International Risk Sharing During the Globalization Era

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(Preliminary; please do not circulate)

Abstract

In theory, international risk sharing should improve with financial globalization, but existing measures have been unable to detect such improvement. We develop a simple measure of international risk sharing that captures how far countries are from the ideal of perfect risk sharing. Our measure shows that international risk sharing has, indeed, improved during globalization. We also find that improved risk sharing comes mostly from the convergence in consumption growth rates among countries rather than from synchronization of consumption at the business cycle frequency.


Keywords: international risk sharing, incomplete market, globalization.

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I. INTRODUCTION

In theory, globalization can enhance growth and risk sharing. Economists, however, have been surprised by the inability of existing measures to detect improved risk sharing during the globalization era. Globalization should make it easier for individuals to diversify insurable risks—those that are minimized by sharing in large groups. Residents of different countries should be able to trade financial assets in a way to insure themselves against country-specific risks that affect the amount of goods and services they consume. But standard measures of consumption smoothing suggest risks are shared poorly internationally. More surprisingly, these measures suggest also that globalization has not led to any discernable increase in the amount of risk sharing. We develop a new measure of international risk sharing. While this new measure confirms that international risk sharing is far from perfect, it does indicate, consistent with theory and intuition, that international risk sharing has been improving over time.

Lucas (1987) observed that the welfare gain from a slightly higher average output growth rate can make up for the welfare loss from small increases in business cycle fluctuations. In the context of international risk sharing, closer average consumption growth rates are far more important than synchronization of consumption at the business cycle

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Surprisingly, existing measures that evaluate international risk sharing ignore differences in average consumption growth rates. Our new measure does not.

Existing measures are well designed to test the null hypothesis of perfect risk sharing. They are, however, poorly suited to gauge the degree of international risk sharing once the null is rejected. Our new measure, in contrast, can assess how far countries are from the ideal and how that distance has evolved over time. Our new measure is the variance of the log share of individual-country consumption in world consumption. Under perfect risk sharing, this variance is zero. In a model we present below, the larger the variance, the farther a country is from the ideal of perfect risk sharing.

Taking this new measure to data, we find that international risk sharing has improved during the globalization period for industrial countries and to some extent in emerging markets also. The improvement, however, shows up in terms of convergence of consumption growth rates between countries, not in terms of short-term consumption smoothing at the business-cycle frequency. We show also that the risks due to consumption growth differences are about twice the size of business-cycle frequency risks for industrial countries and larger also for emerging-market countries. Convergence of these growth differences since 1965 has been dramatic for industrial countries. We emphasize that while we study both industrial and emerging economies, our finding of long-term improvement in risk sharing applies to the industrial countries only. The emerging countries have poorer risk sharing than the industrial countries and have shown indications of improvement only over the last 10 years of our 1960-2004 sample period.

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4 Becker and Hoffmann (2006), and Artis and Hoffmann (2006, 2007) are among the few papers emphasizing long-run risk sharing. van Wincoop (1999) provides a supporting argument for us. He shows that gains from international risk sharing are small if countries’ growth rates are cointegrated and big if they are a random walk.
The rest of the paper is organized as follows. In Section II we present the basic theory of international risk sharing and derive some testable implications of that theory. Section III reviews two existing measures of international risk sharing and explains why they have been unable to uncover improved risk sharing under globalization. In Section IV, we discuss our new measure and provide an example in which our measure is representative of agents’ welfare. Section V takes our new measure to data and shows how international risk sharing has evolved over the 1960-2004 period. Section VI concludes.

II. Theory

Let $C_i^t$ be the date $t$ consumption of the representative country $i$ individual. The individual maximizes the following objective function:

$$E_0 \sum_{t=1}^{\infty} \beta^t u(C_i^t, \theta_i^t),$$ \hspace{1cm} (1)

where $\theta_i^t$ are elements other than consumption that affect individual $i$’s utility.\(^5\) The specification is standard and we assume identical discount factors for all agents. Let

$$u(C_i^t, \theta_i^t) = \frac{C_i^t \gamma}{1 - \gamma} \exp(\theta_i^t).$$

Optimal risk sharing, which can be achieved if there is a full set of Arrow-Debreu securities implies:

