Insuring College Failure Risk

Satyajit Chatterjee

*Federal Reserve Bank of Philadelphia*

Felicia Ionescu

*Colgate University*

Abstract

Under current law, participants in (college) student loan program must repay their loan in full regardless of whether they complete college. Dropout rate among college students from low-income background is anywhere between 33 to 50 percent. The combination of lack of family resources, unconstrained access to student loans and high dropout rates means that for a substantial fraction of low-income students the attempt to acquire a college degree ends in low earnings and high indebtedness. In this paper we examine whether the student loan program can gainfully offer insurance against college failure risk. We argue that such an insurance scheme is administratively feasible and provide conditions under which such insurance can be gainfully offered taking into account the constraints imposed by moral hazard. We show that the provision of such insurance will raise enrollment rates and failure rates. We calibrate the model to US data on college enrollment, early leaving and completion rates as well as the average indebtedness of program participants and quantify the effects on college enrollment, dropout rates, and welfare. Results suggest that no insurance can be offered early during college years, whereas up to 22 percent of college cost can be offered later in college. An insurance scheme that recognizes a fair amount of shirkers (1.8 percent) is optimal, inducing a welfare improvement of 2.7 percent relative to the case where no insurance is offered.
1 Introduction

Recent research in the education literature provides support for the fact that financial con-
straints during college-going years are not crucial for college enrollment (Carneiro and Heckman
(2002), Cameron and Taber (2001)). Rather, it is student’s characteristics such as learning
ability that determine their decision to enroll. Ionescu (2009) finds that given the generosity
of the student loan program, funds are ready available and eligible high school graduates
decide to invest in college if their returns to college education are high enough to pay for the
forgone earnings for college years.

This does not mean, however, that people’s college enrollment decisions are necessarily so-
cially optimal. There is considerable financial risk in taking out a student loan to pay for
college. For instance, using the 1990 PSID, Restuccia and Urrutia (2004) document that
50 percent of people who enroll in college drop out. Similarly, using the NCES data and
surveys, we find that 36.8 percent and 35.2 percent of students enrolled in 1989-90 and 1995-
96, respectively, do not possess a degree and are not enrolled five years after their initial
enrollment. For these people, the foregone earnings and out-of-pocket expenses incurred
while enrolled in college yield little or no financial return. This financial risk may discourage
some people from taking out a loan and enrolling in college. Thus, even though prospective
students do not appear to be credit constrained, a mechanism to share the college failure
risk might improve social welfare and encourage more people to enroll for college.\footnote{Given a
dramatic shift against unskilled labor in the U.S. labor market between 1980-1990 (decline in
wages, labor activity, longer unemployment spells for low skilled), Heckman (1999) has argued
that increasing the supply of college labor can reverse this trend. Specifically, for 1990 workforce
of 120 million, about 5.4 million would have to become college equivalents to reverse the
1980-1990 erosion of real wage, and about 1 million additional skilled persons would need to
be added to the workforce each year on top of the once and for all change of 5.4 million.}

The current operation of the student loan program suggests that it administratively feasible
to offer some insurance against college failure risk. Under the current system, a borrower can
choose from a menu of fairly sophisticated repayment options (standard, graduated, income-
contingent and extended repayment). Nevertheless, under each of these payment options,
the borrower is required to repay the entire loan and associated interest expenses regardless of whether he or she completed college.\footnote{The borrower is permitted to default on the loan only if a repayment effort over 25 years does not fully cover all obligations.}

The objective of this paper is to study, theoretically and quantitatively, whether the student loan program can gainfully offer insurance against college failure risk. We conduct this analysis under two important constraints on the provision of this sort of insurance. First, we assume that any proposed insurance scheme cannot redistribute resources from people with high probability of completing college to people with low probability of completing college (and vice versa). Formally, this requires that the insurance program be self-financing with respect to each person who chooses to participate. The current programs enforce this self-financing constraint regardless of whether the program participant actually graduates from college. In contrast, we permit dropouts to pay less than graduates but each participant pays the full cost of college in expectation. Second, we assume that the insurance program must guard against moral hazard, that is, the possibility that provision of insurance against college failure risk may increase the risk of failure. We derive conditions under which these constraints leave open the possibility of some insurance against college failure risk.

The quantitative section of the paper studies the impact of insurance against college failure risk on college enrollment decisions of individuals. We focus on the college enrollment decisions of individuals from low-income families – the ones for whom this sort of insurance is most relevant. These are individuals for whom the risk of college failure appears to be the highest currently.\footnote{See Manski (1992) and Stinebrickner and Stinebrickner (2007).} Furthermore, these are also the individuals for whom, conditional on dropping out of college, the burden of a student loan can be substantial. We calibrate the model to US data on college enrollment, early leaving and completion rates as well as the average indebtedness of program participants and quantify the effects on college enrollment, leaving, dropout rates, and welfare. Results suggest that no insurance can be offered early during college years, whereas insurance offered later in college is feasible. The indemnity
offered in case of failure increases in the ability level of students: the lowest ability group is forgiven 21.5 percent of their total college cost, whereas the highest ability group is forgiven 22.4 percent of the total college cost. We find that an insurance scheme that recognizes a fair amount of shirkers (1.8 percent) is optimal. This insurance scheme induces an increase in enrollment rates of 14 percentage points and an increase in completion rates of 1.5 percentage points. Even though the insurance scheme increases the risk of failing, fewer students drop out from college voluntarily, thus inducing a higher completion rate. The insurance scheme improves welfare by 2.7 percent relative to the case where no insurance is offered.

The rest of the paper is organized as follows: Section 2 presents the environment; Section 3 describes the theoretical results in the benchmark economy where no insurance is offered and Section 4 describes the results under the insurance scheme; the calibration procedure is given in Section 5; the quantitative results are described in Section 6; and Section 7 concludes.

2 Environment

Time is discrete and indexed by \( t = \{0, 1, 2, \ldots\} \). There is a unit measure of individuals indexed by ability \( a \in [0, 1] \). We think of \( a \) as an observable signal of college preparation such as SAT scores and high school grades. The distribution of ability in the population has CDF \( A(a) \). Periods 0 and 1 will correspond years of college education, where for simplicity we assume (in the theory) that to acquire a college degree takes 2 years.

In period 0, all individuals make a one-time decision to enroll in college or not. If they do not enroll, they work in some low-paid job with disutility of effort \( \theta \geq 0 \) and starting in period 1 earn \( y_N \geq 0 \). We assume that \( \theta \) is drawn independently from a distribution \( F(\theta) \) and that \( y_N \) is drawn independently from a distribution \( H(y_N) \). At the time of the enrollment decision, people know their \( \theta \) but not the realization of their income in period 1 onward if they chose to not enroll.

If the individual chooses to enroll in college in period 0, she gets to learn the disutility of
making effort in college. For simplicity we assume that effort, \( e \), is a binary choice variable taking the values 0 (no effort) or 1 (effort). We assume that effort put in college is not directly observable to others\(^4\). The disutility of making effort in college is denoted \( \gamma \) and each student independently draws \( \gamma \geq 0 \) from a distribution \( G(\gamma|a) \). After she learns \( \gamma \), the individual decides whether to continue in college or not. If she chooses to leave, she incurs the disutility of effort at the low-paid job \( \theta \) and starting in period 1 earns \( y_N \). She incurs some (low) college expenses \( x \). Again, at the time of making the decision to continue in college, she know \( \gamma \) and \( \theta \) but not her earnings starting in period 1.

If the student continues in college she incurs annual college expenses of \( x > \underline{x} \). A continuing student must choose between putting in effort or not. If she chooses to shirk, we assume that she fails with probability 1 but does not incur any disutility cost in period 0 and starts life in period 1 with an earnings draw \( y_N \) from the distribution \( H(y_N) \) and debt of \( x \). If the student chooses to put in effort \( (e = 1) \), she may fail to complete her first year in college successfully with probability \( 1 - \pi(a) \), in which case she starts period 1 with an earnings draw \( y_N \) from \( H(y_N) \) and debt \( x \). We assume that \( \pi(\cdot) \) is an increasing and continuous function of ability \( a \) with \( \pi(1) = 1 \) and \( \pi(0) = 0 \). If she completes her first year successfully (with probability \( \pi(a) \)) she begins period 1 as a college student with one more year to go and debt of (no interest accumulates on the debt as long the student continues in college).

Figure 1 illustrates this timing of decisions in the model. For exposition purposes, we present a simplified picture for the decisions in period 0: leaving or continuing college, shirking or putting in effort in college. In the case students succeed in period 0 in college, they face a similar decision during period 1, which we describe next. Also, to simplify notations, we suppress the dependence on \( a \) of the probability of success from college, \( \pi \).

In period 1, a student with one more year to go has to choose again whether to continue in college or not. If she does not continue, she incurs the disutility \( \theta \) of working in a low paid job and starts period 2 with debt \( x \) and an earnings draw \( y_N \) from the distribution \( H(y_N) \).

