INFECTIOUS LEVERAGE

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Abstract

This paper develops a theory of “infectious leverage” in financial markets in which the lender’s payoff depends heavily on the value of the collateral backing the loan. There are two main results. First, leverage is an “infectious” phenomenon in that high leverage among borrowers also leads to high leverage among lenders. There is thus a multiplicative effect of leverage on the fragility of the financial system. Second, there is an externality in bank leverage in that a bank’s exposure to credit risk depends on the leverage decisions of other banks and banks choose privately-optimal leverage ratios that are higher than what is socially optimal. The model has an overlapping generations setting in which banks make one-period mortgage loans in the first period that are secured by collateral, and the analysis solves for the equilibrium in the housing market, so the current house price and the distribution of future house prices are both endogenous. The prices of houses and hence the second-period values of collateral depend on the supply of loans in the second period, which then determines the ability of second-generation borrowers to raise financing to purchase houses in the second period. Thus, if banks are hit with second-period liquidity shocks that diminish aggregate credit availability in the second period, collateral values fall, thereby increasing the bank’s credit risk. This creates an interesting interaction between credit and liquidity risks for lenders. Moreover, it leads to the result that an idiosyncratic shock suffered by a subset of banks can become a systematic shock for the whole banking system, and the likelihood of this increases with banks’ leverage choices. Although the model is developed in the context of the housing market, we argue that it is applicable in any borrower-lender setting in which collateral values depend on the aggregate availability of credit, and credit risk in turn depends significantly on collateral values. Empirical and policy implications of the analysis are drawn out.
Introduction

It is by now well understood that high leverage ratios of banks make the financial system more fragile and increase the likelihood of financial crises (see, for example, Allen and Gale (2008))\textsuperscript{1} Indeed, much of prudential capital regulation of banks is based on this fundamental premise (e.g., Bhattacharya and Thakor (1993) and Freixas and Rochet (1997)). The issues of financial leverage and bank capital have gained special prominence in light of recent events. The current subprime lending crises is a striking example of the alacrity with which a high-leverage financial system can find itself beset with a crisis that further erodes capital and sets in motion forces that exacerbate the crisis.

However, our knowledge of the dynamics of financial-system leverage is rather limited. That is, under what circumstances do banks become more highly levered, outside of crises periods in which exogenous shocks impose losses on banks, drain capital and cause leverage ratios to spike up? In other words, if more highly levered banks make the financial system more fragile, what causes banks to be so? A related issue that is especially noteworthy in the current financial crisis is that borrower leverage ratios have also increased substantially prior to the crisis (e.g., Gerardi, Lehnert, Sherland and Willen (2008)). Was this just a coincidence or was it in any way related to the leverage ratios of banks themselves?

In this paper we develop a theoretical model that explores the relationship between the leverage decisions of borrowers and banks, in the context of the home mortgage market. We consider a two-period overlapping-generations economy in which first-period homebuyers with limited wealth endowments need bank loans to finance house purchases. Borrowers’ leverage decisions are driven by first-period house prices that dictate the amounts they need to borrow. Higher house prices lead to larger bank loans and higher borrower leverage. Since house prices in the first period depend on expected house prices in the second period, banks react to high first-period house prices by lowering their expectation of low second-period house prices. This, in turn, lowers their assessment of the probability that the borrower will default on the loan.

\textsuperscript{1}There is a vast literature on financial crises that we will not review here. See, for example, Allen and Carletti (2006, 2008), Allen and Gale (1998, 2000a, 2000b), and Boyd, Kwak and Smith (2005).
when the loan repayment is expected to be financed by the sale of the house in the second period to second-period homebuyers. Banks thus keep lower capital in the first period when first-period house prices are higher. This phenomenon, whereby the leverage ratios of borrowers and banks move in unison, is what we call “infectious leverage.” It implies that in markets in which banks make secured loans and collateral is the principal source for loan repayment, leverage propagates in the manner of an infection from borrowers to banks.

An essential element of the analysis is that along with borrower and bank capital structures, house prices in both periods are also endogenously determined. This introduces an interesting source of fragility for the credit market. More highly-levered banks find it costlier to raise equity capital in the second period conditional on default by first-period borrowers. This reduces the aggregate supply of credit for homebuyers in the second period and leads to a lower equilibrium house price in the second period. Thus, higher first-period house prices make the banking system and the housing market more vulnerable to negative shocks in the future that precipitate housing price declines.

The endogeneity of house prices allows us to examine bank leverage externality. In particular, we establish four results. First, an increase in first-period bank leverage leads to lower second-period house prices. Second, as a bank increases its leverage, the sensitivity of its credit risk exposure to the leverage choices of other banks goes up. Third, an increase in leverage in the banking system elevates the likelihood that an idiosyncratic shock experienced by a subset of banks will become a systematic shock experienced by all banks. Fourth, each bank’s privately-optimal leverage ratio is higher than the social optimum. The reason for the divergence of the private and social optima is that the bank does not take into account the negative externality on future house prices created by its own leverage decision, and house prices are important because houses serve as collateral for bank loans. This suggests that bank capital requirements – that attempt to move bank leverage ratios closer to the social optimum – may have a hitherto-unexplored role in affecting the dynamics of real-asset prices that, in turn, have systematic risk implications for banks.
We also show that in markets like this, idiosyncratic shocks suffered by a subset of banks could become systematic risk and the likelihood of this becomes stronger as bank leverage increases. Moreover, the model allows us to compare the possible effects of capital requirements relative to ex post capital infusions by the government when banks are hit with a negative shock.

There is a vast literature on bank capital that we will touch upon briefly here because its relationship to our work is mostly tangential. For example, there is a significant theoretical literature on bank capital requirements and their effects (e.g., Caminal and Matutes (2002), Furlong and Keeley (1990), Hellmann, Murdock and Stiglitz (2000), Kim and Santomero (1988), Koehn and Santomero (1980), and Thakor (1996)). Empirically, some of the predictions of these models have been tested and the effects of changes in the capital requirements regime on banks’ portfolios have been examined (e.g., Bernanke and Lown (1991), and Thakor (1996)). Hancock, Laing and Wilcox (1995) study the dynamic response to shocks in the capital of U.S. banks and show that these banks adjust their capital ratios faster than they adjust their loan portfolios. See Rochet (2008) for a review. What distinguishes our paper from this literature is that our focus is not on the determination of regulatory capital requirements. What we are interested in is how bank leverage ratios respond to changes in borrowers’ leverage ratios when loans are secured by collateral whose future value is dependent on aggregate bank credit supply.

This paper is also related to the literature on real estate and household finance. Stein (1995) theoretically examines how down payment restriction affects both the house prices and trading volume in the real estate market. Lamont and Stein (1999) provide empirical evidence for the Stein (1995) model and find that house prices in U.S. cities where more homeowners are highly levered (i.e., take loans with high loan-to-value ratios) are more sensitive to city-specific shocks. Ortalo-Magné and Rady (2006) extend Stein’s (1995) analysis to a life-cycle model of the housing market and demonstrate a correlation between house prices and the incomes of young households that are eager to climb a property ladder but are credit-constrained. Kiyotaki and Moore (1997) develop a dynamic model in which lenders cannot force borrowers to repay unless

\[\text{There are also numerous papers on the capital structure decisions of banks (e.g., Diamond and Rajan (2000), and Inderst and Mueller (2008)). However, these too are not directly related to our paper because they do not examine the interaction between banks’ and borrowers’ leverage dynamics.}\]
debts is secured. Consequently, durable assets serve not only as factors of production but also as collateral for loans. The dynamic interaction between credit limits and asset prices leads to temporary shocks to technology or income becoming persistent shocks to asset prices. Bernanke, Gertler and Gilchrist (1996) provide evidence that at the onset of a recession borrowers facing high agency costs receive a relatively lower share of the credit extended and hence account for a proportionally greater part of the decline in economic activity. Chen (2001) develops a dynamic general equilibrium model which explains why banking crises so often coincide with depressed prices in asset markets. Fostel and Geanakoplos (2008) study how leverage cycles can cause contagion, flight to collateral and issuance rationing in a so-called “anxious economy.” Elul (2008) shows how a drop in the value of the underlying collateral in secured borrowing may help stabilize aggregate fluctuations in the housing market. None of them, however, studies the relationship between borrower leverage and bank leverage or bank leverage externality as we do.

