Reconsidering the Microeconomic Foundations of Price-Setting Behavior

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Board of Governors of the Federal Reserve System
April 2009

Abstract
Although the Dixit-Stiglitz aggregator is the workhorse specification of monopolistic competition, this framework and related variants abstract from a key stylized fact from empirical studies of consumer behavior and product marketing, namely, that the price elasticity of demand for a given brand is primarily determined by the extensive margin (i.e., changes in the number of customers purchasing that product) rather than the intensive margin (i.e., changes in the specific quantity purchased by each individual customer). In this paper, we begin by analyzing household scanner data to confirm the salient empirical results. We then proceed to formulate a new dynamic stochastic general equilibrium modelling framework that captures both the intensive and extensive margins of demand, and we investigate the implications of this framework under two alternative sources of firm-level heterogeneity. First, in the case of idiosyncratic productivity shocks, we obtain analytic results for consumer demand and firms’ price-setting behavior and we show that the implications of the model are consistent with the key stylized facts. Second, in the case of staggered nominal price contracts, we show that the presence of customer search has important consequences for the welfare costs of inflation variability and hence for the design of monetary policy.

JEL classification: E30; E31; E32

Keywords: Customer Search; Product Differentiation; Extensive and Intensive Margin; Quasi-Kinked Demand Curve

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1 Introduction

Over the past several decades, monopolistic competition has played a crucial role in pushing out the frontiers of analytical and empirical work across a wide range of economic fields. In nearly all of those studies, monopolistic competition has been represented in terms of the household preference aggregator introduced by Dixit and Stiglitz (1977) or more recent variants.\(^1\) In particular, the Dixit-Stiglitz aggregator has been a key building block in the development of New Keynesian economics; prominent examples include Rotemberg (1982), Blanchard and Kiyotaki (1987), Dornbusch (1987), Benhabib and Farmer (1994), Rotemberg and Woodford (1997, 1999), McCallum and Nelson (1999), and Khan, King, and Wolman (2003).\(^2\) This specification has also been used in a number of seminal studies in international trade, endogenous growth, and economic geography; see Krugman (1980, 1991), Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995), Bernard et al. (2003), and Melitz (2003).

In recent years, the increased accessibility of highly disaggregated economic data has contributed to a growing interest in developing models of price-setting behavior that can provide closer links between the theory and the empirical evidence.\(^3\) In macroeconomics, of course, that interest also reflects the ongoing quest to address the Lucas (1976) critique by building models with sufficiently deep micro foundations that are reasonably invariant to changes in the policy regime; moreover, the specific characteristics of the micro foundations can turn out to be crucial in determining the features of the welfare-maximizing policy and in assessing the welfare costs of alternative regimes.\(^4\) Even in very recent studies, however, the Dixit-Stiglitz aggregator has continued to serve as the workhorse specification of monopolistic competition; see Broda and Weinstein (2006), Golosov and Lucas (2007), Klenow and Willis (2007), Gertler and Leahy (2008), and Mackowiack and Wiederholt (2008a,b).

Nevertheless, the Dixit-Stiglitz specification and related variants are fundamentally inconsistent with a key stylized fact from empirical studies of consumer behavior and product marketing, namely, that the price elasticity of demand for a given brand is primarily determined by the extensive margin (i.e., changes in the number of customers purchasing that

\(^1\) Such variants include the generalized preference aggregator introduced by Kimball (1995) and the deep habits formulation of Ravn, Schmitt-Grohe, and Uribe (2007).

\(^2\) See also Yun (1996) and Erceg, Henderson, and Levin (2000).


\(^4\) See Levin et al. (2005), Schmitt-Grohe and Uribe (2005), Levin, Lopez-Salido and Yun (2007), and Levin et al. (2008).
product) rather than the intensive margin (i.e., changes in the specific quantity purchased by each individual customer). Indeed, the extensive margin is completely absent from the Dixit-Stiglitz framework, which assumes that every household purchases output from every producer and hence that the elasticity of demand faced by the firm is identical to the own-price demand elasticity of households. Moreover, the Dixit-Stiglitz framework requires a high value of this elasticity to match a low steady-state markup of price over marginal cost, whereas consumer demand studies indicate that own-price elasticities of demand are very low—even for extremely narrow product categories—and the empirical marketing literature has shown that the elasticity of demand for a firm's product mainly depends on consumers' choices over competing brands within a narrow product category.\(^5\)

In this paper, we begin by analyzing household scanner data to confirm several key stylized facts, namely, consumers typically purchase a single brand within each narrow product category, and the own-price elasticity of household demand is very low compared with the demand elasticity for each brand within the product category. We then proceed to formulate a new dynamic stochastic general equilibrium (DSGE) modeling framework that captures both the intensive and extensive margins of demand, and we investigate the implications of this framework under two alternative sources of firm-level heterogeneity. First, in the case of idiosyncratic productivity shocks, we obtain analytic results for consumer demand and firms' price-setting behavior and we show that the implications of the model are consistent with the key stylized facts. Second, in the case of staggered nominal price contracts, we show that the presence of customer search has important consequences for the welfare costs of inflation variability and hence for the design of monetary policy. Finally, we discuss how this new approach holds substantial promise for future research on price-setting behavior, not only in enhancing the linkages between micro data and macro models but in building stronger connections to ongoing research in marketing and consumer behavior.\(^6\)

Our theoretical framework involves a two-dimensional product space and incomplete household information, giving rise to an equilibrium price distribution with customer search.\(^7\) The first dimension of the product space represents the set of narrow categories of consumer goods and services—such as bathsoap, breakfast cereal, toothpaste, haircuts, etc.—while the second dimension captures the extensive margin of consumer behavior, specifically the brand choices made by each consumer.\(^7\)

\(^5\)The classic studies of consumer demand may be found in Stone (1954), Theil (1965), and Deaton and Muellbauer (1980).

\(^6\)The need for such connections has recently been emphasized by Rotemberg (2008).

\(^7\)Guimarães and Sheedy (2008) also formulate a DGE model with a two-dimensional product space that provides important insights about firms' motives for posting temporary sale prices.
and automotive repairs—and the second dimension represents the set of producers within each category. Moreover, consistent with the typical pattern observed in the micro data, we assume that each household purchases items from only a single producer within a given product category. Finally, households have an intrinsic search motive because they have full knowledge of the distribution of prices within each product category but do not have ex ante knowledge of the characteristics of individual producers.

In an environment where firms are subject to idiosyncratic productivity shocks, we can derive analytical expressions for the equilibrium distribution of prices and for the optimal price-setting behavior of each producer by assuming a uniform distribution of productivity shocks across firms and a uniform distribution of search costs across household members. We show that each firm faces a continuous downward-sloping demand curve, where the steady-state elasticity of demand can be represented as the sum of the elasticity at the intensive margin (which is determined by the household's own-price elasticity of demand for that product category) and the elasticity at the extensive margin (which is determined by the distribution of search costs). With an empirically reasonable calibration, our framework matches both the relatively low demand elasticity of households and the relatively high demand elasticity faced by each producer.

Although the analysis of customer search has been relatively quiescent in recent years, our investigation builds directly on the earlier work of MacMinn (1980), who analyzed sequential search in a partial-equilibrium setting with firm-specific idiosyncratic shocks, heterogeneous search costs, and completely inelastic demand by each consumer (and hence no intensive margin). Moreover, the role of customer search in generating a kinked or quasi-kinked demand curve for each firm was emphasized by Stiglitz (1979, 1987). Finally, a large but now-dormant literature studied the incidence of relative price dispersion in search models with nominal rigidities.\(^8\)

Our analysis is also reminiscent of the burgeoning literature on the role of search in labor markets, although that literature has mainly focused on the extensive margin of demand. In the case of customer search, of course, there are no direct parallels to the incidence of unfilled job vacancies or of workers who remain unemployed while searching for a new job; hence, specifying an aggregate matching function for customers and firms might provide a useful approximation for analyzing inflation dynamics, as in Hall (2008), but might not be as useful for interpreting micro evidence on price-setting behavior. Finally,

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while the assumption of an exogenous separation rate may be useful for interpreting U.S. labor market data over the past couple of decades, specifications that incorporate on-the-job search and endogenous separation may well have greater potential for cross-fertilization with the analysis of customer search.\footnote{For recent analysis of labor market separation patterns, see Shimer (2005), Hall (2006), and Davis et al. (2006).}

Finally, it is interesting to compare our approach with other recent studies that have formulated extensions of the Dixit-Stiglitz framework, including the generalized aggregator of Kimball (1995), the deep habits model of Ravn, Schmitt-Grohe, and Uribe (2005), and the nested CES aggregator of Atkeson and Burstein (2008). Each of those specifications maintains the assumption that every differentiated product is consumed by every household, implying that the elasticity of demand is determined solely at the intensive margin. While the absence of an extensive margin of demand or a role for customer search may be innocuous for some purposes, our analysis suggests that capturing these features of price-setting behavior in a DSGE model can be crucial for characterizing the welfare costs of inflation and the optimal design of monetary policy.

The remainder of this paper is organized as follows. In sections 2 and 3, we analyze the disaggregated household scanner data and the aggregate data for peanut butter to document empirical evidence on extensive and intensive margins of demand, respectively. Section 4 specifies our analytical framework. In section 5, we briefly summarize the implications of this framework for demand analysis. Section 6 contains welfare implications of inflation. Section 7 concludes.
2 Empirical Evidence on the Extensive Margin of Demand

In this section, we confirm the key stylized fact that the price elasticity of demand for a given brand is primarily determined by changes in the number of customers rather than changes in the specific quantity purchased by each individual.

The ERIM data set tracks information for each UPC at several stores in two markets (Sioux Falls, South Dakota and Springfield, Missouri) between January 1985 and June 1987 for narrowly defined product categories such as ketchup, canned tuna, peanut butter, stick margarine and toilet tissue. Using this data, we document the following stylized facts regarding the extensive margin of demand:

- Each consumer faces a multiplicity of producers (brands) of each specific good/service, but typically purchases the item from a single producer (e.g., haircuts, toothpaste).
- Firms face relatively high demand elasticities (consistent with low markups), reflecting the dominant role of the extensive margin (number of customers) rather than the intensive margin (quantity purchased by each customer).

