Efficient Recapitalization *

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June 2010

Abstract

We analyze public interventions to alleviate debt overhang among private firms when the government has limited information and limited resources. We compare the efficiency of buying equity, purchasing assets, and providing debt guarantees. With symmetric information, all the interventions are equivalent. With asymmetric information between firms and the government, buying equity dominates the two other interventions. We also solve for the optimal public intervention and show how it can be implemented with preferred stock and warrants.

*We thank seminar participants at the 2010 American Finance Association Meeting, Baruch College, the CEPR Bank Crisis Prevention and Resolution Conference, the CEPR Gerzensee Corporate Finance Meeting, the Federal Reserve Chicago Bank Structure and Competition Conference, Harvard University, the NBER Monetary Economics Program Meeting, New York University, the New York Federal Reserve Liquidity Working Group, Princeton University, the UBC Summer Finance Conference, the University of Chicago, the University of Rochester (Simon) and our discussants Marco Becht, Ron Giammarino, Adriano Rampini, Gustav Sigurdsson, Juan Sole, and Luigi Zingales, for helpful comments and suggestions.

†NBER and CEPR.
It has been well understood since the seminal work of Myers (1977) that debt overhang can lead to under-investment. Firms in financial distress find it difficult to raise capital for new investments because the proceeds from these new investments end up increasing the value of the existing debt instead of the value of equity.

Debt overhang can be resolved by quick and efficient renegotiations between equity and debt holders. In practice, however, renegotiations can be hampered by free-rider problems among dispersed creditors and by contract incompleteness (Bulow and Shoven (1978), Gertner and Scharfstein (1991), and Bhattacharya and Faure-Grimaud (2001)). Hence, a costly bankruptcy procedure might be required to achieve the desired outcome. Indeed, a large body of empirical research has shown the economic importance of renegotiation costs for firms in financial distress (Gilson, John, and Lang (1990), Asquith, Gertner, and Scharfstein (1994), Hennessy (2004)). Moreover, from a theoretical perspective, one should expect renegotiation to be costly for at least two reasons. First, the covenants that protect debt holders from risk shifting (Jensen and Meckling (1976)) are precisely the ones that can create debt overhang. Second, debt contracts are able to discipline managers only because they are difficult to renegotiate (Hart and Moore (1995)).

In addition to these microeconomic issues, negative externalities can arise when renegotiations occur in the midst of a financial crisis. Default by one firm can trigger creditor runs among other firms. The social costs of financial distress are therefore likely to exceed the private costs, and there might be room for an intervention by the government. By directly providing capital to firms, the government can alleviate the debt overhang problem without triggering a wave of defaults. This, however, raises the question of how to intervene efficiently.

The goal of our paper is to characterize the optimal form of government interventions against debt overhang. In our model firms differ along two dimensions: the quality of their existing assets and their investment opportunities. Asset quality determines the severity of debt overhang and missed investment opportunities generate welfare losses.

A crucial element of our analysis is the information structure at the time when firms decide whether to participate in a government program. We consider two cases. In the symmetric information case, the government and the private sector only know the joint distribution of investment opportunities and asset quality. In the asymmetric information
case, the private sector knows more than the government about quality and opportunities. Mitchell (2001) reviews the evidence from past financial crises and explains why both cases are relevant in practice.\footnote{Note that we do not consider asymmetric information among private investors. The market failure in our model comes from debt overhang, not from adverse selection. For an analysis of optimal interventions in Lemon markets, see Philippon and Skreta (2009).}

The objective of the government is to increase socially valuable investments while minimizing the deadweight losses from raising new taxes. Different interventions lead to different payoff structures for the equity holders, the debt holders and the government, and it is in general difficult to understand how these affect the firms’ incentives to invest. Our strategy is to compare three specific programs (equity injections, asset purchases, and debt guarantees), before solving for the optimal (second best) intervention.

Our analysis delivers three main insights. The first result is that the form of intervention is irrelevant under symmetric information. The intuition for this result is the following. The cost to the government equals the transfer to debt holders minus shareholders’ gains from future investment opportunities. Under symmetric information, the government holds equity holders to their reservation utility, and we can show that all interventions reduce debt overhang to the same extent as long as they provide the same amount of financing. The three interventions are therefore equivalent.

Our second result is that buying equity dominates the two other interventions under asymmetric information. The intuition for this result is that an efficient program should limit informational rents. Under asymmetric information, firms can take advantage of the program even though they would be able to invest by themselves, and the shareholders of participating firms are typically not held to their reservation utility. Compared to other interventions, buying equity reduces informational rents because firms with good assets and good investment opportunities have to share their surplus with the government.

Our third result is that an optimal intervention can be implemented by injecting capital in exchange for preferred stock and warrants. The preferred stock is bought at a price above market value and provides a subsidy to equity holders to encourage new investment. The warrants’ strike price is set at the firm’s initial book equity value, which allows the government to extract the entire surplus from future investment opportunities.\footnote{We also show that the preferred stock-warrant intervention is equivalent to the optimal intervention in Lerner (2004).}
We study two extensions of the model. We first show that heterogeneity among assets makes asset purchase programs less attractive because firms choose to sell their worst assets to the government. The second extension deals with deposit insurance. Deposit insurance decreases the cost of intervention because the government is partly reducing its own expected insurance payments, but deposit insurance does not alter our results on the relative efficiency of the different interventions.

This paper relates to the theoretical literature on government bailouts, which focuses mostly on financial institutions. Gorton and Huang (2004) argue that the government can bail out banks in distress because it can provide liquidity more effectively than private investors. Diamond and Rajan (2005) show that bank bailouts can backfire by increasing the demand for liquidity and causing further insolvency. Diamond (2001) emphasizes that governments should only bail out the banks that have specialized knowledge about their borrowers. Aghion, Bolton, and Fries (1999) show that bailouts can be designed so as not to distort ex-ante lending incentives. Bebchuk and Goldstein (2009) study bank bailouts in a model where banks may not lend because of self-fulfilling credit market freezes. Farhi and Tirole (2009) examine bailouts in a setting in which private leverage choices exhibit strategic complementarities due to the monetary policy reaction. Corbett and Mitchell (2000) discuss the importance of reputation in a setting where a bank’s decision to participate in a government intervention is a signal about asset values, and Philippon and Skreta (2009) provide a general analysis of optimal interventions in Lemon markets. Mitchell (2001) analyzes interventions when there is both hidden actions and hidden information. Landier and Ueda (2009) provide an overview of policy options for bank restructuring.

The paper also relates to the empirical literature on bank bailouts. Allen, Chakraborty, and Watanabe (2009) provide evidence consistent with the main predictions of our model: they find that interventions work best when they target equity injections into the banks that have material risks of insolvency. Giannetti and Simonov (2009) find that bank recapitalizations result in positive abnormal returns for the clients of recapitalized banks as predicted by our debt overhang model. Glasserman and Wang (2009) develop a contingent claims

\[ \text{a setting where asset values and investment opportunities are perfectly known to the government. We note that the government cannot simply use observed market prices to implement the intervention because its intervention may in turn affect prices (see Bond, Goldstein, and Prescott (2010) and Bond and Goldstein (2010)).} \]
framework to estimate market values of securities issued during bank recapitalizations such as preferred stock and warrants.

Three other theoretical papers share our focus on debt overhang. Kocherlakota (2009) analyzes a model where it is the insurance provided by the government that generates debt overhang. He analyzes the optimal form of government intervention and finds an equivalence result similar to our symmetric information equivalence theorem. Our papers differ because we focus on debt overhang generated within the private sector and we consider the problem of endogenous selection into the government’s programs. In Diamond and Rajan (2009) as in our model, debt overhang makes banks unwilling to sell their toxic assets. In effect, refusing to sell risky assets for safe cash is a form of risk shifting. But while we use this initial insight to characterize the general form of government interventions, Diamond and Rajan (2009) study its interactions with trading and liquidity. In their model, the reluctance to sell leads to a collapse in trading which increases the risks of a liquidity crisis. Bhattacharya and Nyborg (2010) examine bank bailouts in a model with debt overhang and heterogeneity in the support of future distributions of bank asset values. Similar to our paper, they analyze the optimal government intervention in their setting. They show that the optimal intervention may consist of a menu of government interventions. We note that menus may also be optimal in our setting.

Our results can shed light on government actions during the financial crisis of 2007-2009. In October 2008, the US government decided to inject capital into the nine largest US banks under the Trouble Asset Relief Program. Attempts to set up an asset purchase program failed and, after various iterations, the intervention was eventually implemented using preferred stock plus warrants. This is qualitatively similar to the optimal intervention derived in our model.

The paper proceeds as follows. Section 1 provides an example of recapitalization, which conveys the main idea of our analysis. Section 2 sets up the formal model. Section 3 solves for the decentralized equilibrium with and without debt overhang. Section 4 describes the government interventions. Section 5 compares the interventions under symmetric information. Section 6 compares the interventions under asymmetric information. Section 7 discusses optimal interventions. Section 8 describes two extensions to our baseline model. Section 9 discusses the relation of our results to the financial crisis of 2007-2009. Section
10 concludes.

1 An example of recapitalization

We use a simple numerical example to illustrate our three main results. Let us start with one bank (bank A) that holds risky securities, which pay off either 100 or 0 with equal probability. The bank is financed by equity and debt with a face value of 90. Assuming investors are risk neutral, debt value is \(1/2 \times 90 = 45\) and equity value is \(1/2 \times (100 - 90) = 5\). The bank can invest in a safe project, which requires an outlay of 5 and yields a discounted value of 6. The NPV of the project is one and it should therefore be undertaken.

Let us first illustrate the debt overhang issue. The bank needs to raise 5 from new lenders to pay for the project. If the bank invests, debt value increases to \(1/2 \times (90 + 6) = 48\) since in the good state debt holders still receive 90, and in the bad state they now receive 6. The new lenders must receive an expected payment of 5 to break even: they get 10 in the good state. Equity value, however, declines to \(1/2 \times (100 - 90 - 10 + 6) = 3\). Investing is not in the interest of shareholders even though the project has a positive NPV. This is the debt overhang problem analyzed by Myers (1977).

First result

Our first main result is an irrelevance theorem under symmetric information. Consider government interventions to buy back assets or inject equity with voluntary participation by shareholders. The government can offer to purchase assets with a face value of 6 for a cash payment of 5. The cash injection covers the cost of investment, therefore conditional on participation, shareholders strictly prefer to invest. After investing, debt value is 48 and equity value is \(1/2 \times (94 - 90 + 6) = 5\). The shareholders are therefore willing to participate in the program. The net expected cost to the government is \(5 - 1/2 \times 6 = 2\).