A paper that is more closely related to ours is Shleifer and Vishny (1992), where collateral value depends on other industry peers’ ability to buy the asset. The similarity is that in our model the future house price depends on a future borrower’s ability to purchase the house. However, our focus differs substantially in that we link the future borrower’s ability to purchase the house to the bank’s future ability to lend, and examine how this, in turn depends on the capital structure decisions of banks in the current period. Another related paper is Holmstrom and Tirole (1997) in which the capital of the bank interacts with the capital of the borrower. Moral hazard prevents low-capital borrowers from being able to raise unmonitored finance. Banks can provide monitoring and hence not only extend credit to these borrowers but also enhance their ability to obtain credit from elsewhere. However, banks need to have sufficient capital of their own to have incentives to monitor. Thus, access to credit may depend on capital in both banks and borrowers. While our paper too examines the interaction between the capital levels of banks and borrowers, the objectives of the two papers are completely different. Thus, unlike Holmstrom and Tirole (1997), we focus on how borrower leverage covaries with bank leverage, i.e., how the dynamics of these two leverage processes are related.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 has the main analysis, and derives the infectious leverage result. Section 4 examines how first-period
leverage affects second-period house prices. Section 5 concludes with a discussion of the empirical implications. All proofs are in the Appendix.

2 The Model

Consider a three-date \((t = 1, 2, \text{ and } 3)\) economy with universal risk neutrality and a zero riskless interest rate. There are two goods in the economy, money and houses, where money is the numeraire good and can be stored costlessly over time, and houses are indivisible. There is a continuum of atomistic and identical houses available in the market at \(t = 1\) with Lebesgue measure of \(S \in (0, 1)\). There are no new houses built after \(t = 1\). We call the period between \(t = 1\) and \(t = 2\) the first period, and the period between \(t = 2\) and \(t = 3\) the second period.

There is also a continuum of atomistic consumers in each period. Consumers within a given period are identical, but they may differ across periods. A consumer in period \(i \in \{1, 2\}\) is born at \(t = i\) without a house but with a monetary endowment \(M_i > 0\), and will earn an income \(X_i\) at \(t = i + 1\). She maximizes her expected utility at \(t = i\) as given by:

\[
U_i = h_i B_i + C_i + \mathbb{E}(C_{i+1}),
\]

where \(h_i\) is an indicator variable that equals to 1 if the consumer owns a house in period \(i\) and zero otherwise, \(B_i > 0\) is the consumer’s utility from home ownership in period \(i\), and \(C_i\) and \(C_{i+1}\) are, respectively, the consumer’s monetary consumptions at \(t = i\) and \(t = i + 1\). \(\mathbb{E}(\cdot)\) is an expectation operator.

The measure of consumers in each period exceeds the total housing supply, \(S\), in that period (i.e., there are more consumers than houses). Since houses are indivisible, consumers compete to buy houses in each period. First-period consumers who buy houses at \(t = 1\) sell their houses to some second-period consumers at \(t = 2\), who in turn sell theirs to some (unmodeled) third-period consumers at \(t = 3\), and so on. The house price at \(t = 3\), denoted as \(P_3 > 0\), is exogenously given but random when viewed at \(t = 1\) and \(2\). At \(t = 1\), the (first-period) consumer’s utility from home ownership in the first period, \(B_1\), is known to everyone, whereas the (second-period) consumer’s utility from home ownership in the second period, \(B_2\), is a random variable that realizes its value
at $t = 2$. House prices at $t = 1$ and 2, $P_1$ and $P_2$, are determined by competition among the first-period and second-period consumers for buying the fixed housing supply, $S$, at $t = 1$ and 2, respectively.\(^3\) Buying and selling houses involve no transaction costs. It is clear that in the absence of wealth and credit constraints, the house price at $t = 2$ will be $P_2 = B_2 + \mathbb{E}(P_3)$, and the house price at $t = 1$ will be $P_1 = B_1 + \mathbb{E}(B_2 + P_3)$.

However, we make:

**Assumption 1.** $0 < M_1 < B_1 + \mathbb{E}(B_2 + P_3)$, and $0 < M_2 < B_2 + \mathbb{E}(P_3)$,

so that consumers’ consumption and housing purchases are constrained by their monetary endowments; spending cannot exceed a consumer’s endowment and negative cash balances are precluded.

We also assume that consumers cannot directly borrow and lend money to each other: they can only borrow from banks by taking mortgage loans.\(^4\) There is a continuum of atomistic and identical banks with a measure of $S$. Conditional on not defaulting at the end of the first period, each bank stays in business in both periods, lending to both the first-period and second-period consumers who want to purchase houses. Banks compete in each period, which determines the amount to lend and the equilibrium loan interest rate. We assume that the credit market in which banks make mortgage loans is perfectly competitive, leading to the competitive outcome of zero expected profits for all banks in equilibrium. Since a measure $S$ of consumers buy houses in each period in equilibrium (otherwise, the housing market will not clear), each bank provides one loan to one consumer.\(^5\) Bank loans made at date $t$ ($t = 1$ or 2) must be repaid at date $t + 1$.

\(^3\)We assume that housing supply is inelastic because we want to focus on the demand for housing and how it interacts with credit availability. It enables us to get the result that housing prices fall as credit constraints worsen. This result obtains, for example, in Stein (1995). Vigdor (2006) shows that this result holds even when housing supply is elastic.

\(^4\)This assumption can be justified on the basis of the special role of banks in screening out borrowers who are not creditworthy (e.g., Allen (1990), and Ramakrishnan and Thakor (1984)).

\(^5\)The assumption that there are equal numbers of banks and consumers in each period is made for expositional simplicity. Our results will not change if the number of banks and the number consumers are not equal so each bank may lend to more than one consumer or several banks lend to one consumer.
Banks extend loans using the funds they raise through equity capital and deposits. Each bank independently chooses the amount of equity and deposits on its balance sheet. Deposits are in elastic supply and fully insured, so the deposit interest rate is zero. Raising equity capital is costly; this cost may arise due to capital-market frictions of various sorts such as asymmetric information (e.g., Myers and Majluf (1984)) and the transactions costs of raising equity.\textsuperscript{6} We assume that for a bank with deposits $D$ and equity $E$, the cost of equity capital is $\Lambda(E)$, an increasing and convex function that satisfies the Inada conditions.\textsuperscript{7} Finally, since capital is costly and banks do not have any other investment opportunities, each bank will raise no more funds than needed to meet its loan requirements.

A bank defaults when it cannot repay its depositors in full. In this case, deposit insurance covers the shortfall. However, default is costly to the bank. Each dollar of shortfall in the bank’s repayment to depositors (that is covered by deposit insurance) imposes a cost of $1 + \delta$ dollars on the bank, where $\delta > 0$ is a constant. Two interpretations of this bankruptcy cost are possible. One is that deposit insurance is free to the bank and $1 + \delta$ is merely the ex post cost of default, possibly due to the expected loss of the bank’s charter value upon bankruptcy and consequent closure. Alternatively, $1 + \delta$ may be viewed as the ex ante expected bankruptcy cost, which consists of partly the fair price of deposit insurance ($1$ for each $1$ of expected loss) plus some other dissipative ex post costs of financial distress that equal $\delta$ times the amount of deposit repayment shortfall.

The consumer can choose whether or not to repay her loan. To minimize the risk of default as well as the loss given default, banks require that the loans used to purchase houses be secured by these houses. If a consumer does not repay the loan in full, the bank can seize her house (i.e., foreclosure). We assume that the loan is without recourse, i.e., the bank has no legal claim on the borrower’s other assets or income, and similarly the borrower has no legal claim on the bank’s proceeds from the sale of the foreclosed house.

\textsuperscript{6}Despite costly equity, the relationship between bank capital and value may be positive in the cross-section; see Mehran and Thakor (2009).

\textsuperscript{7}Because there are no scale economies for the bank on either side of its balance sheet, the positive-equilibrium-profits outcomes derived by Yannelle (1997) do not obtain here.
3 The Analysis

In this section, we present our main analysis of the model. We examine how house prices are determined in equilibrium, how the bank determines its capital structure, how interest rates are determined in equilibrium, and how the optimal leverage decisions of borrowers and banks co-vary in equilibrium.

3.1 House Price

We now consider how house prices are determined. The two periods share the same trading mechanisms, so the following analysis will be carried out for a generic period $i$, where $i$ can take value of 1 or 2.