\(^4\)Clearly this is a simplification because grades provide some signal of effort. However grades are not a perfect signal of effort because a student may work hard and yet do poorly in the college.
If she continues, she incurs another year of college expense $5x/4$. And, as in period 0, she a
continuing student must choose between putting effort or shirk. If she shirks, she fails with
probability 1 but does not incur any disutility cost in period 1 and starts life in period 2
with an earnings draw $y_N$ from the distribution $H(y_N)$ and debt of $2x$. If the student chooses
to put in effort, she completes college with probability $\pi$, in which case the student starts
period 2 with debt of $2x$ and earnings $y_C$ drawn from a distribution $M(y_C)$, where $M(y_C)$
first order stochastically dominates the distribution $H(y_N)$. If the student chooses to put in
effort but fails to complete college (with probability $1 - \pi$), the student starts period 2 with
an earnings draw $y_N$ from $H(y_N)$ and debt of $2x$.

In order to describe individuals decision problems in period 0 and 1 (these are the only
periods in which there are decisions to be made), we will start with describing the utility
(payoffs) to students at the start of period 1 (students that have one more year of college to
go).
1. A student who drops out gets

\[ V_1^N(x) = \int U(y_N - 5x/4)dH(y_N). \]

2. A student who continues but shirks gets

\[ V_1^S(x) = \beta \int U(y_N - 2x)dH(y_N). \]

3. A student who continues and puts in effort gets

\[ V_1^E(\pi, x, \gamma) = -\gamma + \beta \left[ \pi \int U(y_C - 2x)dM(y_C) + (1 - \pi) \int U(y_N - 2x)dH(y_N) \right]. \]

Turning to period 0, the payoffs are as follows

1. Individuals who do not enroll get

\[ W(\theta) = -\theta + \beta \int U(y_N)dH(y_N). \]

2. Individuals who enroll but leave get

\[ V_0^N(x, \theta) = -\theta + \int U(y_N - x/4)dH(y_N). \]

3. Individuals who enroll, do not leave and shirk get

\[ V_0^S(x, \theta) = -\beta \theta + \beta \int U(y_N - x)dH(y_N). \]

4. Individuals who enroll, do not leave and put in the effort get

\[ V_0^E(\pi, x, \gamma) = -\gamma + \beta \left[ \pi \max[V_1^E(\pi, x, \gamma), V_1^S(x), V_1^N(x)] + (1 - \pi) \int U(y_N - x)dH(y_N) \right]. \]
There is a social planner (student loan program administrator) who enforce intertemporal contracts at zero cost. We make the following assumptions:

**Assumption 1:** \( \lim_{c \to 0} U(c) = -\infty \) and \( \lim_{c \to 0} U'(c) = \infty \).

**Assumption 2:** \( \beta^2 \int U(y_C - 2x) dM(y_C) > \int U(y_N) dH(y_N) \) (college degree is profitable financial investment)

**Assumption 3:** The distribution \( M(y_C) \) dominates the distribution \( H(y_N) \) in the first-order stochastic sense.

**Assumption 4:** \( \int U(y_N - 5x/4) dH(y_N) > \beta \int U(y_N - 2x) dH(y_N) \) (the discount factor \( \beta \) is sufficiently close to 1).

### 3 College Enrollment and Failure Under the Current System

As noted in the introduction under the current system any student wishing to enroll for college can obtain financing to do so. If they leave college at the beginning of period 0, they incur some low college cost, \( x \). If they avail themselves of the financing, but they do not successfully complete their first year in college, they will owe \( x \) starting in period 1. In case students successfully complete their first year in college, they will owe \( x \) starting in period 2, in case they choose to leave at the beginning of the second year in college. In case they choose to continue they will owe \( 2x \) starting in period 2, regardless of whether they acquire a college degree.

Students in our model have to make 3 sets of choices in period 0. First, they must choose to work or enroll in college. Second, those who choose to enroll learn \( \gamma \) and then must choose to continue or leave. Third, those who choose to continue must choose to exert effort in college or not. In period 1, students face 2 sets of choices. First, they must choose to continue or leave. Second, those who choose to continue must choose to exert effort in college or not.

We start by studying the choices in period 1. Then, the following result shows that if the
effort required in college is too high, students decide to drop out.

**Proposition 3.1.** In period 1, there is a cut-off \( \gamma_1(x, \pi) \) such that for \( \gamma > \gamma_1(x, \pi) \), students drop out and for \( \gamma \leq \gamma_1(x, \pi) \), they continue with effort. Furthermore, \( \gamma_1(x, \pi) \) is increasing in \( \pi \).

**Proof.** By Assumption 4, dropping out is better than shirking in period 1. Therefore, the student needs to choose between continuing with effort or dropping out. Consider

\[
V^E_1(x, \pi, \gamma) - V^N_1(x) = -\gamma + \beta \pi \left[ \int U(y_C - 2x) dM(y_C) - \int U(y_N - 2x) dH(y_N) \right] + \beta \int U(y_N - 2x) dH(y_N) - \int U(y_N - 5x/4) dH(y_N).
\]

Since \( \pi(0) = 0 \) and \( \pi(1) = 1 \), it follows from Assumption 2 that \( V^E_1(x, 0, 0) - V^N_1(x) < 0 \) and from Assumptions 3 and 4 that \( V^E_1(x, 1, 0) - V^N_1(x) > 0 \). Thus, when no effort in school is required, \( \gamma = 0 \), there exists \( \gamma_1 \) such that \( V^E_1(x, \pi, 0) - V^N_1(x) = 0 \) and for all \( \pi > \gamma_1 \), \( V^E_1(x, \pi, 0) - V^N_1(x) > 0 \). Since \( V^E_1(x, \pi, \gamma) \) is decreasing in \( \gamma \) and \( \gamma \) is unbounded it follows that for all \( \pi > \gamma_1 \), there exists a \( \gamma_1(x, \pi) > 0 \) such that \( V^E_1(x, \pi, \gamma_1(x, \pi)) - V^N_1(x) = 0 \). This threshold is given by

\[
\beta \pi \left[ \int U(y_C - 2x) dM(y_C) - \int U(y_N - 2x) dH(y_N) \right] + \beta \int U(y_N - 2x) dH(y_N) - \int U(y_N - 5x/4) dH(y_N),
\]

which is increasing in \( \pi \). Furthermore, it follows that for all \( \pi \leq \gamma_1(x, \pi) \) students who enroll in college will leave in period 1 for any realization of \( \gamma \).

Next, we study the choices in period 0. The following two results determine the decisions to drop, to shirk, and to put in effort for students who decide to go to college.

**Proposition 3.2.** In period 0, there exists a cut-off \( \overline{\theta}_0(x) > 0 \) such that dropping out gives the same utility as shirking. For any \( \theta > \overline{\theta}_0(x) \) shirking is strictly preferred to dropping out.
Proof. Consider the function $V_0^N(x, \theta) - V_0^S(x, \theta) = -\theta(1 - \beta) + \int U(y_N - x/4) dH(y_N) - \beta \int U(y_N - x) dH(y_N)$, which is decreasing in $\theta$. We have that $V_0^N(x, 0) - V_0^S(x, 0) = \int U(y_N - x/4) dH(y_N) - \beta \int U(y_N - x) dH(y_N) > 0$. Since $\theta$ has unbounded support, it follows that there exists $\overline{\theta}_0(x) > 0$ such that $V_0^N(x, \overline{\theta}_0(x)) = V_0^S(x, \overline{\theta}_0(x))$. For any effort above this cut-off, shirking is preferred to dropping out.

Proposition [3.2] shows that students have the incentive to shirk in school when the disutility from effort in the low-paid job is sufficiently high. If working is sufficiently unpleasant people would rather spend some time in college even without putting in any effort and thus guaranteeing failure from college.

**Proposition 3.3.** In period 0, there is locus $\overline{\gamma}_0(x, \pi, \theta)$ such that for $\gamma \leq \overline{\gamma}_0(x, \pi, \theta)$, students put in effort in period 0. Furthermore, $\overline{\gamma}_0(x, \pi, \theta)$ is increasing in $\pi$ and is decreasing in $\theta$.

Proof. We consider the function $V_0(x, \pi, \gamma, \theta) = V_0^E(x, \pi, \gamma, \theta) - \max[V_0^N(x, \theta), V_0^S(x, \theta)]$. By Propositions [3.2] the function $V_0(x, \pi, \gamma, \theta)$ is given by $V_0^E(x, \pi, \gamma, \theta) - V_0^N(x, \theta)$, if $\theta \leq \overline{\theta}_0(x)$ and $V_0^E(x, \pi, \gamma, \theta) - V_0^S(x, \theta)$, if $\theta > \overline{\theta}_0(x)$. We will consider each of these cases.

