the proportion of searchers in the consumer population is sufficiently small, the following strategies make up an equilibrium when searchers have no credible information about whether the local price is discounted: (i) each firm sets its price to extract surplus fully from their inert consumers, i.e., a firm chooses \( p = V_i \) when its realized local demand is \( V_i \), and (ii) searchers buy immediately if the local price satisfies \( p \leq V_L \) and otherwise they travel to the remote firm.

**Proof.** We have already shown that this search rule is optimal given the claimed price choice by firms. To see that firms optimally price in the stated way given this consumer search rule whenever the proportion of searchers is sufficiently small, argue as follows. Suppose the number of inert consumers is \( N \) and the number of searchers is \( n \). Suppose for instance that the market state is \( \{V_L, V_M\} \) and a firm’s demand realization is \( V_M \). If the firm follows the stated strategy and sets price \( p = V_M \), its expected profit is \( V_M(\frac{1}{2}N + \frac{1}{4}n) \) since its \( \frac{1}{2}N \) inert consumers will buy and the \( \frac{1}{4}n \) searchers local to the rival firm will buy from it if the rival price is also \( V_M \), which occurs with probability \( \frac{1}{2} \). (The firm’s local searchers
will never buy from it.) If the firm deviates to price $p = V_L$, its profit is $V_L(\frac{1}{2}N + \frac{3}{4}n)$, since now the firm’s local searchers will buy from it as well. The latter profit is below the former when $\frac{n}{n+N}$ is small. Another potentially profitable deviation is to set price $p = V_M - s$, which will induce all searchers to buy from it in the event the rival price is $p = V_M$, and so generates profit $(V_M - s)(\frac{1}{2}N + \frac{1}{2}n)$. This is below $V_M(\frac{1}{2}N + \frac{1}{4}n)$ whenever the proportion of searchers satisfies $\frac{n}{n+N} < \frac{2s}{V_M}$. Similar arguments apply in other situations. ■

Note that if searchers could observe whether a firm’s price was discounted or not, the equilibrium outcome would be that a searcher buys immediately if and only if the local price was discounted, and firms continue to set prices to reflect local demand conditions.$^{11}$ Thus, a simplifying feature of this particular framework is that equilibrium prices are not affected by policy towards misleading pricing.

Suppose the market initially operates in a regime where nothing except the current price is revealed to consumers. When does a firm have an incentive to reveal more details about its pricing policy? The firm’s aim is simple: regardless of its current price state, it wishes to deter its local consumers from travelling to the remote firm. Suppose first that a firm can only make truthful claims about its prices. When the market state is $\{V_M, V_H\}$, a firm will announce that its price is discounted when $p = V_M$, as this will induce searchers to buy immediately (while otherwise they would have travelled to the other firm). Consumers are better off if they know when the local price is discounted, as this helps to refine their search strategy.

However, a firm has an incentive to mislead consumers, and falsely to claim its regular price is discounted. If firms are free to do so without penalty, savvy consumers will treat any claimed discount as cheap talk—they recognize that a firm will claim a price $p = V_M$ is discounted, regardless of whether the market state is $\{V_L, V_M\}$ or $\{V_M, V_H\}$—and so the outcome is as if consumers do not know whether or not the good is on sale. If instead consumers are more gullible, they believe a firm’s false claims whenever such claims are possible. In this framework, this implies that when the market state is $\{V_L, V_M\}$ and a seller’s price is $p = V_M$, the firm can claim its price is discounted (i.e., that the market state is $\{V_M, V_H\}$) and induce gullible consumers to buy immediately. (However, these consumers are not so gullible that they believe a firm’s claim that its price $p = V_H$ was discounted.)

