International Capital Flows and Credit Market Imperfections: 

a Tale of Two Frictions*

Alberto Martin†
CREI, UPF and CEPR

Filippo Taddei‡
Collegio Carlo Alberto and CeRP

November 2010

Abstract

The financial crisis of 2007-08 has underscored the importance of adverse selection in financial markets. This friction has been mostly neglected by macroeconomic models of financial frictions, however, which have focused almost exclusively on the effects of limited pledgeability. In this paper, we fill this gap by developing a standard growth model with adverse selection. Our main results are that, by fostering unproductive investment, adverse selection: (i) leads to an increase in the economy’s equilibrium interest rate, and; (ii) it generates a negative wedge between the marginal return to investment and the equilibrium interest rate. Under financial integration, we show how this translates into excessive capital inflows and endogenous cycles. We also explore how these results change when limited pledgeability is added to the model. We conclude that both frictions complement one another and argue that limited pledgeability exacerbates the effects of adverse selection.

Keywords: Limited Pledgeability, Adverse Selection, International Capital Flows, Credit Market Imperfections

JEL Classification: D53, D82, E22, F34

*We are grateful to Andrea Canidio, Luca Dedola, Paolo Epifani and Paolo Pesenti for very helpful discussions. We also thank Fernando Broner, Nobu Kiyotaki, Guido Lorenzoni, and Jaume Ventura for their comments and suggestions, as well as participants at the ECB-JIE “What Future for Financial Globalization?” and CEPR-Magyar Nemzeti Bank “Understanding Financial Frictions” conferences and seminar participants at Collegio Carlo Alberto, CREI-UPF, LUISS, Università Bocconi, Università di Bologna and Università de Montreal. Martin acknowledges financial support from the Spanish Ministry of Science and Innovation through grants ECO2008-01666, CONSOLIDER-INGENIO 2010 (CSD2006-00016) and Ramon y Cajal (RCY-2009-04524), the Generalitat de Catalunya-DIUE (2009SGR1157) and the Barcelona GSE Research Network.

†CREI, Ramon Trias Fargas, 25-27, Barcelona, Spain. E-mail: amartin@crei.cat
‡Collegio Carlo Alberto, Via Real Collegio 30, Moncalieri (TO), Italy. E-mail: filippo.taddei@carloalberto.org
1 Introduction

In recent years, two important developments have spurred renewed interest in the macroeconomic effects of financial frictions: global imbalances and the financial crisis of 2007-08. In the case of global imbalances, financial frictions have been invoked to account for the large and persistent capital flows from Asia to the United States and other developed economies (e.g. Caballero et al. 2008). According to this explanation, the ultimate reason behind these capital flows is that — being subject to financial frictions — Asian financial markets have been unable to supply the assets required to channel their high savings towards productive investment. Hence, these savings have flowed to developed financial markets in which these assets could be supplied. In the case of the financial crisis of 2007-08, financial frictions have also been invoked to explain the run-up to the crisis and the unfolding of events during the crisis itself (e.g. Bernanke 2009, Brunnermeier 2009). In most of these explanations, however, financial frictions are cast in an entirely different light: instead of constraining the supply of assets, thereby limiting the amount of resources that can be channeled towards productive investment, they are portrayed as the source of an excessive supply of assets that has channeled too many resources towards unproductive investment. Which of these views of financial frictions is correct?

The answer is that they both are, although each of these views has a different type of financial friction in mind. On the one hand, underprovision of assets and limited investment is typically attributed to some form of pledgeability constraint, which limits the amount of resources that creditors can seize from debtors in the event of default. On the other hand, overprovision of assets is typically attributed to some form of adverse selection, which fuels investment by unproductive or inefficient individuals. Since financial markets in the real world are jointly characterized by some measure of limited pledgeability and some degree of adverse selection, both views are useful to understand reality. But how do they complement one another? How does, for example, the presence of adverse selection affect the size and direction of capital flows in the presence of pledgeability constraints? How do these capital flows in turn affect the inefficiencies associated to adverse selection? Answering these questions is essential for gaining a thorough understanding of recent events. They are hard to address with existing macroeconomic models of financial frictions, however, which are mostly concerned with the effects of limited pledgeability while neglecting those of adverse selection. In this paper, we fill this gap by bringing adverse selection to the foreground.

To do so, we develop a standard growth model in which individuals need to access credit markets
to invest in capital accumulation. In particular, individuals are endowed with some resources and
an investment project for producing capital and they must decide whether (i) to undertake their
project and become entrepreneurs, in which case they demand funds from credit markets or (ii) to
forego their project and become savers, in which case they supply their resources to credit markets.
Crucially, it is assumed that the quality of investment opportunities differs across individuals, so
that it is in principle desirable for the most productive among them to become entrepreneurs and for
the least productive among them to become savers. To give adverse selection a central role in credit
markets, however, we also assume that an individual’s productivity is private information and thus
unobservable by lenders. What are the main consequences of this assumption for macroeconomic
outcomes?

The first-order implication of asymmetric information in credit markets is that, by preventing
lenders from distinguishing among different types of borrowers, it induces cross-subsidization be-
tween high- and low-productivity entrepreneurs. The reason for this is simple. Precisely because
lenders cannot observe individual productivities, all borrowers must pay the same contractual in-
terest rate in equilibrium. This implies that high-productivity entrepreneurs, who repay often,
effectively face a higher cost of funds than low-productivity entrepreneurs, who repay only seldom.
It is this feature that gives rise to adverse selection by providing some low-productivity individuals,
who would be savers in the absence of cross-subsidization, with incentives to become entrepreneurs.
There are thus two clear macroeconomic implications of adverse selection: (i) by boosting equi-
librium borrowing and investment, it leads to an increase in the economy’s equilibrium interest
rate, and; (ii) by fostering inefficient entrepreneurship, it generates a negative wedge between the
marginal return to investment and the equilibrium interest rate.

We show that both of these implications have important consequences for capital flows when
we allow the economy to borrow from and/or lend to the international financial market. First,
through its effect on the equilibrium interest rate, adverse selection induces the economy to attract
more capital flows than it otherwise would: relative to the full-information economy, then, the
presence of adverse selection boosts net capital inflows from the international financial market.
A second and related consequence is that, since the marginal return to investment lies below the
equilibrium interest rate, these capital inflows can be welfare-reducing: in the presence of adverse
selection, then, there is scope for optimal intervention in the form of government controls on capital
inflows. Finally, since the extent to which it distorts individual incentives depends on the state of
the economy, adverse selection exacerbates the volatility of capital flows, capital accumulation and
output.

This last point warrants some discussion. In our economy, for a given interest rate, the incentives of less productive individuals to become entrepreneurs are strongest when the capital stock and income are low: it is precisely in this case that they are most heavily cross-subsidized by productive entrepreneurs, since a substantial fraction of investment needs to be financed through borrowing. Under these conditions, then, adverse selection exerts a strong boost on investment, capital accumulation and capital inflows. As the economy’s capital stock and income increase, however, the extent of cross-subsidization decreases: individuals become wealthier, an increasing fraction of their investment must be financed with their own resources and entrepreneurship loses its appeal for less productive individuals. Economic growth therefore softens the overinvestment induced by adverse selection and its impact on investment, capital accumulation and capital inflows languishes. We show how, through this mechanism, adverse selection generates endogenous boom-bust cycles in which capital inflows fuel periods of positive capital accumulation and high growth that are followed by periods negative capital accumulation and economic contraction.

These findings on the effects of adverse selection are the exact opposite of the ones stressed by the literature for the case of limited pledgeability. The latter is the standard friction in existing models, which assume that there is a limit on the resources that creditors can appropriate in the event of a default because borrowers are capable of diverting part of the project’s proceeds. There are two clear macroeconomic implications that are recurrent in the literature: (i) by constraining equilibrium borrowing and investment, limited pledgeability leads to a decrease in the economy’s equilibrium interest rate, and; (ii) by preventing efficient investment from being undertaken, limited pledgeability generates a positive wedge between the marginal return to investment and the equilibrium interest rate. Clearly, the contrast between these implications of limited pledgeability and our findings for the case of adverse selection extend to the open economy as well. Our results thus complement the existing literature and provide a more accurate picture of the relationship between financial frictions and the macroeconomy.

Real-world credit markets are not characterized solely by adverse selection or by limited pledgeability, however, but rather by a mixture of the two. It is important then to know whether our findings regarding the effects of adverse selection are robust to the inclusion of limited pledgeability: after all, if one friction tends to boost investment while the other one tends to constrain it, one could think that they somehow offset one another. To address this question, we extend our baseline model to encompass both frictions. We find that there is a sense in which limited pledgeability and
adverse selection exacerbate one another so that, if anything, the inclusion of the former makes the consequences of the latter more severe.

The reason for this “complementarity” between both frictions is that their interaction prevents the interest rate from attaining market clearing. On the one hand, pledgeability constraints require the interest rate to be low in order for lenders to break even; on the other hand, a low interest rate decreases the returns to savings and induces unproductive individuals to become entrepreneurs, exacerbating adverse selection. The ultimate result is the combination of a low interest rate and a large and relatively unproductive pool of potential borrowers, which requires rationing to attain market-clearing. The interaction of both frictions is therefore more harmful than either one of them on its own, which either boosts or constrains total investment but does not affect the order in which projects are financed. The combination of both frictions instead does, so that – for each given level of investment – the average productivity of financed projects falls: the reason is that, due to credit rationing, those projects actually financed are randomly selected out of a larger pool of potential borrowers.