Case 1: $\theta \leq \overline{\theta}_0(x)$

$$V_0(x, \pi, \gamma, \theta) = -\gamma + \beta[\pi \max[V_1^E(x, \pi, \gamma), V_1^N(x)] + (1 - \pi) \int U(y_N - x) dH(y_N)] - V_0^N(x, \theta)$$

At $\gamma = 0$, $V_0(x, \pi, 0, \theta) = \beta \pi V_1^E(x, \pi, \gamma) + (1 - \pi) \int U(y_N - x) dH(y_N) - V_0^N(x, \theta)$. At $\pi = 0$, $V_0(x, 0, 0, \theta) = \beta \int U(y_N - x) dH(y_N) + \theta - \int U(y_N - x/4) dH(y_N) < 0$ for low enough $\theta$. At $\pi = 1$, $V_0(x, 1, 0, \theta) = \beta^2 \int U(y_C - 2x) dM(y_C) + \theta - \int U(y_N - x/4) dH(y_N) > 1$. Thus, when no effort in school is required, $\gamma = 0$, there exists $\overline{\pi}_0$ such that $V_0^E(x, \overline{\pi}_0, 0) - V_0^N(x, \theta) = 0$ and for all $\pi > \overline{\pi}_0$, $V_0^E(x, \pi, 0) - V_0^N(x, \theta) > 0$. Since $V_0^E(x, \pi, \gamma)$ is decreasing in $\gamma$ and $\gamma$ is unbounded it follows that for all $\pi > \overline{\pi}_0$, there exists a $\overline{\gamma}_0(x, \pi, \theta) > 0$ such that $V_0^E(x, \pi, \overline{\gamma}_0(x, \pi, \theta)) - V_0^N(x, \theta) = 0$.

Note that this threshold is increasing in $\pi$: let $\overline{\pi}_0 < \pi_1 < \pi_2$. It follows that $0 = V_0(x, \pi_1, \overline{\gamma}_0(x, \pi_1, \theta), \theta) < V_0(x, \pi_2, \overline{\gamma}_0(x, \pi_1, \theta), \theta)$. Since $V_0(x, \pi, \gamma(x, \pi, \theta), \theta)$ is decreasing
in $\gamma$, it results that $\overline{\gamma}_0(x, \pi_1, \theta) < \overline{\gamma}_0(x, \pi_2, \theta)$. Furthermore, it follows that for all $\pi \leq \overline{\pi}_0$ students who enroll in college will leave in period 0 for any realization of $\gamma$. In addition, the threshold $\overline{\gamma}_0(x, \pi, \theta)$ is decreasing in $\theta$, which follows from the fact that $V_0(x, \pi, \gamma(x, \pi, \theta), \theta)$ is decreasing in $\theta$ and in $\gamma$.

Next, we evaluate the function $V_0(x, \pi, \gamma, \theta)$ at $\gamma = \overline{\gamma}_1(x, \pi)$ to determine the relation between the effort level that induces students to put effort in period 0, $\overline{\gamma}_0(x, \pi, \theta)$, and the effort level that induces students to put effort in period 1, $\overline{\gamma}_1(x, \pi)$:

$$V_0(x, \pi, \overline{\gamma}_1(x, \pi), \theta) = -\overline{\gamma}_1(x, \pi) + \beta \int U(y_N - 5x/4)dH(y_N) + \theta - \int U(y_N - x/4)dH(y_N) < 0$$

for small enough $\theta$. This implies that $\overline{\gamma}_0(x, \pi, \theta) < \overline{\gamma}_1(x, \pi)$. It remains to determine the threshold $\overline{\gamma}_0(x, \pi, \theta)$.

**Case 2:** $\theta > \overline{\theta}_0(x)$

$$V_0(x, \pi, \gamma, \theta) = -\gamma + \beta[\pi \max[V_1^E(x, \pi, \gamma), V_1^N(x)] + (1 - \pi) \int U(y_N - x)dH(y_N)] - V_0^S(x, \theta).$$

At $\gamma = 0$, $V_0(x, \pi, 0, \theta) = \beta \pi V_1^E(x, \pi, \gamma) + (1 - \pi) \int U(y_N - x)dH(y_N)] - V_0^S(x, \theta)$. At $\pi = 0$, $V_0(x, 0, 0, \theta) = \beta \int U(y_N - x)dH(y_N) + \beta \theta - \beta \int U(y_N - x)dH(y_N) = \beta \theta > 0$. Thus, when no effort in school is required, $\gamma = 0$, for all $\pi \geq 0$, $V_0^E(x, \pi, 0) - V_0^S(x, \theta) > 0$. Since $V_0^E(x, \pi, \gamma)$ is decreasing in $\gamma$ and $\gamma$ is unbounded it follows that there exists a $\overline{\gamma}_0(x, \pi, \theta) > 0$ such that $V_0^E(x, \pi, \overline{\gamma}_0(x, \pi, \theta)) - V_0^S(x, \theta) = 0$.

As in the previous case, we evaluate the function $V_0(x, \pi, \gamma, \theta)$ at $\gamma = \overline{\gamma}_1(x, \pi)$ to determine the relation between the effort level that induces students to put effort in period 0, $\overline{\gamma}_0(x, \pi, \theta)$, and the effort level that induces students to put effort in period 1, $\overline{\gamma}_1(x, \pi)$:

$$V_0(x, \pi, \overline{\gamma}_1(x, \pi), \theta) = -\overline{\gamma}_1(x, \pi) + \beta \int U(y_N - 5x/4)dH(y_N) + \beta \theta - \beta \int U(y_N - x)dH(y_N).$$

Note that $V_0(x, \pi, \overline{\gamma}_1(x, \pi), \theta) \approx -\overline{\gamma}_1(x, \pi) + \beta \theta$. This implies that for $\theta \approx \overline{\gamma}_1(x, \pi)/\beta$, $\overline{\gamma}_0(x, \pi, \theta) = \overline{\gamma}_1(x, \pi)$, for $\theta < \overline{\gamma}_1(x, \pi)/\beta$, $\overline{\gamma}_0(x, \pi, \theta) < \overline{\gamma}_1(x, \pi)$, and for $\theta > \overline{\gamma}_1(x, \pi)/\beta$, $\overline{\gamma}_0(x, \pi, \theta) > \overline{\gamma}_1(x, \pi)$. As before, this threshold is increasing in $\pi$ and is decreasing in $\theta$, which follow from the fact that $V_0^E(x, \pi, \gamma)$ is increasing in $\pi$ and is decreasing in $\gamma$ and
$V_0^S(x, \theta)$ is decreasing in $\theta$. \hfill \Box

These Propositions can be conveniently seen in Figures 2. Consider an ability level, $a$ with the debt level, $x(a)$ and the probability of success, $\pi(a)$. The left (right) figure presents the choices that the student of ability $a$ makes in period 0 (period 1) in terms of the effort levels required on the job, $\theta$ and the effort level required in college, $\gamma$.

**Figure 2: Choices in periods 0 and 1**

Next we will determine who enrolls in college. Observe that since enrolling in college and then leaving gives people about the same utility as working, there is a very small cost to a student to enroll in college and learn her $\gamma$. However, if the student’s probability of success is sufficiently low, she may choose not to enroll because regardless of the value of $\gamma$ she will find it optimal to leave rather than continue with college. Similarly, for a student of a given probability of success, if the effort in the low paid job is sufficiently high, she may choose not to enroll. We make the following assumption:

**Assumption 5:** The highest ability student, $a = 1$ (with probability of success, $\pi(1) = 1$) enrolls in college for any $\theta \geq 0$, while the lowest ability student, $a = 0$ (with probability of success, $\pi(0) = 0$), does not enroll in college unless $\theta \to \infty$ (college is better for the highest ability student even if zero effort on the job is required, whereas working is better for the lowest ability student, even if the job requires a very high effort level).

The following proposition gives the cut-off value of effort required on the job that makes the student indifferent between working and enrolling in college. For every effort less than that,
the student strictly prefers not to enroll.

**Proposition 3.4.** In period 0, there exists a cut-off \( \overline{\theta}_c(x, \pi) > 0 \) such that working gives the same utility as enrolling. For any \( \theta > \overline{\theta}_c(x, \pi) \) enrolling is strictly preferred to working. Furthermore, \( \overline{\theta}_c(x, \pi) \) is decreasing in \( \pi \).

**Proof.** We consider the function \( V_C(x, \pi, \theta) = \max[V_0^E(x, \pi, \gamma), V_0^N(x, \theta), V_0^S(x, \theta)] - W(\theta) \) evaluated at \( \theta = 0 \). Note that \( V_0^N(x, 0) > V_0^S(x, 0) \). Thus, \( V_C(x, \pi, 0) = \int_{\overline{\gamma}}{V_0^E(x, \pi, \gamma)dG(\gamma)} + \int_{\gamma_0}^{\infty}{V_0^N(x, 0)dG(\gamma)} - W(0) \). Also, by Proposition 3.3 it follows that \( \overline{\gamma}_0(x, \pi, 0) < \overline{\gamma}_1(x, \pi) \), which implies that effort is preferred to dropping out in period 1. Thus,

\[
V_C(x, \pi, 0) = \int_{\overline{\gamma}_0}^{0} \{-(1 + \beta\pi)\gamma + \beta\pi[\beta\pi \int U(yC - 2x)dM(yC) + \beta(1 - \pi) \int U(yN - 2x)dH(yN)] \\
\quad + \beta(1 - \pi) \int U(yN - x)dH(yN)\}dG(\gamma) \\
\quad + \int_{\overline{\gamma}_0}^{\infty} -0 + \int U(yN - x/4)dH(yN)dG(\gamma) \\
\quad + 0 - \beta \int U(yN)dH(yN).
\]

