Mortgage Innovation and the Foreclosure Boom

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Abstract
We present a model where households select from a set of possible mortgage contracts and choose whether to default on their mortgage payments given idiosyncratic realizations of non-insurable income and housing price shocks. The mortgage menu consists of traditional fixed rate mortgages (FRM) which require a 20% downpayment and nontraditional mortgages (LIP) that became popular in 2005 characterized by zero downpayment and backloaded payment of principal. The mortgage market is competitive and each contract, contingent on household earnings and assets at origination, must earn zero expected profits. To assess whether issues of mortgage selection were an important factor in the rise of foreclosures we calibrate the benchmark economy to pre-2006 US data and then run a counterfactual with only FRM mortgages. We show that many low income, low asset households would be excluded from the housing market. But since low income, low asset households also default more, nontraditional mortgages account for a disproportionate fraction of default rates. Our model predicts that while almost all foreclosures occur in a state of negative net equity, that state is not sufficient to predict foreclosure. To assess how intermediaries try to separate households on the basis of risk we run a counterfactual where mortgages are not made conditional on income and asset characteristics of the household at point of origination. In that case, there is a sizeable increase in default rates. Finally, we introduce an unanticipated aggregate shock to house prices of a magnitude consistent with the US evidence since 2006. We show that an unanticipated 20% price decline generates a 60% rise in foreclosure rates during a transition to the new steady state, which means the model can account for 40% of the rise in foreclosures in the data.

Preliminary and incomplete, comments welcome.

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1 Introduction

Between 2003 and 2006, the composition of the stock of outstanding residential mortgages in the United States changed in several important respects. First, the fraction of mortgages with fixed payments (FRMs) fell from 85% to under 75% (see figure 1.) At the same time, the fraction of “subprime” mortgages (mortgages issued to borrowers perceived to be high-default risks) rose from 5% to nearly 15%. Recent work (see e.g. Gerardi et al., 2009, figure 3) has revealed that many of these subprime loans are characterized by high leverage at origination, and non-traditional amortization schedules. These features cause payments from the borrowers to the lender to be backloaded compared to loans with standard downpayments and standard amortization schedules. By lowering payments initially, these innovations made it possible for more households to obtain the financing necessary to purchase a house and in other papers (e.g. Chambers, et. al. (forthcoming)) have been associated with the rise in homeownership.

Our objective is to quantify the importance of mortgage innovation for the recent flare-up in foreclosure rates. Specifically, we answer the following questions. First, how much of the rise in foreclosures can be attributed to innovation in mortgage contracts? How much does mortgage innovation magnify the effect of downturns in house values on default rates? What is the welfare gain associated with mortgage innovation?

To answer these questions, we first describe a model that mimics salient features of the US housing market before the flare-up in foreclosure rates in the middle of 2006. We consider an economy where households value both consumption and housing services and move stochastically through several stages of life. For simplicity, agents who are young are constrained to obtain housing services from the rental market and split their remaining income between consumption and the accumulation of liquid assets. Given idiosyncratic income shocks, despite the fact that households begin life ex-ante identical in our model, there is an endogenous distribution of assets among the set of people who turn middle aged.

When agents become mid-aged, they have the option to purchase one of two possible quantities of housing capital: a small house or a large house. We assume they must finance house purchases via a mortgage drawn from a set of contracts with properties like those available in the United States. Standard fixed-rate mortgages (FRMs) feature a 20% downpayment and fixed payments until maturity. Agents can opt instead for a mortgage (which we term LIP) with no-downpayment and delayed amortization. We think of this second mortgage as capturing the backloaded nature of the mortgages that became popular after 2005 in the United States (one popular example is an interest-only-mortgage).

Mortgage holders can terminate their contract before maturity, in which case the house is immediately sold and the borrower receives any proceeds in excess of the outstanding loan principal, and transaction costs.\footnote{Here we are assuming the default law is consistent with antideficiency (as in California for example) where the defaulting household is not responsible for the deficit between the proceeds from the sale of the property and the outstanding loan balance.} We consider a house sale to be a foreclosure if it occurs in...
a state where the house value is below the mortgage’s balance (that is, the agent’s net home equity is negative) or where the agent’s income realization is such that they cannot make the mortgage payment they would owe for the period. Terminations of the mortgage contract in these cases incur transaction costs consistent with loss incidence estimates in the data.

Our model predicts that almost all foreclosures involve negative net equity. This is because most agents with positive equity who are at a high risk of finding themselves unable to meet their mortgage payments sell before reaching that state in order to avoid transaction costs. On the other hand, many agents with negative equity choose to continue meeting their mortgage obligations to avoid losing their homes. These predictions are also consistent with the growing empirical literature on the determinants of foreclosure.\(^2\)

Foreclosures are costly for lenders because of the associated transactions costs and because they occur in most cases when home equity is negative. As a result, intermediaries demand higher yields from agents whose asset and income position make foreclosure more likely. In fact, intermediaries do not issue loans to some agents because their default risk is too high or because the agents are too poor to make a downpayment. In particular, our model is consistent with the fact that agents at lower asset and income positions are less likely to become home-owners, face more expensive borrowing terms, and are more likely to default on their loan obligations.

Since high initial payments are prohibitively costly for asset and income poor agents, there is a natural role to play in our economy for mortgage innovation in the form of contracts with low initial payments. We find that in an economy calibrated to match key aspects of the US housing market prior to the flare up of foreclosure rates, removing the option to issue non-standard contracts causes a significant fall in both home-ownership rates and default rates.

In particular, we find contracts with low initial payments are necessary for asset and income poor households (those who could be interpreted as subprime) to become home-owners. At the same time, the availability of these contracts cause default rates to be higher for two complementary reasons, which our environment enables us to make explicit. First, they enable high-default risk households to become home-owners. Second, these contracts are characterized by a much slower accumulation of home equity than FRMs, which makes default due to a fall in house prices much more likely, even at equal asset and income household characteristics.

We also find in the economy where both types of contracts are available, an aggregate, unanticipated price shock of a magnitude similar to what the US has experienced since mid-2006 causes an increase in default rates that represents about 40% of the empirical behavior of foreclosure rates during the same period. When only FRMs are available, the increase in foreclosure rates associated with the same aggregate price shock is significantly lower.

These findings have a number of implications for how one might interpret current events. Mortgage innovation serves an important purpose and can raise welfare by expanding the

\(^2\)See, among many other papers, Foote et al. (2008a,b), Gerardi et al. (2007), Sherlund (2008), Danis and Pennington-Cross (2005), and Deng et al. (2000).
range of choices for a number of households, particularly agents at the bottom of the asset and income distributions. The nature of these innovations, however, make an increase in default rates more likely since agents are much slower to accumulate home equity. They also greatly magnify the impact of negative aggregate housing price shocks on default rates.

Our paper is closely related to several studies of the recent evolution of the US housing market and mortgage choice. Chambers et al. (forthcoming) argue that the development of mortgages with gradually increasing payments has had a positive impact on participation in the housing market. The idea that mortgage innovation may have implications for foreclosures is taken up in Garriga and Schlagenhauf (2009). They quantify the impact of aggregate house price shocks on default rates where there is cross-subsidization of mortgages within but not across mortgage types (e.g. FRM or LIP). A key difference between our paper and theirs is that we consider a menu of different terms on contracts both within and across mortgage types. Effectively, Garriga and Schlagenhauf (2009) apply the equilibrium concept in Athreya (2002) while we apply the equilibrium concept in Chatterjee et al. (2007). This enables us to build a model that is consistent with the heterogeneity of foreclosure rates and mortgage terms across wealth and income categories which we document using the Survey of Consumer Finance. In fact, our model enables us to quantify the importance of contracts that try to separate risky households on the basis of observable characteristics for key housing statistics. In particular, if we counterfactually impose that yields should be equal on all mortgages of a given type, long-run default rates rise by 12%. This suggests that the ability of financial intermediary to condition mortgage terms on household’s financial characteristics at origination matters significantly for the magnitude of foreclosure rates in equilibrium. Along this separation dimension our paper is more closely related to Guler (2008) where intermediaries offer a menu of FRMs at different possible downpayment rates without cross-subsidization. Guler, however, assumes certain household characteristics are unobservable and studies the impact of an innovation to the screening technology on default rates. Our paper also builds on the work of Stein (1995) and Ortalo-Magné and Rady (2006) who study housing choices in overlapping generation models where downpayment requirements affect ownership decisions and house prices. Our framework shares several key features with those employed in these studies, but our primary concern is to quantify the effects of various mortgage options, particularly the option to backload payments, on foreclosure rates.

Section 2 lays out the economic environment. Section 3 describes optimal behavior on the part of all agents and defines an equilibrium. Section 4 describes our parameterization under the assumption that the housing technology is constant returns to scale (which pins down

\[3\] There are numerous other housing papers which are a bit less closely related. Campbell and Cocco (2003) study the microeconomic determinants of mortgage choice but do so in a model where all agents are homeowners by assumption, and focus their attention on the choice between adjustable rate mortgages and standard FRMs with no option for default. Rios-Rull and Sanchez-Marcos (2008) develop a model of housing choice where agents can choose to move to bigger houses over time. A different strand of the housing literature (see e.g. Gervais, 2002, and Jeske and Krueger, 2005) studies the macroeconomic effects of various institutional features of the mortgage industry, again where there is no possibility of default. Davis and Heathcote (2005) describe a model of housing that is consistent with the key business cycle features of residential investment.
the housing price) and then presents our main quantitative results. Section 5 considers the case where technology is decreasing returns so that housing prices also depend on demand. Section 6 concludes.

2 The environment

We study an economic environment where time is discrete and infinite. The economy is populated by a continuum of households and by a financial intermediary. Each period a mass one of households is born. Over time, households move stochastically through four stages of life: young (Y), middle-aged (M), old (O) and dead. At the beginning of each period, young households become middle-aged with probability $\rho_M$, middle-age households become old with probability $\rho_O$, while old households die with probability $\rho_D$. We assume that the population size is at its unique invariant value, and that the fraction of households of each type obeys a law of large numbers.

Each period, as long as they are young or middle-aged, households receive stochastic income shocks denominated in terms of the unique consumption good. These shocks are i.i.d. across households and evolve stochastically according to a stationary transition matrix $\pi$. Agents begin life at an income level $y \in \{y_L, y_M, y_H\}$ drawn from the unique invariant distribution associated with $\pi$. When old, agents earn a fixed, certain amount of income denoted $y^O$.

All households alive at date $t$ receive the same profit distribution $\iota_t$ from the intermediary. We will interpret the profits associated with the production of housing capital as returns to a fixed factor (land, presumably.) Allocating profits uniformly across agents amounts to endowing all agents with the same quantity of that fixed factor.$^4$

Until they become old, households can save in one-period bonds that earn rate $1 + r_t \geq 0$ at date $t$ with certainty. When old, households can buy annuities that pay rate $\frac{1 + r^O}{1 - \rho_D}$ in the following period provided they are alive, and pay nothing otherwise. Returns are annuitized in the last stage of a household’s life in order to rule out accidental bequests.

Households value both consumption and housing services. They order non-negative processes $\{c_t, s_t\}_{t=0}^{\infty}$ according to:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, s_t)$$

where $U$ satisfies standard assumptions.

Households can obtain housing services from the rental market or from the owner-occupied market. On the first market, they can rent quantity $h_1 > 0$ of housing services at unit price $R_t$ at date $t$. In the period when agents move from youth to middle-age – and only in that period – agents can choose instead to purchase quantity $h \in \{h_2, h_3\}$ of housing capital for $^4$Because profits are very small under our calibration strategy, different allocations of profits are not likely to alter our results. In fact under our first parameterization of the housing technology, the profits are zero.
unit price \( q_t \), where \( h_3 > h_2 > h_1 \). We refer to this asset as a house. A house of size \( h \) initially delivers \( h\theta \) of housing services every period with \( \theta \geq 1 \).