Expected expenditure from the searchers in the various regimes can be calculated as

\[\text{Expected expenditure} = \begin{cases} \frac{1}{2}N + \frac{3}{4}n & \text{if } p = V_L \\ \frac{1}{2}N + \frac{1}{4}n & \text{if } p = V_M \\ \frac{1}{2}N + \frac{1}{2}n & \text{if } p = V_M - s \\ \frac{1}{2}N + \frac{1}{4}n & \text{if } p = V_M - s \end{cases}\]

\[\text{Expected expenditure} = \begin{cases} \frac{1}{2}N + \frac{3}{4}n & \text{if } p = V_M - s \\ \frac{2s}{V_M} \frac{1}{2}N + \frac{1}{4}n & \text{if } p = V_M \end{cases}\]

\[\text{Expected expenditure} = \begin{cases} \frac{1}{2}N + \frac{3}{4}n & \text{if } p = V_H \end{cases}\]

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\[\text{Expected expenditure} = \begin{cases} \frac{1}{2}N + \frac{3}{4}n & \text{if } p = V_H \\ \frac{2s}{V_M} \frac{1}{2}N + \frac{1}{4}n & \text{if } p = V_M \end{cases}\]

\[\text{Expected expenditure} = \begin{cases} \frac{1}{2}N + \frac{3}{4}n & \text{if } p = V_H \\ \frac{2s}{V_M} \frac{1}{2}N + \frac{1}{4}n & \text{if } p = V_M \end{cases}\]
follows. In the regime where searchers do not know when a price is discounted, a searcher’s expected outlay (including search costs where incurred) is\(^\text{12}\)

\[
\frac{3}{8}V_L + \frac{1}{2}V_M + \frac{1}{8}V_H + \frac{7}{8}s .
\]  

(12)

Likewise, when a searcher knows when a price is discounted, her expected outlay is

\[
\frac{3}{8}V_L + \frac{1}{2}V_M + \frac{1}{8}V_H + \frac{1}{2}s
\]

(13)

since she searches less often (although she makes exactly the same purchase decision). Finally, if the consumer is more gullible and always believes the price \(p = V_M\) is discounted, her outlay is

\[
\frac{1}{4}V_L + \frac{5}{8}V_M + \frac{1}{8}V_H + \frac{1}{4}s .
\]

(14)

Here, relative to the other regimes, the consumer searches too little and ends up with a more expensive product on average.

In sum, in this stylized framework a policy which prevents firms from making misleading claims about discounts is good for consumers. With such a policy, a firm will always reveal when its price is discounted, and this enables consumers to improve their search strategy. Absent the policy, a firm will always claim its product is on sale, and consumers will be worse off: savvy consumers will disregard the permanent sale signs and search in ignorance of whether the local price is discounted or not; more gullible consumers will fall victim to the sale signs and too rarely search for a lower price. Industry profits are not affected by policy when consumers are savvy, as consumers make exactly the same purchase decisions in either regime. However, if consumers are more gullible, policy which prevents misleading price claims will reduce profits, as consumers are more likely to search for a better deal.

In general, the impact of policy on welfare depends on the underlying process of price determination, i.e., on whether profit margins are higher or lower when price is high or low. However, the impact is easy to understand in this framework where unit costs do not vary, since the prices paid by consumers are merely a transfer to firms and have no impact on welfare. Welfare is then inversely related to how much search occurs in the various regimes. By inspecting expressions (12)–(14), we see that welfare is highest when consumers are gullible and firms mislead them with false sales, for then search is rare. If instead consumers are savvy and disregard false sale signs, then policy to prevent misleading

\(^{12}\)For instance, in this regime a consumer will pay the lowest price \(V_L\) when the market state is \(\{V_L, V_M\}\) and at least one of the two firms has price \(p = V_L\), which together occur with probability \(\frac{3}{8}\). The consumer makes a costly trip with probability \(\frac{7}{8}\), since she searches when the local price is not \(V_L\) and she makes two trips when the local price is \(V_M\) and the remote price is \(V_H\).
sales signs reduces the intensity of search and so boosts welfare. In sum, while the impact of policy on consumers alone is clear-cut in this model, the impact on overall welfare is more complex and depends on the presumed gullibility of consumers.

