Are Macroeconomic Variables Useful for Forecasting the Distribution of US Inflation?

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Abstract

Much of the US inflation forecasting literature deals with examining the ability of macroeconomic indicators to predict the mean of future inflation, and the overwhelming evidence suggests that the macroeconomic indicators provide little or no predictability. In this paper, we expand the scope of inflation predictability and explore whether macroeconomic indicators are useful in predicting the distribution of future inflation. To incorporate macroeconomic indicators into the prediction of the conditional distribution of future inflation, we introduce a semi-parametric approach using conditional quantiles. The approach offers more flexibility in capturing the possible role of macroeconomic indicators in predicting the different parts of the future inflation distribution. Using monthly data on US inflation, we find that unemployment rate, housing starts, and the term spread provide significant out-of-sample predictability for the distribution of core inflation. Importantly, this result is obtained for a forecast evaluation period that we intentionally chose to be after 1984, when current research shows that macroeconomic indicators do not add much to the predictability of the future mean inflation. This paper discusses various findings using forecast intervals and forecast densities, and highlights the unique insights that the distribution approach offers, which otherwise would be ignored if we relied only on mean forecasts.

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1 Introduction

Forecasting the behavior of inflation plays a central role in the conduct of monetary policy due to the lagged impact of the central bank actions on economic activity. It is thus important to accurately predict the effect of the many shocks that hit the economy on the future dynamics of inflation. The standard approach for forecasting inflation has been the Phillips curve (PC) model that, in its expectation-augmented version, assumes a trade-off between unexpected inflation and unemployment, or more generally, indicators of real economic activity. Despite its long-time success, recent empirical evidence on the effectiveness of PC models is far from unanimous. Stock and Watson (1999) provide a detailed study on the out-of-sample forecast accuracy of the PC by using an extensive set of macroeconomic variables. Using the forecast evaluation period January 1970 - September 1996, their conclusion is that PC models have better forecasting performances (compared to univariate time series models) using the unemployment rate as well as other leading indicators of economic activity (e.g., output gap and capacity utilization). They also find that combining information or models might provide better results than simply relying on few indicators. However, Atkenson and Ohanian (2001) provide an opposite empirical evidence, albeit a different forecast evaluation period January 1984 - November 1999, where they report that PC models are no better than the naïve model, which assumes that the expected inflation over the next 12 months is equal to inflation over the previous 12 months. For a comprehensive survey as well as discussion of the outstanding issues in inflation forecasting, see Stock and Watson (2008).

While the contrasting findings on inflation predictability cannot be directly compared, they may, nevertheless, suggest that the relationship predicted by the PC models might have been unstable over time due to a possible shift in the dynamics of inflation. A phenomenon that is typically suggested to have caused a regime shift is the change in monetary policy that took place when Paul Volcker became Chairman of the Federal Reserve Board in August 1979. The effect of the stricter monetary policy was fully incorporated into the inflation process after 1984 and since then inflation has been low and stable (compared
to the 1970s). Fisher et al. (2002) conduct a systematic comparison of the forecasting accuracy (one-year ahead) of the naïve and PC models in different sub-periods: January 1977 to December 1984, January 1985 to December 1992 and January 1993 to December 2000. They find that the PC forecasts outperform the naïve forecasts only in the first sub-period for most of the inflation measures that they consider. The issue of model instability is also examined by Clark and McCracken (2006) and their results suggest that while model instability cannot be ruled out, the bulk of the findings of unpredictability could also be the result of the low power of out-of-sample forecast comparison tests. Inoue and Kilian (2004) also question the practice of evaluating a model based on its out-of-sample performance (as opposed to its in-sample fit).

Notwithstanding the causes (e.g., regime change), most of the current empirical evidence suggests that indicators of economic activity are weak predictors of inflation. This is especially true in the most recent years (post 1984) when forecasting inflation has become increasingly harder in the sense of providing forecast gains over time series models (Stock and Watson, 2007). Despite the availability of extensive literature on inflation forecasting, little or no attention has been paid to examining whether indicators of economic activity carry useful information about the dynamics of higher moments, beyond the mean. For example, having some idea on the conditional second-order moment of future inflation can be vital in assessing the risk to inflation stability due to macroeconomic shocks. Greenspan (2004) discusses this issue in the following terms: “Given our inevitably incomplete knowledge about key structural aspects of an ever-changing economy and the sometimes asymmetric costs or benefits of particular outcomes, a central bank needs to consider not only the most likely future path for the economy, but also the distribution of possible outcomes about that path. The decision-makers then need to reach a judgment about the probabilities, costs, and benefits of the various possible outcomes under alternative choices for policy” (p. 37). While average future inflation may signal the direction of the economy, it cannot help policy makers to evaluate the risks of deviations from the most likely path and the cost for the economy of such deviations. In a recent paper, Kilian and Manganelli (2008) introduce a model in which the monetary policy maker is viewed as a risk manager trying
to balance the risks to inflation and output stability. In this framework, if the preferences of the policy maker are assumed to be quadratic and symmetric, then the only relevant moment (of the inflation and output distributions) is the conditional mean. However, they provide evidence of departure of the preferences from such a benchmark. All the above elements point to the suggestion that forecasting the distribution of inflation represents a relevant tool in the conduct of monetary policy. In fact, the Bank of England has been publishing for quite some time the so-called “fan charts” that represent the subjective forecasts of the Bank about the future distribution of inflation.

In this paper, departing from the existing focus on conditional mean forecasting, we explore whether leading indicators of economic activity are useful in predicting the distribution of future inflation. To incorporate macroeconomic variables into the prediction of the conditional distribution of future inflation, we introduce a semi-parametric method using conditional quantiles. The approach considers several conditional quantiles of future inflation, and by doing so, it offers more flexibility (than, for example, the conventional PC models) in capturing the possible role of macroeconomic indicators in predicting the different parts of the inflation distribution. For instance, one may be able to investigate if some periods of low or high inflation are driven by some macroeconomic indicators. Surely such information cannot be delivered by PC-type models that deal only with predicting the average or typical inflation. We specify the conditional mean of future inflation to follow a univariate time series process and assume that the leading indicators are the driving factors in explaining the dynamics of the distribution of the forecasting errors. Our set-up that the conditional mean follows a time series process is motivated by the overwhelming empirical evidence that, at least for the post-1984 period, incorporating macroeconomic information in the conditional mean does not deliver superior forecasts compared to pure time series models. To estimate the conditional distribution of the forecasting errors we use linear quantile regression that relates the quantiles of the errors to the economic indicators. To assess the benefit of conditioning on the leading indicators, we compare the out-of-sample forecast densities of the proposed method with some benchmark (time series) models that assume independent forecasting errors. We also explore whether density forecasts that use
individual leading indicators can be combined to deliver better forecast accuracy. A great number of studies on average inflation forecasting report that combining forecasts tends to outperform individual forecasts.