Our paper is related to the large body of research that studies the macroeconomic effects of financial frictions. This literature, which goes back to the contributions of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), stresses the role of borrowing constraints for macroeconomic outcomes. Of this literature, we are closest in interest and focus to the branch that has extended the analysis to open economies, studying the effects of contracting frictions on the direction and magnitude of capital flows. Most of these papers illustrate how contracting frictions can restrict an economy’s ability to borrow from the international financial market, thereby generating capital outflows even in capital-scarce economies. Gertler and Rogoff (1990), Boyd and Smith (1997), Matsuyama (2004) and Aoki et al. (2009) fall within this category. Similar models have been used recently to account for global imbalances. In Caballero et al. (2008), for example, high-growing developing economies may experience capital outflows due to pledgeability constraints that restrict their supply of financial assets. In Mendoza, Quadrini, and Rios-Rull (2007), it is instead the lack of insurance markets in developing economies that fosters precautionary savings and the consequent capital outflows. To the best of our knowledge, however, we are the first to analyze the implications of adverse selection for capital flows as well as its interaction with pledgeability constraints.

In its modeling of asymmetric information, our paper is related to the work on adverse selection by Bester (1985, 1987), DeMeza and Webb (1987), and Besanko and Thakor (1987). Of these, our model is closest to DeMeza and Webb (1987), in which adverse selection also fosters overinvestment.
In the implications of adverse selection for volatility our model is related to Martin (2008), who also shows how this type of friction can give rise to endogenous cycles.\footnote{In this regard, our paper is also related to the endogenous cycle literature, albeit less directly. Martin (2008) provides a brief discussion of this literature. Of these papers, perhaps the ones closest to ours are Reichlin and Siconolfi (2004) and Aghion, Bachetta and Banerjee (2004), the last of which stresses the link between financial frictions and volatility in small-open economies.}

The paper is organized as follows. Section 2 presents the basic setup. Section 3 studies the dynamics of the closed economy when credit markets are characterized by adverse selection and it extends these results to the inclusion of limited pledgeability. Section 4 studies the dynamics of the economy under financial integration, doing it first for the case of pure adverse selection and then extending these results to the inclusion of limited pledgeability. Finally, Section 5 concludes.

\section{Basic setup}

Consider an economy inhabited by overlapping generations of young and old, all with size one. We use $J_t$ to denote the set of individuals born at time $t$. Time starts at $t = 0$ and then goes on forever.

All generations maximize the expected consumption when old so that $U_t = E_t c_{t+1}$; where $U_t$ and $c_{t+1}$ are the welfare and the old-age consumption of generation $t$.

The output of the economy is given by a Cobb-Douglas production function of labor and capital:

$$y_t = F(l_t, k_t) = l_t^{1-\gamma} \cdot k_t^\gamma$$

with $\gamma \in (0, 1)$, and $l_t$ and $k_t$ are the economy’s labor force and capital stock, respectively. All generations have one unit of labor which they supply inelastically when they are young, i.e. $l_t = 1$. The stock of capital in period $t+1$ is produced through the investment made by generation $t$ during its youth.\footnote{That is, we assume that that capital fully depreciates in production. We also assume that the first generation found some positive amount of capital to work with, i.e. $k_0 > 0$.} In order to ensure that financial markets have an important role to play, we assume that individuals differ in their ability to produce capital.

In particular, individuals in each generation are indexed by $j \in J_t$ and they are uniformly distributed over the unit interval. Each of them is endowed with an investment project of fixed size, which requires $I$ units of output at time $t$. The project of individual $j \in J_t$ succeeds with probability $p_j = j \in [0, 1]$, in which case it delivers $\alpha \cdot I$ units of capital in period $t+1$. With probability $1 - p_j$, the project of individual $j \in J_t$ fails and it delivers nothing.

In this setting, the capital stock at $t+1$ depends not only on the total investment made at time $t$, but also on the productivity of such investment. In particular, if we let $E(p_t)$ denote the expected probability of success among investment projects undertaken at time $t$, we can define
\( A_t = A(p_t) = \alpha \cdot E(p_t) \) as the average productivity of such investment.\(^3\) Then, we can write the law of motion of capital as:

\[
k_{t+1} = A_t \cdot s_t \cdot k_t^\gamma, \tag{1}\]

where \( s_t \) is the investment rate, i.e. the fraction of output that is devoted to capital formation. Markets are competitive and factors of production are paid the value of their marginal product:

\[
w_t = w(k_t) = (1 - \gamma) \cdot k_t^\gamma \quad \text{and} \quad q_t = \gamma \cdot k_t^{\gamma-1}, \tag{2}\]

where \( w_t \) and \( q_t \) are the wage and the rental rate of capital, respectively.

To solve the model, we need to find the investment rate and the expected productivity of investment. In our economy the investment rate is straightforward: the old do not save and the young save all their income. What do the young do with their savings? The young as a group can only use them to build capital. This means that the investment rate equals the savings of the young. Since the latter equal labor income, which is a constant fraction \( 1 - \gamma \) of output, the investment rate is constant as in the classic Solow (1956) model:

\[
s_t = 1 - \gamma. \tag{3}\]

For a given initial capital stock \( k_0 > 0 \), a competitive equilibrium of our economy is thus a sequence \( \{k_t\}_{t=0}^\infty \) satisfying Equations (1) and (3). A full characterization of such an equilibrium clearly requires an understanding of the way in which \( A_t \) is determined: this depends on the workings of credit markets, which intermediate resources among the young in each generation. To save for old age, each young individual must choose between (i) becoming an entrepreneur and undertaking an investment project, which requires credit whenever \( I > w_t \), and; (ii) lending his wage to other individuals who want to become entrepreneurs in exchange for an interest payment. We assume that all such borrowing and lending is intermediated through banks. Banks are finite in number, risk neutral and competitive. They act as intermediaries that collect deposits from individuals to offer loan contracts to active entrepreneurs. On the deposit side, they take the gross interest factor on deposits \( r_{t+1} \) as given and they compete on the loan market by designing contracts that take the following form:

\[^3\text{That is, } A_t \text{ denotes the average units of capital produced per unit invested in such projects.}\]
Definition 1 Entrepreneurs and banks sign a contract defined by the couple \((L_t, R_{t+1})\), where \(L_t\) is the amount lent to entrepreneurs for investment at time \(t\) and \(R_{t+1}\) is the gross contractual interest rate on the loan at time \(t + 1\). In the event of success, entrepreneurs pay back the amount borrowed adjusted by the interest factor. Otherwise, they default and the bank gets nothing.

This implies that the expected profit that individual \(j \in J_t\) obtains from loan contract \((L_t, R_{t+1})\) in the event that he chooses to become an entrepreneur is

\[
\pi_t(p_j, L_t, R_{t+1}) = p_j \cdot [q_{t+1} \cdot \alpha \cdot I - R_{t+1} \cdot L_t].
\]  

Since competition among banks is usually crucial in determining the types of contracts that are offered in equilibrium, it is important to specify how we model it. We follow the traditional model of Rothschild and Stiglitz (1976) and model competition in the credit market as a two-stage game of screening. In the first stage, banks design a menu of loan contracts and, in the second stage, individuals that want to become entrepreneurs apply to the contract that they find most attractive. It is assumed that each bank gets the same share of total deposits and, if they design the same contract, they get the same share and composition of loan applications.

3 Equilibria in the closed economy

The key driving force behind the dynamics of our economy lies clearly in the production of capital and hence in the functioning of credit markets. We now analyze the competitive equilibrium of the economy under different assumptions regarding these markets. To do so, we first consider the case of frictionless credit markets, which will provide a useful benchmark that we can turn to throughout the paper. We then analyze the case in which credit markets are characterized by the presence of asymmetric information, and we contrast it to the more familiar one of limited pledgeability.

Regardless of the particular credit-market friction that is imposed, there are two features that any equilibrium must satisfy. First, all contracts offered must satisfy a zero-profit condition for banks: clearly, no equilibrium contracts can yield negative profits to intermediaries, and – due to perfect competition – no equilibrium contracts can yield positive profits either. Second, investment in equilibrium must satisfy a “participation constraint”: since all individuals care only about old-age consumption, they will only choose to become entrepreneurs if the return of doing so exceeds that of being a depositor in the banking system.
3.1 The frictionless economy

In the absence of any friction, the equilibrium of our economy is straightforward. Given that any individual must borrow $L_t = I - w_t$ to become an entrepreneur, the only relevant degree of heterogeneity among individuals is the probability of success of the investment project. Individual $j \in J_t$ will decide to become an entrepreneur if and only if the expected profits from starting the project (Equation (4)) exceeds the revenues from depositing his funds in the banks. Formally, the participation constraint is given by:

$$\pi_t(p_j, L_t, R_{t+1}) = p_j \cdot [q_{t+1} \cdot \alpha \cdot I - R_{t+1}(j) \cdot (I - w(k_t))] \geq r_{t+1} \cdot w(k_t), \quad (5)$$

where $R_{t+1}(j)$ denotes the contractual interest rate faced by individual $j \in J_t$.