Homeowners face a risk that their house will devalue. Specifically, every period, a fraction \( \lambda > 0 \) of agents who own a house of size \( h = h_3 \) see the quantity of capital they own fall to \( h_2 > 0 \). Likewise, a fraction \( \lambda \) of agents who own a house of size \( h = h_2 \) see the quantity of capital they own fall to \( h_1 \). Furthermore, houses of size \( h_1 \) generate quantity \( h_1 \) of housing services, rather than \( h_1\theta \), whether owned (following a devaluation) or rented. We will interpret the devaluation shock as an idiosyncratic house price shock. In the absence of such shocks, households would never find themselves with negative equity in a steady state equilibrium.

There are several possible interpretations for this devaluation shock. One could for instance think of as a neighborhood shock which makes house in a given location less valuable. Note that while we assume that devaluation shocks satisfy a law of large numbers (the fraction of houses that devalue in each period is \( \lambda \)) we do not need to assume that these shocks are independent across households. Alternatively, one could consider introducing more heterogeneity in houses and modeling taste shocks that render certain house types less valuable. Our devaluation shocks are a tractable way to capture the possibility of microeconomic events that affect house values and are difficult to insure against.

Since devalued houses of size \( h_1 \) provide no advantage over rental units, no agent who becomes middle-aged would strictly prefer to purchase a house of that size, and all homeowners whose housing capital fall to that level are at least as well off selling their house and becoming renters as they would be if they keep their house.

Owners of a house of size \( h \in \{h_1, h_2, h_3\} \) bear maintenance costs \( \delta h \) in all periods where \( \delta > 0 \). We assume that maintenance costs, denominated in terms of the consumption good, must be paid by any homeowner. In that case, a house does not physically depreciate (other than through a devaluation shock), which in turn maintains the low cardinality of the housing state space. Once agents sell or foreclose their house, they are constrained to rely on the rental market for the remainder of their life. We also assume that in the period in which agents become old, they must sell their house immediately and become renters for the remainder of their life. We assume house sales due to the old age shock do not entail foreclosure costs (and hence they do not get counted in foreclosures for an arbitrary reason).

The financial intermediary holds household savings. The intermediary can store savings at exogenously given return \( 1 + r_t \) at date \( t \). It can also transform quantities \( k \geq 0 \) of consumption good (i.e. deposits) into quantity \( Ak^\alpha > 0 \) of housing capital, where \( \alpha \in (0, 1] \).

Housing capital can be rented at rate \( R_t \) at date \( t \). The intermediary incurs maintenance cost \( \delta \) on each unit of housing capital rented, measured in terms of the consumption good. At date \( t \), each unit of consumption good rented thus earns net return \( R_t - \delta \). The intermediary

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5 This is similar to Jeske and Krueger (2005).

6 In fact, independence across agents is essentially incompatible with assuming that a law of large numbers holds. See Feldman and Giles (1985).

7 Arbitrage implies that the present value of renting housing services each period is the same as purchasing a depreciated house. Selling the depreciated house, however, can relax an agent’s budget constraint.
can also sell housing capital as houses to eligible households, at unit price \( q_t \). Note that the fact that each agent’s housing choice set is discrete does not impose an integer constraint on the intermediary since it deals with a continuum of households.

We assume that households that purchase a house of size \( h \in \{ h_2, h_3 \} \) at a given date are constrained to finance this purchase with one of two possible types of mortgage contracts. The first contract (which we design to mimic the basic features of a standard fixed-rate mortgage, or FRM) requires a downpayment of size \( \nu h q_t \) at date \( t \) where \( \nu \in (0, 1) \) and stipulates a yield \( r^{FRM,t}(a_0, y_0, h) \) that depends on the household wealth and income characteristics \((a_0, y_0)\) at the date \( t \) of origination of the loan, and on the selected house size. Given this yield, constant payments \( m^{FRM,t}(a_0, y_0, h) \) and a principal balance schedule \( \{b^{FRM,t}(a_0, y_0, h)\}_{n=0}^{T} \) can be computed using standard calculations, where \( T \) is the maturity of the loan.

Specifically, suppressing the initial characteristics for notational simplicity,

\[
m^{FRM,t} = \frac{ r^{FRM,t} }{ 1 - (1 + r^{FRM,t})^{-T} } (1 - \nu) h q_t
\]

and, for all \( n \in \{0, T - 1\} \),

\[
b^{FRM,t}_{n+1} = b^{FRM,t}_n (1 + r^{FRM,t}) - m^{FRM,t}
\]

where \( b^{FRM,t}_0 = (1 - \nu) h q \). Standard calculations show that \( b^{FRM,t}_T = 0 \).

The second contract (low initial payments mortgage, or LIP) stipulates yield \( r^{LIP,t}(a_0, y_0, h) \), no down-payment, low constant payments \( m^{LIP,t}(a_0, y_0, h) \) for \( n < n^{LIP} < T \) followed by higher payments when \( n \in \{n^{LIP}, ..., T\} \).

One example of such a backloaded mortgage that became popular in the middle of 2000 is the ”interest-only” mortgage. In that case,

\[
m^{LIP,t}_n = \begin{cases} h q r^{LIP,t} \\ \frac{ r^{LIP,t} }{ 1 - (1 + r^{LIP,t})^{-T} } h q \end{cases} \text{ if } n < n^{LIP} \text{ } \text{ and } \text{ if } n \geq n^{LIP}
\]

and, for all \( n \in \{0, T - 1\} \),

\[
b^{LIP,t}_{n+1} = b^{LIP,t}_n (1 + r^{LIP,t}) - m^{LIP,t}_n
\]

where \( b^{LIP,t}_0 = h q \), and, once again, \( b^{LIP,t}_T = 0 \). Notice that for \( n < n^{LIP} \), \( b^{LIP,t}_{n+1} = b^{LIP,t}_0 \) so that the principal remains unchanged for \( n^{LIP} \) periods.

Alternative mortgages, therefore, have two main characteristics: low downpayment and delayed amortization. These are two of the salient features of the mortgages that became highly popular after 2005 in the United States see Gerardi et al., 2007.) Naturally, delayed amortization can take many forms. Subprime mortgages, for instance, often feature balloon payments rather than interest-only periods. We use an IOM structure for concreteness only, any device that delays amortization would yield similar results.
Figure 2 shows typical mortgage payment schedules for both mortgage types. The chart assumes a yield of 15.75% and a loan size of 0.75, a maturity of 10 periods, and an interest-only phase of 3 periods for LIPs. Payments due on LIP mortgages jump once the interest-only phase ends, while FRM mortgages feature constant payments.

Mortgages are issued by the financial intermediary. The intermediary incurs service costs which we model as a premium $\phi > 0$ on the opportunity cost of funds loaned to the agent for housing purposes.

The household can terminate the contract at the beginning of any period, in which case the house is sold. We will consider a termination to be a foreclosure when the outstanding principal exceeds the house value, or when the agent’s state is such that they cannot meet their mortgage payment in the current period. In the event of foreclosure, fraction $\chi > 0$ of the house sale value is lost in transaction costs. If the mortgage’s outstanding balance at the time of default is $b$, the intermediary collects $\min\{(1 - \chi)qh, b\}$, while the household receives $\max\{(1 - \chi)qh - b, 0\}$.

Agents may also choose to sell their house even when they can meet the payment and have positive equity, for instance because they are borrowing constrained in the current period. We also assume that agents sell their house when they become old. Those contract terminations, we assume, do not impose transaction costs on the intermediary.

The timing in each period is as follows. At the beginning of the period, agents discover whether or not they have aged, and receive a perfectly informative signal about their income draw. Middle-aged agents who own homes also observe the realization of their devaluation shock at the beginning of the period, hence the market value of their home. These agents then decide whether to remain home-owners or to become renters either via selling their house or through foreclosure. Agents who just became middle-aged also make their home-buying and mortgage choice decisions at the beginning of the period, after all uncertainty for the period is resolved. At the end of the period, agents receive their income and profit distribution, mortgage payments are made, and consumption takes place.

3 Equilibrium

We initially study equilibria in which all prices are constant. For notational simplicity, we now drop all time markers.

3.1 Agent’s problem

We state the household problem recursively. In general, the household value functions will be written as $V_{\text{age}}(\omega)$ where $\omega \in \Omega_{\text{age}}$ is the state facing an agent of $\text{age} \in \{Y, M, O\}$.
3.1.1 Old agents

For old agents, the state space is $\Omega_O = \mathbb{R}_+$ with typical element $\omega \equiv a \geq 0$. The value function (that is, the expected present value of future utility) for an old agent with assets $a \in \mathbb{R}_+$ solves
\[
V_O(a) = \max_{a' \geq 0} \{ U(c, h_1) + \beta(1 - \rho_D)V_O(a') \}
\]
s.t.
\[
c = a(1 + r) + y^O - h_1R - a' \geq 0
\]

3.1.2 Mid-aged agents

For mid-aged agents, the state space is $\Omega_M = \mathbb{R}_+ \times \{y_L, y_M, y_H\} \times \{0, 1\} \times \{h_1, h_2, h_3\} \times \mathbb{N} \times \{\{FRM, LIP\} \times \mathbb{R}_+ \times \{h_2, h_3\}\} \cup \{\emptyset\}$ with typical element $\omega = (a, y, H, h, n; \kappa)$. Here, $H = 1$ denotes that the household begins the period as a homeowner, while $H = 0$ if they begin as renters. Further, $h \in \{h_1, h_2, h_3\}$ denotes the quantity of housing capital that the household owns at the start of a given period once the devaluation shock has been revealed.\(^8\) We write $n \in \{0, 1, \ldots\}$ for the number of periods the agent has been mid-aged, hence the age of their mortgage when they have one.

The final argument, $\kappa$ denotes the type of mortgage chosen by a homeowner - that is, $\kappa \equiv (\zeta, \kappa', h_0) \in \{FRM, LIP\} \times \mathbb{R}_+ \times \{h_2, h_3\}$ which lists the agent’s mortgage and house choice when they just become mid-aged. In equilibrium, the yield on a given loan will depend on the agent’s wealth-income position $(a_0, y_0)$ and house size choice $h_0$ at origination. For agents who enter a period as renters, the current house size and mortgage type arguments are undefined, and so we simply let $\kappa = \emptyset$.

Working backwards, we begin with the case where the household has already made its home purchase decision (i.e. $n \geq 1$).

**Case 1: $n \geq 1$**

If the household enters the period as renters (i.e. $H = 0$), they must remain renters:
\[
V_M(a, y, 0, h_1, n; \emptyset) = \max_{c, a'} U(c, h_1) + \beta E_{y'|y}[(1 - \rho_O)V_M(a', y', 0, h_1, n + 1; \emptyset) + \rho_OV_O(a')]
\]
s.t. $c + a' = y + (a + \iota)(1 + r) - Rh_1$.

If, on the other hand, the household owns a home (i.e. $H = 1$), they first have to decide whether to remain homeowners or to become renters. We will write $H'(\omega) = 1$ if they choose to remain home-owners and $H'(\omega) = 0$ if they become renters.

\(^8\)We need both $H$ and $h$ to differentiate a renter from a homeowner whose size $h_2$ received a shock down to $h_1$. 