We provide new empirical evidence of predictability of US core monthly inflation (core inflation is measured after excluding food and energy), for which we find that indicators of economic activity are useful in forecasting its distribution, especially when using unemployment rate, housing starts, and the term spread. Interestingly, the empirical findings apply to a forecast evaluation period that is intentionally chosen to be post 1984, when the existing literature shows that macroeconomic indicators are not relevant to predict future average inflation. We attribute this result to the ability of the semi-parametric method to account for the differing relationship between inflation and indicators at different quantiles of inflation. For some indicator variables, we also find an asymmetric effect in the sense that an indicator is more relevant on the lower part of the forecasting distribution than the upper part (and viceversa, depending on the indicator considered). These observed quantile effects take place far away from the center of the distribution, making them difficult to be detected with approaches (like PC-type models) that solely focus on evaluating the relevance of these variables in predicting the conditional mean. To illustrate the aforementioned effects, we consider the late 1990s when inflation was at historically low levels despite the possible rising inflation signaled by most indicators of economic activity. During this period, unemployment rate was decreasing, and went even below 4% at the beginning of 2000. Our analysis shows that when unemployment rate is incorporated into forecasting the future inflation distribution, the “expected” negative relationship between unemployment and inflation was mostly at work in the tails of the inflation distribution, and more so at lower quantiles. This observation is of practical relevance to policy makers when assessing the possibility of such events as deflation, which was the case at the beginning of 1998.

A few, yet increasing, research exists that relates to our work in the sense of dealing with distributional aspects of inflation. Robertson et al. (2005) forecast the distribution of inflation based on a VAR specification. In addition, they propose a methodology to “twist”
the forecasting distribution in order to incorporate theoretical restrictions (e.g., a Taylor rule). Cogley et al. (2005) propose a Bayesian VAR model where both the conditional mean and variance are time varying. They forecast inflation for the UK and illustrate their method by comparing interval forecasts from their model to the fan charts of the Bank of England. Corradi and Swanson (2006) evaluate the performance of time series and PC models in forecasting one-month ahead inflation using different distributional assumptions for the error term. Amisano and Giacomini (2007) forecast the distribution of inflation at the one-month horizon using a Markov Switching model and find that the forecasts from the nonlinear specification are more accurate compared to a linear one.

The rest of the paper is organized as follows. In Section (2) we introduce a quantile based semi-parametric approach for predicting the inflation distribution. Section (3) outlines a test of predictive accuracy that is used to evaluate the conditional distribution models discussed in Section (2). In Section (4), we present (with discussion) the empirical findings of the paper. Finally, Section (5) concludes.

2 Models

We denote the annualized inflation over a $h$-month period by $Y^h_t = (1200/h)[\log P_t - \log P_{t-h}]$ and the one-month annualized inflation by $Y_t = 1200[\log P_t - \log P_{t-1}]$ where $P_t$ is the level of the price index in month $t$. Also let $X^i_t$ be some indicator of real economic activity such as unemployment rate. A baseline specification often used in forecasting inflation is the PC model, although in recent years different studies have questioned its predictive power (see Atkenson and Ohanian, 2001, and Fisher et al., 2002). In this paper we consider the generalized PC model of Stock and Watson (1999) that postulates that changes in $h$-month inflation, $Y_{t+h}^h$, depend on recent changes in one-month inflation and past and present values of a candidate economic leading indicator,

$$Y_{t+h}^h - Y_t = \mu_0 + \sum_{j=0}^{p-1} \beta_j \Delta Y_{t-j} + \sum_{k=0}^{q-1} \gamma_k X^i_{t-k} + U_{t+h}$$

(1)
where the error term $U_{t+h}$ has a zero conditional mean. Note that the above specification assumes that $Y_t$ has a unit root. There is no consensus yet on the stationarity of inflation (see the recent work by Stock and Watson, 2007, and Ang et al., 2006, for opposite views). The monthly frequency considered in this paper is likely to provide more persistence compared to the quarterly frequency that is used in most studies.

In evaluating the forecasting performance, the PC model is often compared with two time series models: the autoregressive (AR) model and the naïve or random walk model. Although simple, these two time series models are very competitive benchmarks. For example, Atkinson and Ohanian (2001) find that various PC specifications do not outperform the naïve model for the period 1984-1999. The naïve model specifies that the expected inflation over the next $h$ months is equal to inflation over the previous $h$ months, i.e.,

$$Y_{t+h}^h - Y_t^h = U_{t+h}. \quad (2)$$

The AR model is a special case of the PC model where no information on present and past values of $X_i^t$ are included, i.e.,

$$Y_{t+h}^h - Y_t = \mu_0 + \sum_{j=0}^{p-1} \beta_j \Delta Y_{t-j}^h + U_{t+h}. \quad (3)$$

As mentioned earlier, much of the focus in the literature has been on predicting the mean of future inflation using PC models, and the evidence suggests that indicators of economic activity are weak predictors of its mean dynamics. This is especially true in the most recent years where forecasting mean inflation has become increasingly harder in the sense that PC models are unable to provide forecast gains over time series models (naïve or AR models), see for example Stock and Watson (2007). On the other hand, little or no attention has been paid to examine whether indicators of economic activity carry useful information about the dynamics of higher moments, and hence help improve the accuracy of density forecasts of inflation. The implicit assumption is that the error $U_{t+h}$ in the PC specification in Equation (1) is independent of the past and present values of the economic indicator ($X_i^t$). In other words, the effect (if any) of the macroeconomic variable on the
conditional distribution of $Y_{t+h}$ is only limited to the conditional mean.

In this paper we do not restrict ourselves to just the first moment (the conditional mean) and instead consider the estimation of the forecast density of $Y_{t+h}$ conditional on the available information set at time $t$. Let the past and present values of a particular indicator variable, $X^i$, be denoted by the vector $\tilde{X}^i_t=(X^i_t, \cdots, X^i_{t-q+1})$. We also assume that a time series model is the true model for predicting the conditional mean of $Y_{t+h}$. In particular, we specify $Y_{t+h}$ as a naïve model in (2). This implies that $\tilde{X}^i_t$ does not carry any relevant information for predicting the mean, which is consistent with the existing empirical evidence after 1984. We argue that $\tilde{X}^i_t$ may have an effect on higher-order moments of $Y_{t+h}$ and, more generally, on the conditional density of $Y_{t+h}$, which is not permitted in a PC-type specification.

