International Portfolios, Capital Accumulation and Foreign Assets Dynamics

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Abstract

Despite the liberalization of capital flows among OECD countries, equity home bias remains sizable. We depart from the two familiar explanations of equity home bias: transaction costs that impede international diversification, and terms of trade responses to supply shocks that provide risk sharing, so that there is little incentive to hold diversified portfolios. We show that the interaction of the following ingredients generates a realistic equity home bias: capital accumulation and international trade in stocks and bonds. In our model, domestic stocks are used to hedge fluctuations in local wage income. Terms of trade risk is hedged using bonds denominated in local goods and in foreign goods. In contrast to related models, the low level of international diversification does not depend on strongly counter-cyclical terms of trade. The model also reproduces the cyclical dynamics of foreign asset positions and of international capital flows.

JEL classification: F2, F3, G1.

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1 Introduction

Cross-country capital flows have increased greatly, since the liberalization of international capital markets two decades ago (e.g., Lane and Milesi-Feretti (2003, 2005, 2006)). Equity home bias, while less severe than in earlier decades, remains sizable and is observed in all industrialized countries (see French and Poterba (1991) for early evidence and Sereu and Vanpée (2007) for a recent survey). There are broadly two classes of explanations for the persisting equity home bias. The first one centers on transaction costs and informational barriers in cross-border financial transactions and suggests that international risk sharing is insufficient.\(^1\) The second one focuses on the possibility that terms of trade changes in response to supply shocks may provide international insurance against these shocks, so that even a portfolio with home bias delivers efficient international risk sharing (Cole and Obstfeld (1991), Helpman and Razin (1978)).

Both types of explanations are helpful but are not without problems. Several authors have argued that frictions would have to be large to fully explain the equity home bias (French and Poterba (1991), Tesar and Werner (1995), Warnock (2002)). In order to interpret terms of trade changes as providing insurance (rather than a source of risk), the terms of trade would have to improve strongly after a negative supply shock. However, empirically the terms of trade are only weakly correlated with output (e.g., Backus, Kehoe and Kydland (1994)).

Using a two-country general equilibrium model with fully integrated financial markets, this paper shows that the interaction of the following ingredients is key for generating realistic equity home bias, without requiring strongly countercyclical terms of trade: capital accumulation and international trade in stocks and bonds denominated in local and foreign goods.\(^2\)

By contrast, other recent general equilibrium models of international equity holdings (see Devereux and Sutherland (2006a,b) for references) have mostly assumed *endowment economies*, i.e. economies without production or capital accumulation – Heathcote and Perri (2007) is a notable exception discussed below. In such economies, households trade in international financial markets solely for consumption smoothing and risk sharing purposes so that the equity portfolio is structured to sustain net imports.


\(^{2}\)Pavlova and Rigobon (2007a and b), Engel and Matsumoto (2006) and Coeurdacier, Kollmann and Martin (2007) have previously analyzed equity portfolio choice in general equilibrium models with trade in bonds.
in states of nature where local production is low; this leads to local equity bias if relative Home equity returns rise (compared to Foreign returns) when the Home terms of trade improve and the Home real exchange rate appreciates, in response to a drop in the Home output.³ This condition however is not met in the data: empirically, the correlation between relative equity returns and the real exchange rate is low (van Wincoop and Warnock (2006)).

We consider a model with capital accumulation because, as discussed by Obstfeld and Rogoff (1996), one of the key functions of international financial markets is to finance physical investment; empirically, productive investment is a key driver of fluctuations in net imports (Sachs (1981), Backus, Kehoe and Kydland (1994)). With two stocks and two bonds, and two types of (Home and Foreign) technology shocks, markets are effectively complete, up to a first order (linear) approximation of the model. In addition to standard TFP (total factor productivity) shocks, the model here assumes shock to investment efficiency (as in Greenwood, Hercowitz and Krusell (1997, 2000), Fisher (2002, 2006)), because recent empirical research suggests that those shocks are an important source of fluctuations in real activity (Justiniano and Primiceri (2006), Justiniano et al. (2007)).

The equilibrium portfolio is structured to optimally hedge fluctuations in the real exchange rate and in labor incomes.⁴ Specifically, bonds are used for real exchange rate hedging, since the difference between the pay-offs of bonds denominated in Home and Foreign goods is correlated with the real exchange rate. Fluctuations in labor incomes are hedged through the equity portfolio. The key mechanism here is that fluctuations in investment generate a negative co-movement between Home capital income (net of investment) and Home labor income (relative to their Foreign counterparts). A Home investment boom lowers Home payments to shareholders (to finance investment) and raises Home output and wage incomes (relative to foreign wages), under the realistic assumption (made here) that there is a local bias in investment spending. Thus, local equity offers a good hedge against movements in local labor incomes associated with investment fluctuations – which explains why equilibrium equity portfolios exhibit home bias. The predicted equity home bias only depends on the degree of home bias in investment spending, and on the labor share. In particular, it is independent of preference parameters.⁵ Importantly, the

⁵Coeurdacier and Gourinchas (2008) provide a general discussion of the conditions under which equity portfolios are
optimal portfolio does not hinge on the presence of investment efficiency shocks. These shocks help to
explain the countercyclical nature of the trade balance and the acyclicality of the terms of trade but our
portfolio results would also hold in a model with TFP shocks and a range of other (domestic and foreign)
shocks to output and/or investment.

The closest paper to ours is Heathcote and Perri (2007) [HP henceforth] who were the first to inves-
tigate the importance of physical investment for equity portfolios. Trade in bonds, and the shocks to
investment efficiency assumed here are the main difference between our model and HP. The HP model
only generates realistic equity home bias if the terms of trade respond strongly to TFP shocks (or,
equivalently, if preferences are "close enough" to log-separability between the two goods, as in a Cole
and Obstfeld (1991) economy). Our model does not require strong terms of trade effects of productivity
shocks—nevertheless, there is sizable equity home bias. This is important since the empirical evidence
concerning the response of the terms of trade to technology shocks is mixed.\footnote{Corsetti, Dedola and Leduc (2006) argue that, empirically, a positive technology shocks triggers a terms of trade appreciation; Acemoglu and Ventura (2002) and Kollmann (2006c) provide evidence that higher productivity depreciates the terms-of-trade.}

Another paper close to ours is Engel and Matsumoto (2006) who analyze international equity portfolio
choices in a model with money, sticky prices and trade in bonds, but without capital accumulation. Under
price stickiness, the short run level of output is demand determined, so that a positive productivity shock
leads to a fall in employment and labor income, but an increase in profits. Ownership of local equity
is thus an effective hedge against labor income risk. In our model, local equity has a similar hedging
property—but that property is driven by physical investment shocks (without requiring price stickiness).

A key contribution of the paper here is to explore the \textit{quantitative} implications of the model regarding
the \textit{dynamics} of external asset positions and international capital flows. Gourinchas and Rey (2005), Tille
(2005) and Lane and Milesi-Ferretti (2006) document empirically that fluctuations in the value of domestic
and foreign assets induce external capital gains/losses that have a substantial effect on countries’ net
foreign asset positions (NFA). We show that the present model generates sizable international valuation
effects. Here, fluctuations in a country’s NFA are driven by asset price changes—NFA is thus predicted
to have the time series properties of asset prices; in particular, the first difference of a country’s NFA
is predicted to be highly volatile and to have low serial correlation. We show that these predictions are

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consistent with the data. When there is a positive TFP or investment efficiency shock, net imports are predicted to rise on impact (due to a strong short run rise in investment), and to fall thereafter. As NFA equals the present value of current and future net imports, the NFA drops, on impact. Thus, the change in NFA is predicted to be countercyclical, which is likewise consistent with the data. Finally, the model generates sizable asset trades.\footnote{For other related recent empirical and theoretical analysis of international portfolios and external valuation effects, see e.g. Lewis (1999), Han and Rey (2004), Siourounis (2004), Kraay and Ventura (2005), Devereux and Saito (2005), Ghironi, Lee and Rebucci (2005), Obstfeld (2006), Kollmann (2006a), and Matsumoto (2007). Evans and Hnatkovska (2005,2007) and Hnatkovska (2005) also discuss a world with capital accumulation and portfolios; those papers do not analyze the hedging logic that is central to our paper, and have a different empirical focus. Cantor and Mark (1988) provided an early theoretical discussion of the role of equity price changes for current accounts, based on a one-good model with equities trade (their model predicts full portfolio diversification).}

We also show that our model has several interesting business cycle features. The investment efficiency shocks assumed here generate net exports and real exchange rate volatility that is larger—and thus closer to the data—than the volatility induced by TFP shocks. In the model here, a positive shock to a country’s TFP raises that country’s output while worsening its terms of trade; a country-specific shock to investment efficiency likewise raises output, but (on impact) it improves the terms of trade (the shock raises investment spending which is biased towards local inputs; hence it raises the relative price of those inputs). As a result, with the combined two types of shock, the terms of trade (and the real exchange rate) are less cyclical than in standard RBC models that are driven just by TFP shocks (e.g., Backus, Kehoe and Kydland (1994)). The presence of investment efficiency shocks also generates a cross-country correlation of consumption that is lower, and closer to the low correlations seen in the data. However, investment efficiency shocks generate cross-country correlations of investment, and within country correlations between investment and consumption that are too small when compared to the data.\footnote{In simultaneous and independent work, Raffo (2008) also studies the effect of investment efficiency shocks on international business cycles.}

The paper is structured as follows. In section 2, we present the model set-up. In section 3, we derive equilibrium equity and bond portfolios, and we provide empirical support for the key condition that drives equity home bias in the model. In section 4, we provide stylized facts on the dynamics of external asset positions in G7 countries; we present simulation results that show that the model quantitatively captures key dynamic stylized facts.

\footnote{In simultaneous and independent work, Raffo (2008) also studies the effect of investment efficiency shocks on international business cycles.}
2 The model

There are two (ex-ante) symmetric countries, Home (H) and Foreign (F), each with a representative household. Country \(i = H, F\) produces one good using labor and capital. There is trade in goods and in financial assets (stocks and bonds). All markets are perfectly competitive.

2.1 Preferences

Country \(i\) is inhabited by a representative household who lives in periods \(t = 0, 1, 2, \ldots\). The household has the following life-time utility function:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \frac{l_{i,t}^{1+\omega}}{1+\omega} \right),
\]

with \(\omega > 0\). \(C_{i,t}\) is \(i\)'s aggregate consumption in period \(t\) and \(l_{i,t}\) is labor effort. Like much of the macroeconomics and finance literature, we take the coefficient of relative risk aversion to be greater than unity: \(\sigma > 1\).

\(C_{i,t}\) is a composite good given by:

\[
C_{i,t} = a^{1/\phi} (c_{i,t}^i)^{(\phi-1)/\phi} + (1-a)^{1/\phi} (c_{i,t}^j)^{(\phi-1)/\phi} \right)^{\phi/(\phi-1)}, \text{ with } j \neq i,
\]

where \(c_{i,t}^j\) is country \(i\)'s consumption of the good produced by country \(j\) at date \(t\). \(\phi > 0\) is the elasticity of substitution between the two goods. In the (symmetric) deterministic steady state, \(a\) is the share of consumption spending devoted to the local good. We assume a preference bias for local goods, \(1/2 < a < 1\).

The welfare based consumer price index that corresponds to these preferences is:

\[
P_{i,t} = a (p_{i,t})^{1-\phi} + (1-a) (p_{j,t})^{1-\phi} \right]^{1/(1-\phi)}, \text{ with } j \neq i,
\]

where \(p_{i,t}\) is the price of good \(i\).

2.2 Technologies and firms

In period \(t\), country \(i\) produces \(y_{i,t}\) units of good \(i\) according to the production function

\[
y_{i,t} = \theta_{i,t} (k_{i,t})^\kappa (l_{i,t})^{1-\kappa},
\]
with $0 < \kappa < 1$. $k_{i,t}$ is the country’s stock of capital. Total factor productivity (TFP) $\theta_{i,t} > 0$ is an exogenous random variable. The law of motion of the capital stock is:

$$k_{i,t+1} = (1 - \delta)k_{i,t} + \chi_{i,t}I_{i,t}$$  \hspace{1cm} (5)

where $0 < \delta < 1$ is the depreciation rate of capital. $I_{i,t}$ is gross investment in country $i$ at date $t$. $\chi_{i,t} > 0$ is an exogenous shock to investment efficiency (see Fisher (2002, 2006), Greenwood, Hercowitz and Krusell (1997), Justiniano, Primiceri and Tambalotti (2007)). The stochastic properties of the exogenous shocks are symmetric across countries.

In both countries, gross investment is generated using Home and Foreign inputs:

$$I_{i,t} = \left[ a_I^{1/\phi_i} \left( i_{i,t}^j \right)^{(\phi_j - 1)/\phi_j} + (1 - a_I)^{1/\phi_i} \left( i_{j,t}^i \right)^{(\phi_j - 1)/\phi_j} \right]^{\phi_j/(\phi_j - 1)}, \ j \neq i,$$  \hspace{1cm} (6)

where $i_{j,t}^i$ is the amount of good $j$ used for investment in country $i$. We assume a local bias in investment spending, $\frac{1}{2} < a_I < 1$. Home bias and the substitution elasticity between domestic and imported inputs may be different for investment and consumption (i.e. we allow for the possibility that $a_I \neq a, \phi_I \neq \phi$). The associated investment price index is:

$$P_{i,t}^I = \left[ a_I (p_{i,t})^{1-\phi_i} + (1 - a_I) (p_{j,t})^{1-\phi_j} \right]^{1/(1-\phi_i)}, \ j \neq i.$$  \hspace{1cm} (7)

There is a (representative) firm in country $i$ that hires local labor, accumulates physical capital and produces output, using the technology (4),(5); it maximizes the present value of dividend payments, taking prices and wage rates as given.