2.1 Selection of Brands

In order to see the first stylized fact, we investigate how frequently a typical household changes producers each month for narrow product categories. It is the case in the A.C. Nielsen ERIM data (Chicago GSB) that a wide diversity of brands exist in narrow product categories. For example, creamy peanut butter includes a variety of brands such as Arrowhead, Peter Pan, Billy Boy, Robb Ross, Elam’s, Skippy, Hallam’s, Smucker’s, Home Brand, Sun Gold, JIF, Superman, and 26 chain-specific “private label” brands. Nevertheless, as shown in Figure 1, more than 90 percent of households who purchased creamy and chunky peanut butters 18 oz. buy the item from a single producer. We also see essentially the same brand choice behavior of households for ketchup 32 oz. and tuna 6.5 oz. as well.
Figure 1: Consumer Choices in Narrow Product Categories

Note: this figure shows how many different brands households on average each month between January 1985 and June 1987.

2.2 The Price Elasticity of Demand at the Extensive Margin

The ERIM data suggest that firms face relatively high demand elasticities, consistent with low markups. In order to show this, we follow the approach of Hausman, Leonard and Zona (1994) that takes account of a three stage demand system in estimating demand for differentiated products. For example, the top level correspond to overall demand for the product such beer. The middle level corresponds different segments for the product. The bottom level of the demand system corresponds to competition among brands in a given segment.

We begin with the bottom level of the demand system. The demand specification for

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10The household data were collected from a panel of households that were issued magnetic ID cards to be presented at the checkout counter in participating stores. Purchases made by households were scanned along with their ID numbers. Sales from the participating stores were collected for all universal product code (UPC) products through scanners at the stores’ checkout counters.
each product within a market segment is given by

\[ s_{int} = \alpha_i + \omega_{int}d_{nt} + \beta_i \log(y_{Gnt}/P_{nt}) + \sum_{j=1}^{J} \gamma_{ij} \log p_{jnt} + \epsilon_{int} \] (1)

where \( s_{int} \) is the revenues share of total segment expenditure of the \( i \)th brand in city \( n \) in period \( t \), \( d_{nt} \) is the dummy variable for city \( n \), \( y_{Gnt} \) is overall segment expenditure, \( P_{nt} \) is the price index and \( p_{jnt} \) is the \( j \)th brand in city \( n \).

It is worthwhile to mention the identification of coefficients for individual prices in the demand equation specified above because prices are likely to be correlated with error terms. Following the work of Hausman and Taylor (1981), Hausman et al. (1994) use prices of one city as instrument variables for prices of another city.\(^{11}\) The idea is that prices in one city (after elimination of city and brand effects) are driven by underlying costs that can be used as instrument variables. Specifically, they assume that the price for a brand \( i \) in city \( n \) at time \( t \) is determined by the following equation:

\[ \log p_{jnt} = \delta_j \log c_{jt} + \alpha_{jn} + w_{jnt} \]

where \( p_{jnt} \) is the \( j \)th brand in city \( n \) and \( \alpha_{jn} \) is a city specific brand differential that accounts for transportation costs or local wage differentials. In addition, they assume that a mean zeros stochastic disturbance \( w_{jnt} \) is uncorrelated with \( \epsilon_{int} \) if \( n \neq m \).

In the analysis of peanut butter data, we consider two distinct segments of the peanut butter market, known as “creamy” and “chunky”.\(^{12}\) Although there are a large number of competing brands of peanut butter, three national brands have a dominant share of the market; thus, in our empirical analysis, we compute weighted averages for all of the regional and store-specific brands and treat this group as a single brand (commonly referred to as the “control” brand).

\(^{11}\) Alternatively one can use factor price instruments such as wages and prices materials that do not affect unobserved demand shocks. It is also possible to construct a time-series of dummies that can be used as an instrument variable if the aggregate price index of a particular product shifts independent of demand shocks.

\(^{12}\) A brief explanation for non-afficianados of peanut butter: the “creamy” label indicates that the peanut butter is completely smooth and homogeneous, whereas the “chunky” label indicates that small fragments of roasted peanuts are mixed in with the butter.
Table 1: The Demand for Competing Brands of Peanut Butter

<table>
<thead>
<tr>
<th></th>
<th>Creamy, 18 oz.</th>
<th>Chunky, 18 oz.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jif</td>
<td>PPan</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.02</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.29)</td>
</tr>
<tr>
<td>log(Y/P)</td>
<td>0.05</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>log(P_{Jif})</td>
<td>-0.93</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>(0.31)</td>
<td>(0.40)</td>
</tr>
<tr>
<td>log(P_{PPan})</td>
<td>0.22</td>
<td>-0.81</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>log(P_{Skippy})</td>
<td>0.63</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.27)</td>
</tr>
<tr>
<td>log(P_{cntrl})</td>
<td>0.20</td>
<td>-0.30</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.22)</td>
</tr>
<tr>
<td>City Dummy</td>
<td>0.01</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.26</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Note: For each column, the dependent variable is the share of the specified national brand (Jif, Peter Pan, or Skippy) as a fraction of total expenditures in that product category (creamy or chunky, 18 oz. container), and the explanatory variables include a constant, the logarithm of real expenditures, log(Y/P), the price of each national brand, and the average price of all other regional and store-specific brands. Standard errors are given in parentheses.
Table 2: The Extensive Margin of Demand for Peanut Butter  
*(Conditional Own-Price Elasticities of National Brands)*

<table>
<thead>
<tr>
<th>Brand Name</th>
<th>Creamy, 18 oz.</th>
<th>Chunky, 18 oz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jif</td>
<td>-5.2</td>
<td>-5.8</td>
</tr>
<tr>
<td></td>
<td>(1.4)</td>
<td>(1.2)</td>
</tr>
<tr>
<td>Peter Pan</td>
<td>-3.2</td>
<td>-3.7</td>
</tr>
<tr>
<td></td>
<td>(0.5)</td>
<td>(0.8)</td>
</tr>
<tr>
<td>Skippy</td>
<td>-7.0</td>
<td>-6.1</td>
</tr>
<tr>
<td></td>
<td>(1.6)</td>
<td>(1.4)</td>
</tr>
</tbody>
</table>

Note: The regression equation for each brand share implies that the own-price elasticity \((\epsilon_i)\) and the own-price coefficient \((\beta_i)\) in the regression equation for brand \(i\) should satisfy \(\epsilon_i = \beta_i \left( \frac{1}{T} \sum_{t=1}^{T} s_{it}^{-1} \right) - 1\), where \(s_{it}\) is the expenditure share of brand \(i\) at period \(t\). Standard errors are given in parentheses.

Furthermore, as for many retail items, each manufacturer typically makes its products in a variety of different sizes that are appealing to distinct groups of consumers. In the ERIM dataset, for example, each national brand of peanut butter is available in three sizes: 18 ounces, 28 ounces and 40 ounces. Although not shown here, we have documented that most households purchase a single size of peanut butter and that choice of size does not vary significantly over time; that is, the household typically continues to purchase the same size container even when switching to a different brand. In effect, each size of peanut butter should be viewed as a distinct product category. Since this is the case, it is important to have segments of products appropriately defined in this type of empirical analysis. For example, if the definition of a segment is too narrow, it tends to create high substitution between different segments.

Turning to estimation results, Table 1 shows estimation results of brand share equations for Jif, Peter Pan and Skippy. The estimates of own-price coefficients are all significantly negative, which fall into a range of -0.8 to -1.2. These estimates indicate that the market share of each brand are likely to fall by around 1 percent as the logarithm of its price rises.

\(^{13}\) Accordingly, each segment amounts to a narrow product category that helps define a product customers might search for a seller who provides the lowest price or the best quality. It is important to have segments of products appropriately defined in this type of empirical analysis. For example, if the definition of a segment is too narrow, it incurs substantially high substitution between different segments.
Table 3: The Extensive Margin of Demand for Beer

(*Estimates from Hausman et al., 1994*)

<table>
<thead>
<tr>
<th></th>
<th>Conditional Elasticity</th>
<th>Popular Brands</th>
<th>Conditional Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Premium Brands</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Budweiser</td>
<td>-3.5 (0.1)</td>
<td>Old Milwaukee</td>
<td>-4.8 (0.1)</td>
</tr>
<tr>
<td>Molson</td>
<td>-5.1 (0.2)</td>
<td>Genesee</td>
<td>-3.8 (0.1)</td>
</tr>
<tr>
<td>Labatts</td>
<td>-4.3 (0.3)</td>
<td>Milwaukee’s Best</td>
<td>-5.8 (0.2)</td>
</tr>
<tr>
<td>Miller</td>
<td>-4.2 (0.3)</td>
<td>Busch</td>
<td>-5.7 (0.3)</td>
</tr>
<tr>
<td>Coors</td>
<td>-4.6 (0.2)</td>
<td>Piels</td>
<td>-4.0 (0.5)</td>
</tr>
<tr>
<td><strong>Popular Brands</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The estimates of conditional own-price elasticities for each brand of premium beer (left panel) and each brand of popular beer (right panel) are taken from Hausman et al. (1994). Although not shown in the table, their study also obtained corresponding estimates for five brands of light beer: Genesee Light, -3.2; Coors Light, -4.2; Lite Beer, -4.8; Molson Light, -5.7; and Old Milwaukee Light, -5.9.

by one percent. In addition, the adjusted \( R^2 \) of brand share equations covers a range of 0.3 to 0.4.

To gauge the extensive margin of demand, we now use these regression results to compute the conditional own-price elasticity of demand. Using the multi-stage demand system, it is possible to allow for conditional own-price elasticities and total own-price elasticities depending on whether to include the effect of a change in a particular brand’s price on the expenditure of the market segment that the brand belongs to. The conditional own-price elasticity is defined as the own-price elasticity holding constant the expenditure of market segment. For the creamy peanut butter, the conditional own elasticities are -3.2 to -7.0. For example, Jif has a conditional own elasticity of -5.2, Peter Pan is -3.2 and Skippy is -7.0. Similarly, estimates of the chunky peanut butter covers a range of -3.7 and -6.1. As a result, the ERIM data indicate that firms face relatively high demand elasticities consistent with low markups. These estimates are comparable to those shown
in Hausman et al. (1994). For example, the conditional own-price elasticities are in the range of -3.5 to -5.0 for the premium beer.

3 Empirical Evidence on the Intensive Margin of Demand

We now consider the magnitude of the intensive margin of household demand for a given product category, which can be gauged in terms of the own-price elasticity of demand for that product. In particular, we analyze the intensive margin of demand for peanut butter, thereby facilitating comparison with our foregoing estimates regarding the extensive margin of demand.

3.1 Estimation using Instrumental Variables

In estimating the slope of demand for a given product, of course, one faces the classic econometric challenge of identifying exogenous supply shocks that effectively trace out the shape of the demand curve. The standard approach—taken in Hausman et al. (1994) and numerous other studies—is to estimate the demand equation via two-stage least-squares (2SLS), using national time series data on average retail prices, consumption, and disposable income, with input prices of labor and raw materials serving as instruments.