9
The event \( H'(\omega) = 0 \) entails a sale of the house and hence a termination of the mortgage contract. As explained in the previous section, we think of that termination as a foreclosure in two cases. First, if it is not budget feasible for the household to meet its mortgage payment \( m(n; \kappa) \), that is if,

\[
y + (a + \iota)(1 + r) - m(n; \kappa) - \delta h < 0, \tag{3.1}
\]

the household is constrained to become renters. We call this event an involuntarily default and in that case write \( DI(\omega) = 1 \), while \( DI(\omega) = 0 \) otherwise. A second form of default occurs when the household can meet their mortgage payment (i.e. \( (3.1) \) does not hold) but the household chooses nonetheless to become renters and

\[
qh - b(n; \kappa) < 0, \tag{3.2}
\]

i.e. home net equity is negative. We call this event a voluntary default (the household is better off turning the house over to the intermediary in that case) and write \( DV(\omega) = 1 \).

If neither \( (3.1) \) nor \( (3.2) \) holds but the household decides to sell their house and become renters, we write \( S(\omega) = 1 \), while \( S(\omega) = 0 \) otherwise. In that case, the household simply sells their house, pays their mortgage balance, and their asset position is augmented by the value of their home net equity.

Note that

\[
1 - H'(\omega) = S(\omega) + DI(\omega) + DV(\omega).
\]

In other words, \((S, DI, DV)\) classify a mortgage termination into three mutually exclusive events: a simple sale (in which the intermediary need not get involved), a voluntary default, or an involuntary default.

Equipped with this notation, we can now define the value function of a homeowner (i.e. a household whose \( H = 1 \)):

\[
V_M(a, y, 1, h, n; \kappa) = \max_{c \geq 0, a' \geq 0, (H', DI, DV, S) \in \{0, 1\}^4} U(c, (1 - H')h_1 + H'(1_{h=h_1} + \theta 1_{h \neq h_1})h)
+ (1 - H')\beta E_{y' | y} \left[ (1 - \rho_O)V_M(a', y', 0, h_1, n + 1; 0) + \rho_O V_O(a') \right]
+ H'\beta E_{(y', h') | (y, h)} \left[ (1 - \rho_O)V_M(a', y', 1, h', n + 1; \kappa) + \rho_O V_O(a' + \max \{qh - b(n + 1; \kappa), 0\}) \right]
\]

subject to:

\[
c + a' = y + (1 + r)(a + \iota + (1 - H') \max((1 - (DI + DV)\chi)qh - b(n; \kappa), 0)) - H'(m(n; \kappa) + \delta h) - (1 - H')Rh_1
\]

\[
DI = 1 \text{ if and only if } (3.1) \text{ holds}
\]

\[
DV = 1 \text{ if } H' = 0 \text{ and } (3.2) \text{ holds}
\]

\[
S = 1 - H' - DI - DV
\]
There are several things to note in the statement of the household’s problem. Starting with the objective, housing services (s) depend on the household’s housing status, and the size of the house they occupy. Second, recall that we assumed that housing sales due to the old age shock do not entail foreclosure costs. Third, the right-hand side of the budget constraint depends on whether or not the household chooses to keep its house. When they become renters (i.e. when \( H' = 0 \)) their asset position is increased by the value of the house net of their outstanding principal and net, in the event of default, of transaction costs. Their housing expenses are the sum of mortgage and maintenance payments if they keep the house, or the cost of rental otherwise. The final constraint states that selling the house without incurring default costs is only possible if the household is able to meet its mortgage obligations and has positive equity.

The house devaluation shock is part of the conditional expectation operator \( E_{(y', h')}|(y, h) \) in the problem’s statement. Given \( h \in \{h_1, h_2, h_3\} \) and the assumptions we made on the devaluation process, next period’s house value evolves according to a Markov Chain with transition matrix

\[
P(h'|h) = \begin{bmatrix}
1 & 0 & 0 \\
\lambda & 1 - \lambda & 0 \\
0 & \lambda & 1 - \lambda
\end{bmatrix}.
\]

**Case 2: \( n = 0 \) (The agent just became mid-aged)**

Agents who become mid-aged at the start of a given period must decide whether or not to buy a house, and in the event they become homeowners, what mortgage to use to finance their house purchase. Write \( K(\omega_0) \) for the set of mortgage contracts available to a household that becomes mid-aged in state \( \omega_0 \). The set \( K(\omega_0) \) has typical element \( \kappa = (\zeta, r^\zeta, h_0) \). The household’s value function solves:

\[
V_M(a, y, 1, h, 0; \emptyset) = \max_{c \geq 0, a' \geq 0, H' \in \{0, 1\}, \kappa \in K(\omega_0)} U(c, (1 - H')h_1 + H'\theta h_0) + \beta E_{y'}[(1 - \rho_O)V_M(a', y', 0, h_1, 1; \emptyset) + \rho_O V_O(a')] + H'\beta E_{(y', h')}|(y, h_0) \left[ (1 - \rho_O)V_M(a', y', 1, h', 1; \kappa) + \rho_O V_O(a' + \max \{q h_0 - b(1; \kappa), 0\}) \right]
\]

subject to:

\[
c + a' = y + (1 + r)(a + \iota - H'\nu 1_{\{\zeta = \text{FRM}\}q h_0}) - H'(m(0; \kappa) + \delta h_0) - (1 - H')Rh_1
\]

\[
a + \iota \geq H'\nu 1_{\{\zeta = \text{FRM}\}q h_0}
\]

Households who choose to become homeowners (\( H' = 1 \)) choose the contract \( \kappa^* \in K(\omega_0) \)
that maximizes their future expected utility. We will write $\Xi(\omega_0) = \kappa^*$ for this part of the household’s choice, while $\Xi(\omega_0) = \emptyset$ if $H' = 0$. Note that included in the choice of the contract is the size of the house $h_0$.

3.1.3 Young agents

For young agents, the state space is $\Omega_Y = \mathbb{R}^+ \times \{y_L, y_M, y_H\}$ with typical element $\omega = (a, y)$. The value function $V_Y : \Omega_Y \mapsto \mathbb{R}$ for a young agent with assets $a$ and income $y$ solves

$$V_Y(a, y) = \max_{c \geq 0, a' \geq 0} \left\{ U(c, h_1) + \beta E_{y' | y} [(1 - \rho_M)V_Y(a', y') + \rho_M V_M(a', y', 0, h_1, 0; \emptyset)] \right\}$$

s.t. $c + a' = y + (a + \iota)(1 + r) - Rh_1$.

3.2 Intermediary’s problem

All possible uses of loanable funds must earn the same return for the intermediary. This implies, first, that given the price $q$ of houses, the intermediary chooses the quantity $k$ it invests in producing housing capital to solve:

$$\max A k^\alpha q - k$$

so that,

$$k = (Aq)^{\frac{1}{1-\alpha}}. \quad (3.3)$$

Note that the opportunity cost of housing capital investment is $k$ rather than $k(1 + r)$ as a result of our assumption that house purchases are made at the beginning of the period. Note also that if $\alpha = 1$, profit maximization implies that the price of housing capital $q = 1/A$ is pinned down exogenously.

Arbitrage between renting and selling houses also requires that:

$$q = \sum_{t=1}^{+\infty} \frac{R - \delta}{(1 + r)^t}$$

$$\iff R = rq + \delta. \quad (3.4)$$

Note in particular that a change in $q$ must be associated with a change in $R$ in this environment. This simple observation can play a role in the analysis of the consequences of mortgage innovation for welfare in the case where $\alpha < 1$. A bit of algebra also shows that conditions (3.3) and (3.4) imply that the returns to turning a marginal unit of deposits into housing capital and renting that capital ad infinitum is the same as the returns to storing that marginal unit of deposit.

Arbitrage also requires that for all mortgages issued at a given date, the expected return on the mortgage net of expected foreclosure costs cover the opportunity cost of funds, which by assumption is the returns to storage plus the servicing premium $\phi$.

To make this precise, denote the value to the intermediary of a mortgage contract $\kappa$ held by a mid-aged agent in state $\omega \in \Omega_M$ by $W^\kappa(\omega)$. Again, we need to consider several cases.
• If the homeowner’s mortgage is not paid off, so that $\omega = (a, y, 1, h, n; \kappa)$ with $n \in (0, T - 1]$, then:

$$W^\kappa(\omega) = \left( D^I(\omega) + D^V(\omega) \right) \min\{(1 - \chi)qh, b(n; \kappa)\} + S(\omega)b(n; \kappa)$$

$$+ \left( 1 - D^I(\omega) - D^V(\omega) - S(\omega) \right) \left( \frac{m(n; \kappa)}{1 + r + \phi} + E_{\omega'|\omega} \left[ \frac{W^\kappa(\omega')}{1 + r + \phi} \right] \right)$$

• If the household just became mid-aged and her budget set is not empty so that $\omega_0 = (a_0, y_0, 0, h_1, 0)$ and, for some contract $\kappa$,

$$y_0 + (a_0 + \iota - \nu qh_0 \cdot 1_{\{\zeta = FRM\}}) (1 + r) - m(0; \kappa) - \delta h_0 \geq 0,$$

then

$$W^\kappa(\omega_0) = \frac{m(0; \kappa)}{1 + r + \phi} + E_{\omega'|\omega_0} \left[ \frac{W^\kappa(\omega')}{1 + r + \phi} \right]$$

• In all other cases, $W^\kappa(\omega) = 0$.9

Then, the expected present discounted value of a loan contract $\kappa = (\zeta, r^\kappa, h_0)$ offered to a household that just turned mid-age with state $\omega_0 = (a_0, y_0, \ldots)$ is $W^\kappa(\omega_0)$. The zero profit condition on a loan contract $\kappa$ can then be written as

$$W^\kappa(\omega_0) - (1 - \nu 1_{\{\zeta = FRM\}})qh_0 = 0. \quad (3.5)$$

In equilibrium, the set $K(\omega_0)$ of mortgage contracts available to an agent who becomes mid-aged in state $\omega_0$ is the set of contracts that satisfy condition (3.5).

### 3.3 Distribution of agent states

The household’s problem yields decision rules for a given set of prices. In turn, these decision rules imply in the usual way transition probability functions across possible agent states. In the next section we study equilibria in which the distribution of agent states is invariant under those probability functions. This section makes this notion precise.

In our environment, the transition matrix across ages is given by:

$$
\begin{bmatrix}
(1 - \rho_M) & \rho_M & 0 \\
0 & (1 - \rho_O) & \rho_O \\
\rho_D & 0 & 1 - \rho_D
\end{bmatrix}
$$

9Specifically, this is the case when:

1. the agent just turned mid-aged and her budget set is empty;
2. the agent is a renter;
3. the agent has been mid-aged for more than $T$ periods.
since the old are immediately replaced by newly born young people. Let \((n_Y, n_M, n_O)\) be the corresponding invariant distribution of ages. The invariant mass of agents born each period is then given by

\[ \mu_0 \equiv n_O \rho_D. \]

With this notation in hand, we can define invariant distributions over possible states at each demographic stage.

### 3.3.1 The young

The invariant distribution \(\mu_Y\) on \(\Omega_Y\) solves, for all \(y \in \{y_L, y_M, y_H\}\) and \(A \subset \mathbb{R}_+\):

\[ \mu_Y(A, y) = \mu_0 1_{\{0 \in A\}} \pi^*(y) + (1 - \rho_M) \int_{\omega \in \Omega_Y} 1_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) \mu_Y(d\omega) \]

where \(\pi^*(y)\) is the mass of agents born with income \(y\) (in other words, \(\pi^*\) denotes the invariant distribution associated with our Markov process for income), \(a'_Y : \Omega_Y \mapsto \mathbb{R}_+\) is the saving decision rule for young agents, and, abusing notation somewhat, \(\Pi(y|\omega)\) is the likelihood of income draw \(y \in \{y_L, y_M, y_H\}\) in the next period given current state \(\omega \in \Omega_Y\).

