The Heterogeneity and Dynamics of Individual Labor Supply over the Life Cycle: Facts and Theory

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Abstract

This paper shows that the neoclassical growth model with “balanced growth preferences,” as used in standard business cycles and growth studies, can be reconciled with the micro evidence on labor supply. We first document various facts about the labor supply decisions of male workers in the US over their life-cycle. We then develop a theory of individual life-cycle labor supply that builds on the neoclassical growth model. The theory features heterogeneous agents, incomplete markets, nonlinear wages, and variation in labor supply along the extensive and intensive margin. All of these features create a disconnection between the theoretical and the empirical elasticity of labor supply. While the model economy is calibrated without explicitly targeting the facts on hours worked, the predictions of the theory on labor supply are quantitatively close to the observations documented in the data. We show that in the calibrated model economy there is a disconnect between the theoretical elasticity of labor supply

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and the one that is recovered from model data using standard econometric techniques. While the theoretical elasticity of leisure is $-0.60$, the estimated empirical elasticity in the simulated data is $-0.29$ if the econometrician is able to perfectly purge the data from measurement error. If, on the other hand, the econometrician cannot remove all the measurement error, then the estimated empirical elasticities of leisure are much smaller (in absolute value) and can even be positive. Moreover, when we simulate the model for different preference parameters we find that the empirical elasticity virtually does not change, a finding that underscores that the empirical elasticity of labor supply is not a good target for identifying preference parameters in our model economy. Our findings point that time aggregation and the extensive margin play an important role in generating the disconnect: When we use quarterly data and consider individuals that do not work, the empirical elasticity of leisure increases by a factor of 4 relative to the baseline estimates with annual data and is even higher than the theoretical elasticity.

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1 Introduction

The aggregate response of labor supply to macroeconomic shocks is the subject of a heated debate among economists. The workhorse in macroeconomics analysis is the representative agent neoclassical growth model calibrated to fit aggregate time series data. Macroeconomists typically assume that preferences are such that the income and substitution effects of permanent wage changes cancel out, rendering a constant aggregate labor supply in an economy with sustained technological progress. This research has led to numerous papers in macroeconomics making the case of a large aggregate labor supply elasticity.\(^1\) Empirical studies based on micro level data, however, find a much smaller labor supply elasticity (MaCurdy (1981) and Altonji (1986)). Hence, recently, macroeconomists modeling heterogeneous agents and individual labor supply assume away “balance growth preferences” and technical progress (see, Castañeda, Díaz-Giménez, and Ríos-Rull (2003) and Conesa, Kitao, and Krueger (2008)). Despite these recent contributions, the main objective of our paper is to show that the neoclassical growth model with “balanced growth preferences,” as used in standard business cycles and growth studies, can be reconciled with the micro evidence on labor supply. We show that an heterogeneous agent model with preferences consistent with balance growth can go along way in capturing salient features of labor supply over the life-cycle for male workers. Moreover, using simulated data from the model economy, we find that the empirical elasticity of labor supply is not a good calibration target for pinning down preference parameters in our model economy.

In this paper we first document various facts about the labor supply decisions of male workers in the US over their life-cycle. For cohorts of college and non-college individuals in the PSID, we study the life-cycle profiles of average hours worked, the fraction of individuals with positive hours worked during the year, and the coefficient of variation of hours. In addition, we analyze the persistence in labor force participation and hours worked over the life-cycle. Then, we develop a theory of individual labor supply that builds on the neoclassical growth model. The theory models life-cycle behavior to better relate the model predictions to the data. We also model heterogeneous agents, incomplete markets, nonlinear wages, and variation in labor supply along the extensive and intensive margin. All of these features, as discussed in our literature review, can create a disconnection between the theoretical and the empirical elasticity of labor supply. Heterogeneity is introduced by assuming that individuals

\(^1\)Kydland and Prescott (1982) and Prescott (1986) find that aggregate labor supply is very responsive to business cycle shocks while Prescott (2004) finds a large response to a change in taxes.
are subject to uninsured labor productivity risk. The period utility function is a CES function of consumption and leisure, which allows for balance growth and for variation in labor supply along the intensive and extensive margin. To model nonlinear wages, we follow Hornstein and Prescott (1993). We also model social security and endogenous retirement. To calibrate the model, we estimate an annual wage process on the PSID data and, given that we use a model period of a quarter, we find a \textit{quarterly} stochastic process on labor productivity that makes the predictions of the model economy consistent with the annual wage process estimated in the data. While the model economy is calibrated without explicitly targeting the facts on hours worked, the predictions of the theory on labor supply are quantitatively close to the observations documented on the data.

We show that in the calibrated model economy there is a disconnect between the theoretical elasticity of labor supply and the one that is recovered from model data using standard econometric techniques. While the theoretical elasticity of leisure is $-0.60$, the estimated empirical elasticity in the simulated data is $-0.29$. Moreover, when we simulate the model for different preference parameters we find that the empirical elasticity virtually does not change, a finding that underscores that the empirical elasticity of labor supply is not a good target for identifying preference parameters in our model economy. Since the model economy allows for measurement error, we emphasize that we have assumed that the econometrician is able to perfectly purge the data from measurement error in estimating empirical elasticities in the simulated data. If, on the other hand, the econometrician cannot remove all the measurement error, then the estimated empirical elasticities of leisure are much smaller (in absolute value) and can even be positive.

We find that an operative extensive margin in labor supply at the quarterly frequency and time aggregation play an important role in driving the disconnect between the empirical and theoretical elasticities in our model economy. While the empirical elasticity of leisure estimated on annual data is $-0.29$, it is $-0.45$ when using quarterly data. Intuitively, the elasticity is smaller in the annual data because the annual data gives a noisy measure of the returns to work faced by individuals during the year. In particular, while temporary low wage shocks may induce individuals to not work in a quarter, there are no traces of these low wage (labor productivity) shocks on the annual data. Moreover, the variation in hours and wage data is also truncated in the quarterly data. To illustrate this point, we estimated in the quarterly simulated data the elasticity of leisure as follows: We use labor productivity as a measure of the returns to work and we set leisure to 100% when individuals do not work. In this way, the estimated elasticity of leisure uses data from all
individuals alive in the economy, and not just those who work. The finding could not be more striking: We obtain an elasticity of leisure of $-0.91$, which is bigger (in absolute value) than the theoretical elasticity of $-0.60$. In understanding this finding, note that the theoretical elasticity only describes labor supply responses at the intensive margin, as its derivation assumes an interior solution in the labor supply decision. Further, in an experiment which eliminates the social security system, we find that the changes in aggregate labor supply are unrelated to the individual labor supply elasticity. Altogether, our findings point that the empirical elasticity understates the theoretical elasticity by a factor of two. When the extensive margin is considered, the empirical elasticity using annual data understates the true labor supply response by a factor of 4.

Several important recent contributions have argued that the micro elasticity of labor supply need not be related to the aggregate labor supply response. In a theory of a representative agent with indivisible labor, Rogerson (1988) is the first one to show that individual and aggregate labor supply elasticities are effectively unrelated. By modeling heterogeneity, Chang and Kim (2006) go one step further and show that the slope of the aggregate labor supply schedule is determined by the distribution of reservation wages rather than by the willingness to substitute leisure intertemporally, establishing that when the extensive margin is operative aggregation plays a crucial role in determining aggregate labor supply responses. Moreover, borrowing constraints (Domeij and Flodén (2006)) and nonlinear wage functions (Rogerson and Wallenius (2006)) have been shown to generate a disconnect between the individual labor supply elasticity and the theoretical labor supply elasticity implied by preference parameters. Despite the insights provided by these recent contributions, each of them is usually consistent with only a small set of facts regarding labor supply at the micro level. This motivates us to build a theory of labor supply of an economy with heterogeneous agents which is consistent with micro level data and captures salient features of labor supply over the life cycle in several important dimensions. This important for disciplining the aggregation from individual to aggregate labor supply responses, and for proceeding with some confidence to evaluate the aggregate labor supply response to macroeconomic shocks. Perhaps, the closest paper to ours is Imai and Keane (2004). These authors use a life-cycle model disciplined by micro level data to show that the empirical labor supply elasticity is unrelated to the preferences parameters when agents accumulate human capital accumulation. Relative to these authors, we contribute by focusing on the role of the extensive margin, time aggregation, and borrowing constraints in accounting for the disconnect. Moreover, we study
an economy with preferences consistent with balance growth. Finally, Kimmel and Kniesner (1998) provide empirical evidence that the interaction of time aggregation and the extensive margin plays an important role in accounting for the low estimated empirical elasticities of leisure in the empirical literature.