In the rest of the paper, we use the notation AO-$U|X^i$ to denote a predictive model where the conditional mean follows the naïve model of Atkinson and Ohanian (2001) (denoted by AO) and the error, $U_{t+h}$, is dependent on $\tilde{X}^i_t$. Let the conditional density of $U_{t+h}$ be

$$h(u|\tilde{X}_t^i) = \frac{d}{du}H(u|\tilde{X}_t^i)$$

where $H(\cdot|\cdot)$ is the conditional CDF of $U_{t+h}$. Then, we define the forecast density of $Y_{t+h}$ implied by AO-$U|X^i$ as

$$f^i_{t+h|t}(Y^i_{t+h}) = h(U_{t+h}|\tilde{X}^i_t).$$

In a similar logic of notation, the naïve model with errors independent of $\tilde{X}^i_t$ is denoted AO-$U$ (Equation 2), the generalized PC model of Stock and Watson (1999) (Equation 1) is

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1The choice of the naïve model for the conditional mean of the inflation process is motivated by the overwhelming evidence that this specification outperforms both AR and PC-type models in out-of-sample forecasting (at least for the post-1984 period). As we will see in Section (4), the forecast evaluation period is intentionally chosen to be post 1984.

2Note that the $h$-step ahead forecast errors $U_{t+h}$ will follow a moving average process of order $h-1$, $MA(h-1)$. With a slight abuse of terminology and notation, in the rest of the paper we refer to the forecast errors as “independent” when the MA-filtered forecast errors are assumed independent of the indicators, and as “dependent” when the filtered forecast errors are modeled conditional on the $\tilde{X}_t^i$. Hence, when referring to the forecast errors as (in)dependent, we are actually pointing to the filtered forecast errors, that is, after the $MA(h-1)$ structure has been removed.
denoted by PC-U and the standard AR model by AR-U (Equation 3). For the models AO-U, AR-U and PC-U, the respective \( h \)-step forecast densities are estimated via a smoothed empirical distribution of the forecasting errors \( U_{t+h} \).

We now outline a simple approach to estimate the forecast density of AO-U|\( X_i \), i.e. \( f_{t+h|t}(Y^h_{t+h}) \). Note from (5) that it is sufficient to estimate \( h(u|\tilde{X}_t) \). One possible method of estimation may be to use Hansen (1994) by assuming a parametric distribution for \( U_{t+h} \), and then allowing the higher-order parameters (such as skewness) to depend upon \( \tilde{X}_t \). For example, Hong et al. (2007) use this approach to estimate forecast densities of (high frequency) exchange rates by assuming a generalized skewed-\( t \) for the standardized error distribution, and allow the skewness and kurtosis follow an autoregressive process. Ideally, if \( h(u|\tilde{X}_t) \) can be represented by a few dimensional parametric distribution, Hansen’s approach can be useful to identify which higher-order dynamics (variance, skewness or kurtosis) are affected by the macroeconomic indicator. Although this direction is worth investigating, we instead use a quantile regression approach which is direct and does not require any parametric assumption.\(^3\)

Denote the \( \alpha \in (0, 1) \) conditional quantile of \( U_{t+h} \) conditional on \( \tilde{X}_t \) by \( Q_{t+h}(\alpha|\tilde{x}_i^t) \). We estimate \( Q(\alpha|\tilde{x}_i^t) \) using a linear quantile regression model (Koenker and Bassett, 1978),

\[
Q_{t+h}(\alpha|\tilde{x}_i^t) = \delta_{0,\alpha} + \sum_{k=1}^{q} \delta_{k,\alpha} x_{i,t-k+1}^t.
\]

Although the local effect of \( x_{i,t-k+1}^t \) on the \( \alpha \)-quantile is assumed to be linear, the model is very flexible because each slope coefficient \( \delta_{k,\alpha} \) is allowed to differ across quantiles. This is a useful property since it provides guidance as to where in the distribution of \( Y^h_{t+h} \) the indicator \( X_i^t \) has a significant effect. Of course, the effect of a macro variable \( X_i^t \) may well be non-linear. Possible non-linearity can be easily entertained by extending (6) to additive models; see for example, de Gooijer and Zerom (2003), among others. Additive quantile models can be estimated with no added difficulty over their linear counterpart. We think

\(^3\)In a recent paper, Cenesizoglu and Timmermann (2008) use quantile regression to investigate predictability of the distribution of stock returns.
that linear quantiles are already flexible enough to capture higher order features of the
forecast errors under our set-up. We estimate $h(u|\tilde{X}_i^t)$ using (4) where

$$
\hat{H}(u|\tilde{X}_i^t) = \int_0^1 1\left(Q_{t+h}(\alpha|\tilde{X}_i^t) \leq u\right) d\alpha
$$

(7)

with $1(A)$ denoting an indicator function of set $A$. An advantage of (7) is that even when the
conditional quantile $Q_{t+h}(\alpha|\tilde{X}_i^t)$ may not be monotonic in $\alpha$, the conditional distribution
$\hat{H}(u|\tilde{X}_i^t)$ stays monotonic in $u$, see Chernozhukov et al. (2006) for more details.

In an ever evolving macroeconomic environment, a particular prediction model might out-
perform alternative models in one period and not in others. Thus, averaging different
forecasts may provide superior performance over time. In fact, the literature on condi-
tional mean forecasting has documented that combining forecasts from different models
typically achieves better performance compared to the (best) individual models, see for
example, Stock and Watson (1999) and Ang et al. (2006). In addition, simple combination
schemes such as averaging forecasts, achieves better performance than more sophisticated
schemes. For an extensive survey of the empirical evidence and the motivation for com-
bining forecasts, see Timmermann (2006). To explore whether the idea of combining also
extends to density forecasts\footnote{An example in the context of density forecasts is Mitchell and Hall (2005) that investigate whether combining density forecasts (for UK inflation) produced by the Bank of England and the NIESR achieves better results compared to using the individual forecasts.}, we average the forecast densities from various economic indi-
cators, i.e. $f_{t+h|t}(Y_{t+h}^i)$ for $i = 1, \ldots, I$ where $I$ is the total number of economic indicators.