Due to the Cobb-Douglas technology, a share $1 - \kappa$ of output is paid to workers. Thus, the country $i$ wage bill is:

$$w_{i,t}l_{i,t} = (1 - \kappa)p_{i,t}y_{i,t},$$  \hspace{1cm} (8)

where $p_{i,t}$ is the price of the country $i$ good and $w_{i,t}$ is the country $i$ wage rate.

For simplicity, we consider a baseline model specification in which investment is financed out of retained earning; a share $\kappa$ of the country’s output, net of physical investment spending is thus paid out as a dividend $d_{i,t}$ to shareholders:

$$d_{i,t} = \kappa p_{i,t}y_{i,t} - P_{i,t}^I I_{i,t},$$  \hspace{1cm} (9)
Below, we also discuss a model variant in which firms issue debt to finance investment spending; the household’s equilibrium equity portfolio in that variant is the same as in the baseline specification.

The firm chooses \( I_{i,t} \) to equate the expected future marginal gain of investment to the marginal cost. This implies the following first-order condition:

\[
1 = E_t \left[ \frac{\chi_{i,t}}{P_{i,t}^t} \left[ p_{i,t+1} \theta_{i,t+1} \kappa k_{i,t+1}^{\kappa - 1} I_{i,t+1}^{1-\kappa} + (1 - \delta) \frac{p_{i,t+1}^I}{X_{i,t+1}} \right] \right],
\]

(10)

where \( \phi_{i,t+1} \equiv \beta(C_{i,t+1}/C_{i,t})^{-\sigma} (P_{i,t}/P_{i,t+1}) \) is a pricing kernel used at date \( t \) to value date \( t+1 \) payoffs. Note that we assume that \( \phi_{i,t+1} \) equals the intertemporal marginal rate of substitution of the country \( i \) household.\(^9\) The firm chooses the Home and Foreign investment inputs \( i_{H,t}^i, i_{F,t}^j \) that minimize the cost of generating \( I_{i,t} \). That cost minimization problem has the following first-order conditions:

\[
i_{i,t}^i = a_I \left( \frac{p_{i,t}}{P_{i,t}^t} \right)^{-\phi_i} I_{i,t}, \quad i_{j,t}^i = (1 - a_I) \left( \frac{p_{j,t}}{P_{i,t}^t} \right)^{-\phi_i} I_{i,t}, \quad j \neq i.
\]

(11)

2.3 Financial markets, household decisions, market clearing

There is international trade in stocks and bonds. The country \( i \) firm issues a stock that represents a claim to its stream of cash-flows \( \{d_{i,t}\} \). The supply of each share is normalized at unity. There is a bond denominated in the Home good, and a bond denominated in the Foreign good; buying one unit of the Home (Foreign) bond in period \( t \) gives one unit of the Home (Foreign) good in all future periods. Both bonds are in zero net supply. Each household fully owns the local stock, at birth, and has zero initial foreign assets.\(^10\) Let \( S_{j,t+1} \) denote the number of shares of stock \( j \) held by the country \( i \) household at the end of period \( t \), while \( b_{j,t+1}^i \) represents claims held by \( i \) (at the end of \( t \)) to future unconditional payments of good \( j \). At date \( t \), the country \( i \) household faces the following budget constraint:

\[
P_{i,t} C_{i,t} + p_{i,t}^S S_{i,t+1}^i + p_{j,t}^S S_{j,t+1}^j + p_{i,t}^b b_{i,t+1}^i + p_{j,t}^b b_{j,t+1}^j
\]

\[
= w_i i_{i,t} + (p_{i,t}^S + d_{i,t}) S_{i,t}^i + (p_{j,t}^S + d_{j,t}) S_{j,t}^j + (p_{i,t}^b + p_{i,t}) b_{i,t}^i + (p_{j,t}^b + p_{j,t}) b_{j,t}^j, \quad j \neq i,
\]

(12)

\(^9\) When the Home and Foreign households’ Euler equations for Home/Foreign stocks shown below hold (see (14)), then (10) holds also for a pricing kernel that equals the intertemporal marginal rate of substitution of the country \( j \neq i \) household.

\(^10\) We also assume that initial capital stocks and productivities are identical across countries: \( K_{H,0} = K_{F,0}, \theta_{H,0} = \theta_{F,0} \), and \( \chi_{H,0} = \chi_{F,0} \). This ensures that both countries have equal wealth at birth, and preserves the (ex ante) symmetry of the two countries.
where $p_{i,t}^S$ is the price of stock $i$ and $p_{i,t}^b$ is the price of the good-$i$ bond.

Each household selects portfolios, consumptions and labor supplies that maximize her life-time utility (1) subject to her budget constraint (12) for $t \geq 0$. Ruling out Ponzi-schemes, the following equations are first-order conditions of household $i$’s decision problem:

$$c_{i,t} = a \left( \frac{p_{i,t}}{P_{t,t}} \right)^{-\phi} C_{i,t}, \quad c_{j,t} = (1-a) \left( \frac{p_{j,t}}{P_{t,t}} \right)^{-\phi} C_{i,t}, \quad l_{i,t} = \left( \frac{w_{i,t}}{P_{t,t}} \right) C_{i,t}^{-\sigma}$$

$$1 = E_{t} \ddot{q}_{i,t+1} R_{j,t+1}^S, \quad 1 = E_{t} \ddot{q}_{i,t+1} R_{j,t+1}^b \quad \text{for } j = H, F,$$

with $R_{j,t+1}^S = \frac{p_{j,t+1}^S + d_{j,t+1}^j}{p_{j,t}^S}$, $R_{j,t+1}^b = \frac{p_{j,t+1}^b + p_{b,t+1}^j}{p_{j,t}^b}$.

$R_{j,t+1}^S$ and $R_{j,t+1}^b$ are the gross returns of stock $j$, and of the good-$j$ bond, respectively (between periods $t$ and $t+1$). (13) represents the optimal allocation of consumption spending across goods, and the labor supply decision. (14) shows Euler equations with respect to the two stocks and the Home and Foreign good bonds.

**Market-clearing** in goods and asset markets requires:

$$c_{H,t} + c_{F,t} + i_{H,t}^H + i_{F,t}^F = y_{H,t}, \quad c_{F,t} + c_{H,t} + i_{H,t}^F + i_{F,t}^H = y_{F,t},$$

$$S_{H,t}^H + S_{F,t}^F = S_{F,t}^H + S_{H,t}^H = 1, \quad b_{H,t}^H + b_{F,t}^F = b_{F,t}^H + b_{H,t}^H = 0.$$  

**2.4 Relative consumption and investment demand**

Subsequent discussions will use the following properties of consumption and investment demand. The first-order condition for consumption (13) implies:

$$c_{H,t}^{H} + c_{H,t}^{F} = p_{H,t}^{-\phi} \left[ aC_{H,t} P_{H,t}^{\phi} + (1-a) C_{F,t} P_{F,t}^{\phi} \right], \quad c_{F,t}^{H} + c_{F,t}^{F} = p_{F,t}^{-\phi} \left[ aC_{F,t} P_{F,t}^{\phi} + (1-a) C_{H,t} P_{H,t}^{\phi} \right]$$

Taking the ratio of these expressions gives:

$$y_{C,t} = \frac{c_{H,t}^{H} + c_{H,t}^{F}}{c_{F,t}^{H} + c_{F,t}^{F}} = q_t^{-\phi} \Omega_a \left[ \frac{P_{F,t}}{P_{H,t}} \right]^{\phi} C_{F,t}^{H}, \quad \text{with } \Omega_a(x) = \frac{1 + x(\frac{1}{x} - \phi)}{x + (\frac{1}{x} - \phi)}.$$  

$y_{C,t}$ is the ratio of world consumption of Home goods over world consumption of Foreign goods, while $q_t = p_{H,t}/p_{F,t}$ denotes the country H terms of trade.
The ratio of world demand for Home vs. Foreign goods used for physical investment \( y_{I,t} \) can similarly be expressed as:

\[
y_{I,t} \equiv q^t_i \Omega_{I,t} \left[ \left( \frac{P_{F,t}}{P_{H,t}} \right) \phi^t_i \frac{I_{F,t}}{I_{H,t}} \right]. \tag{19}
\]

### 3 Characterization of (steady state) equilibrium portfolios

Equilibrium portfolio holdings chosen at date \( t \) \((S^i_{i,t+1}, S^j_{j,t+1}, b^i_{i,t+1}, b^j_{j,t+1})\) are functions of predetermined state variables, and of exogenous shocks at \( t \). Devereux and Sutherland (2006a,b) show how to compute Taylor expansion of the portfolio decision rules, in the neighborhood of the deterministic steady state. In this Section, we provide closed form solutions for the Home/Foreign 'zero-order portfolio' (denoted by variables without time subscripts) \( S^i_{i}, S^j_{j}, b^i_{i}, b^j_{j} \), i.e. portfolio decision rules evaluated at steady state values of state variables. That portfolio can be determined by linearizing the model around its steady state.\(^1\)

#### 3.1 Linearization of the model

Henceforth, variables without a time subscript refer to the steady state. \( \tilde{z}_t \equiv (z_t - z) / z \) denotes the relative deviation of a variable \( z_t \) from its steady state value \( z \).

Below we find a zero-order portfolio such that the ratio of Home to Foreign marginal utilities of aggregate consumption, \( C_{H,t}^{\sigma} / C_{F,t}^{\sigma} \), is equated to the consumption-based real exchange rate, \( RER_t \equiv \frac{P_{H,t}}{P_{F,t}} \), up to first order:

\[
-\sigma(\tilde{C}_{H,t} - \tilde{C}_{F,t}) = \tilde{RER}_t. \tag{20}
\]

This is a linearized version of a risk sharing condition that holds under complete markets (Backus and Smith (1993), Kollmann (1991, 1995)). Up to first order, the asset structure here (four assets, in a

\(^1\)Devereux and Sutherland (2006a,b) show that the zero-order equilibrium portfolio has to satisfy a second-order accurate approximation of household Euler equations, expressed in ‘relative’ form: \( 0 = E_t \tilde{q}_{t+1} \tilde{r}^{X}_{t+1} \), where \( \tilde{q}_{t+1} \equiv q_{t+1}^{H} - q_{t+1}^{F} \) is the ‘relative’ IMRS of the two households, while \( \tilde{r}^{X}_{t+1} \equiv (R_{H,t+1}^{S} - R_{H,t+1}^{B}, R_{F,t+1}^{S} - R_{F,t+1}^{B}, R_{H,t+1}^{W} - R_{H,t+1}^{B}, R_{F,t+1}^{W} - R_{F,t+1}^{B}) \) is a vector of excess returns. As \( \tilde{q} = \tilde{r}^{X} = 0 \) in steady state, a second-order accurate approximation is given by \( 0 = E_t (\tilde{q}_{t+1})^{(1)} (\tilde{r}^{X}_{t+1})^{(1)} \), where \((\tilde{q}_{t+1})^{(1)}\) and \((\tilde{r}^{X}_{t+1})^{(1)}\) are first-order accurate. The zero-order portfolio discusses below satisfies a linearized risk sharing condition (see (20)) that entails that \((\tilde{q}_{t+1})^{(1)} = 0\); thus, the zero-order portfolio discussed below ensures that \( 0 = E_t (\tilde{q}_{t+1})^{(1)} (\tilde{r}^{X}_{t+1})^{(1)} \) holds to second-order.
world with four exogenous shocks) is thus (effectively) complete.\footnote{Using the apparatus of Devereux and Sutherland (2006a,b) we confirmed for the model calibration below (and for all of many other calibrations with which we experimented) that the zero-order equilibrium portfolio is unique; there is no zero-order equilibrium portfolio for which the risk sharing condition (20) does not hold, to first order.}

It follows from the definition of Home and Foreign CPI indices (see (3)) that

\[ \overline{RER}_t = \overline{P}_{H,t} - \overline{P}_{F,t} = (2a - 1) \tilde{q}_t. \] (21)

Due to consumption home bias \((a > \frac{1}{2})\), an improvement of the Home terms of trade generates an appreciation of the Home real exchange rate.

When (20) holds, then the relative world consumption demand for the Home good obeys (from (18)):

\[ \overline{y}_{C,t} = -\left[ \phi \left( 1 - 2a \right)^2 (2a - 1)^2 \frac{1}{\sigma} \right] \tilde{q}_t \equiv -\lambda \tilde{q}_t \] (22)

where \(\lambda \equiv \phi(1 - (2a - 1)^2) + \frac{(2a - 1)^2}{\sigma} \). Note that \(\lambda > 0\) (as \(1/2 < a < 1\) implies \(0 < 1 - (2a - 1)^2\)). Thus, an improvement in the Home terms of trade lowers worldwide relative consumption of the Home good.

Linearization of (19) and (7) shows that relative world investment demand for the Home good, \(y_{I,t}\), obeys:

\[ \overline{y}_{I,t} = -\phi_t \left( 1 - 2a_I \right)^2 \tilde{q}_t + (2a_I - 1) \hat{I}_t, \] (23)

where \(I_t \equiv I_{H,t}/I_{F,t}\) is relative real aggregate investment. Holding constant the terms of trade, the relative demand for Home investment goods, \(y_{I,t}\), increases with relative real investment in the Home country, \(I_t\), since Home aggregate investment is biased towards the Home good \((a_I > \frac{1}{2})\).