We rewrite this function as:

\[
V_C(x, \pi, 0) = -(1 + \beta\pi)E(\gamma_{[0, \overline{\gamma}_0]}) + G(\gamma \leq \overline{\gamma}_0)[\beta^2\pi^2 \int U(yC - 2x)dM(yC) \\
\quad + \beta^2\pi(1 - \pi) \int U(yN - 2x)dH(yN)] + \beta(1 - \pi) \int U(yN - x)dH(yN)] \\
\quad + G(\gamma > \overline{\gamma}_0) \int U(yN - x/4)dH(yN) - \beta \int U(yN)dH(yN).
\]

First, we have that

\[
G(\gamma > \overline{\gamma}_0) \int U(yN - x/4)dH(yN) < \beta G(\gamma > \overline{\gamma}_0) \int U(yN - x/4)dH(yN)
\] (1)
Also,
\[ \beta \int U(y_N - x)dH(y_N) \leq \beta \int U(y_N - x/4)dH(y_N) \quad (2) \]

Conditions 1 and 2 imply that
\[ G(\gamma < \bar{\gamma}_0) \beta \int U(y_N - x)dH(y_N) + G(\gamma > \bar{\gamma}_0) \int U(y_N - x)dH(y_N) \]
\[ < G(\gamma < \bar{\gamma}_0) \beta \int U(y_N - x/4)dH(y_N) + G(\gamma > \bar{\gamma}_0) \beta \int U(y_N - x/4)dH(y_N) \]
\[ = \beta \int U(y_N - x/4)dH(y_N). \]

But, we have that \( -\beta \int U(y_N - x)dH(y_N) + \beta \int U(y_N - x/4)dH(y_N) < 0. \) Thus, it follows
that \( V_C(x, \pi, 0) < 0. \) Since \( V_C(x, \pi, \theta) \) is continuous and increasing in \( \theta \) and \( \theta \) has unbounded support, it follows that there exists \( \bar{\theta}_c(x, \pi) > 0 \) such that \( V_C(x, \pi, \bar{\theta}_c(x, \pi)) = 0. \) For any \( \theta \)
above this threshold, the student with probability of success \( \pi \) does not enroll in college
and for any \( \theta \) below this threshold she enrolls in college.

The threshold is decreasing in \( \pi \): let \( \pi_1 < \pi_2. \) It follows that \( V_C(x, \pi_2, \bar{\theta}_c(x, \pi_2)) = 0 = V_C(x, \pi_1, \bar{\theta}_c(x, \pi_1)) < V_C(x, \pi_2, \bar{\theta}_c(x, \pi_1)). \) Since \( V_C(x, \pi, \theta) \) is decreasing in \( \theta, \) it results that
\( \bar{\theta}_c(x, \pi_2) < \bar{\theta}_c(x, \pi_1). \)

Figure 3 sums-up the benchmark economy for an individual of ability \( a \) given all the
choices he faces in the model in terms of the two types of effort, \( \theta \) and \( \gamma(a). \)

Next, we study how the cut-offs illustrated above change with the debt level, \( x, \) and the
probability of success in college, \( \pi. \) The purpose is to show that the model is consistent with
the basic qualitative patterns in the data with regarding enrollment, dropout and completion
across ability groups. Proposition 3.4 delivers that the threshold of effort on the job which
induces the enrollment decision, \( \bar{\theta}(x, \pi) \) is decreasing in the probability of success \( \pi. \) But, by
assumption, \( \pi(a) \) is increasing in \( a. \) Thus, for high ability people college is attractive even
Figure 3: Choices in college

![Diagram showing choices in college enrollment.]

If the effort required on job is very low. For low ability people, however, a higher effort on the job is needed to trigger college enrollment. With \([0, \infty)\), the support for \(F(\theta)\), this result implies that high ability people are more likely to enroll in college. Results also show that students with low ability levels have less incentive to put in effort in college. They are more likely to shirk or drop out in period 0 or period 1.

Even though this model of college enrollment and college completion is very simple it is consistent with several important facts. First, it predicts that not every student will enroll in college. We know that students with very low ability are less likely to enroll in college. This is consistent with the empirical fact that enrollments rates are increasing in ability. Second, among those who enroll some will leave college voluntarily or shirk in period 0. These are the students who discover that their disutility from putting in effort in college is higher than \(\gamma_0(x, \pi(a), \theta)\) where \(a\) is their ability level and \(\theta\) the effort required on job. Third, there will be students who continue on in college (and put in effort) in period 0, but fail to graduate, with probability \(1 - \pi(a)\). Fourth, among students who successfully complete period 0 in college, some will leave college voluntarily in period 1. These are the students who discover that their disutility from putting in effort in college is higher than \(\gamma_1(x, \pi(a))\). Fifth, there will be students who continue on in college (and put in effort) in period 1, but fail to graduate, with probability \(1 - \pi(a)\). Thus, dropouts from college include both people...
who leave voluntarily and those who fail. Finally there are students who enroll in college and complete their degrees.

For each ability level \( a \) define the \textit{dropout rate} as the sum of the fraction of students who enroll in college but leave and the fraction that continue in college but fail, i.e.,

\[
d(a) = [1 - G(\bar{g}_0(a, \theta))] + G(\bar{g}_0(a, \theta))\pi(a)[1 - G(\bar{g}_1(a))] + G(\bar{g}_1(a))[1 - \pi(a)].
\]

Then we have the following result.

**Proposition 3.5.** The dropout rate \( d(a) \) is decreasing in \( a \)

**Proof.** Follows from Propositions 3.2 and 3.1 which established that \( \bar{g}_0(x, \pi, \theta) \) and \( \bar{g}_1(x, \pi) \) are increasing in \( \pi \) and the assumption that \( \pi(a) \) is increasing in \( a \).

The above Proposition sheds light on why the dropout rate is high among students from low-income families. There is some evidence (Carneiro and Heckman and Cameron and Taber) that students from low-income families are (on average) less well-prepared for college than students from high-income families. In the context of our model this means that students from low-income background tend to have lower \( a \). As shown in Proposition 3.5, the dropout rate is decreasing in \( a \).

The logic of the model also suggests why the dropout rate is actually quite high. Observe that for a person of ability \( a \)

\[
V_0^E(x, \pi(a), E(\gamma)) - W(\theta) \approx \theta - E(\gamma)(1 + \beta \pi(a))
\]

\[
+ \pi^2(a)\beta^2[\int u(y_C - 2x)dM - \int u(y_N)dH]
\]

\[
+ (1 - \pi(a))\beta \pi(a)\beta[\int u(y_N - x)dH + \int u(y_N - x)dH - \int u(y_N)dH].
\]

The expression is satisfied with equality for \( \beta = 1 \). The first term on the right hand side represents the (net) benefit of enrolling in college that stems from avoiding work in a low-
paid unpleasant job. The second term is the expected future benefit of acquiring a college degree. The third is the expected future cost of failing to acquire a degree after having paid for it. This consists in two terms, representing failure that occurs in the first year in college and in the second year, respectively. For students with high \( a \) (and therefore low \( 1 - \pi(a) \)) the expected cost of failure is negligible and enrolling in college is a “win-win” proposition: there is both a current and a future benefit from doing so. Students continue to enroll for college until the expected future cost of failure is high enough to balance the current and future expected benefits when \( \gamma = 0 \).

Next we show that people with high ability levels earn more, which is consistent with empirical findings.

**Proposition 3.6.** The earnings are increasing in \( a \)

*Proof.* Follows from Proposition 3.4 which established that enrollment rates are increasing in \( a \), Proposition 3.5 which delivered that dropout rates are decreasing in \( a \) and the assumption that the distribution \( M(y_C) \) first order stochastically dominates the distribution \( H(y_N) \).

## 4 Insuring College Failure Risk

Can the student loan program gainfully offer insurance against college failure risk? As noted in the introduction, we wish to answer this question recognizing that the student loan program cannot redistribute resources from observably high ability students (high \( a \)) to low ability students and recognizing that insurance against college failure may encourage shirking (and therefore failure).

It is best to break up the answer in two parts. Consider first the nature of the insurance when loan administrators can observe effort so the moral hazard constraint is not an issue. Also, we will consider the optimal insurance in the second period. Conditional on the student having put in effort, the student loan program gives a transfer \( f_1 \) to a student with ability \( a \) if she fails college and collects a premium \( s_1 \) if she completes college. Since the insurance is
required to be self-financing (no cross-subsidies) we must have

\[-\pi \cdot s_1 + (1 - \pi) \cdot f_1 = 0. \quad (4)\]

Ignoring the \(-\gamma\) term, expected utility given these transfer is then

\[\pi \cdot \int U(y_C - 2x - [(1 - \pi)/\pi] f_1) dM + (1 - \pi) \cdot \int U(y_N - 2x + f_1) dH, \quad (5)\]

where we have used equation (4) to express \(s_1\) in terms of \(f_1\). Maximizing the above expression with respect to \(f_1\) yields the following first-order condition:

\[\int U'(y_C - 2x - [(1 - \pi)/\pi] f_1) dM = \int U'(y_N - 2x + f_1) dH. \]

Hence the value of \(f_1\) that attains the maximum is one that equalizes the marginal utility of consumption following failure and success. Denote this value of \(f_1\) by \(f_1^*\).