For measuring the total volume of peanut butter consumption, there is no published data on retail sales or consumer spending. However, the National Agricultural Statistics Service does produce a regular publication titled “Peanut Stocks and Processing” which reports on the monthly volume of shelled peanut shipments (measured in thousands of pounds) from farms to peanut butter manufacturers.\(^{14}\) Since roasted peanuts and a pinch of salt are the only ingredients used in producing peanut butter, this time series can serve as an excellent proxy for total U.S. peanut butter consumption.\(^{15}\) As for the retail price of peanut butter, we use the national average unit price (measured in dollars per pound) published on a monthly basis by the Bureau of Labor Statistics.\(^{16}\) Finally, our analysis

\(^{14}\)This series does not reflect the magnitude of imported peanuts, which was negligible prior to 1994, increased rapidly to a volume equivalent to about 4 percent of total U.S. farm production by 2001, and then faded away again after the enactment of the U.S. Farm Act of 2002; see figure 4 of Dohlman and Livezey (2005) and the accompanying discussion.

\(^{15}\)We assume that a two-month lag occurs between shipment of the raw peanuts and the retail sale of the resulting peanut butter; this timing assumption reflects the characteristics of the fairly brief production process, transportation lags, and the prevalence of just-in-time inventory management in this market.

\(^{16}\)This series indicates the average unit price of creamy peanut butter across all U.S. cities for all container sizes. For our purposes, it would have been ideal to have retail price data for a specific size (namely, 18 ounces) for each type of peanut butter (creamy and chunky); however, the published BLS series is likely to be an excellent proxy, since our analysis of the ERIM dataset did not reveal any systematic deviations between the prices of creamy vs. chunky peanut butter nor any systematic time variation in the deviations.
Table 4: The Elasticity of Demand at the Intensive Margin

<table>
<thead>
<tr>
<th>Product Category</th>
<th>Own-Price Elasticity</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peanut Butter</td>
<td>-1.0</td>
<td>This Paper</td>
</tr>
<tr>
<td></td>
<td>(0.4)</td>
<td></td>
</tr>
<tr>
<td>Beer</td>
<td>-1.4</td>
<td>Hausman et al. (1994)</td>
</tr>
<tr>
<td></td>
<td>(0.2)</td>
<td></td>
</tr>
<tr>
<td>Canned Tuna</td>
<td>-0.3</td>
<td>Babula and Corey (2004)</td>
</tr>
<tr>
<td></td>
<td>(0.7)</td>
<td></td>
</tr>
<tr>
<td>Butter</td>
<td>-0.4</td>
<td>Deaton &amp; Muellbauer (1985)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

uses two input prices as instruments: the unit price of raw peanuts (as reported by the U.S. Department of Agriculture), and the hourly compensation of employees in the nonfarm business sector. Each price series is deflated by chain-weighted price index for personal consumption expenditures.

Thus, using monthly data over the period January 1985 to June 2008, we obtain the following regression results via 2SLS:

$$
\log C_t = -0.96 \log \tilde{P}_t + 1.47 \log Y_t - 18.5 \log N_t - 75.7 + 0.0004T + \epsilon_t
$$

where $C_t$ denotes the volume of peanut butter consumption, $\tilde{P}_t$ is the relative price of peanut butter, $Y_t$ denotes real disposable income, $N_t$ is total U.S. population, and the standard error of each coefficient is shown in parentheses.\(^{17}\) The value of $R^2$ for this regression is 0.28.

Evidently, the elasticity of peanut butter demand at the intensive margin is very close to unity. Of course, given the sampling uncertainty of the coefficient estimate, the magnitude of the true elasticity might well be nearly twice as large but could also be quite close to zero. In any case, the product-level elasticity is almost surely much smaller than any of the brand-specific elasticities reported above; that is, the elasticity of demand at the intensive

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\(^{17}\) Although not shown in equation 2, the regression also includes seasonal dummies, which are also specified as instruments along with the constant, time trend, and the logarithms of total population, real disposable income, real hourly compensation, and the relative price of raw peanuts.
margin is relatively small compared with the extensive margin.

3.2 Confirmation from a “Natural Experiment”

In the early 2000s the U.S. peanut butter market encountered a large, discrete, transparent, and exogenous supply shock: the enactment of the Farm Act of 2002.18 This legislation eliminated the quotas on peanut farm production that had been in place since 1949, leading to a substantial decline in raw peanut prices that in turn placed downward pressure on the retail price of peanut butter. Indeed, as shown in figure 2, the national average price of peanut butter (in dollars per pound) had been fairly steady at about $1.95 around the beginning of the decade but dropped to about $1.70 by 2005—a decline of about 13 percent. Over this same period, the volume of peanut butter consumption (measured at an annual rate) grew from about 800 million pounds in 2001 to about 950 million pounds in 2005—a rise of about 19 percent. Thus, this “natural experiment” implies that the own-price elasticity of peanut butter demand is about -1.4, a value that is well within the confidence bands of the 2SLS estimate reported above.

18See Dohlman et al. (2004) and Dohlman and Livezey (2005) for a detailed discussion of the peanut-related provisions of the Farm Act of 2002 and its effects on peanut growers’ output and prices.
3.3 Substitution between Narrow Product Categories

We now move onto the analysis of the substitution between narrow product categories. In order to do this, we analyze the demand of peanut butter using the disaggregated household scanner data. In particular, the current analysis corresponds to the middle level within the three stage demand system used in Hausman, Leonard and Zona (1994).

In the estimation of demand equations described below, we consider two segments for peanut butter: creamy peanut butter and chunky peanut butter. The estimation results for product segments can be summarized as follows:

\[
\log q_{crnt} = 1.0 \log y_{Bnt} + 0.9 \log \pi_{crnt} - 0.9 \log \pi_{chnt} - 0.2 - 0.03 d_{crnt} + \epsilon_{crnt}
\]

\[
\log q_{chnt} = 1.0 \log y_{Bnt} - 1.5 \log \pi_{crnt} + 1.4 \log \pi_{chnt} - 0.3 - 0.01 d_{chnt} + \epsilon_{chnt}
\]

where \( q_{mnt} \) is log quantity of the \( m \)th segment in city \( n \) at period \( t \), the expenditure variable \( y_{Bnt} \) is the total peanut butter expenditure, and the \( \pi_{mkt} \) is the price index for segment \( m \), \( d_{mnt} \) is the city dummy. In addition, \( m = cr \) denotes the creamy peanut butter and \( m = ch \) corresponds to the chunky peanut butter. We also construct the price index for each segment, \( \pi_{mkt} \), by calculating a weighted average of average transaction prices of brands in each segment.
4 A New Analytical Framework

In this section, we specify a new analytical framework in which consumers have imperfect information about the characteristics of each producer within a narrow product category.

We begin with the product space of our model. Our product space is two-dimensional. Along the first dimension, there are a lot of different product categories. Each household purchases many different specific types of goods and services (bath soap, breakfast cereal, toothpaste, haircuts, auto repairs). Moreover, there are a lot of producers within each product category. For each specific type of good, each household purchases that item from a single one of many nearly-identical producers.

In order to generate a non-degenerate price distribution for each product category, we assume that each producer with a narrow product category is affected by idiosyncratic technology shocks. These idiosyncratic shocks are identically and independently distributed across different product categories and over time. The production technology of each producer is linear in labor. Labor services are traded in a perfectly competitive labor market and wages are fully flexible.

Each household knows the properties of the distribution of firm-specific characteristics and the distribution of prices for each specific product category. The consumer does not have \textit{ex ante} knowledge of the characteristics of each individual producer nor of the price charged by each producer. Therefore, in the presence of firm-level heterogeneity, each consumer has an incentive to search and thereby obtain information about individual producers.

We assume a continuum of households, each of which is comprised of many individual members; each member is responsible for searching across the producers within a single product category. There is a fixed cost per search which varies randomly across members of the household, and household members are not permitted to communicate with each other during the search process. Under these assumptions, each household member follows a search strategy involving a maximum reservation price. Finally, the household makes its spending decisions given the vector of prices obtained during the search process.

Of course, no firm can charge a price that exceeds consumers’ maximum reservation price. Therefore, since we assume that there are no costs of entry or exit, a firm with productivity below a specific threshold will optimally choose to shut down its production during that period rather than earning negative profits.
4.1 Household Optimization and Search Behavior

Each period is divided into two sub-periods: the first-half is the search stage at which households search for a particular seller within each product category and the second half is the spending stage at which households optimize their total expenditures after a particular seller has been chosen for each of different products.

As a result of this assumption, the optimization problem of households at the spending stage becomes the same as the one that would have been in the presence of perfect knowledge about prices of sellers. Specifically, each household minimizes the total cost of obtaining $C_t$ to determine the intensive margin of demand:

$$C_{ijt}(j) = \frac{P_{ijt}(j) - \theta C_t}{P_t} - \theta C_t,$$

where $P_t$ denotes the Lagrange multiplier of this cost minimization problem: $P_t = \int_{j=0}^{1} (P_{ijt}(j))^{1-\theta} dj$. Here, households aggregate differentiated goods to produce composite goods using the Dixit-Stiglitz aggregator:

$$C_t = \left( \int_{j=0}^{1} C_{ijt}(j)^{2-\theta} dj \right)^{\frac{\theta}{\theta - 1}}, \quad (4)$$

where $C_t$ is the real amount of the composite goods and $C_{ijt}(j)$ is the amount of type $j$ goods that each household purchases from a seller $i_j$. The nominal expenditure of the household ($= L_t$) is then given by $L_t = P_tC_t$. In particular, we note that this Lagrange multiplier is identical across households because search costs are identically and independently distributed across different products and households in each period.

In the spending stage, households also determine demands for composite goods and supplies of labor services, where the preference of each household at period 0 is given by

$$\sum_{t=0}^{\infty} E_0[U(C_t, \bar{H} - H_t)], \quad (5)$$

where $C_t$ is the consumption at period $t$, $\bar{H}$ is the amount of time endowment available for each household, $H_t$ is the amount of hours worked at period $t$. The instantaneous utility function $U(C_t, \bar{H} - H_t)$ is continuously twice differentiable and concave in consumption and leisure.