The market clearing condition for goods (16) implies:

\[ (1 - \Lambda) \overline{y}_{C,t} + \Lambda \overline{y}_{I,t} = \overline{y}_t, \] (24)

where \(y_t \equiv Y_{H,t}/Y_{F,t}\) is relative Home output, while \(\Lambda \equiv \frac{P^H_{I,H}}{P_{W,Y_H}} = \frac{P^F_{I,F}}{P_{W,Y_F}}\) is the steady state investment/GDP ratio.\footnote{Note that, because of symmetry, \(P^H_{I,H}/P_H = P^F_{I,F}/P_F = 1, I_H = I_F, y_H = y_F.\)}

Substituting (22) and (23) into (24) gives:

\[ \overline{y}_t = -\lambda^* \tilde{q}_t + \Lambda (2a_I - 1) \hat{I}_t \] (25)
where \( \lambda^* = (1 - \Lambda)\lambda + \Lambda \phi_1 \left(1 - (2a_I - 1)^2\right) > 0. \)

Not surprisingly, Home terms of trade worsen when the relative supply of Home goods increases, for a given amount of relative Home country investment. Home terms of trade improve when Home investment rises (due to local bias in investment spending), for a given value of the relative Home/Foreign output.

### 3.2 Zero-order portfolios

Ex-ante symmetry implies that the zero-order portfolios have to satisfy these conditions: 
\[
S \equiv S_H^H = S_F^H = 1 - S_H^F = 1 - S_F^F; \quad b \equiv b_H^H = b_F^H = -b_H^F = -b_F^F.
\]

The pair \((S, b)\) thus describes the (zero-order) equilibrium portfolio. Note that \(S\) denotes a country’s holdings of local stock, while \(b\) denotes its holdings of bonds denominated in the local good. There is equity home bias when \(S > \frac{1}{2} \); \(b > 0\) means that a country is long in local-good bonds (and short in foreign-good bonds).

We now show that there exists a unique portfolio \((S, b)\) that satisfies the following 'static' budget constraint, for consumptions that are consistent with the linearized risk sharing condition (20):

\[
P_{i,t}c_{i,t} = w_{i,t}l_{i,t} + Sd_{i,t} + (1 - S)d_{j,t} + b(p_{i,t} - p_{j,t}), \quad \text{for } i = H, F. \tag{26}
\]

According to this constraint, country \(i\) consumption spending at date \(t\) equals date \(t\) wage income, \(w_{i,t}l_{i,t}\), plus the financial income generated by the zero-order portfolio \((S, b)\). We show in Appendix A.1 that when this 'static' budget constraint holds, then the period-by-period budget constraint (12) is likewise satisfied, up to first-order. We here focus on the 'static' budget constraint, as it greatly simplifies the analysis.

Subtracting the 'static' budget constraint of country \(F\) from that of country \(H\) gives:

\[
PH_{i,t}C_{H,i} - PF_{F,i}C_{F,i} = (w_{H,t}l_{H,t} - w_{F,t}l_{F,t}) + (2S - 1)(d_{H,i} - d_{F,i}) + 2b(p_{H,t} - p_{F,t}) \tag{27}
\]

Linearizing this yields:

\[
(1 - \Lambda)(\overline{PH_{i,t}C_{H,i} - PF_{F,i}C_{F,i}}) = (1 - \Lambda)(1 - \frac{1}{\sigma}(2a - 1)\overline{\tilde{q}_t}) = (1 - \kappa)\overline{w_{i}l_{i}} + (2S - 1)(\kappa - \Lambda)\overline{\tilde{d}_i} + 2\overline{b\tilde{q}_t}, \quad \overline{\tilde{b}} \equiv b/y_H, \tag{28}
\]

where \(\overline{w_{i}l_{i}} \equiv w_{H,i}l_{H,i} - w_{F,i}l_{F,i}\) denotes relative Home labor income, while \(\overline{\tilde{d}_i} \equiv d_{H,i} - d_{F,i}\) is the relative Home dividend, and \(\overline{\tilde{b}}\) represents holdings of local-good bonds, divided by steady state GDP.

---

\(^{14}\)When \(\phi_1 = \phi\) and \(a_I = a\) then \(\lambda^* = \phi(1 - (2a - 1)^2) + \frac{1 - \Lambda}{\sigma}(2a - 1)^2\).
The first equality in (28) follows from the linearized risk-sharing condition (20); it shows the efficient reaction of relative consumption spending to a change of the welfare based real exchange rate. This reaction depends on the coefficient of relative risk aversion. A shock that appreciates the real exchange rate of country $H$, induces an increase in country $H$ relative consumption spending when $\sigma > 1$ (as assumed here). (20) shows that when the Home real exchange rate appreciates by $1\%$, then relative aggregate country $H$ consumption $\left(\frac{C_H}{C_F}\right)$ decreases by $1/\sigma \%$. Hence, relative country $H$ consumption spending $\left(\frac{P_HC_H}{P_FC_F}\right)$ increases by $(1 - \frac{1}{\sigma})\%$.

The expression to the right in (28) shows the change in country $H$ income (relative to the income of $F$) necessary to finance the efficient consumption (up to first order). Given $\sigma > 1$, the efficient portfolio has to be such that a real appreciation is associated with an increase in relative Home income.

Since labor income is a constant share of output (see (8)), relative labor income $(\hat{w}_H)$ is given by:

$$\hat{w}_H = \hat{q}_t + \hat{y}_t.$$ (9) and (7) imply that the relative dividend $(\hat{d}_t)$ is given by:

$$(\kappa - \Lambda)\hat{d}_t = \kappa(\hat{q}_t + \hat{y}_t) - \Lambda(P^f_{H,t}I_{H,t} - P^f_{F,t}I_{F,t}) = \kappa(\hat{q}_t + \hat{y}_t) - \Lambda((2a_I - 1)\hat{q}_t + \hat{I}_t).$$ (29)

Substituting (29) into (28) gives:

$$(1 - \Lambda)(1 - \frac{1}{\sigma}) (2a - 1) \hat{q}_t = (1 - \sigma)(\hat{q}_t + \hat{y}_t) + (2S - 1) \{\kappa(\hat{q}_t + \hat{y}_t) - \Lambda((2a_I - 1)\hat{q}_t + \hat{I}_t)\} + 2b\hat{q}_t$$ (30)

Using (25), we can express (30) as:

$$(1 - \Lambda)(1 - \frac{1}{\sigma}) (2a - 1) \hat{q}_t = [(1 - \kappa + \kappa(2S - 1))((1 - \Lambda^*)\hat{q}_t + \Lambda(2a_I - 1)\hat{I}_t) - \Lambda(2S - 1) [(2a_I - 1)\hat{q}_t + \hat{I}_t]] + 2\bar{b}\hat{q}_t$$ (31)

The asset structure supports the full risk sharing condition (20), up to first-order, if (31) holds for all realizations of the two (relative) exogenous shocks $(\hat{q}_t, \hat{I}_t)$. The following portfolio $(S, \bar{b})$ ensures that (31) holds for arbitrary realizations of $(\hat{q}_t, \hat{I}_t)$:

$$S = \frac{1}{2} \left[1 + \frac{(2a_I - 1)(1 - \kappa)}{1 - (2a_I - 1)\kappa}\right] > \frac{1}{2}$$ (32)

$$\bar{b} = \frac{1}{2} \left[(1 - \Lambda)(1 - \frac{1}{\sigma})(2a - 1) + \frac{(1 - \kappa)[\Lambda^* - 1] + \Lambda(2a_I - 1)^2)}{1 - (2a_I - 1)\kappa}\right]$$ (33)

Thus, the model generates equity home bias: $1/2 < S < 1$. Interestingly, the equity portfolio is independent of preference parameters; in particular, $S$ is independent of the substitution elasticity between
Home and Foreign goods, and thus of the strength of the response of the terms of trade to shocks.\(^{15}\) The equity portfolio solely depends on the local bias in investment spending \((a_I)\) and on the capital share \((\kappa)\); equity home bias is increasing in the local spending bias—this prediction is strongly supported by the data (see Heathcote and Perri (2007) and Collard, Dellas, Diba and Stockman (2007)).

The persistence of shocks and their correlation do not matter for the (zero-order) equilibrium portfolio (as long as the shocks are not perfectly correlated). Note that, to solve for the equilibrium portfolio, we do not have to solve for output and investment, as a unique pair of terms of trade and relative real investment \((q_t, \tilde{I}_t)\) is associated with each realization of \((\tilde{\theta}_t, \tilde{\kappa}_t)\). In fact, any other combinations of two types of relative (Home vs. Foreign) shocks that only affect the (linearized) relative budget constraint through their effect on the terms of trade and relative investment generates the same equilibrium portfolio—other potential shocks that would generate the same portfolio are e.g. labor supply shocks, "news" shocks regarding future TFP or investment efficiency, and shocks to the depreciation rate of capital.

Note also that, in contrast to the setting here (with trade in stocks and bonds), general equilibrium models with just trade in stocks (no bonds) predict that the equity portfolio exhibits strong sensitivity with respect to the substitution elasticity between local and imported goods (e.g., Kollmann (2006b), Coeurdacier (2009) and Heathcote and Perri (2007)).\(^{16}\)

In the model here, the bond portfolio does depend on the substitution elasticities \(\phi, \phi_I\) (via \(\lambda^*\)) and on risk aversion \((\sigma)\); however this dependence is 'smooth': in particular, the net local-good bond position \(\tilde{b}\) is a linear function of \(\phi\) and \(\phi_I\).\(^{17}\) Depending on preference parameters, the model can generate a positive or negative value of \(\tilde{b}\). The country will go short in the local-good bond \((\tilde{b} < 0)\) when \(\phi\) is sufficiently low (roughly below unity). When \(\phi\) is low, then the terms of trade respond strongly to shocks; an improvement in the Home terms of trade (induced by a fall in Home TFP and/or an increase in Home investment efficiency) increases Home relative wage plus dividend income (due to the strong terms of trade change); risk sharing requires to compensate this relative income effect by shorting the local good bond (when \(\tilde{b} < 0)\, a\ terms\ of\ trade\ improvement\ lowers\ the\ net\ bond\ income\ received\ by\ Home).\)

\(^{15}\) However it is necessary that good are imperfect substitutes so that the terms of trade show a non-zero response to shocks.

\(^{16}\) In those models, the asset structure cannot support the efficient allocation when Home and Foreign dividends are co-linear, which occurs for a value of the substitution elasticity roughly located between 1 and 2; for substitution elasticities just below or above the critical value, the local equity share takes extremely large positive or negative values.

\(^{17}\) See Coeurdacier, Kollmann and Martin (2007) and Coeurdacier and Gourinchas (2008) for a similar result.
contrast, the country goes long in the local good bond, and short in the foreign good bond, when \( \phi \) is (roughly) greater than unity.

Recent empirical research (Lane and Shambaugh (2007, 2008)) shows that, on average, the advanced countries have negative net foreign-currency debt positions—which is consistent with our model for values of \( \phi \) above unity. However, there is a great deal of cross-country heterogeneity in net foreign/domestic currency debt positions. While net debt positions are small for most advanced countries, some major countries have large negative net domestic-currency debt positions; most notably this is the case for the US. In our theoretical framework, negative net local-good debt positions obtain for a wider range of parameters, if one allows for corporate debt (see below). Finally, one should note that countries can easily alter the effective currency composition of their debt portfolio by taking net positions in the forward currency market; this further complicates the comparison of our theoretical bond positions with their empirical counterparts.

**Debt and Equity Financing**

We have assumed so far that firms are fully financed through equity, and that investment is fully financed through retained earnings. In Appendix A.3, we discuss a model variant in which firms are partly financed through debt. Since the Modigliani-Miller theorem applies in the structure here, corporate debt does not affect the value of firms, physical investment and the equilibrium consumption allocation. As shown in the Appendix, the equilibrium equity portfolio is likewise unaffected by the presence of corporate bonds (i.e. \( S \) continues to be given by (32)). The country i household holds a fraction \( S \) of the corporate debt issued by the local firm, in order to offset the implicit debt position entailed by the household’s local equity position; thus households exhibit home bias for corporate debt, in the same proportion as for stocks. If firms mainly issue local-good debt, this lowers the countries’ overall (household+corporate) net local-good bond position: when the local firm issues one unit of debt denominated in the local good, then the overall net local debt position changes by \( S - 1 < 0 \) units, as a share \( S \) of the new debt will be purchased by the local household, while a share \( 1 - S \) will be bought by the foreign household.

**The hedging roles of bonds and stocks**

We now show that the bond portfolio hedges terms of trade (real exchange rate) risk—as preference parameters affect the response of relative consumption to terms of trade changes, bond holdings depend on
those preference parameters. Equities are used to hedge fluctuations in relative wages and dividends that are orthogonal to the terms of trade. The comovement of relative wages and dividends, at constant terms of trade, depends on $a_I$ and $\kappa$, but not on preference parameters—which explains why the equilibrium value of $S$ is a function of $a_I$ and $\kappa$.\footnote{Coeurdacier and Gourinchas (2008) provide a general discussion of conditions under which international equity portfolios are independent of preferences; they show that an important condition is that bonds exist whose pay-offs perfectly track real exchange rate movements.}

Assume a combination of exogenous shocks ($\tilde{\theta}_t, \tilde{x}_t$) that raises relative country $H$ real investment spending, without altering the terms of trade: $\tilde{I}_t > 0$, $\tilde{q}_t = 0$. From (25), we know that this combination of shocks raises $H$ relative output $\tilde{y}_t$, due to local bias in investment spending ($a_I > 1/2$): $\tilde{y}_t = \Lambda(2a_I - 1)\tilde{I}_t > 0$, when $\tilde{q}_t = 0$. As the real exchange rate is unaffected when $\tilde{q}_t = 0$ (see (21)), efficient risk sharing requires that countries’ relative consumption spending remains unchanged. Hence, the efficient portfolio has to be such that the countries’ relative income too is unaffected. From (30) it can be seen that this requires that:

$$0 = (1 - \kappa)\tilde{y}_t + (2S - 1)\{\kappa\tilde{y}_t - \Lambda\tilde{I}_t\}. \tag{34}$$

$(1 - \kappa)\tilde{y}_t$ and $\{\kappa\tilde{y}_t - \Lambda\tilde{I}_t\}$ respectively represent relative labor income of country $H$ and the relative dividend of stock $H$, for $\tilde{q}_t = 0$. Note that $\kappa\tilde{y}_t - \Lambda\tilde{I}_t = [\kappa(2a_I - 1) - 1]\Lambda\tilde{I}_t$ when $\tilde{y}_t = \Lambda(2a_I - 1)\tilde{I}_t$. Thus, $\kappa\tilde{y}_t - \Lambda\tilde{I}_t < 0$ when $\tilde{I}_t > 0$, $\tilde{q}_t = 0$. In other terms, a combination of shocks that raises $H$ relative investment without affecting the terms of trade induces a rise in $H$’s relative wage income, and a fall in the relative dividend of stock $H$. This makes holding local equity attractive: $S > 1/2$ is needed to ensure that (34) holds.\footnote{To derive the value of $S$ shown in (32), one can substitute $\tilde{y}_t = \Lambda(2a_I - 1)\tilde{I}_t$ into (34); the only value of $S$ for which the resulting expression holds for arbitrary $\tilde{I}_t$ is given by (32).}

Once shocks that do not affect the terms of trade have been hedged by holding local equity, the remaining risk (changes in output/investment that are associated with terms of trade changes) can be hedged (up to a linear approximation) using the bond portfolio; this is so because terms of trade movements are perfectly correlated with the difference between the pay-offs of Home and Foreign good bonds.