We summarize this discussion as:

**Proposition 2** In the oligopoly search setting, when firms provide accurate information about when their price is discounted this benefits consumers relative to the situation where no such information is available. Consumers buy immediately when they see a discounted price. A policy which prevents firms from falsely claiming discounts will benefit consumers regardless of whether or not consumers believe false sales signs. The impact on welfare depends on whether consumers are gullible or savvy.

4 Selling to Bargain-Loving Consumers

In our final model of discount pricing, we suppose that consumers intrinsically like the idea of getting a bargain. Thus, unlike models in sections 2 and 3, here we do not derive why it is that consumers care about receiving a discount, but simply take this as given. The model here, then, is a behavioural model with reference dependence. Unlike recent papers in industrial organization which focus on loss-aversion, we take the less familiar route of supposing consumers also enjoy a benefit if they pay a price below the reference price. In our model, the reference price is simply the average price offered by the firm.\(^{13}\)

Suppose that a monopolist sells to a unit mass of consumers with constant marginal cost \(c\), and chooses its price according to a mixed strategy with c.d.f. \(G(p)\) which has expected value \(\bar{p}\). (The firm offering a deterministic price as a special case of this framework.) Note that a given consumer is offered a single price, and cannot search for additional prices. To be concrete, we might imagine that the firm makes its price contingent on some arbitrary aspect of the consumer (e.g., location) which cannot easily be altered, and so pricing is not strictly random. Suppose a consumer’s “raw” valuation for the item is \(v\), which has smooth distribution function \(F(v)\). If the consumer is given a “rip-off” price \(p \geq \bar{p}\) then she buys if \(v - \lambda_R (p - \bar{p}) \geq p\), where \(\lambda_R \geq 0\) is a parameter which reflects her aversion

\(^{13}\)An important ingredient of any model with reference dependence is how the reference point is determined. Broadly speaking, Heidhues and Köszegi (2005) take the reference price to be the price a consumer expect to pay if she decides to buy, while Spiegler (2011b) takes the reference price to be the expected price offered by the seller (where that expected price is a random price draw from the firm, as might be generated by “word of mouth” for example). Puppe and Rosenkranz (2011) describe a model in which a manufacturer’s non-binding “recommended retail price” acts as the reference price for consumers, while Zhou (2011) studies an oligopoly model in which consumers take the price of one “prominent” seller as their reference price when they evaluate other offers.
to paying above-average prices. If the consumer gets a bargain price $p \leq \bar{p}$ then she buys if $v + \lambda_B(\bar{p} - p) \geq p$, where $\lambda_B \geq 0$ is a parameter which reflects her enjoyment of the bargain.

Consider to start with the case where consumers are accurately informed about the firm’s price policy (in particular, they know the average price $\bar{p}$, which, together with their own price, is what they care about). First, we show that it is always profitable for the monopolist to offer dispersed prices in this context, provided that consumers care more about getting a bargain than they do about avoiding a rip-off:

**Lemma 4** When consumers can observe the firms price policy, the firm prefers to offer dispersed prices than a uniform price when

$$\lambda_B > \lambda_R$$

**(Proof.** Let $p > c$ represent any profitable uniform price (not necessarily the most profitable uniform price). Suppose the firm deviates from this uniform price by offering two prices, $p_L = p - \varepsilon$ and $p_H = p + \varepsilon$ where $\varepsilon > 0$, where each price is offered to half the consumer population. (This modified strategy leaves the average price unchanged at $p$.) The firm’s profit with this new strategy is

$$\pi(\varepsilon) = \frac{1}{2}(p + \varepsilon - c)(1 - F(p + [1 + \lambda_B]\varepsilon)) + \frac{1}{2}(p - \varepsilon - c)(1 - F(p - [1 + \lambda_B]\varepsilon)) .$$

Differentiating this expression with respect to $\varepsilon$ shows that

$$\pi'(0) = \frac{1}{2}(p - c)f(p)[\lambda_B - \lambda_R] > 0 ,$$

where $f(\cdot)$ is the density associated with $F(\cdot)$. Thus, starting from any profitable uniform price, profit is increased by implementing a mean-preserving spread in its prices. ■