In so doing, we use a simple equally weighted averaging as follows,

$$
f_{t+h|t}(Y_{t+h}^{\text{(comb)}}) = \frac{1}{I} \sum_{i=1}^I f_{t+h|t}(Y_{t+h}^i).
$$

Note that the weights are assumed to be constant over time. Other schemes for combining
forecasts have been proposed in the literature in the context of conditional mean forecasts
that can be easily adapted to density forecasts. For example, the weights used in combining
could be varying over time based on the recent accuracy of the density forecasts.
3 Measuring relative predictability of models

In this paper our focus is on forecast distribution of future inflation. Clearly, this raises the question on how to measure the relative accuracy of a particular forecast density as compared to a certain benchmark. Much of the approaches to evaluate forecast densities have mainly focused on their absolute accuracy by developing tests that examine their dynamical and distributional misspecification, see for example Diebold et al. (1998) and Hong et al. (2007). However, it is very likely that empirical forecasting models are, to some extent, almost always misspecified. In this sense, an “absolute” evaluation measure of one or more forecast densities would not be that informative. On the other hand, we might be willing to accept a possibly misspecified model if it provides a more accurate forecast density relative to another model. This is the approach we take to evaluate the relative accuracy of a particular forecast density. Let’s assume there are two forecasting methods used to estimate the density forecast of the $h$-month ahead inflation, $Y_{t+h}^h$, where one of them is the benchmark model. As benchmark model, we consider AO-U or AR-U, and we denote the benchmark forecast density by $f_{t+h|t}^0(Y_{t+h}^h)$ where $0 \in \{\text{AO}, \text{AR}\}$. These benchmark forecast densities are separately compared against alternative models that incorporate the effect of macroeconomic indicators. For the latter, we consider density forecasts from PC-U (denoted by $f_{t+h|t}^{\text{PC}}(Y_{t+h}^h)$), AO-U$|X^i$ ($f_{t+h|t}^{i}(Y_{t+h}^h)$) and the combined density forecasts ($f_{t+h|t}^{(\text{comb})}(Y_{t+h}^h)$). For the purpose of this section, let’s denote these alternative models by $f_{t+h|t}^1(Y_{t+h}^h)$ where $1 \in \{\text{PC}, i, \text{comb}\}$.

We adopt a rolling window approach when generating the out-of-sample density forecasts. Let $T$ be the total number of available observations and $t_0$ be the first forecast base. This means that there are $t_0$ observations up to and including the $t_0$-th observation. By rolling it is meant that the forecast base $t$ extends as far as $T - h$ where $h$ is the forecast horizon. Hence, we have $t = t_0, t_0 + 1, \ldots, T - h$. The goal is to compare the relative accuracy of the two forecast densities even if both models may be misspecified. In other words, which forecast density provide better predictability. We use an intuitively simple metric introduced by Giacomini and White (2006) and Amisano and Giacomini (2007) although
other similar suggestions can also be used, see Mitchell and Hall (2005) and Bao et al. (2007). This metric is based on the average logarithm score of two competing forecast densities defined as follows

\[
\frac{1}{N} \sum_{t=t_0}^{T-h} \log f^0_{t+h|t}(Y^h_{t+h}) \quad \text{and} \quad \frac{1}{N} \sum_{t=t_0}^{T-h} \log f^1_{t+h|t}(Y^h_{t+h})
\]

where \( N = T - h - t_0 + 1 \). Note that the forecast densities are evaluated at the realized \( h \)-step true value, \( Y^h_{t+h} \). Then, the forecast density (\( f^0 \) or \( f^1 \)) with higher average logarithm score is said to be relatively more accurate. Based on this idea, Amisano and Giacomini (2007) introduce a test procedure to evaluate the null hypothesis of equal density forecast accuracy. Let

\[
W_{LR_t} = \log f^0_{t+h|t}(Y^h_{t+h}) - \log f^1_{t+h|t}(Y^h_{t+h}), \quad t = t_0, t_0 + 1, \ldots, T - h
\]

where the null hypothesis of the test is

\[
H_0 : \quad E(W_{LR_t}) = 0.
\]

Note that \( W_{LR_t} \) can also be weighted if one is interested to focus only on a certain aspect of the distribution (such as the center or the tails). In this paper, we compare forecast densities in the complete range of variation of the variable and hence weighting is not applied. The test for equal accuracy of the density forecasts is based on the AG (Amisano and Giacomini) test statistics,

\[
AG = \frac{\overline{W_{LR}}_N}{\hat{\sigma}_N / \sqrt{N}}
\]

where \( \overline{W_{LR}}_N \) is the sample average of \( W_{LR_t} \) and the variance of the test statistic is of the HAC type to correct for heteroskedasticity and autocorrelation. The AG statistic is asymptotically standard normal distributed. Rejections that occur for \( AG < 0 \) indicate that \( f^1(\cdot) \) provides more accurate density forecasts relatively to \( f^0(\cdot) \), and vice-versa for \( AG > 0 \). One can also examine the pattern of \( W_{LR_t} \) to see if the predictive ability of a particular economic indicator varies over time.

In order to put our contribution in the context of the existing empirical evidence on US
inflation forecasting, it is also necessary to evaluate the relative accuracy of models in terms of their point forecasts. In so doing, we use the Root Mean Square Prediction Error (RMSPE) of the conditional mean forecasts of PC-\(U\) (where \(X^i\) is allowed to affect the conditional mean) relative to the RMSPE of AO-\(U\)\(^5\). By construction, the relative RMSPE of AO-\(U|X^i\) is 1 because the conditional mean of the latter model is the same as AO-\(U\). Post 1984, the overwhelming majority of empirical studies on forecasting inflation suggests that indicators of economic activity do not carry much relevant information about the conditional mean of the inflation process. When this is the case, we expect the relative RMSPE of PC-\(U\) to be greater than or equal to 1. The forecast density of AO-\(U\) is also expected to be more accurate than that of PC-\(U\) because the only part of the conditional distribution that is allowed to depend on \(\tilde{X}_t\) is the conditional mean. Thus, for the PC-\(U\) model, both the AG-test and relative RMSPE are likely to lead to the same conclusion. On the other hand, if the forecast density of AO-\(U|X^i\) is more accurate than AO-\(U\) as reflected in the AG-test, this suggests that \(\tilde{X}_t\) carries relevant information for moments beyond the conditional mean. So, in summary, a particular macroeconomic indicator is said to have a dynamic effect on higher-order conditional moments of \(h\)-step future inflation when the following occur: (a) The relative RMSPE of PC-\(U\) \(\geq 1\) and (b) The AG-test shows that the forecast density of AO-\(U|X^i\) is more accurate than AO-\(U\).