The paper proceeds as follows. Section 2 presents empirical facts on labor supply using data from the PSID. Section 3 develops the framework a life-cycle theory of individual labor supply with heterogeneous agents. The calibration of the model economy is discussed in Section 4. Section 5 discusses the performance of the baseline economy in accounting for the facts documented on labor supply and discusses the disconnect between the empirical and theoretical elasticities of labor supply. Section 6 concludes.

2 Empirical Facts

2.1 The Data

The main dataset used in our analysis is the Michigan Panel Study of Income Dynamics (PSID) for the period 1968-1997.2 The sample is restricted to males between the ages of 18 and 65. We do not place other restrictions on the sample. In particular, note that we do not restrict to heads of household - we use the information on annual hours worked provided by the PSID for those males who are listed as “wives” as well as the information on annual hours worked, whenever available in the individual files, on males who are dependents. This allows us to provide a more representative overview of the facts on labor supply as compared to the related literature which has mainly focused on male workers with strong labor market attachment.3 Appendix I provides a detailed description of the construction of the dataset and the variables used in the analysis.

The analysis is focused on the labor supply of men. A cohort is defined to consist of all individuals who turn 18 years old in a given year — for example, the 1967 cohort consists of all individuals who turn 18 years old in 1967. Since the PSID is a relatively small dataset, we grouped our sample into age and cohort groups. By age, individuals are grouped into 12 age groups each consisting of four ages — for example, age 18 on the graphs include individuals between the age of 18 and 21, while age 22 includes all individuals between the ages of 22

2 We have performed a similar empirical analysis using the Survey of Income and Program Participation (SIPP). The results obtained on the SIPP data are largely consistent with those obtained on the PSID data. These are available from the authors upon request.

3 See for example Storesletten, Telmer, and Yaron (2001), Heathcote, Storesletten, and Violante (2004), Badel and Huggett (2007).
and 25. We have 17 cohort groups each consisting of three cohorts — for instance, the 1976 cohort group includes cohorts 1976, 1977, and 1978 while the 1985 cohort includes cohorts 1985, 1986, and 1987. We drop all cohorts smaller than 1940 and all cohorts greater than 1990.\footnote{When we conduct the analysis by education groups, our last cohort is 1985 in order to be able to classify individuals as either high school or college.}

We use PSID sample weights in the analysis.

As the data suggests, cohort effects are not significant in the case of men. Figures 1-5 show the following labor supply statistics over the life-cycle for various cohorts of men and women: mean annual hours worked, mean annual hours worked for those with positive hours, the fraction reporting positive annual hours, the variance of log annual hours, and the coefficient of variation of annual hours. While more recent cohorts of women have dramatically different labor supply behavior than older cohorts of women, that is not the case for men — while we do observe some differences across cohorts the labor supply behavior of more recent cohorts does not differ much from that of older cohorts. As a result, at this point we do not take out cohort effects for men.

Next we proceed with the empirical analysis and document a wealth of facts regarding the labor supply of men over the life-cycle. The patterns that we see in the data will be motivating the main features which will be introduced in the model. The most important patterns are as follows:

- We see a very pronounced life-cycle pattern in the labor supply behavior of men. We see the life-cycle trend in the mean annual hours worked, the participation rate, the dispersion of annual hours, the persistence in annual hours worked, and in the persistence of labor market participation.

- There is a substantial dispersion of annual hours worked at every point in the life-cycle.

- For most individuals, and for most ages during the life-cycle, annual hours are quite persistent.

- The labor supply behavior of high school and college graduates is different enough to warrant a separate analysis for each of these groups.\footnote{In this version of the paper, we consider an individual to be high school if he or she has at most 13 years of education while those with 14 years of education or more are considered to be college graduates. A sensitivity analysis with respect to the education cut-off separating high-school and college graduates indicates the current partition is a sensible one.}
2.2 Facts on the Life-Cycle Labor Supply of Men

Figures 6-13 and Tables 1-5 provide various facts on the life-cycle labor supply of men.

2.2.1 Average Annual Hours over the Life-Cycle

Figures 6 shows that mean annual hours worked clearly exhibit an inverted U-shape over the life-cycle – they increase early in life until the late 20s, stay constant after that until the late 40s, and decline monotonically after the age of 50. The second panel shows that college and non-college graduates have different life-cycle profiles – college graduates initially work less (while studying) while working more after the age of 26. In addition, the mean annual hours of high-school workers start declining earlier at the age of 50.

Figures 7 and 8 illustrate the intensive and extensive margin of the above mentioned facts. The extensive margin matters early in life until the age of 26, but is especially quantitatively important late in life after the age of 50. Furthermore, it is interesting to point out that the participation rate of those with high-school starts declining in the late 40s while the participation rate of those with college start declining significantly only in the late 50s.

2.2.2 Dispersion of Annual Hours over the Life-Cycle

Figures 9 and 10 display the dispersion of annual hours over the life-cycle as measured by the log of annual hours and the coefficient of variation of annual hours. Three facts are of particular importance. First, the dispersion in annual hours is U-shaped – it is high early in the life-cycle until the age of 26, then declines and is constant until the late 40s, and increases substantially after the age of 50. Second, the degree of dispersion is quite substantial. Third, even though the dispersion of those with high-school and college is U-shaped over the life-cycle, the dispersion of those with college is higher initially (while they are studying in college) and lower afterwards.

2.2.3 Persistence in Annual Hours Worked

In this section, we investigate the extent to which annual hours worked are persistent over the individual’s life. For that purpose each year we divide individuals into four groups: 1 – those with annual hours less than 100; 2 – those with annual hours between 100 and 1500; 3 – those with annual hours between 1500 and 2800; and 4 – those with annual hours greater...
than 2800. We then construct transition matrices where cell \( i j \) indicates the fraction of all individuals in cell \( i \) in year \( t \) who moved to cell \( j \) in year \( t + 1 \). We document the facts for all men as well as for high school graduates and college graduates.

Table 1 presents the transition matrix and the relative size of each group for men in three age groups: young workers between the ages of 18 and 29, middle-aged workers between the ages of 30 and 54, and old workers between the ages of 55 and 65. We found it useful to present graphically some of these results. In particular, Figure 11 graphs the relative size of each of the four groups as well as the fraction of workers who stay in each of these groups in two consecutive years (i.e. the diagonal elements from the transition matrix). Note that this graphical representation makes it easy to consider 12 age groups rather than the 3 age groups considered in Table 1.

Three important findings are worth pointing out. First, the group of full-time workers with annual hours between 1500 and 2800 is by far the largest, with the exception of the first and very last years of the life-cycle, and exhibiting very high persistence in annual hours worked — over 70% of men are in this labor supply group and more than 80% of those who are in this group in year \( t \) remain in it in year \( t + 1 \). Table 1 further shows that, between the ages of 30 and 54, most of those who move out of this group move temporarily into the group with large labor supply and work more than 2800 hours. That indicates that for the most part of the life-cycle, especially between the ages of 30 and 50, annual hours worked are quite persistent for most men. Second, the fraction of men who work less than 100 hours is quite small throughout the life-cycle, but starts increasing gradually after the age of 46. Furthermore, with age, this group becomes an absorbing state — after the age of 46, more than 80% of men who are in this group in year \( t \) will be there in year \( t + 1 \). Furthermore, as Table 1 shows, those who move out of it later in life, move temporarily into the part-time labor supply group. Third, the other two groups — those working between 100 and 1500 hours and those working more than 2800 hours — do exhibit a life-cycle pattern but are relatively small. In addition, each of these two groups seem to represent a temporary state in one’s labor market history since the probability of remaining there is not very high.