The intuition for this result is clear. Relative to a uniform price strategy, adding a small amount of noise to prices reduces demand from those consumers offered above-average prices and boosts demand from those who get a bargain, and given (15) the latter effect dominates. We deduce that the firm has an incentive to offer at least two prices when consumers are more bargain-loving than loss-averse. Clearly, if only a fraction of consumers had these preferences (while the rest were “rational” and cared only about their own price), the firm would still have an incentive to pursue this dispersed pricing policy. If instead consumers were more loss-averse than bargain-loving, so $\lambda_B < \lambda_R$, then the firm has no (local) incentive to disperse its prices. In sum, the presence of bargain-loving consumers
gives the firm an incentive to offer distinct prices to otherwise identical consumers: in order to satisfy a “demand for bargains”, the firm creates bargains by artificially dispersing its prices.

If we assume that the demand curve $1 - F$ is weakly concave, one can show that the firm will use only two prices in its optimal pricing policy. In order to derive this optimal policy, we suppose that the firm is restricted to offer prices which are sometimes accepted by consumers. (Or equivalently, that consumers ignore any price which is so high that demand at that price is zero when they calculate the average price.) Let $v_{\text{max}}$ be the maximum valuation in the support of $v$. (Since the demand curve is concave, we know there is such a valuation.) Stated precisely, the firm is restricted to choose a price policy such that

\[ p_{\text{max}} + \lambda R(p_{\text{max}} - \bar{p}) \leq v_{\text{max}}, \]  

where $p_{\text{max}}$ is the firm’s maximum offered price and $\bar{p}$ is its expected offered price. This assumption rules out a strategy in which the firm offers arbitrarily high prices to a tiny fraction of consumers, which are not accepted, which would then make $\bar{p}$ arbitrarily large without significant cost to the firm.\(^\text{14}\)

**Lemma 5** Suppose consumers have a preference for bargains in the sense that (15) holds and can observe the firm’s price policy. If demand $1 - F(v)$ is weakly concave and the firm chooses prices which satisfy (16), the firm wishes to use exactly two prices in its pricing scheme.

**Proof.** To avoid technicalities, suppose the firm offers a finite number of distinct prices (at least two in number), where price $p_i$ is offered to a fraction $\alpha_i > 0$ of consumers and average price is $\bar{p} = \sum_i \alpha_i p_i$. Clearly, at least one price is strictly above the mean and one price is strictly below the mean.

Note first that it cannot be optimal for the firm to set any price below cost. (If some prices were below $c$, then profit is strictly increased by adjusting such prices to equal $c$: this adjustment increases $\bar{p}$ and so boosts demand from all consumers with $p_i \geq c$, and it clearly increases profit from these hitherto loss-making consumers.) So suppose that all prices satisfy $p_i \geq c$.

Next, we claim that the firm optimally offers only one price which is strictly above the mean. (The following argument is essentially an instance of Jensen’s Inequality.) Suppose,\(^\text{14}\)A more satisfying solution to this problem would be for consumers to construct the “average price” in terms of the average accepted price among the consumer population instead of the firm’s average offered price. However, this alternative approach is substantially more complex to solve.
to the contrary, there are at least two distinct prices, say \( p_1 \) and \( p_2 \), where \( p_1 > p_2 > \bar{p} \). Suppose we reduce \( p_1 \) by \( \varepsilon > 0 \) and increase \( p_2 \) by \( \frac{\alpha_1}{\alpha_2} \varepsilon \), where \( \varepsilon \) is small enough that both prices remain above \( \bar{p} \) and that (16) continues to hold. By construction, the average price \( \bar{p} \) is not affected by this change, and so the profits obtained from all other prices \( p_i \notin \{p_1, p_2\} \) are unaffected. If we write \( \pi(\varepsilon) \) for the firm’s expected profits as a function of \( \varepsilon \), then

\[
\pi'(0) \equiv [F(p_1 + \lambda_R(p_1 - \bar{p})) - F(p_2 + \lambda_R(p_2 - \bar{p}))]
+(1 + \lambda_R) \left[ (p_1 - c)f(p_1 + \lambda_R(p_1 - \bar{p})) - (p_2 - c)f(p_2 + \lambda_R(p_2 - \bar{p})) \right].
\]