4 US inflation density forecasts

This Section presents results on the application of models introduced in Section (2) in order to explore the out-of-sample predictability of the distribution of US inflation using leading indicators of economic activity. We use four measures of the monthly price index (\(P_t\)): Consumer Price Index for all items (CPI), CPI excluding food and energy (core-CPI), Personal Consumption Expenditure deflator (PCE), and the PCE excluding food and energy (core-PCE). We follow the inflation forecasting literature (see Stock and Watson, 1999, and Ang et al., 2006) and include six of the indicators of economic activity that are often

\(^5\)As discussed earlier, the literature indicates that the AO model is the best performing model and should be considered as the benchmark in evaluating alternative models. Instead, using the AR would give the idea that alternative approaches are indeed useful only because of the sub-optimal choice of benchmark.
considered as predictors of inflation: the civilian unemployment rate (UNEM), the index of industrial production (IP), real personal consumption expenditure (INC), employees on non-farm payrolls (WORK), housing starts (HS), and the term spread (SPREAD) defined as the yield on the 5-year Treasury bond minus the 3-month Treasury bill. In terms of the notation used in Sections (2) and (3), the activity variables are denoted by $X_i^t$ where $i \in \{\text{UNEM, IP, WORK, HS, INC, SPREAD}\}$. All the data were gathered from the Federal Reserve Bank of Saint Louis database FRED and the sample period spans from January 1959 until December 2007\textsuperscript{6}. Some of the leading indicators (i.e., IP, INC, and WORK) are not stationary. We thus consider these variables in gap form where the long-run trend is modeled using a Hodrick and Prescott (1997) filter (HP) with parameter equal to 14400 (typically used for monthly data)\textsuperscript{7}. The trend is estimated only on information available at the time the forecast is made. For the estimation we use a rolling window scheme as described in Section (3). Our first forecast is January 1985 and the models are estimated on the window 1959:1 to 1984:12 minus the forecasting horizon $h$ (equal to 6 and 12 months\textsuperscript{8}). The next forecast is for February 1985 and so on. The size of the rolling window is kept constant by dropping one observation at the beginning of the sample. We report forecasting results for two sub-periods, 1985:1 to 1995:12 and 1996:1 to 2007:12. This allows us to evaluate whether there is any significant change in predictability of the macroeconomic variables.

4.1 Results

We report results for CPI and PCE in Table (1) and Table (2), that correspond to $h = 12$ and $h = 6$, respectively. Similarly, we give results of core-CPI and core-PCE in Table (3) and Table (4).

\textsuperscript{6}The macroeconomic series consists of revised data available at the January 2008 vintage due to the lack of a comprehensive real-time dataset at the monthly frequency.

\textsuperscript{7}We also considered a quadratic trend as in Ang et al. (2006) but the results are very similar to the HP filter. To conserve space we decided to report only the results of the HP filter.

\textsuperscript{8}We also used a one quarter horizon. However, the results were largely similar to the semi-annual and annual horizon and decided not to report them in this paper.
CPI and PCE

For both $h = 12$ and $h = 6$, the ratio of the RMSPE of AR-$U$ to the AO-$U$ is larger than one, indicating that the naïve predictor of the conditional mean outperforms the AR model. The same result also holds when the PC-$U$ (conditional mean) forecasts are compared to the naïve forecasts, confirming the earlier results in the literature on the inability of activity indicators to predict mean inflation. With regard to the density forecasts, we focus on the AG statistic that uses the benchmark (time series) models AR-$U$ and AO-$U$. When this statistic is negative and significant, it indicates that the alternative model (PC-$U$ or AO-$U|X^i$) provides more accurate density forecast compared to the benchmark model. Examining the AG test, we observe that none of the macroeconomic variables have any predictive power for CPI. For PCE, there seems to be some evidence of predictive ability using housing starts (HS) and the term spread (SPREAD), but this occurs only in the first sub period, 1985-1995. Further, when combining the semi-parametric densities of AO-$U|X^i$, the resulting density outperforms that from AO-$U$ although this improvement is restricted once again to the first sub-period.

Core-CPI and Core-PCE

Considering the results for forecasting the mean process, we find that using the activity indicators, as in the PC-$U$, does not improve forecasts compared to simple time series models, which is consistent with existing empirical evidence. On the density forecasts, the negative and convincingly significant AG statistics values show that the semi-parametric method outperforms the AR-$U$ density forecasts at both horizons and sub-periods. When the benchmark time series model is AO-$U$, the evidence is mixed. At the annual horizon ($h = 12$), a large number of activity indicators are useful in providing more accurate forecasts of the distribution of inflation for core-PCE (in particular in the second sub-period). For predicting core-CPI at $h = 12$, unemployment rate, housing starts, and the term spread are found to be useful in the first sub-period while only unemployment is able to extend its relevance to the second sub-period. When considering core-CPI at $h = 6$, all indicator variables are found to be significant as reflected in their respective AG values.
(compared to AO-U) in the first sub-period. The usefulness of half of the indicators also extends to the second sub-period. For core-PCE and at $h = 6$, predictive relevance of the indicator variables appears to be concentrated in the second sub-period. For both core-CPI and core-PCE, combining forecast densities of individual indicators is able to deliver better accuracy at both horizons.

4.2 A closer look

One interesting conclusion that emerges from our out-of-sample predictability evidence is that activity indicators provide improvements in accuracy of the density forecasts for core-CPI and core-PCE, although they seem to be uninformative when forecasting CPI and PCE. We now provide a more detailed analysis using forecast intervals and forecast densities in order to gain some understanding of the reasons underlying this result. The discussion also highlights the unique insights that the distribution approach offers, which otherwise would be ignored if we only focused on conditional mean based forecasts. The analysis will be restricted to PCE and core-PCE inflation measures. The conclusions that can be drawn from the results of CPI and core-CPI are qualitatively similar.