These broad patterns are observed also for each of the two education groups — high-school and college men. As one can expect, after the age of 26, the group of full-time workers with annual hours between 1500 and 2800 (i) is bigger for the college men than the high-school

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6 The cut-offs were chosen in order to capture four broad patterns of labor market behavior — no labor market participation (group 1), part-time labor supply (group 2), full-time labor supply (group 3), and very high labor supply (group 4). Slight changes in these cut-offs do not significantly change the main patterns documented here.
men, (ii) starts declining gradually for high-school men at the age of 46 while for college men declines only after the age of 62, and (iii) is more persistent for college than for high-school men.

2.2.4 Lifetime Labor Supply

The dispersion in lifetime labor supply is another useful statistic which is closely related to the persistence in individual's labor supply over time. Due to the nature of the PSID dataset, we do not observe individuals throughout all their life — some of them have already been in the labor market for some time when the survey starts in 1968 while those who enter the labor market in 1968 at the age of 18 are only in their 40s in 1997. Nevertheless, we can learn a lot even if we followed individuals for shorter periods. We choose to follow individuals for periods of 10 years at different stages in their life-cycle: ages 26-35, 36-45, 46-55, and 56-65. We drop all individuals who have a missing observation during the relevant ten years and sum the hours worked for each individual during the whole ten years. Then we compute the dispersion in this cumulative measure of hours worked. Considering two extreme examples is useful for illustrating how to interpret the results. Consider a particular group, e.g. the group between the ages of 36 and 45, and suppose that all individuals work the same number of hours throughout the whole period as at the beginning at the age of 36. In that case, the coefficient of variation of the cumulative hours worked throughout the whole period would be the same as the coefficient of variation (cross-sectionally) at the age of 36 (or any other age in the period). Alternative, suppose that individual hours fluctuate a lot over the period and those who work a lot in one year work very little the year after that. In that case, workers would end up working quite similar cumulative hours over the period, and the coefficient of variation of the cumulative hours worked throughout the whole period would be quite small and substantially lower than the coefficient of variation (cross-sectionally) at the age of 36 (or any other age in the period).

Table 4 reports the coefficient of variation of the cumulative hours worked for the four age groups defined above. We report this statistic for all men as well as for high-school and college graduates. Comparing the results with those on Figure 10, which plots the coefficient of variation of cross-sectional hours worked at each age, we conclude that hours are quite persistent, especially over the ages of 25 and 55. This analysis provides us with two important findings. First, the dispersion in cumulative hours is quite substantial, indicating that individuals tend to be quite persistent in their labor supply behavior. This is consistent with the mobility matrices discussed in section 2.2.3. Second, the dispersion of cumulative
hours is smaller than the cross-sectional dispersion at any age in the 26-65 interval. This implies that workers do sometimes change their hours worked. This is also consistent with the mobility matrices discussed in section 2.2.3 since — as seen in the middle panel of Table 1 for those between the ages of 30 and 54 — the diagonal elements of the mobility matrices are not zero and we do observe workers who switch across the hours categories.

3 The Model

We develop a life-cycle theory of the labor supply of individuals. For simplicity, we abstract from the labor supply decisions of women and model males only. We consider a small open economy facing a fixed interest rate and exhibiting a constant rate of technological progress. We follow Hornstein and Prescott (1993), in modeling a production technology that give rise to a competitive equilibrium with non-linear wages.

3.1 Population, preferences, and endowments

We consider an economy populated by overlapping generations of individuals. Individuals face uncertain lifetimes and can live, at most, $J$ periods. They differ in terms of their education (college versus non-college) and labor-productivity. The date-$t$ utility function takes the form

$$u_t = u(c_t, l_t) = \frac{[c_t^{\alpha} l_t^{1-\alpha}]^{1-\sigma} - 1}{1-\sigma},$$

where $c_t$ is consumption and $l_t$ denotes leisure. This assumption on preferences ensure that labor supply remains constant over time as the economy grows, and it is motivated by the observation that there are no important cohort effects in the labor supply of men. It also allows the theory to be consistent with the fact that there are large permanent differences in labor productivity across individuals (fixed effects) but not in their lifetime labor supply.

Individuals maximize lifetime expected utility

$$E_t \sum_{j=t}^{J} \beta^{t-j} u(c_j, l_j),$$

where $E_t$ denotes expectations at date-$t$. Individuals face uncertainty regarding their labor productivity $z$ and mortality shock. An individual’s time endowment in each period is one. The amount of time that can be allocated to work is $h_j = 1-l_j$. The college decision is
exogenous. The education type of an individual determines the stochastic processes driving the mortality and labor productivity shocks.

3.2 Technology

There are a large number of plants and each plant is a collection of jobs. We assume that plants can operate jobs at zero costs. The production function of a job is given by

\[ f(K, h, A_t z) = h^\varepsilon K^{1-\theta}(A_t z)^{\theta}, \quad \text{with } \theta \leq \varepsilon \leq 1. \]

where \( h \) denotes the workweek, \( K \) is the amount of capital for the job, and \( A_t z \) is effective labor in the job. Effective labor in the job is given by the product of the worker productivity \( z \) and the level of technology \( A_t \) which grows at a rate \( g \). Note that, for a fixed workweek, the job technology exhibits constant returns to scale in capital and effective labor. Moreover, as discussed in Osuna and Ríos-Rull (2003), when \( \varepsilon = \theta \) the job technology reduces to the standard Cobb-Douglas technology where total hours of effective labour is what matters. When \( \varepsilon > \theta \) the composition between hours of work and effective labour matters. When \( \varepsilon = 1 \) the technology is linear in hours and corresponds to the case where workers are not subject to fatigue.

3.3 The plant’s problem

The plant’s production plan is given by the choice of hours of operation \( h \), capital \( K \), and effective labor \( N \). The plant takes as given the wage schedule \( \tilde{w}(h, N) \) and the interest rate \( r \). In equilibrium, the wage schedule is a non-linear function of the workweek \( h \) and a linear function of effective labor \( N \). To show this point, consider a plant operating \( h \) hours and hiring \( N \) units of effective labor. The optimal amount of capital \( K \) solves

\[ \pi = \max_K h^\varepsilon K^{1-\theta}N^{\theta} - K(r + \delta) - \tilde{w}(h, N). \]

The solution to this problem implies

\[ \frac{K}{N} = k^*(h, r) = \left[ \frac{(1 - \theta)h^\varepsilon}{r + \delta} \right]^{1/\theta}. \]

Next, notice that plants will only operate if profits are non-negative. Free entry, and the fact that plants can be created at zero costs, imply that in equilibrium plants will make
zero profits (will not extract economic rents from workers). Hence, competition for workers implies that the wage bill $\tilde{w}(h, N)$ is determined from

$$\pi = h^\varepsilon [Nk^*(h, r)]^{1-\theta} N^\theta - N k^*(h, r)(r + \delta) - \tilde{w}(h, N) = 0,$$

which gives

$$\tilde{w}(h, N) = w(h) N, \text{ where } w(h) \equiv (r + \delta) \frac{\theta}{1 - \theta} \left[ \frac{(1 - \theta) h^\varepsilon}{r + \delta} \right]^{1/\theta}.$$

It follows that the wage schedule $\tilde{w}(h, N)$ is linear in effective labor $N$ and non-linear in hours of work $h$. When $\varepsilon = \theta$ earnings are also linear in $h$. When $\varepsilon > \theta$ earnings increase with $h$. In this case, households would be better off by selling employment lotteries to firms (Hornstein and Prescott (1993)). However, we rule out this possibility by assuming that households cannot commit to work when the realization of the employment lottery imply that they should work.