**Comparison with Heathcote and Perri (2007)**

Our equity portfolio (32) corresponds to that obtained by Heathcote and Perri (2007) [HP], for a special case of their model with a unit risk aversion coefficient ($\sigma = 1$) and a unit elasticity of substitution
between domestic and foreign good \((\phi = 1)\). HP assume a two-country world with capital accumulation, with just trade in stocks (no bonds), and just TFP shocks. In their model, the equity portfolio is sensitive to slight changes in \(\sigma\) and \(\phi\); extreme home or foreign equity bias occurs for values of \(\sigma\) and \(\phi\) in a plausible range above unity.\(^{20}\)

Here we have shown that this sensitivity of portfolio choices disappears once we allow for trade in bonds, and an additional source of uncertainty on the production side (here shock to investment efficiency). This robustness is due to the fact that, in our model, terms of trade risk is hedged by the bond portfolio. This result is important, as there is considerable uncertainty regarding the value of the substitution elasticity between domestic and foreign goods: estimates from aggregate macro data are scattered around unity, but estimates from sectoral trade data are above 4 (see Imbs and Méjean (2008) for a detailed discussion).

The reason why the HP model delivers equity home bias when \(\sigma = \phi = 1\) is that, for that parameterization, the two countries’ efficient relative consumption spending is constant, while a country’s relative wage income is (perfectly) negatively correlated with the relative dividend of the stock issued by the country \((\text{Corr}(\hat{w_t}l_t, \hat{d_t}) < 0)\), which implies that local equity is a good hedge for labor income risk. As documented below, the correlation between relative wage income \((\hat{w_t}l_t)\) and the relative dividend \((\hat{d_t})\) is positive, for G7 countries. Thus, the key mechanism that generates equity home bias in the HP model is rejected empirically.

### 3.3 The role of the correlation between relative wage incomes and relative ‘dividends’

In our model, the unconditional correlation \(\text{Corr}(\hat{w_t}l_t, \hat{d_t})\) per se is irrelevant for the equilibrium equity portfolio. What matters is the correlation between the components of \(\hat{w_t}l_t\) and \(\hat{d_t}\) that are orthogonal to the terms of trade, \(\tilde{q}_t\): there is equity home bias when that correlation is negative. To see this, project equation (28) on \(\tilde{q}_t\). This gives:

\[
(1 - \Lambda)(1 - \frac{1}{\sigma})(2a - 1)\tilde{q}_t = (1 - \kappa)P[\hat{w_t}l_t|\tilde{q}_t] + (2S - 1)(\kappa - \Lambda)P[\hat{d_t}|\tilde{q}_t] + 2b\tilde{q}_t; \quad (35)
\]

\(^{20}\)Castello (2007) considers a model of portfolio choice with capital accumulation close to HP; in her model too, equity portfolios are highly sensitive to preference parameters.
where $P[w_t I_t | \hat{q}_t]$ is the (linear) projection of $w_t I_t$ on $\hat{q}_t$. (NB $\hat{q}_t = P[\hat{q}_t | \hat{q}_t]$.) Subtracting (35) from (28) gives:

$$0 = (1 - \kappa)\{w_t I_t - P[w_t I_t | \hat{q}_t]\} + (2S - 1) (\kappa - \Lambda)\{d_t - P[d_t | \hat{q}_t]\}. \tag{36}$$

Thus, the equity portfolio has to hedge the components of $w_t I_t$ and $d_t$ that are orthogonal to the terms of trade $\hat{q}_t$. (36) implies that

$$S = \frac{1}{2} - \frac{1}{2} \frac{1}{\kappa - \Lambda} \text{Cov}(w_t I_t, d_t), \tag{37}$$

with $\text{Cov}(w_t I_t, d_t) = E\{w_t I_t - P[w_t I_t | \hat{q}_t]\} \{d_t - P[d_t | \hat{q}_t]\}$, $\text{Var}(d_t) = E\{d_t - P[d_t | \hat{q}_t]\}^2$. \footnote{To see this, multiply (36) by $\{d_t - P[d_t | \hat{q}_t]\}$ and take expectations; solving the resulting equation for $S$ gives (37).} Hence there is equity home bias if and only if $\text{Cov}(w_t I_t, d_t) < 0$. \footnote{The steady state investment/GDP ratio is $\Lambda = \kappa/[(1/\delta)(1 - \beta)/\beta + 1]$, hence $\kappa > \Lambda$. This ensures that dividends are strictly positive in steady state.} In the model here, $\text{Cov}(w_t I_t, d_t) = (\kappa - \Lambda)(2a_t - 1)/[(\kappa(2a_t - 1) - 1)] < 0$. Empirically, $\text{Cov}(w_t I_t, d_t) < 0$, for G7 countries, as documented below.

A similar condition is derived by Engel and Matsumoto (2006) who show, in a model with trade in equity and nominal forward currency contracts, that the equilibrium equity position depends on the conditional covariance between wage income and dividends, conditional on the nominal exchange rate.

Note also that $(1 - \kappa)P[w_t I_t | \hat{q}_t] + (2S - 1) (\kappa - \Lambda)P[d_t | \hat{q}_t] = \gamma \hat{q}_t$ for some coefficient $\gamma$. Hence, (35) can be expressed as: $(1 - \Lambda)(1 - \frac{1}{\sigma}) (2a_t - 1) \hat{q}_t = \gamma \hat{q}_t + 2b \hat{q}_t$. The bond position is set at the value for which this condition holds for any realization of $\hat{q}_t$: $b = \frac{1}{2}[(1 - \Lambda)(1 - \frac{1}{\sigma}) (2a_t - 1) - \gamma]$. Thus, the optimal bond position ensures that terms of trade fluctuations induce movements in the two countries’ relative incomes (given the optimal equity portfolio) that track optimal relative consumption spending. For this to be the case, relative bond payments need to track the real exchange rate. In the data, domestic-versus foreign-currency bond return differentials are tightly linked to real exchange rate changes (see Coeurdacier and Gourinchas (2009) and van Wincoop and Warnock (2006)).

**Equilibrium equity portfolios for countries of different size**

In order to permit empirical analysis of the determinants of equity home bias, we now briefly consider a two-country model with countries of unequal size, due to different steady state TFP (and/or population). Assume that all preference and technology parameters are the same across countries. Then $S_t^I$ is given...
by:

\[ S_i^t = \mu_i - (1 - \mu_i) \left( 1 - \kappa \frac{Cov_{q_i}(\hat{w}_t, \hat{d}_t)}{Var_{q_i}(\hat{d}_t)} \right). \]  

(38)

were \( \mu_i \equiv p_i y_i / (p_H y_H + p_F y_F) \) is the (steady state) share of country i’s GDP in world GDP.\(^{23}\) Again, there is equity home bias \( (S_i^t > \mu_i) \) when \( Cov_{q_i}(\hat{w}_t, \hat{d}_t) < 0 \). We now show that this condition holds empirically.

**Empirical evidence on the correlation between relative wage income and relative 'dividends' in G7 countries**

For each G7 country, we obtained annual time series on aggregate nominal wage incomes of domestic households, and on profits of domestic firms.\(^{24}\) An empirical counterpart to the 'dividend' of the country \( i \) firm \( d_i \) is constructed by subtracting gross investment spending from profits. Note that the series computed this way correspond exactly to the 'dividends' assumed in the model. This variable obviously differ from actual dividends, in a world in which firms issue debt to finance investment. However this is the relevant measure for testing the theory, as dividend policy has no impact on equity portfolios in our framework, because the Modigliani-Miller theorem holds here.

We divide each country’s nominal wage income (dividend) by total wage income (dividend) in the remaining G7 countries (using nominal exchange rates). We log and linearly de-trend the resulting series to construct the relative wage income and dividend variables \( \hat{w}_t \hat{d}_t \) and \( \hat{d}_t \). All series are annual and pertain to 1984–2004. Unconditional empirical correlations \( Corr(\hat{w}_t \hat{d}_t, \hat{d}_t) \) are given in Panel (a) of Table 1. For six of the G7 countries \( Corr(\hat{w}_t \hat{d}_t, \hat{d}_t) \) is positive, and significantly different from zero.

Panels (b) and (c) of Table 1 shows estimates of the conditional correlation \( Corr_{q_i}(\hat{w}_t \hat{d}_t, \hat{d}_t) \). In the model, the terms of trade correspond to the relative price of Home-produced and Foreign-produced goods. We consider two empirical measures of the country \( i \) terms of trade: the first measure is the ratios of \( i \)'s GDP deflator to a geometric weighted average (based on GDP weights) of the GDP deflators of the remaining G7 countries (expressed in country \( i \) currency using nominal exchange rates); the second measure is \( i \)'s export price index relative to a weighted average of the export prices of the remaining G7 countries. The resulting terms of trade measures are logged and linearly detrended to construct \( \hat{q}_i \).\(^{25}\)

\(^{23}\)When \( \kappa, \delta, \beta \) are identical across countries, then the steady state investment spending/GDP ratio \( A \) is likewise identical.

\(^{24}\)Series: 'Compensation of employees' and 'Gross operating surplus and gross mixed income' from OECD Annual National Accounts. Source of other data used in this Section: OECD Annual National Accounts and International Financial Statistics.

\(^{25}\)Note that in the model here, the terms of trade and the real exchange rate are perfectly positively correlated. Mendoza
For each country, we regressed its relative wage income \( \hat{w}_t \) and dividend \( \hat{d}_t \) on \( \hat{q}_t \); the correlation between the residuals of those regressions is our estimate of \( \text{Corr}_{\hat{q}}(\hat{w}_t, \hat{d}_t) \), for that country. For all countries, the two (detrended) terms of trade measures are highly positively correlated (average correlation: 0.86), and the implied values of \( \text{Corr}_{\hat{q}}(\hat{w}_t, \hat{d}_t) \) are thus mostly similar across the two terms of trade measures. Panel (b) of Table 1 shows that, when the first terms of trade measure is used, the conditional correlation \( \text{Corr}_{\hat{q}}(\hat{w}_t, \hat{d}_t) \) is negative, for all G7 countries. The second terms of trade measure yields \( \text{Corr}_{\hat{q}}(\hat{w}_t, \hat{d}_t) < 0 \) for all G7 countries, with the exception of Italy (see Panel (c)).

**Implied equity portfolios**

Across G7 countries, the average capital share is \( \kappa = 0.4 \); the average ratio of gross physical investment spending to GDP is \( \Lambda = 0.22 \). The mean values (1984-2004) of the G7 countries’s shares in total G7 GDP are: 0.44 (US), 0.19 (Japan), 0.11 (Germany), 0.08 (France), 0.06 (UK), 0.06 (Italy) and 0.04 (Canada), respectively. Using these values for \( \kappa, \Lambda \) and \( \mu_i \), and estimates of \( \text{Cov}_{\hat{q}}(\hat{w}_t, \hat{d}_t)/\text{Var}_{\hat{q}}(\hat{d}_t) \), we compute locally held equity shares \( S^i \) predicted by the model (see (38)). The results are likewise shown in Table 1. The predicted degrees of equity home bias \( S^i - \mu_i \) are mostly sizable and highly statistically significant. For example, based on the first terms of trade measure, the equity home bias ranges between 14% (US, Canada) and 84% (Japan); the implied locally held equity share \( S^i \) ranges between 18% (Canada) and 103% (Japan).

### 3.4 Returns and the equilibrium portfolio

The preceding discussion shows that the zero-order local equity position \( S \) depends on the conditional covariance between relative (Home vs. Foreign) wage incomes and dividend payments. As shown in Appendix A.2., \( S \) can equivalently be expressed as a function of the covariance between components of relative (Home vs. Foreign) human capital returns and equity returns that are orthogonal to the return differential between the Home-good and Foreign-good bonds; equity home bias is optimal when this conditional covariance is negative.

(1995) and Obstfeld and Rogoff (2000) report high positive correlations between real exchange rates and the terms of trade, for G7 countries. Below, we use a CPI based real exchange rate as our empirical RER measure. That RER measure is highly positively correlated with our two empirical terms of trade measures (average correlation with first [second] t.o.t. measure: 0.99 [0.85]).
Looking at stock returns and estimated returns on human capital for G7 countries, Coeurdacier and Gourinchas (2009) provide evidence that the conditional covariance between the two is very different from the unconditional one.\textsuperscript{26} The former being negative when significant (for the US and Japan and to a lesser extent UK and Canada) while the latter is unambiguously positive (very much alike our estimates based on on income payments). Quantitatively, they find that for the US and Japan, the conditional covariance between returns on human capital (relative to other G7 countries) and (relative) returns on stocks is negative enough to match the observed equity home bias for these countries.\textsuperscript{27}

4 The dynamics of external financial positions

This Section describes the dynamics of the external financial positions of G7 countries; we then show that our model captures key aspects of the observed dynamics.