Since all consumers are ex ante identical, an equilibrium in which a mass $S$ of consumers purchase houses while the rest do not is possible only if consumers are indifferent between purchasing a house and not purchasing it. A consumer who purchases a house at $t = i$ will borrow an amount $L_i = P_i - M_i$ from a bank. Let $R_i > 1$ be the gross interest rate charged by the bank (which will be endogenously determined later). Note that this rate is independent of the identity of the consumer or the bank because of our assumption of identical consumers and identical banks. The expected utility of a consumer from buying a house at $t = i$ is:

$$B_i + X_i + \mathbb{E}(\max(P_{i+1} - R_i L_i, 0)).$$

(2)

The expected utility of a consumer who does not buy a house is:

$$M_i + X_i.$$  

(3)
In equilibrium, the expected utilities in (2) and (3) must be equal:

\[ M_i = B_i + \mathbb{E}(\max(P_{i+1} - R_i L_i, 0)) \]
\[ = B_i + \mathbb{E}(\max(P_{i+1} - [R_i][P_i - M_i], 0)). \]  

This equation is the consumer’s indifference condition that will in part determine the equilibrium house price \( P_i \). Note that the left-hand-side (LHS) of (4) is the monetary consumption that the consumer gives up in buying a house, and its right-hand-side (RHS) consists of the utility from owning a house in period \( i \), \( B_i \), and the expected gain from house price appreciation in period \( i \) after paying off the bank loan, \( \mathbb{E}(\max(P_{i+1} - [R_i][P_i - M_i], 0)) \). It is clear that a higher loan interest rate (higher \( R_i \)) increases the consumer’s purchasing cost, and hence will decrease \( P_i \).

We now analyze the banks’ problem that determines the equilibrium \( R_i \).

### 3.2 Bank Capital Structure

A bank lends \( L_i \) to a consumer buying a house at \( t = i \). Let equity and deposits be \( E_i \) and \( D_i \), respectively, so that \( E_i + D_i = L_i \). We assume for now that a bank’s capital structure choices are independent across the two periods, so that each problem can be solved in isolation. We later consider a situation in which a bank’s capital structure choice in the first period affects its cost of capital in the second period, so that the bank must take second-period cash flows into account while choosing its capital structure in the first period.

The bank maximizes expected loan repayment proceeds to shareholders net of the cost of equity capital and the expected cost of bankruptcy. Thus, the bank chooses \( E_i \) and \( D_i \) to maximize the following objective function, which is the net expected payoff to shareholders at \( t = i \):

\[ \mathbb{E}(\max(\min(P_{i+1}, R_i L_i) - D_i, 0)) - [1 + \delta]\mathbb{E}(\max(D_i - \min(P_{i+1}, R_i L_i), 0)) - E_i - \Lambda(E_i). \]  

Note that \( \min(P_{i+1}, R_i L_i) \) is what the consumer repays the bank at \( t = i + 1 \). This expression can be interpreted as follows. When the house price is higher than bank loan repayment obligation,
i.e., \( P_{i+1} \geq R_i L_i \), the consumer repays the loan in full, so the bank receives \( R_i L_i \). When the house price is lower than the bank loan repayment obligation, i.e., \( P_{i+1} < R_i L_i \), the consumer forfeits her house to the bank, which then sells the foreclosed house but has no legal claim over the consumer’s income, so in this state the bank collects \( P_{i+1} \). Thus, the bank receives \( \min(P_{i+1}, R_i L_i) \) at \( t = i + 1 \) for the loan it extended at \( t = i \). The above expression can be simplified to:

\[ E(\min(P_{i+1}, R_i L_i) - D_i) - \delta E(\max(D_i - \min(P_{i+1}, R_i L_i), 0)) - E_i - \Lambda(E_i) \]

which further simplifies to:

\[ E(\min(P_{i+1}, R_i L_i)) - \delta E(\max(L_i - E_i - P_{i+1}, 0)) - L_i - \Lambda(E_i). \]

Thus, the first-order-condition (FOC) for a profit-maximizing capital structure for the bank is:

\[ \Lambda'(E_i) = \delta \Pr(P_{i+1} < D_i) = \delta \Pr(P_{i+1} < L_i - E_i). \]  

The bank’s optimal capital structure choice equates the marginal cost of equity capital, \( \Lambda'(E_i) \), to the marginal cost of deposits, \( \delta \Pr(P_{i+1} < D_i) \). The latter is the marginal bankruptcy cost net of the deposit insurance benefit. Note that the Inada conditions for \( \Lambda(\cdot) \) guarantee interior solutions to (8): at \( E_i = 0 \), we have \( \Lambda'(E_i) = 0 < \delta \Pr(P_{i+1} < D_i) \), whereas at \( D_i = 0 \) we have \( \Lambda'(E_i) > 0 = \delta \Pr(P_{i+1} < D_i) \). Thus, \( E_i > 0 \) and \( D_i > 0 \).

The following result is immediate:

**Lemma 1.** A higher house price at \( t = i + 1, P_{i+1} \), ceteris paribus results in the bank raising less equity capital at \( t = i, E_i \).

The economic intuition is as follows. A higher house price in the future means a higher collateral value of the loan. This reduces the bank’s probability of falling into bankruptcy and

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8If the consumer doesn’t repay the loan in full, she will lose her house to the bank, whose market value is higher than the loan repayment obligation in this case. How the consumer repays the loan from her total wealth at \( t = i + 1, P_{i+1} + X_i \), is irrelevant.

9The first step of the simplification follows from the observation that \( E(\max(\min(P_{i+1}, R_i L_i) - D_i, 0)) = E(\max(D_i - \min(P_{i+1}, R_i L_i), 0)) = E(\min(P_{i+1}, R_i L_i) - D_i) \). To understand the second step of the simplification, note that \( R_i L_i > L_i \geq D_i \), so \( E(\max(D_i - \min(P_{i+1}, R_i L_i), 0)) = E(\max(D_i - P_{i+1}, 0)) \).
consequently diminishes the marginal value of equity capital as a cushion against bankruptcy, without impacting the marginal cost of equity capital. Since equity capital is costly, the bank optimally chooses less equity capital in equilibrium at $t = i$ in response to a higher anticipated house price at $t = i + 1$.

### 3.3 Interest Rate

The equilibrium interest rate is determined by the zero-profit condition for banks (recall that the mortgage loan market is perfectly competitive). This condition is:

$$\mathbb{E}(\min(P_{i+1}, R_i L_i)) = L_i + \delta \mathbb{E}(\max(D_i - P_{i+1}, 0)) + \Lambda(E_i).$$

(9)

This condition will, together with the consumers’ indifference condition (4), determine the equilibrium house price $P_i$ and gross loan interest rate $R_i$. Note that the left-hand-side (LHS) of (9) is what the bank expects to receive from the consumer’s loan repayment, and its right-hand-side (RHS) consists of the amount of the loan extended, the expected bankruptcy cost, and the cost of equity capital.

For a given distribution of house prices at $t = i + 1$, $P_{i+1}$, the equilibrium house price $P_i$, gross loan interest rate $R_i$ and bank’s equity capital $E_i$ are jointly determined by the consumer’s indifference condition (4), the bank’s optimal capital structure condition (8), and the bank’s zero-profit condition (9). The following simple example is used to illustrate the equilibrium as well as to serve as a preamble to the core idea of the paper, infectious leverage, which we will analyze more formally in the following section.

**An example showing that higher house prices increase consumer and bank leverage:**

Suppose $P_2$ is a binary random variable: with probability $\theta$ it is $Y > 0$ which is assumed to be sufficiently large to cover the contractually stipulated loan repayment, and with probability $1 - \theta$ it is zero. We assume $\Lambda(E_i) = \lambda E_i^2 / 2$, where $\lambda > 0$ is a constant. Note that the first-period consumers’ indifference condition becomes:

$$M_1 = B_1 + \theta \{Y - [R_1][P_1 - M_1]\},$$

(10)
the banks’ optimal capital structure condition becomes:

$$\lambda E_1 = \delta[1 - \theta],$$

and the banks’ zero-profit condition becomes:

$$\theta R_1[P_1 - M_1] = P_1 - M_1 + \delta[1 - \theta][P_1 - M_1 - E_1] + \frac{\lambda E_1^2}{2}.$$  (12)

With three equations: (10), (11) and (12), and three unknowns: $P_1$, $R_1$ and $E_1$, we can solve for the equilibrium:

$$P_1 = \frac{\theta Y + B_1 + \delta M_1[1 - \theta] + 0.5\delta^2[1 - \theta]^2/\lambda}{1 + \delta[1 - \theta]},$$

(13)

$$E_1 = \frac{\delta[1 - \theta]}{\lambda},$$

(14)

$$R_1 = \frac{[Y + B_1/\theta - M_1/\theta] \{1 + \delta[1 - \theta]\}}{\theta Y + B_1 - M_1 + 0.5\delta^2[1 - \theta]^2/\lambda}.$$  (15)

Note that we must require $L_1 = P_1 - M_1 > E_1$, otherwise the bank raises more equity capital than the funds needed for the loan. That is,

$$\lambda[\theta Y + B_1 - M_1] > \delta[1 - \theta] + \frac{\delta^2[1 - \theta]^2}{2}.$$  (16)