### 3.4 Government, annuity, and credit market

The government taxes consumption, capital income, and labor income. The tax revenue is used to finance government expenditures. Individuals can insure mortality risk in fair annuity markets. Denoting by $R$ the gross interest rate net of capital income taxes $\tau_k$, the gross interest rate faced by an individual $j$ years old with education $e$ is given by

$$R^e_j = R/\pi^e_j, \text{ where } R = 1 + r(1 - \tau_k),$$

and $\pi^e_j$ is the conditional probability that an age $j - 1$ individual with education $e$ survives to age $j$. We assume that individuals can’t borrow.

**Social Security.** The government also administers a pay as you go social security system. To finance pensions for retired individuals, the government uses a payroll tax $\tau_{ss}$. Individuals can apply for social security benefits after their 62nd birthday. Social security benefits depend on the average earnings made by individuals over the 35 highest years of earnings. Denoting this average earnings by $\overline{w}$, social security benefits can be expressed as $b_s(\overline{w}, j)$, where $j$ denote the age of individuals. As discussed in French (2005), Social security encourage individuals to retire from the labor market by age 65. One reason is that, after
35 years of work, an extra year of work will only lead to an increase of pension benefits if the earnings during this last year is higher than average past earnings. A second reason is that the social security benefit formula penalizes individuals that retire after age 65. After this age, for every year that individuals postpone retirement their benefits rise by 3% but this is actuarially unfair. A third reason is that the Social Security earnings test taxes labor income for retired individuals at a very high rate. In a nutshell, if a person perceiving social security benefits earns labor income above a threshold level of 31% of average earnings in the economy, earnings are taxed at a 50% rate (on top of state and federal income taxes and payroll taxes) until all benefits from social security have been taxed away.

Social security benefits are a function of the Average Indexed Monthly Earnings (AIME) over the 35 highest earnings years. Given that the model period is a quarter, for computational simplicity we compute average quarterly earnings over the $35 \times 4$ highest earnings quarters as follows

\begin{align}
\bar{w}_{j+1} &= \bar{w}_j + zw(h_j)/(35 \times 4) \quad \text{for } j \leq 35 \times 4, \quad (1) \\
\bar{w}_{j+1} &= \bar{w}_j + \max\{0, \min\{zw(h_j), \hat{y}\} - \bar{w}_j\}/(35 \times 4) \quad \text{for } j > 35 \times 4 \text{ and } j < 40 \times 4 \quad (2) \\
\bar{w}_{j+1} &= \bar{w}_j + \max\{0, \min\{\frac{zw(h_j)}{(1+g)^{j-40\times4}}, \hat{y}\} - \bar{w}_j\}/(35 \times 4) \quad \text{for } j \geq 40 \times 4, \quad (3)
\end{align}

where $\hat{y}$ is the maximum taxable earnings by the social security administration, which is set at 2.47 the average earnings in the economy). In computing AIME, the Social Security Administration indexes past earnings to reflect the change in general wage levels that occur during the workers careers. The equation (3) reflects that earnings after age 60 are not indexed by yearly wage growth. To expressed (1)-(3) we write:

\begin{equation}
\bar{w}' = \Gamma_{ss}(\bar{w}, zw(h), j) \quad (4)
\end{equation}

At retirement, the Social Security Administration computes the Primary Insurance Amount (PIA) which is the sum of three portions of the Average Indexed Monthly Earnings (AIME). The bend points in the PIA formula are 0.2 and 1.24 of the average earnings in the economy when individuals file for social security($\bar{W}$).\footnote{$\bar{W}$ is the average earnings in the economy in the year when the individual becomes 62 years old.}
The monthly retirement benefit derived from PIA may be higher or lower than the PIA. At the normal retirement age of 65, benefits are equal to PIA. Individuals can file for social security benefits after they become 62 years old. For each year that an individual file social security taxes before the normal retirement age of 65, individuals loose about 6.7% of pension benefits, which according to French (2005) is actuarially fair. After age 65, individuals who postpone retirement for a year increase their pension benefits by 3%.

To avoid carrying the age of retirement as a state variable, it is convenient to invert (5) in order to map the decrease/increase in pension benefits associated with different retirement ages into an implied change in $\bar{w}$. This is summarized by the relationship

$$\bar{w}' = \Gamma_{ret}(\bar{w}, j).$$  

### 3.5 The individual’s problem

We use the recursive language to describe the problem of an individual. To simplify the notation, we abstract from the fact that the education type of an individual determines his earnings and mortality processes. The state of an individual is given by his age $j$, assets $a$, average lifetime earnings $\bar{w}$, and earnings shock $z$.

Since individuals live at most $J$ periods, we set $B_{J+1}(x) = V_{J+1}(x) = 0$. When a person is retired (has applied for social security benefits) his value is given by
\[ B_j(a, b_s, z) = \max\{u(c, l) + \beta\pi_{j+1}E[B_{j+1}(a, b_s', z')]|\} \]
\[ (1 + g)a_{j+1} = (1 - \tau_{ss} - \tau_h + 0.5\tau_{ss}\tau_h) \min\{zw(h), \hat{y}\} + (1 - \tau_h) \max\{zw(h) - \hat{y}, 0\} \]
\[ -I_{SS} \min\{0.5zw(h), b_s\} + R_ja - c(1 + \tau_c), \]
\[ b_s' = \frac{b_s}{(1 + g)} \]
\[ a_{j+1} \geq 0, \text{ and } l + h = 1. \]

where \( I_{SS} \) is an indicator function that takes the value of 1 if labor earnings are above the social security earnings test (31\% of average earnings in the economy). In this case, each dollar of labor earnings is taxed at a rate of 50\% (on top of payroll and income taxes) until all social security benefits \( b_s \) received during the current period are taxed away. Using the linearity of the wage function \( w(h, N) \) the earnings of an individual working \( h \) hours and supplying \( z \) units of labor can be expressed as \( zw(h) \). Note that the Social Security Administration does not tax earnings above \( \hat{y} \). Half of the social security taxes are paid by the employer and are not subject to personal income tax \( \tau_h \).

The value of a person that has not retired is

\[ V_j(a, \overline{w}, z) = \max\{u(c, l) + \beta\pi_{j+1}E[V_{j+1}(a, \overline{w}', z')]|\} \]

subject to
\[ (1 + g)a_{j+1} = (1 - \tau_{ss} - \tau_h + 0.5\tau_{ss}\tau_h) \min\{zw(h), \hat{y}\} + (1 - \tau_h) \max\{zw(h) - \hat{y}, 0\} \]
\[ ... + R_ja - c(1 + \tau_c), \]
\[ a_{j+1} \geq 0, \]
\[ \overline{w}' = \Gamma_{ss}(\overline{w}', zw(h), j), \text{ and } l + h = 1. \]

The individual takes as given the wage schedule \( w(h) \), the function \( \Gamma_{ss} \) in (4) determining the evolution of average lifetime earnings.

At age 62 * 4, and above, individuals that have not applied for social security benefits have the option to do so. If the individual has not applied for social security benefits, he faces the following retirement decision

\[ v_j(a, \overline{w}, z) = \max\{B_j(a, b_s(\overline{w}, j), z), V_j(a, \overline{w}, z)\} \]
where \( b_s(\overline{w}, j) \) gives pension benefits as a function of the \( AIME \) and the retirement age. The first term in the Max operator represents the value of retirement (collect social security benefits) and the second term is the value of not retiring.

4 Calibration

The calibration for most of the parameters in the Baseline Economy is quite standard so that we fix these parameters using available estimates in the literature. The crucial task in our calibration is the parameterization of the stochastic process on labor productivity. To model variation in employment within a year, the calibration sets the model period to 1 quarter. In calibrating a quarterly stochastic process on labor productivity, one difficulty is that the PSID only reports earnings and hours of work at an annual frequency. Moreover, in using wage data to calibrate an stochastic process on labor productivity we need to take a stand on how hours of work affect labor productivity and we need to consider that the data only report wages for individuals that work. To deal with these problems, we proceed as follows:

1. Estimate an annual wage process for college and non-college workers from the PSID data.

2. Use estimates from Aaronson and French (2004) on nonlinear wages to pin down the parameter \( \varepsilon \) determining how hours of work affect labor productivity in the model economy.