Because of the zero-profit condition of banks, we know that the contractual interest rate will vary across borrowers according to their probability of success. In particular, it must be true in equilibrium that

$$p_j \cdot R_{t+1}(j) = r_{t+1} \iff R_{t+1}(j) = \frac{r_{t+1}}{p_j}, \quad (6)$$

which allows us to rewrite Equation (5) as:

$$p_j \cdot [q_{t+1} \cdot \alpha \cdot I - \frac{r_{t+1}}{p_j} \cdot (I - w(k_t))] \geq r_{t+1} \cdot w(k_t). \quad (7)$$

Equation (7) determines a critical probability of success $\hat{p}_t$, below which individuals prefer to deposit their wage in banks instead of becoming entrepreneurs. Formally,

$$\hat{p}_t = \frac{r_{t+1}}{q_{t+1}} \cdot \frac{1}{\alpha}, \quad (8)$$

which has a very natural interpretation. In the absence of financial frictions, only those projects that yield a rate of return that is higher than the interest rate will be undertaken, i.e. those projects for which $p_j \cdot q_{t+1} \cdot \alpha \geq r_{t+1}$. By increasing the opportunity cost of becoming an entrepreneur, higher interest rates on deposits raise the threshold productivity $\hat{p}_t$ and lower aggregate investment; on the contrary, by increasing the return of becoming an entrepreneur, a higher future price of capital.

---

4In a setting with uncertainty, the participation constraint at time $t$ would be a function of the expected return to capital at time $t + 1$: in our environment, there is perfect foresight and hence the participation constraint depends directly on $q_{t+1}$. Naturally, the addition of uncertainty to our economy would be straightforward since all individuals are risk neutral.
or productivity of investment $\alpha$ both lower the threshold probability of success $\hat{p}_t$ and expand aggregate investment.

Any equilibrium in the credit market must therefore satisfy Equation (8). But it must also satisfy a market clearing condition, since the supply of savings has to be matched by an equal internal demand for investment in equilibrium. This condition, which also depends on the productivity of the marginal investor, can be expressed as follows:

$$\hat{p}_t \cdot w(k_t) = (1 - \hat{p}_t)(I - w(k_t))$$

$$\Leftrightarrow \hat{p}_t = 1 - \frac{w(k_t)}{I}.$$  \hspace{1cm} (9)

Equations (8) and (9) jointly determine the credit-market equilibrium of our economy, characterized by a pair $(\hat{p}_t, r_{t+1})$.\(^5\) It follows directly that, in equilibrium, the average productivity of investment at time $t$ is given by

$$A(\hat{p}_t) = \alpha \cdot \left[1 - \frac{w(k_t)}{2 \cdot I}\right],$$  \hspace{1cm} (10)

which is decreasing in wages. Intuitively, as the economy grows and wages increase, so does investment and less productive projects are therefore undertaken. Equations (8) and also provide the equilibrium interest rate for this economy:

$$r_{t+1} = q_{t+1} \cdot \alpha \cdot \left(1 - \frac{w(k_t)}{I}\right).$$  \hspace{1cm} (11)

Finally, the law of motion of this economy follows from replacing Equations (3) and (10) into Equation (1):

$$k_{t+1} = \alpha \cdot \left[1 - \frac{(1 - \gamma) \cdot k_{t-1}^\gamma}{2 \cdot I}\right] \cdot (1 - \gamma) \cdot k_{t-1}^\gamma,$$  \hspace{1cm} (12)

which can be shown to be increasing and concave as long as wages do not exceed the size of investment projects $I$, which is clearly the case of interest to us. We assume that this holds throughout.\(^6\)

\(^5\)The rental price of capital $q_{t+1}$, which is also endogenous, depends ultimately on $\hat{p}_t$.

\(^6\)A sufficient condition for wages to always lie below $I$ is that

$$I > \left(\frac{\alpha}{2}\right)^\frac{1}{1 - \gamma}. \hspace{1cm} (11)$$

This comes from considering that the maximum steady-state level of capital of this economy can never exceed

$$\left[\frac{\alpha}{2(1 - \gamma)}\right]^\frac{1}{1 - \gamma},$$

and making sure that even at this steady-state wages do not exceed the size of investment projects $I$.  \hspace{1cm} (12)
3.2 Adverse selection

Consider now that we modify the previous setup by introducing a friction in credit markets. In particular, we initially focus on a type of friction that has allegedly been at the heart of the recent turmoil in financial markets: adverse selection. Relative to the model of Section 3.1, the only modification that we make is to assume that individual $j$’s probability of success is private information and is thus unobservable to banks.\(^7\) Because this is the only dimension along which projects differ from one another, banks will now offer one “pooling” contract that will be accepted by individuals that differ in their probability of success.\(^8\)

If we use $(\hat{L}_t, \hat{R}_{t+1})$ to define the pooling loan contract offered by banks under adverse selection, the participation constraint of an individual $j \in J_t$ is now given by:

$$\pi(p_j, \hat{L}_t, \hat{R}_{t+1}) = p_j \cdot \left[ q_{t+1} \cdot \alpha \cdot I - \hat{R}_{t+1} \cdot (I - w(k_t)) \right] \geq r_{t+1} \cdot w(k_t),$$

which is essentially the same as Equation (5) with the difference that the contractual interest rate $\hat{R}_{t+1}$ is now independent of the individual’s probability of success $p_j$. We can use Equation (13) to obtain the marginal investor, i.e. the investor that is indifferent between applying to a loan contract or depositing his savings in the bank. We use $\hat{p}_{AS,t}$ denote the probability of success of this investor, where the subscript $AS$ indicates the presence of adverse selection. The zero-profit condition of banks takes this into account because, since banks must break even on average, the contractual interest rate $\hat{R}_{t+1}$ must reflect the average quality in the pool of borrowers. Formally, it must hold in equilibrium that:

$$E_j [p_j | p \geq \hat{p}_{AS,t}] \cdot \hat{R}_{t+1} - r_{t+1} = 0 \iff \hat{R}_{t+1} = \frac{r_{t+1}}{E_j [p_j | p \geq \hat{p}_{AS,t}]} = 2 \cdot \frac{r_{t+1}}{1 + \hat{p}_{AS,t}}$$

\(^7\)In our setup, in which this is the only dimension along which projects differ from one another, the debt/loan contracts analyzed in the previous section cannot be improved upon by banks.

\(^8\)In this sense, our environment is similar to DeMeza and Webb (1987).
By combining Equations (13) and (14) we can obtain the equivalent of Equation (8):

\[
\hat{p}_{AS,t} \cdot \left[ q_{t+1} \cdot \alpha \cdot I - 2 \cdot \frac{r_{t+1}}{1 + \hat{p}_{AS,t}} \cdot (I - w(k_t)) \right] = r_{t+1} \cdot w(k_t)
\]

\[
\iff r_{t+1} = \frac{\hat{p}_{AS,t} \cdot q_{t+1} \cdot \alpha \cdot I}{w(k_t) + 2 \cdot \frac{\hat{p}_{AS,t}}{1 + \hat{p}_{AS,t}} \cdot (I - w(k_t))},
\]

(15)

which defines an increasing relationship between \( \hat{p}_{AS,t} \) and \( r_{t+1} \) that must be satisfied in equilibrium. Together with the market clearing condition of Equation (9), this relationship determines the credit-market equilibrium of the economy \((\hat{p}_{AS,t}, r_{t+1})\) which is characterized as follows:

\[
\hat{p}_{AS,t} = 1 - \frac{w(k_t)}{I},
\]

(16)

\[
r_{t+1} = q_{t+1} \cdot \alpha \cdot \frac{[2I - w(k_t)] \cdot [I - w(k_t)]}{I^2 + [I - w(k_t)]^2}.
\]

(17)

A direct comparison of Equations (9) and (16) reveals that \( \hat{p}_t = \hat{p}_{AS,t} \), so that the introduction of asymmetric information does not change the average productivity of projects undertaken in the closed economy. This follows from two special assumptions in our model: (i) since savings are inelastic, investment must equal the total wage bill of the economy at all times, regardless of whether there is asymmetric information or not, and; (ii) since projects are of fixed size, the total investment undertaken in the economy is simply equal to this wage bill divided by the size of each project \( I \). This means that the presence of asymmetric information does not affect the law of motion of capital in our economy, which is still given by Equation (12). At the end of the day, all savings must be invested in capital formation and the presence of adverse selection does not affect the order in which projects are financed. Although none of our qualitative results depend on it, we find this to be a desirable feature of our model because it will allow us to isolate (i) the economic effects of the interaction between adverse selection and limited pledgeability, which we address in Section 3.4, and; (ii) the economic effects of adverse selection under financial integration, which we address in Section 4.