This expression is strictly positive: the first term \([\_\_\_\_]\) is strictly positive since \( F(.) \) is strictly increasing over this range, and the second term \([\_\_\_\_]\) is strictly positive from the assumption that \( 1 - F \) is weakly concave. We deduce that the original prices cannot be optimal, and so the firm chooses exactly one price above the average price in its optimal policy.

A similar argument shows that the firm’s optimal policy also involves a single price which is weakly below the mean. □

At least with concave demand, we deduce that the firm uses exactly two prices and so pursues a “high-low” price policy. It is then a simple matter to derive the firm’s optimal price policy. If the firm offers the full price \( p_H \) with probability \( \alpha \) and the discounted price \( p_L < p_H \) with probability \( 1 - \alpha \), its profit is

\[
(1 - \alpha)(p_L - c)[1 - F(p_L - \lambda_B \alpha(p_H - p_L))] + \alpha(p_L - c)[1 - F(p_H + \lambda_R(1 - \alpha)(p_H - p_L))].
\]

Consider the example where \( v \) is uniform on \([0, 1] \), \( c = 0 \) and \( \lambda_R = 0 \). Here, the most profitable uniform price is \( p^* = \frac{1}{2} \). One can check from (17) that the optimal pricing strategy is

\[
p_H = \frac{\sqrt{\lambda_B + 1} + 3}{8 - \lambda_B}; \quad p_L = \frac{p_H}{\sqrt{\lambda_B + 1}}; \quad \alpha = \frac{\sqrt{\lambda_B + 1} - 1}{\lambda_B}.
\]

This policy satisfies \( p_H > p^* = \frac{1}{2} > p_L \), so that the high price is above, and the low price is below, the optimal uniform price \( p^* = \frac{1}{2} \). This solution requires \( \lambda_B \) to lie in the range \( 0 < \lambda_B < 3 \) to satisfy (16). The policy converges to the optimal uniform price as \( \lambda_B \) becomes small. When \( \lambda_B = 1 \) the approximately optimal policy involves \( p_L = 0.44 \) and \( p_H = 0.63 \), and the full price is offered to 41% of consumers. Note that the average price here (\( \bar{p} \approx 0.52 \)) is higher than it would be if the firm charged a uniform price (for instance, because consumers did not exhibit reference dependence, so \( \lambda_B = 0 \)).\(^{15}\)

\(^{15}\)Spiegler (2011a, section 9.1.2) shows that in a model where loss-aversion is the dominant force average price falls relative to the standard case.
firm’s profit with this policy is about 0.26 and aggregate consumer surplus, taking their reference-dependent preferences at face value, is 0.15.

There are at least two ways to relax the strong assumption that consumers observe the firm’s full pricing policy, and instead observe only the price they themselves are offered. First, savvy consumers could hold equilibrium beliefs about the average price; second, consumers might be more gullible and believe the firm’s claims about its average price.\textsuperscript{16}

Consider first the situation where consumers hold equilibrium beliefs about the firm’s entire pricing strategy, even though they observe only their own price. That is to say, from a consumer’s viewpoint, the firm’s prices to other consumers are “secret”. If all consumers believe the average price is $P$, the firm’s expected profit when it offers price $p$ to a given consumer is $(p - c)(1 - F(p - \lambda_B(P - p)))$ if $p \leq P$ and $(p - c)(1 - F(p + \lambda_R(p - P)))$ otherwise.\textsuperscript{17} Thus, when (15) holds the firm faces a demand curve with an “inward” kink at the reference price $P$. In this case we have the following result.\textsuperscript{18}

**Lemma 6** Suppose consumers observe only their own price, and that the demand curve $1 - F(\cdot)$ is logconcave.\textsuperscript{19} If (15) holds then (i) there is no equilibrium in which the firm offers a uniform price, and (ii) there exists an equilibrium in which the firm offers exactly two prices, $p_L$ and $p_H$, where both of the these prices are below the most profitable uniform price $p^*$.