Furthermore, there is a complete financial market in which all agents can participate. In addition, wages are fully flexible in a perfectly competitive market. Given these assumptions, the period budget constraint of each household can be written as

$$\int_{0}^{1} (P_{ijt}(j)C_{ijt}(j) + z_t(j)X_t(j)) dj + E_t[Q_{t,t+1}B_{t+1}] \leq W_t^N H_t + B_t + \Phi_t, \quad (6)$$

where $P_{ijt}(j)$ is the dollar price at period $t$ of good $j$ at seller $i_j$, $X_t(j)$ is the number of search that an individual household has made in order to determine a seller, $z_t(j)$ is
the nominal cost of each visit to a seller, \( Q_{t,t+1} \) is the stochastic discount factor used for computing the dollar value at period \( t \) of one dollar at period \( t+1 \) so that \( E_t[Q_{t,t+1}B_{t+1}] \) is the nominal price at period \( t \) of an asset whose payoff at period \( t+1 \) is \( B_{t+1} \), \( B_t \) is the nominal payoff of the household’s financial portfolio held between periods \( t-1 \) and \( t \), \( W_t^N \) is the nominal wage rate, and \( \Phi_t \) is the dividend distributed to households.

Given specification of the budget constraint, the utility maximization of the household leads to the following optimization conditions; the labor supply decision yields

\[
U_2(C_t, \bar{H} - H_t) = (W_t^N/P_t)U_1(C_t, \bar{H} - H_t)
\]

and the inter-temporal substitution of consumption between periods \( t \) and \( t+1 \) leads to

\[
Q_{t,t+1} = \beta U_1(C_{t+1}, \bar{H} - H_{t+1})P_t/(U_1(C_t, \bar{H} - H_t)P_{t+1})
\]

where \( U_1(C_t, \bar{H} - H_t) \) is the marginal utility of consumption and \( U_2(C_t, \bar{H} - H_t) \) is the marginal utility of leisure.

Having described the spending decision, we now explain how households determine reservation prices for their sequential searches. At the beginning of each period, a shopping member of the representative household is endowed with a realized value of a random variable \( z \). Each visit to a seller incurs a nominal amount of cost that is proportional to the total expenditure of the household: \( zL_t \). The random variable \( z \) is identically and independently distributed across households and product categories: a uniform distribution on a bounded interval between \( \underline{z} \) and \( \bar{z} \).

Turning to the determination of the reservation price, we will pick a representative product category, so that we do not use any subscript for different product categories when we describe the search decision of each household. Each shopping member stops searching for any price observation \( P_{it} \leq R_t(z) \), while for any \( P_{it} \geq R_t(z) \), they continue to search. In order to describe the determination of a reservation price in the context of dynamic programming, we let \( V(P_{it}) \) represent the value function of search cost when the

\[19\] In our model, we assume that a particular level of search cost is randomly assigned to each household for each narrow product category in each period. In additions, a continuum of different products exist in the economy. Therefore, we can rely on the law of large numbers in order to have an identical level of expected total search cost across households. It is also not difficult to see that households have incentives to find a seller that gives the lowest price for each narrow product category. The partial derivative of the consumption expenditure function with respect to the relative price of an individual price is positive: \( \partial L_t/\partial P_{it}(j) = (P_{it}(j)/P_t)^{-\theta}C_t \). Hence, to the extent that \( \theta \) is not very big and search does not require an arbitrary large amount of costs, we can find that households have incentive to search for sellers with the lowest price for each type of goods. As a result, the standard Dixit-Stiglitz model without search can be viewed as implicitly assuming that search costs are arbitrarily large so that no consumer wants to search.
nominal price of his or her first visit is \( P_{it} \). Then, the optimization of a shopping member whose objective is to find a seller with the lowest price can be written as

\[
V(P_{it}) = \min\{L_t(P_{it}), zP_{it}C_t + \int_0^{P_{\max,t}} V(K) dF(K)\}. \tag{7}
\]

The reservation price then satisfies the following condition:

\[
zP_t^{1-\theta} = \int_0^{R_t(z)} \{P_{it}^{\theta}F(P_{it})\}dP_{it}. \tag{8}
\]

The left-hand side of (8) corresponds to the cost of an additional search, while the right-hand side is its expected benefit. Meanwhile, it is noteworthy that, given the reservation price strategy, the fraction of the aggregate search cost in the aggregate consumption is 

\[
\frac{\int_{\bar{z}}^{\bar{z}} \int \frac{z}{F(R_t(z))} dz}{\bar{z} - \bar{z}}.
\]

4.2 Matching between Customers and Sellers and the Resulting Demand Curve

Having described the search behavior of the representative household, we now turn to the demand curve facing each seller within a narrow product category. Moreover, we continue to describe the demand curve facing an individual seller without any subscript to specify each product category.

We first describe the matching between customers and sellers. Consumers whose reservation price is \( R_t(z) \) are randomly distributed to a set of sellers whose prices are less than \( R_t(z) \). Thus, the measure of consumers with reservation price \( R_t(z) \) who are assigned to a group of sellers whose nominal price is \( P_{it} \) is given by \( f(P_{it})/(F(R_t(z))(\bar{z} - \bar{z})) \). Therefore, the total expected number of consumers who purchase their products at a shop with its nominal price \( P_{it} \) is given by

\[
\frac{1}{\bar{z} - \bar{z}} \int_{\bar{P}_t}^{\bar{R}_t} \frac{f(P_{it})}{F(R_t(z))} dz
\]

where \( \bar{R}_t \) is the maximum reservation price at period \( t \). In order to rewrite the total expected number of consumers in terms of prices, we note that the impact of an infinitesimal increase in search cost on reservation price is \( dz = P_{it}^{\theta-1}\{F(R_t(z))R_t(z)^{-\theta}\}dR_t(z) \). Substituting this equation into the integral specified above, we find that the expected number of consumers for each individual seller whose nominal price is \( P_{it} \) can be written as

\[
N_t(P_{it}) = (\bar{z} - \bar{z})^{-1}(\bar{R}_t^{1-\theta}/1-\theta - P_{it}^{1-\theta}/1-\theta)P_{it}^{\theta-1}.
\]

Up to this point, we have described how the demand facing an individual firm is affected by the demand of an individual consumer and the number of consumers who
decides to purchase. We call the former the demand at the intensive margin and the latter at the demand at the extensive margin. We then combine both the extensive and intensive margins in order to have the following demand curve of each seller:

\[ D_t(\tilde{P}_{it}) = \tilde{P}_{it}^{-\theta} \left( \tilde{R}_{11}^{-\theta} - \frac{\tilde{P}_{it}^{-\theta}}{1-\theta} \right) Y_t / (\tilde{z} - \tilde{z}) \],

(9)

where \( D_t(\tilde{P}_{it}) \) is the demand function at period \( t \) when a seller sets its relative price at \( \tilde{P}_{it} \) and \( \tilde{R}_t \) is the relative price of the maximum reservation price.

An immediate implication of the demand curve (9) is that the elasticity of demand for each good, denoted by \( \epsilon(\tilde{P}_{it}) \), depends on its relative price. The main reason for this is associated with the presence of the maximum relative price. In particular, the expected number of consumers turns out to be nil when relative price of each firm exceeds the maximum reservation price. Thus, the logarithm of the expected number of consumers is not linear in the logarithm of the relative price. Specifically, the elasticity of demand can be written as follows:

\[ \epsilon(\tilde{P}_{it}) = \theta + \frac{\tilde{P}_{it}^{1-\theta}}{\tilde{R}_t^{1-\theta}/(1-\theta) - \tilde{P}_{it}^{1-\theta}/(1-\theta)} \].

(10)

The demand elasticity reflects both of the elasticity of demand at the intensive margin and the elasticity of demand at the extensive margin. For example, the first-term in the right-hand side of this equation is the elasticity of demand at the intensive margin, while the second-term corresponds to the elasticity of demand at the extensive margin. In particular, the relative size of these two elasticities indicates which source is more important in the determination of the monopoly powers of firms. When \( \theta \) is slightly above one, product differentiation is not a major contribution to the monopoly power of firms in the model. Rather, firms can have substantial monopoly power because of search costs together with the imperfect information of consumers about the location of prices. As a result, the introduction of customer search is useful in matching a low level of the markup even when \( \theta \) is less or closely greater than one.\(^{20}\)

Now we turn to the discussion of the aggregate price level and the aggregate production relation. In the first place, we note that the GDP deflator is defined as the ratio of the aggregate nominal output to the real output. Along with the fact that nominal outputs of individual firms should sum up to the nominal aggregate output, the definition of the

\(^{20}\)In order to see the relation between search cost and demand elasticity that is implied by the elasticity equation specified above, we note that the price dispersion across firms becomes so small as \( \tilde{z} \) gets close to zero. In this case, the elasticity of demand approaches infinity so that all firms act like perfect competitors.
GDP deflator implies the following relation:

\[
\frac{\bar{P}_t}{P_t} = \frac{P_t^{2(\theta-1)}}{\bar{z} - \bar{z}} \int_{P_{\min,t}}^{P_{\max,t}} P_{it}^{1-\theta} (\frac{R_{it}^{1-\theta}}{1-\theta} - \frac{P_{it}^{1-\theta}}{1-\theta})dF(P_{it})
\]

where \(\bar{P}_t\) is the price index of output.

The resulting aggregate social resource constraint becomes \(Y_t = (P_t/\bar{P}_t) C_t(1 + X_t)\). We now point out that this form of the aggregate social resource constraint does not hold true for all cases. First of all, it would be plausible to allow for the possibility that firms can temporarily shut down their production activities when they are inefficient, especially in a general equilibrium model with customer search. In this case, the value of output differs from that of consumption when a fraction of firms shut down their production activities temporarily because of inefficient productivity shocks. But this difference disappears when all firms are willing to participate in production activities because the maximum reservation price is higher than the maximum actual price. When all firms voluntarily continue to produce, the value of output should be identical to that of consumption: \(\bar{P}_t = P_t\). As a result, we have the following equilibrium condition:

\[
P_t^{2(1-\theta)} = \frac{1}{\bar{z} - \bar{z}} \int_{P_{\min,t}}^{P_{\max,t}} P_{it}^{1-\theta} (\frac{R_{it}^{1-\theta}}{1-\theta} - \frac{P_{it}^{1-\theta}}{1-\theta})dF(P_{it}).
\]

(11)

In this case, the aggregate social resource constraint also becomes \(Y_t = C_t(1 + X_t)\).