3. Feed a quarterly labor productivity process into the model economy.

4. Simulate the model economy to obtain quarterly data on employment, hours of work, and earnings.

5. Aggregate the quarterly data to an annual period.

6. Estimate an annual wage process for college and non-college workers in the model generated data.

7. Feed a new quarterly labor productivity process (go back to step 3), until the ”same” annual wage process is obtained in the model and in the data.

Below we describe the calibration in detail. We first discuss the calibration of the “macro” parameters. We then discuss the calibration of the labor productivity process and how we deal with the possibility of measurement error in hours and earnings in the PSID data.
4.1 Calibration of preferences, technology, and macro parameters.

Since the calibration of most of these parameters is standard, we follow the study of Huggett and Ventura (1999) who also calibrate a life-cycle model.

Preference parameters and mortality rates. The discount rate $\beta$ is set so that the implied annual discounting is $.96$, the consumption share $\alpha$ is fixed at $.4$, and the coefficient of risk aversion $\sigma$ is set to $3$. The mortality risk for college and non-college individuals is taken from Bhattacharya and Lakdawalla (2006).

Technology parameters. The labor share $\theta$ is set to $.64$. The depreciation rate $\delta$ and the growth rate of technology $g$ are set to that the implied annual depreciation is $.04$ and the implied annual growth rate of labor productivity is $.021$. To calibrate the parameter $\varepsilon$, we use the fact that the equilibrium wage rate in our theory satisfies

$$\frac{w(h)}{h} = \text{constant} \quad h^{\varepsilon/\theta-1},$$

Note that the elasticity of the wage rate to a change in hours of work is given by $\varepsilon/\theta - 1$. In an empirical study, Aaronson and French (2004) estimate this elasticity to be slightly above $.40$. We thus set $\varepsilon = 1.4*\theta$.

Real interest rate, tax rates, and social security. The real interest rate $r$ is fixed at $.06$. The tax rate on consumption, capital income and labor income are set so that $\tau_c = .055$, $\tau_k = .40$, $\tau_w = .27$. The social security tax rate is set to $\tau_{ss} = 0.12$, and the cap $\hat{y}$ on social security taxation is fixed at $2.47$ of average earnings in the economy.

4.2 Calibration of labor productivity

We use a GMM procedure to estimate the following annual wage process in the PSID data for college and non-college individuals:

$$\ln w_{ij} = x_j \kappa + \alpha_i + u_j + \varepsilon_j, \quad (7)$$

where $x_j$ is a cubic polynomial in age, $\kappa$ is a vector of coefficients, $\alpha_i \sim N(0, \sigma_\alpha^2)$ is a fixed effect determined at birth, $\varepsilon_j \sim N(0, \sigma_\varepsilon^2)$ is an idiosyncratic transitory shock, and $u_j$ follows a first-order autoregression:

$$u_j = \rho u_{j-1} + \eta_j, \quad \eta_j \sim N(0, \sigma_\eta^2), u_0 = 0. \quad (8)$$
While the parameters \((\kappa, \rho, \sigma_\alpha^2, \sigma_\eta^2)\) vary across education types, this is omitted in the notation to avoid clutter. The estimated wage processes are reported in Table 6. The empirical findings show that the variance of fixed effects is quite large for both education types, with values of .06 and .08, for the non-college and the college types. Both wage processes exhibit high autocorrelation, with a value of .97 for non-college individuals and .92 for college individuals. The variance of the innovation of the autoregressive process is .013 and .029. The estimates reveal that both education types exhibit quite high transitory shocks to their wages, with variances of .097 and .079.

To calibrate the model economy, we need to find a *quarterly* stochastic process on labor productivity that is consistent with the annual wage process estimated in the data (equations (7)-(8)). To do this, we assume that labor productivity is the sum of an annual autoregressive process and a quarterly transitory shock.\(^8\) Specifically, while the transitory shock is drawn every quarter, the persistent shock is only drawn at the first quarter of each year (age). To make these assumptions operational, we discretize all shocks by considering, for each education type, 15 values for the autoregressive shocks, 4 values for temporary shocks, and 2 values for fixed effects. The transition probabilities of the persistent shock are computed using a Tauchen routine.

The empirical literature has stressed the importance of measurement error in hours and earnings in household survey. Moreover, our empirical findings are suggestive of the importance of measurement error since the estimated variation in the transitory component of wages seem implausibly large. To get a quantitative sense of the magnitude of these variance, we have calibrated the model economy assuming that there is no measurement error in hours and earnings. We find that to match the estimated variance of transitory shocks to wages in the data the calibration requires a temporary shock process that implies that with 20\% percent chance labor productivity can either increase by a factor of 15 or decrease by a factor of 15. Moreover, in the presence of such large temporary shocks, 50\% of individuals in the model economy work less than 3 quarters, an implication that is grossly counterfactual.

We thus need to take seriously measurement error in the data. To this end, we assume that the transitory shock \(\epsilon_j\) in the empirical model is the sum of a (true) temporary wage shock and measurement error in hours \((m_H)\) and earnings \((m_E)\), with measurement error in hours and earnings being normally distributed with mean zero. The estimated transitory

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\(^8\)We have tried an specification that allows for an autoregressive process at the quarterly level. In this case, however, we were not able to recover the stochastic process estimated in the data. When labor productivity follows an autoregressive process at the quarterly level, there is no reason to expect the logarithm of the sum of quarterly earnings to be well approximated with an autoregressive process.
variation in "observed" wages $\sigma_\varepsilon$ is then the sum of the variances of transitory true wages ($\sigma_T$), measurement error in earnings $\sigma_E^2$, and measurement error in hours $\sigma_H^2$:

$$\sigma_\varepsilon^2 = \sigma_T^2 + \sigma_E^2 + \sigma_H^2.$$  \hfill (9)

We use the implications of our theory to take a stand on the relative importance of ($\sigma_T^2, \sigma_E^2, \sigma_H^2$) in accounting for the estimated variance in “observed” transitory wages ($\sigma_\varepsilon^2$). To this end, we assume that annual hours and earnings are measured with error in the model economy. To calibrate the variance of (true) transitory wages $\sigma_T^2$, we note that in our theory this variance has important effects on the probability that individuals work in each quarter: The larger $\sigma_T^2$ the less likely individuals will work four quarters in a year. We thus use this statistic as a calibration target. Using data from SIPP we find that about 70% of individuals aged 18-65 work 4 quarters in a year (more than 56 weeks). We note that the SIPP allows us to have more relatable measures of labor force participation at the quarterly frequency than the PSID as it interviews individuals three times in a year (rather than once a year as in the PSID).