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\(^5\) For example, these elements could be preference shocks or leisure. See Obstfeld (1994b) and Canova and Ravn (1996).
\[
\left( \frac{C_i^t}{C_j^t} \right)^{-\gamma} \exp \left( \theta_i^t - \theta_j^t \right) = k^{ij}, \quad \forall t
\]  \hfill (2)

or, taking logs,

\[
\ln C_i^t - \ln C_j^t = \kappa^{ij} + \frac{\theta_i^t}{\gamma} - \frac{\theta_j^t}{\gamma},
\]  \hfill (3)

where \(k^{ij}\) and \(\kappa^{ij} = -\frac{1}{\gamma} \ln k^{ij}\) are constants that depend on initial wealth or the weight a social planner might attach to the utility of the agent in a country.\(^6\) If we assume that the \(\theta_i^t\) are constant, then equation (2) implies that the consumption ratio between any two countries is constant over time when there is perfect risk sharing.

Two implications follow. The most widely studied one is that consumption growth rates are equalized across countries under perfect risk sharing. This relationship, holding for any country pair, holds equally well for any one country relative to the rest of the world. Thus under perfect risk sharing, a country’s per capita consumption is a fixed share of average world per capita consumption. Further, since this share is constant, its variance is zero.

III. EXISTING MEASURES OF INTERNATIONAL RISK SHARING

The existing empirical consumption-risk sharing literature follows from the theoretical observation that if countries share risks perfectly, their consumption ratios will be constant and a country’s share in world consumption will be constant also. Perfect risk sharing also implies consumption growth rates will be equal across countries and that

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\(^6\) See Lewis (1996).
deviations from constant consumption ratios or differences in consumption growth rates –
both supposedly zero - should be uncorrelated with other variables such as incomes.

The empirical literature on international consumption risk sharing was launched when
Backus, Kehoe and Kydland (1992) documented the “consumption correlation puzzle,” the
finding that international consumption correlations are lower than international output
correlations. Obstfeld (1994b, 1995) confirmed the puzzle, but unlike many subsequent
studies, found evidence of increasing correlations after 1973 among industrial countries.7
Tesar and Stockman(1995) offered some possible reasons for the puzzle, such as presence of
preference shocks, and pointed out that if the empirical puzzle held up to scrutiny, it implied
that agents were using international financial markets to destabilize consumption. Lewis
(1996) did regression tests confirming that consumption risk sharing is imperfect, laying the
blame on non-traded goods and capital controls.8

Following these seminal papers, the empirical literature extended and studied the
relationships between countries’ consumptions and their incomes. The literature focused
primarily on two types of measures, correlation measures, which we call $\rho$ measures, and
regression measures, which we call $\beta$ measures. The $\rho$ measures normally come from
computing correlation coefficients of cross-country consumption aggregates measured in
levels or growth rates. It is thought such correlations should be unity when risks are shared
perfectly. The $\beta$ measures are usually obtained from regressing consumption growth rates
on idiosyncratic output growth or other things such as world consumption growth rates. The
$\beta$ coefficient attached to world consumption growth should be unity and that attached to

7 Obstfeld (1986) also finds that international capital markets are more integrated after 1973.

8 Canova and Ravn (1997) claim that short run risk insurance is almost complete among pairs of industrial
countries while medium to long run risk insurance is not.
idiosyncratic output growth rates should be zero when risks are shared perfectly as idiosyncratic shocks should not affect consumption growth.

III. A. ρ Measures

Correlation measures are studied by Devereux, Gregory, and Smith (1992), Obstfeld (1994,1995), Canova and Ravn (1997), Pakko (1998),9 Heathcote and Perri (2003), Ambler, Cardia and Zimmerman (2004), and Kose, Prasad and Terrones (KPT) (2003a), among others. If consumption risks are insured perfectly, the ratio of individual-country consumption to world consumption is constant and the correlation coefficient between any two consumption-growth measures, therefore, should be unity. If the correlation coefficient between any two countries’ consumption growth rates turns out to be significantly different from unity, then that is a rejection of perfect risk sharing between those two aggregates.

The above-mentioned studies almost uniformly find individual-country consumption growth to be relatively poorly correlated with world consumption growth. KPT, for example find the correlation between average industrial country consumption growth and world consumption growth to be 0.45 with a standard error of 0.06 – economically and statistically well below unity. For developing countries, KPT find the correlation to be even lower, 0.02 with a standard error of 0.04.