Figure (1) (mid-plot) gives a 90% forecast interval (at $h = 12$) of PCE inflation for AO-U and AO-U$|X^{UNEM}$ (the latter is the semi-parametric model that conditions on the unemployment rate). In Figure (1) (bottom-plot), we report $WLR_t$ (see Equation 8) where we smooth it to eliminate short-run fluctuations for better exposition. The shaded areas in the Figure represent the months when $WLR_t$ is negative, indicating that AO-U$|X^{UNEM}$ is more accurate compared to the AO-U model. Note that for a large part of the forecasting evaluation period, the semi-parametric densities outperform the naive density forecasts. A significant difference between the implied forecast intervals of the above models occurs in the late 1990s when the historically low levels of unemployment (shown in the top part of the Figure) predict an increase in inflation, in particular at the lower end of the inflation distribution. This feature seems to indicate that the expected negative relationship between

\footnote{We shifted the variable to be aligned with the forecasting target date. So, the value, e.g. in January 1985, actually refers to January 1984 and represents the value of the activity indicator used in producing the forecast for the target date.}
the two variables might be more relevant at low quantiles of the inflation distribution rather than at the median or high quantiles. We will return to this issue later, since this effect is more pronounced (and statistically relevant) for the core inflation measures. From the $WLR_t$ plot, it can be observed that positive values of $WLR_t$ (meaning that AO-$U$ is more accurate than the AO-$U|X^{UNEM}$) are mainly associated with periods of rapid change in inflation. An example of this occurs in the months during and after the recession of 1990-1991, when PCE inflation experienced a rapid decline, and the period between 1997 and 2003, when PCE inflation fluctuated between below 1% and peaked (at the beginning of 2000) around 3%. In part, this can explain the earlier findings that, on average (see the AG test result), the semi-parametric method does not (significantly) outperform the naïve random walk model with independent forecast errors. Although AO-$U|X^{UNEM}$ delivers more accurate forecasts in a large fraction of the forecasting sample, the forecasting gain is wiped out during the periods of sudden change of the inflation rate.

Figure (2) is similar to Figure (1) except that the conditioning variable for the semi-parametric method is housing starts (HS). We have seen earlier that (see Table 1) AO-$U|X^{HS}$ outperforms the naïve model in the period 1985-1995, but not in 1996-2007. For the period 1985-1995, the Figure shows that the 90% forecast interval from the semi-parametric distribution is narrower than that for AO-$U$. The slow and steady decrease in housing starts that occurred from the mid-1980s to January 1990 exerts a downward pressure on the inflation rate. However, this effect seems to take place mostly at higher quantiles of the inflation distribution and less so at low quantiles. Also notice the gaps between the forecast interval bounds of AO-$U$ and AO-$U|X^{HS}$. For example, the maximum difference between the intervals is reached in January 1992\textsuperscript{10} when the AO-$U|X^{HS}$ upper bound is 1.38% lower compared to the corresponding bound for AO-$U$ and the lower bound was 0.52% below. To further evaluate the different implications of the methods, we show in Figure (3) the one-year ahead forecast densities of AO-$U$ and AO-$U|X^{HS}$ based on the information available in January 1991. Note that conditioning on the level of housing starts leads to the narrowing of the spread of the AO-$U|X^{HS}$ distribution compared to the naïve.

\textsuperscript{10}Which corresponds to 12 months after the housing starts indicator reached its bottom.
It also results in a shift in central location due to the significance of housing starts for the conditional median. Although, for both distributions, the realization of inflation is far in the left tail, it seems that the $AO-U|X^{HS}$ provides a more precise prediction of the distribution of PCE inflation.

Moving to the period 1996-2007, the result of the AG statistic (Table 1) indicates that there is no significant difference in accuracy between the two forecast densities considered. But, even in this sub-period, housing starts plays a significant role in changing the forecasting distribution (compared to $AO-U$). Because this period is characterized by higher levels of housing starts, the outcome is an upward pressure on the inflation distribution that is visible in the upper limits of the 90% forecasting interval. This turn out to be useful at the end of 2004 and beginning of 2005 when the realization of inflation happens to be outside the forecasting interval of $AO-U$, but within the interval for the semi-parametric method conditional on housing starts. Figure (4) shows the density forecasts for October 2004 (based on information available 12 months earlier). The foregoing analysis shows that even when statistical tests (AG test) may not show better accuracy for $AO-U|X^i$, it may still be the case that a particular indicator (for example, unemployment rate and housing starts) provide useful information for a forecaster. For example, the forecaster can use it to assess the possible outcomes that macroeconomic events might have on the future evolution of inflation.

Another related issue, in light of the empirical findings, is the relative importance of the various economic activity indicators. It may be that some episodes of increase or decrease in inflation might be driven by different factors. To illustrate this point we report the out-of-sample $WLR^i_t$ (of the six indicators that we consider) in Figure (5). From the Figure, there appears to be a marked difference between the sub-periods 1985-1995 and 1996-2007. In the first sub-period, the performance of the semi-parametric method (compared to the naïve model) varies significantly depending on the indicator used. On the other hand, it seems to be more synchronized in the second sub-period, when their performance (against $AO-U$) broadly follows a similar pattern. For example, looking at the performance in the
period after the 1990-91 recession, some variables (UNEM, IP GAP, WORK GAP, and SPREAD) were less accurate compared to the naïve model, although HS and INC GAP outperformed the AO-U model. Interestingly, the dispersion (or lack of) in performance has implications for the potential gains from aggregating densities through combination. While in the first sub-period there is a clear advantage in combining (rather than relying on just an individual indicator) due to the different performance of the indicators, in the second sub-period all the variables provide similar performances, hence compromising the usefulness of combining. This explains the result for the combined density forecasts in Table (1) where the AG statistic was significantly negative in the first sub-period, but insignificant in the latter.

Finally, we consider core-PCE and, to save space, only focus on the unemployment rate as indicator variable. As given in Figure (6), for the period 1985-1995, the semi-parametric method that uses the unemployment rate as conditioning variable (AO-U|X^{UNEM}) provides forecast intervals that are shifted downward compared to AO-U. In addition, starting in 1996 there is a remarkable difference between the forecast intervals for the two methods, in particular at the lower quantile of the forecasting distribution. This result may be attributed to the upward pressure on inflation deriving from the persistent decrease in the unemployment rate in the late 1990s. The fact that unemployment was at historically low levels and approached 4% at the beginning of 2000 explains the shift of the forecast distribution to higher levels (compared to the naïve model), mostly at low quantiles of the inflation distribution. In Figure (7), we show the forecast densities for AO-U and AO-U|X^{UNEM} based on information available in December 1997 for the target date December 1998. We chose this specific date because at the end of 1997 and beginning of 1998 a debate on the possibility that the U.S. economy could enter a period of deflation was started. The Figure shows that a forecaster using the AO-U model would have assigned a 9.18% probability to such an event, while the probability of deflation based on the AO-U|X^{UNEM} density forecast was 0.074%. In Figure (8) we report the time series of the (predictive)

\footnote{The debate was also sparked by a speech by the Federal Reserve Board Chairman on January 3rd, 1998 (see Greenspan, 1998). Figure (7) show the density that a forecasters would have produced based on the information available in December 1997.}
probability of deflation for the two methods. A forecaster using the AO-U model would have predicted the probability of deflation to be larger than 5% for the period between 1998 and June 2000 and from the end of 2004 to the beginning of 2005. The largest probability of the event occurring was 18.2% for the target date of June 1999. On the other hand, conditioning the density forecasts on the unemployment rate delivered a forecast of the probability of deflation for June 1999 of 1.18% (the highest probability in the forecast sample). This example supports the relevance of forecasting the distribution of inflation in evaluating events of interest and the usefulness of considering economic indicators in providing more accurate predictions.