To distinguish between $\sigma_E^2$ and $\sigma_H^2$, we need a second target. This is done by comparing the variance of transitory wages in two alternative estimations of the wage process in (7)-(8): The first specification regresses observed wages while the second specification regresses wages net of the effect of hours of work on wages. Identification come from the fact that measurement error in hours and in earnings affect differently the variance of transitory earnings in the two specifications of the regression. To develop this point, we start by noticing that when wages are a non-linear function of hours, the observed wage rate is given by

$$w(h) = \frac{e^w h^\varepsilon e^{m_E}}{h e^{m_H}} = e^w h^{\varepsilon-1} e^{m_E-m_H},$$ \hfill (10)

where $w$ is the logarithm of labor productivity, $\varepsilon$ is the parameter determining the elasticity of earnings to hours of work, and $(m_E, m_H)$ are measurement error in (log) earnings and (log) wages. In the absence of measurement error, the wage rate net of the effect of hours on wages would be uncovered by taking logs and subtracting $(\varepsilon - 1) \ln h$ from both sides of (10)

$$\ln w(h) - (\varepsilon - 1) \ln h = \ln w.$$
In practice, though, hours are observed with error. Subtracting \((\varepsilon - 1) \ln(h e^{m_H})\) from both sides of equation (10) to “clean” wages from the effect of (observed) hours gives

\[
\ln w(h) - (\varepsilon - 1) \ln(h e^{m_H}) = \ln w + (\varepsilon - 1) \ln h + m_E - m_H - (\varepsilon - 1) \ln(h e^{m_H}),
\]

which can be re-arranged as

\[
\ln w(h) - (\varepsilon - 1) \ln(h e^{m_H}) = \ln w + m_E - \varepsilon m_H
\]

If \(\ln w\) follows the empirical model in (7)-(8), we obtain the following empirical model for “clean” wages:

\[
\ln w(h) - (\varepsilon - 1) \ln(h e^{m_H}) = x_j \kappa + \alpha_i + u_j + \varepsilon_j + m_E + \varepsilon m_H
\]

The transitory variation in “clean wages” is then given by

\[
\text{VAR}(\varepsilon_j + m_E + \varepsilon m_H) = \sigma^2_T + \sigma^2_E + (\varepsilon)^2 \sigma^2_H.
\]  

(11)

When wages are a non-linear function of hours of work \((\varepsilon > 1)\) and the wage process is estimated net of the effect of hours on wages, measurement error in hours lead to an increase in the estimated transitory variation of wages. Intuitively, the estimated transitory variation of wages increase because we are not using the ”correct” hours to clean the wage data. Comparing (11) with (9), the increase in the variance of transitory wages is given by

\[
\Delta \text{VAR}_T = [(\varepsilon)^2 - 1] \sigma^2_H.
\]

For the calibrated value of \(\varepsilon\), we have that \([(\varepsilon)^2 - 1] \simeq 1\) so that \(\Delta \text{VAR}_T = \sigma^2_H\). Thus, for each education type, the variance of measurement error in hours \(\sigma^2_H\) is obtained as the increase in the temporary variance in wages when the wage data is clean with hours data. When we run the two specifications of the regression in the PSID data we found that the variance of temporary wages increase by .045 for non-college individuals and .031 for college individuals. We thus obtain that measurement error in hours for non-college and college individuals is given by \(\sigma^2_H = .045\) and \(\sigma^2_H = .031\). We introduce these values for measurement error in hours into the model economy. We then run the two specifications of the wage regression with model simulated data. Reassuringly, for each education type, when the regression is run with clean wages the variance of the transitory component in wages increases by an amount approximately equal to the measurement error in hours.
5 Quantitative Findings

5.1 The Facts on Labor Supply: The Performance of the Model

Figures 14-20 present the performance of the model in accounting for the facts on labor supply at the micro level. Overall, the model captures most of the salient features of labor supply. Even though the facts on labor supply were not explicitly targeted, the model quantitatively accounts for a large fraction of these facts, indicating that the features included in the analysis are major determinants of individuals’ labor supply decisions.

Figure 14 displays mean annual earnings over the life-cycle in the model and for various cohorts in the data. The model captures very well the pattern observed in the data. The model, however, fails early in the life-cycle, especially for college graduates: while in the data annual hours are increasing until about the age of 25, in the model they start high and decline slightly. An obvious reason for this failure of the model is that individuals in the model enter without any initial assets. The motive to build precautionary savings is so strong that everyone works long hours initially. In addition, the borrowing constraint in the benchmark could be too tight, and relaxing it might allow those with low productivity shocks to work less hours.

Figures 15 and 16 display the labor supply in the model at the intensive and extensive margin, while Figures 17 and 18 show the dispersion of annual hours worked over the life-cycle both in the model and in the data. Overall, the model does an excellent job in accounting for the patterns observed in the data. As mentioned earlier, the model only has trouble accounting for the facts early in the life-cycle.

Finally, we investigate the persistence of annual hours worked in the model. We apply the same procedure as in the data and divide individuals into four groups: 1 — those with annual hours less than 100; 2 — those with annual hours between 100 and 1500; 3 — those with annual hours between 1500 and 2800; and 4 — those with annual hours greater than 2800. Figure 19 shows the relative size of each of these groups over the life-cycle both in the model and in the data. Two observations stand out. First, the model captures the fact that Group 3 (those working between 1500 and 2800 hours) is by far the largest group with a share which declines significantly only after the age of 55. Second, the model captures the fact that Group 1 (those working between 0 and 100 hours) is very small initially and increasing substantially only after the age of 50.

Figure 20 shows the persistence in annual hours worked both in the model and in the data. First, the model captures the fact that those who are working between 1500 and 2800

hours (Group 3) in a given year, with a very high probability will be in the same group the year after. Second, the model also captures the fact that early in the life cycle those who find themselves not working in a given year tend not to stay in the same group the year after. Later in the life cycle, however, this becomes an absorbent group — those who end up in it tend to stay in it with a very high probability. Finally, the other two groups are not very big and tend to be transitory — individuals end up in those group every now and then, but tend to quickly exit them.

5.2 The Elasticity of Labor Supply

We estimate the following regression on data generated from the baseline economy:

\[ \Delta \ln h^*_t = \beta_0 + \beta_1 \Delta \ln w^*_t + \varepsilon_{it}, \]  

(12)

where the ”∗” means that the variables in the regression are measured without error. The regression coefficient \( \beta_1 \) is the empirical labor supply elasticity predicted by the model economy. We run the regression with changes in “true” hours and wages because we are ultimately interested in measuring the response of labor hours to genuine wage changes. Moreover, because the empirical studies typically control for measurement error, this procedure allow us to compare our results with the findings in the empirical literature. In practice, of course, it is not obvious how to fully purged the measurement error from actual data. Hence, we later evaluate the sensitivity of the results to the assumption that some measurement error can’t be purged from the simulated data.

Our assumptions of CES preferences and non-linear wages imply that the theoretical elasticity of leisure to a change in wages is given by (see appendix)

\[ \eta^l = \frac{1 - \alpha(1 - \sigma)}{\sigma}. \]

It is standard to convert this elasticity into a labor supply elasticity by setting \( \eta^h = \frac{(1-h)\eta^l}{h} \). Our goal is to compare the theoretical elasticities with the empirical elasticities obtained from simulated data. To compare results for economies with different values of the theoretical elasticity, we also simulate model economies with \( \sigma = 2 \) and \( \sigma = 3 \) and we keep constant the rest of the parameters in the model. The findings are reported on Table 8.

We find that the empirical elasticity of leisure in the Baseline Economy is .30, well within the range of [0, 0.5] in the empirical literature. Interestingly, the theoretical elasticity is equal to .60, twice the value implied by the empirical elasticity. Moreover, while the theoretical
elasticity varies with $\sigma$ the empirical elasticity remains roughly constant. Another way to evaluate these results is as follows: Suppose that a researcher does not know the preference parameter $\sigma$ used to generate the model data. If this researcher were to use simulated data and the expression for the theoretical elasticity to back up a value of $\sigma$, the researcher would obtain $\sigma = 6$ for all the economies considered. We thus conclude that the simulated data is uninformative about the preference parameter $\sigma$. To put it differently, the empirical elasticity of labor supply is not a good calibration target for the preference parameters in our model economy.

### 5.2.1 Sensitivity

We now explore the sensitivity of our results to the amount of transitory variation in wages and the assumption of non-linear wages. To this end, we consider the following variations of our Baseline Economy ($\sigma = 3$):

1. We consider an economy with high transitory shocks on wages by increasing the variation of these shocks by a factor of 2.5. We also consider an economy with low transitory shocks by reducing the transitory variation of wage shocks by a factor of 2. Measurement error in earnings is changed so that the implied variation of transitory wages in the measured data remains constant. All other parameters remain fixed.

2. For each of the experiments just described, we also compute an economy with linear wages ($\varepsilon = 1$).

The findings are reported in Table 9. The transitory variation in wages has a very small effect on the estimated wage-elasticity of leisure. Moreover, the wage-elasticity of leisure is higher in the economy with linear wages. We conclude that our main finding is robust to these changes in the specification of the model economy.