4.1 External position dynamics: empirical evidence

Table 2 reports standard deviations and (auto)correlations of annual financial/macroeconomic variables for the G7 countries, during the period 1984-2004. All statistics pertain to series that were detrended using the Hodrick-Prescott filter (smoothing parameter: 400). GDP, physical investment and real exchange rate series (CPI-based) were logged, before applying the filter.

The Table reports properties of the first difference (annual change) of countries’ Net Foreign Assets (NFA) at market prices; see rows labeled "ΔNFA" (NFA changes are normalized by domestic nominal GDP).\textsuperscript{28} For 6 of the G7 countries, ΔNFA is more volatile than GDP; the mean standard deviations of ΔNFA and GDP across the G7 countries are 3.23\% and 2.07\%, respectively (see last Column of Table 2).

\textsuperscript{26}Mixed evidence on the unconditional correlation between returns on human wealth and returns on stocks is provided by Baxter and Jermann (1997), Bottazzi et al. (1996), Palacios-Huerta (2001) and Julliard (2004). Note that, in a setting with trade in domestic and foreign currency bonds (as assumed here), the unconditional correlation between these returns is irrelevant for the equilibrium equity portfolio. Only the conditional correlation matters.

\textsuperscript{27}Coeurdacier and Gourinchas (2009) estimate the empirical counterpart of equation (56) in Appendix A.2. They find that the covariance of (relative) stock returns with (relative) returns on human wealth divided by the variance of stock returns (conditional on relative bond returns) is -0.26 (resp. -0.33) for the US (resp. Japan). For the Euro zone countries, results are much less conclusive. For these countries, it is possible that stock returns are not a good proxy for returns on firm value as a smaller share of capital is sold on equity markets.

\textsuperscript{28}The NFA data are from Lane and Milesi-Ferretti (2006). Unless stated otherwise, the other data in this Section are from OECD National Accounts (macroeconomic aggregates, price indices) and International Financial Statistics (exchange rates).
NFA changes are countercyclical and essentially serially uncorrelated (mean correlation with domestic GDP: -0.22; mean autocorrelation: -0.01)

As our model assumes trade in stocks and in bonds, we decompose the change of each country’s NFA into the change of its net foreign equity assets and into the change of its net foreign bond assets, at market prices (normalized by domestic GDP). Equities and bonds both contribute noticeably to NFA changes: the average standard deviations of (normalized) changes of net foreign equity assets and of net foreign bond assets are 2.97% and 2.20%, respectively. Changes in net foreign equity assets and net foreign bond assets are negatively correlated (mean correlation: -0.27). Like NFA changes, the changes of net foreign equity and net foreign bond positions have weak serial correlation; the changes of net foreign bond assets are countercyclical, while the changes of net foreign equity assets have no clear cyclical pattern.

The changes in the net foreign equity/bond assets at market prices reflect asset price (and exchange rate) changes, as well as net foreign asset acquisitions. The net foreign asset acquisition by a country, in a given period, is measured by its current account (CA). In contrast to the first difference of NFA (at market prices), the CA does not take into account external capital gains/losses (on assets acquired in the past). Table 2 reports time series properties of the CAs of the G7 countries; we also disaggregate the CAs into 'Net equity purchases' from the rest of the world and 'Net bond purchases'. (CA and net equity/bond purchases series are normalized by domestic GDP). The CAs are only about a third as volatile as the NFA change (the mean standard deviation of CA $\Delta NFA$ is 1.11% [3.23%]). Thus, NFA changes are largely driven by valuation changes. Net equity purchases (mean standard deviation: 1.38%) and net bond purchases (mean standard deviation: 1.71%) are only slightly more volatile than the CA. Net equity/bond purchases are less volatile than changes in net foreign equity/bond positions at market prices; the difference is especially noticeable for equities—which suggests that, valuation effects are more important for stocks than for bonds. Interestingly, net equity purchases are highly negatively (statistically significantly) correlated with net debt purchases (mean correlation: -0.68).

Finally, we note that net exports (normalized by GDP) are less volatile than GDP, while physical

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29 We measure a country's net equity as the sum of its net portfolio equity and net FDI positions; the net bond position is the sum of net debt and net bank loans (data from Lane and Milesi-Ferretti (2006)).
30 Data source: IMF Balance of Payments Statistics. Our Net equity purchases variable is constructed as outflows minus inflows of 'Portfolio investment equity securities' plus 'Direct investment'. Our Net debt purchases series represents outflows-inflows of 'Portfolio investment debt securities' inflows + 'Other investment'.
31 Kollmann (2006b) previously documented that CAs are less volatile than NFA changes, for a sample of 18 OECD countries. Faruquee and Lee (2007) confirm this empirical result for a sample of 100 countries.
investment and the real exchange rate (CPI-based) are more volatile than GDP (mean standard deviations of GDP, NX and RER: 2.07%, 1.14% and 8.38%, respectively). Net exports are negatively correlated with domestic GDP (in 6 of the G7 countries), while the real exchange rate has no clear cyclical pattern (mean correlation with GDP: 0.12).

4.2 External position dynamics: model predictions

We now study the predictions of the model for the dynamics of foreign asset positions, capital flows and of key macroeconomic variables. We compare these predictions to the stylized facts described in the previous section.

4.2.1 Model calibration

We adopt a model calibration that closely follows the International Real Business Cycle literature (e.g. Backus, Kehoe and Kydland (1994), Kollmann (1996, 1998)). Like Backus et al., we set the degrees of consumption and investment home bias at \( a = a_I = 0.85 \), which implies that the trade share (imports/GDP ratio) is 15% in the (deterministic) steady state.

The risk aversion coefficient, the labor supply elasticity, the substitution elasticity between domestic and foreign goods and the labor share (ratio of wage earnings to GDP) are set at \( \sigma = 2 \), \( 1/\omega = 2 \), \( \phi = \phi_I = 2 \) and \( 1 - \kappa = 0.6 \), respectively; these parameter values are well in the range of empirical parameter estimates, for G7 countries (see Coeurdacier, Kollmann and Martin (2007) for a detailed justification).

The model is calibrated to annual data. As is standard in annual macro models, we set the subjective discount factor and the depreciation rate of capital at \( \beta = 0.96 \) and \( \delta = 0.1 \), respectively. This implies that, in steady state, the return on equity is about 4.16% p.a., the capital-output ratio is 2.82, and 28% of GDP is used for investment.

We assume that the exogenous variables follow AR(1) processes:

\[
\log(\theta_{i,t}) = \rho^\theta \log(\theta_{i,t-1}) + \varepsilon_{i,t}^\theta, \quad (39)
\]

\[
\log(\chi_{i,t}) = \rho^\chi \log(\chi_{i,t-1}) + \varepsilon_{i,t}^\chi \quad \text{for } i = H, F. \quad (40)
\]
We fitted (39) to detrended annual (log) TFP series, for each G7 country (1972-2004). The estimates of $\rho^\theta$ range between 0.64 (US) and 0.80 (Canada); the mean autocorrelation (across G7 countries) is 0.75. The standard deviation of $\varepsilon^{\theta}_{i,t}$ ranges between 1.01% (France) and 1.48% (Japan), with a mean of 1.20%. TFP is positively correlated across countries; for each G7 country, we constructed a measure of 'foreign' TFP, by taking a weighted average (using GDP weights) of (log) TFP in the remaining G7 countries; we then fitted (39) to (linearly detrended) 'foreign' (log) TFP. The correlation of domestic-foreign productivity innovation ranges between 0.29 (UK) and 0.70 (Germany), with an average correlation of 0.45. We thus set $\rho^\theta = 0.75$, $Std(\varepsilon^{\theta}_{H,t}) = Std(\varepsilon^{\theta}_{F,t}) = 1.20\%$, $Corr(\varepsilon^{\theta}_{H,t},\varepsilon^{\theta}_{F,t}) = 0.45$.  

When $a = a_I$ holds, one unit of the country $i$ aggregate investment good in efficiency units is worth $1/\chi_{i,t}$ units of the aggregate consumption good in that country. The literature on investment specific technology shocks has used the ratio of the CPI to the price deflator of investment spending as an estimate of investment specific technology shocks (see Fisher (2006)). We follow that literature. For each G7 country, we computed annual time series of $\chi_{i,t} \equiv CPI/(investment$ deflator), for the period 1972-2004 (data source: OECD National Accounts). The autocorrelations of (linearly detrended) log$(\chi_{i,t})$ range between 0.54 (Japan) and 0.93 (US); the mean autocorrelation is 0.79. The standard deviations of $\varepsilon^{\chi}_{i,t}$ ranges between 1.18% (US) and 2.48% (UK), with a mean of 1.73%. Innovations to investment efficiency in country $i$ and in a rest-of-G7 aggregate are only weakly correlated (mean correlation: 0.19). Empirically, log$(\chi_{i,t})$ is thus roughly as persistent as log$(\theta_{H,t})$, but more volatile, and less correlated across countries. Based on this evidence, we set $\rho^\chi = 0.79$, $Std(\varepsilon^{\chi}_{H,t}) = Std(\varepsilon^{\chi}_{F,t}) = 1.73\%$, $Corr(\varepsilon^{\chi}_{H,t},\varepsilon^{\chi}_{F,t}) = 0.19$.

The correlations between TFP and investment efficiency innovations ($\varepsilon^{\theta}_{i,t},\varepsilon^{\chi}_{i,t}$) are close to zero (mean correlation: 0.0003). In the calibration, we thus assume that TFP and investment efficiency shocks are independent.

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32Our estimate of country i TFP (in logs) is: log(TFP$_{i,t}$) = log($y_{i,t}$) − (1 − $\kappa_i$) log($L_{i,t}$), with 1 − $\kappa_i$ : i’s mean labor share during the sample period; $L_{i,t}$ : total hours worked (from OECD Productivity Database). No capital stocks were used, due to the absence of consistent capital data in G7 countries, during sample period.

33We also estimated VARs in home and foreign TFP: log($\theta_{H,t},\theta_{F,t}$) = R(log($\theta_{H,t-1},\theta_{F,t-1}$)) + $\epsilon_t^\theta$ where $R$ is a 2x2 matrix. We find that the off-diagonal elements of $R$ are generally not statistically significant; the mean value (across G7 countries) of the off-diagonal elements is zero. The simulations thus assume univariate technology processes with innovations that are correlated across countries.

34The empirical literature on investment specific technology shocks has focused on the US. It documents a secular fall in the real price of investment goods (relative to the CPI). Our data show that a similar downward trend exists in the remaining G7 countries. In 1972-2004, the average annual rates of decline of the relative price of investment were: 0.99% (US), 0.84% (Japan), 0.52% (Germany) 0.35% (France), 0.66% (UK), 0.32% (Italy), 1.33% (Canada).
4.2.2 Numerical solution method

We numerically solve for first-order accurate prices, quantities and (time-varying) equilibrium portfolios, building on Devereux and Sutherland (2006b). 35 The numerical solution expresses portfolios held at the end of period t as a linear function of endogenous predetermined variables set at t, and of exogenous variables realized at t: 

\[ S_{i,t+1} = S_{i} + \gamma_{i}(Z_{t} - Z), \quad b_{i,t+1} = b_{i} + \gamma_{i}(Z_{t} - Z) \]

for \( i, j = H, F \), with \( Z_{t} \equiv (K_{H,t+1}, K_{F,t+1}, NFA_{H,t+1}, \theta_{H,t}, \theta_{F,t}, \chi_{H,t}, \chi_{F,t}) \), where \( NFA_{H,t} \) is the Home country net foreign asset position at the end of period t (see below). The coefficients \( \gamma_{i} \) of these linear portfolio decision rules can be computed using a third-order accurate approximation of the household Euler equations, and a second-order accurate approximation of the remaining equilibrium conditions. We use the Sims (2000) algorithm for that purpose.

4.2.3 Numerical results

Table 3 shows predicted moments of (first-order accurate) Home country variables. Net foreign assets, net foreign equity/bond assets, the current account, net equity/bond purchases and net exports are normalized by Home GDP. Statistics for (real) GDP and physical investment pertain to logged series. All statistics are based on Hodrick-Prescott filtered variables (smoothing parameter: 400). 36 That normalization/filter parallels the normalization/filter applied to the empirical series in Table 2.

The theoretical counterparts to the empirical financial variables considered in Table 2 are defined as follows: The Home country’s net foreign equity assets (at market prices) at the end of period t are

\[ p_{F,t}S_{F,t+1}^{H} - p_{H,t}S_{H,t+1}^{F} \]

while Home net foreign bond assets (at market prices) are:

\[ p_{H,t}b_{H,t+1}^{H} + p_{F,t}b_{F,t+1}^{H} \]

Net Foreign Assets are the sum of net foreign equity assets and net foreign bond assets. The Home net equity purchase in period t is \( p_{F,t}\Delta S_{F,t+1}^{H} - p_{H,t}\Delta S_{H,t+1}^{F} \) (with \( \Delta x_{t+1} = x_{t+1} - x_{t} \)); the Home net bonds purchase is \( p_{H,t}\Delta b_{H,t+1}^{H} + p_{F,t}\Delta b_{F,t+1}^{H} \). The current account is the sum of net bond and equity purchases. Up to the linear approximation used here, the change in a country’s NFA equals the current account plus the change in market value of steady state external stock and bond holdings:

\[ \Delta NFA_{H,t+1} = p_{F,t}\Delta S_{F,t+1}^{H} - p_{H,t}\Delta S_{H,t+1}^{F} + p_{H,t}\Delta b_{H,t+1}^{H} + p_{F,t}\Delta b_{F,t+1}^{H} \]

35 Devereux and Sutherland (2006b) compute dynamic portfolios in an economy with two assets; we extend their method to the case with more than two assets.