While studying consumption growth correlations allows one to construct a logical test for perfect risk sharing, the correlation coefficients themselves are not a particularly good

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9 Pakko (1998) casts doubt on the reliability of the correlation measure since the empirical results change with different detrending method. See Lewis (1996) and Matsumoto (2007) on how nontraded goods and nonseparability can affect correlations under perfect risk sharing.
vehicle for measuring the deviation of a particular country from perfect risk sharing either at a point in time or over time. The reason can be illustrated in an example as follows.

Suppose log consumptions of the world \((W)\) and country \(i\) are following random processes:

\[
\ln C_{W,t} = g_1 + \ln C_{W,t-1} + \varepsilon_t
\]

\[
\ln C_{i,t} = g_2 + \ln C_{i,t-1} + \lambda \varepsilon_t
\]

where \(g_1\), \(g_2\) and \(\lambda\) are positive constants, \(\varepsilon_t\) is a mean zero iid shock.

If \(g_1 \neq g_2\), then the ratio of country \(i\)'s consumption to world consumption changes with time. Our measure is positive in this situation and growing with the length of the sample period.

The correlation measure based on consumption growth rates, however, wrongly suggests perfect risk sharing. The correlation is:

\[
\rho(\Delta \ln C_{W,t}, \Delta \ln C_{i,t}) = \frac{\text{cov}(\varepsilon_t, \lambda \varepsilon_t)}{\sqrt{\text{var}(\varepsilon_t)}, \sqrt{\text{var}(\lambda \varepsilon_t)}} = 1,
\]

which ignores the difference in growth rates. The measure ignores also possible differences in the size of shocks in each period – the \(\lambda\)'s cancel in equation (4). Even if \(g_1 = g_2\), our measure will not conclude that country \(i\) is sharing risk perfectly as over a sample period of length \(T\),

\[
\text{Var}_T(\ln(C_{ji} / C_W)) = T \times (1 - \lambda)^2 \times \sigma^2_{\varepsilon}, \text{ where } \sigma^2_{\varepsilon} \text{ is the variance of } \varepsilon
\]

**III. B. \(\beta\) Measures**

Another way to study risk sharing is with regression methods. Obstfeld (1994b, 1995), for example, estimates variants of the following equation:

\[
\Delta \ln C_{it} = \beta_0 + \beta_1 \Delta \ln C_{Wt} + \beta_2 \Delta \ln \text{GDP}_{it} + \varepsilon_{it},
\]
where $\Delta \ln C_{it}$ is the growth of country $i$ per-capita country consumption (from period $t-1$ to $t$), $\Delta \ln C_{Wt}$ is the growth rate of per-capita world consumption, and $\Delta \ln GDP_{it}$ is the growth rate of country $i$ per-capita output; $\epsilon_{it}$ is a residual. If risk sharing is perfect, $\beta_1 = 1$ and $\beta_0 = \beta_2 = \sigma^2_\epsilon = 0$, where $\sigma^2_\epsilon$ is the variance of $\epsilon$. Breaking his data into two periods, 1951-72 and 1973-88, Obstfeld finds that low estimates of $\beta_1$ and high values of $\beta_2$ lead to a rejection of perfect risk sharing in the 1951-1972 period for industrial countries, but he cannot reject perfect risk sharing for industrial countries based on the estimates obtained in the 1973-1988 period.

Obstfeld’s results are an indication that risk sharing may have improved between the two periods, but his results do not settle the question of improved risk sharing since the variance of the output-growth regressor may have risen and/or $\sigma^2_\epsilon$ might have risen between the two periods. Both factors could contribute to an increase, or at least to no decrease, in the variance of the ratio of $\ln(C_{it} / C_{Wt})$.

Kalemli-Ozcan, Sorensen, and Yosha (2003) interpret the beta coefficient from the following panel regression as a degree of consumption risk sharing among a group of areas in the panel.\(^{10}\) The regression specification is

$$
\Delta \ln C_{it} - \Delta \ln C_{Wt} = f_t + (1-\beta)(\Delta \ln GDP_{it} - \Delta \ln GDP_{Wt}) + \epsilon_{it},
$$

where $\Delta \ln GDP_{it}$ and $\Delta \ln GDP_{Wt}$ are the per capita GDP growth rates of area $i$ and the world respectively, $C_{it}$ is consumption, and $f_t$ are time fixed effects. While this specification may be appropriate for a group of countries, it cannot be used to measure how well each

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\(^{10}\) Lewis (1996) used this specification to test for complete risk sharing, which requires $\beta = 1$. With perfect risk sharing, a country’s idiosyncratic consumption growth should be uncorrelated with its idiosyncratic output growth. Bai and Zhang (2005) also study risk sharing using a cross section variant.
individual country shares risk. In addition, the presence of nontraded goods can make the interpretation of beta difficult when it is different from unity.