Furthermore, a price distortion can arise when there is a price dispersion, namely relative price distortion. Specifically, the relative price distortion is defined as the part of output that is foregone because of price dispersion. In particular, the relative price distortion turns out to be

\[
\Delta_t = \frac{P_t^{2\theta - 1}}{\bar{z} - \bar{z}} \int_{P_{\min,t}}^{P_{\max,t}} P_{it}^{\theta} (\frac{R_{it}^{1-\theta}}{1-\theta} - \frac{P_{it}^{1-\theta}}{1-\theta})dF(P_{it}).
\]

(12)

Therefore, the aggregate production function can be written as \(Y_t = H_t/\Delta_t\) where \(H_t\) is the aggregate amount of hours worked.
5 Implications for Demand Analysis

Having described the model, we discuss implications of customer search models for the shape of demand curves facing individual sellers. We first show that the demand curve of the search model can mimic a quasi-kinked demand curve locally around an average price. Second, we allow for the possibility that a group of consumers do not search at all because of high search costs. In this case, demand elasticities can be lower at high prices than at low prices.

5.1 Locally Quasi-Kinked Demand Curve

In order to generate an equilibrium price dispersion for each product, we introduce idiosyncratic productivity shocks into the model. Specifically, firm $i$ produces its output using a production function of the form: $Y_{it} = H_{it}/A_{it}$, where $A_{it}$ is the firm-specific shock at period $t$, $H_{it}$ is the amount of labor hired by firm $i$, and $Y_{it}$ is the output level at period $t$ of firm $i$. In addition, we assume that $A_{it}$ is an i.i.d. random variable over time and across individual firms and its distribution is a uniform distribution whose support is $[A, \bar{A}]$.

Given the demand curve and the production function specified above, we write the profit maximization condition in terms of nominal price as follows:

$$AW_t^N(\theta \bar{R}_t^{1-\theta} - (2\theta - 1)P_{it}^{1-\theta}) = (1 - \theta)P_{it}(2P_{it}^{1-\theta} - \bar{R}_t^{1-\theta}).$$

(13)

where the realized value at period $t$ of the idiosyncratic shock is $A_{it} = A$. Thus, we can use this representation of profit maximization condition to characterize the distribution of nominal prices denoted by $F(P_{it})$. For example, suppose that firm-specific shocks are uniformly distributed over a compact interval, as we did above. Then, the resulting distribution of prices can be written as follows:

$$F(P_{it}) = \frac{P_{it} - P_{\text{min},t}}{P_{\text{max},t} - P_{\text{min},t}},$$

(14)

where $P_{\text{max},t}$ is the price that satisfies the profit maximization condition when $A = A_{\text{max}}$ and $P_{\text{min},t}$ is the price that satisfies the profit maximization condition when $A = A_{\text{min}}$.

Turning to numerical examples of the model, Figure 3 shows that the search model’s demand curves match closely those derived from the Dotsey-King’s aggregator with its curvature parameter $\psi = -1.1$, a degree of curvature that is roughly consistent with recent estimates from firm-level scanner data.\footnote{\cite{Dossche et al. (2006).}} But since our comparison focuses on the
Figure 3: Comparing the Curvature of Demand under Customer Search vs. Kimball-Style Preferences

Note: this figure compares quasi-kinked demand curves that are derived from the Dotsey-King aggregator and the search model analyzed in this paper. In order to compare three different demand curves, we adopt a normalization to make the logarithm of demand in the search model become 100 when the logarithm of real price is zero.

In the neighborhood of an average price, this figure does not indicate that the customer search model can mimic the quasi-kinked demand curve globally. In addition, we note that these two models build on different mechanisms to obtain quasi-kinked demand curves. For example, the Dotsey-King aggregator requires that a satiation level should exist for each type of differentiated goods in order to generate quasi-kinked demand curves, which holds true only when the parameter $\psi$ takes a negative value. On the other hand, the customer search model relies on the sensitivity of customers with respect to changes in prices in the generation of quasi-kinked demand curves.\footnote{In order to generate this figure, we assume that the range of cost shocks covers only a very small interval around one: $A = 0.95$ and $\bar{A} = 1.05$. In addition, we choose a somewhat wide range of search costs: $z_{\text{max}} = 0.20$ and $z_{\text{min}} = 0.001$ for the support of search cost parameter. We do this so as to guarantee that the maximum reservation price is higher than the maximum actual price in our numerical examples.}

Having shown that the customer search model can produce a locally quasi-kinked demand curve, we also point out the possibilities that demand elasticities can be lower at
high prices than at low prices. For example, we consider the case in which the maximum reservation price is higher than the actual maximum price. Specifically, when the maximum reservation price is higher than the actual maximum price, the following equation holds for the maximum reservation price:

\[ \tilde{z} P_t^{1-\theta} = \frac{\tilde{R}_t^{1-\theta}}{1-\theta} - \frac{P_{\text{max},t}^{1-\theta}}{1-\theta} + \int_0^{P_{\text{max},t}} \{ P_{it}^{-\theta} F(P_{it}) \} dP_{it}. \]

In addition, there are consumers whose reservation price is the actual maximum price:

\[ \check{z}_t P_t^{1-\theta} = \int_0^{P_{\text{max},t}} \{ P_{it}^{-\theta} F(P_{it}) \} dP_{it}, \]

where \( \check{z}_t \) is the fraction of the search cost for the consumers whose reservation price is the actual maximum level. Given the uniform distribution of the search costs, the fraction of households who do not search at all is \( (\tilde{z} - \check{z}_t)/(\tilde{z} - \check{z}) \). Because of these consumers, the demand curve can have lower elasticities at high prices such as the actual maximum price.

### 5.2 Life-Cycle Prices

In the model analyzed above, we have assumed that search costs are randomly distributed over different products for each household in each period. We do this in order to make households identical each period, though individual households end up with different prices for a particular product. But search costs of an individual consumer, especially time opportunity costs of search, may change over his or her life-cycle. In particular, Aguiar and Hurst (2007) document substantial heterogeneity in prices paid for identical goods for the same area and time, with older households shopping the most and paying the lowest prices. For example, doubling shopping frequency lowers a good’s price by 7 to 10 percent.

In this vein, it would be interesting to incorporate cross-sectional heterogeneous aspects of search costs into the current modeling framework.

Now, we briefly highlight consequences of the possibility that when customers have different opportunity costs of time, a customer with a lower opportunity cost can reduce the market price for a given basket of goods by shopping more intensively. First, this extension of our model can help explain substantial heterogeneity in prices paid for identical goods for the same area and time with older households shopping the most and paying the lowest prices, documented by Aguiar and Hurst (2007). Second, the presence of households with extremely high search costs can make sellers face inelastic demands at high prices, though their demand curves are still kinked at lower prices.
Figure 4: Demand Curve with Heterogenous Agents

![Demand Curve with Heterogenous Agents](image)

Note: This figure depicts the demand curve when each household contains members with two different distributions of search costs.

For example, we consider a representative household that consists of young and old family members. The total number of family members is normalized to be one in each period and the fraction of the young is $a$. Furthermore, the young cohort has a higher range of search costs than that of the old generation because the young members have a higher level of wage. Hence, the old search more actively for shoppers with low prices. As a result, when a seller chooses a price within a range of prices greater than the maximum reservation price of the old generation, the seller’s demand curve is given by

$\left\{ \frac{aP^{\theta-1}_{it}}{(z_y - z_y)W_{y,t}} \left( \frac{\bar{R}_{yt}^{1-\theta}}{1-\theta} - \frac{P^{1-\theta}_{it}}{1-\theta} \right) \right\}$

and for a range of prices lower than the maximum reservation price of the old generation, the demand curve can be written as

$\left\{ \frac{(1-a)P^{\theta-1}_{it}}{(z_o - z_o)W_{o,t}} \left( \frac{\bar{R}_{ot}^{1-\theta}}{1-\theta} - \frac{P^{1-\theta}_{it}}{1-\theta} \right) + \frac{aP^{\theta-1}_{it}}{(z_y - z_y)W_{y,t}} \left( \frac{\bar{R}_{yt}^{1-\theta}}{1-\theta} - \frac{P^{1-\theta}_{it}}{1-\theta} \right) \right\} \left( \frac{P_{it}}{P_t} \right)^{-\theta} Y_t$

where $z_h$ and $\bar{z}_h$ are maximum and minimum search costs for generation $h$, $\bar{R}_{h,t}$ is the maximum reservation price of generation $h$ and $W_{h,t}$ is the real time cost of generation $h$. 

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for $h = \text{young and old}$. In this case, the elasticity of demand increases with price when the price is below the maximum reservation price of the old, while it becomes less elastic and remains constant as long as the price is above the maximum reservation price of the old.

### 6 Implications for Welfare and Monetary Policy

As an application of our benchmark analysis, we incorporate customer search in an otherwise standard New Keynesian framework. We then demonstrate that the welfare cost of inflation is greater in the presence of customer search than those shown in models with the Kimball-style aggregator including the Dotsey-King aggregator.\(^{23}\)

#### 6.1 Analytic Framework

We now abstract from idiosyncratic productivity shocks to simplify the characterization of equilibrium conditions. In addition, we focus on the case in which the average long-run inflation is positive. Moreover, the aggregate exogenous shocks are not large and thus the most recently updated prices are higher than predetermined prices. As a result of these two assumptions, all sellers are willing to produce in each product market, even though their prices are predetermined.

Turning to the specification of the model, a fraction of firms $1 - \alpha$ change their prices in each period, whereas the other fraction do not. Under the Calvo-type staggered price-setting, we still can use the same demand curve derived in the previous section. Furthermore, when the inflation rate is positive and exogenous shocks are sufficiently small, firms are likely to increase their nominal prices over time so that $P^{*}_{t-n-1} \leq P^{*}_{t-n}$ for $n = 0, 1, \cdots, \infty$ where $P^{*}_{t-n}$ is the nominal price set at period $t-n$. Since the Calvo-type price-setting rule generates a set of restrictions for the cumulative distribution of nominal prices, the

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\(^{23}\)In addition, it would be worth the comparison with the recent model of the pricing to market by Atkeson and Burstein (2008). Atkeson and Burstein consider a nested CES set-up for the aggregation of different goods in which a bunch of industries exist, while each industry has multiple firms. The demand curves from this nested CES set-up imply that the substitution elasticity across industries can be lower than the substitution elasticity among firms within industries. Hence, one might expect that the pricing behavior of firms in their model can be similar with that of the customer search model analyzed above. However, the required information of an individual firm differs in the two models when the firm set a price. Under the nested CES set-up, individual firms are concerned about only the difference between their prices and the average price when they set prices. In contrast, firms in the customer search model pay attention to not only the difference between their prices and the average price but also the distribution of prices. For the information of customers on the price distribution affects their search behavior and thus demand curves facing individual firms.
cumulative distribution at period $t$ of nominal prices can be written as

$$F(P) = \alpha^n \text{ if } P_{t-n}^* \leq P < P_{t-n+1}^*$$

for $n = 1, 2, \cdots, \infty$ and $F(P_t^*) = 1$.