### 3.3.2 The mid-aged

The invariant distribution \(\mu_M\) on \(\Omega_M\) solves, for all \(y \in \{y_L, y_M, y_H\}\), \(A \subset \mathbb{R}_+\) and \((H, h, n; \kappa) \in \{0, 1\} \times \{h_1, h_2, h_3\} \times \mathbb{N} \times \{\{FRM, LIP\} \times \mathbb{R}_+ \times \{h_2, h_3\}\} \cup \{\emptyset\}\):

\[ \mu_M(A, y, H, h, n; \kappa) = \rho_M \int_{\Omega_Y} 1_{\{(H, h, n)=(0, h_1, 0)\}} 1_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) \mu_Y(d\omega) \]

\[ + \ (1 - \rho_0) \int_{\Omega_M} 1_{\{(H', \omega) = H, n(\omega) = n-1, a'_M(\omega) \in A\}} \Pi(y|\omega) P(h|\omega) \mu_M(d\omega) \]

\[ \times \ \{1_{\{n(\omega)=0, \Xi(\omega)=\kappa\}} + 1_{\{n(\omega)>0, \kappa=\kappa(\omega)\}}\} \]

where \(a'_M : \Omega_M \mapsto \mathbb{R}_+\) is the optimal saving policy for mid-aged agents, \(n(\omega)\) extracts the contract age argument of \(\omega\), \(\kappa(\omega)\) extracts the contract type argument of \(\omega\), and \(P(h|\omega)\) is the likelihood of a transition from state \(\omega\) to a state where the house size is \(h\).

The first term corresponds to agents who age from young to mid-aged, while the second integral corresponds to agents who were mid-aged in the previous period and do not get old. The indicator functions reflect the fact that agents make their mortgage choice in the first period they become mid-aged but cannot revisit that choice in subsequent periods.

### 3.3.3 The old

The invariant distribution \(\mu_O\) on \(\Omega_O \equiv \mathbb{R}_+\) solves, for all \(A \subset \mathbb{R}_+\):
\[ \mu_O(A) = (1 - \rho D) \int_{\Omega_O} 1_{\{a'_O(\omega) \in A\}} \mu_O(\omega) + \rho \int_{\Omega_M} 1_{\{a'_M(\omega) + \max\{H'(\omega)[qh(\omega) - b(n+1, \kappa)], 0\} \in A\}} \mu_M(\omega) \]

where, for \( \omega \in \Omega_M \), \( h(\omega) \) extracts the house size argument of \( \omega \), while \( b(n + 1, \kappa) \) is the principal balance on a mortgage of type \( \kappa \) after \( n + 1 \) periods. Recall that we assumed that housing sales due to the old age shock do not entail foreclosure costs.

### 3.4 Housing market clearing

The housing market capital clearing condition can be stated in simple terms, after some algebra. The total for housing (whether rented or owned) in each period is given by:

\[
\int_{\Omega_Y} h_1 d\mu_Y + \int_{\Omega_O} h_1 d\mu_O + \int_{\Omega_M} h_1 \{H' = 0\} d\mu_M + \int_{\Omega_M} h_1 \{H' = 1, h(\omega) = h\} d\mu_M
\]

The first two terms give the demand for housing by the young and old agents, who, by assumption, are renters. The third term is demand from mid-aged agents who choose to be renters. The last integral captures mid-aged agents who choose to be homeowners. Their use of housing capital depends on the size of the house that they own (i.e. \( h(\omega) = h \)).

Similarly, the total quantity of housing available in a given period is the sum of the housing agents carry over from the past period and of the new capital produced by the intermediary. It can be stated formally as:

\[
Ak^\alpha + \int_{\Omega_Y} h_1 d\mu_Y + \int_{\Omega_O} h_1 d\mu_O + \int_{\Omega_M} h_1 \{H' = 0\} d\mu_M + \int_{\Omega_M} h_1 \{H' = 1, h(\omega) = h\} d\mu_M
\]

But the laws of motion for agent states in our economy imply that:

\[
\int_{\Omega_M} h_1 \{H' = 1, h(\omega) = h\} d\mu_M = \int_{\Omega_M} h' \{H' = 1\} P(h' | \omega) d\mu_M
\]

where \( P(h' | \omega) \) is the likelihood that the agent’s house size will be \( h' \in \{h_1, h_2, h_3\} \) in the next period given current state \( \omega \in \Omega_M \).

It follows that the market for housing capital clears provided

\[
\int_{\Omega_M} h_1 \{H' = 1, h(\omega) = h\} d\mu_M - \int_{\Omega_M} h' \{H' = 1\} P(h' | \omega) d\mu_M = Ak^\alpha
\]

This condition has a very intuitive interpretation. It says that in equilibrium the production of new housing capital must equal the housing capital lost to devaluation. Note that because arbitrage condition (3.4) holds in equilibrium, this implies that both the rental and the owner-occupied markets clear since the intermediary is willing to accommodate any allocation of total housing capital.
3.5 Definition of a steady state equilibrium

Equipped with this notation, we may now define an equilibrium. A steady-state equilibrium is a set \( K : \Omega_M \mapsto \{\text{FRM, LIP}\} \times \mathbb{R}^+ \times \{h_2, h_3\} \) of mortgages available to households conditional on any possible state upon entering mid-age, a pair of housing capital prices \((q, R) \geq (0, 0)\), a value \( k > 0 \) of investment in housing capital, per-capita profits \( \iota \), agent value functions \( V_{\text{age}} : \Omega_{\text{age}} \mapsto \mathbb{R} \) for \( \text{age} \in \{Y, M, O\} \), saving policy functions \( a'_{\text{age}} : \Omega_{\text{age}} \mapsto \mathbb{R}^+ \), a mortgage choice policy function \( \Xi : \Omega_M \mapsto K(\omega_0) \), a housing policy function \( H' : \Omega_M \mapsto \{0, 1\} \), mortgage termination policy functions \( D_I, D_V, S : \Omega_M \mapsto \{0, 1\} \), and distributions \( \mu_{\text{age}} \) of agent states on \( \Omega_{\text{age}} \) such that:

1. Household policies are optimal given all prices.
2. The intermediary’s output of new housing capital in each period is optimal given \( q \). In other words, \( k \) solves (3.3).
3. Per capita profits associated with housing capital production are \( \iota \).
4. The allocation of housing capital to rental and the owner-occupied market is optimal for the intermediary. That is, condition (3.4) holds.
5. The market for housing capital clears every period (i.e. (3.7) holds).
6. The intermediary expects to make zero profit on all mortgages. In other words, condition (3.5) holds for all \( \omega_0 \in \Omega_M \) and all mortgages in \( K(\omega_0) \).
7. The distribution of states is invariant given pricing functions and agent policies.

The next section simulates this economy under various calibrations. We will be particularly interested in the fraction of agents who choose to terminate their mortgages early. As we have discussed, this may occur for voluntary or involuntary reasons.

4 Quantitative analysis with Constant Returns to Scale

Our goal is to quantify the importance of contracts with low initial payments for default patterns in the US housing market and for the recent rise in foreclosure. To do so, we select parameters so that the economy we have laid out makes predictions for key statistics that match their US counterparts prior to the collapse in house prices in 2006. We then ask a number of quantitative questions about steady state statistics. In particular, can our economy, so calibrated, account for default rates and patterns across mortgage types? What would be the consequences (on home-ownership rates, default rates, saving decisions...) of precluding agents from choosing LIPs?

Next, we subject the economy to an aggregate house price shock of a magnitude similar to what has transpired in the United States since 2006, and trace out the effects of this shock.
along the transition to a new steady state. We find that our economy can explain two-thirds of the recent increase in foreclosure rates, while an economy where only FRMs are available would only explain half of this increase.

4.1 Parameterization

We choose our benchmark set of parameters so that our economy matches the relevant features of the US economy prior to the housing price collapse. As the top left panel of Figure 1 shows, FRMs accounted for about 75% of the mortgage shock when prices peaked in 2006.

We will think of a model period as representing 3 years. We specify some parameters directly via their implications for certain statistics in our model. These include the parameters governing the income and demographic processes. The other parameters will be selected jointly to match a set of moments with which we want our benchmark economy to be consistent.

We set demographic parameters to \((\rho_M, \rho_0, \rho_D) = (0.25, 0.1, 0.125)\) so that, on average, agents are young for 12 years starting at 20, middle-aged for 30 years, and retired for 24 years. The income process for agents in the first two stages of their life, allowing for the possibility that the process may differ across life stages, are calibrated from the Panel Study of Income Dynamics (PSID) survey. We consider households in each PSID sample whose head is between 20 and 31 years of age to be young, and households between 32 and 61 years to be mid-aged. Each demographic group in the 1996 and 1999 PSID surveys is then split into income terciles. The support for the income distribution is the average income in each tercile in the two surveys, after normalizing the intermediate income value for mid-aged agents to 1. This yields a support for the income distribution of young agents of \(\{0.2664, 0.7218, 1.6306\}\), while the support for mid-aged agents is \(\{0.2737, 1, 2.4057\}\). We assume that income in old age is 0.4. This makes retirement income 40% of median income among the mid-aged, which is consistent with standard estimates of replacement ratios.

We then equate the income transition matrix for each age group to the frequency distribution of transitions across terciles for households who appear in both the 1996 and the 1999 survey and remain in their age category. The resulting transition matrix for young agents is:

\[
\begin{bmatrix}
0.6788 & 0.2363 & 0.0849 \\
0.2503 & 0.5055 & 0.2442 \\
0.0709 & 0.2583 & 0.6709
\end{bmatrix}
\]

while, for mid-aged agents, it is:

\[
\begin{bmatrix}
0.7845 & 0.1781 & 0.0374 \\
0.1647 & 0.6607 & 0.1746 \\
0.0508 & 0.1612 & 0.7880
\end{bmatrix}
\]

The economywide cross-sectional variance of the logarithm of income implied by the resulting
distribution is near 0.45, while the autocorrelation of log income is about 0.76. \(^{10}\)

We let the (three-year) risk-free rate be \(r = 0.12\), and choose the maintenance cost \((\delta)\) to 7.5% to match the yearly gross rate of depreciation of housing capital, which is 2.5% annually according to Haring et al., 2007.

The stipulations of FRM contracts are set to mimic the features of common standard fixed-rate mortgages in the US. The down-payment ratio \(\nu\) is 20% while the maturity \(T\) is 10 periods, or 30 years. Our \(LIP\) contracts have \(n^{LIP} = 3\) and \(T = 10\) so that agents make no payment toward principal for 9 years, and make fixed payments for the remaining 7 contract periods (or 21 years) unless the contract is terminated before maturity.

Housing choices depend on the substitutability of consumption and housing services, and on the owner-occupied premium. We specify, for all \((c,h) \neq (0,0)\),

\[
U(c,s) = \psi \log c + (1 - \psi) \log s.
\]

The intertemporal discount rate, likewise, plays a key role in our model by affecting asset accumulation. Preferences are fully described by \((\theta, \psi, \beta)\). We select these parameters in our joint calibration, to which we now turn.

We need to set the following, ten remaining parameters: the owner-occupied premium \((\theta)\), households’ discount rate \((\beta)\), housing TFP \((A)\), rental unit size \((h_1)\), house sizes \((h_2, h_3)\), the mortgage service premium \((\phi)\), the foreclosure cost \((\chi)\), the utility weight on consumption \((\psi)\), and the house shock probability \((\lambda)\). We select those parameters jointly to target: home-ownership rates, the average ex-housing to income ratio among homeowners, the average loan-to-income ratio at mortgage origination, the average ratio of rents to income in personal consumption expenditures across all households, the average rent-to-income ratio for poor renters, the average housing spending share for homeowners, the average yields on FRMs, the average loss severity rates on foreclosed properties, the average foreclosure rates prior to the flare up, and the fraction of FRMs in the stock of mortgage contract.