Alternatively, the government can offer to buy \(3/8\) of the bank equity for a cash payment of 5. If the bank participates and invests, debt value is 48, equity value is \(5/8 \times 1/2 \times (100 - 90 + 6) = 5\), and government’s cost is \(5 - 3/8 \times 1/2 \times (100 - 90 + 6) = 2\). Asset purchases and equity injections are therefore equivalent: this is our first main result.

Second result

Our second main result emphasizes the efficiency of equity injections under asymmetric
information between the banks and the government. Let us therefore introduce a second bank (bank B) with assets that pay 100 with probability $3/4$ and 0 with probability $1/4$. Debt value is 67.5 and equity value is 7.5. Bank B can invest in a safe project with an outlay of 5 for a discounted value of 7. After investing, asset value becomes 77, debt value is $3/4 \times 90 + 1/4 \times 7 = 69.25$ and equity value is 7.75. Investing increases shareholder value, but by only 0.25 out of a NPV of 2 because shareholders are diluted by new investors. Therefore, even though bank B would invest alone, it might decide to take advantage of the government’s program.

Assume that the government cannot distinguish bank types, and therefore offers the same programs (described above) to all the banks. We already know that bank A participates in both. Under the asset purchase program, bank B’s equity value becomes $3/4 \times (94 - 90 + 7) = 8.25$, and government’s cost is $5 - 3/4 \times 6 = 0.5$.

Under the equity injection program, bank B’s equity value becomes $5/8 \times 3/4 \times (10 + 7) \approx 7.97$, and government’s cost is $5 - 3/8 \times 3/4 \times (10 + 7) \approx 0.22$. The equity injection program implements the same investment (by both types) at a lower cost to the government: this is our second main result.

**Third result**

Our third main result is that an optimal intervention can be implemented with preferred stock and warrants. Suppose that the government still injects 5 in the banks, but now asks for preferred stock with face value 6 plus unlimited warrants at a strike price of 10. Bank A still participates, but bank B does not because its equity value would be $3/4 \times \max(10, 100 - 90 + 7 - 6) = 7.5$ which is lower than its outside option of 7.75. The preferred stock-warrant program therefore implements efficient investment without opportunistic participation and achieves the minimum cost for the government: this is our third main result.

To summarize, a good program improves efficiency along two dimensions: it selects the right banks (the extensive margin) and it minimizes the transfers conditional on participation (the intensive margin). In the remaining of the paper, we will provide a formal comparison of asset purchase, equity injection, and debt guarantee programs, and we will show that the preferred stock-warrant program is the optimal intervention among all interventions that do not involve defaulting on existing debt contracts.

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3Note that $69.25 + 8.25 - 0.5 = 77$ which is the NPV under investment.
2 Model

Our model is applicable to financial and non financial firms but for simplicity and concreteness we refer to all of them as banks. The model has a continuum of banks of measure one and three dates. Banks are ex-ante identical. At time 1, banks become heterogenous: they learn about the quality of their initial assets and they receive new investment opportunities. Banks can borrow from outside investors. The payoffs from the initial assets and the new investments are realized at time 2. All agents are risk neutral, there is no aggregate uncertainty, and we normalize the discount rate to zero. Figure 1 summarizes the timing, technology, and information structure of the model.

2.1 Technology and information

At time 0 banks have both assets and liabilities in place. All banks are ex-ante identical. On the liabilities side, banks have long term debt. Long term debt is due at time 2. Let $D$ be the face value of long-term debt outstanding.

On the asset side, banks have three types of assets: cash, safe long-term assets, and risky long-term assets. Cash is liquid and can be used for investments or for lending at time 1. Let $c_t$ be cash holdings at the beginning of time $t$. All banks start time 0 with $c_0$ in cash. Cash holdings cannot be negative:

$$c_t \geq 0 \text{ for all } t.$$

Safe long-term assets deliver payoff $A$ at time 2. Risky long-term assets deliver random payoff $a = A$ or $a = 0$ at time 2.\footnote{We focus on the binary outcome model because it delivers the main insights while simplifying the algebra. We can extend our equivalence theorem to a general distribution for $a$. Note that any binary asset payoff can be modeled using the risky/safe asset model. For example, suppose that the payoffs are $A^H$ in the good state and $A^L$ in the bad state. To get back to the risky/safe model, we simply define $A = A^L$ and $A = A^H - A^L$.} We define the probability of a good outcome as

$$p \equiv \Pr (a = A).$$

At time 1 private investors learn the value of $p \in [0, 1]$ for each bank.

Banks also receive information about their investment opportunities at time 1. All new investments cost the same fixed amount $x$ at time 1 and deliver income $v \in [0, V]$ at time 2. The payoff $v$ is heterogenous across banks.
A bank’s type is therefore defined by $p$, the quality of risky assets, and $v$, the quality of investment opportunities. From the perspective of time 0, the joint distribution of $p$ and $v$ is $F(p,v)$ defined over $T = [0,1] \times [0,V]$. Let $\bar{p}$ be the unconditional mean of $p$:

$$\bar{p} \equiv \int_0^1 p \times dF(p,v).$$

Banks and private investors learn $(p,v)$ for each bank at time 1, but the government does not.

### 2.2 Assumptions

To make the problem interesting, we assume that individual banks do not have enough cash to finance investment projects. To study debt overhang, we assume that debt is risky such that long term debt $D$ is in default when $a = 0$, but not when $a = A$. We also assume that the income $v$ from new investment is not sufficient to cover long term debt $D$.

**Assumption A1:**

$$c_0 < x < V < D - A < A$$

Assumptions A1 is maintained throughout the paper. Borrowing and lending at time 1 can be among banks, or between banks and outside investors. We assume risk neutral investors and we normalize the risk free rate to 0.

**Assumption A2:** Safe assets $A$ are protected by debt covenants

Assumptions A2 protects debt holders from expropriation by equity holders. It is well known that equity holders have incentives to engage in risk shifting at the expense of debt holders. For instance, shareholders might decide to sell the safe assets and invest the proceeds in risky projects. Debt covenants protect debt holders and play an important role when we discuss asset purchase programs.

### 3 Equilibrium without intervention

In this section, we study the equilibrium without government intervention. We characterize the first best outcome, and the debt overhang equilibrium.
3.1 Investor payoffs

Since $x > c_0$, banks must borrow in order to invest. Let $l$ be the amount borrowed at time 1 and let $r$ be the gross interest rate. At time 2 total bank income $y$ is:

$$y = A + a + c_2 + v \cdot i,$$

where $i$ is one if investment took place, and zero otherwise. Let $y^D$, $y^l$, and $y^e$ be the payoffs to long term debt holders, new lenders, and shareholders, respectively. There are no direct deadweight losses from bankruptcy. Under the usual seniority rules, the payoffs are:

$$y^D = \min(y, D); \quad y^l = \min(y - y^D, rl); \quad y^e = y - y^D - y^l.$$

If $a = A$, all liabilities are fully repaid ($y^D = D$ and $y^l = rl$) and equity holders receive $y^e = y - D - rl$. If $a = 0$, under assumption A1, long term debt holders receive all income ($y^D = y$) and other investors receive nothing: $y^l = y^e = 0$. Figure 2 summarizes the payoffs to investors.

3.2 First best

Without intervention, the banks simply carry their cash holdings from period 0 to period 1, so $c_1 = c_0$. The first best assumption is that banks choose investments at time 1 to maximize firm value $V_1 = A + E_1 [a] + c_2 + v \cdot i - E_1 [y^l]$, subject to the time 1 budget constraint

$$c_2 = c_1 + l - x \cdot i. \quad (1)$$

The break even constraint for new lenders is:

$$E_1 [y^l] = l. \quad (2)$$

Using (1), this implies that $V_1 = A + E_1 [a] + c_1 + (v - x) \cdot i$. Therefore, investment takes place in the domain

$$I^* \equiv \{(p, v) \mid v > x\}. \quad (3)$$

Figure 3 depicts the first best investment region.

**Proposition 1** The first best solution is for investment to take place at time 1 if and only if $v > x$, irrespective of the value of $p$. 

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It is important to understand the connection between maximizing equity value and maximizing firm value. We can always write \( V_1 = E_1 \left[ y - y^f \right] = E_1 \left[ y^e + y^D \right] \). The maximization program for firm value is equivalent to the maximization of equity value \( E_1 \left[ y^e \right] \) as long as we allow renegotiation and transfer payments between equity holders and debt holders.

### 3.3 Debt overhang

Debt overhang occurs when managers maximize shareholder value and debt contracts cannot be renegotiated efficiently, possibly because of contract incompleteness or free-riding among dispersed creditors.\(^5\) Without renegotiation, shareholders get nothing if the bad state realizes at time 2, and if the good state realizes they get \( c_2 + A + A + v \cdot i - D - rl \). The bank maximizes shareholder value subject to budget constraint (1) and break even constraint for new lenders (2).\(^6\) The condition for investment becomes

\[
v - x > (r - 1) l.
\]

This is the investment condition under debt overhang. Recall that the first best investment rule was simply \( v - x > 0 \). The difference with the first best investment rule comes from two critical properties. First, the outside investors ask for a risk premium because they know that lending is risky. Hence \( r > 1 \). Second, shareholders perceive a high cost of capital because they do not get the returns of the investment project in the bad state.

A constrained firm would always choose to invest its own cash first, so \( c_2 = 0 \), and \( l = x - c_1 \). Since \( c_1 = c_0 \), equation (4) becomes \( pv + (1 - p) c_0 > x \) and we get the investment domain:

\[
I^0 \equiv \{(p, v) \mid L^0(p, v) > 0\}, \tag{5}
\]

where we define

\[
L^0(p, v) \equiv pv + (1 - p) c_0 - x. \tag{6}
\]

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\(^5\)Bhattacharya and Faure-Grimaud (2001) show that renegotiation does not achieve the first-best level of investment if investment is non-contractible. Gertner and Scharfstein (1991) argue that it is not profitable for small debt holders to accept a face value lower than \( D \) because of the free-riding problem. These conditions are particularly relevant for banks because their debt holders are usually widely dispersed and their investment opportunities are difficult to observe.