**Proof.** (i) If to the contrary $P$ is an equilibrium uniform price, anticipated by consumers, the firm cannot make greater profit by choosing $p < P$, so that

$$1 - F(P) - (1 + \lambda_B)(P - c)f(P) \geq 0,$$

and neither can the firm make greater profit by choosing $p > P$, so that

$$1 - F(P) - (1 + \lambda_R)(P - c)f(P) \leq 0.$$

These two inequalities are inconsistent if (15) holds.

\textsuperscript{16}In this paper we assume that the firm either makes all its prices public or none. An interesting variant is to suppose that the firm can selectively reveal its price policy to consumers, in which case it might reveal the average price to those consumers who get a bargain, but keep those who pay a high price in the dark.

\textsuperscript{17}Here, we assume consumers have “passive beliefs” about the average price, and the price $p$ a consumer is offered does not alter her anticipated $P$.

\textsuperscript{18}In formal terms, this result resembles the analysis in Zhou (2011). Like us, he finds that a seller faces demand with an inward kink and chooses prices according to a mixed strategy with exactly two prices; in his case, the prominent seller uses “sales” to influence a loss-averse consumer’s reference point when she evaluates the rival offer, while our firm uses “sales” to satisfy a consumer’s demand for bargains.

\textsuperscript{19}If $1 - F$ is weakly concave it is also logconcave.
(ii) We construct the “high-low” equilibrium as follows. Let consumers anticipate the average price \( P \). If the firm chooses a price strictly above \( P \), this price \( p_H \) must (locally) maximize \((p - c)(1 - F(p + \lambda_R(p - P)))\), and when demand is logconcave there is at most one such price, which is determined for given \( P \) by the first-order condition

\[
p_H = c + \frac{1 - F(p_H + \lambda_R(p_H - P))}{(1 + \lambda_R)f(p_H + \lambda_R(p_H - P))}.
\] (19)

Likewise, if the firm chooses a bargain price below \( P \), this price \( p_L \) must maximize \((p - c)(1 - F(p - \lambda_B(P - p)))\), which is uniquely determined for given \( P \) by the first-order condition

\[
p_L = c + \frac{1 - F(p_L - \lambda(P - p_L))}{(1 + \lambda_B)f(p_L - \lambda(P - p_L))}.
\] (20)

The firm must be indifferent between choosing the two prices \( p_L \) and \( p_H \), so that

\[
(p_L - c)(1 - F(p_L - \lambda_B(P - p_L))) = (p_H - c)(1 - F(p_H + \lambda_R(p_H - P))).
\] (21)

Finally, in equilibrium consumer expectations of the average price are fulfilled, so that

\[
P = \alpha p_H + (1 - \alpha) p_L
\] (22)

where \( \alpha \) is the fraction of consumers who pay \( p_H \). The four tariff parameters \( p_L, p_H, P \) and \( \alpha \) then solve the four equations (19)–(22).

To see that a solution to these four equations exists, argue as follows. First note that if we can find \( p_L, p_H \) and \( P \) satisfying (19)–(21) such that \( p_L < P < p_H \), then we can find an \( 0 < \alpha < 1 \) which satisfies (22). Therefore, we look for \( p_L, p_H \) and \( P \) satisfying (19)–(21) such that \( p_L < P < p_H \). Since \( 1 - F(\cdot) \) is logconcave, we can check that \( p_H \) in (19) is above \( P \) if and only if \( P \) is sufficiently small, and the threshold \( P \) which makes the firm choose \( p_H = P \) in (19) is

\[
P_H = c + \frac{1}{1 + \lambda_R} \cdot \frac{1 - F(P_H)}{f(P_H)}.
\]