Given these conditions, the relative price distortion at period $t$ turns out to be

$$\Delta_t = \frac{1}{\bar{z}} \left( \frac{\tilde{P}_{1-t}^{1-\theta}}{1-\theta} \Delta_{1t} - \frac{\Delta_{2t}}{1-\theta} \right)$$

where $\Delta_{1t}$ and $\Delta_{2t}$ are defined as

$$\Delta_{1t} = (1-\alpha)(\tilde{P}_{1-t}^{*})^{-\theta} + \alpha \Pi_{t}^{\theta} \Delta_{1t-1}, \quad \Delta_{2t} = (1-\alpha)(\tilde{P}_{1-t}^{*})^{1-2\theta} + \alpha \Pi_{t}^{2\theta-1} \Delta_{2t-1}.$$ 

In addition, we note that the relative price distortion disappears when $\theta = 0$. In this case, each household purchases only one unit of each product even if prices of goods are different across different sellers and products. As a result, the presence of price dispersion across different sellers does not generate the aggregate inefficiency between the aggregate demand and production.

Furthermore, we show in the appendix that the fraction of aggregate search cost in the aggregate income takes a simple non-linear AR(1) representation of the form:

$$X_t - \frac{\bar{z}}{2} = \frac{1-\alpha}{2\alpha \bar{z}} \left( \frac{\tilde{P}_{1-t}^{1-\theta}}{1-\theta} - \Pi_{t}^{\theta-1} \tilde{P}_{1-t-1}^{1-\theta} \right)^2 + \alpha \Pi_{t}^{2(\theta-1)} (X_{t-1} - \frac{\bar{z}}{2})$$

where $\Pi_t (= P_t/P_{t-1})$ is the ratio of the general price level at period $t$ to its lagged value and a price index $\tilde{P}_{1t}$ is defined as

$$\tilde{P}_{1t}^{1-\theta} = \alpha (\tilde{P}_{1t}^{*})^{1-\theta} + \alpha \Pi_{t}^{\theta-1} \tilde{P}_{1t-1}^{1-\theta}.$$ 

6.2 The New Keynesian Phillips Curve

In this section, we present the log-linear approximation of the profit maximization condition of firm around the deterministic steady state with zero inflation. We demonstrate in the appendix that the linearized Phillips curve equation can be written as

$$\pi_t = \gamma \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} mc_t + \beta E_t[\pi_{t+1}]$$

where $\pi_t$ is the inflation rate at period $t$, $mc_t$ is the logarithmic deviation of the real marginal cost from its steady state value, and $\gamma$ measures the degree of real rigidity in the presence of customer search:

$$\gamma = \frac{(1+\theta\bar{z})(1-(1-\theta)\bar{z})}{(1+\theta\bar{z})(1-(1-\theta)\bar{z}) + (1+\bar{z}(1-\theta))}$$

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The resulting Phillips curve equation is observationally equivalent with that of the Calvo model with the Kimball-style aggregator. Specifically, setting $\theta = 0.2$ and $\bar{z} = 0.2$ gives a roughly similar magnitude of the slope as the one that can be derived when we set $\psi = -1.1$ and $\theta = 6$ in the model with the Dotsey-King aggregator.\footnote{The measure of real rigidity in the New Keynesian model with the Dotsey-King aggregator is given by $\gamma = 1/(1 - \mu \psi)$ where $\mu = \theta / (\theta - 1)$ is the steady-state markup. In the Dotsey-King specification, setting $\theta = 11$ and $\theta = 6$ distinctly gives $\gamma = 0.45$ and $0.43$ given a value of curvature parameter $\psi = -1.1$. Hence, setting $\theta = 1$ and $\bar{z} = 0.2$ gives a somewhat higher value of $\gamma = 0.55$ than the value of $\gamma = 0.43$ obtained by setting $\psi = -1.1$ and $\theta = 6$ in the Dotsey-King specification. But it does not mean that the customer search model cannot attain the value of $\gamma$ in the Dotsey-King specification. For example, setting $\theta = 0.2$ and $\bar{z} = 0.17$ leads to $\gamma = 0.44$ ($\mu = 1.2$), while setting $\theta = 0.2$ and $\bar{z} = 0.1$ leads to $\gamma = 0.46$ ($\mu = 1.1$).}

### 6.3 Welfare Costs of Inflation Variability

We now move onto the second-order approximation to the welfare of the household. In order to do this, we compute the second-order Taylor expansions of the aggregate search cost and the relative price distortion around the deterministic steady state. It is shown in the appendix that the second-order Taylor expansion to the aggregate search cost around the steady-state with zero inflation rate yields

$$X_t - \frac{\bar{z}}{2} \approx \frac{\alpha}{(1 - \alpha)} (1 - \alpha \bar{z}) \pi_t^2 + \alpha (X_{t-1} - \frac{\bar{z}}{2}),$$

while the second-order approximation to the relative price distortion can be written as

$$\delta_t \approx \frac{\alpha}{1 - \alpha} \theta (1 + \frac{1}{\bar{z}}) \pi_t^2 + \alpha \delta_{t-1}$$

Hence, these two second-order approximations indicate that the aggregate search cost and the relative price distortion increase as the variability of the aggregate inflation rises. Substituting these two second-order approximations into the second-order approximation to the utility function of the representative household, the resulting quadratic loss function can be written as

$$L_0 = \nu \sum_{t=0}^{\infty} \beta^t E_0 \left[\frac{\alpha}{(1 - \alpha)(1 - \alpha \beta)} \left( \theta + \frac{\theta}{\bar{z}} + \frac{1}{\bar{z}(1 + \bar{z}/2)} \pi_t^2 + (\sigma + \chi) \pi_t^2 / 2 \right) \right]$$

where $\nu$ is the marginal utility of consumption at the deterministic steady state.

Compared with the New Keynesian model with the Dotsey-King aggregator, the welfare cost of inflation in this model increases by a factor of around 2. In order to see this, notice that setting $\theta = 0.2$ and $\bar{z} = 0.2$ in the customer search model and $\psi = -1.1$ and $\theta = 6$ in the New Keynesian model with the Dotsey-King aggregator generates an observationally equivalent linear Phillips curve. Given these parameterizations, the coefficient of
the aggregate inflation in the quadratic loss function of the customer search model is more than one and half as large as the one in the New Keynesian model with the Dotsey-King aggregator.

7 Directions for Future Research

We have used household scanner data to document that most of households chooses select only one brand on average each month. Much of marketing or industrial organization literature has discussed household’s selection of brands theoretically and empirically. For example, Anderson et al. (2001) examine a duopolistic market wherein brands repeatedly compete for heterogeneous consumers: the loyal consumers buy from only one of the brands; the semi-loyal consumers consider both brands but are willing to pay a premium for the brand purchased last period; the switchers value the two brands identically and buy the lowest priced brand. Villas-Boas (2004) also argues that consumers can gain further information regarding how well a product fits their preferences only by experiencing it after purchase, which could then generate loyalty for the products tried first. In sum, price promotion such as temporary sales would be closely associated with selection and loyalty of brands. In this regard, we expect that the customer search framework can be used to analyze micro foundations of temporary sales.

Finally, our search model framework can help develop deeper micro foundations for nominal price rigidity, allowing for household uncertainty about the current distribution of prices within each narrow product category and about the influence of aggregate shocks on the relative price of that item.
Appendix A

In this appendix, we derive evolution equations of the aggregate search cost and the relative price distortion. In doing so, we make two assumptions. First, we abstract from idiosyncratic productivity shocks to simplify the characterization of equilibrium conditions. Second, we focus on the case in which the average long-run inflation is positive. But the aggregate exogenous shocks are not large and thus the most recently updated prices are higher than predetermined prices. This assumption guarantees that all sellers are willing to produce in each product market, even though their prices are predetermined. Third, we set $z = 0$.

Distribution of Prices  When the inflation rate is positive, the Calvo-type price-setting rule generates a set of restrictions for the cumulative distribution of nominal prices:

$$F(P^*_t - n) = \alpha^n$$

where $F(P^*_t - n)$ is the measure of firms whose nominal prices are less than or equal to $P^*_t - n$. The cumulative distribution of nominal prices of individual sellers are written as

$$F(P_i) = \alpha^n \text{ if } P^*_t - n \leq P_i < P^*_t - (n+1)$$

for $n = 1, 2, \cdots, \infty$ and $F(P^*_t) = 1$.

Determination of Reservation Prices  The reservation price of consumers whose search cost is $zL$ is given by

$$zP^{1-\theta} = \int_0^{R^1} P_i^{-\theta} F(P_i) dP_i$$

We now define a set of search costs whose corresponding reservation prices are $\{P^*_t\}_{n=0}^{\infty}$ in the following way:

$$z_{nt} P^{1-\theta}_t = \int_0^{P^*_t} P_i^{-\theta} F(P_i) dP_i$$

Hence, the difference between two adjacent search costs can be written as

$$(z_{nt} - z_{(n+1)t}) P^{1-\theta}_t = \alpha^{n+1} \int_{P^*_t}^{P^*_t - (n+1)} P_i^{-\theta} dP_i = \alpha^{n+1} (\frac{(P^*_t)^{1-\theta}}{1-\theta} - \frac{(P^*_t - (n+1))^{1-\theta}}{1-\theta})$$

for $n = 0, 1, \cdots, \infty$. Summing up these equations, the maximum reservation price is determined as follows:

$$zP^{1-\theta}_t = (\frac{P^{1-\theta}_t}{1-\theta} - \frac{(P^*_t)^{1-\theta}}{1-\theta}) + (\frac{P^{1-\theta}_{t-1}}{1-\theta} - \frac{P^{1-\theta}_{t-1}}{1-\theta})$$
where \( P_{1t} \) is defined as
\[
P_{1t}^{1-\theta} = \sum_{n=0}^{\infty} \alpha^{n+1}(P_{t-n}^*)^{1-\theta}
\]

Rewriting this equation in terms of real terms leads to
\[
\bar{z} = (\tilde{P}_{1t}^{1-\theta} - (\tilde{P}_t^*)^{1-\theta}) + (\tilde{P}_{1t}^{1-\theta} - \Pi_{1t}^{\theta-1}\tilde{P}_{1t}^{1-\theta})
\]

where \( \tilde{P}_{1t} \) is defined as
\[
\tilde{P}_{1t}^{1-\theta} = \alpha^{n}(\tilde{P}_t^*)^{1-\theta} + \alpha(1-\Pi_{t}^{\theta-1})\tilde{P}_{1t}^{1-\theta}.
\]

Furthermore, the shares of search costs whose corresponding reservation prices are \( \{P_{t-n}^*\}_{n=0}^{\infty} \) can be written as
\[
z_{nt} = \alpha^{n}(\prod_{k=0}^{n-1} \Pi_{t-k}^{\theta-1}(\tilde{P}_{1t-n}^{1-\theta} - \Pi_{1t-n}^{\theta-1}\tilde{P}_{1t-n-1}^{1-\theta})) \quad \text{for} \quad n = 1, 2, \cdots, \infty
\]

and \( z_{0t} = (\tilde{P}_{1t}^{1-\theta} - \Pi_{t}^{\theta-1}\tilde{P}_{1t-1}^{1-\theta})/(1-\theta) \).