### 5.3 Decomposition

Our main finding is that there is a strong disconnect between the theoretical and the empirical elasticity of labor supply: While in our Baseline Economy the former is .60, the latter is only .29. We now consider some experiments to shed light on the factors driving the disconnect between the theoretical and the empirical elasticities of labor supply.

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9The above calculation assumes that $\alpha = 0.4$ is known. However, the conclusions would be roughly the same had we followed the macro literature in using data on the fraction of hours of work to pin down $\alpha$. 

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The theoretical elasticity of labor supply is about the substitution effect of a wage change. One way to measure this elasticity is to evaluate the response to an unanticipated wage change, keeping wealth constant. However, empirically this response is not easy to isolate. In the data, wage changes confound information about transitory and permanent (unanticipated) wage shocks, and the latter can have important wealth effects. To control for wealth effects, some empirical studies used measures of wealth or of food consumption but the measures of wealth and consumption in household surveys are far from perfect. Other empirical studies control for wealth effects by “first-differencing” the data but, again, it is not obvious that this approach works when agents face uninsurable idiosyncratic risk. Moreover, the data only report wages at an annual frequency and wages (earnings) are only observed when individuals work. Thus, time aggregation together with the fact that the extensive margin can be operative within a year make observed annual wages a noisy measure of the returns to work faced by individuals during the year. Econometric theory points that regressing changes in log leisure on changes in log wages will underestimate the true elasticity if the explanatory variable is measured with noise.

To control for the effects of time aggregation on the estimation of empirical elasticity of labor supply, we run the regression (12) using quarterly data simulated from the model economy. Moreover, we only consider wage changes in quarters two, three, and fourth. Recall that in our calibrated model economy wage shocks in the last three quarters are temporary. By dropping the first quarter, we can better isolate the substitution effect of wage changes as temporary wage changes at the quarterly level should have small wealth effects. Table 10 reports that the empirical elasticity of leisure in our Baseline Economy increases from .29 to .45 when computed with quarterly data. We conclude that aggregation matters a lot.

Observed wages are an imperfect measure of the returns to work of individuals when wages are a non-linear function of hours of work. If instead of using data on wages, we use data on labor productivity \( z \) to run the regression (12) the estimated elasticity increases to .48. Thus, non-linear wages contribute to the “disconnect” between the theoretical and the empirical elasticities. To evaluate the contribution of the extensive margin (leading to truncation of the data on how labor supply responds to “unobserved” shocks), we run the regression (12) including all individuals alive in each quarter, regardless of whether they had worked or not. In particular, when individuals do not work their leisure time is set to 100% of their time endowment and that their labor productivity is given by the realized value of \( z \) in that quarter. We found that the empirical elasticity of leisure is now .91, a value much higher than the theoretical elasticity. In understanding this striking result note that the theoretical
elasticity only describes labor supply responses at the intensive margin, as its derivation assumes an interior solution in the labor supply decision. We later show that this finding is important for reconciling “empirical” elasticities and “aggregate” elasticities. Summing up, our findings point that the empirical elasticity understates the theoretical elasticity by a factor of 2. When the extensive margin is considered, the empirical elasticity using annual data understates the true labor supply response by a factor of 4.

6 Conclusion
Table 1: Transition Matrix across Annual Hours Cells, in Percent, PSID, 1968-1996, Men.

<table>
<thead>
<tr>
<th>Ages 18-29</th>
<th>From</th>
<th>000-100</th>
<th>100-1500</th>
<th>1500-2800</th>
<th>2800+</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>50.11</td>
<td>32.20</td>
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<td>1.17</td>
<td>6.42</td>
<td></td>
</tr>
<tr>
<td>100-1500</td>
<td>7.55</td>
<td>46.68</td>
<td>42.53</td>
<td>3.24</td>
<td>22.00</td>
<td></td>
</tr>
<tr>
<td>1500-2800</td>
<td>0.81</td>
<td>9.73</td>
<td>81.34</td>
<td>8.11</td>
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<tr>
<td>2800+</td>
<td>0.32</td>
<td>4.15</td>
<td>48.78</td>
<td>46.75</td>
<td>9.39</td>
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<table>
<thead>
<tr>
<th>Ages 30-54</th>
<th>From</th>
<th>0-100</th>
<th>100-1500</th>
<th>1500-2800</th>
<th>2800+</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
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<tr>
<td>100-1500</td>
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<td>1500-2800</td>
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<td>2.61</td>
<td>38.78</td>
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<table>
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<th>Ages 55-65</th>
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<th>1500-2800</th>
<th>2800+</th>
<th>Size</th>
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<tbody>
<tr>
<td>0-100</td>
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<td>1.37</td>
<td>4.61</td>
<td>40.52</td>
<td>53.51</td>
<td>8.72</td>
<td></td>
</tr>
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Note - Authors’ calculations from the PSID.
Table 2: Transition Probability across Annual Hours Cells, PSID, Men, High-School.

### Ages 18-29

<table>
<thead>
<tr>
<th>From</th>
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<th>100-1500</th>
<th>1500-2800</th>
<th>2800+</th>
<th>Relative Size</th>
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<td>2.33</td>
<td>12.50</td>
</tr>
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<td>48.14</td>
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### Ages 30-54

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<th>1500-2800</th>
<th>2800+</th>
<th>Relative Size</th>
</tr>
</thead>
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<td>8.43</td>
<td>0.36</td>
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<td>17.83</td>
<td>30.20</td>
<td>50.03</td>
<td>1.94</td>
<td>5.40</td>
</tr>
<tr>
<td>1500-2800</td>
<td>1.18</td>
<td>3.69</td>
<td>90.19</td>
<td>4.94</td>
<td>78.83</td>
</tr>
<tr>
<td>2800+</td>
<td>0.24</td>
<td>2.07</td>
<td>42.72</td>
<td>54.97</td>
<td>9.51</td>
</tr>
</tbody>
</table>

### Ages 55-65

<table>
<thead>
<tr>
<th>From</th>
<th>To 0-100</th>
<th>100-1500</th>
<th>1500-2800</th>
<th>2800+</th>
<th>Relative Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-100</td>
<td>92.35</td>
<td>5.30</td>
<td>2.35</td>
<td>0.0</td>
<td>25.55</td>
</tr>
<tr>
<td>100-1500</td>
<td>42.23</td>
<td>31.75</td>
<td>25.42</td>
<td>0.60</td>
<td>9.88</td>
</tr>
<tr>
<td>1500-2800</td>
<td>4.26</td>
<td>9.33</td>
<td>83.72</td>
<td>2.68</td>
<td>59.94</td>
</tr>
<tr>
<td>2800+</td>
<td>2.59</td>
<td>3.73</td>
<td>44.33</td>
<td>49.35</td>
<td>4.62</td>
</tr>
</tbody>
</table>

Note - Authors’ calculations from the PSID.
Table 3: Transition Probability across Annual Hours Cells, PSID, 1968-1996, Men, College.