That the parameters we need to specify are closely related to these targets should be intuitively clear, with the exception perhaps of the fraction of FRMs. This is a moment we consider important since a satisfactory accounting of the role non-FRM contracts play in default patterns should be carried out in an environment that captures their frequency. Experiments reveal that the pair of house sizes are the parameters that affect this frequency most directly. As these values rise, down-payment requirements become more taxing for agents, and LPMs become more appealing. As a result, the pair \((h_2, h_3)\) jointly affects average (owner-equivalent) housing spending shares, and the fraction of FRMs.

There only remains to elaborate on our approach to measuring target values. Since our model only gives agents a one-time option to become homeowners when they just become mid-aged, we choose to target the ownership rate among households whose head is between 32 and 40. That rate is roughly \(\frac{2}{3}\). The model’s counterpart to that number is the rate of

\(^{10}\)Krueger and Perri (2005) report estimates for the cross-sectional variance of log yearly income of roughly 0.4 and for the autocorrelation of log income in the \([0.80 - 0.95]\) range. These numbers imply that log 3-year income has an autocorrelation in the \([0.92 - 0.97]\) range and variance in the \([0.2 - 0.38]\) range.
ownership among agents who have been mid-aged for three periods or fewer. This is the rate we will report throughout the paper.

The average non-housing assets to yearly income ratio we choose to target is based on Survey of Consumer Finance (SCF) data. The average ratio of non-housing assets to income among homeowners whose head age is between 32 and 61 in the 2004 survey is 2.09, which corresponds to a ratio of assets to three-year worth of income of roughly 0.7.

The mortgage loan at origination \((1 - \nu)hq\) for FRMs and \(hq\) for LPMs, where \(h \in (h_2, h_3)\) is the initial house size. Evidence available from the American Housing Survey (AHS) suggests that prior to 2005 the ratio of this original loan size to yearly income is around 2.5 on average in the US, or 0.83 in three-year terms.

According to the evidence discussed by Kahn (2008), the ratio of housing expenditures (in imputed rent terms for owners) to overall expenditures is near 20%, and we make this our fourth target. Turning to the rent-to-income ratio for poor renters, Green and Malpezzi (1993, p11) calculate that poor households who are renter spend roughly 40% of their income on housing. On the other hand, according to the 2004 Consumer Expenditure Survey, expenditures on owned dwellings account for 16% of the expenditures of home-owners.

Next, we choose to target an average FRM-yield of 5.5% yearly, or 17.5% over a three-year period. This was the average contract rate on conventional, fixed rate mortgages between 2003 and 2005 according to Federal Housing Finance Board data.

The loss severity rate is the present value of all losses on a given loan as a fraction of the default date balance. As Hayre and Saraf (2008) explain, these losses are caused both by transaction and time costs associated with the foreclosure process, and by the fact that foreclosed properties tend to sell at a discount relative to other, similar properties. Using a dataset of 90,000 first-lien liquidated loans, they estimate that loss severity rates range from around 35% among recent mortgages to as much as 60% among older loans. Based on these numbers we choose parameters so that in the event of default and on average,

\[
\min\left(\frac{(1 - \chi)qh, b}{b}\right) = 0.5
\]

where \(b\) is the outstanding principal at the time of default and \(qh\) is the house value. In other words, on average, the intermediary recovers 50% of the outstanding principal it is owed on defaulted loans.

We target a three-year default rate of 3.75% which is near the average foreclosure rate among all mortgages during the 1990s in the Mortgage Bankers Association’s National Delinquency survey.

Finally, we target an FRM fraction of originations between 35 and 50%. According to the Mortgage Origination Survey, traditional FRMs accounted for roughly 50% of all originations.

---

11 Because agents only have one asset in the model, it is interpreted as net assets. The net assets do not include housing-related assets or debt, such as home equity or mortgages. Since agents are not allowed to have negative assets in the model, households who have negative non-housing assets are assumed to have zero assets in the calculation.
in 2005 (the first year of the survey.) It is not clear that all other mortgages issued were of the low-payment sort. However, the fact that FRMs account for a stable 85% of the mortgage stock prior to 2005 suggests that the origination rate of FRMs was of the order of 85% before then. It seems safe to assume that 35% fraction drop in FRM originations owes for the most part to the popularity of high LTV, non-traditional amortization loans. The FRM fraction of originations can be directly mapped into another important moment commonly used in the literature, the average loan to value ratio at origination, because the downpayment requirements are the same for contracts of the same mortgage type.\textsuperscript{12} The 35 to 50% range for the FRM fraction is equivalent to a LTV range of 0.87 to 0.90.

### 4.2 Steady state results on mortgage innovation

The benchmark economy has both FRM and LIP mortgages available. The economy in which only FRM mortgages are available will serve as a counter-factual experiment to study the impact of LIPs on long-run aggregate statistics. Table 2 presents some key steady state equilibrium aggregate statistics for both environments.

The table shows that the presence of LIPs has two main consequences on steady state statistics: home-ownership rates and average default rates are much higher when LIPs are available than when they are not. When only FRMs are available, a large number of agents are unable to become owners because they can’t afford a large downpayment.

Default rates, for their part, are higher when LIPs are present as a result of two complementary factors. First, LIPs enable agents at the bottom of the asset and income distributions to become home-owners. These agents are high-default risk agents because they are more likely to find themselves unable to meet their mortgage payments at some point over the life of the contract. Second, even at equal asset/income conditions at origination, LIPs are associated with higher default rates because agents build up home equity at a much slower rate. The remainder of this section makes these ideas precise.

#### 4.2.1 Selection

This section describes the distribution of contracts in equilibrium. In particular, we show that asset and income poor agents tend to select LIPs.

The distribution of assets at house purchase time has direct consequences on the distribution of mortgage choices. Conversely, making LIPs available impacts the equilibrium distribution of wealth at purchase time, since a major incentive to save in the economy with FRMs only is the need to put downpayments on houses.

Figure 3 plots the endogenous distribution of assets among agents that just turned middle-aged. In the FRM-only experiment, the upper panel shows that, quite intuitively, low income

\textsuperscript{12}In our model, the loan to value for FRMs is $1 - \alpha$, while the loan to value for LIPs is 1. The average loan to value ratio (LTV) with $x\%$ FRM mortgage is then $x(1 - \alpha) + (1 - x)$ In another words, the FRM fraction is $\frac{1 - \text{LTV}}{\alpha}$. 

20
Figure 1: Recent trends in US housing

Sources: Haver analytics, National Delinquency Survey (Mortgage Bankers Association), and Statistical Abstract of the United States.

Table 1: Benchmark parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_M )</td>
<td>Fraction of young agents who become mid-aged</td>
<td>0.25</td>
<td>12 years of earnings on average prior to home purchase</td>
</tr>
<tr>
<td>( \rho_O )</td>
<td>Fraction of mid-aged agents who become old</td>
<td>0.10</td>
<td>30 years on average between home purchase and retirement</td>
</tr>
<tr>
<td>( \rho_D )</td>
<td>Fraction of old agents who die</td>
<td>0.125</td>
<td>24 years of retirement on average</td>
</tr>
<tr>
<td>( r )</td>
<td>Storage returns</td>
<td>0.12</td>
<td>3-year risk-free rate</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Maintenance rate</td>
<td>7.5%</td>
<td>Residential housing gross depreciation rate</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Downpayment on FRMs</td>
<td>0.20</td>
<td>Average Loan-to-Value Ratio</td>
</tr>
<tr>
<td>( T )</td>
<td>Mortgage maturity</td>
<td>10</td>
<td>30 years</td>
</tr>
<tr>
<td>( n^{LIP} )</td>
<td>Interest-only period for LIPs</td>
<td>3</td>
<td>9-years interest-only</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Owner-occupied premium</td>
<td>5</td>
<td>Homeownership rates</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Housing shock probability</td>
<td>0.085</td>
<td>Foreclosure rates</td>
</tr>
<tr>
<td>( A )</td>
<td>Housing technology TFP</td>
<td>0.8</td>
<td>Average Loan-to-income ratio at origination</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount rate</td>
<td>0.8</td>
<td>Average ex-housing asset-to-income ratio</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Mortgage service cost</td>
<td>0.03</td>
<td>Average mortgage yields</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Foreclosing costs</td>
<td>0.35</td>
<td>Loss-incidence estimates</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Utility share on consumption</td>
<td>0.8</td>
<td>Average housing spending share</td>
</tr>
<tr>
<td>( h_1 )</td>
<td>Size of rental unit</td>
<td>0.4</td>
<td>Rent-to-income ratio for low-income agents</td>
</tr>
<tr>
<td>( h_2 )</td>
<td>Size of regular house</td>
<td>0.75</td>
<td>Fraction of FRMs</td>
</tr>
<tr>
<td>( h_3 )</td>
<td>Size of luxury house</td>
<td>1.5</td>
<td>Owners’ housing spending share</td>
</tr>
</tbody>
</table>
agents tend to have low assets, and vice-versa. The lower panel shows the change in the distribution when LIPs are introduced. There is a noticeable shift to the left in the distribution as many agents anticipate that they may resort to the LIP option and no-longer need to accumulate assets to meet downpayment requirements. In fact, the average assets of agents who just became mid-aged in the economy with LIPs is lower by one third than its counterpart in the economy with FRMs only (0.1823 vs. 0.1855.)

Table 3 displays contract selection patterns in steady state. It shows, first, that when LIPs are no longer available, many agents are constrained to rent because they cannot meet the down-payment imposed by mortgages and/or cannot make the first payment. This is true in particular of agents whose assets \( a_0 \) are low when they become mid-aged. The presence of LIPs enables some agents at the bottom of the asset distribution to become home-owners instead of renting, as the bottom panel of the table shows. This is true, in fact, of all agents except those at the bottom of the income distribution. The table also shows that house size choices increase with both assets and income.

Figure 4 displays the correlation between housing choices and asset and income level. When the LIP option is removed (as we go from the top to the bottom panel of the figure), agents at the bottom of asset distribution are no longer able to become home-owners. The figure also shows that LIPs are the contract of choice for agents at the bottom of the asset distribution, whereas wealthier agents take an FRM (to take advantage of lower rates, as the next section will discuss.)

All told, the availability of LIPs cause home-ownership rates to rise by giving agents more financing options. The fraction of newly mid-aged agents who enter housing markets and buy smaller houses falls from 39.12% to 18.21% when the LIP option is removed. In addition, the fraction of agents who buy large houses falls from 33.32% to 22.36% as a result of the tighter
Table 2: Steady state statistics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Benchmark</th>
<th>FRM only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homeownership rate</td>
<td>67.00</td>
<td>70.73</td>
<td>40.11</td>
</tr>
<tr>
<td>Avg. ex-housing asset/income ratio</td>
<td>0.74</td>
<td>0.74</td>
<td>0.78</td>
</tr>
<tr>
<td>Avg. loan to income ratio</td>
<td>0.85</td>
<td>0.91</td>
<td>0.72</td>
</tr>
<tr>
<td>Avg. homeowner housing expenditure share</td>
<td>0.20</td>
<td>0.20</td>
<td>0.19</td>
</tr>
<tr>
<td>Rents to income ratio for renters</td>
<td>0.40</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>Avg housing spending share for homeowners</td>
<td>0.16</td>
<td>0.25</td>
<td>0.27</td>
</tr>
<tr>
<td>Avg. mortgage yields (FRMs, LIPs)</td>
<td>(17.5,NA)</td>
<td>(16.69,19.09)</td>
<td>(16.72,NA)</td>
</tr>
<tr>
<td>Loss-incidence estimates</td>
<td>0.50</td>
<td>0.52</td>
<td>0.56</td>
</tr>
<tr>
<td>Foreclosure rates</td>
<td>3.75</td>
<td>3.29</td>
<td>2.45</td>
</tr>
<tr>
<td>FRM fraction (LTV)</td>
<td>[50-65] ([0.87-0.90])</td>
<td>52.8 (0.89)</td>
<td>100.0 (0.80)</td>
</tr>
</tbody>
</table>

financial requirements imposed by FRMs.