\(^6\)We note that using short-term debt does not solve the debt overhang problem because the debt has to be rolled over before the end of the investment project. By backwards induction, the debt overhang problem therefore reemerges, even if the bank borrows short-term (Myers (1977)).
Figure 4 depicts the investment region in the debt overhang equilibrium. If $L^o(p,v) < 0$, no investment takes place. If $L^o(p,v) > 0$, investment takes place using the free cash $c_0$ and the additional borrowing $x - c_0$. The function $L^o(p,v)$ measures the value for shareholders of undertaking a new investment under debt overhang, given the quality of the existing assets $p$, the available cash $c_0$, and the fundamental value of new investment $v$. From the perspective of shareholders, the NPV of the investment is $pv - x$. Internal cash $c_0$ has a lower opportunity cost than external financing because from the equity holder’s perspective internal cash has an expected value of $p$ but external financing has an expected cost of $r \times p = 1$.

### 3.4 Shareholder value and welfare losses

We repeatedly use the time 0 and time 1 equity value to compute equity holder’s optimal investment and participation decisions. The equity value at time 1 is

$$E_1[y^e|p,v] = p(N + c_0) + L^o(p,v)1_{(p,v) \in I^o} \quad (7)$$

where

$$N = A + A - D.$$ 

Equity value at time 1 is the sum of two terms. The first term is the equity holder’s expected value of long term assets and cash minus senior debt. The value is multiplied by probability $p$ because equity holders only receive a payment in the high-payoff state. The second term is the equity holder’s value of new investment opportunities $L^o(p,v)$ as defined above.

Taking expectations at time 0, the equity value is:

$$E_0[y^e] = \bar{p}(N + c_0) + \int_{I^o} L^o(p,v) dF(p,v) \quad (8)$$

The first term is the expected equity value of long term assets and cash minus senior debt using the unconditional probability of solvency $\bar{p}$. The second term is the time 0 expected value of new investment opportunities. The domain $I^o$ is defined in Equation (5). Since investment is chosen optimally, the value of new investment opportunities $L^o(p,v)$ is zero on the border of $I^o$. 

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Social welfare under debt overhang depends on the set of implemented investment projects \( I^o \). We define \( W(.) \) as the social welfare function, so that welfare under debt overhang is

\[
W(I^o).
\]

As long as the second best investment set \( I^o \) is strictly smaller than the first best investment set \( I^* \), there is a welfare loss. In the banking context, these deadweight losses are missed trading and lending opportunities. We assume the social welfare function incorporates deadweight losses to both banks and borrowers. Hence, the welfare function is independent of how the benefits of investment projects are shared among banks and borrowers.

Note that equation (9) assumes that investment projects are bank specific. This assumption is justified by the large literature in banking which argues that one of the main functions of financial intermediaries is to generate private information about their borrowers (see for instance Diamond (1984)), and that it is costly for borrowers to switch to other banks or other sources of funds.

4 Description of government interventions

We consider three government interventions: asset purchases, cash against equity, and debt guarantees. We first discuss the government’s objective function and then briefly describe each intervention. We consider the case where participation in the program is decided at time 0, and the case where it is decided at time 1. The difference matters because banks learn about the value of their existing assets and about their new investment opportunities at time 1. Interventions at time 1 are therefore subject to adverse selection, while interventions at time 0 are not. The two cases are empirically relevant, and we therefore analyze both.

4.1 Government objective function and constraints

The objective of the government is to minimize the welfare losses from missed investment opportunities and the costs of intervention. Let \( \Psi \) be the expected cost of a government intervention. Let \( \chi \) be the marginal deadweight losses associated with raising taxes and administering government interventions. The objective function of the government is

\[
\max_{\Gamma} W(I(\Gamma)) - \chi \Psi(\Gamma)
\]
where \( \Gamma \) describes the specific intervention. For simplicity, we assume that the marginal cost \( \chi \) is constant. This means that the government only cares about total net expected costs.

We place the same constraints on the government as on the private investors. The government must follow the rule of law and cannot break private contracts. These restrictions rule out forced bankruptcy, forced asset sales, and forced debt equity swaps. We also assume that the government cannot force the banks to invest. On the other hand, we assume that the government can restrict dividend payments to shareholders. Otherwise banks could simply pay out proceeds from government interventions as a dividend to shareholders.

### 4.2 Description of asset purchase program

The asset purchase program is parameterized by \( Z \) and \( p^z \). The government announces at time 0 that it is willing to purchase risky assets up to an amount \( Z \) at a per unit price of \( p^z \) in exchange for cash. If a bank decides to participate and sell \( z < Z \), long term assets become \( A_1 = A - z \) and cash \( c_1 = c_0 + zp^z \). Note that the government can only buy risky but not safe long term assets. The reason is that under Assumption A2 debt covenants prevent equity holders from selling safe assets.\(^7\)

When participation is decided at time 0, we can without loss of generality consider programs where all banks participate because they are identical and the government can always set \( Z = 0 \). The expected cost of the time 0 asset purchase program is

\[
\Psi^a_0(Z, p^z) = z_0(p^z - \bar{p}) \text{ with } z_0 \leq Z
\]

where \( z_0 \) is the face value of assets purchased by the government. The government pays out \( z_0p^z \) at time 0 and receives \( z_0 \) in the high-payoff state with probability \( \bar{p} \).

At time 1, the cost of the asset purchase program is different because banks learn the value of investment opportunity \( v \) and the value of long term assets \( p \) before deciding whether to participate. The expected cost is therefore

\[
\Psi^a_1(Z, p^z) = \int \int z_1(Z, p^z; v, p) \cdot (p^z - p)dF(v, p)
\]

\(^7\)This assumption is important because, as we show below, equity holders can extract rents from debt holders by selling safe assets. The intuition is that safe asset sales change the priority structure of financial claims and effectively give equity holders priority over debt holders.
where \( z_1 \) is the face value of risky long term assets sold under the program. This formulation allows for adverse selection because banks may participate in the program depending on their type \((v, p)\).

### 4.3 Description of equity injection program

Equity injection programs are parameterized by \( m \) and \( \alpha \). The government announces at time 0 that it is willing to offer cash \( m \) against a fraction \( \alpha \) of equity returns. Similar to the asset purchase program, the government can offer banks to participate in this program at time 0 or time 1. If a bank decides to participate, its cash position becomes \( c_1 = c_0 + m \).

The expected cost of the program at time 0 is

\[
\Psi_0^e (m, \alpha) = m - \alpha E_0 [y^e (m)]
\]

where \( E_0 [y^e (m)] \) is the expected equity return at time 0 conditional on cash injection \( m \).

In words, the government pays out \( m \) at time 0 and receives a share \( \alpha \) of equity returns \( y^e \) at time 2. There are no constraints on that program, except \( m \geq 0 \) and \( \alpha \in [0, 1] \). The expected cost of the time 1 program is

\[
\Psi_1^e (m, \alpha) = \int \int \delta^e (m, \alpha; v, p) \cdot (m - \alpha E_1 [y^e (m) | v, p]) dF(v, p)
\]

where \( \delta^e \) is an indicator variable whether a bank participates in the program, and \( E_1 [y^e (m) | v, p] \) is the expected equity return at time 1 conditional on cash injection \( m \) and and bank type \((v, p)\). Similar to the asset purchase program, this formulation allows for adverse selection depending on bank type \((v, p)\).

### 4.4 Description of debt guarantee program

Debt guarantee programs are parameterized by \( S \) and \( \phi \). The government announces at time 0 that it is willing to guarantee new bank debt up to a face value of \( S \) and charges banks a fee \( \phi \) per unit of lending. There are several equivalent ways to define the parameters \( S \) and \( \phi \). In our notation, the fee is paid up-front and the upper bound applies to the face value of new bank debt. Let \( s \) be the face value of new bank debt issued under the program and let \( r_s \) be the interest rate on debt issued under the program. The amount of money
raised at time is therefore \( s/r_s - \phi s \) and the constraint is \( s < S \) (we will see shortly that \( r_s = 1 \) in equilibrium). At time 0, the expected cost to the government is

\[
\Psi^g_0 (S, \phi) = s_0 (1 - \phi - \bar{p}).
\]

The expected cost to the government is the probability of the low-payoff state \((1 - \bar{p})\) minus the guarantee fee \(\phi\).

At time 1, the expected cost of the government is

\[
\Psi^g_1 (S, \phi) = \int \int s_1 (S, \phi, v, p) (1 - p - \phi) dF(v, p). \tag{11}
\]

Similar to the other programs, the time 1 debt guarantee allows for adverse selection depending on bank type \((v, p)\).

5 An irrelevance theorem for interventions under symmetric information

Our first main result is that the form of the intervention is irrelevant under symmetric information. Formally, we will say that two interventions are equivalent when they implement the same level of investment at the same expected cost to the government.

To provide intuition for this result, we first discuss the case of providing a free subsidy to banks. A free subsidy means that the government gives cash \(m\) to each bank, without asking for anything in return. This case is a useful benchmark because it illustrates how additional cash affects the investment region.\(^8\) We first define the investment domain \(I(m)\) which plays a key role in our discussion:

**Definition 1** Let the domain \(I(m)\) be defined by

\[
I(m) \equiv \{(p, v) \mid L^o (p, v) + (1 - p) m > 0\}. \tag{12}
\]

The following lemma characterizes the impact of a free subsidy provided by the government:

**Lemma 1** A free subsidy leads to the following welfare function for the government

\[
W(I(m)) - \chi m
\]

\(^8\)In terms of the interventions discussed above, a free subsidy is equivalent to an equity injection \(m\) with equity share \(\alpha = 0\), an asset purchase program with face value \(Z \rightarrow 0\) and cash injection \(p^* Z = m\), or a debt guarantee program with face value \(S = m\) and guarantee fee \(\phi = 0\).
Proof. Suppose the government injects \( m \) in each bank so that the cash balance becomes \( c_1 = c_0 + m \). From equation (5) and (6), we see that the investment domain becomes \( I(m) \) and the total cost is \( m \) since the number of banks is normalized to one. \( \square \)

Figure 5 shows the effect of a free subsidy on the investment region \( I(m) \). The cash injection \( m \) relaxes the investment constraint and therefore expands the investment domain. A cash injection large enough to cover the entire financing need would eliminate debt overhang entirely, i.e., \( I(x - c_0) = I^* \) defined in equation (3).