Likewise, from (20) we can see that \( p_L \) is below \( P \) when \( P \) is sufficiently large, and the threshold \( P \) which makes the firm choose \( p_L = P \) in (20) is

\[
P_L = c + \frac{1}{1 + \lambda_B} \cdot \frac{1 - F(P_L)}{f(P_L)}.
\]

Given the logconcavity of \( 1 - F \) and assumption (15), it follows that \( P_L < P_H \). Thus, for any \( P \) in the range \( P_L < P < P_H \), the firm’s high price in (19) is above \( P \) and the firm’s discounted price in (20) is below \( P \). Note that both \( P_L \) and \( P_H \) are below \( p^* \), the optimal uniform price.
It remains to show that we can find $P$ in the range $P_L < P < P_H$ such that (21) holds. Consider the lower boundary $P = P_L$. By construction, when $P = P_L$ then $p_L = P_L$ in (20) in which case the firm’s profit when it chooses $p = p_L$ is $(P_L - c)(1 - F(P_L))$. But when $P = P_L$, the firm’s profit when it chooses $p_H$ in (19) is strictly higher than this, since the firm could have chosen $p_H = P_L$ which yields the same profit $(P_L - c)(1 - F(P_L))$. Thus, when $P = P_L$ the firm makes strictly greater profits by choosing $p_H$ in (19) than it does by choosing $p_L$ in (20). A similar argument establishes that when $P = P_H$, the firm does strictly better by choosing the lower price $p_L$ in (20) than by choosing $p_H$ in (19). By continuity, there exists at least one $P$ in the range $P_L < P < P_H$ where the firm is indifferent between choosing $p_L$ in (20) and $p_H$ in (19). This completes the proof.

In the same example where $v$ is uniform on $[0, 1]$, $c = 0$ and $\lambda_R = 0$, the equilibrium pricing policy in the regime where consumers observe only their own price can be shown from expressions (19)–(22) to be

$$p_H = p^* = \frac{1}{2} ; \quad p_L = \frac{p_H}{\sqrt{\lambda_B} + 1} ; \quad \alpha = P = \frac{\sqrt{\lambda_B} + 1 - 1}{\lambda_B} .$$

(23)

Note that the high price in this example is equal to the optimal uniform price, and from (19) this is true whenever $\lambda_R = 0$ so that consumers do not care when they pay an above-average price. When $\lambda_B = 1$, the firm’s profit as a function of its price $p$ offered to any particular consumer, given that the consumer believes average price is $P = \sqrt{2} - 1$, looks as shown on Figure 1. This figure illustrates the bimodal nature of profit with bargain-loving consumers, and the equilibrium is constructed so that the height of the two peaks coincides.

![Figure 1: Monopolist’s profit as function of $p$](image)

This price policy in (23) is qualitatively the same as in the case in (18) where a consumer can observe the firm’s prices for all consumers; in particular the percentage discount $p_L/p_H$
is the same and the likelihood of getting a bargain is the same. However, prices are now shifted downwards. Of course, quite generally, the firm’s profits here are lower compared to when consumers see the full range of prices, since the firm could choose the pricing policy seen with secret deals as its policy when its prices are public. In this linear demand example, aggregate consumer surplus is now higher, at about 0.2, and total welfare is higher when the firm’s prices are privately observed. Intuitively, when the firm makes secret deals with each consumer, the firm has a greater incentive to undercut the average price since other consumers do not observe, and cannot react to, the price cut.\footnote{The effect is analogous to the “secret deals” problem in vertical contracting, discussed in Rey and Tirole (2007), in which an upstream manufacturer who sells to two competing retailers has an opportunistic incentive to boost supply to a retailer when the other does not observe the deal.}

Suppose that the firm is able to make false claims about its average price. If consumers are savvy, they foresee that the firm has an incentive to exaggerate its average price to boost its demand from bargain-loving consumers, and so consumers discount its claims and behave as if they cannot observe the average price. In such a situation, a policy which enables the firm credibly to reveal its average price will help the firm and, at least in the linear demand example, harm consumers. If policy forces the firm to publish accurate information about its prices and the proportion of prices which are discounted, then any price-cut targeted at particular individuals reduces demand from other consumers, and so blunts the firm’s incentive to discount.\footnote{Again, this is similar to the impact of policy on the secret deals problem in vertical contacting, where a requirement to make the supplier’s deal to one retailer observed by another will boost supplier profits and harm final consumers.}