**Determination of Average Search Costs** Recall that the definition of the aggregate search cost is \( X = (1/\bar{z})\int_{\bar{z}}^{\infty} (z/F(R(z)))dz \). Given the definition of the cumulative distribution of nominal prices of sellers, the following relation holds:
\[
X = \frac{1}{\bar{z}} \sum_{n=0}^{\infty} \int_{z_n}^{\infty} \frac{z}{F(R(z))} dz = \frac{1}{2\bar{z}} \{(\bar{z}^2 - z_0^2) + \sum_{n=1}^{\infty}(z_{n-1}^2 - z_n^2)\}/\alpha^n
\]

Rewriting the above equation into an infinite series of \( z_{nt} \) and then substituting the equation of \( z_{nt} \) into this infinite series, we have
\[
X_t = \frac{1}{2\bar{z}} \{z^2 + \frac{1-\alpha}{\alpha}P_t^{2(\theta-1)} \sum_{n=0}^{\infty} \alpha^n(P_{1t-n}^{1-\theta} - P_{1t-n-1}^{1-\theta})^2\}
\]

We now derive an AR(1) representation for the aggregate search cost from this infinite series:
\[
X_t - \frac{\bar{z}}{2} = \frac{1-\alpha}{2\alpha \bar{z}} (\tilde{P}_{1t}^{1-\theta} - \Pi_{t}^{\theta-1}\tilde{P}_{1t}^{1-\theta}/1-\theta)^2 + \alpha(1-\Pi_{t}^{\theta-1})^{1-\theta}(X_{t-1} - \frac{\bar{z}}{2})
\]

The steady-state level of the aggregate search cost can be written as
\[
X = \frac{\bar{z}}{2} + \frac{1-\alpha}{2\alpha \bar{z}} (1-\Pi_{t}^{\theta-1})\tilde{P}_{1t}^{1-\theta}/1-\theta)^2 + \frac{1}{1-\alpha(1-\Pi_{t}^{\theta-1})}
\]
**Relative Price Distortion**  The relative price distortion is defined as follows:

\[ \Delta_t = \frac{1}{z} \left( \frac{\tilde{R}_1^{1-\theta}}{1-\theta} \Delta_{1t} - \frac{\Delta_{2t}}{1-\theta} \right) \]

where \( \Delta_{1t} \) and \( \Delta_{2t} \) are defined as

\[ \Delta_{1t} = P_t^{\theta} \sum_{n=0}^{\infty} (1-\alpha)^n (P^*_t)^{-\theta} \]

\[ \Delta_{2t} = P_t^{2\theta-1} \sum_{n=0}^{\infty} (1-\alpha)^n (P^*_t)^{1-2\theta} \]

The two sub-measures of relative price distortion defined above have recursive representations:

\[ \Delta_{1t} = (1-\alpha)(\tilde{P}_t^*)^{-\theta} + \alpha \Pi_t^\theta \Delta_{1t-1} \]

\[ \Delta_{2t} = (1-\alpha)(\tilde{P}_t^*)^{1-2\theta} + \alpha \Pi_t^{2\theta-1} \Delta_{2t-1} \]

**Price Level Equation**  When all firms are willing to produce in the product market, \( P_t = \Lambda_t \). As a result, we have the following equilibrium condition:

\[ 1 = \frac{1}{z} \left( \frac{\tilde{R}_1^{1-\theta}}{1-\theta} S_{1t} - \frac{S_{2t}}{1-\theta} \right) \]

where \( S_{1t} \) and \( S_{2t} \) are defined as

\[ S_{1t} = P_t^{\theta-1} \sum_{n=0}^{\infty} (1-\alpha)^n (P^*_t)^{1-\theta} \]

\[ S_{2t} = P_t^{2(\theta-1)} \sum_{n=0}^{\infty} (1-\alpha)^n (P^*_t)^{2(1-\theta)} \]

These two sub-measures have recursive representations:

\[ S_{1t} = (1-\alpha)(\tilde{P}_t^*)^{1-\theta} + \alpha \Pi_t^{\theta-1} S_{1t-1} \]

\[ S_{2t} = (1-\alpha)(\tilde{P}_t^*)^{2(1-\theta)} + \alpha \Pi_t^{2(\theta-1)} S_{2t-1} \]

**Optimization Conditions**  The first condition can be written as

\[ (1-\theta)\tilde{P}_t^* F_{1t} - 2(1-\theta)(\tilde{P}_t^*)^{2-\theta} F_{2t} + \theta F_{3t} + (1-2\theta)(\tilde{P}_t^*)^{1-\theta} F_{4t} = 0 \]

where \( F_{1t}, F_{2t}, F_{3t}, \) and \( F_{4t} \) are defined as

\[ F_{1t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha \beta)^k MU_{t+k} Y_{t+k} \tilde{R}_1^{1-\theta} \left( \frac{P_{t+k}}{P_t} \right)^{\theta-1} \]

\[ F_{2t} = \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha \beta)^k MU_{t+k} Y_{t+k} \left( \frac{P_{t+k}}{P_t} \right)^{2(\theta-1)} \]
\[
F_{3t} = E_t[\sum_{k=0}^{\infty} (\alpha \beta)^k MU_{t+k} Y_{t+k} MC_{t+k} \tilde{R}_{t+k}^{1-\theta} (\frac{P_{t+k}}{P_t})^\theta]
\]
\[
F_{4t} = E_t[\sum_{k=0}^{\infty} (\alpha \beta)^k MU_{t+k} Y_{t+k} MC_{t+k} (\frac{P_{t+k}}{P_t})^{2\theta-1}]
\]

Appendix B

The log-linear approximation of the first-order condition of the firm’s price-setting around the deterministic steady-state with zero inflation is given by

\[
\tilde{p}_t^* = (1 - \alpha \beta)(\chi_R \tilde{r}_t + \chi_{mc} mc_t) + \alpha \beta E_t[\tilde{p}_{t+1}^* + \pi_{t+1}]
\]

where \(\tilde{p}_t^*\) is the real optimal price at period \(t\), \(\tilde{r}_t\) is the log deviation of the real maximum reservation price from its steady-state value, \(mc_t\) is the log deviation of the real marginal cost from its steady state value, and \(\pi_t\) is the inflation rate at period \(t\). The coefficients \(\chi_R\) and \(\chi_{mc}\) measures the responses of the optimal real price with respect to changes in the maximum reservation price and the real marginal cost and their sum is equal to one:

\[
\chi_R = \frac{1 + \tilde{z}(1-\theta)}{(1+\tilde{z})(1-(1-\theta)\tilde{z}) + (1+\tilde{z}(1-\theta))}; \quad \chi_{mc} = \frac{(1+\tilde{z})(1-(1-\theta)\tilde{z})}{(1+\tilde{z})(1-(1-\theta)\tilde{z}) + (1+\tilde{z}(1-\theta))}
\]

In addition, as \(\tilde{z}\) approaches to the infinity, \(\chi_R\) converges to zero and \(\chi_{mc}\) gets closer to one, so that the first-order condition becomes identical to that of the Calvo model without customer search.

Furthermore, the steady-state version of the first-order condition is

\[
(1-\theta)\tilde{R}_{1-\theta} - 2(1-\theta) + \theta MC\tilde{R}_{1-\theta} + (1-2\theta)MC = 0.
\]

This condition along with the equation of the maximum reservation price implies that the real marginal cost at the steady state with zero inflation is

\[
MC = \frac{1 - (1-\theta)\tilde{z}}{1+\theta\tilde{z}} = \frac{\epsilon}{\epsilon - 1}; \quad \epsilon = \theta + (1/\tilde{z})
\]

where \(\epsilon\) is the demand elasticity at the steady state with zero inflation. The maximum reservation price is given by

\[
\tilde{R}_{1-\theta} = 1 + (1-\theta)\tilde{z}
\]

We now show that the real maximum reservation price can be zero up to the log-linear approximation around the steady-state with zero inflation. The log-linear approximations of \(S_{1t}\) and \(S_{2t}\) are given by

\[
s_{1t} = (1-\theta)((1-\alpha)p_t^* - \alpha\pi_t) + \alpha s_{1t-1}, \quad s_{2t} = 2(1-\theta)((1-\alpha)p_t^* - \alpha\pi_t) + \alpha s_{2t-1}
\]

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The log-linear approximation to the price level equation defined above becomes

\[(1 - \theta)\bar{R}^{1-\theta} + (\bar{R}^{1-\theta} - 2)s_{1t} = 0\]

It follows from these three equations that \(\bar{r}_t = s_{1t} = s_{2t} = 0\) could be a solution. In this case, the real optimal price of firms depends on the aggregate inflation rate:

\[p_t^* = (\alpha/(1 - \alpha))\pi_t\]

Substituting these conditions into the linearized first-order condition of the price-setting, the linearized Phillips curve turns out to be

\[\pi_t = \chi mc(1 - \alpha)/(1 - \alpha)mc_t + \beta E_t[\pi_{t+1}]\]

We now move onto the second-order approximation to the welfare of the household. First, the second-order Taylor expansion to the aggregate search cost around the steady-state with zero inflation rate is

\[S_t - \bar{z}^2 \approx \alpha(1 - \alpha)\bar{z}^2/2 + \alpha(S_{t-1} - \bar{z}^2)/2\]

Second, the distortion of the relative price equation can be written as

\[\delta_t \approx (1 - \theta)\bar{z} + 1/\bar{z}(1 - \theta)\delta_{1t} - 1/\bar{z}(1 - \theta)\delta_{2t}\]

The evolutions of \(\delta_{1t}\) and \(\delta_{1t}\) are given by

\[\delta_{1t} \approx \alpha\delta_{1t-1} + \alpha\theta \pi_t^2/2; \quad \delta_{2t} \approx \alpha\delta_{2t-1} + \alpha(1 - 2(1 - \theta)) \pi_t^2/2\]

Combining these equations, we have the following evolution equation for the relative price distortion:

\[\delta_t \approx \alpha\delta_{t-1} + \alpha\epsilon \pi_t^2/2\]

Furthermore, the utility function of the household is given by

\[U_t = C_t^{1-\sigma} - 1 - H_t^{1+\chi}/1 + \chi\]

where \(C_t\) is the consumption level at period \(t\) and \(H_t\) is the hours worked at period \(t\). The aggregate production function and the social resource constraint imply

\[C_t = Y_t/(1 + S_t); \quad H_t = Y_t\Delta_t\]
We can rewrite this equation as follows:

\( C_t^{1-\sigma} = (C'^*_t)^{1-\sigma}(X_t)^{1-\sigma}(\frac{1+S_t}{1+S})^{\sigma-1} \)

where \( C'^*_t \) is the consumption level of a flexible-price equilibrium and \( X_t \) is the ratio of the output level of the sticky price model to the output level of a flexible-price equilibrium.