When effort is not observable, however, generally this insurance cannot be offered. The reason being that it attenuates the incentive to put in effort in college. The easiest way to see this to assume that \(u(c) = -exp^{-Bc}\). For this utility function, total utility is proportional to marginal utility. Hence, optimal insurance when effort is observable implies

\[\int U(y_C - 2x - s_1^*) dM = \int U(y_N - 2x + f_1^*) dH. \quad (6)\]

Therefore \(V_1^E(x, \pi, \gamma) - V_1^S(x) = -\gamma + \beta \pi [\int U(y_C - 2x - s_1^*) dM - \int U(y_N - 2x + f_1^*) dH]\) is simply \(-\gamma\). Hence for any \(\gamma > 0\), a student will have a strict incentive to shirk. Since failure will then happen with probability 1 the resource balance condition (4) will be violated. Thus it is not feasible to offer this sort of insurance when effort is not observable. This logic carries over when utility is not exponential.

How much insurance can be offered against risk of failure in the second period of college? We can show that as long as insurance is offered at the price \(\pi/(1 - \pi)\), providing as much
insurance (up to $f_1^*$) as possible improves welfare. Without any insurance a student’s next best option in the second period is leaving rather than shirking because (as long as $\beta$ is not too different from 1) we have that $V_1^N(x) > V_1^S(x)$. As long as insurance does not make shirking better than leaving, it is optimal to offer the maximum amount of insurance consistent with this requirement. So, this amount of insurance is

$$\int U(y_N - x)dH(y_N) = \beta \int U(y_N - 2x + \bar{f}_1)dH(y_N) \quad (7)$$

However, we do not know for sure whether $f_1^*$ is bigger or smaller than $\bar{f}_1$. So, we will say that the max insurance that can be offered is $\min\{f_1^*, \bar{f}_1\}$.

We will pose the general insurance problem in the following way. Insurance against failure in college will be offered in both period 0 and period 1. We will denote the indemnity (in payment received in the event of failure in period 0) as $f_0$ and the payment in case of success as $s_0$. Similarly for period 1 – the indemnity is $f_1$ and the premium is $s_1$. We will assume that students who succeed pay their premium when they leave college. Given this arrangement, the value from being a student in good standing in period 1 is now a function of $s_0$. Thus we have (ignoring $a$):

1. A student who drops out gets

$$V_1^N(x, s_0) = \int U(y_N - 5x/4 - s_0)dH(y_N).$$

2. A student who continues but shirks gets

$$V_1^S(x, s_0, f_1) = \beta \int U(y_N - 2x - s_0 + f_1)dH(y_N).$$
3. A student who continues and puts in effort gets

\[ V_1^E(x, \pi, \gamma, s_0, s_1, f_1) = -\gamma + \beta \pi \int U(y_C - 2x - s_0 - s_1) dM(y_C) + (1 - \pi) \int U(y_N - 2x - s_0 + f_1) dH(y_N). \]

Turning to period 0, the payoffs are as follows

1. Individuals who do not enroll get

\[ W(\theta) = -\theta + \beta \int U(y_N) dH(y_N). \]

2. Individuals who enroll but leave get

\[ V_0^N(x, \theta) = -\theta + \int U(y_N - x/4) dH(y_N). \]

3. Individuals who enroll, do not leave and shirk get

\[ V_0^S(x, \theta, f_0) = \beta(f_0 - \theta) + \beta \int U(y_N - x) dH(y_N). \]

4. Individuals who enroll, do not leave and put in the effort get

\[ V_0^E(\pi, x, \gamma, f_0, s_0, s_1) = -\gamma + \beta \max[V_1^E(\pi, x, \gamma, s_0, s_1, f_1), V_1^S(s_0, f_1), V_1^N(s_0)] + (1 - \pi) \int U(y_N - x + f_0) dH(y_N)]. \]

The above equations describe the payoffs from the various actions when we have the possibility of insurance against failure, recognizing that program administrators cannot tell the difference between genuine failures and those who fake failure by shirking. In what follows we will consider insurance schemes that do not induce shirking. In other words, we will consider
optimal insurance subject to the no-shirking constraint. If such an insurance scheme exists then the actuarially fair insurance is possible because failures will occur with probability $1 - \pi$. With this in mind, the optimal insurance program can be stated as:

$$\max_{\{s_0, f_0, s_1, f_1\}} \int_{\theta} \left[ \int_{\gamma} \max \{W(\pi, x, \theta, \gamma, s_0, f_0, s_1, f_1), W(\theta)\} dG(\gamma) \right] dF(\theta)$$  \hspace{1cm} (8)

subject to:

$$V^N_0(x, \theta) - V^S_0(x, f_0) \geq 0 \text{ for all } \theta$$

$$V^N_1(x, s_0) - V^S_1(x, s_0, f_1) \geq 0$$

$$s_0 \pi - f_0(1 - \pi) = 0$$

$$s_1 \pi - f_1(1 - \pi) = 0$$

Our first result about the optimal insurance scheme is that no insurance against failure can be provided in the first period.

**Proposition 4.1.** In period 0, $f_0 = s_0 = 0$.

**Proof.** Consider the incentive constraint in the first period. This constraint requires that

$$-\theta(1 - \beta)/\beta - f_0 + \left[ \int U(y_N - x) dH(y_N) - \beta \int U(y_N - x) dH(y_N) \right] \geq 0$$

Assume that the term in square brackets is strictly positive (which requires that $\beta$ be sufficiently close to 1). Then, for any $f_0 > 0$ there exists a $\theta(f_0)$ such that inequality holds exactly. The distribution $F(\theta)$ has unbounded support. Then for all $\theta > \theta(f_0)$ the constraint is violated. Thus it is not possible to offer any insurance in the first period. \hfill \Box

We turn now to the insurance in period 1. Here we are assuming that $\int U(y_N -$
We have the following result:

**Proposition 4.2.** In period 1, the maximum amount of insurance that can be offered without inducing shirking is the value $f_1$, denoted, $\bar{f}_1$, that satisfies

$$\int U(y_N - x)dH(y_N) - \beta \int (y_N - 2x)dH(y_N) > 0.$$  

**Proof.** First note that with this amount of insurance, people are indifferent between shirking and dropping out. Furthermore, if we offer more insurance that $f_1$, then the person who is indifferent between dropping out and putting in effort in college will be induced to shirk. This is because the gain from insurance when putting effort comes only with failure whereas the gain from insurance when shirking occurs with probability 1. It remains to argue that there is a person who is indifferent between putting in effort and dropping out. This is the person whose $\gamma$ is not too high and not too low. It is not low enough to merit effort for all through college but it is not high enough to drop out in period 0. These are people who put in effort in period 0 in order to avoid $-\theta$ in period 1. So offering more insurance than $\bar{f}_1$ above will induce all these students to shirk.

It remains to show that offering insurance all the way up to $\min\{\bar{f}_1, f_1^*\}$ is optimal. But this follow from the argument we made earlier (which relied on the distribution $M$ first-order stochastically dominates $H$).

Next we discuss an issue that arise if $\min\{\bar{f}_1, f_1^*\} = \bar{f}_1$. Our insurance scheme assumes that students who are indifferent between shirking and dropping out, actually drop out. We would like to consider the possibility that all those who wish to drop out actually shirk. Then, there will be failures due to shirking. Thus the failure rate will be higher than $1 - \pi$. In order to satisfy the resource constraint, the students who succeed must pay a higher premium. Thus the cost of insurance will rise above the actuarially fair level and this will reduce the benefit of insurance for students who put in effort. A lower benefit in turn will increase the fraction
of students who choose to shirk and this will further increase in the failure rate and the premium that needs to be charged to students who succeed. This raises two issues. First, it may be the case that offering an insurance level of $f_1$ is no longer feasible because of the positive feedback between higher insurance cost and the measure of shirkers. Second, even if $f_1$ is feasible to offer it may be too costly and therefore not optimal. In either case, some lower insurance level would be feasible and better.

In order to explore these concerns, define $\gamma(\pi, s_1)$ such that $V^E_1(\pi, x, \bar{f}_1, s_1) = V^S_1(x, \bar{f}_1) = V^N_1(x)$. For all $\gamma > \gamma(\pi, s_1)$ shirking is preferred to putting in effort. Therefore, this sort of insurance will induce a failure rate given by $[1 - G_1\gamma(\pi, x, s_1)] + G_1(\gamma(\pi, x, s_1))[1 - \pi]$. Then there will be more claims and not enough premiums paid and so we must collect more than $[(1 - \pi)/\pi] f_1$. Denote the additional amount collected by $\tau$. Then, $s_1 = [(1 - \pi)/\pi] \bar{f}_1 + \tau = b + \tau$, where $b$ is the “base premium.” With this notation, we can denote the cut-off $\gamma$ by $\gamma(\pi, x, \bar{f}_1, b + \tau)$. This cut-off value satisfies the equation

$$V^E_1(\pi, x, \gamma(\pi, x, \bar{f}_1, b + \tau), \bar{f}_1, b + \tau) - V^S_1(x, \bar{f}_1) = 0.$$  

Then, for the insurance level $\bar{f}_1$ to be feasible, we must find $\tau$ such that

$$\tau G(\gamma(\pi, x, \bar{f}_1, b + \tau) \pi = [1 - G(\gamma(\pi, x, \bar{f}_1, b + \tau))]\bar{f}_1$$

The question is whether we can find such a $\tau$. In the quantitative part of the paper we show that such a $\tau$ exists. Moreover, the insurance scheme will attract more people to college and will result in fewer people dropping from college. The mass of shirkers is small, and thus overall the insurance scheme induces an increase in the completion rates. Figure 4 presents the changes in thresholds of effort levels that induce these choices in the economy where insurance is provided relative to the benchmark economy. In Section 6 of the paper we quantify these effects as well as welfare implications across people of different ability levels.
5 Calibration

First we present the values of the parameters in the model and then we present the estimation procedure for the probability of completion, $\pi(a)$ and the distributions of the effort levels, $F(\theta)$ and $G(\gamma)$.