Each consumer’s leverage is:

$$\frac{P_1 - M_1}{M_1} = \frac{\theta Y + B_1 - M_1 + 0.5\delta^2[1 - \theta]^2/\lambda}{M_1 \{1 + \delta[1 - \theta]\}},$$

(17)

and each bank’s leverage is:

$$\frac{D_1}{L_1} = 1 - \frac{\delta[1 - \theta]}{\lambda} \times \frac{1 + \delta[1 - \theta]}{\theta Y + B_1 - M_1 + 0.5\delta^2[1 - \theta]^2/\lambda} = 1 - \frac{\delta[1 - \theta]}{\lambda} \times \frac{1}{P_1 - M_1}.$$  (18)

Note that consumer leverage is increasing in the probability of a higher future house price, as shown below:

$$\frac{\partial}{\partial \theta} \left( \frac{P_1 - M_1}{M_1} \right) \propto \{1 + \delta[1 - \theta]\} \{\lambda Y - \delta^2[1 - \theta]\} + \delta \left\{ \lambda[\theta Y + B_1 - M_1] + \frac{\delta^2[1 - \theta]^2}{2} \right\}$$

$$> \{1 + \delta[1 - \theta]\} \{\lambda Y - \delta^2[1 - \theta]\} + \delta\{\delta[1 - \theta] + \delta^2[1 - \theta]^2\}$$

$$= \lambda Y \{1 + \delta[1 - \theta]\}$$

$$> 0.$$  (19)
It is also straightforward to show that bank leverage is increasing in the probability of a higher future house price, as shown below:

\[
\frac{\partial}{\partial \theta} \left( \frac{D_1}{L_1} \right) > 0.
\]  

(20)

That is, as \( \theta \) increases and elevates the expected house price at \( t = 2 \), both the consumer’s leverage and the bank’s leverage increase. This is the infectious leverage result that we pursue in this paper. In what follows, we will analyze infectious leverage in a more formal way by endogenizing \( P_2 \). This endogenization will also help us pursue another key result of the paper: bank leverage externality.

### 3.4 Infectious Leverage

We are interested in understanding the relationship between house prices, borrower leverage, and bank leverage at \( t = 1 \). All of these depend on common beliefs about the probability distribution of the future house price, \( P_2 \). The price \( P_2 \), in turn, will depend on the second-period consumer’s utility, \( B_2 \), from owning a house in the second period. Both \( B_2 \) and \( P_2 \) are random variables at \( t = 1 \), but consumers can infer what \( P_2 \) will be for a given realization of \( B_2 \) at \( t = 2 \).

The following intermediate result is useful:

**Lemma 2.** The house price at \( t = 2 \), \( P_2 \), is a concave and increasing function \( \psi(\cdot) \) of the second-period consumer’s utility from home ownership in the second period, \( B_2 \).

This lemma says that equilibrium house price at \( t = 2 \), \( P_2 \), is increasing in each realized value of \( B_2 \), the second-period consumers’ utility from home ownership. This is because a higher utility from home ownership *ceteris paribus* increases the second-period consumer’s willingness to buy a house at \( t = 2 \), which in turn increases the equilibrium house price at \( t = 2 \). The intuition for the concavity result is that a higher second-period house price requires a second-period consumer to take a larger bank loan and hence pay more interest. This partially weakens the consumer’s increased desire to purchase a house due to a higher \( B_2 \), resulting in \( P_2 \) to be a concave function of \( B_2 \).
We model the uncertainty at $t = 1$ about the house price at $t = 2$ by assuming that $B_2$ is a random variable (viewed at $t = 1$) whose probability distribution depends on a parameter $\theta$, where $\theta$ captures common expectations about the future house price, with a higher value of $\theta$ being associated with a higher price. For tractability, we assume in the rest of the analysis that $B_2$ follows a binary distribution and takes a high value $B_{2h}$ with probability $\theta$, and a low value $B_{2l} < B_{2h}$ with probability $1 - \theta$.

Let $P_{2h} = \psi(B_{2h})$ and $P_{2l} = \psi(B_{2l})$ be the corresponding values of $P_2$ as determined by Lemma 2. If the first-period consumers have sufficiently high monetary endowment at $t = 1$, their bank loans will be small enough so that the loans will be riskless. In particular, if $M_1 \geq \theta [P_{2h} - P_{2l}] + B_1$, then from (4), we get $R_1 L_1 \leq P_{2l}$, which means the loan is riskless. A bank can finance it with riskless deposits of $D_1 = L_1$. We assume away this uninteresting case. Instead, we assume that the bank loan at $t = 1$ is risky by assuming that the consumer’s monetary endowment $B_1 < M_1 < \theta [P_{2h} - P_{2l}] + B_1$, so that $P_{2l} < R_1 L_1 < P_{2h}$ and hence the bank loan is risky;\footnote{To see these more clearly, note that (i) $M_1 < \theta [P_{2h} - P_{2l}] + B_1 = \max(P_2 - P_{2l}, 0) + B_1$, and comparing it with (4) shows $R_1 L_1 > P_{2l}$; and (ii) $M_1 > B_1 = \max(P_2 - P_{2h}, 0) + B_1$, and comparing it with (4) shows $R_1 L_1 < P_{2h}$.} note that in this case when $B_2 = B_{2l}$ is realized at $t = 2$, the house price $P_2$ is too low to cover the contractually stipulated loan repayment, $R_1 L_1$. This also requires that the loan amount $L_1 > P_{2l}$. This is because if $L_1 \leq P_{2l}$ were to hold, a bank could finance the loan completely with riskless deposits and earn positive expected profit. By contrast, when $B_2 = B_{2h}$ is realized, $P_{2h}$ is sufficiently high to cover the loan repayment. That is, what we assume here is that $P_{2l}$ (and hence $B_{2l}$) is sufficiently low (insufficient to cover the contractually stipulated loan repayment), whereas $P_{2h}$ (and hence $B_{2h}$) is sufficiently high (sufficient to cover the contractually stipulated loan repayment), so that the bank loan in the first period has risk that depends on the realized value of $B_2$, the second-period consumer’s utility from home ownership.

Summarizing, we make:\footnote{To see how Assumption 2 ensures $B_1 < M_1 < \theta [P_{2h} - P_{2l}] + B_1$, note that $P_{2h} - P_{2l} = \int_{B_{2l}}^{B_{2h}} \frac{\partial P_2}{\partial B_2} dB_2 = \int_{B_{2l}}^{B_{2h}} \frac{1}{1 + \lambda'(P_{3h} - M_{2l})} dB_2 > \int_{B_{2l}}^{B_{2h}} \frac{1}{1 + \lambda'(P_{3h} - M_{2h})} dB_2 = \int_{B_{2l}}^{B_{2h}} \frac{1}{1 + \lambda'(P_{3h} - M_{2l})} dB_2 \geq \frac{M_1 - B_{1}}{\theta} = \frac{M_1 - B_{1}}{\theta} \int_{B_{2l}}^{B_{2h}} \frac{1 + \lambda'(P_{3h} - M_{2h})}{1 + \lambda'(P_{3h} + B_{2h} - M_{2h})}} 0 \implies M_1 > B_1.$}
Assumption 2. $0 < [M_1 - B_1][1 + \Lambda'(\mathbb{E}(P_3) + B_{2h} - M_2)] < \theta[B_{2h} - B_{2l}]$.

We now analyze the relationship between house prices, consumer leverage, and bank leverage. Consumer leverage in period $i$ is defined as $\alpha_i \equiv L_i / M_i$, and bank leverage in period $i$ is defined as $\beta_i \equiv D_i / L_i$.

**Proposition 1.** Consumer leverage $\alpha_1$, bank leverage $\beta_1$, and house price $P_1$ are all increasing in $\theta$.

This is the infectious leverage result. It asserts that when borrowers are more highly levered, so are banks. The economic intuition is as follows. The size of the consumer’s loan is increasing in the price of the house being financed, so a consumer with a fixed initial endowment ends up with personal leverage that is increasing in the price she pays for her house. The bank recognizes that current house prices are predicated on an expectation of future house prices, so a loan repayment that is based on the future value of the house as collateral has credit risk that is decreasing in the current price of the house. This induces banks to keep lower capital precisely when borrowers are more highly leveraged, generating the infectiousness of leverage.

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12 That is, the expected return from investing in a house is assumed to be independent of the level of the current house price.