5 Conclusion

Most studies on U.S. inflation forecasting have focused on predicting the mean inflation using time series and PC models. The findings indicate that using real economic indicators (such as unemployment or the output gap) improve out-of-sample forecasting performance during the late 1970s and the first half of the 1980s. But after 1985, PC based forecasts do not lead to forecasting gains vis-á-vis time series models (autoregressive and random walk models) when the latter models became a lot harder to outperform. This paper examines whether indicators of economic activity carry relevant information about the dynamics of higher moments of inflation, and hence help improve the accuracy of the distribution of inflation. We forecast (out-of-sample) the distribution of inflation for 6 and 12 months ahead for the period 1985:1 to 2007:12, and evaluate the performance of the various models in two sub-samples (1985:1 to 1995:12 and 1996:1 to 2007:12). Our results show that for the core inflation measures, conditioning the dynamics of the inflation distribution on the leading indicators provides more accurate forecasts relative to time series models. In particular, we show that the activity indicators are relevant in driving the low and high quantiles of the inflation distribution. The latter result can be particularly important for policy makers interested in evaluating the probability of certain events, such as whether inflation will be above or below a certain level in the future.
References


### Table 1: $h = 12$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Method</th>
<th>CPI</th>
<th>PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSPE</td>
<td>AG test</td>
<td>RMSPE</td>
</tr>
<tr>
<td></td>
<td>ratio</td>
<td>AR-U</td>
<td>AO-U</td>
</tr>
</tbody>
</table>

| AR-U     | 1.131 | 1.414 | 1.029 | 0.504 | 1.202 | 3.826 | 1.05 | 0.484 |

| UNEM     | PC-U   | 0.992 | -0.107 1.651 | 1.050 | 0.786 | 0.805 | 1.010 | -0.549 | 3.986 | 1.033 | 0.513 | 0.573 |
|          | AO-U| 0.047 | -0.898 | 1.253 | -0.156 | 0.261 | -2.112 | 0.162 | -0.875 | -0.512 |

| IP GAP   | PC-U   | 1.061 | 0.898 | 1.637 | 0.991 | -0.294 | 0.365 | 1.061 | 1.298 | 3.569 | 0.952 | -0.020 | 0.520 |
|          | AO-U| 0.047 | -0.898 | 1.253 | -0.156 | 0.261 | -2.112 | 0.162 | -0.875 | -0.512 |

| INC GAP  | PC-U   | 1.085 | 1.274 | 1.913 | 0.951 | -1.99 | -0.276 | 1.101 | 0.870 | 2.803 | 0.989 | -0.443 | 0.368 |
|          | AO-U| 0.047 | -0.898 | 1.253 | -0.156 | 0.261 | -2.112 | 0.162 | -0.875 | -0.512 |

| WORK GAP | PC-U   | 1.022 | 0.062 | 1.295 | 1.049 | 0.186 | 0.548 | 1.039 | 0.775 | 3.450 | 1.044 | 0.642 | 0.741 |
|          | AO-U| 0.047 | -0.898 | 1.253 | -0.156 | 0.261 | -2.112 | 0.162 | -0.875 | -0.512 |

| HS       | PC-U   | 1.141 | 1.444 | 1.756 | 0.963 | 0.046 | -0.595 | 1.065 | -0.012 | 2.352 | 0.952 | -0.080 | 0.260 |
|          | AO-U| 0.047 | -0.898 | 1.253 | -0.156 | 0.261 | -2.112 | 0.162 | -0.875 | -0.512 |

| SPREAD   | PC-U   | 1.072 | 1.528 | 1.726 | 0.988 | -0.447 | 0.435 | 1.032 | 1.956 | 3.981 | 0.993 | -0.431 | 0.404 |
|          | AO-U| 0.047 | -0.898 | 1.253 | -0.156 | 0.261 | -2.112 | 0.162 | -0.875 | -0.512 |

| COMBINED | AO-U| 0.047 | -0.898 | 1.253 | -0.156 | 0.261 | -2.112 | 0.162 | -0.875 | -0.512 |

AR-U = AR model with $i.i.d$ forecasting errors (order selected by AIC), AO-U = Atkinson-Ohanian random walk model and $i.i.d$ forecasting errors, PC-U = Phillips curve model with $i.i.d$ errors, AO-U|Xq = AO conditional mean and semi-parametric error distribution conditional on the leading indicator ($q=1$). Combined refers to the forecasting distribution resulting from the combination of the AO-U|Xq for $\in$ {UNEM, IP, WORK, HS, INC, SPREAD}. RMSPE Ratio indicates the ratio of the RMSPE of a model forecasts compared to the RMSPE of AO-U. AG test indicates the Amisano-Giacomini test with null benchmark models AR-U and AO-U. In bold are denoted the (one-sided) rejections of the hypothesis of higher accuracy of the alternative model compared to the benchmark (at 5% significance level).
<table>
<thead>
<tr>
<th>Variable</th>
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<th>PCE</th>
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<td>X</td>
<td>0.092 &amp; 0.063</td>
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AR-U = AR model with i.i.d forecasting errors (order selected by AIC), AO-U = Atkerson-Ohanian random walk model and i.i.d forecasting errors, PC-U = Phillips curve model with i.i.d errors, AO-U|X | = AO conditional mean and semi-parametric error distribution conditional on the leading indicator (q=1). Combined refers to the forecasting distribution resulting from the combination of the AO-U|X | for ∈ {UNEM, IP, WORK, HS, INC, SPREAD}. RMSPE Ratio indicates the ratio of the RMSPE of a model forecasts compared to the RMSPE of AO-U. AG test indicates the Amisano-Giacomini test with null benchmark models AR-U and AO-U. In bold are denoted the (one-sided) rejections of the hypothesis of higher accuracy of the alternative model compared to the benchmark (at 5% significance level).
Table 3: $h = 12$