On the other hand, if consumers are more gullible and believe its claims, the firm’s profits are increased when it is able to make misleading claims. It can then obtain the benefit of boosting demand from perceived “bargains” without the cost of sometimes having to set inefficiently high prices. It would like to claim average price was as high as possible, so that it could then set high actual prices without cutting demand.\footnote{In this case, the welfare impact of a policy banning false discounts is more complicated, and depends on how one views a consumer’s utility from getting a “false bargain”.}

Summarizing our discussion of this model, we have:

\textbf{Proposition 3} \begin{itemize}
\item (a) Suppose consumers have an intrinsic preference for bargains. Then the monopolist will offer distinct prices to identical consumers. If demand $1 - F(p)$ is weakly concave, the firm will adopt a “high-low” pricing strategy and offer exactly two prices to the population of consumers. The firm’s profit is higher when consumers can observe its average price compared to when they have no information about its average price.
\end{itemize}
(b) Suppose demand is linear. A policy which prevents the firm from making false claims about its average price helps the firm and harms consumers and welfare if consumers are savvy and foresee the firm will exaggerate its average price. The same policy will harm the firm if consumers are gullible and believe its claims about average price.

5 Conclusion

This paper has explored some economic effects of discount pricing. We suggest two reasons why a discounted price—as opposed to a merely low price—may make a rational consumer more willing to buy. First, the information that the product was initially sold at a high price may indicate the product is high quality. Second, a discounted price can indicate that the product is an unusual bargain, and that there is little point searching for alternative, lower prices. We also discuss discount pricing with behavioural consumers. If consumers have an intrinsic preference for bargains, a seller has an incentive to offer different prices to identical consumers, so that a proportion of its consumers will enjoy a bargain. Information about discounts in this case assures consumers how good their deal is relative to the average, which boosts their willingness to purchase.

Because of their incentive to mislead customers, in some—but not all—of the situations we discuss, there is a potential role for policy to prevent sellers advertising false discounts. In all models, if consumers are gullible and believe—rather than merely ignore—a firm’s false claims, such a policy will help consumers and harm the firm. In most cases, the overall impact on welfare of a policy which combats false discounting is positive.23 If consumers are savvier, matters are more nuanced. In our model where the initial price serves to signal the choice of high quality, a ban on misleading claims will actually benefit the firm, as it makes it easier to signal its quality. In the model with oligopoly search, such a policy benefits consumers as they then learn when an offered price is a discounted price and can reduce their search effort. Finally, in our model of bargain-lovers, when consumers are savvy a ban on misleading price claims will help the firm but harm consumers. Policy which helps the firm make public its pricing policy overcomes its “secret deals” problem, to the detriment of consumers.

In any case, the potential benefit from regulatory policy can be realized only if it is effectively enforced. Indeed, weakly enforced policy may be worse than no policy: it may make consumers gullible and act on a firm’s false discounts, and it may harm honest sellers.

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23 The exception is the model of oligopoly search in section 3, where permanent sale signs induce gullible consumers to buy more often from their local seller, which reduces search costs.
who follow the letter of policy. As discussed by Muris (1991) and Rubin (2008), it is hard to enforce, or perhaps even coherently to formulate, policy towards misleading pricing. A basic problem is how to determine how few sales need to occur at the full price, or for how short a time the full price is available, for a sales campaign stating “was $200, now $100” to be classified as misleading. Sellers have a strong motive to make their customers feel they are getting a special deal, and they have myriad ways to achieve this. It is unrealistic and undesirable to suppose that regulation can address all forms of false discounting without unduly restricting a seller’s marketing abilities, and regulators should focus only on flagrant examples of deception.

**References**