13 The infectious leverage result can be generalized. Proposition 1 showed that bank leverage is high when consumer leverage is high with the assumption of a binary probability distribution for the second-period house price $P_2$ (coming from a binary probability distribution assumption for $B_2$). We believe the result is more general. To illustrate this point, we prove the result now for a uniform distribution of $P_2$ with a quadratic cost of equity capital for tractability. Suppose $P_2$ is uniformly distributed between 0 and $\theta$ (the upper bound could be any positive number that is increasing in $\theta$). The cost of equity capital $\Lambda(\cdot)$ is such that $\Lambda'(E)/E$ is non-decreasing in $E$, a property that is true for quadratic (where $\Lambda'(E)/E$ is a constant) as well as many other cost functions. Equation (A6) in the Appendix does not depend on specific functional form of $P_2$ or cost of capital $\Lambda(\cdot)$ and still holds, so $P_1$ as well as consumer leverage $\alpha_1$ is still increasing in $\theta$. The optimal capital structure condition (8) reduces to $\Lambda'(E_1) = \delta \Pr(P_2 < D_1) = \delta D_1/\theta$. If $E_1$ increases with $\theta$, then bank leverage $\beta_1 = D_1 / L_1 = \theta \Lambda'(E_1) / [\delta E_1 + \theta \Lambda'(E_1)]$ increases with $\theta$. To see this, we examine $1/\beta_1 = 1 + [\delta/\theta][E_1/\Lambda'(E_1)]$. Note that $E_1/\Lambda'(E_1)$ is non-increasing in $E_1$ (since $\Lambda'(E_1)/E_1$ is assumed to be non-decreasing in $E_1$) and hence non-increasing in $\theta$. It is then clear that $1/\beta_1$ is decreasing in $\theta$ and hence $\beta_1$ is increasing in $\theta$. If $E_1$ decreases with $\theta$, note that $D_1 = P_1 - M_1 - E_1$ increases with $\theta$, so bank leverage $\beta_1 = D_1 / L_1$ still increases with $\theta$. 

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15
4 Bank Leverage Externality

The purpose of this section is to show the social inefficiency of the capital structure decisions of individual banks. We prove four main results. First, an increase in bank leverage in the first period leads to lower house prices in the second period. Second, an individual bank’s loan credit exposure depends on the leverage decisions of other banks in the first period. Third, a small negative shock to a subset of banks in the first period could become a systematic shock that spreads to all the banks. Fourth, banks choose lower privately-optimal capital levels than a social planner would.

4.1 The Link Between First-Period Bank Leverage and Second-Period House Prices

We now assume that lower bank profits in the first period increase the cost of equity capital for the bank in the second period. We assume for the remaining analysis that the cost of equity capital in the second period for a bank is \( \Lambda(E_2) \times z(\Pi) \), where \( \Pi \) is the realized return to the bank’s shareholders in the first period. We assume \( z(\Pi) = 1 \) for \( \Pi \geq 0 \), and \( z'(\Pi) < 0 \) for \( \Pi < 0 \). This implies that the cost of capital for the bank increases when the bank incurs losses and it is increasing in the amount of bank losses. The rationale for this assumption is that banks incurring larger losses may find it more difficult to raise capital to replace what is wiped out by these losses.\(^{14}\) In the extreme case, a bank may fail and be unable to lend at all. We also assume that new banks cannot arise in the short run at \( t = 2 \) to replace poorly-performing banks. Thus, intuitively bank losses in the first period adversely impact aggregate lending in the second period and this, in turn, depresses house prices at \( t = 2 \). This depression in house prices further hurts the profits of banks because houses serve as collateral for their loans. This is what we want to show formally in the following analysis.

\(^{14}\)See Acharya, Shin and Yorulmazer (2009) for an analysis of why banks that have their capital wiped out by losses are unable to raise funds to replace this capital.
We now analyze the housing market and bank lending in the second period while allowing each bank’s cost of capital to depend on its profits from the first period. Since all banks are ex ante identical, each will have the same equilibrium first-period profit. However, in order to determine the first-period equilibrium, we need to consider out-of-equilibrium behavior in which banks may differ in their first-period capital structure choices and hence also in the profits realized at the end of the first period, which will result in different lending behaviors for different banks in the second period.

Consider bank $i$. For its second-period lending, suppose bank $i$ extends $\omega_i$ loans at $t = 2$ where each loan is of size $L_2$, so bank $i$ lends in total $\omega_i L_2$. The bank raises $\omega_i E_{2i}$ of equity capital and $\omega_i D_{2i}$ of deposits such that $E_{2i} + D_{2i} = L_2$.\(^{15}\) We assume that if a borrower’s loan is financed by multiple banks, a bank’s fraction of the loan repayment equals the fraction of the borrower loan that the bank provided. Thus, bank $i$’s expected profit in the second period is:

\[
[\omega_i] \left[ \mathbb{E}(\min(P_3, R_2 L_2)) - L_2 - \delta \mathbb{E}(\max(D_{2i} - P_3, 0)) \right] - z(\Pi_i) \Lambda(\omega_i E_{2i}),
\]

(21)

where $\Pi_i$ is bank $i$’s (realized) profit in the first period given by:

\[
\Pi_i = \min(P_2, R_1 L_1) - L_1 - \delta \max(D_{1i} - P_2, 0) - \Lambda(L_1 - D_{1i}),
\]

(22)

and $D_{1i}$ is the bank $i$’s first-period deposit. The expression in (21) can be understood as follows. Bank $i$’s expected profit is equal to the amount of loans, $\omega_i$, times the expected profit per loan, $\mathbb{E}(\min(P_3, R_2 L_2)) - L_2 - \delta \mathbb{E}(\max(D_{2i} - P_3, 0))$, net of the cost of equity capital to support the loans, $z(\Pi_i) \Lambda(\omega_i E_{2i})$. The bank’s problem in the second period can be seen as the choice of the number of loans ($\omega_i$), and the choice of its optimal capital structure mix ($D_{2i}$ and $E_{2i}$) given the number of loans extended and the size of each loan. The capital structure choice (for given $\omega_i$ and $L_2$) minimizes the cost of financing $\omega_i$ loans of size $L_2$, denoted by $K(\omega_i, L_2, \Pi_i)$:

\[
K(\omega_i, L_2, \Pi_i) = \min_{\{D_{2i} + E_{2i} = L_2\}} \omega_i \delta \mathbb{E}(\max(D_{2i} - P_3, 0)) + z(\Pi_i) \Lambda(\omega_i E_{2i}).
\]

(23)

\(^{15}\)Note that the size of each loan, $L_2$, is independent of the identity of the individual lending bank. This is because $L_2$ is determined by the amount of money that a second-period homebuyer needs to borrow and hence the house price at $t = 2, P_2$, which does not depend on a specific individual bank since each individual bank is atomistic. Also, $\omega_i < 1$ is possible, in which case bank $i$ finances only a fraction of a loan.
In a competitive second-period equilibrium, banks must earn zero expected profit almost surely. A positive measure of banks cannot earn positive expected profits in equilibrium; otherwise these banks can increase their profits by slightly lowering their second-period loan interest rate at \( t = 2 \) (they can afford to do so because of their positive profits), however, even a tiny deduction in the loan interest rate offered by these banks will attract all the second-period homebuyers, and this breaks the equilibrium.\(^{16}\) Thus, for bank \( i \), we have:

\[
\mathbb{E}(\min(P_3, R_2 L_2)) - L_2 = K(\omega_i, L_2, \Pi_i)/\omega_i. \tag{24}
\]

The second-period equilibrium requirement is that the supply of loans equals the demand for loans:

\[
\int \omega_i di = S. \tag{25}
\]

We can now establish the following result which shows that an increase in the first-period leverage of some banks can increase the risk of their losses and thereby lead to lower house prices at \( t = 2 \). Before we move on, it is useful to clarify here that the following result is an out-of-equilibrium result. In the first-period equilibrium, all banks choose the same leverage, have the same first-period profit and hence the same second-period cost of equity. Thus, we have \( \omega_i = 1 \ \forall i \) in equilibrium, i.e., each bank also lends to exactly one borrower in the second period.

**Proposition 2.** (i) Banks with higher leverage in the first period extend fewer loans in the second period. (ii) An increase in the leverage of a positive measure of banks in the first period causes a decline in the second-period house price \( P_2 \), and a larger increase in the leverage by these banks or a larger measure of such banks causes a larger decline in \( P_2 \).

The intuition for this result is that an increase in leverage by a positive measure of banks in the first period increases the volatility of their profits at \( t = 2 \). To see why this happens, note that an increase in leverage causes a bank’s profit to increase conditional on a positive shock (\( B_2 = B_{2h} \)) because higher leverage replaces equity with cheaper deposits which are not associated with any cost increase due to the positive shock. However, an increase in leverage also

\(^{16}\)No bank will earn negative expected profit because a bank can earn zero expected profit by not lending at all.
causes banks’ losses to increase when the house price suffers a negative shock at $t = 2$ ($B_2 = B_2$).