Overall, LIPs turn out to be selected by roughly 47% of newly mid-aged households, and tend to be selected by households whose assets are low. The next section argues that, holding contract terms fixed, poor agents are more likely to default than other agents. In addition, it shows that LIPs, holding initial asset to income position fixed, are inherently more prone to default. Combined, these facts imply that LIP-holders account for a disproportionate share of overall default rates, and explains why default rates are higher in the economy where the LIP option is present than in the economy where only FRMs are available.

4.2.2 Default

Figure 5 shows the link between initial income positions and default rates by mortgage age at the median level of assets at origination for houses of size $h_2$. These graphs provide similar information as hazard rates (i.e. the probability of default at time $n$ conditional on not having defaulted earlier.)\(^{13}\)

Agents who at origination time are at the lowest income level experience a big increase in sales rates in the second mortgage period. Other agents tend to remain as homeowners for a longer time.

If the household experiences a house devaluation shocks (to rental unit size), home equity becomes negative unless the mortgage has fewer than four periods left to maturity. As a result, there is no voluntary default after the fourth period. Because smaller houses are inexpensive, the associated mortgage payments-to-income ratios are low, and involuntary default does not

\(^{13}\)Specifically, if $x_n$ for $n$ in $\{1, 2, ..., T-1\}$ is the value in Figure 5, then the hazard rate is $\frac{x_n}{1 - \sum_{t=1}^{T} x_n}$. 

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occur. However, households with lower initial assets tend to sell their houses earlier than those with higher initial assets.

Figure 6 shows how default rates are affected by initial asset positions (for the median level of income). Agents with higher initial assets have a later spike for sales. Rates of voluntary defaults are similar across asset categories. Once again, no involuntary default happens for households who purchase smaller houses.

The evolution of the principal balance and home equity as a function of maturity for each type of contract is displayed on figure 7. While FRM contracts feature a progressive reduction of mortgage debt and a corresponding increase in home equity, LIP contracts only begin this process after three periods. The result is a much greater risk that agents will find themselves with negative equity following a devaluation shock. The dotted lines at the bottom of the figure show home equity following a devaluation from $h_2$ to $h_1$ as a function of maturity. The shock cause equity to become much more negative at any given maturity for LIPs than for FRMs.

Figure 8 illustrates the impact of contract choices on overall default hazard rates for an agent whose initial asset-income position when she becomes mid-aged is $(a_0, y_0) = (0.28, 1)$, the median values of both arguments. If agents experience housing devaluation, home equity becomes negative unless only two contract periods remain on LIPs. For FRM holders, equity only becomes negative following the same shock on contracts less than half-way to maturity.

Our model also predicts that properties should sell at a discount following default. This
is because, as we discuss below, some default is caused by devaluation shocks as agents walk away from houses in which they have negative equity. In the study we discussed in the parameterization section, Hayre and Saraf (2008) estimate that foreclosed properties sell a discount relative to their appraised value that ranges from 10% among properties with appraisal values over $180,000 to 45% among properties with appraisal values near $20,000. Other studies of foreclosure discounts (see Pennington-Cross, 2004, for a review) typically find discount rates near one quarter, with some exceptions. Our model predicts that houses sold following default should sell at a discount of 40% relative to other houses.

Selection and home-equity accumulation effects imply that, in equilibrium, the frequency of default is much higher among LIP-holders than it is among FRM-holders. Table 4 provides a breakdown of default frequencies by contract type across experiments. Each entry gives the fraction of mortgages of each type that go into default in steady state in each of the two economies we consider.\textsuperscript{14}

\textsuperscript{14}In the notation we introduced in section 3.1.2, involuntary and voluntary default rates on a FRM contracts.
The table shows first that voluntary default rates are twice as high on LIPs than on FRMs. Voluntary defaults occur following a devaluation shock when the house value falls below the outstanding principal. Since agents with LIP contracts are much slower to accumulate equity in their house than agents with FRMs, they are also more likely to find themselves with negative equity following a devaluation shock.

For their part, involuntary defaults – defaults triggered by an income shock such that the borrower can no longer meet their mortgage payments – are nearly eight times as high on LIPs as on FRMs. There are two main reasons for this. First, agents who select LIPs tend to be asset and income poor which makes payment difficulties more likely. Second, because principal payments are concentrated over few periods on LIPs, payments suddenly jump towards the end of the contract, adding to the risk that agents will not be able to meet their obligations.

are given by, respectively:

\[
\frac{\int_{\Omega_M} D^I(\omega)1_{\{\zeta=FRM,n \leq T,H=1\}}d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta=FRM,n \leq T,H=1\}}d\mu_M(\omega)}
\]

and

\[
\frac{\int_{\Omega_M} D^V(\omega)1_{\{\zeta=FRM,n \leq T,H=1\}}d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta=FRM,n \leq T,H=1\}}d\mu_M(\omega)}.
\]

Similar expressions give default rates for LIPs.

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Since LIPs are characterized by much higher default rates than FRMs, they account for a disproportionate fraction of the overall default rate. Table 5 shows the contributions of each contract type to each type of default rate in each of the scenarios we consider. The table shows that LIPs account for nearly 60% of overall default rates even though they only represent one third of all mortgages.

The table also shows that involuntary defaults account for a relatively small fraction (17.3%) of overall defaults. Most defaults, in other words, involve agents who could meet their obligations but choose to terminate their contract in a position of negative equity. In fact, even in the case of involuntary default (where default is the only option because the household cannot meet their mortgage payment), 86.6% of foreclosures involve negative equity. This is because most households who are at a high risk of being in a state of involuntary default in the following period and have positive equity tend to sell their house in order to avoid transaction costs.

Several steady state statistics illustrate this behavior. Consider for instance the set of

\[ \left( \int_{\Omega_M} 1_{\{\zeta=FRM,n\leq T,H=1\}} d\mu_M(\omega) \right) \times \left( \int_{\Omega_M} D^I(\omega) 1_{\{\zeta=FRM,n\leq T,H=1\}} d\mu_M(\omega) \right). \]

For instance, the contributions of FRM contracts to involuntary default rates is given by the share of FRM mortgages in the total stock of mortgages in steady state times the rate of involuntary default on FRMs.
households who, should they choose to keep their house, face a positive probability of being in an involuntary default situations in the next period. Almost 93% of these high-risk households choose to sell their house, while selling rates are below 10% among other mortgage holders. Conversely, among agents who choose to sell in a given period, the probability that they would be in an involuntary default situation in the next period should they choose not to sell is 78.28%. Among households who do not sell, that probability is 35.91%. In other words, almost all households at a risk of imminent involuntary default choose to sell. Since households with positive equity stand to lose the most by defaulting for involuntary reasons, it is not surprising that most households who do end up in a situation of involuntary default have negative equity.

All told then, the vast majority of foreclosures involve negative home equity. On the other hand, many households (roughly 33%) with negative home equity choose to keep their house and continue meeting their mortgage obligations. While defaulting would entail a net worth gain for these households, they would be forced to rent a smaller housing unit and would forego the ownership premium.

These model predictions are consistent with the empirical literature on the determinants of foreclosure (see, e.g., Gerardi et al., 2007). Available data suggest that most foreclosures involve negative equity but that, at the same time, most households with negative equity choose not to foreclose. Our model captures the fact that most foreclosures involve a combination of negative equity and adverse income shocks.
4.2.3 Interest rates

A distinguishing feature of our model is that mortgage terms depend not only on mortgage types but also on the initial asset and income position of borrowers.

Figure 9 plots the equilibrium FRM and LIP rates agents can obtain from the intermediary when they become middle-aged, depending on the house size they opt for and their asset-income position at origination.

Note first that all schedules are left-truncated because agents whose income and assets are low do not get a mortgage in equilibrium. This occurs for several reasons. First, asset and income poor agents cannot meet the down-payment requirement and/or mortgage payments. Second, these agents are more likely to default, hence receive less favorable borrowing terms. In some cases in fact, there is no yield such that the intermediary would expect to break even on the mortgage, even when the agents have the means to finance the initial down-payment.\footnote{In that period (i.e. when $n = 0$), the budget set is empty when $c = a' = 0$ and

$$m(0; \kappa) > y_0 + (a_0 + \tau - vq_h \cdot 1_{\{\zeta = FRM\}})(1 + r).$$

Since $m(0; \kappa)$ is strictly increasing in $r^\kappa$, we know there is an interest rate $r^\kappa$ that depends on $y_0$ and $a_0$ such that for any $r > r^\kappa$ the bank cannot break even.}

Among agents who do receive a mortgage offer, yields fall both with assets and income. This prediction accords well with the well-documented mortgage industry practice of including...
overall debt-to-income ratios in their rate sheets. It is also borne out by the statistical evidence available from the Survey of Consumer Finance. Figure 9 also shows that conditional on a given asset-income position at origination, yields are higher for agents who opt for large houses than agents who opt for small houses. This prediction of our model is consistent with the well-documented fact that mortgage rates rise with borrowers’ loan-to-income ratio.

When LIPs are not available, agents face only the FRM interest rate schedule. Yields offered on FRMs are unchanged. Several facts are immediately apparent. First, a glance at the vertical scale of the figure reveals that LIP rates exceed FRM rates at all possible asset-income positions. This is because LIPs entail a greater risk of default since home equity is slower to rise. The likelihood that an agent will find herself with negative equity in her home is higher when she holds an LIP than when she holds an FRM. Furthermore, LIP payments are concentrated over a lower number of periods hence become large after three contract periods, which makes involuntary default a greater possibility.

Figure 10 graphs the distribution of equilibrium interest rates by mortgage type. If we define subprime as the bottom 15% of households who obtained the highest mortgage interest rate, then there are prime FRMs, prime LIPs and subprime LIPs held in equilibrium. The average return is 16.69% on prime FRMs and 17.20% on prime LIPs, while the average return on subprime LIPs is 19.98%.