But how is it that, despite the presence of asymmetric information, no individual with \( p_j < \hat{p}_{AS,t} \) is tempted to become an entrepreneur? The answer, as can be seen by comparing Equations (11) and (16),

---

9 Once again, we do not include the rental price of capital \( q_{t+1} \) in the definition of equilibrium because it is determined by \( \hat{p}_{AS,t} \) (see Footnote 5).
and (17), is that the equilibrium interest rate increases in order to discourage this type of entry. In the presence of adverse selection, less productive individuals are effectively cross-subsidized in their loan payments by their more productive peers: consequently, for any given interest rate on deposits, the demand for credit is larger than it would be in the frictionless economy, i.e. the productivity of the marginal investor is lower. In other words, under adverse selection, the productivity of the marginal investor lies below the market interest rate. In the closed economy, in which the total amount of investment must necessarily equal the total wage bill, this leads to an increase in the interest rate in order to restore equilibrium. This increase in the interest rate relative to the frictionless economy of Section 3.1 is the sole consequence of adverse selection.10

3.3 Limited pledgeability

In analyzing the macroeconomic effects of financial frictions, the recent literature has focused predominantly on the effects of limited pledgeability.11 The severity of this friction, which arises when borrowers are capable of diverting part of their ex-post resources away from the reach of creditors, is believed to be a good indicator of the quality of financial institutions in an economy.12 Because of this, we want to understand how our analysis of adverse selection extends to the case in which credit markets are also characterized by some form of limited pledgeability. To do so, we first show briefly how the introduction of this friction affects – by itself – the equilibrium of the frictionless economy of Section 3.1.13 We will then analyze the equilibrium of our economy when both frictions are simultaneously present.

Consider then that we modify the frictionless economy by assuming that, in the event of default, lenders can seize at most a fraction \( \lambda \in [0, 1] \) of the resources of borrowers. In this case, the set of

\footnotesize

10 Once again, this result depends on our assumptions regarding the perfectly inelastic supply of total savings and the fixed size of investment projects. It is because of these two features that adverse selection does not affect the amount or the productivity of investment. If total savings were increasing on the interest rate, for example, adverse selection would lead to an increase in the equilibrium level of investment. If projects did not have a fixed size, adverse selection would also affect the composition of investment. These modifications would complicate the exposition without adding much to our results.

11 See, for example, Aoki et al. (2009), Caballero and Krishnamurthy (2001), Matsuyama (2004) and Lorenzoni (2008).

12 This is true both in the theoretical and in the empirical literature. In the latter, the quality of financial institutions is usually proxied with the creditor rights index based on La Porta et al. (1998). This index, which is the leading “institutional” predictor of credit market development around the world, measures the powers of secured lenders in bankruptcy and it essentially reflects the ability of these lenders to seize assets in the event of default.

13 This has the advantage of allowing us to contrast the findings of the previous section with those that arise most commonly in the literature.
loan contracts \((L_t, R_{t+1})\) that can be implemented are those for which:

\[
R_{t+1}(j) \cdot L_{t+1} = R_{t+1}(j) \cdot (I - w(k_t)) \leq \lambda \cdot q_{t+1} \cdot \alpha \cdot I. \tag{18}
\]

Equation (18) introduces an additional constraint that, in addition to the participation constraint of entrepreneurs in Equation (5) and the zero-profit condition of banks in Equation (6), loan contracts must satisfy in equilibrium. Note that, when the pledgeability constraint of Equation (18) binds in equilibrium, the participation constraint of Equation (5) is slack. In this case, some individuals that would invest in the frictionless economy cannot do so in the presence of limited pledgeability because they cannot commit to a repayment that would allow the bank to break even. Hence, under a binding pledgeability constraint, the marginal investor for a given interest rate becomes an individual with a higher probability of success relative to the frictionless economy. Formally, if we use \(\hat{p}_t(\lambda)\) to denote the probability of success of the marginal investor in this economy, we have that:

\[
\hat{p}_t(\lambda) = \frac{r_{t+1}}{q_{t+1}} \cdot \frac{1}{\alpha} \max \left\{1, \frac{I - w(k_t)}{\lambda I}\right\}. \tag{19}
\]

Equation (19) illustrates the two types of equilibria that may arise under limited pledgeability. On the one hand, if \(\lambda > 1 - \frac{w(k_t)}{I}\), the pledgeability constraint does not bind in equilibrium and we are back in the frictionless case: these are economies in which wages are high relative to the size of investment, so that leverage is low and pledgeability is not a concern. On the other hand, if \(\lambda \leq 1 - \frac{w(k_t)}{I}\), the pledgeability constraint binds in equilibrium and investment is constrained relative to the frictionless economy: these are economies in which wages are low relative to the size of investment and the required level of leverage is too high given the institutional constraints.

To determine the credit-market equilibrium of the economy \((\hat{p}_t(\lambda), r_{t+1})\), we can combine Equation (19) with the credit market clearing condition of Equation (9).\(^{14}\) A first result that emerges is that \(\hat{p}_t(\lambda) = \hat{p}_t\) in equilibrium, so that limited pledgeability has no effect on the average productivity of projects that are undertaken: as was the case in the economy under adverse selection, ultimately the totality of labor income must be directed towards investment in the closed economy. Hence, the law of motion of capital is still given by Equation (12) and pledgeability constraints

\[^{14}\text{We have already stressed that, when the pledgeability constraint of Equation (18) is binding, the participation constraint of Equation (5) is slack. Hence, as in the previous sections, an equilibrium is characterized in this case by two equations (the participation constraint and the market-clearing condition) and two unknowns (\(\hat{p}_t(\lambda)\) and \(r_{t+1}\)).}\]
have an effect only through the equilibrium interest rate. Indeed, the equilibrium interest rate is given by,

\[
r_{t+1} = \begin{cases} 
q_{t+1} \cdot \alpha \cdot \left(1 - \frac{w(k_t)}{I}\right) & \text{if } w(k_t) < (1 - \lambda) \cdot I \\
q_{t+1} \cdot \alpha \cdot \lambda & \text{if } w(k_t) \geq (1 - \lambda) \cdot I 
\end{cases}
\] (20)

Equation (20) illustrates the basic workings of this economy. A binding pledgeability constraint implies that, for each given level of the interest rate, investment is lower than it would be in the frictionless economy, i.e. the productivity of the marginal investor is higher. Under limited pledgeability, the productivity of the marginal investor thus raises above the interest rate. In the closed economy, in which total investment equals the total wage bill, the interest rate must fall to restore equilibrium. Economies with more severe credit frictions, i.e. with lower \(\lambda\), therefore display lower equilibrium interest rates. The severity of credit market frictions, however, does not affect the law of motion of the closed economy: once again, it affects neither total investment nor the order in which projects are financed.\(^{15}\)

### 3.4 A tale of two frictions

We now extend our analysis of adverse selection to an economy in which credit markets are also characterized by limited pledgeability as modeled in the previous section. In this economy, banks can neither directly observe an entrepreneur’s probability of success at the time of granting credit nor can they fully seize an entrepreneur’s resources in the event of default.

The economy is formally similar to the one analyzed in Section 3.2. Equation (13) still identifies the marginal investor, i.e. the individual that is indifferent between applying to a loan contract and depositing his savings in the bank: let \(\hat{p}_{AS,t}(\lambda)\) denote the probability of success of this investor in the economy with both adverse selection and limited pledgeability as captured by \(\lambda\). The zero-profit condition of banks is still given by Equation (14), so that the contractual interest rate must allow banks to break even given the average probability of success within the pool of borrowers:

\[
\hat{R}_{t+1} = \frac{r_{t+1}}{E_t[p_j | p \geq \hat{p}_{AS,t}(\lambda)]} = 2 \cdot \frac{r_{t+1}}{1 + \hat{p}_{AS,t}(\lambda)}
\] (21)

Since the participation constraint and the zero-profit condition of banks are as before, so is the relationship between the \(r_{t+1}\) and \(\hat{p}_{AS,t}(\lambda)\) captured by Equation (15).

\(^{15}\)This feature of the model, which closely mirrors Matsuyama (2004), is of course due to the particular assumptions of our model (see Footnote 10).
Besides Equation (15) and the market-clearing condition of Equation (9), an equilibrium \((\tilde{p}_{AS,t}(\lambda), r_{t+1})\) in the presence of limited pledgeability must also satisfy:

\[
\tilde{R}_{t+1} \cdot (I - w(k_t)) \leq \lambda q_{t+1} \cdot \alpha \cdot I, \quad (22)
\]

which is the equivalent of Equation (18) for the economy with adverse selection. The novelty in Equation (22) is that it is independent of \(j\), so that it either binds for all borrowers or for none of them. This is not the case when the only friction is limited pledgeability and the contractual interest rate can vary with the borrower’s probability of success. As we saw in Section 3.3, the pledgeability constraint is then binding for some borrowers \(j \in J_t\) but not for others. Once adverse selection is introduced, however, this is no longer possible: the contractual interest rate must necessarily be the same for all borrowers, and this means that the pledgeability constraint will either bind or not bind for all of them simultaneously.

There are clearly two types of equilibria in this economy. In the first one, the pledgeability constraint is not binding, Equation (22) is slack and – exactly as in Section 3.2 – the equilibrium is fully described by Equations (15) and (16). In the second one, which is the one of interest here, the credit constraint binds and the equilibrium must also satisfy Equation (22) with an equality. But this poses a problem: this equilibrium must satisfy one more equation but it is apparently described by the same two variables, \(\tilde{p}_{AS,t}(\lambda)\) and \(r_{t+1}\). In other words, Equations (15) and (22) jointly determine in this case the combination of \(\tilde{p}_{AS,t}(\lambda)\) and \(r_{t+1}\) that satisfy the participation constraint of individuals and the zero-profit condition of banks. But how do we know that this pair also satisfies market clearing?