We now compare government programs under symmetric information and obtain our first main result:

**Theorem 1** *Equivalence under symmetric information.* A time 0 risky asset purchase program \( (Z, p^*) \) is equivalent to a time 0 debt guarantee program with \( S = Z \) and \( p^* = 1 - \phi \). It is also equivalent to a time 0 equity injection program \( (m, \alpha) \), where \( m = Zp^* \) and \( p^* \) and \( \alpha \) are chosen such that at time 0 all banks are indifferent between participating and not participating in the program. All programs implement the same investment set \( I(m) \) and have the same expected cost:

\[
\Psi_0(m) \equiv m \int_{T \setminus I(m)} (1-p) dF(p,v) - \int_{I(m) \setminus I^o} L^o(p,v) dF(p,v), \tag{13}
\]

where \( T = [0,1] \times [0,V] \) is the state space. The function \( \Psi_0(m) \) is always positive.

Proof. See Appendix. \( \square \)

The key to this irrelevance theorem is that banks decide whether to participate before they receive information about investment opportunities and asset values. The government thus optimally chooses the program parameters such that banks are indifferent between participating and not participating. The cost to the government is thus independent of whether banks are charged through assets sales, debt guarantee fees, or equity.

The government’s cost under symmetric information has a natural interpretation in terms of the two terms on the right-hand side of equation (13). The first term reflects the transfer of wealth from the government to the debt holders of banks that do not invest: debt
value simply increases by \((1 - \bar{p}) m\) over the domain \(T \setminus I (m)\). The second term measures the subsidy needed to induce investment over the expanded domain \(I (m)\) compared to the initial debt overhang domain \(I^o\). By definition \(L^o (p, v)\) is negative over the domain \(I (m) \setminus I^o\), and \(\Psi_0 (m)\) is therefore always positive.

Finally, note the symmetric information case can alternatively be interpreted as a program with compulsory participation subject to the constraint that the program be acceptable to the average bank (for instance, a well diversified equity investor would accept the offer on behalf of all the banks). This interpretation seems relevant in light of actual interventions during financial crises.\(^9\)

6. Comparison of interventions under asymmetric information

Our second main result is that equity injections dominate asset purchases and debt guarantees under asymmetric information. Dominance between two interventions means that they implement the same level of investment but the dominant one has a lower cost.

To build intuition for this result, we first discuss the structure of debt guarantees under asymmetric information, i.e. at time 1. Banks benefit from a debt guarantee because it allows them to issue riskless debt. The equilibrium interest rate on riskless debt is \(r_s = 1\) and the equilibrium interest rate on unsecured debt \(r_u = 1/p\). The budget constraint (1) becomes

\[
c_2 = c_0 + l_u + (1 - \phi) s - x, \tag{14}
\]

and the investment condition (4) becomes

\[
L^o (p, v) + s (1 - \phi - p) > 0. \tag{15}
\]

We make two observations. First, it is clear from the bank’s budget constraint (14) that it cannot be optimal for the government to provide a debt guarantee \(S\) that is larger than the financing need \(x - c_0\). Second, suppose the government imposes a ‘no inefficient participation’ constraint (NIP from now on), such that no bank would participate without

\(^9\)“In Mexico, Japan and Thailand banks could voluntarily apply for programs while in Korea and Malaysia some coercion seems to have been applied.” Corbett and Mitchell (2000). Some coercion also seems to have been applied in the US during the Fall of 2008.
investing. We can use budget constraint (14) to derive the NIP constraint. Banks receive 
\((1 - \phi)s\) at time 1 and have to repay \(s\) in the high-payoff state. Assuming a bank does not 
invest, the bank does not participate if \((1 - \phi)s < s\) or
\[
\phi > 0. \tag{16}
\]
Hence, if the government charges a positive fee for the debt guarantee, all banks that 
choose to participate also invest. Conversely, a debt guarantee program with fee \(\phi \leq 0\) 
cannot improve over a free subsidy program in which all banks participate. We summarize 
this result in the following lemma.

**Lemma 2** It is enough to consider debt guarantees such that \(S \in [0, x - c_0]\) and \(\phi > 0\).

Under the debt guarantee, banks have a choice between secured borrowing from the government and unsecured borrowing from other lenders. Banks choose secured borrowing over unsecured borrowing if and only if the net benefit of secured borrowing \(1 - \phi\) is higher than the net benefit of unsecured borrowing \(p\), that means if \((1 - \phi) > p\). This condition defines an upper-bound schedule \(U^g_1(p, v; S, \phi) < 0\) for participation in the debt guarantee program:
\[
U^g_1(p, v; S, \phi) \equiv p + \phi - 1. \tag{17}
\]
Hence, banks participate only if asset quality \(p \in [0, 1 - \phi]\). In the case of debt guarantees, 
this upper-schedule is vertical (it does not depend on \(v\)), but it will depend on both \(p\) and \(v\) in the case of equity injections.

Participation also requires banks to have sufficiently profitable investment opportunities. It is clear that participating banks choose to borrow the maximum guarantee \(s = S\) 
because the per-unit payoff \((1 - \phi)\) is linear in \(s\). This implies that unsecured borrowing 
of participating banks is \(l_u = x - (1 - \phi)S - c_0\). Using the investment equation (15), we 
derive the lower bound schedule for investment and participation, \(L^g_1(p, v; S, \phi)\):
\[
L^g_1(p, v; S, \phi) \equiv L^o(p, v) + (1 - \phi - p)S \tag{18}
\]
Banks participate and invest only if \(L^g_1(p, v; S, \phi) > 0\), or equivalently if the sum of the equity holder’s share of the investment payoff \(L^o(p, v)\) and the net subsidy \((1 - \phi - p)S\) is positive.
The lower- and the upper-schedules define the participation set:

$$\Omega_1^g (S, \phi) = \{(p, v) \mid L_1^g (p, v; S, \phi) > 0 \land U_1^g (p, v; S, \phi) < 0\} \quad (19)$$

The investment domain under the debt guarantee program $I_1^g$ is the combination of the investment set $I^o$ (banks that would invest without government intervention) and the participation set $\Omega_1^g$:

$$I_1^g (S, \phi) = I^o \cup \Omega_1^g (S, \phi). \quad (20)$$

Note that the overlap between the two sets, $I^o \cap \Omega_1^g (S, \phi)$, represents opportunistic participation. Opportunistic participation is inefficient, because the government provides a subsidy to banks that would have invested even without the intervention.

The participation set determines the expected cost of the government intervention. Using equation (11), the expected cost of the debt guarantee program is

$$\Psi_1^g (S, \phi) = S \int \int_{\Omega_1^g (S, \phi)} (1 - p - \phi) dF (p, v). \quad (21)$$

In words, the government’s cost is the net subsidy $S (1 - p - \phi)$ for all banks in the participation set.

Figure 6 shows the investment and participation sets for debt guarantees under asymmetric information. The figure distinguishes three regions of interest: efficient participation, opportunistic participation, and independent investment. The efficient participation region comprises the banks that participate in the intervention and that invest because of the intervention. The opportunistic region comprises the banks that participate in the intervention but would have invested even in the absence of the intervention. The independent investment region comprises the banks that invest without government intervention. As is clear from the figure, the government’s trade-off is between expanding the efficient participation region and reducing the opportunistic participation region.

Comparing equations (13) and (21), we see that debt guarantees are less costly than free subsidies for three reasons. First, the independent investment region reduces opportunistic participation without reducing investment. Second, the program fee $\phi > 0$ excludes banks that would not invest. Third, banks repay $S$ in the high-payoff state which lowers the government’s cost without affecting investment.
Let us now compare debt guarantees to asset purchases:

**Theorem 2** *Equivalence of asset purchases and debt guarantees under asymmetric information.* An asset purchase program \((Z, p^z)\) with participation at time 1 is equivalent to a debt guarantee program with \(S = Z\) and \(p^z = 1 - \phi\).

**Proof.** See Appendix.  

The equivalence of debt guarantees and asset purchases comes from the fact that both programs make participation contingent on asset quality \(p\) but not investment opportunity \(v\). To see this result, consider the upper-bound schedule. If \(p^z = 1 - \phi\), banks with asset quality \(p \in [1 - \phi, 1]\) choose not to participate. Hence, asset purchase program and debt guarantees have the same upper-bound schedule. Next, note that the net benefit of asset purchases is \((p^z - p)\), whereas the net benefit of debt guarantees is \((1 - \phi - p)\). Hence, asset purchases and debt guarantees have the same lower bound schedule. The NIP constraint for asset purchases is \(p < 1\), which is equivalent to \(\phi > 0\). The last step is to show that both asset purchases and debt guarantees have the same cost to the government, which is true since they yield the same net benefit to participants.

We can finally compare debt guarantees and asset purchases to equity injections:

**Theorem 3** *Dominance of equity injection under asymmetric information.* For any asset purchase program \((Z, p^z)\) with participation at time 1, there is an equity program that achieves the same allocation at a lower cost for the government.

**Proof.** See Appendix.  

The dominance of equity injection over debt guarantees and asset purchases comes from the fact the equity injections are dependent both on asset quality \(p\) and investment opportunity \(v\). To understand this result, it is helpful to define the function \(X(p; m, \alpha)\) as the net benefit of participating in the equity injection \((m, \alpha)\):

\[
X(p; m, \alpha) \equiv (1 - \alpha) m - \alpha p (N + c_0).
\]
In words, a participating bank receives net cash injection \((1 - \alpha) m\) and gives up share \(\alpha\) of the bank’s expected equity value \(p (N + c_0)\).

To compare equity injections with other programs, start by choosing an arbitrary asset purchase program. Then choose the net benefit of participation \(X (p; m, \alpha)\) such that the lower-bound schedule of the asset purchase program coincides with the lower-bound schedule of the equity injection program. Under both programs, equity holders at the lower-bound schedule receive no surplus and are indifferent between participating and not participating. Both programs offer the same net cash injection.

The main difference between the equity injection program and the asset purchase program is that, for given level of asset quality \(p\), the cost of participation for banks with a good investment opportunity \(v\) is higher under the equity injection program than under the asset purchase program. The reason is that under the equity injection program, the government receives a share in both the existing assets and future investment opportunities but under the asset purchase program the government only receives a share in the existing assets. The benefit of participation is the same under both programs. As a result, there is less opportunistic participation under the equity injection than under the asset purchase program.

Figure 7 shows the participation and investment regions under the equity injection program. The increase in cost of participation relative to the asset purchase program has two effects. First, conditional on participation, the cost to the government is smaller because the government receives a share in the investment opportunity \(v\). Second, there is less opportunistic participation because participation is more costly. As a result, equity injections and asset purchases implement the same level of investment but equity injections are less costly to the government relative to asset purchases.