The higher volatility of profits accompanying the higher leverage increases the cost of capital for these banks. As a consequence, these banks reduce their lending in the second period. That is, for a fixed loan size $L_2$, these banks reduce the number of loans extended ($\omega_i$ decreases). To clear the loan market, the banks that did not increase their leverage ratios in the first period need to increase their lending in the second period to compensate, but this compensation is only partial because their lending is also constrained by their own cost of financing in the second period. Thus, the fact that the marginal cost of capital increases for some banks – that deviate from the equilibrium and increase their leverage in the first period – means that aggregate lending declines. To induce the banks to extend enough loans to clear the housing market at $t = 2$, the second-period loan interest rate must increase. But this lowers the second-generation consumers’ demand for loans to finance house purchases, causing the equilibrium house price to decline at $t = 2$ in order to clear the housing market. Moreover, this adverse effect will be stronger when a larger number of banks increase their first-period leverage and when the leverage increase is greater. And as this adverse effect strengthens, there is a larger decline in $P_2$.

### 4.2 The Link Between a Bank’s Credit Risk Exposure and the Leverage Choices of Other Banks

Next, we consider an extension of Proposition 2. An individual bank $i$’s expected bankruptcy cost at $t = 2$ when it makes a deposit choice of $D_{1i}$ in the first period, defined as its “credit exposure,” is:

$$[1 + \delta]\mathbb{E}(\max(D_{1i} - \min(P_2, R_1L_1), 0))$$

$$= \Pr(\min(R_1L_1, P_2) < D_{1i}) \times [1 + \delta] \max(D_{1i} - \min(P_2, R_1L_1), 0).$$

(26)

Note that $R_1L_1$ is the stipulated contractual repayment, so the amount the bank receives from the first-period homebuyer depends crucially on the house price at $t = 2$, $P_2$, which, in turn, depends on the leverage choices of all other banks in the first period (see Proposition 2). This means that bank $i$’s credit exposure depends on the collective leverage choices of all the other
The fact that all the banks’ loans are backed by the same collateral (i.e., houses) engenders interconnectedness between otherwise-independent banks. Summarizing, we have:

**Proposition 3.** Each individual bank’s loan credit exposure at \( t = 2 \) depends on the leverage choices of other banks in the first period, and the credit exposure becomes larger when other banks choose higher leverage ratios.

### 4.3 Can Idiosyncratic Risks Become Systematic?

As another extension of Proposition 2, consider the following variation of the model. Suppose each bank’s realized profit for the first period, \( \Pi_i \), also has an idiosyncratic component, \( \varepsilon_i \), where \( \varepsilon_i \) can take upon two possible values: zero and a negative shock, \( -\varepsilon \). We assume that the \( \varepsilon_i \)’s are identical and independently distributed, with \( \varepsilon_i = 0 \) and \( \varepsilon_i = -\varepsilon \) being equiprobable. Thus, viewed at \( t = 1 \), the probability that all banks suffer a negative shock, \((1/2)^S\), approaches zero as \( S \) approaches infinity. To illustrate the key idea, we assume here that there is a denumerable number of banks, say \( S \) in number (instead of the measure \( S \)), where \( S \) is sufficiently large. We also assume that the shock is sufficiently negative, so that when \( \varepsilon_i = -\varepsilon \) is realized at \( t = 2 \), bank \( i \)’s profit will become negative even if \( B_2 = B_{2h} \) is realized.\(^{17}\)

Consider now the event that \( B_2 = B_{2h} \) and \( \hat{S} < S \) banks suffer the negative shock, whereas the remaining \( S - \hat{S} \) banks do not. It is clear that the \( \hat{S} \) banks will realize negative profit in the first period, which will impact their abilities to lend in the second period and hence decrease the equilibrium house price at \( t = 2 \), \( P_2 \). This decrease in \( P_2 \) will also diminish the profits of the remaining \( S - \hat{S} \) banks. But as long as the profits of those \( S - \hat{S} \) banks remain positive, their lending capacity in the second period will be unaffected. The interesting case is when \( \hat{S} \) is sufficiently large, so that the housing price drop at \( t = 2 \) due to the negative shock suffered by the \( \hat{S} \) banks is so severe that the remaining \( S - \hat{S} \) banks lose money in the first period despite having escaped the initial negative shock. This will cause those \( S - \hat{S} \) banks to curtail their lending in the second period, which will exert further downward pressure on the house price at \( t = 2 \), generating a downward housing price spiral. In that case, an idiosyncratic negative shock

\(^{17}\)The house price \( P_2 \) would be reasonably high in the absence of the shock.
suffered by a subset of the banks can become a systematic shock that propagates through the entire banking system.

The key to this conversion of idiosyncratic risk into systematic risk is that the number of banks hit by the negative shock, $\hat{S}$, should be sufficiently large. Note that when $\hat{S}$ is sufficiently large, the ex ante probability that all of these banks will suffer a negative shock, $(1/2)^{\hat{S}}$, becomes very small. However, when bank leverage is high, it takes a relatively small drop in house price to cause the $S - \hat{S}$ banks to start losing money. That is, when leverage is high, the critical number ($\hat{S}$) of banks that need to experience a negative shock to convert idiosyncratic risk into systematic risk becomes small. Thus, the higher the leverage of banks, the more likely it is that idiosyncratic shocks will be transformed into systematic risk for the banking system.

**Proposition 4.** The conversion of idiosyncratic risk into systematic risk becomes more likely for a larger $\theta$, i.e., when the common expectation of the future house price becomes higher.

To understand this result, note that when the common expectation of the future house price becomes higher, banks take higher leverage in the first period (see Proposition 1). This choice of higher leverage makes the entire banking system more vulnerable to the idiosyncratic negative shocks suffered by a subset of the banks, causing idiosyncratic risk to become more likely to be converted into systematic risk.

**4.4 Privately-Optimal Leverage Decisions of Banks and the Social Optimum**

The above results describe a channel through which a bank’s actions can impose an externality on other banks. We now show that ignoring this externality may cause banks to take excessive risk as compared to the social optimum. Let $D^*$ be the equilibrium first-period bank debt, and $D^{**}$ be the bank debt that is socially optimal. That is, $D^{**}$ is the amount of debt the banks will choose if they coordinate their capital structure choices while acting competitively. We can also view $D^{**}$ as the bank debt a social planner would prescribe to minimize the total cost of
borrowing (or maximize house prices). Let \( \Pi(D, \hat{D}) \) be the expected first-period profit of a bank if it chooses debt \( D \) while all other banks choose debt \( \hat{D} \). Thus,

\[
\Pi(D, \hat{D}) = \mathbb{E}(\min(P_2(\hat{D}), R_1 L_1)) - \delta \mathbb{E}(\max(D_1 - P_2(\hat{D}), 0)) - L_1 - \Lambda(L_1 - D_1),
\]

where possible dependence of the second-period house price \( P_2 \) on the leverage choices of other banks is explicitly recognized. The expression in (27) can be understood as follows. Note that: (i) \( \mathbb{E}(\min(P_2(\hat{D}), R_1 L_1)) \) is what the bank expects to receive as loan repayment from the borrower, where the atomistic bank’s own choice of \( D \) does not impact the equilibrium house price, \( P_2(\hat{D}) \), which is determined by the collective equilibrium choice of \( \hat{D} \) by all the other banks; and (ii) \( \delta \mathbb{E}(\max(D_1 - P_2(\hat{D}), 0)) \) is the bank’s expected default cost, \( L_1 \) is the amount loaned, and \( \Lambda(L_1 - D_1) \) is the cost of equity. We can now establish the fourth main result of this section.

**Proposition 5.** The (privately-optimal) equilibrium level of bank debt \( D^* \) is higher than the socially-optimal bank debt \( D^{**} \).

The intuition for the result is as follows. When banks compete with each other and each chooses its capital structure while taking other banks’ actions as given, each bank maximizes its own expected profit while taking the probability distribution of the second-period house price, \( P_2 \), as given. This is rational because even though a bank’s choice of capital structure will impact its profit and hence its lending ability in the second-period, the bank anticipates having no effect on the house price because it is atomistic. Further, the bank’s choice of capital structure does not influence the capital structure choices of other banks. Of course, each bank thinks so, so banks collectively fail to recognize that the effect of higher leverage by all banks in the first period is to make second-period lending more costly for all banks in case of a negative shock at \( t = 2 \) and thereby generating a lower second-period house price. A social planner takes the effect of high bank leverage on future house prices into account and chooses lower bank debt in order to maximize aggregate expected bank profits.