The correlation of yields on various contracts with assets and income is qualitatively consistent with the statistical evidence available from the Survey of Consumer Finance, as
Table 5: Share of overall default rates

<table>
<thead>
<tr>
<th></th>
<th>voluntary</th>
<th>involuntary</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRM</td>
<td>1.32</td>
<td>0.00</td>
<td>1.32</td>
</tr>
<tr>
<td>LIP</td>
<td>1.91</td>
<td>0.06</td>
<td>1.97</td>
</tr>
<tr>
<td>Total</td>
<td>3.47</td>
<td>0.57</td>
<td><strong>3.29</strong></td>
</tr>
<tr>
<td><strong>FRM only</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRM</td>
<td>2.45</td>
<td>0.00</td>
<td><strong>2.45</strong></td>
</tr>
</tbody>
</table>

Table 6: Contract terms moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CV(yield)$ for FRMs</td>
<td>0.153</td>
<td>0.023</td>
</tr>
<tr>
<td>$CV(yield)$ for other</td>
<td>0.341</td>
<td>0.056</td>
</tr>
<tr>
<td>$\rho(yield, income)$ on FRMs</td>
<td>-0.12</td>
<td>-0.997</td>
</tr>
<tr>
<td>$\rho(yield, income)$ on other</td>
<td>-0.18</td>
<td>-0.969</td>
</tr>
<tr>
<td>$\rho(yield, net worth)$ on FRMs</td>
<td>-0.023</td>
<td>-0.451</td>
</tr>
<tr>
<td>$\rho(yield, net worth)$ on other</td>
<td>-0.141</td>
<td>0.289</td>
</tr>
</tbody>
</table>

Table 6 shows. To compute these moments in the data, we looked at all the mortgages issued within the two years prior to the 2004 survey. We restrict the sample to recently issued mortgages so that current income and assets are reasonable proxies for their counterparts at origination time. We also restrict our attention to households whose head age is between 30 and 45 since mortgages are only issued at the middle-age stage in our model. These data show that origination yields are negatively correlated with both net worth and income, particularly with income.\(^{17}\) Restricting the sample to FRMs reduces the correlation with assets, but the correlation with income remains strong.

Notice that the model predicts less volatility in yield in the FRM sample than in the LIP sample, despite the fact that the range of FRM yields is much broader than its counterpart for LIPs. One reason for this (see the bottom panel of figure 4) is that the distribution of wealth is more highly concentrated among agents who hold an LIP than among agents who hold an FRM.

\(^{17}\)We define net worth as liquid assets, CDs, stocks, bond, vehicles, primary residence, real estate investment, business interest minus housing debts, credit card, installment debts, and line of credits. This notion of net worth includes housing equity because we observe agents shortly after the mortgage origination. Housing equity, at that time, reflects mainly the down-payment made at origination by the borrower. That downpayment, in turn, was part of assets prior to the origination.
The table shows however that our model understates the variation in yields suggested by these data, and overstates the degree to which income and yields are correlated. A key reason for both findings is that the SCF sample of both FRMs and other mortgages are characterized by much heterogeneity in maturity and initial loan-to-value ratios which we do not model, and which SCF data do not enable one to control for. This heterogeneity raises the volatility of yields and reduces the correlation with asset and income for reasons which our model cannot replicate.

Given the monotonicity of rates and mortgage availability in asset and income, ownership rates are also monotonic in assets and income, as the top panel of table 3 illustrates. Overall, home ownership-rates are near 72% as the third column of table 2 shows.

4.2.4 Separation matters

In this subsection, we conduct a counterfactual experiment to examine the importance of allowing intermediaries to offer mortgage contracts that separate households on the basis of income and asset characteristics at the time the household takes out the loan (as we do in this paper) rather than offering only noncontingent or “pooling” FRM and LIP contracts (as in the paper by Garriga and Schlagenhauf (2009)). In order to answer this question, we construct an equilibrium by taking our parameterization from the benchmark model but restricting the set of available mortgages to be only an FRM or an LIP, both independent of income and
asset position at the time of the loan. The equilibrium mortgage rate is then determined by a zero profit condition across all possible households selecting into that contract (and hence the distribution of households directly affects the calculation of mortgage rates). In this case, the households with high incomes and assets (who have low probability of default on the given contract) are indirectly subsidizing households with low incomes and assets (who have high probability of default on the contract). Such cross-subsidization is unlikely to survive in competitive environments since an intermediary can simply offer a contract with lower interest rate to households with observable high income and/or assets and skim those good customers away from the pooled contract.

The most important difference between the benchmark equilibrium and this counterfactual “pooling” equilibrium is that the economywide default rate increase by 12%, from 3.29 to 3.68. This should be expected since there is no way for intermediaries to separate out the good from the bad risks. In response to the higher default frequency, the equilibrium “pooled” interest rate on LIP contracts rises to 26%. Therefore, not only is the pooling equilibrium inconsistent with the data (i.e. zero dispersion of mortgage rates within FRM and non-FRM contracts), but such misspecification has a big impact on aggregate default probabilities.

### 4.2.5 Welfare

The removal of the LIPs unambiguously reduces the welfare of all agents when $\alpha = 1$ because they lose a financing option without any offsetting price change. However, the welfare consequences of innovation are bound to differ across agents. Agents whose homeownership
Table 7: Steady state statistics

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>No separation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homeownership rate</td>
<td>70.73</td>
<td>68.18</td>
</tr>
<tr>
<td>Avg. ex-housing asset/income ratio</td>
<td>0.74</td>
<td>0.75</td>
</tr>
<tr>
<td>Avg. loan to income ratio</td>
<td>0.91</td>
<td>0.77</td>
</tr>
<tr>
<td>Avg. housing expenditure share</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Rents to income ratio for renters</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>Avg housing spending share for homeowners</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Avg. mortgage yields (FRMs, LIPs)</td>
<td>(16.69,19.09)</td>
<td>(17.5,26.6)</td>
</tr>
<tr>
<td>Loss-incidence estimates</td>
<td>0.52</td>
<td>0.49</td>
</tr>
<tr>
<td>Foreclosure rates</td>
<td>3.29</td>
<td>3.68</td>
</tr>
<tr>
<td>FRM fraction (LTV)</td>
<td>52.8 (0.89)</td>
<td>59.7 (0.88)</td>
</tr>
</tbody>
</table>

Prospects at birth are not significantly reduced by the removal of LIPs will not be affected much, while agents whose ownership prospects do drop significantly are likely to see their welfare drop markedly. This section verifies this intuition.

Our welfare measure is the fraction of lifetime consumption households in the benchmark economy are willing to give up to keep the LIP option. To calculate this consumption-equivalent welfare value of LIPs under our benchmark parameterization, consider agents born with income at birth \( y_i \) where \( i \in \{L, M, H\} \) and let \( U_{\text{Benchmark}}(y_i) \) and \( U_{\text{FRM-only}}(y_i) \) denote the lifetime utility they expect at birth in the benchmark and FRM-only economies, respectively. Denote the optimal consumption and housing service plans in the benchmark economy by \( \{c_{t,i}^{\text{Benchmark}}, s_{t,i}^{\text{Benchmark}}\} \) for an agent born with initial income \( y_i \).

Then, let \( 1 + k_i \) be the multiple one has to apply to the consumption of agents born in the benchmark economy to make their welfare equal the same agents born in an economy with only FRMs. That is, \( k_i \) solves for all \( i \in \{L, M, H\} \):

\[
U_{\text{FRM-only}}(y_i) = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_{t,i}^{\text{Benchmark}}(1 + k_i), s_{t,i}^{\text{Benchmark}}) \right] \\
= E_0 \left[ \sum_{t=0}^{\infty} \beta^t \{ \ln(c_{t,i}^{\text{Benchmark}}) + \ln(1 + k_i) + \ln(s_{t,i}^{\text{Benchmark}}) \} \right] \\
= U_{\text{Benchmark}}(y_i) + \ln(1 + k_i) \frac{1}{(1 - \beta)}
\]

It follows that:

\[
(1 - \beta) \left[ U_{\text{FRM-only}}(y_i) - U_{\text{Benchmark}}(y_i) \right] = \ln(1 + k_i) \\
\implies k_i = \exp \left( (1 - \beta)[U_{\text{FRM-only}}(y_i) - U_{\text{Benchmark}}(y_i)] - 1 \right)
\]

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We calculate that, on average:

\[ k_L = -0.0274 e^{-3} \]
\[ k_M = -0.1142 e^{-3} \]
\[ k_H = -0.0012 e^{-3} \]

making the average welfare loss associated with removing the LIP option less than 0.01% in consumption-equivalent terms. Agents who receive median incomes at birth suffer the highest welfare loss. This is because these agents are the most likely to use the LIP option when they become mid-aged. Since income is quite persistent, households who are born poor know that they are most likely to remain renters their entire life. Likewise, households born rich know that they are most likely to opt for an FRM when they become mid-aged.

We should emphasize that these magnitudes are likely to change quite a bit once \( \alpha < 1 \) because housing prices (both on the owner-occupied and rental markets) respond endogenously to the available mortgage types.

### 4.3 Unanticipated price shocks

The fact that foreclosure rates started picking up almost exactly as house prices began their free-fall in mid-2006 (see figure 1) makes it clear that the two phenomena are intimately related. It is equally natural, however, to think that the increased importance of contracts with back-loaded payments magnified the impact of the price shock on default rates. The purpose of this section is to quantify this effect.

When \( \alpha = 1 \), the way to cause home-prices to fall in our model is to assume that the productivity of the housing technology rises. Assume for instance that \( A \) rises in such a way that the price of a given home falls by 20%, a drop of a magnitude similar to the fall in home prices the US has experienced since prices peaked in 2006. Assume further that this drop in prices catches agents as a complete surprise so that, at the time of the shock, the distribution of states across agents is in steady state. This pushes a number of agents with contracts written prior to the shock (when houses were expensive) into negative equity territory. Furthermore, until agents with old contract disappear from the sample of active mortgages, devaluation shocks become more likely to make equity negative on a given house.

Because the intermediary is also surprised by the shock, it experiences unforeseen revenue and capital losses. To see this, note that steady state net profits on the intermediary’s mortgage activities are given by:

\[
\int b(\omega) (1 - D_f(\omega) - D_v(\omega) - S(\omega)) \frac{(r^c(\omega) - (r + \phi))}{(1 + r + \phi)^{n(\omega)+1}} d\mu_M(\omega) \\
- \int (b(\omega) - \min\{(1 - \chi)qh(\omega), b(\omega)\} (D_f(\omega) + D_v(\omega)) \frac{1 + r + \phi}{(1 + r + \phi)^{n(\omega)+1}} d\mu_M(\omega)
\]
The first term gives the net return on active mortgages that are not terminated in the current period, while the second term is the cost (direct capital losses and opportunity cost) associated with the capital lost in the event of foreclosure.

In steady state, those profits are zero. However, the unexpected drop in prices causes default rates to rise which reduces revenues and raises foreclosure losses, causing profits to become negative until contracts written before the price shock disappear. One has to be explicit about who bears these losses. We assume that constant lump-sum taxes are imposed on all agents following the price shock in such a way as to exactly cover the intermediary’s losses in present value terms. Computationally, this involves guessing a value for the constant and permanent tax, solving for the new steady state equilibrium and the transition to this new state, evaluating the present value of the intermediary’s transitory losses, and updating the permanent tax level until losses and tax revenues match.\footnote{See the computational appendix for details. There are obviously many possible ways to redistribute the intermediary’s losses. Per capita losses are negligible in practice and barring extremely concentrated tax schemes, their exact distribution cannot have large effects on the results we present.}

Figure 11 plots the intermediary’s losses during the transition in the FRM-only economy, and in the benchmark economy. In both economies, losses are steepest on impact but eventually die down as mortgages issued prior to the shock become less prevalent. More interestingly, losses are much steeper when LIP are present because more households find themselves with negative equity following the shock.

The top panel of Figure 12 shows the evolution of default rates during the transition both in the benchmark and in the FRM-only economy, and in the benchmark economy. On impact, as expected, default rates spike up as many agents with contracts written before the shock find themselves with negative equity. Default rates more than double in both economies, but increase by a higher proportion when LIPs are present. The bottom panel of Figure 12 shows the evolution of homeownership rates during the transition. We can see the homeownership rates first drop due to high foreclosures and then climb up to the final steady state.