Kose et al. (2007) run a time series regression of similar form:

$$\Delta \ln C_{it} - \Delta \ln C_{wt} = \alpha_i + (1 - \beta_i) (\Delta \ln GDP_{it} - \Delta \ln GDP_{wt}) + \varepsilon_{it},$$

(7)

where $\beta_i$ is a country-specific risk sharing measure. This regression is harder to understand, even $\beta = 1$ does not have much meaning unless the intercept, $\alpha$, is zero, as otherwise the consumption growth rate of country $i$ is different from the world growth rate during the sample period.

Other regressions tests, such as those by Artis and Hoffmann (AH) (2006), work with consumption levels instead of consumption growth rates and thus incorporate low frequency risk sharing. Yet the tests done by AH are better designed to reject perfect risk sharing than to measure how well risks are being shared. Consider the following AH regression:

$$\ln C_{it} - \ln C_{wt} = f_t + (1 - \beta_i) (\ln GDP_{it} - \ln GDP_{wt}) + \varepsilon_{it},$$

(8)

where $\ln C_{it}$ and $\ln C_{wt}$ are the logs of per capita consumption in period $t$ for country $i$ and the world, respectively; $\ln GDP_{it}$ and $\ln GDP_{wt}$ are the logs of per capita output in period $t$ for country $i$ and the world; and $\varepsilon_{it}$ is an error term.

In this levels regression, perfect risk sharing requires $\beta_i = 1$ and $\sigma^2 = 0$. AH find that sequential cross-section estimates of $\beta_i$ are statistically different from unity but become closer through time, particularly for EU and EMU countries. While the drift of $\beta_i$, by itself,
is an indicator that risk might be shared better, it does not make the case tightly because it could be that the variance of $\ln GDP_n - \ln GDP_{W_t}$ and $\sigma^2$ are rising.\[11\]

**III C. Growth rate volatility**

Kose, Prasad and Terrones (2003b) study the volatility of the consumption growth rate, income growth rate, and output growth rate of each country. They infer the degree of risk sharing from these measures. However, this is quite a difficult task as theoretical impacts of trade and/or financial integration on these volatilities are ambiguous, as they noted. For example, suppose there are two countries, one with a constant endowment and the other with a volatile endowment. Optimal risk sharing will reduce consumption volatility of one country but increase that of the other, while output volatility will not change. They measure risk sharing using the ratio of the volatility of the total consumption growth rate to that of the income growth rate.\[12\] Complete Arrow-Debreu securities allow national income and its growth to be insured over time and state as well as consumption.\[13\] Using the ratio of the volatility of the total consumption growth rate to income growth, therefore, is not correct from a theoretical point of view.

**IV. A NEW MEASURE OF RISK SHARING ($\sigma$)**

We wish to measure how close countries come to the benchmark of perfect risk sharing. We do so by computing over different time intervals the squared deviations of the

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\[11\] This is a fine point. In fact, AH interpret their results as showing that risk sharing has been improving for industrial countries at low frequencies, which is consistent with our results.

\[12\] Kose et. al. use GNP refined by the terms of trade as an income measure.

\[13\] Indeed, Asdrubali, Sorensen and Yoshia (1996) find that income risk sharing contributes most in terms of interstate consumption risk sharing in the US.
log of a country’s share of world consumption from its average over the time period. Below and in Appendix I we study our measure in example world economies. In the text example, where consumption growth rates are the same and there is one type of shock, our measure unambiguously matches perfectly with individual and world welfare.

Over some time interval, the variance of country i’s log share of world consumption is

\[
\hat{\sigma}_{i,t}^2 = \frac{1}{T} \sum_{t=0}^{T} (X_{i,t} - \bar{X})^2
\]  

(9)

where \( X_{i,t} = \ln \left( \frac{C_{i,t}}{C_{W,t}} \right) \) and \( \bar{X} \) is the sample mean of \( X_{i,t} \) for the corresponding period, and \( \tau \) is the end of the sample period. In the benchmark case of perfect risk sharing, \( X_{i,t} = \bar{X} \) for all \( t \) where \( \bar{X} \) is a constant related to initial wealth or a social planner’s weights as in equation (3).