The final step in the proof is to show that the no-inefficient participation constraint is the same under both program. This is true because the equity injection provides lower rents to participating banks than the asset purchase programs. Hence, if the no-efficient participation holds under the asset purchase program, it also holds under the equity injection program.\(^{10}\)

\(^{10}\)We can also show that equity programs at time 1 cannot be improved by mixing them with a debt guarantee or asset purchase program. Pure equity programs always dominate. The proof is available upon request.
Finally, we can compare interventions with voluntary participation (at date 1 in our model) and interventions with compulsory participation (equivalent to time 0 interventions, as discussed above). Compulsory participation is costly because some banks participate and do not invest, as shown in equation (13). However, compulsory participation allows the government to extract the expected rents of future investment projects. Compulsory interventions are therefore attractive when most banks have profitable investment opportunities and when the government wants to implement a large investment set. On the other hand, voluntary interventions are attractive when many banks do not have valuable new projects and when the desired scale of intervention is small.\footnote{A formal proof of these comparative statics can be found in our working paper.}

7 Optimal interventions

Our third main result is that optimal interventions can be implemented with warrants and preferred stock. Optimal interventions dominate asset purchases, equity injections, and debt guarantees both under symmetric and asymmetric information.

To derive the optimal intervention, we first characterize the minimum cost of a government intervention. Under the optimal intervention, the government injects cash $m$ at time 1 in exchange for state contingent payoffs at time 2. New lenders at time 1 must break even and we can without loss of generality restrict our attention to the case where the government payoffs depend on the residual payoffs $y - y^D - y'$. We also maintain the restrictions explained in Section 4.1.

As in previous sections, we will examine cost minimization for a given investment set.\footnote{In general, the government can offer a menu of contracts to the banks in order to obtain various investment sets. The actual choice depends on the distribution of types $F(p,v)$ and the welfare function $W$ but we do not need to characterize it. We simply show how to minimize the cost of implementing any particular set.} Let us start with a general characterization of the minimum cost for any intervention:

**Lemma 3** Any program with voluntary participation of equity holders over the set $\Omega$ has a minimum cost of

$$\psi^{\text{min}}(\Omega) = - \int_{\Omega} L^o(p,v) \, dF(p,v)$$
**Proof.** Voluntary participation means that equity holders must get at least \(p (N + c_0)\). The government and old equity holders must share the residual surplus whose value is

\[p (N + c_0) + L^o (p, v)\]

Hence the expected net payments to the government must be \(\int_{\Omega} L^o (p, v) dF (p, v)\). These are negative as long as \(\Omega\) extends the debt overhang investment set, hence the positive minimum cost.

A simple way to understand this result is to imagine what would happen if the government could observe the types and write contracts contingent on investment. For the shareholders of type \((p, v)\), the value of investment is \(L^o (p, v)\), which is negative outside the debt overhang investment region \(I^o\). If the government had full information, it could offer a contract with a type-specific payment contingent on investment. The minimum the government would have to offer type \((p, v)\) would be \(-L^o (p, v)\).

However, it is far from obvious whether the government can reach this lower bound without full information.\(^{13}\) The surprising result is that it can do so with warrants and preferred stock.

**Theorem 4** Consider the program \(\Gamma = \{m, h, \varepsilon\}\) where the government provides cash \(m\) at time 1 in exchange for preferred stock with face value \((1 + h) m\) and a portfolio of \((1 - \varepsilon) / \varepsilon\) warrants at the strike price \(N + c_0\). This program implements the investment set

\[I (\Gamma) = I^g_1 (S, \phi),\]

where we identify the cash injection \(m = (1 - \phi) S\) and the interest rate \(h = \phi / (1 - \phi)\). In the limit \(\varepsilon \to 0\), opportunistic participation disappears and the program achieves the minimum cost:

\[
\lim_{\varepsilon \to 0} U (\Gamma) = L^o, \\
\lim_{\varepsilon \to 0} \Psi (\Gamma) = \Psi^{\min} (I (\Gamma) \setminus I^o) .
\]

\(^{13}\)Note that the government cannot simply overcome asymmetric information by learning about asset values and investment opportunities from observed asset prices because assets prices also reflect the perceived likelihood of future government interventions, so the government cannot use the prices to learn about what asset values would be without intervention. Credit default swap prices of US banks during the financial crisis of 2007 to 2009 provide clear evidence of this issue.

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Proof. See Appendix. ■

Figure 8 shows the investment and participation region under the optimal intervention. Under the optimal intervention, the initial shareholders receive the following payoff in the high-payoff state:

$$f(y^*) = \min (y^*, N + c_0) + \varepsilon \max (y^* - N - c_0, 0)$$

Shareholders are full residual claimants up to the face value of old assets $N + c_0$ and $\varepsilon$ residual claimants beyond. When $\varepsilon$ goes to zero, the entire increase in equity value over the no-investment case is extracted by the government via warrants. As a result, the opportunistic participation region disappears and only the banks that really need the capital injection to invest participate in the program.\(^{14}\)

Four properties of this optimal program are worth mentioning. First, we use preferred stock because it is junior to new lenders at time 1 and senior to common equity, but the program could also be implemented with a subordinated loan. Second, it is important that the government also takes a position that is junior to equity holders. The warrants give the upside to the government, which limits opportunistic participation. Third, the use of warrants limits risk shifting incentives since the government owns the upside, not the old equity holders (see, for instance, Green (1984)). Fourth, the use of warrants may allow the government to credibly commit to protecting new equity holders. This may be important for reasons outside the model if investors worry about outright nationalization of the banks.

We also note that our results describe the optimal intervention for a given investment set. The optimal investment set is the solution to the government objection function \((10)\) and depends on the distribution of asset values and investment opportunities $F(p, v)$, the social welfare function $W(I(m))$, and the deadweight losses of taxation $\chi$. We note that implementing the optimal investment set may require a menu of programs.

\(^{14}\)In practice, there might be a lower bound on $\varepsilon$ because the government might not want to own the banks. An approximate optimal program could then be implemented at this lower bound $\varepsilon$. Similarly, the rate $h$ is chosen to rule out inefficient participation (the NIP constraint). In theory, any $h > 0$ would work, but in practice, parameter uncertainty could prevent $h$ from being too close to zero.
8 Extensions

In this section we present two extensions to our baseline model. We consider the consequences of heterogeneous assets within banks and of deposit insurance.\footnote{Other extensions to continuous asset distributions and the sale of safe assets are available upon request. They do not generate new insights that justify their inclusion in this section.}

8.1 Heterogeneous assets within banks

We consider an extension of our model to allow for asset heterogeneity within banks. Suppose that the face value of assets at time 0 is $A + A'$. All these assets are ex-ante identical. At time 1, the bank learns which assets are $A'$ and which assets are $A$. The $A$ assets are just like before, with probability $p$ of $A$ and $1-p$ of 0. The $A'$ assets are worth zero with certainty. The ex-ante problems are unchanged, so all programs are still equivalent at time 0.

The equity and debt guarantee programs are unchanged at time 1. So equity still dominates debt guarantee. But the asset purchase program at time 1 is changed. For any price $p^z > 0$ the banks will always want to sell their $A'$ assets. This will be true in particular of the banks without profitable lending opportunities.

**Proposition 2** With heterogeneous assets inside banks, there is a strict ranking of programs: equity injection is best, debt guarantee is intermediate, asset purchase program is worse.

The main insight from this extension is that adverse selection across banks is different from adverse selection across assets within banks. Adverse selection within banks increases the cost of the asset purchase program but does not affect the other programs.

8.2 Deposit insurance

Suppose long term debt consists of two types of debt: deposits $\Delta$ and unsecured long term debt $B$ such that

$$D = \Delta + B.$$
Suppose that the government provides insurance for deposit holders and that deposit holders have priority over unsecured debt holders. Then the payoffs are are:

\[ y^\Delta = \min(y, \Delta); y^B = \min(y - y^\Delta, B) \]

Deposits are safe if \( \Delta < A + c_0 \), and risky if \( \Delta \geq A + c_0 \).

**Proposition 3** With safe deposits, the cost and benefits of both time 0 and time 1 programs remain unchanged. With risky deposits, the costs of time 0 and time 1 programs decrease. The equivalence results and ranking of both time 0 and time 1 programs remain unchanged.

**Proof.** See Appendix. ■

If deposits are safe, banks always have sufficient income to repay deposit holders. Hence, the expected cost of deposit insurance is zero independent of whether there is a government intervention. As a result, the costs and benefits of all programs remain unchanged.

With risky deposits, the government has to pay out deposit insurance in the low-payoff state. Hence, every cash injection lowers the expected cost of deposit insurance in the low-payoff state one-for-one. As a result, the government recoups the cash injection both in the high- and low-payoff state. Put differently, a cash injection represents a wealth transfer to depositors and, because of deposit insurance, a wealth transfer to the government. Hence, the equivalence results and the ranking of interventions remain unchanged.

9 Discussion of financial crisis of 2007/09

The financial crisis of 2007-2009 has underlined the importance of debt overhang. There is agreement among many observers that debt overhang is an important reason for the decline in lending and investment during the crisis (see Allen, Bhattacharya, Rajan, and Schoar (2008) and Fama (2009), among others).

For example, Ivashina and Scharfstein (2008) show that new lending was 68% lower in the three-month period around the Lehman bankruptcy relative to the three-month period before the Lehman bankruptcy. Using cross-sectional variation in bank access to deposit financing, the authors show that the reduction in lending reflects a reduction in credit supply by banks rather than a reduction in credit demand by borrowers.
The crisis has also shown the difficulty of finding effective solutions to the debt overhang problem. Several experts have expressed concerns that existing bankruptcy procedures for financial institutions are insufficient for reorganizing the capital structure. As an alternative, Zingales (2008) argues for a law change that allows for forced debt-for-equity swaps. Coates and Scharfstein (2009) suggest to restructure bank holding companies instead of bank subsidiaries. Ayotte and Skeel (2009) argue that Chapter 11 proceedings are adequate if managed properly by the government. Assuming that restructuring can be carried effectively, these approaches reduce debt overhang at low cost to the government. However, Swagel (2009) argues that the government lacks the legal authority to force restructuring and that changing bankruptcy procedures is politically infeasible once banks are in financial distress.