<table>
<thead>
<tr>
<th></th>
<th>0-100</th>
<th>100-1500</th>
<th>1500-2800</th>
<th>2800+</th>
<th>Relative Size</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ages 18-29</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From</td>
<td>0-100</td>
<td>100-1500</td>
<td>1500-2800</td>
<td>2800+</td>
<td></td>
</tr>
<tr>
<td>0-100</td>
<td>48.75</td>
<td>29.99</td>
<td>20.14</td>
<td>1.13</td>
<td>5.65</td>
</tr>
<tr>
<td>100-1500</td>
<td>7.99</td>
<td>42.97</td>
<td>47.47</td>
<td>1.57</td>
<td>15.78</td>
</tr>
<tr>
<td>1500-2800</td>
<td>0.65</td>
<td>5.77</td>
<td>88.08</td>
<td>5.49</td>
<td>72.64</td>
</tr>
<tr>
<td>2800+</td>
<td>0.38</td>
<td>2.07</td>
<td>54.50</td>
<td>43.05</td>
<td>5.93</td>
</tr>
<tr>
<td><strong>Ages 30-54</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From</td>
<td>0-100</td>
<td>100-1500</td>
<td>1500-2800</td>
<td>2800+</td>
<td></td>
</tr>
<tr>
<td>0-100</td>
<td>68.47</td>
<td>13.82</td>
<td>16.90</td>
<td>0.81</td>
<td>1.65</td>
</tr>
<tr>
<td>100-1500</td>
<td>8.31</td>
<td>31.72</td>
<td>55.52</td>
<td>4.44</td>
<td>3.27</td>
</tr>
<tr>
<td>1500-2800</td>
<td>0.44</td>
<td>2.28</td>
<td>91.42</td>
<td>5.86</td>
<td>84.55</td>
</tr>
<tr>
<td>2800+</td>
<td>0.24</td>
<td>0.98</td>
<td>49.03</td>
<td>49.76</td>
<td>10.54</td>
</tr>
<tr>
<td><strong>Ages 55-65</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From</td>
<td>0-100</td>
<td>100-1500</td>
<td>1500-2800</td>
<td>2800+</td>
<td></td>
</tr>
<tr>
<td>0-100</td>
<td>93.69</td>
<td>4.48</td>
<td>1.83</td>
<td>0.0</td>
<td>14.13</td>
</tr>
<tr>
<td>100-1500</td>
<td>30.56</td>
<td>37.05</td>
<td>29.42</td>
<td>2.97</td>
<td>21.30</td>
</tr>
<tr>
<td>1500-2800</td>
<td>2.32</td>
<td>7.70</td>
<td>85.90</td>
<td>4.08</td>
<td>73.00</td>
</tr>
<tr>
<td>2800+</td>
<td>0.0</td>
<td>2.94</td>
<td>62.06</td>
<td>35.0</td>
<td>5.70</td>
</tr>
</tbody>
</table>

Note - Authors’ calculations from the PSID.
Table 4: The Coefficient of Variation of Lifetime Hours, PSID, 1968-1996, Men.

<table>
<thead>
<tr>
<th>Age</th>
<th>All</th>
<th>High School</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-35</td>
<td>0.27</td>
<td>0.30</td>
<td>0.22</td>
</tr>
<tr>
<td>35-45</td>
<td>0.26</td>
<td>0.30</td>
<td>0.21</td>
</tr>
<tr>
<td>45-55</td>
<td>0.37</td>
<td>0.40</td>
<td>0.28</td>
</tr>
<tr>
<td>55-65</td>
<td>0.64</td>
<td>0.71</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Note - Authors’ calculations from the PSID.

Table 5: The Variance of Log Lifetime Hours, PSID, 1968-1996, Men.

<table>
<thead>
<tr>
<th>Age</th>
<th>All</th>
<th>High School</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>25-35</td>
<td>0.20</td>
<td>0.29</td>
<td>0.09</td>
</tr>
<tr>
<td>35-45</td>
<td>0.17</td>
<td>0.27</td>
<td>0.07</td>
</tr>
<tr>
<td>45-55</td>
<td>0.38</td>
<td>0.48</td>
<td>0.17</td>
</tr>
<tr>
<td>55-65</td>
<td>1.21</td>
<td>1.65</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Note - Authors’ calculations from the PSID.

Table 6: PSID: Stochastic Process of Hourly Wages.

<table>
<thead>
<tr>
<th></th>
<th>Non-college</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var(α)</td>
<td>0.063</td>
<td>0.083</td>
</tr>
<tr>
<td>ρ</td>
<td>0.972</td>
<td>0.921</td>
</tr>
<tr>
<td>Var(η)</td>
<td>0.013</td>
<td>0.029</td>
</tr>
<tr>
<td>Var(λ)</td>
<td>0.097</td>
<td>0.079</td>
</tr>
</tbody>
</table>

Notes - .
Table 7: Stochastic Process of Hourly Wages: Model vs. Data

<table>
<thead>
<tr>
<th></th>
<th>Non-college</th>
<th></th>
<th>College</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Var((\alpha))</td>
<td>0.063</td>
<td>0.041</td>
<td>0.083</td>
<td>0.081</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.972</td>
<td>0.973</td>
<td>0.921</td>
<td>0.929</td>
</tr>
<tr>
<td>Var((\eta))</td>
<td>0.013</td>
<td>0.018</td>
<td>0.029</td>
<td>0.030</td>
</tr>
<tr>
<td>Var((\lambda))</td>
<td>0.097</td>
<td>0.098</td>
<td>0.079</td>
<td>0.081</td>
</tr>
</tbody>
</table>

Notes - .

Table 8: The Elasticity of Labor Supply

<table>
<thead>
<tr>
<th></th>
<th>(\sigma = 2)</th>
<th>(\sigma = 3)</th>
<th>(\sigma = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\eta^h)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Micro</td>
<td>0.52</td>
<td>0.47</td>
<td>0.44</td>
</tr>
<tr>
<td>Theoretical</td>
<td>1.24</td>
<td>0.98</td>
<td>0.86</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\eta^l)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Micro</td>
<td>0.31</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td>Theoretical</td>
<td>0.70</td>
<td>0.60</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Notes - .

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Table 9: Elasticity of Leisure: Robustness

<table>
<thead>
<tr>
<th>Variance</th>
<th>Transitory Shock</th>
<th>Non-linear</th>
<th>Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>.30</td>
<td>.21</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>.29</td>
<td>.17</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>.27</td>
<td>na</td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Elasticity of Leisure: Decomposition

<table>
<thead>
<tr>
<th>Variance</th>
<th>Transitory Shock</th>
<th>Theoretical</th>
<th>Annual</th>
<th>Quarterly -1</th>
<th>Labor Prod.</th>
<th>Extensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>0.60</td>
<td>.30</td>
<td>.46</td>
<td>.51</td>
<td>.93</td>
<td></td>
</tr>
<tr>
<td>Baseline Economy</td>
<td>0.60</td>
<td>.29</td>
<td>.45</td>
<td>.48</td>
<td>.91</td>
<td></td>
</tr>
</tbody>
</table>

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Figure 1: Mean Annual Hours Worked, 1968-1996, PSID.

Figure 2: Mean Annual Hours Worked, Workers with Positive Hours, 1968-1996, PSID.
Figure 3: Participation Rate, Fraction of Workers with Positive Annual Hours, 1968-1996, PSID.

Figure 4: Variance of Log Annual Hours, 1968-1996, PSID.
Figure 5: Coefficient of Variation of Annual Hours, 1968-1996, PSID.

Figure 6: Mean Annual Hours Worked, 1968-1996, PSID, Men, All and by Education.
Figure 7: Mean Annual Hours Worked, Workers with Positive Hours, 1968-1996, PSID Men, All and by Education.

Figure 8: Participation Rate, Fraction of Workers with Positive Annual Hours, 1968-1996, PSID Men, All and by Education.
Figure 9: Variance of Log Annual Hours, 1968-1996, PSID Men, All and by Education.

Figure 10: Coefficient of Variation of Annual Hours, 1968-1996, PSID Men, All and by Education.
Figure 11: Persistence of Annual Hours, 1968-1996, PSID, Men.

Figure 12: Persistence of Annual Hours, 1968-1996, PSID, Men, High-School.
Figure 13: Persistence of Annual Hours, 1968-1996, PSID, Men, College.
Figure 14: Mean Annual Hours Worked, Model vs. Data.

Figure 15: Mean Annual Hours Worked, Workers with Positive Hours, Model vs. Data.
Figure 16: Participation Rate, Fraction of Workers with Positive Annual Hours, Model vs. Data.

Figure 17: Variance of Log Annual Hours, Model vs. Data.
Figure 18: Coefficient of Variation of Annual Hours, Model vs. Data.

Figure 19: Annual Hours Groups, Model vs. Data.
Figure 20: Persistence of Annual Hours, Model vs. Data.
References


APPENDICES

I  The PSID Dataset: Variable Description.