The variance measure in (9) is our measure of risk sharing. As the measure approaches zero, the benchmark for perfect risk sharing, country i increasingly shares risk internationally. In implementation below, we average our measure within country groups.

IV A. A Simple Approximation Example in which Welfare is Monotonic in \( \sigma^2 \)

The point of this example is to illustrate an economy where our measure of risk sharing is utility and social-welfare based. In this example we study our measure’s relation to welfare in a two-agent, one shock model. Let social welfare be:

\[
S = \sum_{t=1}^{T} \left( \frac{1}{2} \ln C_{1,t} + \frac{1}{2} \ln C_{2,t} \right)
\]
We assume countries 1 and 2 are initially equally wealthy and face the same uncertain future endowment process. Given this assumption, the optimal allocation of consumption is

$$C_{1,t}^* = C_{2,t}^* = C_{W,t}$$

regardless of the realization of the endowment.

It follows that optimal social welfare is:

$$S^* = \sum_{t=1}^{T} \ln(C_{W,t})$$

Now we evaluate the actual allocation of consumption. Let the actual consumption be:

$$C_{1,t} = (1 + \nu_t)C_{W,t},$$

$$C_{2,t} = (1 - \nu_t)C_{W,t}.$$

Using this series of allocations,

$$S = \sum_{t=1}^{T} \left[ \ln(C_{W,t}) + \frac{1}{2} \ln(1 + \nu_t) + \frac{1}{2} \ln(1 - \nu_t) \right]$$

Taking a second-order approximation of $\ln(.)$, we get

$$S - S^* = \sum_{t=1}^{T} \frac{1}{2} \left[ \nu_t^2 \frac{1}{2} \nu_t^2 \right] + \frac{1}{2} \left[ -\nu_t - \frac{1}{2} \nu_t^2 \right] = \sum_{t=1}^{T} \left[ \frac{1}{2} \nu_t^2 \right]$$

This implies that maximizing social welfare is equivalent to minimizing $\sum_{t=1}^{T} \nu_t^2$, which is our measure as

$$-\nu_{1,t} \approx \ln(C_{W,t}) - \ln(C_{i,t}).$$

V. TAKING THE NEW MEASURE TO DATA

We construct our risk sharing measure, using data from the Penn World Tables, Version 6.2 (Heston, Summers and Aten 2006). We create our world consumption index by accumulating weighted average growth rates of countries regarded as the world. The
definition of ‘world’ in our study is simply the rest of the countries in our sample. Different definitions of ‘world’ do not significantly change our results. We use data on private consumption, but the results are very similar when we use total (private plus public) consumption.

Figure 1 and Figure 2 depict the within-group averages of our measure of risk sharing for three groups of countries: Industrial Countries (“Industrial”), More Financially Integrated Emerging Market Countries (MFIE), and Less Financially Integrated Emerging Countries (LFIE), rolling over time. The measures plotted in the figures, $\hat{\sigma}_{t,i,15}$ and $\hat{\sigma}_{t,i,20}$, are the simple averages of the standard deviations of relative consumption for each country group computed in rolling windows of length 15 and 20 years, respectively. The windows end at date t and pertain to country group i. $\hat{\sigma}_{2003, MFIE, 15}$, for example, is the simple cross-country average standard deviation of $\log(C_{i,t} / C_{w,t})$ for the MFIE country group computed in a 15-year window ending in 2003.

Figures 3, 4 and 5 attach 90% bootstrapped confidence intervals to the estimates in Figure 1. (We have not completed finding similar confidence intervals for other $\sigma$ measures.)

We find that emerging countries, MFIE and LFIE, did, indeed, improve (point estimate) risk-sharing during the recent globalization era, since about 1995, after having experienced a worsening in the early sample period. Industrial countries, on the other hand, improved risk sharing significantly in the 1970s and early 1980s but have not shown much change thereafter. For the entire sample period, regardless of the length of the window, we

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14 Kose et al (2007) define ‘world’ as the set of industrial countries. Relative to a particular industrial country, the ‘world’ is the rest of the industrial countries.