The answer is that, in general, it does not. When the pledgeability constraint is binding, it can be shown that the pool of borrowers is too large and there is an excess demand for credit.\(^{16}\) The only way to restore market-clearing in such a situation is by rationing some borrowers in equilibrium, in the sense that not all of those who wish to become entrepreneurs will actually get credit. If we use \(\varepsilon_t\) to denote the probability of receiving a loan, the relevant market-clearing condition for this

\(^{16}\)It can be shown that, in the presence of adverse selection, the pledgeability constraint is binding in equilibrium whenever

\[
\lambda < \frac{2 \cdot (I - w(k_t))^2}{I^2 + (I - w(k_t))^2}
\]

In this case, it also follows that the demand for funds exceeds the total supply, i.e. \(\tilde{p}_{AS,t}(\lambda) < 1 - \frac{w(k_t)}{I}\).
case can be expressed as:

\[
\begin{align*}
\left[\hat{p}_{ALS,t}(\lambda) + (1 - \hat{p}_{ALS,t}(\lambda)) \cdot (1 - \varepsilon_t)\right] \cdot w_t(k_t) &= (1 - \hat{\rho}_{ALS,t}(\lambda)) \cdot (I - w(k_t)) \cdot \varepsilon_t \\
\iff \varepsilon_t = \frac{w(k_t)}{I} \cdot \frac{1}{1 - \hat{p}_{ALS,t}(\lambda)} = \frac{1 - \hat{p}_{ARS,t}}{1 - \hat{p}_{ARS,t}(\lambda)},
\end{align*}
\]

(23)

where in the last step we have substituted \( \hat{p}_{ARS,t} \) from Equation (16). Equation (23) provides an intuition of how the likelihood that a loan applicant is denied credit, \((1 - \varepsilon_t)\), changes. The probability of rationing is decreasing in the difference between \( \hat{p}_{ARS,t} \) and \( \hat{p}_{ARS,t}(\lambda) \), i.e. between the productivities of the marginal investor in the pure adverse-selection economy and in the economy with both frictions.

Equation (23) completes our characterization of the equilibrium. When the pledgeability constraint binds, the interest rate on deposits must decrease to guarantee repayment: in the presence of adverse selection, however, this provides incentives for less productive individuals to become entrepreneurs, thereby increasing the demand for funds. Indeed, Equations (15) and (22) jointly imply that decreases in \( \lambda \) lead to lower equilibrium levels of both \( r_{t+1} \) and \( \hat{p}_{ARS,t}(\lambda) \). In order to restore market-clearing, borrowers must therefore be rationed. As Equation (23) shows, this rationing will increase with the severity of pledgeability constraints, i.e. with the difference between \( \hat{p}_{ARS,t} \) and \( \hat{p}_{ARS,t}(\lambda) \).\(^{17}\)

This discussion highlights an interesting implication of our model. In our setup, neither limited pledgeability nor adverse selection per se have an effect on the law of motion of the economy. When considered separately, we have seen that each of them affects the equilibrium interest rate but not the productivity of projects that are financed in equilibrium: one way to think about this is that they do not affect the order in which projects are financed. When both frictions are combined, however, this is no longer true: since \( \hat{p}_{ARS,t}(\lambda) \leq \hat{p}_{ARS,t} \), the average productivity of investment

\(^{17}\)In this equilibrium, we can think of banks as offering a lottery, i.e. a contract \( \left( \hat{L}_{t+1} = I - w(k_t), \hat{R}_{t+1} \right) \) coupled with a probability of actually getting the loan equal to \( \varepsilon_t \). To see that rationing is compatible with equilibrium, note that no bank can gain by offering a contract that entails a lower probability of rationing in exchange for a higher contractual interest rate: as long as the pledgeability constraint is binding, \( R_{t+1} \) cannot be increased and hence no profitable deviations are feasible. Another deviation that might seem attractive for banks is to offer a similar contract with an application fee that, in the event of being rationed, entrepreneurs actually lose. Since less productive individuals have less to gain from entrepreneurship, this might seem to discourage them from applying to the contracts. Simple computation reveals that any such deviation will be equally attractive to all entrepreneurs and it cannot therefore be profitable. In fact, if we can not lure low quality entrepreneurs away with a contract that requires them to invest all their wage in the project, losing it if this turns to be a failure, then it is even less likely to lure them away by charging an application fee that everybody lose with the same probability.
actually falls in equilibrium. The reason is that, if lenders are to break even, limited pledgeability requires the interest rate to be low; a low interest rate, in turn, decreases the returns to savings and induces unproductive individuals to become entrepreneurs, thus exacerbating adverse selection. This tension results in a low rate of interest and a large and relatively unproductive pool of potential borrowers, which is why rationing is required to attain market clearing. This is the sense in which both frictions exacerbate and “complement” one another so that, while each one of them does not affect the order in which projects are financed, their interaction does: the average quality of projects that are financed falls relative to the frictionless economy, thereby slowing down capital accumulation and growth. Formally, the law of motion is now given by:

\[
k_{t+1} = \alpha \cdot E_t \left[ p_j | p \geq \hat{p}_{AS,t}(\lambda) \right] \cdot w(k_t) \leq \alpha \cdot E_t \left[ p_j | p \geq \hat{p}_t \right] \cdot w(k_t)
\]

\[
k_{t+1} = A(\bar{\lambda}_t(\lambda)) \cdot (1 - \gamma) \cdot k_t^\gamma = \begin{cases} 
\alpha \cdot \left( \frac{1 + \hat{p}_{AS,t}(\lambda)}{2} \right) \cdot (1 - \gamma) \cdot k_t^\gamma & \text{if } \lambda \geq \frac{2 \cdot (I - w(k_t))^2}{I^2 + (I - w(k_t))^2} , \\
\alpha \cdot \left( \frac{1 + \hat{p}_{AS,t}(\lambda)}{2} \right) \cdot (1 - \gamma) \cdot k_t^\gamma & \text{if } \lambda < \frac{2 \cdot (I - w(k_t))^2}{I^2 + (I - w(k_t))^2}
\end{cases}
\]

which lies below the law of motion of Equation (12) as long as the pledgeability constraint is binding. As \( \lambda \) increases and the pledgeability constraint is relaxed, banks are able to raise the contractual interest rate and this discourages inefficient entry; consequently, there is an increase in the average productivity of projects undertaken and the law of motion of this economy approaches that of previous sections.

4 The open economy: capital flows and financial frictions

We now consider that our economy opens its financial markets to the rest of the world, so that individuals \( j \in J_t \) can borrow from and/or lend to the international financial market. Throughout, we assume that this market is willing and able to borrow or lend any amount at an expected gross return of \( r^* \). Hence, the underlying assumption is that our economy is small in relation to this market and the analysis is thus restricted to the case of a small open economy.

In the closed economy, aggregate investment is constrained by the availability of domestic resources and – ultimately – by the domestic capital stock. In the open economy, this is no longer
the case because investment can be financed with foreign resources: in principle, the determinant of investment is the international interest rate $r^\ast$. To reflect this, we use $\tilde{p}^\ast$ (in all its variations) throughout to denote the probability of success of the marginal project undertaken, where the apex (*) signals that the variable refers to the open economy. Once the value of $\tilde{p}^\ast$ is determined in equilibrium, it follows that total investment in the economy equals $(1 - \tilde{p}^\ast) \cdot I$. Keeping this in mind, we now characterize the equilibrium of our economy under international financial integration and for different assumptions regarding the functioning of its credit market.

4.1 The frictionless economy

In the absence of financial frictions, the equilibrium of the open economy is straightforward. Given the international interest rate $r^\ast$, the level of investment is immediately determined by the analog of Equation (8):

$$\tilde{p}^\ast(r^\ast) = \frac{r^\ast}{q(k^\ast)} \cdot \frac{1}{\alpha}, \quad (25)$$

where $q(\cdot)$ denotes the rental price of capital and we have dropped time-subscripts to reflect the fact that there are no state variables in this economy. Equation (25) illustrates that, in the absence of financial frictions, capital flows between the small open economy and the rest of the world until the return to domestic investment equals the international interest rate. From the perspective of each generation $t$, then, total consumption is maximized when capital flows between them and the international financial market at time $t$ are unrestricted in any way.\(^{18}\)

Given $r^\ast$, there is a unique value of $\tilde{p}^\ast$ that satisfies Equation (25). Once this value is determined, so is the steady-state level of capital $k^\ast$, which is formally given by:

$$k^\ast = A(\tilde{p}^\ast(r^\ast)) \cdot (1 - \tilde{p}^\ast(r^\ast)) \cdot I = \alpha \cdot E[p_j | p \geq \tilde{p}^\ast(r^\ast)] \cdot (1 - \tilde{p}^\ast(r^\ast)) \cdot I \quad \Leftrightarrow \quad (26)$$

$$k^\ast = \alpha \cdot I \cdot \left[ \frac{1 - (\tilde{p}^\ast(r^\ast))^2}{2} \right] = \frac{\alpha I}{2} \cdot \left[ 1 - \left( \frac{r^\ast}{q(k^\ast) \cdot \alpha} \right)^2 \right].$$

In the open economy, the credit-market equilibrium can thus be found simply by determining the value of $\tilde{p}^\ast(r^\ast)$ that satisfies individual rationality and the zero-profit condition of banks. The fact that $\tilde{p}^\ast(r^\ast)$ depends only on the international interest rate and that it is independent of the

\(^{18}\)From an intergenerational perspective, however, the issue is more complicated. The reason is the usual one in this class of models: greater capital accumulation today, even if costly for the current generation, benefits future generations through higher wages. Although certainly interesting, a full analysis of welfare implications would exceed the scope of this paper and we therefore leave it for future research.
economy’s capital stock $k_t$ reflects a well-known feature of small open economies in the absence of financial frictions: they converge immediately to the steady state and there are no dynamics to speak of. This result is standard and we shall not dwell on it. We turn instead to the more interesting implications of financial frictions for capital flows.