Moreover, concerns for systemic risk and contagion make it difficult to restructure financial balance sheets in the midst of a financial crisis. Aside from the costs of its own failure, the bankruptcy of a large financial institution may trigger further bankruptcies because of counterparty risks and runs by creditors. For example, Heider, Hoerova, and Holthausen (2008) emphasize the role of counterparty risk in the interbank market.

The government may therefore decide to avoid restructuring because there is a positive probability of a breakdown of the entire financial system. Even if the government decides to let some institutions restructure, the government also has to address debt overhang among the financial institutions that do not restructure. In fact, even proponents of restructuring suggest to rank banks based on their financial health and only restructure banks below a cutoff. Hence, independent of whether the government restructures some banks, the optimal form of government intervention outside restructuring remains an important question.

Surprisingly however, while there is at least some agreement regarding the diagnostic (debt overhang), there is considerable disagreement about the optimal form of government intervention outside restructuring. The original bailout plan proposed by former Treasury Secretary Paulson favors asset purchases over other forms of interventions. Stiglitz (2008) argues that equity injections are preferable to asset purchases because the government can participate in the upside if financial institutions recover. Soros (2009a) also favors equity injections over asset purchases because otherwise banks sell their least valuable assets to the government. Diamond, Kaplan, Kashyap, Rajan, and Thaler (2008) argue that the optimal
government policy should be a combination of both asset purchases and equity injections because asset purchases establish prices in illiquid markets and equity injections encourage new lending. Bernanke (2009) suggests that in addition to equity injections and debt guarantees the government should purchase hard-to-value assets to alleviate uncertainty about bank solvency. Geithner (2009) argues that asset purchases are necessary because they support price discovery of risky assets.

Other observers have pointed out common elements among the different interventions without necessarily endorsing a specific one. Ausubel and Cramton (2009) argue that both asset purchases and equity injections require to put a price on hard-to-value assets. Bebchuk (2008) argues that both asset purchases and equity injections have to be conducted at market values to avoid overpaying for bad assets. Soros (2009b) argues that bank recapitalization has to be compulsory rather than voluntary. Kashyap and Hoshi (2008) compare the financial crisis of 2007-2009 with the Japanese banking crisis and argue that in Japan both asset purchases and capital injections failed because the programs were too small. Scharfstein and Stein (2008) argue that government interventions should restrict banks from paying dividends because, if there is debt overhang, equity holders favor immediate payouts over new investment. Acharya and Backus (2009) suggest that public lender of last resort interventions would be less costly if they borrowed some of the standard tools used in private contracts for lines of credit.

We believe our results in this paper make three contributions to this debate. First, we believe an analytical approach to this question is helpful because it allows the government to implement a principled approach in which financial institutions are treated equally and government actions are predictable. This approach is preferable to a trial-and-error approach in which the government adjust interventions depending on current market conditions and tailors interventions to requests of individual financial institutions. In fact, such a trial-and-error approach may create more uncertainty for private investors, which makes them even less willing to invest. Uncertainty also generates an option to wait for future interventions, which further undermines private recapitalizations. Moreover, tailor-made interventions are more likely to be influenced and distorted by powerful incumbents (see Hart and Zingales (2008), Johnson (2009)).

Second, we distinguish the economic forces that matter from the ones that do not by
providing a benchmark in which the form of government interventions is irrelevant. Under symmetric information, all interventions implement the same level of lending at the same expected costs. In contrast, under asymmetric information buying equity dominates other forms of intervention because buying equity reduces the extent of adverse selection across banks. Our analysis also shows how the government can use warrants to minimize the expected cost to taxpayers, an important element which has not been emphasized in the public debate on the financial crisis. Interestingly, Swagel (2009) notes that the terms of the Capital Purchase Program, the first round of government intervention, consisted of providing a loan in exchange for preferred stock and warrants. This structure is qualitatively consistent with the optimal intervention.

Third, our analysis clarifies why government interventions are costly. Under symmetric information, equity holders are held to their participation constraint but debt holders receive an implicit transfer. Hence, the same economic force that generates debt overhang in the first place, also generate the cost to the government. Under asymmetric information, participating banks receive informational rents because otherwise they would choose not to participate. Hence, under asymmetric information government interventions are costly because the government has to recapitalize at above market rates.

Our analysis focuses on one specific market failure, debt overhang, and its negative consequences on lending and investment. We have emphasized the importance of asymmetric information between the government and the private sector, but we have maintained the assumption of symmetric information within the private sector. Philippon and Skreta (2009) solve for the optimal form of intervention when the market failure is adverse selection among private agents. They find that debt guarantees are optimal and that the government should always aim for pooling interventions where all banks participate.

Finally, we note that our analysis does not address why the banking system entered financial distress and whether government bailouts affect future bank actions. In our model, we take debt overhang as given and rely on other research that links the financial crisis to securitization (Mian and Sufi (2008), Keys, Mukherjee, Seru, and Vig (2010)) and the tendency of banks to become highly levered (Adrian and Shin (2008), Acharya, Schnabl, and Suarez (2009), Kacperczyk and Schnabl (2009)). Regarding the impact of government interventions on future bank actions, we recognize that bailouts can create expectations
of future bailouts which may cause moral hazard. However, if the government decides to intervene, then it is optimal for the government to choose the intervention with the lowest costs. Also, the optimal intervention minimizes rents to equity and debt holders, so the optimal intervention also minimizes moral hazard conditional on the decision to intervene.

10 Conclusion

In this paper we study the efficiency and welfare implications of different government interventions in a standard model with debt overhang. We consider asset purchases, equity injections, and debt guarantees. We find that under symmetric information, all interventions are equivalent. Under asymmetric information, equity injections dominate both asset purchases and debt guarantees. We also solve for the optimal mechanism, and find that it can be implemented with preferred stock and warrants.
Proof of Theorem 1

We show the equivalence result in the case of a binary distribution for asset value \( a \).

Cash against equity

The government offers cash \( m \) against fraction \( \alpha \) of equity capital. The investment domain \( I(m) \) is the same as in the case of pure cash injections. At time 0, shareholders participate in the intervention if

\[
(1 - \alpha) E_0 [y^e (m)] \geq E_0 [y^e (0)]. \tag{22}
\]

The cost of the program to the government is

\[
\Psi^c_0 (m, \alpha) = m - \alpha E_0 [y^e (m)].
\]

Because the investment domain does not depend on \( \alpha \), the government chooses equity share \( \alpha \) such that the participation constraint (22) binds. Expected shareholder values at time 0 are

\[
E_0 [y^e] = \bar{p} (N + c_0) + \int \int_{I^o} L^o (p, v) \, dF (p, v),
\]

\[
E_0 [y^e (m)] = p (N + c_0 + m) + \int \int_{I(m)} (L^o (p, v) + (1 - p) m) \, dF (p, v).
\]

Using the participation constraint (22) to eliminate \( \alpha \) from the cost function yields

\[
\Psi^c_0 (m, \alpha) = m - (E_0 [y^e | m] - E_0 [y^e])
\]

with

\[
E_0 [y^e (m)] - E_0 [y^e] = \bar{p} m + m \int \int_{I(m)} (1 - p) \, dF (p, v) + \int \int_{I(m) \setminus I^o} L^o (p, v) \, dF (p, v).
\]

The expected cost of the optimally designed program is \( \Psi_0 (m) \) defined in equation (13).

Asset purchase

Under the asset purchase program, we have \( c_1 = c_0 + Z p^z \). Hence the investment domain becomes \( I (Z p^z) \). The expected shareholder value at time 0 is

\[
E_0 [y^e (z, p^z)] = \bar{p} (N + c_0 - (1 - p^z) Z) + \int \int_{I (Z p^z)} (L^o (p, v) + (1 - p) Z p^z) \, dF (p, v).
\]

Shareholders participate in the program if

\[
E_0 [y^e (Z, p^z)] \geq E_0 [y^e (0, 0)] . \tag{23}
\]
Because the investment domain only depends on the total repurchase amount $Z p^z$, the
government chooses to satisfy participation constraint (23) with equality. As a result, the
participation constraint (23) yields

$$\tilde{p} (1 - p^2) Z = Z p^z \int_{I(Z p^z)} (1 - p) dF(p, v) + \int_{I(Z p^z) \setminus I^0} L^o(p, v) dF(p, v).$$

Therefore the cost to the government is

$$\Psi_0^o(Z, p^z) = Z p^z - Z \tilde{p}$$

$$= (1 - \tilde{p}) Z p^z - Z p^z \int_{I(Z p^z)} (1 - p) dF(p, v) - \int_{I(Z p^z) \setminus I^0} L^o(p, v) dF(p, v)$$

$$= \Psi_0(Z p^z).$$

The program is equivalent to cash against equity when $m = Z p^z$.

Debt guarantee

Under the program $c_1 = c_0 + (1 - \phi) S$. Hence the investment domain becomes $I((1 - \phi) S)$. The expected shareholder value at time 0 is

$$E_0 [y^e(S, 1 - \phi)] = \tilde{p} (N + c_0 - \phi S) + \int_{I((1 - \phi)S)} (L^o(p, v) + (1 - p) (1 - \phi) S) dF(p, v)$$

This is equivalent to the asset purchase program if we set $S = Z$ and $p^z = 1 - \phi$.

Proof of Theorem 2

We first analyze the asset purchase program at time 1. To prove the theorem, we must show
equivalence along four dimensions: (i) the NIP constraint, (ii) the upper schedule, (iii) the
lower schedule, and (iv) the cost function.

Upon participation and investment, equity value is

$$E_1[y^e(z, p^z) | p, v, i = 1] = p (N + c_0 - z) + L^o(p, v) + p^z z.$$  

Participation without investment yields

$$E_1[y^e(z, p^z) | p, v, i = 0] = p (N + c_0 - z + p^z z).$$

Now consider the three constraints:

- NIP: $E_1[y^e(z, p^z) | p, v, i = 0] < E_1[y^e(0, 0) | p, v, i = 0]$ or
  $$p^z < 1.$$  

- Upper schedule: $E_1[y^e(0, 0) | p, v, i = 1] > E_1[y^e(z, p^z) | p, v, i = 1]$ or
  $$p > p^z.$$  

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• Lower schedule: $E_1[y^e(z, p^z)|p, v, i = 1] > E_1[y^e(0, 0)|p, v, i = 0]$ or

\[ L_1^o(p, v; z, p^z) \equiv L_1^o(p, v) + (p^z - p) z > 0. \]

Note that banks therefore set $z$ either to 0 or to $Z$. Using the notations of the debt guarantee section, the participation set is

\[ \Omega_1^g(Z, 1 - p^z) \]

where $\Omega_1^g$ is defined above in equation (19). The expected cost of the program is

\[ \Psi_1^o(Z, p^z) = Z \int \int_{\Omega^o(Z, 1 - p^z)} (p^z - p) dF(p, v) = \Lambda_1^o(Z, 1 - p^z) \]

and the investment domain is

\[ I^o \cup \Omega_1^o(Z, 1 - p^z). \]

Now if we set $S = Z$ and $p^z = 1 - \phi$, we see that the NIP constraint, the upper and lower schedules, and the cost function are the same for the asset purchase program as for the debt guarantee program. The two programs are therefore equivalent.