5.1 Parameter Values

The utility function is CRRA with coefficient $\sigma$. We calibrate the benchmark model to the U.S. economy. We calibrate the distribution of ability, $A(a)$ to the SAT scores of all students in a senior year in 1999 in the U.S. The mean is 1016 and standard deviation 226 (College Board 2007). In our model, this translates to 0.51 for the mean and 0.18 for the standard deviation. Parameters of the model are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Coef of risk aversion</td>
<td>2</td>
<td>standard</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.97</td>
<td>avg rate of 3%</td>
</tr>
<tr>
<td>$H(y_N)$</td>
<td>Noncollege earnings distribution</td>
<td>1.07(0.5)</td>
<td>earnings HS graduates</td>
</tr>
<tr>
<td>$M(y_C)$</td>
<td>College earnings distribution</td>
<td>1.69(0.8)</td>
<td>earnings college graduates</td>
</tr>
<tr>
<td>$2x$</td>
<td>College debt</td>
<td>0.047</td>
<td>college cost/College Board</td>
</tr>
<tr>
<td>$A(a)$</td>
<td>Ability distribution</td>
<td>0.51(0.18)</td>
<td>SAT scores - seniors</td>
</tr>
</tbody>
</table>

Table 1: Parameter Values
We assume that the distributions of earnings for college and non-college are normal with known mean and variance. Statistics for lifetime earnings are based on earnings data from the CPS for 1969-2002 with synthetic cohorts. We divide education based on years of education completed with 12 years and more, but less than 16 years of completed schooling for the no college group, and those with 16 years of completed schooling for college graduates. Mean of earnings for high school graduates and college graduates are given by $\mu(y_N) = 1.07$ and $\mu(y_C) = 1.69$ and the standard deviations by $\sigma(y_N) = 0.5$ and $\sigma(y_C) = 0.8$. In our model, the ratio between the two education groups represents the earnings premium for an individual that completes college, relative to the same individual in the counterfactual situation of not attending college. This cannot be directly mapped into the rates of return for college education obtained from standard Mincer regressions, since these rates do not typically correct for the ability bias implied by the selection of individuals between college and non-college choices. Willis and Rosen (1979), adjusting for selection in ability, find a lifetime rate of return of attending college of 9.9% per year. Later research estimates the rate of return on college to be between 8% and 13% per year (see Willis (1986) and Card (2001)). This high return to college education also implies very large differences in lifetime income. We use a lifetime college premium of 1.58 (as in Murphy and Welch (1992)), which is consistent with the estimates of 13% rate of return per year. The life-cycle earnings for high-school graduates who do not go to college in the CPS data are 1.07 million dollars (1999 constant dollars). This low outcome is normalized to 1.07.

The average loan amount is set to $2x = 0.047$. In our model, the loan amount that the government collects is computed as the fixed payment for a console with the discounted present value that equals the weighted average college cost for private and public institutions. The cost for college is $20706/\text{year}$ for private universities and $8275/\text{year}$ for public institutions. 

---

5For each year in the CPS, we use earnings of heads of households age 25 in 1969, age 26 in 1970, and so on until age 58 in 2002. We consider a five-year bin to allow for more observations, i.e., by age 25 at 1969, we mean high school graduates in the sample that are 23 to 27 years old. Real values are calculated using the CPI 1982-1984. There are an average of 5000 observations in each year’s sample.

6Restuccia and Urrutia (2004) use the 10% rate of return that corresponds to a lifetime college premium of 1.5.
universities in 1999. Among the students who borrowed for their education, 67% went to public and 33% to private universities. The enrollment-weighted average cost is $49508 in constant 1999 dollars (College Board (2001)). In Section 6 of the paper, we consider heterogeneous cost of college. Using the same enrollment-weighted procedure, we estimate college costs across ability groups. The set of ability, \( a \) (preparation for college) has 5 elements, \( a_i \) with \( i = 1, 2, \ldots, 5 \) corresponding to the following 5 groups of SAT scores: < 700, 700 – 900, 901 – 1100, 1101 – 1250, > 1250. Using data for college costs from the U.S. News ranking and average SAT for accepted students across colleges from the Princeton Review, we estimate the following college costs for the 5 groups of ability levels in the model: $32545, $36812, $41016, $59165, and $77269.

5.2 Completion Probability and Distribution of Effort Levels

In order to estimate the function \( \pi(a) \) and the distributions of effort levels, \( F(\theta) \) and \( G(\gamma) \), we use the National Education Longitudinal Study (NELS:88) that follows eighth-graders in 1988 and the Beginning Postsecondary Students Longitudinal Study (BPS) that follows students who enroll in a postsecondary institution for the first time in 1996. The first data set (NELS:88) collects information about postsecondary education for senior students in 1992. We use this data set for enrollment rates by ability groups as measured by SAT scores. We restrict the sample to high school graduates in 1992 who have enrolled in college without delay after high-school graduation and who are fully enrolled in 2-year and 4-year colleges in October 1992. The second data set (BPS) is designed specifically to collect data related to persistence in college and completion of postsecondary education programs for students who enroll in college in 1996. These same students are surveyed 2- and 5-years later through BPS. We use this data set for early leaving rates and completion rates by ability groups as measured by SAT scores for students beginning at 2-year and 4-year institutions. As before, we restrict the sample to students who have enrolled in college without a delay after high-school graduation and are fully enrolled. For early leaving rates we consider all students who were last enrolled in college in 1998 and have not returned to (any) college to earn a college degree. For completion rates, we consider all students who obtained a college
degree by 2001. Data are presented in the Table 2.

<table>
<thead>
<tr>
<th>SAT scores</th>
<th>≤ 700</th>
<th>701 – 900</th>
<th>901 – 1100</th>
<th>1101 – 1250</th>
<th>≥ 1251</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enrollment rates</td>
<td>71.5</td>
<td>81.2</td>
<td>88.8</td>
<td>95</td>
<td>95.6</td>
</tr>
<tr>
<td>Early leaving rates</td>
<td>16.7</td>
<td>9.6</td>
<td>4.7</td>
<td>1.6</td>
<td>1.4</td>
</tr>
<tr>
<td>Completion rates</td>
<td>37.2</td>
<td>62.7</td>
<td>69.6</td>
<td>78.2</td>
<td>86.5</td>
</tr>
</tbody>
</table>

We first identify the probability of success from college \( \pi(a) \). For each ability group we know the fraction that completes college and the fraction of early leavers. Thus we use the last two rows of Table 2 and compute the fraction of students who complete college out of all students who decide to continue in college. These rates directly pin down \( \pi(a_i) \). The probabilities of success from college across ability groups are given by 44.66, 69.36, 73.03, 79.47 and 87.73. Here we are assuming that if the student has not graduated after 5 years, the student will not graduate at all.

Second, we identify the distributions \( F(\theta) \) and \( G(\gamma|a) \). We calibrate these distributions to match the rates across ability levels described in Table 2. We proceed as follows:

1. We compute the enrollment rates, early leaving rates and completion rates in the model for an initial grid of effort levels.

2. We choose the distributions of the effort levels to best replicate the properties of U.S. data documented in Table 2. Using a parametric approach, we search over the vector of parameters that characterize the two distributions to minimize the distance between the model and the data moments: enrollment, early leaving and completion rates.