### 4.2 Adverse selection

In our analysis of adverse selection of Section 3.2, we found that it fell upon the interest rate to ensure market-clearing in the closed economy. We concluded then that, in the presence of adverse selection, the equilibrium interest rate had to increase up to the point at which the marginal investor was the same as in the frictionless economy, i.e. $\hat{p}_{AS,t} = \hat{p}_t$ for all $t$. But how does adverse selection affect the direction and magnitude of capital flows when our economy becomes integrated with the international financial market?

As before, we begin by focusing our attention on the probability of success of the marginal investor $\tilde{p}_{AS,t}$. From Equation (15), we know that the relationship between $r^*$ and $\tilde{p}_{AS,t}$,

$$r^* = \frac{\hat{p}_{AS,t} \cdot I_{t+1} \cdot \alpha \cdot I}{w(k_t) + 2 \cdot \frac{\tilde{p}_{AS,t}}{1 + \tilde{p}_{AS,t}} \cdot (I - w(k_t))},$$

must hold in equilibrium in order to satisfy the participation constraint of entrepreneurs while allowing banks to break even. Equation (27) implicitly defines $\tilde{p}_{AS,t}(r^*, k_t)$, the probability of success that the marginal investment project must have in equilibrium given the international interest rate $r^*$. Our choice of notation already points to an important modification relative to the analysis of the previous section: once adverse selection is introduced, $\tilde{p}_{AS,t}$ is no longer independent of $k_t$ and dynamics are therefore influenced by the state of the economy. The reason, of course, is that the capital stock affects wages and thus the incentive of individuals to become entrepreneurs.

Although Equation (27) does not deliver a closed-form expression for $\tilde{p}_{AS,t}$, we can differentiate it to establish that – in equilibrium and for a given value of $r^*$ – $\tilde{p}_{AS,t}$ is increasing in $w(k_t)$ so that total investment is decreasing in the economy’s capital stock.\(^{19}\) Taking this into account, the law

\(^{19}\)When differentiating Equation (27) it must be kept in mind that the following holds in equilibrium:

$$\alpha \cdot q \geq \frac{r^*}{E_{p_j|p \geq \hat{p}_{AS,t}(r^*, k_t)}},$$

Differentiation of Equation (27) also reveals that, given $k_t$, $\tilde{p}_{AS,t}$ is increasing in $r^*$ so that domestic investment is decreasing in the international interest rate.
of motion of the economy is given by,
\[ k_{t+1} = A(p_{AS,t}^*(r^*, k_t)) \cdot \left( 1 - \bar{p}_{AS,t}^*(r^*, k_t) \right) \cdot I = \alpha \cdot I \cdot E \left[ p_j | p \geq \bar{p}_{AS,t}^*(r^*, k_t) \right] \cdot \left( 1 - \bar{p}_{AS,t}^*(r^*, k_t) \right) \]
\[ k_{t+1} = \frac{\alpha \cdot I}{2} \cdot \left[ 1 - \left( \bar{p}_{AS,t}^*(r^*, k_t) \right)^2 \right], \]
which is depicted graphically in Figure 1.

![Figure 1 - Adverse Selection](image)

The thick line in Figure 1 illustrates a representative law of motion for the capital stock in the small open economy under adverse selection, where \( k^* \) denotes the steady-state level of capital in the absence of financial frictions. Two important features stand out: (i) the law of motion lies everywhere above the corresponding law of motion for the frictionless economy, and; (ii) it is downward-sloping.\(^{20}\) We now discuss each of these features separately.

By fostering the cross-subsidization of less productive individuals, adverse selection exacerbates investment. In the closed economy, we have seen how this excess investment can be counterbalanced by an increase in the equilibrium interest rate. In the open economy, in which the interest rate is given and equals \( r^* \), there is no such countervailing force. Consequently, the adverse-selection

\(^{20}\)On the horizontal axis, the figure depicts values of \( k_t \) for which \( w(k_t) < I \), so that the adverse selection problem is binding throughout. Once this ceases to be the case, the law of motion naturally coincides with that of the frictionless economy.
economy overinvests relative to the frictionless economy, which explains why the law of motion lies everywhere above $k^*$. This is of course true in steady state as well, so that if we let $k^*_{AS}$ denote the steady-state level of capital in this economy, it must necessarily hold that $k^*_{AS} > k^*$ as depicted in the figure. In equilibrium, adverse selection thus leads the economy to undertake investment projects with returns that are lower than the international interest rate so that – contrary to common results in the literature – this type of friction introduces a negative wedge between the marginal return to investment and the international interest rate. Thus, from the perspective of generation $t$, total consumption would be clearly maximized by raising the domestic interest rate at time $t$ so as to eliminate this wedge, effectively taxing domestic investment and subsidizing domestic savings.\footnote{For a given capital stock $k_t$, the total consumption of generation $t$ can be expressed as follows:}

$$c_{t+1} = \gamma \cdot (k_{t+1})^\gamma - r^* \cdot \left[1 - \tilde{p}_{AS,t}(r^*, k_t) \cdot I - (1 - \gamma) \cdot (k_t)^\gamma \right],$$

where: $k_{t+1}$ is a function of $\tilde{p}_{AS,t}(r^*, k_t)$ as in Equation (28); the first term represents the total capital income of the economy in period $t + 1$, and; the second term represents the net interest payments made to the international financial market at time $t + 1$. Since $\tilde{p}_{AS,t}(r^*, k_t) < r^*$ in equilibrium, maximization of $c_{t+1}$ requires effectively raising the domestic interest rate above $r^*$ so as to reduce domestic capital accumulation and decrease net interest payments to the international financial market.

The second important feature of the law of motion depicted in Figure 1 is that it is downward sloping. The reason for this is that, as we mentioned above, $\tilde{p}_{AS,t}$ is increasing in $w(k_t)$. When the capital stock and wages are low, less productive individuals have a strong incentive to become entrepreneurs: since they need to borrow most of the investment from banks, they will be heavily cross-subsidized by the more productive individuals. As the capital stock and wages increase, however, the extent of cross-subsidization decreases and entrepreneurship loses its appeal for less productive individuals. This raises $\tilde{p}_{AS,t}$, depressing investment and capital accumulation.

This last discussion points to an interesting implication of adverse selection in the context of a small open economy: it generically exacerbates economic volatility. Whereas in financial autarky the economy converges monotonically to its steady state, the open economy necessarily displays oscillatory behavior.\footnote{The steady state of this economy can in principle be either stable or unstable. Although the economy displays fluctuations in both cases, in the case of stability it fluctuates while converging to the steady state.}
discourages investment by unproductive individuals and brings about a reduction in output that restarts the economic cycle.

We have thus designed a model in which adverse selection has no real effects under financial autarky. When individuals are allowed to borrow from and/or lend to the international financial market, however, the picture is drastically different. Adverse selection exacerbates investment and capital accumulation and, even in the absence of any type of uncertainty, generates volatility. As we now show, this is very different from the standard role attributed to financial frictions in the open economy, which we analyze next.

4.3 Limited pledgeability

Just as we did for the case of the closed economy, we are want to assess how the introduction of limited pledgeability affects our results regarding adverse selection. To do so, we first isolate the implications of limited pledgeability in the small open economy.

Relative to the frictionless case of Section 4.1, consider that the economy is subject to a pledgeability constraint as in Equation (18). Using \( \hat{p}_t^*(\lambda, r^*, k_t) \) to denote the probability of success of the marginal investor in the open economy, we have that,

\[
\hat{p}_t^* = \frac{r^*}{q_t + 1} \cdot \max \left\{ 1, \frac{1}{\lambda} \cdot \frac{I - w(k_t)}{I} \right\},
\]

must be satisfied in equilibrium. Equation (29) provides, for each level of \( r^* \), the probability of success of the marginal investor that is consistent with both the participation constraint of individuals and the zero-profit condition of banks given the pledgeability constraint. A first observation that emerges from it is that, as in the case of adverse selection, the introduction of limited pledgeability implies that \( \hat{p}_t^* \) is no longer independent of \( k_t \) and dynamics are therefore influenced by the state of the economy. The reason is that, through its effect on wages, the capital stock affects the extent to which investors are leveraged and thus the extent to which the pledgeability constraint binds in equilibrium.