36 Predicted moments are computed by applying the Hodrick-Prescott filter to the moving average representation of the linearized model solution (using formulae for Hodrick-Prescott filter weights derived by McElroy (2008)).
Similarly, the change in net foreign equity [bond] assets (at market prices) equals net equity [bond] purchases, plus the change in the market value of the steady state stock [bond] holdings. In Table 3 (as in the model) we define the real exchange rate as the relative price of Home aggregate consumption in terms of Foreign aggregate consumption; see (21) (thus, a fall in $RER_t$ represents a depreciation of the Home real exchange rate).

**Zero-order portfolio**

The zero-order equilibrium portfolio is: $S = 0.79$, $\bar{b} = 0.26$. Thus, 79% of a country’s capital stock is predicted to be held by local investors.

**Predicted dynamic properties**

Col. 1 of Table 3 reports predicted statistics for the model, with the two types of shocks. In order to assess the relative importance of each type of shock, we also report predicted statistics for variants with only TFP shocks (Col. 2) and with only investment efficiency shocks (Col. 3). Col. 4 reports average historical statistics for the G7 countries.

Table 3 shows that the model generates sizable fluctuations in equity and bond holdings: the predicted standard deviations of a country’s net bond purchases and of its net equity purchases are both 3.23% (the mean empirical standard deviations of net bond and net equity purchases are 1.71% and 1.38%, respectively); the $\theta$ and $\chi$ shocks both account for roughly the same share of the variance of net equity/bond purchases. However, up to a first order approximation, these net asset purchases do not affect the current account: in each period, the value of net equity purchases equals the value of net bond sales (see proof in Appendix A.1). The model thus predicts that net bond purchases are perfectly negatively correlated with net equity purchases. Empirically, net bond purchases and net equity purchases are strongly negatively correlated (mean correlation: -0.68).

The model captures the fact that the persistence of net equity/bond purchases is much lower than the persistence of GDP: the predicted autocorrelations of net equity and net bond purchases are close to zero (0.07). This is due to the fact that equity and bond holdings at the end of period $t$ $(S_{i,t+1}^s, b_{i,t+1}^b)$

\[CA_{H,t} + (\Delta p_{S,F,t}^S - \Delta p_{S,H,t}^S)(1 - S) + (\Delta p_{b,H,t}^b - \Delta p_{b,F,t}^b)b.\]  

\[NF_{A,H,t+1} = \Delta NF_{A,H,t} + \Delta S_{F,t+1}^H + \Delta b_{F,t+1}^H - \Delta S_{H,t+1}^H - \Delta b_{H,t+1}^H,\]  

\[\Delta NF_{A,H,t+1} = \Delta S_{F,t+1}^H + \Delta b_{F,t+1}^H - \Delta S_{H,t+1}^H - \Delta b_{H,t+1}^H.\]  

\[\Delta S_{F,t+1}^H = (\Delta p_{S,F,t}^S - \Delta p_{S,H,t}^S)(1 - S) + (\Delta p_{b,H,t}^b - \Delta p_{b,F,t}^b)b.\]  

\[\Delta b_{F,t+1}^H = (\Delta p_{b,H,t}^b - \Delta p_{b,F,t}^b).\]
are functions of state variables (capital stocks, Net Foreign Assets and exogenous variables) that are highly persistent; thus, the first difference of net equity/bond holdings (i.e. net equity/bond purchases) has little serial correlation.

The predicted standard deviation, correlation with GDP and autocorrelation of the first difference of Net Foreign Assets (with simultaneous two types of shocks) are 2.21%, -0.26 and 0.12, respectively (corresponding average empirical statistics: 3.23%, -0.22, -0.01). Consistent with the data, the model predicts thus that the change of NFA is more volatile than GDP; countercyclical and basically serially uncorrelated. As the current account is predicted to be zero (up to first order), NFA changes are solely driven by movements in asset prices. NFA is thus predicted to have the time series properties of asset prices. When there is a positive TFP or investment efficiency shock, net imports are predicted to rise on impact (due to a strong short run rise in investment), and to fall thereafter; the present value of future net imports drops, on impact. As the country’s NFA equals the present value of its current and future net imports, the NFA drops too on impact. This explains why the change in NFA is predicted to be countercyclical.

With just TFP shocks [just investment efficiency shocks] the predicted standard deviation of the change in NFA is 1.72% [1.39%] (see Cols., 2 and 3, Table 3). Thus, investment efficiency shocks contribute noticeably to the model’s ability to generate volatile NFA changes.

The changes of net foreign equity assets and of net foreign bond assets too are predicted to exhibit sizable fluctuation (predicted standard deviations with the two simultaneous types of shocks: 5.32% and 3.13%, respectively). As in the data, the changes of net equity assets and of net bond assets are negatively correlated.

The model (with simultaneous $\theta$ and $\chi$ shocks) matches closely the observed volatility of GDP, and it captures the fact that investment spending is markedly more volatile than GDP (predicted standard deviations of GDP and investment: 1.87%, 8.26%). The predicted standard deviation of net exports (1.07%) is close to the empirical standard deviation (1.14%). The model predicts that net exports are weakly countercyclical. Cols. 2 and 3 show that TFP shocks are the main source of output fluctuations, but that investment and net exports are mainly driven by investment efficiency shocks ($\chi$). $\chi$ also generates countercyclical fluctuations of net exports. The model captures the fact that the fluctuations
of GDP, investment and net exports are persistent.

The high volatility of the real exchange rate is one of the key puzzles of international macroeconomics. Standard RBC models (driven just by TFP shocks) underpredict sharply the observed volatility of real exchange rates; those models also predicts that the real external value of a currency (and the terms of trade) is highly countercyclical (see, e.g., Backus et al. (1994)). The model here, with the two types of shocks, performs somewhat better. With just TFP shocks, the predicted standard deviation of the real exchange rate and its correlation with domestic GDP are 0.60% and -0.52, respectively. Adding the investment efficiency shock more than doubles the predicted standard deviation of the real exchange rate (to 1.38%), and raises the predicted correlation with GDP to -0.22 (the empirical correlation is 0.08). However, quantitatively the model is still far from accounting for observed real exchange rate volatility.

When there are just \( \chi \) shocks, the real exchange rate is essentially acyclical. On impact, a positive Home investment efficiency shock appreciates the Home country’s terms of trade and its real exchange rate (the increase in Home investment triggered by the shock raises relative demand for the H good, as there is local bias in investment spending); however, in subsequent periods, the real exchange rate depreciates (once Home has increased its capital stock, the supply of the H good is above its pre-shock level, and thus its relative prices is lower). This explains why, with the simultaneous two types of shock, the real exchange rate is much less cyclical than with just TFP shocks.

The presence of investment efficiency shocks lowers the predicted cross-country correlations of output, consumption and investment. This brings the predicted cross-country consumption correlation (0.58) closer to the empirical correlation (0.44); however, the predicted cross-country output correlation and (especially) the cross-country investment correlation are too low when there are investment efficiency shocks. Also, investment efficiency shocks generate a predicted within-country correlation between consumption and GDP (0.38) that is lower than the empirical correlation (0.78). Raffo (2008), following Jaimovich and Rebelo (2009), addresses these issues by introducing variable capital utilization and Greenwood, Hercowitz, Huffman (1988) preferences into an international business cycle model with investment-specific technology shocks. Variable capital utilization would leave the equilibrium portfolio unchanged. The GHH preferences are non-separable in consumption and leisure; these preferences imply zero wealth effect of shocks on hours worked, while making consumption more responsive to wealth changes; this allows to
generate a consumption-investment correlation that is larger, and thus closer to the data. However, due to the assumed non-separability between consumption and leisure, a new hedging motive in portfolio choice appears, namely the hedging of the relative price of leisure. We leave for future research the study of the impact of these non-separable preferences between leisure and consumption on international portfolios holdings.

5 Conclusion

This paper departs from the two familiar explanations of equity home bias: transaction costs that impede international diversification, and the possibility that terms of trade responses to supply shocks provide international risk sharing, so that households have little incentive to hold diversified portfolios (Cole and Obstfeld (1991), Helpman and Razin (1978)). We study a two-country/two-good RBC model with frictionless international trade in stocks and in bonds; there are shocks to total factor productivity and to the efficiency of physical investment. In the setting here, domestic stocks are used to hedge fluctuations in local wage income triggered by shocks to investment spending. Terms of trade risk is hedged using bonds denominated in local goods and in foreign goods. In contrast to related models, the low level of international diversification does not depend on strongly countercyclical terms of trade.
APPENDIX

A.1. Verifying that the period-by-period household budget constraint (12) is satisfied and that the current account is zero, to first order

In deriving the zero-order equilibrium portfolio \((S, b)\), we replaced the period-by-period household budget constraint (12) by the `static' constraint (26). We now show that when a first-order approximation of the `static' constraint holds at all dates, then a first-order approximation of (12) holds likewise. Thus it is sufficient to consider the `static' constraint (26) when solving for \((S, b)\).

Following Devereux and Sutherland (2006a,b), we express the period \(t\) budget constraint (12) of country \(i\) as

\[
NFA_{i,t+1} = NX_{i,t} + NFA_{i,t}R_{i,t}^b + \xi_{i,t}, \quad \text{with}
\]

\[
NFA_{i,t+1} = p^S_{i,t}S_{i,t+1} + p^b_{i,t}b^i_{i,t+1}, \quad j \neq i,
\]

\[
NX_{i,t} = p_{i,t}y_{i,t} - P_{i,t}C_{i,t} - P^I_{i,t}I_{i,t},
\]

\[
\xi_{i,t} = S^i_{i,t}p^S_{i,t-1}(R^S_{i,t} - R^b_{i,t}) - S^i_{i,t}p^S_{i,t-1}(R^S_{i,t} - R^b_{i,t}) + b^i_{i,t}p^b_{i,t-1}(R^b_{i,t} - R^b_{i,t}), \quad j \neq i.
\]

\(NFA_{i,t+1}\) are country \(i\)'s net foreign assets at the end of period \(t\), and \(NX_{i,t}\) are \(i\)'s net exports. \(R^S_{i,t}, R^S_{j,t}, R^b_{i,t}, R^b_{j,t}\) are gross equity/bond returns between \(t - 1\) and \(t\) (see (15)). \(\xi_{i,t}\) is the "excess return" on the country's net foreign assets (between \(t - 1\) and \(t\)) relative to the return on the good-\(i\) bond. 39

As before, variables without time indices represent (deterministic) steady state values, and \(\xi_{i,t} \equiv (z_{i,t} - z_i)/z_i\). Note that \(NFA_i = 0\), \(NX_i = 0\), \(p^S \equiv p^S_H = p^S_F\), \(p^b \equiv p^b_H = p^b_F\), \(d \equiv d_H = d_F\), \(p \equiv p_H = p_F\), due to the symmetric structure of the two countries; also, \(R^S_H = R^S_F = R^b_H = R^b_F = 1/\beta\).

A linear approximation of (41) around the steady state yields thus:

\[
NFA_{i,t+1} = NX_{i,t} + NFA_{i,t}/\beta + S^ip^S_{i,t}(R^S_{i,t} - R^0_{i,t}) - S^i_jp^S_{i,t}(R^S_{j,t} - R^0_{i,t}) + b^i_jp^b_{i,t}(R^b_{j,t} - R^0_{i,t}), \quad j \neq i
\]

39Note that \(\xi_{i,t} = S^i_{j,t}(d_{i,t} + p^S_{i,t}) - S^i_{i,t}(d_{i,t} + p^S_{i,t}) + (p_{i,t} + p^S_{i,t})b^i_{i,t} + (p_{j,t} + p^S_{j,t})b^j_{i,t} - NFA_{i,t}R^b_{i,t}\). Thus, \(\xi_{i,t}\) is the difference between country \(i\)'s net external wealth (including net dividend and coupon income) at the beginning of period \(t\), minus the hypothetical value of \(i\)'s net external wealth at the beginning of \(t\) that would obtain if \(i\) fully invested her net external wealth at the end of \(t - 1\) in the good-\(i\) bond.
where $S_j, b_j$ and $b_j$ are zero-order portfolio holdings. Symmetry implies $S_j = S_i$ for $j \neq i$. Hence:

$$NF A_{i,t+1} = NX_{i,t} + NF A_{i,t}/\beta + (S - 1)p^S \frac{1}{\beta} (R^S_{i,t} - R^S_{j,t}) + b^S \frac{1}{\beta} (R^S_{i,t} - R^S_{j,t}), \ j \neq i.$$  \hspace{1cm} (43)

Solving the Euler equations (14) forward gives $p^S_{i,t} = E_t \sum_{\tau \geq 1} \delta_{i,t+\tau} d_{i,t+\tau}$ and $p^b_{i,t} = E_t \sum_{\tau \geq 1} \delta_{i,t+\tau} p_{i,t+\tau}$. Up to first order, the relative stock and bond prices and returns obey thus:

$$\tilde{p}^S_{i,t} - \tilde{p}^S_{j,t} = \frac{1 - \beta}{\beta} E_t \sum_{\tau \geq 1} \beta^\tau (d_{i,t+\tau} - d_{j,t+\tau}), \ \tilde{p}^b_{i,t} - \tilde{p}^b_{j,t} = \frac{1 - \beta}{\beta} E_t \sum_{\tau \geq 1} \beta^\tau (\tilde{p}_{i,t+\tau} - \tilde{p}_{j,t+\tau}), \ j \neq i.$$  \hspace{1cm} (44)

$$\tilde{R}^S_{i,t} - \tilde{R}^S_{j,t} = (1 - \beta) \tilde{E}_t \sum_{\tau \geq 1} \beta^\tau (d_{i,t+\tau} - d_{j,t+\tau}), \ \tilde{R}^b_{i,t} - \tilde{R}^b_{j,t} = (1 - \beta) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\tilde{p}_{i,t+\tau} - \tilde{p}_{j,t+\tau}), \ j \neq i.$$  \hspace{1cm} (45)

with $\tilde{E}_t \equiv E_t z - E_{t-1} z$ (revision of expectation between $t - 1$ and $t$). Thus, $E_t (\tilde{R}^S_{i,t+\tau} - \tilde{R}^S_{j,t+\tau}) = E_t (\tilde{R}^b_{i,t+\tau} - \tilde{R}^b_{j,t+\tau}) = 0$ for $\tau > 0$ : up to first order, the expected value of future excess returns is zero.