The second-order approximation to the utility function for consumption is

\[
\frac{C_t^{1-\sigma} - 1}{1-\sigma} = \frac{(C'^*_t)^{1-\sigma} - 1}{1-\sigma} + (C'^*_t)^{1-\sigma}((X_t-1)-\frac{\sigma}{2}(X_t-1)^2) - \frac{(C'^*_t)^{1-\sigma}(S_t-S)}{1+S} + t.i.p. + O(||\xi||^3)
\]

where \( O(||\xi||^3) \) is the approximation error higher than the third order of \( \xi \), exogenous shocks, and \( t.i.p. \) denotes the terms independent of the output gap and the inflation rate.

Following Woodford (2003), one can express second-order approximations to \( X_t \) in terms of their logarithmic deviations from their steady state values as follows:

\[
X_t - 1 = x_t + \frac{1}{2}x_t^2 + O(||\xi||^3)
\]  \hspace{1cm} (15)

where \( x_t = \log X_t \) denotes the output gap. Substituting this equation into the second-order approximation of the utility function, we have

\[
\frac{C_t^{1-\sigma}}{1-\sigma} = \frac{(C'^*_t)^{1-\sigma}}{1-\sigma} + (C'^*_t)^{1-\sigma}(x_t + \frac{1-\sigma}{2}x_t^2) - \frac{(C'^*_t)^{1-\sigma}(S_t-S)}{1+S} + t.i.p. + O(||\xi||^3)
\]

Similarly, the second-order approximation to the utility function for hours is

\[
\frac{H_t^{1+\chi}}{1+\chi} = \frac{H^{1+\chi}}{1+\chi} + H^{1+\chi}[x_t + \delta_t + \frac{1+\chi}{2}x_t^2] + t.i.p. + O(||\delta,\xi||^3)
\]

The following relation holds at the deterministic steady-state

\( (C'^*_t)^{1-\sigma} = H^{1+\chi} \)

Hence, the second-order approximation to the period utility function can be written as

\[ U_t = U - (C'^*_t)^{1-\sigma}(\frac{\sigma+\chi}{2}x_t^2 + \delta_t + \frac{S_t-S}{1+S}) + O(||\delta,\xi||^3) \]

Based on the evolution equations of the relative price distortion and the aggregate search cost, we now derive a second-order approximation to the expected discounted sum of the period utility functions. The resulting quadratic loss function of the social planner can be written as

\[
L_0 = \nu \sum_{t=0}^{\infty} \beta^t E_0\left[ \frac{\alpha}{(1-\alpha)(1-\alpha\beta)}(\theta + \frac{\theta}{\bar{z}} + \frac{1}{\bar{z}(1+\bar{z}/2)}\frac{\pi_t^2}{2} + (\sigma + \chi)x_t^2) \right]
\]
Finally, we note that the deterministic steady state with zero inflation is non-distorted because of the employment subsidy. In order to compute the level of subsidy that attains the non-distorted steady state, notice that steady-state equilibrium conditions are

\[ C^{\sigma} H^X = \tau MC; \quad H = (1 + S)C; \quad MC = \frac{\epsilon}{\epsilon - 1} \]

where \( \tau \) is the employment subsidy. In addition, we want the following condition to hold at the steady state:

\[ C^{1-\sigma} = H^{1+\chi}. \]

As a result, we get the following relation:

\[ \tau = \frac{\epsilon}{(\epsilon - 1)(1 + S)}. \]

Here, we see that this turns out to be the same as the one in the case of the Dixit-Stiglitz if \( \bar{z} = \infty \) and \( S = 0 \).

**Appendix C**

The reservation price of old customers whose search time is \( z \) satisfies the following equation:

\[ zW_o^o = P^\theta Y \int_0^{R(z)} P_i^{-\theta} F(P_i) dP_i \]

where \( W_o^o \) is the nominal opportunity cost of time for old customers.

In the similar way as is done in the text, the expected number of old customers for each individual seller whose nominal price is \( P_i \) can be written as

\[ N(P_i) = ((1 - a)/(\bar{z} - \bar{z}))(\bar{R}^{1-\theta} \frac{P_i^{1-\theta}}{1 - \theta} - \frac{\bar{P}_i^{1-\theta}}{1 - \theta})(P^{\theta-1}/W^o) \]

where \( W^o \) is the real opportunity cost of time for old customers and \( \bar{R} \) is the maximum reservation price of old customers. Here, it should be noted that the real opportunity cost of time is defined as the fraction of the nominal opportunity cost of time in the total expenditure: \( W^o = W_o^o/(PY) \). The reason why we do this is that it might be convenient to express the real opportunity cost of time as a fraction of the total expenditure when old customers are retired. In the meantime, young customers do not search at all. They are randomly distributed across different sellers. In addition, the fraction of young customers for each seller is \( a \). As a result, we have the following demand curve:

\[ D(P_i) = \{((1 - a)/(\bar{z} - \bar{z}))(\bar{R}^{1-\theta} \frac{P_i^{1-\theta}}{1 - \theta} - \frac{\bar{P}_i^{1-\theta}}{1 - \theta})(P^{\theta-1}/W^o) + a\}(P_i/P)^{-\theta}Y \]
We now describe how the intensive margin of demand is determined in this model. In the same way as we did in the text, each household already has finished its searches during the first-half of each period. Given the perfect information on prices of sellers that the household wants to purchase, each household minimizes the total cost of obtaining \( C \) to determine the intensive margin of demand:

\[
C_i(j,o) = \left( \frac{P_i(j,o)}{P} \right)^{-\theta} \quad \text{for old customers}
\]

\[
C_i(j,y) = \left( \frac{P_i(j,y)}{P} \right)^{-\theta} \quad \text{for young customers},
\]

where \( P_i(j,o) \) is the nominal price for \( j \) type goods that old customers choose, \( P_i(j,y) \) is the nominal price for \( j \) type goods that young customers choose, \( C_i(j,o) \) is the amount of type \( j \) goods that old customer purchase, and \( C_i(j,y) \) is the amount of type \( j \) goods that young customer purchase. In this model, each household aggregates differentiated goods using the following aggregator:

\[
C = \{ (1 - a) \int_0^1 C_i(j,o) \frac{\theta - 1}{\theta} dj + a \int_0^1 C_i(j,y) \frac{\theta - 1}{\theta} dj \}^{\frac{\theta}{\theta - 1}}
\]

In addition, the price level is defined as

\[
P = \{ (1 - a) \int_0^1 P_i(j,o)^{1-\theta} dj + a \int_0^1 P_i(j,y)^{1-\theta} dj \}^{\frac{1}{1-\theta}}.
\]

Given these definitions of the aggregator and the price level, the nominal expenditure of each household can be written as

\[
PC = (1 - a) \int_0^1 P_i(j,o) C_i(j,o) dj + a \int_0^1 P_i(j,y) C_i(j,y) dj
\]

Next, we move onto the description of the demographic structure within a representative household. Each member in the household lives only \( T \) periods. During each period, \((1/T)\) fraction of members are born and at the same time \((1/T)\) fraction of members die; the total number of each household is constant over time. We also assume that each member works for \((T - 1)\) periods after the member is born and then retires in its last period before the member dies. Hence, each member becomes an old customer for only one period. Given this demographic structure, the fraction of young customers is \(1 - a = (T - 1)/T\) and the fraction of old customers is \(a = (1/T)\) in each period.

We turn to the optimization behavior of the representative household. During each period, there is a time constraint for the representative household:

\[
H_t + S_t + L_t \leq \bar{H}
\]

where \( L_t \) is the amount of time for leisure, \( \bar{H} \) is the time endowment for each household and \( S_t \) is the aggregate time amount of search: \( S = (\bar{z} - \bar{z})^{-1} \int_{\bar{z}}^{\bar{z}} (z/F(R(z)))dz. \)
Given this framework described above, we can assume that the preference of each household at period 0 is represented by the following utility function:

$$\sum_{t=0}^{\infty} E_0[U(C_t, \bar{H} - H_t - S_t)]$$

where $C_t$ is the consumption at period $t$, $\bar{H}$ is the amount of time endowment available for each household, $H_t$ is the amount of hours worked at period $t$, and $S_t$ is the amount of time devoted to search. The utility level of each household is affected by the level of the composite goods. Furthermore, each household has the following flow-budget constraint:

$$(1 - a) \int_0^1 P_{i,t}(j,o)C_{i,t}(j,o) dj + a \int_0^1 P_{i,t}(j,y)C_{i,t}(j,y) dj \leq W_N^t H_t + \Phi_t$$

where $P_{i,t}(j,o)$ is the nominal price for $j$ type goods that old customers choose, $P_{i,t}(j,y)$ is the nominal price for $j$ type goods that young customers choose, $C_{i,t}(j,o)$ is the amount of type $j$ goods that old customer purchase, $C_{i,t}(j,y)$ is the amount of type $j$ goods that young customer purchase, $W_N^t$ is the nominal wage, and $\Phi_t$ is the aggregate nominal profit.\(^{25}\)

\(^{25}\)In this appendix, the budget constraint does not contain the total nominal opportunity cost of search, $SW^o PY$, because we assume that old customers consist of only those who are retired and their times are used for nothing. Moreover, if home production is introduced into the model, the opportunity cost of search will have impact on the substitution between market labor and non-market labor.
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