3. For any trial of the vector describing the initial distribution, we calculate these three rates by ability groups. If the histogram that best matches the data puts strictly positive weight on the right hand side of the grids of effort levels, then the upper bounds are increased and steps 1-3 are repeated.
In order to carry out the procedure in step 1, we compute the enrollment rates, early leaving rates and completion rates in the model as follows: According to our theory, for each ability group, $a_i$, there is a cutoff of effort on the job, $\theta^c_i$ that will make people within that group indifferent between enrolling and not enrolling (and ignoring application costs). Then, for each ability group we have that the student enrolls in college when $\theta \geq \theta^c_i$, i.e. when

$$V^E_0(a_i, E(\gamma)) - W(\theta) = -E(\gamma)(1 + \beta\pi(a_i)) + \theta + \pi^2(a_i)\beta^2[\int u(y) - 2x(a_i))dM - \int u(y_N - 2x(a_i))dH] + \pi(a_i)\beta[\int u(y_N - x(a_i)) - u(y_N)dH] \geq 0.$$  

(11)

Note that the enrollment choice takes into account the expected value of effort level in college, $E(\gamma)$. We use Equation (11) to compute enrollment rates by ability groups in the model. Furthermore, according to our theory, a student leaves college if the $V^E_0(a_i, \gamma) - V^N_0(\theta)$ is strictly negative, i.e.

$$V^E_0(a_i, \gamma) - V^N_0(\theta) = \theta - \gamma(1 + \pi(a_i))\beta + \pi^2(a_i)\beta^2[\int u(y) - 2x(a_i))dM - \int u(y_N - 2x(a_i))dH] + \pi(a_i)\beta[\int u(y_N - x(a_i)) - u(y_N)dH] + \beta \int u(y_N - x(a_i)) - u(y_N)/4)dH < 0.$$  

(12)

Additionally, we consider the case where students prefer leaving to shirking, i.e. $V^S_0(\theta) - V^N_0(\theta) < 0$. The model delivers, however, that no student prefers shirking when no insurance is provided. We use Equation (12) to compute the early leaving rates by ability groups. Finally, we compute the completion rate in the model, recognizing that some people may choose to shirk in college, and thus the probability of completing college for them is 0, whereas for people who decide to continue and put in effort, the probability of completion is $\pi(a_i)$.

Once we compute all three rates by ability levels for an initial guess of the distributions
of effort levels in the model, we find the best distributions of effort levels (step 2 in the procedure). We restrict the initial distribution of effort during work to be normally distributed and the distribution of effort during college to be of the exponential type. We consider a general specification with different distributions of effort during college by ability levels, \( G_i(\gamma) \). This formulation is motivated by the fact that students of different ability levels may require a different level of effort during college. The first class of distributions is characterized by 2 parameters and the second by one parameter. Thus, the problem reduces to finding the vector of parameters \( \alpha = (\mu_\theta, \sigma_\theta, \mu_{\gamma_1}, \ldots, \mu_{\gamma_5}) \) characterizing these initial distributions to solve the minimization problem

\[
\min_{\gamma} \left( \sum_{j=1}^{5} w_1((e_j - e_j(\alpha))^2 + w_2(l_j - l_j(\alpha))^2 + w_3(c_j - c_j(\alpha))^2) \right),
\]

where \( e_j, l_j \) and \( c_j \) represent the enrollment, early leaving and completion rates in the NELS and BPS data sets, \( e_j(\alpha), l_j(\alpha), \) and \( c_j(\alpha) \) are the corresponding model rates and \( w_i \) are weights assigned to the three rates. Overall we match 15 moments. We use the simplex algorithm in Matlab (the “fminsearch” function) to carry out this step.

We find the distributions \( F(\theta) \sim (0.268, 0.92) \) and \( G_1(\gamma) \sim (0.04), G_2(\gamma) \sim (0.077), G_3(\gamma) \sim (0.0595), G_4(\gamma) \sim (0.038), \) and \( G_5(\gamma) \sim (0.0365) \).

6 Quantitative Results

6.1 Benchmark

The enrollment, leaving and completion rates simulated in our benchmark economy are given in Table 3. There is no shirking in the economy. The enrollment rate in the economy is 88.4%. Out of those who enroll, 4.8% decide to leave college and 70% complete college. Out of people who leave college, most of them decide to do so in the second part of college in the case of low ability people, but the opposite is true for people from high ability groups.
Table 3: Enrollment, Leaving and Completion Rates: Model

<table>
<thead>
<tr>
<th>SAT scores</th>
<th>≤ 700</th>
<th>701 - 900</th>
<th>901 - 1100</th>
<th>1101 - 1250</th>
<th>≥ 1251</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enrollment rates</td>
<td>68</td>
<td>80</td>
<td>89.8</td>
<td>93.1</td>
<td>95.5</td>
</tr>
<tr>
<td>Early leaving rates</td>
<td>16.6</td>
<td>9.6</td>
<td>4.5</td>
<td>1</td>
<td>1.4</td>
</tr>
<tr>
<td>Completion rates</td>
<td>37.3</td>
<td>62.7</td>
<td>69.7</td>
<td>78.3</td>
<td>86.5</td>
</tr>
</tbody>
</table>

6.2 Insurance

We follow the analysis in Section 4. First, we consider the case where effort is observable. The model delivers that the optimal level of insurance (that equates marginal utilities across states), $f^*$ is higher than the cost of college. However, the loan administrators cannot offer more insurance than the college cost. The most that they can do is to forgive the full loan amount in the case of failure. Thus, when we restrict the insurance up to the college cost, it is optimal to offer the full college cost by ability groups, $2x(a_i)$.

We turn now to the case where effort is not observable. Recall that no insurance can be offered in period 0 without inducing shirking, given unbounded support of the effort level on the job, $\theta$. Thus, we first restrict attention to offering insurance in period 1, and then we will reconsider offering insurance in period 0, in the case where there is an upper bound on $\theta$. Recall from Proposition 4.2 that it is optimal to offer insurance up to $f_1$, which satisfies $\int U(y_N - x)dH(y_N) = \beta \int (y_N - 2x + \bar{f}_1)dH(y_N)$. Given the previous discussion about $f^*$, it follows that the $\min\{f^*, \bar{f}_1\} = \bar{f}_1$. Table 4 presents the indemnity offered, $f_1(a_i)$ by ability groups, as well as the premium paid in case of success, $s_1(a_i)$. The table also presents the insurance amounts as percentages of $x$, the cost of the second part of college. Note that the indemnity offered increases in ability. The lowest ability group is forgiven 43% of their college cost in the second part, or 21.5% of their total college cost, whereas the highest ability group is forgiven 54.8% of the college cost in the second part of college, or 22.4% of the total college cost.

When this insurance is offered, the total enrollment rate is 94.48, the early leaving rate
Table 4: Insurance

<table>
<thead>
<tr>
<th>SAT scores</th>
<th>≤ 700</th>
<th>701 – 900</th>
<th>901 – 1100</th>
<th>1101 – 1250</th>
<th>≥ 1251</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indemnity $\bar{f}_1$</td>
<td>0.0055</td>
<td>0.0082</td>
<td>0.0088</td>
<td>0.0147</td>
<td>0.0201</td>
</tr>
<tr>
<td>Perc of $x$</td>
<td>43</td>
<td>46.5</td>
<td>47.6</td>
<td>52.1</td>
<td>54.8</td>
</tr>
<tr>
<td>Base Premium</td>
<td>0.0068</td>
<td>0.0036</td>
<td>0.0033</td>
<td>0.0038</td>
<td>0.0028</td>
</tr>
<tr>
<td>Additional tax $\tau$</td>
<td>0.9577e-04</td>
<td>4.6076e-04</td>
<td>1.2433e-4</td>
<td>0.191e-04</td>
<td>0.3382e-04</td>
</tr>
</tbody>
</table>

Table 5: Enrollment, Leaving and Completion Rates: Insurance

<table>
<thead>
<tr>
<th>SAT scores</th>
<th>≤ 700</th>
<th>701 – 900</th>
<th>901 – 1100</th>
<th>1101 – 1250</th>
<th>≥ 1251</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enrollment rates</td>
<td>80.1</td>
<td>90.35</td>
<td>95.35</td>
<td>97.1</td>
<td>96.75</td>
</tr>
<tr>
<td>Early leaving rates</td>
<td>0.25</td>
<td>6.0</td>
<td>2.56</td>
<td>0.61</td>
<td>0.5</td>
</tr>
<tr>
<td>Completion rates</td>
<td>44.1</td>
<td>65.2</td>
<td>70.96</td>
<td>78.98</td>
<td>87.3</td>
</tr>
<tr>
<td>Indifference rates</td>
<td>1.22</td>
<td>4.24</td>
<td>1.38</td>
<td>0.13</td>
<td>0.17</td>
</tr>
</tbody>
</table>

is 2.69 and the completion rate is 71.56. The enrollment, leaving and completion rates by ability groups in the economy when this insurance scheme is provided are given in Table 5.

Note that the shirking rate and leaving rate is non-monotonic if we account for the lowest ability group. This is because for this group to go to school, the effort required in college, $\gamma$, should be low enough for the same $\theta$ distribution (the effort required on the job). Their completion probability is extremely low compared to the ones for the other groups. The ordering of the $\gamma$ distribution makes sense if we think this is a quality of college issue – the low ability colleges must have lower standards so that the effort required to get a degree is lower too. The completion rate increases, especially for the lower ability groups. With insurance people decide to stay and put in effort rather than dropping out voluntarily.

Another important observation is that when insurance is provided, there is a positive mass of students on average (1.62%) who are indifferent between shirking and dropping out. We now wish to consider the case where these students actually decide to shirk. Under these circumstances, for the insurance level $\bar{f}_1$ to be feasible, we must collect an additional tax, $\tau$ such that
\[\tau G(\gamma(\pi, x, \bar{f}_1, b + \tau))\pi = [1 - G(\gamma(\pi, x, \bar{f}_1, b + \tau))])\bar{f}_1 \] 

(13)

We iterate on Equation (13) to search for \(\tau\), the additional amount that has to be paid in case of success to support the mass of shirkers who receive insurance. As \(\tau\) increases, the amount of shirkers increases, but these increases are incrementally small, and thus we get \(\tau\) by ability groups, as given in Table 4.