How do these increases in default rates compare to the evidence shown in figure 1? Foreclosure rates more than double in the data much like they do in our simulations of both model economies. However, they jump to a quarterly rate of roughly 0.9%, which implies a three year default rate of over 10%. The economy with FRMs only does not come close to generating default rates of that magnitude following a surprise 25% price drop. The economy with LIPs, on the other hand, causes default rates to rise within 3 percentage points of their data counterpart. If one factors in the fact that the data refer to foreclosures started which, in a non-negligible fraction of the cases do not lead to formal default, this suggests that our economy with LIPs can easily account for most of the recent flare-up in foreclosure rates.

4.4 Mortgage innovation and the foreclosure boom

The previous section shows that unanticipated aggregate price shocks can cause large flare-ups in foreclosure rates, particularly when non-traditional mortgages are present. We will
now describe a quantitative experiment designed to evaluate the role of these new mortgages in the foreclosure crisis depicted in figure 1.

Figure 1 suggests that the course of events leading up to the collapsed of house prices and the foreclosure crises can be decomposed into three basic stages. Prior to 2005, the composition of the mortgage stock is stable and traditional mortgages are the dominant form of home financing. In 2005, the composition of the mortgages stocks changes noticeably as non-traditional mortgages start accounting for a high fraction of originations. After mid-2006, prices start collapsing, foreclosure rates rise sharply, and the importance of traditional mortgages begins rising once again as originations of non-traditional mortgages slow to a trickle.\(^\text{19}\)

We will use our model to simulate this course of events and quantify the role of non-traditional mortgages using a three-stage experiment.

We begin by calibrating a steady-state version of our economy where only FRMs are available to pre-2005 data. As figure 1 shows, FRMs account for around 85% of mortgages, and the fraction is mostly stable between 1998 and 2005. Furthermore, evidence available from the American Housing Survey suggests that mortgages with non-traditional amortization

\(^{19}\text{In the most recent Mortgage Origination Survey data, traditional FRMs now account for 90% of all originations.}\)
schedules accounted for a small fraction of the 15% of non-FRMs prior to 2005. Traditional FRMs and traditional (nominally indexed) ARMs account for 95% of all mortgages in the American Housing Sample before then. At the same time, data on available from the Federal Housing Finance Board for fully amortizing loans show no increase in average loan-to-value ratios between 1995 and 2005. These numbers suggest that high-LTV, delayed amortization mortgages accounted for a small fraction of the stock of mortgages and of originations before 2005.

In the second stage of the experiment, we introduce the option for newly mid-aged agents to finance their house purchase with a low initial payment mortgage (LIP.) We assume that this introduction is unanticipated by agents. Our steady state analysis suggest that roughly half of these households will take advantage of that option, which is consistent with the origination data available for 2005-2006.

One period later, in the third stage, we hit the economy with a surprise 20% aggregate price shock, and take away the LIP option. This stage is meant to approximate the state of the US housing market in 2008, a state characterized by home prices 20% below their peak, and the end of the availability of non-traditional mortgages.

This experiment will easily mimic the main patterns displayed in figure 1. The second stage will bring about a sharp increase in the use of non-traditional mortgages for one model.
period. This will cause ownership rates to rise slightly, without causing any significant change in default rates because agents do not default in the first period of the contract in our model. In the following period (stage 3), default rates will rise sharply on impact, the fraction of FRMs in the mortgage stock will begin rising slowly. Home-ownership rates, for their part, should change little because the decisions of newly mid-aged agents are affected by two offsetting forces. On the one hand, houses have become cheaper. On the other hand, the mortgage menu has shrunk.

Our steady state analysis suggests that another aspect of the evidence the experiment will easily capture is the fact that new mortgages are characterized by default rates that are well above average. As in the steady state case, this will occur because of both selection and home-equity effects. But these effects will now be compounded by the fact that non-FRMs are recently issued mortgages, hence are mortgages characterized by zero home-equity when the price shock strikes.

This three-act representation of the foreclosure crisis can then be used to ask the two key quantitative questions that concern us. Can a model with non-traditional mortgages account for the recent doubling of foreclosure rates? Assuming the same aggregate house price shock, how much would foreclosure rates have risen if new mortgages had not been introduced after 2004? The first question is straightforward to answer. The second involves running a counterfactual where the second stage does not occur. Our prior is that this counterfactual will show that the increase in foreclosure rates becomes much lower because the aggregate price hits an economy where young mortgages are characterized by 20% equity rather than zero equity. Our prior, in other words, is that new mortgages account for much of the foreclosure flare-up in the model.

Importantly, this estimate of the role of new mortgages will provide a lower bound on their true contribution to the foreclosure crisis. Indeed, by contributing to the recent credit meltdown, they probably account for part of the recent collapse of housing demand. Our model (with $\alpha = 1$) abstracts from any effect of mortgage innovation on prices.

5 Mortgage innovation and the endogenous response of house prices ($\alpha < 1$)

So far, we have considered exogenous house prices. However, by enabling more agents to purchase houses, and enabling some agent to buy bigger houses than they would if only FRMs were available, mortgage innovation is bound to induce an endogenous response of prices. To quantify this effect, we need to relax the assumption that $\alpha = 1$ and re-calibrate things accordingly.

We let the degree $\alpha$ of strict concavity of the housing capital production function be 0.8. If one interprets the fixed factor inherent in our specification of the housing capital production function as land, this implies that land accounts for 20% of the cost of housing services. This value falls between those used by Davis and Heathcote (2005) and by Kahn (2008). We adjust
all other parameters accordingly to continue matching the same pre-2000 targets as before.

[TO BE COMPLETED]

6 Summary

In our model:

- default frequencies are more than twice as high on LIPs than they are on FRMs;
- LIPs account for nearly 60% of overall default rates, even though they only account for one third of all mortgages;
- a surprise housing price decrease of 25% causes default rates to more than double on impact – which is consistent with the magnitude of the increase in default rates since mid-2006. Default rates remain elevated for almost 10 years.

A Computational appendix

A.1 Steady State Equilibrium

1. The asset space consists of thirty equally spaced asset grid points between 0 and 0.3, thirty equally spaced asset grid points between 0.3 and 1.5, and another ninety equally spaced asset grid points between 1.5 and 30.

2. We use value function iteration to find $V_O(a)$ on the asset grid from which we obtain decision rules $a'_O(a)$ for old agents. The value functions are approximated by using linear interpolation.

3. Given the value functions for old agents, use value function iteration to find $V_M(a, y, 0, \cdot)$ on the asset grid from which we obtain decision rules $a'_M(a, y, 0, \cdot)$ for mid-aged renters for each $y$. The value functions are approximated using linear interpolation.

4. Given the value functions for old agents and mid-aged renters, use value function iteration to find $V_M(a, y, 1, h, n > T; \kappa)$ on the asset grid from which we obtain asset choice decision rules $a'_M(a, y, 1, h, n > T; \kappa)$ and homeownership decisions $H'(a, y, 1, n > T; \kappa)$ for mid-aged homeowners who have paid off their mortgage for each $(y, h)$. The value functions are independent of the original mortgage contract terms $\kappa$. The value functions are approximated using linear interpolation.

5. For every pair of $h_0$ and $(a_0, y_0)$, if a household does not have enough assets to make the downpayment, $\alpha q h_{o}$, no FRM contract will be offered. Set an initial guess of mortgage interest rate for each contract, given the value functions for old agent, mid-aged renters, and mid-aged homeowners with one less period of mortgage payments to

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make $V_M(a, y, 1, h, n = t + 1; \kappa)$, solve for $V_M(a, y, 1, h, n = t; \kappa)$ by backward induction for each $(y, h, t = \{1, \ldots, T\})$. For each path of possible realization of incomes and housing capital given $\kappa$, keep track the household decisions along the path. Calculate the present value according to the decision rules from each path and the probability of this path being realized. If this present value is not equal to the initial loan size, update the interest rate and repeat this step. Otherwise, the equilibrium interest rate is found. The value functions are approximated using linear interpolation.

6. Given the value functions for old and mid-aged agents, use value function iteration to find $V_Y(a, y)$ on the asset grid from which we obtain decision rules $a_Y(a, y)$ and contract selection decisions $(\zeta(a, y), h_0)$ on mortgage terms and initial housing capital. Because of the potential discontinuity caused by the downpayment requirements, the value functions for young agents are solved by grid search.

7. Solve for the equilibrium stationary distribution $\mu$ given the implied law of motion.

A.2 Transition Dynamics

1. Solve for the initial steady state equilibrium with price $q^0$ using the algorithm above with zero lump-sum tax.

2. Start the initial guess of lump-sum tax $\tau_{i=1} = 0$. Solve for the final steady state with a new house price $q^n$ with the lump-sum tax implemented.

3. Solve for the optimization problems for homeowners who have purchased the house before the unanticipated house price shock occurs by backward induction. If households choose to sell their houses, they sell at the new price $q^n$. If households choose to remain a homeowner, they have to follow the original mortgage terms (if they have not paid off their mortgage debts).

- If the agent is a homeowner but it is not budget feasible for her to make her mortgage payment $m^o(n; \kappa)$, which he obtained before the unanticipated price shock, or:

$$y + (a + i)(1 + r) - m^o(n; \kappa) - \delta h - \tau_i < 0,$$

(A.1)

then the value function solves:

$$V_M^o(a, y, 1, h, n; \kappa) = \max_{c, a'} U(c, h_1) + \beta E_{y'}[\delta(Y - \eta h'_1 - b_o(n; \kappa))].$$

- If it is budget feasible for a homeowner to make her mortgage payment then, if the household chooses to sell her house and become a renter (so that $H' = 0$), define
the value function by

\[ V^{o,H'}_{0}(a, y, 1, h, n; \kappa) = \max_{c,a'} U(c, h) + \beta E_{y'|y} [(1 - \rho_O)V^n_{M}(a', y', 0, \ldots) + \rho_O V^n_{O}(a') \] 

s.t. \( c + a' = y + (a + \iota)(1 + r) + \max\{q^n h - b^n(n; \kappa), 0\} - R^n h_1 - \tau_i. \)

- If the agent is able to meet her current mortgage payment and chooses to keep her house \((H' = 1)\), define the value function by

\[ V^{o,H' = 1}_{M}(a, y, 1, h, n; \kappa) = \max_{c,a'} U(c, h) \left[ 1, h + (1 - 1_{h = h_1}) \theta \right] 

+ \beta E_{y'|y} [(1 - \rho_O)V^n_{M}(a', y', 1, h', n + 1; \kappa) + \rho_O V^n_{O}(a' + \max\{q^n h - b^n(n + 1; \kappa), 0\}) \] 

s.t. \( c + a' = y + (a + \iota)(1 + r) - m^n(n; \kappa) - \delta h - \tau_i. \)

4. Select a large integer \( N \) to be the number of periods during transitions. In the first period of transitions, start the economy with the initial steady state distribution. Starting from the second period, apply the decision rules to solve for the distribution one period ahead. For renters, use the decision rules solved for the final steady state. For homeowners, if they purchase the house before the transition starts, use the optimization problems solved in the previous step. If they purchase the house after the transition starts, use the decision rules in the final steady state. Young agents who turn mid-aged during the transition purchase houses at the new price \( q^n \). Continue to solve for the distribution in every period of the transition path.

5. Given the decision rules and distribution along the transition path, calculate the discounted present value of the net profits for the financial intermediary over the transition path. Update the lump-sum tax \( \tau_{i+1} \) such that \( \frac{\tau_i}{r} \) is equal to the discounted present value of the net profits. Return to step 3 and repeat using \( \tau_{i+1} \) until the discounted present value of the net profits equals the discounted present value of the lump-sum tax.

6. Check if the distribution converges to the final steady state in \( N \) periods. If not, increase \( N \) and repeat all the steps above.
Bibliography