**Proof of Theorem 3**

We first analyze the cash against equity program at time 1. Upon participation and investment, equity value (including the share going to the government) is

\[ E_1[y^e(m)|p, v, i = 1] = p(N + c_0) + L^o(p, v) + m \]

Participation without investment yields

\[ E_1[y^e(m)|p, v, i = 0] = p(N + c_0 + m) \]

Now consider the three constraints

• NIP: $(1 - \alpha) E_1[y^e(m)|p, v, i = 0] < E_1[y^e(0)|p, v, i = 0]$ or:

\[ (1 - \alpha) m < \alpha (N + c_0). \]

• Upper schedule: $E_1[y^e(0)|p, v, i = 1] > (1 - \alpha) E_1[y^e(m)|p, v, i = 1]$ or:

\[ \alpha (p(N + c_0) + L^o(p, v)) > (1 - \alpha) m. \]

• Lower schedule: $(1 - \alpha) E_1[y^e(m)|p, v, i = 1] > E_1[y^e(0)|p, v, i = 0]$ or:

\[ (1 - \alpha) (L^o(p, v) + m) > \alpha p (N + c_0). \]
We define the function
\[ X(p; m, \alpha) \equiv (1 - \alpha) m - \alpha p (N + c_0) \]

We can summarize the cash against equity program by:
\[ L_1^e(p, v; m) \equiv (1 - \alpha) L^0(p, v) + X(p; m, \alpha) \]
\[ U_1^e(p, v; m) \equiv \alpha L^0(p, v) - X(p; m, \alpha) \]
\[ NIP: \quad X(1; m, \alpha) < 0. \]

The participation set is
\[ \Omega_1^e(m, \alpha) = \{(p, v) \mid L_1^e(p, v; m, \alpha) > 0 \land U_1^e(p, v; m, \alpha) < 0\}. \]

The cost function is therefore
\[ \Psi_1^e(m, \alpha) = \int \int_{\Omega_1^e(m, \alpha)} (m - \alpha E_1[y^e(m, \alpha) | p, v, i = 1]) dF(p, v). \]

We can rewrite the cost function such that
\[ \Psi_1^e(m, \alpha) = \int \int_{\Omega_1^e(m, \alpha)} X(p; m, \alpha) dF(p, v) - \alpha \int \int_{\Omega_1^e(m, \alpha)} L^0(p, v) dF(p, v). \]

The following table provides a comparison of the three government interventions:

<table>
<thead>
<tr>
<th>Part</th>
<th>Debt guarantee</th>
<th>Asset purchase</th>
<th>Equity injection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participation</td>
<td>$\Omega_1^q(S, \phi)$</td>
<td>$\Omega_1^q(Z, 1 - \phi^2)$</td>
<td>$\Omega_1^e(m, \alpha)$</td>
</tr>
<tr>
<td>Investment $I_1$</td>
<td>$I^0 \cup \Omega_1^q(S, \phi)$</td>
<td>$I^0 \cup \Omega_1^q(Z, 1 - \phi^2)$</td>
<td>$I^0 \cup \Omega_1^e(m, \alpha)$</td>
</tr>
<tr>
<td>NIP constraint</td>
<td>$\phi &gt; 0$</td>
<td>$p^2 &lt; 1$</td>
<td>$X(1, m, \alpha) &lt; 0$</td>
</tr>
<tr>
<td>Cost function</td>
<td>$\Psi_1^q(S, \phi)$</td>
<td>$\Psi_1^q(Z, 1 - \phi^2)$</td>
<td>$\Psi_1^e(m, \alpha)$</td>
</tr>
</tbody>
</table>

Now let us prove that the cash against equity program dominates the other two programs. Take a program $S, \phi$. We are going to construct an equity program that has same welfare gains at lower cost. To get equity with same lower bound graph we need to ensure that:

\[ L_1^e(p, v; m, \alpha) = L_1^q(p, v; S, \phi) \text{ for all } p, v. \]

So we must have
\[ X(p; m, \alpha) = (1 - \alpha) (1 - \phi - p) S \text{ for all } p. \quad (24) \]
It is easy to see that this is indeed possible if we identify term by term: \( \frac{\alpha}{1-\alpha} = \frac{S}{S+\alpha-D} \) and \( m = (1 - \phi)S \). Therefore it is possible to implement exactly the same lower schedules. Formally, we have just shown that:

\[
I^g_1 (S, \phi) = I^e_1 (m, \alpha).
\]

Next notice that the NIP constraints are equivalent since:

\[
X (1, m, \alpha) < 0 \iff \phi > 0.
\]

Now consider the upper bound. Consider the lowest point on the upper schedule of the guarantee program, i.e., the intersection of \( U^g_1 (p, v; S, \phi) = 0 \) with \( L^o_1 (p, v) = 0 \). At that point, we have \( \tilde{p} = 1 - \phi \) and \( \tilde{v} = (x - \phi c_0) / (1 - \phi) \). But from (24), it is clear that \( X (\tilde{p}; m, \alpha) = 0 \), and therefore \( U^e_1 (\tilde{p}, \tilde{v}; m, \alpha) = \alpha L^o_1 (\tilde{p}, \tilde{v}) - X (\tilde{p}; m, \alpha) = 0 \). Therefore the upper schedule \( U^c_1 (p, v; m, \alpha) = 0 \) also passes by this point. But the schedule \( U^c_1 (p, v; m, \alpha) = 0 \) is downward slopping in \((p, v)\), so the domain of inefficient participation is smaller (see Figure 7) than in the debt guarantee case. Formally, we have just shown that:

\[
\Omega^c_1 (m, \alpha) \subset \Omega^g_1 (S, \phi).
\]

As an aside, it is also easy to see that the schedule \( U^c_1 (p, v; m, \alpha) = 0 \) is above the schedule \( L^o_1 (p, v) = 0 \) so it does not get rid completely of opportunistic participation, but it helps.

The final step is to compare the cost functions.

\[
\Psi^g_1 (S, \phi) = S \int_{\Omega^g_1 (S, \phi)} (1 - p - \phi) dF (p, v)
\]

\[
\Psi^c_1 (m, \alpha) = \int_{\Omega^c_1 (m, \alpha)} X (p; m, \alpha) dF (p, v) - \alpha \int_{\Omega^c_1 (m, \alpha)} L^o_1 (p, v) dF (p, v)
\]

By definition of the participation domain, we know that \( L^c_1 (p, v; m, \alpha) > 0 \). Therefore:

\[
- \int_{\Omega^c_1 (m, \alpha)} L^o_1 (p, v) dF (p, v) < \frac{X (p; m, \alpha)}{1 - \alpha} \quad \text{for all } (p, v) \in \Omega^c_1 (m, \alpha)
\]

Therefore:

\[
\Psi^c_1 (m, \alpha) < \frac{1}{1 - \alpha} \int_{\Omega^c_1 (m, \alpha)} X (p; m, \alpha) dF (p, v) = S \int_{\Omega^c_1 (m, \alpha)} (1 - \phi - p) dF (p, v)
\]

Finally, since \( 1 - \phi - p > 0 \) for all \((p, v) \in \Omega^c_1 (m, \alpha)\), and since \( \Omega^c_1 (m, \alpha) \subset \Omega^g_1 (S, \phi) \), we have

\[
\Psi^c_1 (m, \alpha) < \Psi^g_1 (S, \phi).
\]
Proof of Theorem 4

In the good state, the residual payoffs conditional on investment are

\[ N + c_0 + m + \frac{L^o(p, v) + (1 - p) m}{p} \]

The loan is repaid first. Then shareholders receive

\[ y^e = \max \left( N + c_0 + \frac{L^o(p, v) + (1 - p) m}{p} - hm, 0 \right) \text{ if } i = 1 \text{ and } a = A \]

\[ y^e = \max (N + c_0 - hm, 0) \text{ if } i = 0 \text{ and } a = A. \]

As soon as \( y^e > N + c_0 \), the warrants are in the money and the number of shares jumps to \( 1 + \frac{1 - \varepsilon}{\varepsilon} = \frac{1}{\varepsilon} \). So the old shareholders get only a fraction \( \varepsilon \) of the value beyond \( N + c_0 \). Their payoff function is therefore:

\[ f (y^e) = \min (y^e, N + c_0) + \varepsilon \max (y^e - N - c_0, 0). \]

So old shareholders are full residual claimants up to the face value of old assets \( N + c_0 \) and \( \varepsilon \) residual claimants beyond. Now let us think about their decisions at time 1. As usual only the payoffs in the non default state matter. If they do not invest they get \( N + c_0 \). If they invest, they receive more if and only if \( L^o(p, v) + (1 - p) m > phm \). The lower participation constraint is therefore

\[ L^o(p, v) + (1 - (1 + h) p) m > 0. \]

It converges to \( L^o(p, v) + (1 - p) m \) if \( h \to 0 \). We can compare this to the equity injection schedule \( L^1_{\Omega} (p; v; m, \alpha) \), we can identify the same cash injection \( m \), and the dilution factor

\[ \alpha = \frac{m (1 + h)}{N + c_0 + m (1 + h)}. \]

If we compare to debt guarantee \( L^1_{\Omega} (p, v; S, \phi) = L^o(p, v) + (1 - \phi - p) S \). Then

\[ m = (1 - \phi) S \text{ and } h = \frac{\phi}{1 - \phi}. \]

Next consider the upper schedule. Investing alone gets \( N + c_0 + L^o(p, v) / p \) so they opt in if and only if \( L^o(p, v) > \varepsilon (L^o(p, v) + (1 - (1 + h) p) m) \) and therefore

\[ U = L^o(p, v) - m \varepsilon \frac{1 - (1 + h)p}{1 - \varepsilon}. \]

The upper bound converges to \( L^o(p, v) \) when \( \varepsilon \to 0 \). The NIP constraint is simply

\[ h > 0. \]

Finally, the cost of the program is small because the government gets all the upside value of the new projects. The expected payments to the old shareholders converge to \( p(N + c_0) \). So the government receives expected value \( L^o(p, v) + m \) by paying \( m \) at time 1. The total cost is therefore:

\[ - \int \int_{I(1) \setminus I^o} L^o(p, v) dF(p, v) \]

The cost is positive because \( L^o(p, v) < 0 \) for all \( (p, v) \in I(m) \setminus I^o \).
Proof of Proposition 3

If deposits are safe, then the optimization problem from the equity holders perspective remains unchanged because the investment and participation decision only depend on total debt $D$. Now consider the expected cost of deposit insurance. Note that the time 0 expected value of deposits is $\Delta$ because $\Delta \leq A + c_0$. Hence, the cost of government intervention is unchanged and therefore the cost and benefits of both time 0 and time 1 programs are unchanged.