An implication of this is that a rational bank cannot be expected to implement the socially-efficient capital level in the absence of capital-requirement regulations. Moreover, a central planner must take into account the effect of an individual bank’s leverage choice on other banks in determining minimum capital requirements for all banks. Our analysis points out that this
issue is particularly important when bank loans are secured by collateral whose value is highly dependent on the availability of aggregate bank credit in the first place.

Although our paper is not about capital requirements \textit{per se}, the analysis in this section provides a new rationale for regulatory capital requirements. An interesting departure relative to previous analyses is that our analysis reveals that by imposing minimum capital requirements that are higher than banks’ privately optimal capital levels, the regulator can affect the dynamics of future housing prices and \textit{reduce} the systematic risk associated with low future housing prices. That is, the interaction between bank capital levels and real-asset prices sheds new light on the role of bank capital and the potential for capital-requirements-based prudential regulation to control systematic risk in real-asset markets. In the next subsection, we discuss further possible implications of our analysis for alternative forms of government intervention – capital regulation and capital infusion for banks in need.

4.5 Ex Ante Capital Regulation and Ex Post Capital Infusion

We now discuss, within the context of our model, the consequences of ex ante bank capital regulation (at $t = 1$) and ex post capital infusion (at $t = 2$) by a regulator. Recall that bank equity capital acts as a cushion against bankruptcy when the housing market experiences a negative shock ($B_2 = B_{2l}$) at $t = 2$. But when the shock is positive ($B_2 = B_{2h}$), the bank would have been better off if it had chosen a higher leverage ratio at $t = 1$. Ex ante each bank makes its own capital structure decision by trading off the benefit of equity capital against its cost to maximize its own expected profit. But we know from Proposition 5 that a bank’s privately-optimal choice of equity capital is below the social optimum. So now we ask: what is the economic consequence of the regulator imposing a capital-requirement equal to the social optimum? Note that higher bank capital acts as a cushion against the banks’ losses at $t = 2$ when a negative shock occurs, and this reduces banks’ costs of financing in the second period, thereby helping to elevate the equilibrium house price at $t = 2$. However, imposing a capital requirement higher than the bank’s privately optimal choice of equity capital reduces the bank’s ex ante expected profit, which in turn will make the first-period loan more costly for the first-generation
homebuyers. This will reduce the first-generation consumers’ demand for houses, causing the house price at $t = 1$, $P_1$, to decline. Thus, a binding capital requirement is likely to reduce house prices initially but will also lower the likelihood of a further housing price decline.

An alternative to capital requirement is for the regulator to infuse capital into the banks with a non-zero probability at $t = 2$ when the housing market is hit by a negative shock. A capital infusion reduces the banks’ reliance on the capital market to raise (now very costly) equity to finance the second-period loan, which then helps to arrest any house price decline at $t = 2$. In this sense, an ex post capital infusion is similar to a capital requirement. However, these two methods of intervention may have very different ex ante consequences. To see that, note if the first-period homebuyers anticipate the government’s capital infusion will occur ex post with a positive probability in the adverse state, they will attach a lower probability to a housing price decline in the second period. This will cause housing demand to increase in the first period, and as a result the equilibrium house price at $t = 1$, $P_1$, will increase. As for banks, the possibility of an ex post capital infusion reduces the marginal value of bank capital ex ante, and hence banks will choose higher leverage in the first period. However, this will increase the banks’ losses if the housing market is hit by a negative shock at $t = 2$ and the government does not provide an ex post capital infusion. In turn, this increases the attractiveness of an ex post capital infusion from the standpoint of the government. Thus, an ex post government capital infusion could become a self-fulfilling prophecy, contributing to an elevation of banking system leverage and fragility.

5 Empirical Implications and Conclusion

In this paper we have shown that the leverage decisions of borrowers and banks may move in unison – banks choose to become more highly levered when their borrowers are more highly levered. This finding is obtained in a setting in which bank loans are secured and borrowers’ repayment of bank loans depends primarily on the stochastic value of the collateral backing the loan. Moreover, the probability distribution of the value of the collateral is affected by the aggregate lending behavior of banks, which in turn is dependent on their earlier capital structure decisions. Markets such as the one examined in this paper are also characterized by bank leverage
externality – higher leverage ratios chosen by banks at a given point in time tend to lower future housing prices. Because individual banks do not internalize this externality, each bank’s leverage exceeds the social optimum.

What empirical implications can we draw from this analysis? First, the main prediction of the model is that we should find in the data that high house prices, high borrower leverage and high bank leverage should occur together. The recent home mortgage crisis is an example of this. Second, there will be a negative correlation between aggregate bank leverage in a given period and subsequent house prices. That is, house prices in any given period will be decreasing in appropriately-lagged bank capital. Third, because banks wish to operate with less capital during such periods, regulatory capital requirements will tend to be more binding for banks that engage in more secured lending and when the price of collateral is higher for borrowers. Fourth, as leverage increases in the banking system, it is more likely that idiosyncratic shocks will become systematic risks.

Future extensions of the analysis could proceed in many different directions. One would be to endogenize the cost of bank equity capital and examine the resulting interactions to see how robust the findings are to an endogenously-determined cost of bank equity. Another would be to introduce borrower equity and analyze its interaction with bank equity as well as its effect on the analysis.
Appendix

Proof of Lemma 1: Note that when $P_{i+1}$ increases, keeping $E_i$ unchanged, the right-hand-side (RHS) of (8), $\delta \Pr(P_{i+1} < L_i - E_i)$, decreases. If $E_i$ were to increase, the RHS of (8) would decrease further, whereas its left-hand-side (LHS) would increase since $\Lambda(\cdot)$ is an increasing convex function. Thus, we must have $E_i$ decrease in order to maintain the equality in (8) in equilibrium. □

Proof of Lemma 2: Substituting $L_i = P_i - M_i$ into (9), we have:

$$P_i = M_i + \mathbb{E}(\min(P_{i+1}, R_iL_i)) - \delta \mathbb{E}(\max(D_i - P_{i+1}, 0)) - \Lambda(E_i),$$ (A1)

which, combined with (4), yields:

$$P_i = \mathbb{E}(\max(P_{i+1} - R_iL_i, 0)) + B_i + \mathbb{E}(\min(P_{i+1}, R_iL_i)) - \delta \mathbb{E}(\max(D_i - P_{i+1}, 0)) - \Lambda(E_i)$$

$$= \mathbb{E}(P_{i+1}) + B_i - \delta \mathbb{E}(\max(P_i - M_i - E_i - P_{i+1}, 0)) - \Lambda(E_i).$$ (A2)

Letting $i = 2$ and using the Implicit Function Theorem, we have:

$$\frac{\partial P_2}{\partial B_2} = \frac{1}{1 + \delta \Pr(P_3 < D_2)} = \frac{1}{1 + \Lambda'(E_2)} > 0,$$ (A3)

where the last equality follows from (8).

Thus, $P_2$ is an increasing function of $B_2$. Substituting $i = 2$ and $D_2 = P_2 - M_2 - E_2$ into (8) and totally differentiating it with respect to $P_2$, we have:

$$\Lambda''(E_2) \left[ \frac{\partial E_2}{\partial P_2} \right] = \delta \left[ \frac{\partial \Pr(P_3 < D)}{\partial D} \right]_{D=P_2-M_2-E_2} \times \left[ 1 - \frac{\partial E_2}{\partial P_2} \right],$$ (A4)

i.e.,

$$\left[ \frac{\partial E_2}{\partial P_2} \right] \Lambda''(E_2) + \delta \left[ \frac{\partial \Pr(P_3 < D)}{\partial D} \right]_{D=P_2-M_2-E_2} = \delta \left[ \frac{\partial \Pr(P_3 < D)}{\partial D} \right]_{D=P_2-M_2-E_2},$$ (A5)

which shows that $E_2$ is increasing in $P_2$ and hence in $B_2$. The RHS of (A3) then must be decreasing in $B_2$, showing that $P_2$ is an increasing and concave function, call it $\psi(\cdot)$, of $B_2$. □

Proof of Proposition 1: Letting $i = 1$ in (A2), and noting that $P_2$ depends on $\theta$, we have:

$$P_1 = \mathbb{E}(P_2|\theta) + B_1 - \delta \mathbb{E}(\max(D_1 - P_2, 0)|\theta) - \Lambda(E_1).$$ (A6)
We know from Lemma 2 that $P_2$ is increasing in the realized value of $B_2$. Note that as $\theta$ increases, $B_2$ is more likely to take the high value $B_2 h$. Thus, $\mathbb{E}(P_2|\theta)$ must be increasing in $\theta$, and hence $P_1$ is increasing in $\theta$. Borrower leverage $\alpha_1 = (P_1 - M_1)/M_1$ is increasing in $P_1$ and hence in $\theta$.