When this insurance is offered, the total enrollment rate is 92.42, the early leaving rate is 2.79 and the completion rate is 71.456. The enrollment, leaving and completion rates by ability groups in the economy when this insurance scheme is provided are given in Table 6.

<table>
<thead>
<tr>
<th>SAT scores</th>
<th>(\leq 700)</th>
<th>(701 - 900)</th>
<th>(901 - 1100)</th>
<th>(1101 - 1250)</th>
<th>(\geq 1251)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enrollment rates</td>
<td>81</td>
<td>85.8</td>
<td>93.1</td>
<td>97.1</td>
<td>96.8</td>
</tr>
<tr>
<td>Early leaving rates</td>
<td>0.25</td>
<td>6.91</td>
<td>2.48</td>
<td>0.61</td>
<td>0.5</td>
</tr>
<tr>
<td>Completion rates</td>
<td>43.9</td>
<td>64.6</td>
<td>71</td>
<td>78.98</td>
<td>87.3</td>
</tr>
<tr>
<td>Shirking rates</td>
<td>1.71</td>
<td>5.32</td>
<td>1.39</td>
<td>0.13</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Note that for students of higher ability levels the indifference rates are sufficiently small such that there is no additional incentive to shirk induced by the additional tax collected. The enrollment and leaving choices remain the same as in the case no shirking is considered. People of lower ability levels, however are more inclined to shirk when the additional tax \(\tau\) is collected. Thus a higher additional tax must be collected in the case of success. This in turn, affects the value of getting a college degree, and thus the value of going to college declines and fewer low ability students enroll in college when shirking is allowed. The enrollment and completion rates are lower for these groups relative to the case where these people leave rather than shirk and no additional tax is collected.

We turn now to the welfare analysis. We use an aggregate welfare measure equally weighted. The welfare gain for everyone who goes to college under the insurance scheme when shirking is considered and the additional tax \(\tau\) is collected is 2.71% of the welfare in...
the benchmark economy. The gain for those who enrolled in college without insurance is 2.77% and for those who did not enroll is 1.25%. In the case where the students who are indifferent between shirking and leaving decide to leave and thus no extra cost is imposed by the insurance scheme in addition to the base premium, the welfare gain is 3.06% relative to the benchmark economy. The gain for those who enrolled in college without insurance is 3.18% and for those who did not enroll is 1.26%. Thus, an insurance scheme where no shirking occurs in equilibrium is optimal. However, even in the case where shirking occurs, it is optimal to offer insurance all the way up to \( f_1 \) and collect enough tax in case of success to cover the mass of shirkers.

The different incentives to shirk across ability groups deliver different welfare consequences across these groups of students. Tables 7 and 8 present the welfare changes across ability groups under the two environments (with and without shirking) relative to the benchmark economy.

### Table 7: Welfare changes: shirking

<table>
<thead>
<tr>
<th>SAT scores</th>
<th>( \leq 700 )</th>
<th>701–900</th>
<th>901–1100</th>
<th>1101–1250</th>
<th>( \geq 1251 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>All students</td>
<td>2.5</td>
<td>1.74</td>
<td>2.74</td>
<td>3.45</td>
<td>2.74</td>
</tr>
<tr>
<td>College-benchmark</td>
<td>2.69</td>
<td>1.82</td>
<td>2.79</td>
<td>3.52</td>
<td>2.76</td>
</tr>
<tr>
<td>No college-benchmark</td>
<td>1.49</td>
<td>0.7</td>
<td>1.35</td>
<td>1.79</td>
<td>1.36</td>
</tr>
</tbody>
</table>

### Table 8: Welfare changes: no shirking

<table>
<thead>
<tr>
<th>SAT scores</th>
<th>( \leq 700 )</th>
<th>701–900</th>
<th>901–1100</th>
<th>1101–1250</th>
<th>( \geq 1251 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>All students</td>
<td>2.58</td>
<td>2.8</td>
<td>2.97</td>
<td>3.51</td>
<td>2.87</td>
</tr>
<tr>
<td>College-benchmark</td>
<td>2.77</td>
<td>2.99</td>
<td>3.08</td>
<td>3.58</td>
<td>2.89</td>
</tr>
<tr>
<td>No college-benchmark</td>
<td>1.57</td>
<td>1.24</td>
<td>1.06</td>
<td>1.84</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Note that all ability groups present a higher welfare gain in the case shirking is not allowed except for the third ability group - people who do not enroll in college in the benchmark economy. The highest welfare improvement in the case shirking is not allowed relative to the case where it is allowed is for the second group of ability.

Now we want to reconsider the problem of offering insurance in period 0 in the case where
there is an upper limit on $\theta$, $\theta_{\text{max}}$, such that $F(\theta > \theta_{\text{max}}) = 0$. Computationally, we find $\theta_{\text{max}} = 0.6543$. We can offer insurance in period 0, such that $V_0^S(x, f_0, \theta) = V_0^N(x, \theta)$. This gives:

$$-\theta + \int U(y_N - x/4) dH(y_N) = \beta(f_0 - \theta) + \int U(y_N - x) dH(y_N).$$

(14)

To figure out the insurance in period 0, we proceed as follows: using Equation (14) we find the effort level $\theta(0)$, when no insurance is offered in period 0. This is given by ability groups by: 0.877, 1.141, 1.188, 1.671 and 2.062. Since these values are greater than $\theta_{\text{max}}$, then we can offer insurance up to $\bar{f}_0$ that satisfies:

$$-\theta(\bar{f}_0) + \int U(y_N - x/4) dH(y_N) = \beta(\bar{f}_0 - \theta(\bar{f}_0)) + \int U(y_N - x) dH(y_N).$$

(15)

Equation (15) delivers that the indemnity offered in case of failure in period 0 increases in the ability level. The indemnity offered across ability groups (as a percentage of the college cost) is given by: 26.96, 42.76, 44.64, 55.74, and 59.33. Even though this insurance scheme is more generous than the one offered in period 1, it induces less enrollment and completion on average relative to the case where insurance is offered in period 1. Table 9 presents the enrollment, leaving and completion rates across ability groups when insurance is offered in period 0.

<table>
<thead>
<tr>
<th>SAT scores</th>
<th>$\leq 700$</th>
<th>$701 - 900$</th>
<th>$901 - 1100$</th>
<th>$1101 - 1250$</th>
<th>$\geq 1251$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enrollment rates</td>
<td>73.9</td>
<td>85.8</td>
<td>93.1</td>
<td>95.5</td>
<td>98.3</td>
</tr>
<tr>
<td>Early leaving rates</td>
<td>16.35</td>
<td>8.96</td>
<td>3.29</td>
<td>0.62</td>
<td>0.33</td>
</tr>
<tr>
<td>Completion rates</td>
<td>37.36</td>
<td>61.1</td>
<td>67.9</td>
<td>75.45</td>
<td>80.5</td>
</tr>
<tr>
<td>Indifference rates</td>
<td>0</td>
<td>10.06</td>
<td>6.47</td>
<td>5.04</td>
<td>8.25</td>
</tr>
</tbody>
</table>

The enrollment rate is 91.92% and the completion rate is 68%. This is because a significant mass of students decide to leave college (3.84%) and also a significant fraction of
them are indifferent between leaving and shirking (6.76%). Consequently, offering insurance in period 0 is more costly. In the case we consider that the students who are indifferent between leaving and shirking, decide to shirk, a higher additional tax must be collected in case of success to support the amount of shirkers induced by offering insurance in period 0 relative to the case where insurance is offered in period 1. This additional tax, $\tau_0$ is given by: 0, 0.0017, 0.0014, 0.00167, and 0.0039. Finally, there is less welfare improvement relative to the benchmark economy when insurance is offered in period 0 (1.37% on average) relative to the case where insurance is offered in period 1 (see Table 10). We conclude that offering insurance in period 1 while recognizing a fair amount of shirkers is optimal.

7 Conclusion

We develop a theoretical framework that studies whether the student loan program can gainfully offer insurance against college failure risk. We conduct the analysis under two important constraints on the provision of this sort of insurance. First, we assume that any proposed insurance scheme cannot redistribute resources from people with high probability of completing college to people with low probability of completing college. The government loan program does not permit students with superior post-college labor market outcomes to partially pay the college expenses of students with poor post-college labor market outcomes. Second, we assume that the insurance program must guard against moral hazard, that is, the possibility that provision of insurance against college failure risk may increase the risk of failure.

We argue that such an insurance scheme is administratively feasible and provide conditions
under which these constraints leave open the possibility of some insurance against college failure risk. We calibrate the model to US data on college enrollment, early leaving and completion rates as well as the average indebtedness of program participants and quantify the effects on college enrollment, leaving, dropout rates, and welfare. Results suggest that no insurance can be offered early during college years, whereas up to 22 percent of college cost can be offered later in college. Any insurance beyond this level induces shirking. However, the model suggests that an insurance scheme that recognizes a fair amount of shirkers (1.8 percent) is optimal. This insurance scheme is consistent with higher enrollment and failure rates. However, more people complete college, given that fewer students decide to drop out from college voluntarily. This insurance induces a welfare improvement of 2.7 percent relative to the case where no insurance is offered.

**References**