If the deposits are risky, we distinguish two cases depending on the recovery rate on deposits in the low payoff state.

**Time 0 Programs**

**Full transfer:** $A + v < \Delta$

The expected values of deposits at time 1 and time 0 are

$$E_1[y^\Delta(m) | p, v] = p\Delta + (1-p) (A + c_0 + m) \text{ if } (p, v) \in T \setminus I_0(m)$$

$$= p\Delta + (1-p) (A + v) \text{ if } (p, v) \in I_0(m)$$

$$E_0[y^\Delta(m)] = \bar{p}\Delta + (1-\bar{p}) (A + c_0 + m) + \int_{I_0(m)} \int (1-p) (v - c_0 - m) dF(p, v).$$

The expected cost of deposit insurance at time 0 is

$$\Psi_0^F(m) \equiv \Delta - E_0[y^\Delta(m)]$$

$$= (1-\bar{p}) (\Delta - A - c_0 - m) - \int_{I_0(m)} \int (1-p) (v - c_0 - m) dF(p, v)$$

The change in the expected cost of deposit insurance is

$$\Lambda_0^F(m) = \Psi_0^F(m) - \Psi_0^F(0)$$

$$= - (1-\bar{p}) m + m \int_{I_0(m)} \int (1-p) dF(p, v) - \int_{I_0(m) \setminus I^o} \int (1-p) (v - c_0) dF(p, v).$$

The net cost of government intervention is

$$\bar{\Psi}_0(m) + \Lambda_0^F(m) = - \int_{I_0(m) \setminus I^o} (v - c_0) dF(p, v).$$

Note that this term is negative because the benefits of incremental investments accrue to the government.

**Partial Transfer:** $A + c_0 < \Delta < A + v$
The expected values of deposits at time 1 and time 0 are

\[
E_1 [y^\Delta (m) | p, v] = \begin{cases} 
  p\Delta + (1-p) \max (\Delta, A + c_0 + m) & \text{if } (p, v) \in T \setminus I_0 (m) \\
  \Delta & \text{if } (p, v) \in I_0 (m) 
\end{cases}
\]

\[
E_0 [y^\Delta (m)] = \Delta - \int \int_{T \setminus I_0 (m)} (1-p) (\Delta - \max (\Delta, A + c_0 + m)) dF(p, v)
\]

The expected cost of deposit insurance is

\[
\Psi_0^F (m) = \int \int_{T \setminus I_0 (m)} (1-p) (\Delta - \max (\Delta, A + c_0 + m)) dF(p, v).
\]

The change in the expected cost of deposit insurance

\[
\Lambda_0^F (m) = \int \int_{T \setminus I_0 (m)} (1-p) (\Delta - \max (\Delta, A + c_0 + m)) dF(p, v) - \int \int_{T \setminus I^o} (1-p) (\Delta - A - c_0) dF(p, v).
\]

Note that when \( \Delta \to (A + c_0) \), then \( \Lambda_0^F (m) \to 0 \). This means the expected change in the cost of deposit insurance goes to zero as deposits become safe. Also note that when \( \Delta \to (A + v) \), then

\[
\Lambda_0^F (m) \to - (1-p) m + m \int \int_{I_0 (m)} (1-p) dF(p, v) - \int \int_{I_0 (m) \setminus I^o} (1-p) (v - c_0) dF(p, v)
\]

which is the change in expected cost of deposit insurance in the full transfer case. The government cost is \( \Lambda_0^F (m) + \Psi_0 (m) \). The results apply to all programs because all programs have the same cost function at time 0.

**Time 1 programs**

**Full Transfer: \( A + v < \Delta \)**

The expected values of deposits at time 1 and time 0 are

\[
E_1 [y^\Delta (Z, p^z) | p, v] = \begin{cases} 
  p\Delta + (1-p) (A + c_0) & \text{if } (p, v) \in T \setminus (I^o \cup \Omega_1^o (Z, 1 - p^z)) \\
  p\Delta + (1-p) (A + v) & \text{if } (p, v) \in I^o \cup \Omega_1^o (Z, 1 - p^z) 
\end{cases}
\]

\[
E_0 [y^\Delta (Z, p^z)] = \bar{p}\Delta + (1-\bar{p}) (A + c_0) + \int_{I^o \cup \Omega_1^o (Z, 1 - p^z)} (1-p) (v - c_0) dF(p, v)
\]

The expected cost of deposit insurance is

\[
\Psi_0^F (Z, p^z) = (1-\bar{p}) (\Delta - A - c_0) - \int \int_{I^o \cup \Omega_1^o (Z, 1 - p^z)} (1-p) (v - c_0) dF(p, v)
\]
The change in the cost of deposit insurance is

$$\Lambda^F_0 (Z, p^z) = - \int \int_{\Omega^g_1(Z,1-p^z)/I^o} (1 - p) (v - c_0) dF(p, v)$$

Expected government cost is

$$\Psi^g_1 (Z, p^z) = \Lambda (Z, 1 - p^z) - \int \int_{\Omega^g_1(Z,1-p^z)/I^o} (1 - p) (v - c_0) dF(p, v).$$

Partial Transfer: $A + c_0 < \Delta < A + v$

The expected values of deposits at time 1 and time 0 are

$$E_1 \left[ y^\Delta (Z, p^z) \big| p, v \right] = p\Delta + (1 - p) (A + c_0) \text{ if } (p, v) \in T \setminus (I^o \cup \Omega^g_1(Z,1-p^z))$$

$$= \Delta \text{ if } (p, v) \in I^o \cup \Omega^g_1(Z,1-p^z)$$

$$E_0 \left[ y^\Delta (Z, p^z) \right] = \Delta - \int \int_{T \setminus (I^o \cup \Omega^g_1(Z,1-p^z))} (1 - p) (\Delta - A - c_0) dF(p, v)$$

The expected cost of government insurance is

$$\Psi^g_0 (Z, p^z) = \int \int_{T \setminus (I^o \cup \Omega^g_1(Z,1-p^z))} (1 - p) (\Delta - A - c_0) dF(p, v).$$

The change in expected cost of deposit insurance is

$$\Lambda^F_0 (Z, p^z) = - \int \int_{\Omega^g_1(Z,1-p^z)/I^o} (1 - p) (\Delta - A - c_0) dF(p, v).$$

Note that when $\Delta \to (A + c_0)$, then $\Lambda^F_0 (Z, p^z) \to 0$. Also note that when $\Delta \to (A + v)$, then

$$\Lambda^F_0 (Z, p^z) \to - \int \int_{\Omega^g_1(Z,1-p^z)/I^o} (1 - p) (v - c_0) dF(p, v).$$

Total government cost is

$$\Psi^g_1 (Z, p^z) = Z \int \int_{\Omega^g_1(Z,1-p^z)} (p^z - p) dF(p, v)$$

$$= \Lambda (Z, 1 - p^z) - \int \int_{\Omega^g_1(Z,1-p^z)/I^o} (1 - p) (\Delta - A - c_0) dF(p, v).$$

The results also apply to debt guarantees at time 1 because asset purchases and debt guarantees have the same cost function at time 1.
Cash against equity at time 1

Note that we can compute the expected cost of time 1 cash against equity similarly to the time 1 asset purchase program. The only difference is the participation region for cash against equity $\Omega^c_1(m, \alpha)$ and the participation region for asset purchase $\Omega^g_1(Z, 1 - p)$. It turns out that the change in the expected cost of deposit insurance $\Delta F_1^C(m)$ is equivalent under both programs because both in the full and partial transfer case the difference in the participation region cancels out when computing the difference in expected cost of deposit insurance. It follows that the relative ranking of programs is unchanged.
References


Fig 1: Information & Technology

- **t = 0**
- **t = 1**
- **t = 2**

**Existing Assets**

- \( p \)
- \( 1 - p \)

**New Opportunity**

- \(-x\)
- \(v\)

\( A + A \)
\( A + 0 \)
Fig 2: Payoffs

<table>
<thead>
<tr>
<th>t = 1</th>
<th>t = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learn p and v</td>
<td>Senior Debt</td>
</tr>
<tr>
<td>p</td>
<td>D</td>
</tr>
<tr>
<td>c₂ = c₁</td>
<td>A + c₁</td>
</tr>
<tr>
<td>(l - p)</td>
<td>A + c₁</td>
</tr>
<tr>
<td>Keep cash</td>
<td></td>
</tr>
<tr>
<td>Invest</td>
<td></td>
</tr>
<tr>
<td>c₂ = c₁ + l - x</td>
<td>D</td>
</tr>
<tr>
<td>(l - p)</td>
<td>A + c₂ + v</td>
</tr>
<tr>
<td></td>
<td>A + c₂ + v</td>
</tr>
</tbody>
</table>
Fig 3: First Best
Fig 4: Debt Overhang
Fig 5: Cash at time 0

\[ L^0 + (1-p)m = 0 \quad L^0 = 0 \]

\[ I(m) \]
Fig 6: Debt Guarantee at time 1

\[ L^g(S, \varphi) = 0 \quad L^o = 0 \quad U^g(S, \varphi) = 0 \]

Efficient participation

Opportunistic participation

Invest alone
Figure 7: Equity injection at time 1
Figure 8: Efficient Mechanism

$V(\Gamma) = 0$

$U(\Gamma) = 0$

$L(\Gamma) = 0$

$L^o = 0$