Solving (43) forward (imposing the no-Ponzi/transversality condition $\lim_{\tau \to -\infty} E_t \beta^\tau NF A_{i,t+\tau} = 0$) gives the following present value budget constraint:

$$E_t \sum_{\tau \geq 0} \beta^\tau (\tilde{E}_t) (\tilde{N} X_{i,t+\tau}) = NF A_{i,t}/\beta + (S - 1) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau (d_{i,t+\tau} - d_{j,t+\tau}) + b^S \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\tilde{p}_{i,t+\tau} - \tilde{p}_{j,t+\tau}), \ j \neq i,$$  \hspace{1cm} (46)

where we used that fact that $p^S = d\beta/(1 - \beta), \ p^b = p\beta/(1 - \beta)$.

(46) holds if and only if:

$$E_t \sum_{\tau \geq 0} \beta^\tau (\tilde{E}_t) (\tilde{N} X_{i,t+\tau}) = (S - 1) \tilde{d} \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau (d_{i,t+\tau} - d_{j,t+\tau}) + b^S \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\tilde{p}_{i,t+\tau} - \tilde{p}_{j,t+\tau}), \ j \neq i $$  \hspace{1cm} (47)

and

$$E_{t-1} \sum_{\tau \geq 0} \beta^\tau (\tilde{E}_t) (\tilde{N} X_{i,t+\tau}) = NF A_{i,t}/\beta.$$  \hspace{1cm} (48)

(47) shows that, up to first order, date $t$ innovations to the expected present value of current and future country $i$ net imports have to equal innovations to the present value of net dividend and net bond income received by country $i$.

The 'static' budget constraint

In deriving the zero-order equilibrium portfolio, we replaced the period-by-period household budget constraint (12) by the 'static' budget constraint: $P_{i,t} C_{i,t} = w_{i,t} l_{i,t} + S d_{i,t} + (1 - S) d_{j,t} + b (p_{i,t} - p_{j,t})$ (see
where 5 is zero: implying that, to first order, the value of country’s net external assets, evaluated at steady state asset prices. Linearizing the expression using the formulae for relative asset prices (44), we can write this as:

$$S_{zero-order \text{ portfolio holdings}} = \text{holdings from zero-order portfolio holdings. This implies that, when solving for the zero-order portfolio zero-order portfolio (S, b), it is sufficient to consider the 'static' budget constraint (26).}

A restriction on first-order accurate (time-varying) portfolio holdings

Substitution of (49) into (48) yields:

$$NFA_{i,t} = (S - 1)\beta E_{t-1} \sum_{\tau \geq 0} \beta^\tau d(\bar{d}_{i,t+\tau} - \bar{d}_{j,t+\tau}) + b\beta E_{t-1} \sum_{\tau \geq 0} \beta^\tau p(\bar{p}_{i,t+\tau} - \bar{p}_{j,t+\tau}), \quad j \neq i. \quad (50)$$

Using the formulae for relative asset prices (44), we can write this as:

$$NFA_{i,t} = (S - 1)p^S(\bar{p}_{i,t-1} - \bar{p}_{j,t-1}^b) + bp^b(\bar{p}_{i,t-1}^b - \bar{p}_{j,t-1}^b), \quad j \neq i. \quad (51)$$

Linearizing the expression $NFA_{i,t} = p^S(\bar{p}_{i,t-1}^\tau - \bar{p}_{j,t-1}^\tau) + p^b(\bar{p}_{i,t-1}^b - \bar{p}_{j,t-1}^b)$ gives

$$NFA_{i,t} \equiv (S - 1)p^S(\bar{p}_{i,t-1}^\tau - \bar{p}_{j,t-1}^\tau) + p^b(\bar{p}_{i,t-1}^b - \bar{p}_{j,t-1}^b) + (\nabla S_{i,t}^\tau - \nabla S_{j,t}^\tau)p^S + (\nabla b_{i,t}^b - \nabla b_{j,t}^b)p^b, \quad j \neq i, \quad (52)$$

where $\nabla S_{i,t}^\tau \equiv S_{i,t}^\tau - (1 - S)$, $\nabla S_{j,t}^\tau \equiv S_{j,t}^\tau - (1 - S)$, $\nabla b_{i,t}^b \equiv b_{i,t}^b - b$, $\nabla b_{j,t}^b \equiv b_{j,t}^b - (b)$ denote the deviations of portfolio holdings at the end of period $t - 1$ from the zero-order portfolio. (51) and (52) imply that, to first order, the value of country’s net external assets, evaluated at steady state asset prices is zero:

$$\nabla S_{i,t}^\tau - \nabla S_{j,t}^\tau)p^S + (\nabla b_{i,t}^b + \nabla b_{j,t}^b)p^b = (S_{i,t}^\tau - S_{j,t}^\tau)p^S + (b_{i,t}^b + b_{j,t}^b)p^b = 0, \quad j \neq i. \quad (53)$$

The current account

The period $t$ current account of country $i$ is: $CA_{i,t} = (S_{i,t+1}^\tau - S_{j,t+1}^\tau)P_{i,t}^S - (S_{i,t+1}^\tau - S_{j,t+1}^\tau)P_{i,t}^b + (b_{i,t+1}^b + b_{j,t+1}^b)P_{i,t}^b$. Linearization of this expression gives: $CA_{i,t} = \{(S_{i,t+1}^\tau - S_{j,t+1}^\tau) - (S_{i,t+1}^\tau - S_{j,t+1}^\tau)\}P_{i,t}^S + (b_{i,t+1}^b - b_{j,t+1}^b + b_{j,t+1}^b - b_{j,t+1}^b)p^b$. It thus follows from (53) that $CA_{i,t} = 0$, up to first order.

---

40 Subtracting $w_{i,t}d_{i,t}$ and $d_{i,t}$ from both sides of the static constraint gives: $P_{i,t}^c X_{i,t} - w_{i,t}d_{i,t} - d_{i,t} = (S - 1)(d_{i,t} - d_{j,t}) + b(p_{i,t} - p_{j,t})$. The left-hand side of this expression equals $-NX_{i,t}$ (as $d_{i,t} \equiv p_{i,t}d_{i,t} - w_{i,t}d_{i,t} - P_{i,t}^c X_{i,t}$).
A.2. Returns and the equilibrium portfolio

Equation (37) in the main text shows that the zero-order local equity position \( S \) depends on the covariance between components of relative (Home vs. Foreign) wage incomes and dividend payments that are orthogonal to the terms of trade: 
\[
S = \frac{1}{2} - \frac{1}{2} \frac{1 - \kappa}{\kappa - \lambda} \frac{\text{Cov}_{ij}(w_{it}, \tilde{d}_t)}{\text{Var}_{i}(\tilde{d}_t)}.
\]
We now show that \( S \) can equivalently be expressed as a function of the covariance between components of relative (Home vs. Foreign) human capital returns and equity returns that are orthogonal to the return differential between the Home-good and Foreign-good bonds.

As shown above, country \( i \) net imports can be expressed as:
\[
-NX_{i,t} = P_{i,t}C_{i,t} - w_{i,t}l_{i,t} - d_{i,t};
\]
this can be written as:
\[
-NX_{i,t} = py_i(1 - \Lambda) \tilde{P}_{i,t} \tilde{C}_{i,t} - (1 - \kappa)py_iw_{i,t}l_{i,t} - py_i(\kappa - \Lambda) \tilde{d}_{i,t}. \quad 41
\]
Inserting this expression into (47) gives:
\[
(1 - \Lambda) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau P_{i,t+\tau} \tilde{C}_{i,t+\tau} = (1 - \kappa) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau w_{i,t+\tau} \tilde{l}_{i,t+\tau} + S(\kappa - \Lambda) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau \tilde{d}_{i,t+\tau}
\]
\[
+(1 - S)(\kappa - \Lambda) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau \tilde{d}_{j,t+\tau} + \tilde{b} \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\tilde{p}_{i,t+\tau} - \tilde{p}_{j,t+\tau}), \quad j \neq i. \quad (54)
\]
where \( \tilde{b} = b/y_i \) is the local-good bond holding divided by steady state output.

Subtracting the linearized present value budget constraint (54) of country \( F \) from that of country \( H \) yields:
\[
(1 - \Lambda) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau [P_{H,t+\tau} \tilde{C}_{H,t+\tau} - P_{F,t+\tau} \tilde{C}_{F,t+\tau}] = (1 - \kappa) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau [w_{H,t+\tau} \tilde{l}_{H,t+\tau} - w_{H,t+\tau} \tilde{l}_{H,t+\tau} + (2S - 1)(\kappa - \Lambda) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau [\tilde{d}_{H,t+\tau} - \tilde{d}_{F,t+\tau}] + 2\tilde{b} \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau \tilde{q}_{i,t+\tau}, \quad (55)
\]
where \( \tilde{q}_t = p_{H,t}/p_{F,t} \) are the Home terms of trade. Effective market completeness (up to first order) implies that 
\[
P_{H,t} \tilde{C}_{H,t} - P_{F,t} \tilde{C}_{F,t} = (1 - \frac{1}{\sigma}) \tilde{q}_t \quad \text{(see (28)).}
\]
Thus, innovations to the present value of relative consumption spending are perfectly correlated with the return differential between Home-good and Foreign-good bonds (from (45)):
\[
\tilde{E}_t \sum_{\tau \geq 0} \beta^\tau [P_{H,t+\tau} \tilde{C}_{H,t+\tau} - P_{F,t+\tau} \tilde{C}_{F,t+\tau}] = (1 - \frac{1}{\sigma}) \frac{1}{1 - \beta} (R_{H,t}^p - R_{F,t}^p).
\]

\cite{NB} Steady state consumption spending is a fraction \((1 - \Lambda)\) of output, where \( \Lambda \) is the steady state ratio of investment spending to GDP; wage income and dividends account for fractions \( 1 - \kappa \) and \( \kappa - \Lambda \) of output, respectively.
Define the return on country $i$ Human capital as: $R_{i,t}^W = \frac{P_{i,t}^W + \omega_i w_{i,t+\tau}^t}{P_{i,t}^W}$, where $P_{i,t}^W \equiv E_t \sum_{\tau \geq 1} \beta_t \omega_{i,t+\tau}^t w_{i,t+\tau}^t$ is the present value of the country $i$ labor income; linearizing these formulae gives:

$$
\tilde{R}_{i,t}^W - \tilde{R}_{F,t}^W = (1 - \beta) \tilde{E}_t \sum_{\tau \geq 0} \beta^\tau [w_{i,t+\tau}^t l_{i,t+\tau} - w_{F,t+\tau}^t l_{F,t+\tau}].
$$

Using the expression for the cross-country equity return differential shown in (45), we can thus express (55) as:

$$(1 - \Lambda)(1 - \frac{1}{\sigma}) R_b^t = (1 - \kappa) R_t^W + (2S - 1)(\kappa - \Lambda) R_t^S + 2\tilde{b} R_b^t,$$

where $R_b^t \equiv R_{H,t}^b - R_{F,t}^b$, $R_t^W \equiv R_{H,t}^W - R_{F,t}^W$, $R_t^S \equiv R_{H,t}^S - R_{F,t}^S$ are Home-foreign return differentials for bonds, Human capital and equity, respectively. This condition implies:

$$S = \frac{1}{2} - \frac{1}{2} \frac{1 - \kappa \text{ Cov}_{R_b}(R_t^W, R_t^S)}{\text{ Var}_{R_b}(R_t^S)},$$

(56)

with $\text{ Cov}_{R_b}(R_t^W, R_t^S) = E\{R_t^W - P[R_t^W|R_b^t]\} \{R_t^S - P[R_t^S|R_t^W]\}$, $\text{ Var}_{R_b}(R_t^S) = E\{R_t^S - P[R_t^S|R_b^t]\}^2$, where $P[R_t^W|R_b^t]$ is the linear projection of $R_t^W$ on $R_b^t$. Thus, the local equity share can be expressed as a function of the covariance between the components of relative (Home vs. Foreign) human capital returns and (relative) equity returns that are orthogonal to (relative) bond returns: equity home bias arises when that covariance is negative.

The model here generates a negative covariance. In the main text we showed that a combination of exogenous shocks that raises relative Home real investment spending, without affecting the terms of trade has these consequences: relative Home wage income rises, and the relative dividend of the Home firm falls. The same logic also applies directly to capitalized income streams, and thus to returns. Consider a combination of exogenous innovations that raises the present discounted value of relative Home real investment spending, without changing the present value of (current and future) Home terms of trade; that combination of shocks raises the present value of relative Home wage income, while lowering the present value of relative Home dividends; in other terms, such a combination of shocks has no effect on the return differential between Home-good and Foreign-good bonds, and no effect on the present value of efficient relative Home consumption spending; however, it raises the relative return on Home human capital, while reducing the relative return of the Home stock. Holding constant the bond return differential, the relative return on Home human capital co-moves thus negatively with the relative Home stock return: $\text{ Cov}_{R_b}(R_t^W, R_t^S) < 0$. 

34
A.3. Corporate Debt

The following results hold when firms (partly) finance investment spending by issuing debt: (1) The structure of equilibrium equity portfolio is unaffected (i.e. the equity portfolio continues to be given by (32)); (2) Households’s holdings of corporate debt exhibit a home bias in the same proportion as for stocks; (3) The net external bond positions are altered; if firms issue debt denominated in their local good, the range of parameters increases for which a country’s net local-good debt position is negative.