| Variable | Method | core-CPI | | | core-PCE | | |
|----------|--------|----------|----------|----------|----------|----------|
|          |        | RMSPE ratio | AG test | RMSPE ratio | AG test | RMSPE ratio | AG test | RMSPE ratio | AG test | RMSPE ratio | AG test |
| AR-U     |        | 1.064 | 9.765 | 1.175 | 4.578 | 1.248 | 13.47 | 1.287 | 2.637 |
| UNEM     | PC-U   | 0.963 | -6.998 | 7.216 | 1.225 | -0.218 | 4.135 | 1.043 | -0.736 | 6.663 | 1.129 | 0.578 | 1.917 |
| IP GAP   | PC-U   | 1.666 | -0.696 | 4.703 | 1.089 | -1.840 | 3.898 | 1.168 | 1.445 | 8.987 | 1.024 | 0.242 | 2.695 |
|          | AO-U| -6.959 | 0.030 | -4.018 | -0.852 | -6.276 | -0.817 | -7.810 | -1.453 |
| INC GAP  | PC-U   | 1.449 | -3.031 | 5.452 | 0.987 | -3.943 | 3.599 | 1.145 | 1.171 | 15.106 | 1.000 | 0.383 | 2.752 |
| WORK GAP | PC-U   | 1.654 | -0.586 | 3.747 | 1.001 | -5.353 | 2.987 | 1.19 | 0.92 | 6.43 | 1.03 | -0.36 | 2.62 |
|          | AO-U| -7.576 | 0.042 | -4.199 | -1.042 | -6.789 | -0.79 | -8.698 | -4.916 |
| HS       | PC-U   | 1.441 | -3.005 | 7.280 | 0.997 | -2.056 | 1.695 | 1.012 | -0.917 | 7.145 | 0.908 | -1.044 | 1.102 |
| SPREAD   | PC-U   | 0.946 | -6.259 | 7.306 | 1.012 | -0.269 | 4.663 | 0.992 | -1.824 | 7.448 | 0.962 | -1.413 | 2.329 |

AR-U = AR model with i.i.d forecasting errors (order selected by AIC), AO-U = Atkenson-Ohanian random walk model and i.i.d forecasting errors, PC-U = Phillips curve model with i.i.d errors, AO-U|X| = AO conditional mean and semi-parametric error distribution conditional on the leading indicator (q=1). Combined refers to the forecasting distribution resulting from the combination of the AO-U|X| for ∈ {UNEM, IP, WORK, HS, INC, SPREAD}. RMSPE Ratio indicates the ratio of the RMSPE of a model forecasts compared to the RMSPE of AO-U. AG test indicates the Amisano-Giacomini test with null benchmark models AR-U and AO-U. In bold are denoted the (one-sided) rejections of the hypothesis of higher accuracy of the alternative model compared to the benchmark (at 5% significance level).
Table 4: \( h = 6 \)

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AR-U = AR model with \( i.i.d \) forecasting errors (order selected by AIC), AO-U = Atkinson-Ohanian random walk model and \( i.i.d \) forecasting errors, PC-U = Phillips curve model with \( i.i.d \) errors, AO-U/X\(i\) = AO conditional mean and semi-parametric error distribution conditional on the leading indicator \( (q=1) \). Combined refers to the forecasting distribution resulting from the combination of the AO-U/X\(i\) for \( \in \{ \text{UNEM, IP, WORK, HS, INC, SPREAD} \} \). RMSPE Ratio indicates the ratio of the RMSPE of a model forecasts compared to the RMSPE of AO-U. AG test indicates the Amisano-Giacomini test with null benchmark models AR-U and AO-U. In bold are denoted the (one-sided) rejections of the hypothesis of higher accuracy of the alternative model compared to the benchmark (at 5% significance level).
Figure 1: (Top plot) Unemployment rate (shifted forward by $h = 12$ months), (Middle) 90% forecasting interval for AO-$U$ and AO-$U|X^{UNEM}$ with the shaded area denoting the months when the smoothed $WLR_t$ (with null model AO-$U$ and alternative AO-$U|X^{UNEM}$) is negative, and (bottom) the smoothed $WLR_t$. The vertical lines indicate the NBER recessions.
Figure 2: (Top plot) Housing starts (shifted forward by \( h = 12 \) months), (Middle) 90% forecasting interval for AO-U and AO-U|\( X^{HS} \) with the shaded area denoting the months when the smoothed \( WLR_t \) (with null model AO-U and alternative AO-U|\( X^{HS} \)) is negative, and (bottom) the smoothed \( WLR_t \). The vertical lines indicate the NBER recessions.
Figure 3: Predictive densities for PCE inflation using AO-U and AO-U|X^{HS}. The forecasting base is January 1991 and the horizon is 12 months. The vertical line represents the realization of PCE inflation in January 1992.
Figure 4: Predictive densities for PCE inflation using AO-U and AO-U|X^{HS}. The forecasting base is October 2003 and the horizon is 12 months. The vertical line represents the realization of PCE inflation in October 2004.
Figure 5: Smoothed $WLR^*_i$ for PCE inflation ($h=12$) with null model AO-U and alternative model AO-U|X$^i$ for $i \in \{\text{UNEM, IP GAP, INC GAP, WORK GAP, HS, SPREAD}\}$. Negative values indicate that the alternative model outperforms the null model (and vice versa).
Figure 6: (Top plot) the unemployment rate (shifted forward by $h = 12$ months), (Middle) 90% forecasting interval for AO-$U$ and AO-$U|X_{UNEM}^U$ with the shaded area denoting the months in which the smoothed $WLR_t$ (with null model AO-$U$ and alternative AO-$U|X_{UNEM}^U$) is negative, and (bottom) the smoothed $WLR_t$. The vertical lines indicate the NBER recessions.
Figure 7: Predictive densities for core-PCE inflation using AO-U and AO-U|X^{UNEM}. The forecasting base is December 1997 and the horizon is 12 months. The vertical line represents the realization of core-PCE inflation in December 1998. The (predictive) probability of negative inflation is 9.18% for AO-U and 0.074% for AO-U|X^{UNEM}.
Figure 8: Time series of the predictive probability of deflation (defined as negative inflation) for core-PCE using AO-\(U\) and AO-\(U|X\).

\[ \text{Probability of Deflation} \]

\[ \text{AO–U} \quad \text{AO–U|X} \]

\[ 1985 \quad 1988 \quad 1991 \quad 1994 \quad 1997 \quad 2000 \quad 2003 \quad 2006 \]

\[ 0.00 \quad 0.04 \quad 0.08 \quad 0.12 \quad 0.16 \quad 0.20 \quad 0.24 \]