Equation (29) defines a (weakly) decreasing relationship between \( \hat{p}_t^* \) and \( k_t \), so that total investment is increasing in the economy’s capital stock. Taking this into account, the law of motion of the economy is given by,

\[\text{It is important to stress that the existence of these cycles in the presence of adverse selection does not rely on investment projects having a fixed size. In a closely related setting, Martin (2008) shows how similar cycles may arise in an environment in which the size of projects is variable.}\]
\[
k_{t+1} = A(\hat{p}_t^*(\lambda, r^*, k_t)) \cdot (1 - \hat{p}_t^*(\lambda, r^*, k_t)) \cdot I = \alpha \cdot E \left[ p_j | p \geq \hat{p}_t^*(\lambda, r^*, k_t) \right] \cdot (1 - \hat{p}_t^*(\lambda, r^*, k_t)) \cdot I
\]

\[
k_{t+1} = \begin{cases} 
\frac{\alpha \cdot I}{2} \cdot \left( 1 - \left( \frac{r^*}{q(k_{t+1}) \cdot \alpha} \right) \right)^2 & \text{if } w(k_t) < (1 - \lambda) \cdot I \\
\frac{\alpha \cdot I}{2} \cdot \left( 1 - \left( \frac{r^*}{q(k_{t+1}) \cdot \alpha} \right) \right)^2 & \text{if } w(k_t) \geq (1 - \lambda) \cdot I 
\end{cases}
\]

which is illustrated in Figure 2.

Figure 2 - Limited Pledgeability

The thick line in Figure 2 illustrates a representative law of motion for the capital stock in the small open economy under limited pledgeability. The figure, in which \( \kappa \) denotes the capital stock at which the pledgeability constraint ceases to bind, depicts the case of an economy that has one steady state \( k_t^* (\lambda) \) in which investment is constrained. Two important features stand out: (i) as long as the pledgeability constraint is binding, the law of motion lies everywhere below the corresponding law of motion for the frictionless economy, and; (ii) the law of motion is upward-sloping.\(^{24}\)

\(^{24}\)Figure 2 illustrates the law of motion as being strictly concave, which need not be the case. Intuitively, there are two opposing forces that determine the shape of the law of motion: (i) the diminishing marginal productivity of investment and capital, which makes the law of motion concave, and; (ii) the relaxation of the pledgeability constraint as capital and wages increase, which makes the law of motion convex. The exact shape of the law of motion depends on the relative strength of these two forces, which may give rise to multiple steady states in this small-open economy. For a thorough discussion of this point in a related model, see Matsuyama (2004).
When \( w(k_t) > (1 - \lambda) \cdot I \), the pledgeability constraint is not binding and Equation (30) coincides with the law of motion of the frictionless economy. When \( w(k_t) < (1 - \lambda) \cdot I \) and the pledgeability constraint binds, the economy underinvests relative to the frictionless economy and the law of motion lies below \( k^* \): Figure 2 illustrates the case in which the economy has a unique steady state that lies in this range, denoted by \( k^*(\lambda) \). Limited pledgeability thus prevents the economy from undertaking all those investment projects with returns that exceed the international interest rate, so that in equilibrium it introduces a positive wedge between the marginal return to investment and the international interest rate. From the perspective of generation \( t \), therefore, total consumption would be maximized by lowering the domestic interest rate at time \( t \) so as to eliminate this wedge, effectively subsidizing domestic investment and taxing domestic savings.\(^{25}\) This would amount, in practice, to taxing capital outflows (if the economy is a net capital exporter) or to subsidizing capital inflows (if the economy is a net capital importer).

The second important feature of the law of motion depicted in Figure 2 is that it is upward sloping so that, whenever the pledgeability constraint is binding, the economy does not converge immediately to the steady state. The reason for this is clear. Under a binding pledgeability constraint, the productivity of the marginal investor depends on his wages and \( \hat{p}_t^*(\lambda, r^*, k_t) \) is therefore a function of \( k_t \). For any given value of \( r^* \), increases in the capital stock relax borrowing constraints and lead to a decrease in \( \hat{p}_t^*(\cdot) \) and an expansion in investment.

Our simple model thus reproduces a common result in the international finance literature. As captured by a binding pledgeability constraint, a low quality of financial institutions tends to restrict investment. In Section 3.3, we argued that this depresses the equilibrium interest rate under financial autarky. This implies that, under financial integration, economies with low levels of \( \lambda \) will tend to experience greater capital outflows (or lower inflows) than they otherwise would. This summarizes, in a nutshell, the mechanism emphasized by much of the literature to account for the seeming inability of developing economies to attract capital flows despite the high returns to capital accumulation in many of them.\(^{26,27}\) A similar mechanism underlies the “asymmetric financial development” view of global imbalances, according to which the large recent capital flows out of many Asian economies (predominantly China) are due to the inability of these economies of

---

\(^{25}\) The analysis in this case is the mirror image of the one carried out for the adverse-selection economy (see Footnote 21).

\(^{26}\) See, for example, Boyd and Smith (1997) and Matsuyama (2004), among others.

\(^{27}\) Of course, private contracting frictions between borrowers and lenders are not the only reason for which these countries might fail to attract capital. It is commonly believed that opportunistic behavior by the government plays a substantial role as well. For a recent view along these lines, see Broner and Ventura (2010).
supplying financial assets, i.e. of translating a high productivity of physical investment into a high return for lenders.  

As in the closed economy, the implications of limited pledgeability and adverse selection for total investment and for the direction and magnitude of capital flows therefore mirror one another. Real-world credit markets are not characterized solely by adverse selection or by limited pledgeability, however, but rather by a mixture of the two. It is important then to know whether our findings of Section 4.2 regarding the effects of adverse selection are robust to the inclusion of limited pledgeability.

4.4 A tale of two frictions: capital flows in the open economy

Consider the case of a small open economy in which both frictions, limited pledgeability and adverse selection, coexist and interact with one another. In this case, any equilibrium in the open economy must jointly satisfy the participation constraint of Equation (13), the zero-profit condition of banks of Equation (14) and the pledgeability constraint of Equation (22). Whenever the latter is binding, these three equations determine the relationship that \( r^* \), on the one hand, and \( \hat{p}_{AS,t}(\lambda, k_t, r^*) \), on the other, must satisfy in equilibrium:

\[
\hat{p}_{AS,t}(\lambda, k_t, r^*) = \frac{\lambda \cdot w(k_t)}{2 \cdot (I - w(k_t)) - \lambda (2I - w(k_t))}.
\]  

(31)

\[
r^* = q_{t+1} \cdot \alpha \cdot \frac{I \cdot \lambda \cdot (1 - \lambda)}{2 \cdot (I - w(k_t)) - \lambda (2I - w(k_t))},
\]  

(32)

Equation (31) illustrates, once again, that there is a unique value of \( \hat{p}_{AS,t}(\lambda, k_t, r^*) \) that is able to simultaneously satisfy the participation constraint of individuals and the economy’s pledgeability constraint while allowing banks to break even. It shows that, when both frictions are present, \( \hat{p}_{AS,t}(\lambda, k_t, r^*) \) is increasing in \( w(k_t) \) and thus in the capital stock: exactly as the adverse-selection economy of Section 4.2, then, increases in the capital stock discourage relatively unproductive individuals from becoming entrepreneurs and thereby improve the quality of potential borrowers. Equation (32), however, illustrates that this cannot be the whole story. Increases in \( w(k_t) \) and thus in the capital stock raise the right-hand side of the equation: but if \( \hat{p}_{AS,t}(\lambda, k_t, r^*) \) increases at the same time, investment falls and \( q_{t+1} \) must rise as well. This is clearly incompatible with a fixed interest rate \( r^* \).

---

28 Caballero et al. (2008) provide a theoretical framework along these lines.
How can Equations (31) and (32) be jointly satisfied, then? Exactly as in the case of the closed economy, the probability of rationing \((1 - \varepsilon_t)\) does the trick, since – by decoupling the productivity of the marginal investor from total investment – it enables \(\hat{p}_{AS,t}^*(\lambda, k_t, r^*)\) and \(q_{t+1}\) to move in opposite directions.\(^{29}\) Given \(\hat{p}_{AS,t}^*(\lambda, k_t, r^*)\), which determines the quality of potential borrowers, the probability of rationing determines actual investment, the future capital stock \(k_{t+1}\) and thus the rental price of capital \(q_{t+1}\). Taking this into account, the law of motion of the economy can be formally expressed by,

\[
k_{t+1} = \frac{\alpha \cdot I}{2} \left[ 1 - \left( \frac{\lambda \cdot w(k_t)}{2 \cdot (1 - \lambda \cdot w(k_t))} \right)^2 \right] \cdot \varepsilon_t(\lambda, r^*, k_t) + (1 - \lambda) \cdot I
\]

\[
k_{t+1} = \begin{cases} 
\frac{\alpha \cdot I}{2} \cdot \left[ 1 - \left( \frac{\lambda \cdot w(k_t)}{2 \cdot (1 - \lambda \cdot w(k_t))} \right)^2 \right] \cdot \varepsilon_t(\lambda, r^*, k_t) & \text{if } w_t \leq \chi(\lambda, r^*, k_t) \cdot (1 - \lambda) \cdot I \\
\frac{\alpha \cdot I}{2} \cdot \left[ 1 - \hat{p}_{AS,t}^*(r^*, k_t)^2 \right] & \text{if } w_t > \chi(\lambda, r^*, k_t) \cdot (1 - \lambda) \cdot I
\end{cases}
\]

where \(\hat{p}_{AS,t}^*(r^*, k_t)\) is as in as in Section (4.2), i.e. the productivity of the marginal investor once the pledgeability constraint ceases to bind, and \(\chi(\lambda, r^*, k_t) \leq 1\).\(^{30}\) An example of Equation (33) is depicted graphically in Figure 3 below.