To understand these results note that, in the economy here, the Modigliani-Miller theorem holds, i.e. the issuance of corporate debt does not affect firm values or equilibrium consumptions. Assume that firms issue bonds denominated in the good that they produce, and that one unit of a corporate bond pays one unit of output in all future periods; assume also that a constant share \( \mu \) of the net investment of the country \( i \) firm \( P_{i,t}^I (I_{i,t} - \delta K_{i,t}) \) is financed by issuing debt.\(^{42}\) The rest of investment spending \( P_{i,t}^I I_{i,t} - \mu P_{i,t}^I (I_{i,t} - \delta K_{i,t}) \) is financed through retained earnings.

Let \( D_{it} \) denote the outstanding debt of the country \( i \) firm, at the beginning of period \( t \). The price of one unit of the debt is \( p_{it}^D \). Suppose that the country \( i \) household holds a fraction \( S \) \([1 - S]\) of local [foreign] equity and of the outstanding local [foreign] corporate bonds.

This portfolio strategy allows the household to offset the implicit debt position entailed by its equity position. When \( S \) is set at the optimal value in the baseline model (where firms are fully equity financed, \( \mu = 0 \), such a portfolio strategy thus generates the same financial income as the household’s portfolio in the baseline model; hence that portfolio strategy replicates the efficient consumption allocation (up to first order). To see this, note that in the present setting the period \( t \) dividend of the country \( i \) firm equals a share \( \kappa \) of its output, \( \kappa p_{i,t} y_{i,t} \) (NB \( \kappa \) is the capital share) minus retained earnings \( P_{i,t}^I I_{i,t} - \mu P_{i,t}^I (I_{i,t} - \delta K_{i,t}) \), less the coupon payment \( p_{it} D_{it} \) made be the firm (to holders of its bonds).

Thus, the date \( t \) dividend is:
\[
d_{it}(\mu) = \kappa p_{i,t} y_{i,t} - [P_{i,t}^I I_{i,t} - \mu P_{i,t}^I (I_{i,t} - \delta K_{i,t})] - p_{it} D_{it}
\]

The firm’s issuance of new corporate bonds in period \( t \) is given by: \( p_{it}^D (D_{it+1} - D_{it}) = \mu P_{i,t}^I (I_{i,t} - \delta K_{i,t}) \). When the country \( i \) household holds a share \( S \) of local equity and local corporate debt, then that household derives the following income from local equity and corporate debt, in period \( t \) (net of the amount spent

\(^{42}\)Note that results (1) and (2) do not hinge on these assumptions.
at \( t \) to purchase a \( S \) fraction of the newly issued local corporate debt):

\[
Sd_{it}(\mu) + Sp_{it}D_{it} - Sp_{it}D_{it+1} = S (\kappa p_{it} y_{it} - P_{i,t} I_{i,t}) = Sd_{it}(\mu = 0).
\]

This corresponds to the dividend income of the household, from her holdings of local equity, in the baseline model in which firms do not issue debt \( d_{it}(\mu = 0) \). By the same reasoning, the portfolio strategy described above ensures that the household receives an income from her holdings of foreign equity and foreign corporate debt that equals her dividend income from foreign equity, in the baseline model. In order to replicate efficient risk-sharing (up to first order), the household in addition has to hold the same amount of local-good and foreign-good non-corporate debt as in the baseline model.

In summary, the equilibrium holdings of local and foreign equity shares and of non-corporate bonds are the same as in the baseline model, but investors now also hold a share \( S \) of domestic corporate debt and a share \( 1 - S \) of foreign corporate debt. When the country \( i \) firm issues one unit of debt denominated in local good \( i \), then the country’s overall (household+firm) net local good debt position changes by \( 1 - S < 0 \) units, as a share \( S \) of the new debt is purchased by the local household (while a share \( 1 - S \) of the new debt is bought by the foreign household). Thus the presence of local-good corporate debt lowers the country’s overall net local-good debt. When all corporate debt is denominated in the local good, the country’s overall (household+corporate) local-good debt position is \( b + (S - 1)D_{Ht} < b \), where \( b \) is the local household’s holding of local-good non-corporate debt (as discussed above, \( b \) has the same value as in the baseline model without corporate debt).
References


42

Table 1. Correlations between relative labor incomes and dividends, and implied locally held equity shares

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>Japan</th>
<th>Germany</th>
<th>France</th>
<th>UK</th>
<th>Italy</th>
<th>Canada</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Unconditional correlations Corr((\hat{w}_t, \hat{d}_t))</td>
<td>0.73</td>
<td>0.59</td>
<td>0.70</td>
<td>0.77</td>
<td>-0.24</td>
<td>0.89</td>
<td>0.28</td>
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<tr>
<td></td>
<td>(.12)</td>
<td>(.08)</td>
<td>(.14)</td>
<td>(.12)</td>
<td>(.12)</td>
<td>(.02)</td>
<td>(.11)</td>
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<tr>
<td>(b) Conditional correlations based on terms of trade measure given by relative GDP deflators Corr_{\hat{q}_t}((\hat{w}_t, \hat{d}_t))</td>
<td>-0.17</td>
<td>-0.75</td>
<td>-0.69</td>
<td>-0.39</td>
<td>-0.71</td>
<td>-0.26</td>
<td>-0.22</td>
</tr>
<tr>
<td></td>
<td>(.15)</td>
<td>(.05)</td>
<td>(.10)</td>
<td>(.07)</td>
<td>(.11)</td>
<td>(.19)</td>
<td>(.16)</td>
</tr>
<tr>
<td>Implied locally held equity share (S^t_i)</td>
<td>0.58</td>
<td>1.03</td>
<td>0.50</td>
<td>0.51</td>
<td>0.68</td>
<td>0.34</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(.17)</td>
<td>(.16)</td>
<td>(.09)</td>
<td>(.23)</td>
<td>(.13)</td>
<td>(.24)</td>
<td>(.14)</td>
</tr>
<tr>
<td>(c) Correlations conditional based on terms of trade measure given by relative export prices Corr_{\hat{q}_t}((\hat{w}_t, \hat{d}_t))</td>
<td>-0.27</td>
<td>-0.61</td>
<td>-0.22</td>
<td>-0.25</td>
<td>-0.74</td>
<td>0.87</td>
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<tr>
<td></td>
<td>(.18)</td>
<td>(.09)</td>
<td>(.22)</td>
<td>(.15)</td>
<td>(.08)</td>
<td>(.03)</td>
<td>(.13)</td>
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<tr>
<td>Implied locally held equity share (S^t_i)</td>
<td>0.84</td>
<td>1.22</td>
<td>0.41</td>
<td>0.50</td>
<td>2.58</td>
<td>-1.73</td>
<td>1.61</td>
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<tr>
<td></td>
<td>(.31)</td>
<td>(.29)</td>
<td>(.30)</td>
<td>(.35)</td>
<td>(.51)</td>
<td>(.21)</td>
<td>(.59)</td>
</tr>
</tbody>
</table>

Notes

Corr(\(\hat{w}_t, \hat{d}_t\)): correlation between relative labor income and the relative dividend income in a given country (compared to total labor income and total dividend in remaining G7 countries).

Corr_{\hat{q}_t}(\(\hat{w}_t, \hat{d}_t\)): correlation between components of relative labor income and the relative dividend income that are orthogonal to terms of trade.

The data are annual, 1984-2004. Figures in parentheses are standard errors (based on GMM). See text for further explanations.
Table 2. Time series properties of international financial positions (G7 countries)

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>JA</th>
<th>GE</th>
<th>FR</th>
<th>UK</th>
<th>IT</th>
<th>CA</th>
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<tr>
<td><strong>Standard deviations (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>GDP</td>
<td>1.58</td>
<td>2.94</td>
<td>1.67</td>
<td>1.61</td>
<td>2.12</td>
<td>1.76</td>
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<td>Investment</td>
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<td>7.20</td>
<td>5.07</td>
<td>7.65</td>
<td>6.89</td>
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<tr>
<td>Net exports</td>
<td>0.72</td>
<td>0.63</td>
<td>0.85</td>
<td>0.87</td>
<td>1.81</td>
<td>1.61</td>
<td>1.50</td>
<td>1.14</td>
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<tr>
<td>Δ (Net foreign assets)</td>
<td>2.36</td>
<td>2.60</td>
<td>2.63</td>
<td>4.33</td>
<td>5.02</td>
<td>2.67</td>
<td>3.03</td>
<td>3.23</td>
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<tr>
<td>Δ (Net foreign bond assets)</td>
<td>0.85</td>
<td>1.60</td>
<td>2.88</td>
<td>2.37</td>
<td>3.18</td>
<td>2.59</td>
<td>1.91</td>
<td>2.20</td>
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<tr>
<td>Δ (Net foreign equity assets)</td>
<td>2.27</td>
<td>3.20</td>
<td>1.37</td>
<td>4.69</td>
<td>4.20</td>
<td>2.35</td>
<td>2.74</td>
<td>2.97</td>
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<td>Current account</td>
<td>0.77</td>
<td>0.62</td>
<td>1.73</td>
<td>0.75</td>
<td>1.35</td>
<td>1.34</td>
<td>1.17</td>
<td>1.11</td>
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<tr>
<td>Net bond purchases</td>
<td>0.88</td>
<td>1.25</td>
<td>2.49</td>
<td>1.76</td>
<td>2.24</td>
<td>1.92</td>
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<td>1.71</td>
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<td>Net equity purchases</td>
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<td>1.67</td>
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<td>Real exchange rate</td>
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<td>10.51</td>
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<td>7.45</td>
<td>6.47</td>
<td>9.65</td>
<td>6.49</td>
<td>8.38</td>
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|                        |        |        |        |        |        |        |        |        |
| **Correlations with domestic GDP** |        |        |        |        |        |        |        |        |
| Net exports            | -0.38  | -0.56  | -0.29  | -0.52  | -0.56  | -0.48  | 0.01   | -0.39  |
| Δ (Net foreign assets) | -0.27  | -0.00  | -0.24  | -0.18  | -0.28  | -0.10  | -0.46  | -0.22  |
| Δ (Net foreign bond assets) | 0.37 | -0.23  | -0.38  | -0.50  | -0.38  | -0.32  | -0.22  | -0.24  |
| Δ (Net foreign equity assets) | -0.43 | 0.20   | 0.31   | 0.11   | -0.07  | 0.17   | -0.45  | -0.02  |
| Current account        | -0.44  | -0.48  | -0.54  | -0.32  | -0.74  | -0.44  | 0.18   | -0.40  |
| Net bond purchases     | 0.52   | -0.06  | -0.35  | -0.59  | -0.54  | -0.55  | 0.00   | -0.22  |
| Net equity purchases   | -0.77  | -0.08  | 0.07   | 0.55   | 0.04   | 0.20   | -0.22  | -0.03  |
| Real exchange rate     | 0.07   | 0.17   | -0.04  | -0.18  | 0.37   | 0.43   | 0.04   | 0.12   |

|                        |        |        |        |        |        |        |        |        |
| **Autocorrelations**   |        |        |        |        |        |        |        |        |
| GDP                    | 0.68   | 0.84   | 0.68   | 0.72   | 0.76   | 0.71   | 0.78   | 0.74   |
| Δ (Net foreign assets) | 0.03   | -0.42  | 0.00   | -0.13  | -0.14  | -0.01  | 0.58   | -0.01  |
| Δ (Net foreign bond assets) | 0.27 | 0.13   | 0.12   | 0.09   | 0.14   | 0.10   | 0.12   | 0.14   |
| Δ (Net foreign equity assets) | 0.26 | -0.23  | 0.12   | -0.17  | -0.16  | 0.17   | 0.16   | 0.02   |
| Current account        | 0.71   | 0.46   | 0.66   | 0.67   | 0.70   | 0.73   | 0.55   | 0.64   |
| Net bond purchases     | 0.65   | 0.26   | 0.66   | 0.11   | 0.17   | 0.39   | -0.01  | 0.32   |
| Net equity purchases   | 0.58   | -0.00  | 0.41   | 0.35   | -0.17  | 0.50   | -0.08  | 0.23   |

|                        |        |        |        |        |        |        |        |        |
| **Other correlations** |        |        |        |        |        |        |        |        |
| Δ (Net foreign equity assets) & Δ (Net foreign bond assets) | -0.09 | -0.48  | -0.39  | -0.39  | -0.01  | -0.36  | -0.15  | -0.27  |
| Net equity purchases & Net bond purchases | -0.52 | -0.68  | -0.70  | -0.78  | -0.63  | -0.77  | -0.69  | -0.68  |

Notes--Data are annual, 1984-2004, and were Hodrick-Prescott filtered. GDP, investment and real exchange rate series were logged. Real exchange rates are CPI-based. Underlined correlations are statistically significant at a 10% level (two-sided test, GMM based, assuming 4-th order serial correlation in residuals). JA: Japan, GE: Germany, FR: France, IT: Italy, CA: Canada.
## Table 3. Model predictions: dynamic properties

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<th></th>
<th>$\theta_H \theta_F$</th>
<th>$\chi_H \chi_F$</th>
<th>$\theta_H \theta_F$</th>
<th>$\chi_H \chi_F$</th>
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<tr>
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<td>-1.00</td>
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</table>

Notes--The following variables are normalized by domestic GDP: Net exports, $\Delta$ (Net foreign assets), $\Delta$ (Net foreign bond assets), $\Delta$ (Net foreign equity assets), Current account, Net bond purchases, Net equity purchases. GDP, Investment and the Real exchange rate (CPI-based) are logged. All variables are Hodrick-Prescott filtered. The empirical measure of consumption is non-durables plus services expenditures (from OECD National Accounts). The Real exchange rate measure is based on relative prices indices of non-durables and services consumption.