Substituting $i = 1$ in (8) and noting that the loan is risky, $L_1 > P_2 l$, we must have $D_1 > P_2 l$ as otherwise $\Lambda'(E_2) > 0 = \delta \Pr(P_2 < D_1)$, violating (8). Also, we must have $D_1 < P_2 h$ as otherwise the loan always defaults and the bank always loses money. Thus, (8) reduces to:

$$\Lambda'(E_1) = \delta[1 - \theta].$$

(A7)

Totally differentiating with respect to $\theta$, we get:

$$\frac{\partial E_1}{\partial \theta} = \frac{-\delta}{\Lambda''(E_1)} < 0.$$  

(A8)

Thus, bank leverage $\beta_1 = \frac{P_1 - M_1 - E_1}{P_1 - M_1}$ is also increasing in $\theta$. □

Proof of Proposition 2: If $B_2 = B_2 l$ (low house price), then $\Pi_i < \mathbb{E}(\Pi_i) = 0$. Differentiating (22), which now becomes $\Pi_i = P_2 l - \delta[D_{1i} - P_2 l] - L_1 - \Lambda(L_1 - D_{1i})$, with respect to $D_{1i}$, we have $\partial \Pi_i/\partial D_{1i} = -\delta + \Lambda'(E_{1i}) < 0$ where the inequality follows from (8). Then, $\partial z/\partial D_{1i} = z'(\Pi_i) \times \partial \Pi_i/\partial D_{1i} > 0$. If $B_2 = B_2 h$ (high house price), then $\Pi_i > \mathbb{E}(\Pi_i) = 0$, and hence $z(\Pi_i) = 1$ is independent of $D_{1i}$. Thus, we always have:

$$\partial z/\partial D_{1i} \geq 0.$$  

(A9)

From (23), it follows that $\partial K/\partial \omega_i = \delta \mathbb{E}(\max(D_{2i} - P_3, 0)) + z(\Pi_i) E_{2i} \Lambda'(\omega_i, E_{2i})$ is increasing in $D_{1i}$ and also in $\omega_i$ (because $\Lambda(\cdot)$ is convex). From (24), we get:

$$\partial K/\partial \omega_i = \mathbb{E}(\min(P_3, R_2 L_2)) - L_2.$$  

(A10)

Totally differentiating with respect to $D_{1i}$, we get:

$$\frac{\partial \omega_i}{\partial D_{1i}} = -\frac{\partial^2 K/\partial \omega_i \partial D_{1i}}{\partial^2 K/\partial \omega_i^2} \leq 0,$$

(A11)

and this proves part (i).

We now prove part (ii). Substituting $L_2 = P_2 - M_2$ into (24), we have:

$$P_2 = M_2 + \mathbb{E}(\min(P_3, R_2 L_2)) - K(\omega_i, L_2, \Pi_i)/\omega_i,$$

(A12)
which, combined with (4), yields:

\[ P_2 = B_2 + \mathbb{E}(\max(P_3 - R_2 L_2, 0)) + \mathbb{E}(\min(P_3, R_2 L_2)) - K(\omega_i, L_2, \Pi_i)/\omega_i \]

\[ = B_2 + \mathbb{E}(P_3) - K(\omega_i, L_2, \Pi_i)/\omega_i. \tag{A13} \]

Suppose a positive measure, say \( \hat{S} \in (0, S) \), of banks increase their first-period deposits while the remaining banks keep their deposits fixed. The equilibrium will adjust so that these \( \hat{S} \) measure of banks will earn zero expected profit with the increased deposit, and thus the house price \( P_2 \) will follow (A13). For such a bank, say bank \( i \), the increase in leverage increases \( z(\Pi_i) \) (see (A9)), which leads to an increase in the cost per loan \( K(\omega_i, L_2, \Pi_i)/\omega_i \) (see (23)), which in turn leads to a decrease in \( P_2 \) (see (A13)). If bank \( i \) chooses a higher \( D_{1i} \) (i.e., higher leverage), \( z(\Pi_i) \) will increase more, and this will lead to a larger increase in the cost per loan \( K(\omega_i, L_2, \Pi_i)/\omega_i \), thereby causing a larger decline in \( P_2 \). If more banks increase their first-period leverage, i.e., \( \hat{S} \) becomes larger, we can easily see that \( P_2 \) will decrease further by applying the above arguments twice. \( \square \)

**Proof of Proposition 3:** The first part of the proposition is straightforward by examining the mathematical expression of the bank’s credit exposure: \( \Pr(\min(R_1 L_1, P_2) < D_{1i}) \times [1 + \delta]\max(D_{1i} - \min(P_2, R_1 L_1), 0) \). The second part follows from Proposition 2 by noting that higher leverage among other banks depresses \( P_2 \), which in turn increases bank \( i \)’s credit exposure. \( \square \)

**Proof of Proposition 4:** Note that when \( B_2 = B_{2h} \), all the banks lose money in the first period regardless of the idiosyncratic shock. Consider the event when \( B_2 = B_{2h} \) and \( \hat{S} \) banks suffer the negative shock. In this case, these \( \hat{S} \) banks lose money in the first period, and this increases their cost of lending in the second period, which in turn depresses the house price, \( P_2 \); note that the effect of the negative shock is similar to that of a higher first-period leverage choice by the \( \hat{S} \) banks, as both increase the loss suffered by the \( \hat{S} \) banks, so we can apply the argument in the proof of Proposition 2 to conclude that the negative shock depresses \( P_2 \). It also follows from Proposition 2 that a larger \( \hat{S} \) causes a larger drop in \( P_2 \), which in turn causes a larger drop in the remaining \( S - \hat{S} \) banks’ profit:

\[ \hat{\Pi} = \min(P_{2h}, R_1 L_1) - \delta \max(D_1 - P_{2h}, 0) - L_1 - \Lambda(E_1). \tag{A14} \]
Then, there exists a critical cutoff \( \hat{S}^* \), such that \( \hat{H} \) will become negative if \( \hat{S} > \hat{S}^* \). Denote the corresponding house price as \( P_{2h}^* \), i.e., when house price drops below \( P_{2h}^* \), \( \hat{H} \) becomes negative.

Next, we show that when bank leverage in the first period is higher, the cutoff \( \hat{S}^* \) becomes smaller and \( P_{2h}^* \) becomes larger, i.e., it takes a smaller drop in house price to cause the \( S - \hat{S} \) banks to lose money. This can be shown by totally differentiating (A14) and noting that:

\[
\frac{\partial P_{2h}^*}{\partial D_1} = -\frac{\partial \hat{H}/\partial D_1}{\partial \hat{H}/\partial P_{2h}} \geq 0, \tag{A15}
\]

since \( \partial \hat{H}/\partial D_1 \leq 0 \) and \( \partial \hat{H}/\partial P_{2h} > 0 \). The result then follows from Proposition 1, as banks choose higher leverage in the first period when \( \theta \) is larger. □

**Proof of Proposition 5:** First, we take the loan amount \( L_1 \) as fixed. Denote \( \Pi(D_i, D_{-i}) \) as bank \( i \)'s profit when bank \( i \)'s first-period deposit choice is \( D_i \) and other banks choose \( D_{-i} \). The equilibrium debt level \( D^* \) is determined by:

\[
D^* = \arg \max_D \Pi(D, D^*), \tag{A16}
\]

while the socially optimal debt \( D^{**} \) is determined by:

\[
D^{**} = \arg \max_D \Pi(D, D) \tag{A17}
\]

Suppose counterfactually that the result claimed in the proposition is not true and \( D^* \leq D^{**} \). Then,

\[
\Pi(D^*, D^*) \geq \Pi(D^{**}, D^*) \geq \Pi(D^{**}, D^{**}), \tag{A18}
\]

where the first inequality follows from (A16), and the second follows from Proposition 2 and the fact that \( \Pi \) is increasing in \( P_2 \). But (A18) contradicts (A17). Thus, we must have \( D^* > D^{**} \). Finally, the result will hold even if we allow \( L_1 \) to be chosen endogenously, because the socially optimal choice minimizes total cost of financing, and that requires the marginal cost of debt at the socially optimal capital structure to be smaller than the marginal cost of debt in an equilibrium where banks choose capital structure taking other banks’ choices as given. □
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Figure 1: Sequence of Events

- Each first-period consumer chooses whether to buy a house or not.
- Banks compete to lend to first-period homebuyers.
- Each bank chooses its first-period capital structure.
- First-period homebuyers sell their houses.
- Each second-period consumer chooses whether to buy a house or not.
- Bank loans to first-period homebuyers are settled.
- Banks compete to lend to second-period homebuyers.
- Each bank chooses its second-period capital structure.
- Second-period homebuyers sell their houses.
- Banks loans to second-period homebuyers are settled.