\(^{29}\)Relative to the closed economy of Section 3.4, this small-open economy has one less equation (the market-clearing condition) and one less endogenous variable (the interest rate). Hence, exactly as in the case of the closed economy, rationing is required in equilibrium. Formally, the productivity of the marginal investor \(\hat{p}_{AS,t}^*(\lambda, k_t, r^*)\) and the probability of rationing \(1 - \varepsilon_t\) jointly determine the rental price of capital \(q_{t+1}\), which is formally given by:

\[
q_{t+1} = \gamma \left\{ \frac{\alpha \cdot I \cdot 1 - \left[ \hat{p}_{AS,t}^*(\lambda, k_t)^2 \right]}{2} \cdot \varepsilon_t \right\}^{\gamma-1}.
\]

Equations (31) and (32) thus jointly determine \(\hat{p}_{AS,t}^*(\lambda, k_t, r^*)\) and \(\varepsilon_t(\lambda, k_t, r^*)\).

\(^{30}\)When the pledgeability constraint ceases to bind, rationing disappears, \(\varepsilon_t = 1\), and the interest rate depicted in Equation (32) equals the interest rate of the pure adverse-selection economy in Equation (27). A comparison of these two expressions yields that they are equal when \(w(k_t) = \chi(\lambda, r^*, k_t) \cdot (1 - \lambda) \cdot I\), where

\[
\chi(\lambda, r^*, k_t) = \frac{2 \cdot \hat{p}_{AS,t}^*(k_t, r^*)}{2 \cdot \hat{p}_{AS,t}^*(k_t, r^*) + (1 - \hat{p}_{AS,t}^*(k_t, r^*))} \cdot \lambda \leq 1,
\]

for all \(\hat{p}_{AS,t}^*(k_t, r^*) \in [0, 1]\).
The thick line in Figure 3 illustrates a representative law of motion for the capital stock in the small open economy under both limited pledgeability and adverse selection. The economy depicted in the figure has a unique steady state, denoted by \( k_{AS}^* \), in which the pledgeability constraint is no longer binding. Two important features stand out: (i) as long as the pledgeability constraint is binding, the law of motion lies everywhere below the corresponding law of motion for the pure adverse-selection economy, and; (ii) the law of motion is non-monotonic.

When \( w_t > \chi \cdot (1 - \lambda) \cdot I \), the pledgeability constraint is not binding and Equation (33) coincides with the law of motion of the pure adverse-selection economy of Section 4.2. When \( w_t < \chi \cdot (1 - \lambda) \cdot I \) and the pledgeability constraint binds, the economy’s production of capital is instead hindered relative to the pure-adverse selection economy. One might be tempted to think that, in this case, the constraint imposed by limited pledgeability is actually helpful to mitigate the overinvestment induced by adverse selection. This constraint, however, only makes the adverse selection problem worse: by limiting the contractual interest rate that banks can charge entrepreneurs, it provides even greater incentives for inefficient individuals to become entrepreneurs. In this sense, and exactly as we found for the case of the closed economy in Section 3.4, both frictions exacerbate one another and this leads to a fall in the average productivity of investment. Ultimately, limited pledgeability does indeed limit investment relative to the pure adverse-selection economy, but it does so randomly through rationing and not selectively by weeding out relatively unproductive individuals.

From the perspective of generation \( t \), the maximization of total consumption requires a combination of taxes and subsidies. On the one hand, the distortions originating in limited pledgeability can only be dealt with through a decrease in the domestic interest rate that enable banks to break
even: this requires, for example, a subsidy on capital inflows (in the case of a net capital importer) or a tax on capital outflows (in the case of a net capital exporter). On the other hand, the distortions originating in adverse selection can only be dealt with through an increase in the interest rate obtained by domestic savers: this requires, for example, a subsidy to domestic depositors so as to discourage relatively unproductive individuals from becoming entrepreneurs.

The second important feature of the law of motion depicted in Figure 3 is that it is non-monotonic. When the economy’s capital stock is low and the pledgeability constraint is binding, the law of motion is upward sloping. This happens even though increases in the capital stock raise wages and make entrepreneurship less appealing for relatively unproductive individuals, thereby increasing \( \hat{p}_t(\lambda, r^*, k_t) \). But this reduction in the pool of borrowers decreases the need for equilibrium rationing and, ultimately, it is this fall in rationing what makes \( k_{t+1} \) increasing in \( k_t \). Once the pledgeability constraint ceases to bind and rationing disappears altogether, the law of motion coincides with that of Section 4.2 and it becomes downward-sloping.

The introduction of limited pledgeability therefore enriches the dynamic effects of adverse selection as characterized in Section 4.2. First, it slows down capital accumulation not by mitigating the effects of adverse selection for overinvestment, but rather by exacerbating them to bring about a decrease in the average productivity of investment. Second, as we saw in the analysis of Section 4.3, limited pledgeability can give rise to multiple steady states.\(^{31}\) Of these steady states, only one can lie on the downward-sloping part of the law of motion in Figure 3. This implies that institutional reforms that increase \( \lambda \) shift the economy’s law of motion upwards and they eventually eliminate all steady states but \( k_{AS}^* \). In this case, improvements in institutional quality relax borrowing constraints until, in the long run, the pledgeability constraint ceases to bind and the only remaining friction is adverse selection. These increases in \( \lambda \) expand leverage and enhance the productivity of investment but, due to the oscillatory nature of \( k_{AS}^* \), they must also fuel economic volatility in the long-run.

This discussion illustrates two important implications of our model. The first is that the development of financial markets, as well as the set of policies aimed at improving their efficiency, is not necessarily one-dimensional. When both frictions are present, for example, we have seen how they complement one another: this means that, to improve the allocation of resources, policies that improve creditor rights might be just as useful as policies that lower the cost of screening borrowers. The relative efficiency of different policies, however, might vary according to the level

\(^{31}\)See Footnote 24.
of economic development. A second and related implication is precisely that some of the problems associated to adverse selection might surface only when the economy surpasses a certain level of wealth or financial development. During the recent financial crisis, for example, economists were taken aback by the difficulties faced by the seemingly developed financial markets of the United States. How could it be that these markets had done such a poor job of allocating credit? Our model highlights that some of these problems, like excessive investment and the resulting volatility associated with it, can only arise precisely where financial markets surpass a minimum level of economic development. In a world in which a substantial fraction of economies are characterized by poor financial institutions, it may well be that the economy where these institutions work best will end up being most visibly affected by the problems of adverse selection.\textsuperscript{32}

## 5 Conclusion

The financial crisis of 2007-08 has underscored the importance of adverse selection in financial markets. This friction has been mostly neglected by macroeconomic models of financial frictions, however, which have focused almost exclusively on the effects of limited pledgeability. In this paper, we have attempted to fill this gap by developing a standard growth model with adverse selection. Our main results are that, by fostering unproductive investment, adverse selection: (i) leads to an increase in the economy’s equilibrium interest rate, and; (ii) it generates a negative wedge between the marginal return to investment and the equilibrium interest rate. We have shown how, under financial integration, these effects translate into excessive capital inflows and generate endogenous fluctuations in the capital stock and output. We have also explored how these results change when limited pledgeability is added to the model, and we have concluded that there is a sense in which both frictions complement one another: if anything, limited pledgeability exacerbates the consequences of adverse selection on the macroeconomy.

Our analysis is incomplete in two important respects. The first one is that we have stopped short of characterizing the full welfare implications of adverse selection and limited pledgeability. Instead, we have referred exclusively to the contemporaneous effects of these frictions on each generation of savers that is exposed to them. This shortcoming of our analysis is not due to lack

\textsuperscript{32}Imagine, for example, that the world is made up of two economies like the one analyzed in this section, one of which is characterized by a low level of $\lambda$. Under financial integration, capital in this world will tend to flow towards the economy with the most developed markets, which will receive them as a mixed blessing. On the one hand, these inflows will be beneficial because they will lower the cost of financing and allow for an expansion in the capital stock; on the other hand, they may also be costly by fueling inefficient investments and economic volatility.
of interest on our behalf. As we mentioned in the main body of the text, a full welfare analysis is quite involved because it requires the balancing of different effects across generations. There is simply no space for this here.

A second and related shortcoming is that we have restricted our analysis of financial integration to the case of a small open economy. Doing so has been instrumental to simplify the analysis and it has allowed us to portray the effects of adverse selection and limited pledgeability in a very clear manner. It has also, however, prevented us from using the model to directly address the recent turn of events. The prevailing view on global imbalances and financial frictions is that limited pledgeability has been at the heart of capital flows between Asia and the United States. According to this view, the United States has only stood to gain from these inflows. How is this view affected once the importance of adverse selection is acknowledged? Is it possible that, through their effects on the interest rate, these capital inflows exacerbate adverse selection and lead to inefficient investment in the United States? Can the United States ultimately suffer a welfare loss if the rest of the world uses its financial system to intermediate resources? Addressing these questions should be the exciting next step in this research agenda.
References