15 Our country groupings are from Kose et al (2003a) and are listed in Appendix II.

16 Statistical significance of the improvement for LIFE ans MIFE is arguable.
find a robust and intuitive ranking of country groups’ risk sharing - industrial countries share risks best, MIFE second and LFIE last.

Figure 6 and Figure 7 depict scatterplots of $\sigma_{2003,i,15}$ and $\sigma_{1964,i,15}$, on a country-by-country basis, against the logarithm of per-capita country consumption for the last year of the sample period, 2003 or 1964.\(^{17}\) In both figures, there is always the tendency for richer countries to share risks better than poorer ones.

Moreover, the risk-sharing order of countries rarely changes. In Figure 8, we pull out of our aggregated groups the results for India, Japan and the U.S. as examples. The figure shows that the U.S., for most of the period, shares risks better than India and Japan. Japan did not share risks well early in our sample since its growth miracle increased per capita consumption faster than the world average.

Figure 9 and Figure 10 depict group averages and country examples, respectively, of correlations of annual growth rates of per capita consumption with the annual growth rate of world per capita consumption.\(^{18}\) In Figure 9 and Figure 10, higher correlations are regarded as indicating better risk sharing. Figure 9, which shows no long-term increase in the correlations, is often regarded as evidence that emerging countries did not benefit from globalization. In addition, Figure 10 shows the correlation measure is an unreliable indicator of risk sharing since its ranking is sensitive to the sample period. The reason is likely due to the fact that the correlation measure ignores the average growth rate of a country for the sample period and hence does not capture a key component of risk sharing.

Figure 11 and Figure 12 depict group averages and country examples of the rolling time series “beta measures” used by Kose et. al (2007). The value for the Y axis is $\beta_i$ defined

\(^{17}\) We should be cautious about interpreting the earlier sample because some countries, such as Japan, were are categorized as “Industrial” but were middle income.
in equation (7), and higher values may indicate better risk sharing. Counter intuitively, the beta measure graphed in Figure 11 and Figure 12 indicates that less financially integrated countries share risks better than industrial countries. When we use total consumption in the place of private consumption and a rolling window of 10 years instead of 15, then the $\beta_i$ of industrial countries are higher. The $\beta_i$ measure is sensitive to the precise choice of variables, rolling windows length, and definition of 'world'. Consequently, $\beta_i$ is not a robust measure of risk sharing.

We can provide insight into our risk sharing measure by decomposing it into high- and low-frequency components. The high-frequency component is the deviation from sample trend. The low-frequency component is the difference between trends. We now provide the analytics for decomposing our risk measure and then study the decomposition in our sample.

Let $g$ be the average growth rate of $X_t$. Formally,

$$g = \frac{1}{T-1} (X_t - X_{t-T+1})$$

(10)

Then $X_{t-n+1} = X_t - g(n-1)$ and our risk-sharing measure in (9) can be re-written as

$$\frac{1}{T-1} \sum_{t=1}^{T} (X_{t-n+1} - \bar{X})^2 = \frac{1}{T-1} \sum_{n=0}^{T-1} \left[ X_{t-n} - (X_t - gn) + (X_t - gn - \bar{X}) \right]^2$$

$$= \frac{1}{T-1} \sum_{n=0}^{T-1} \left[ X_{t-n} - (X_t - gn) \right]^2$$

$$+ \frac{1}{T-1} \sum_{n=0}^{T-1} \left[ X_t - gn - \bar{X} \right]^2$$

$$+ \frac{1}{T-1} \sum_{n=0}^{T-1} 2 \left[ X_{t-n} - (X_t - gn) \right] \left[ X_t - gn - \bar{X} \right]$$

(11)
The first term, \( \frac{1}{T-1} \sum_{n=0}^{T-1} [X_{t-n} - (X_t - gn)]^2 \), measures the high-frequency component. The second term, \( \frac{1}{T-1} \sum_{n=0}^{T-1} (X_t - gn - \bar{X})^2 \), captures the difference in average growth rates, which is often excluded from existing measures. We call this the low-frequency component, or the growth-difference component. Finally, the third term measures the interaction.

The shaded area in Figure 13 illustrates the key components of our measure. The area between the trend line and \( X_{t-s} \) captures the high-frequency component, or the term

\[
\frac{1}{T-1} \sum_{s=0}^{T-1} [X_{t-s} - (X_t - gs)]^2.
\]

The two triangular areas between the trend line and the average, \( \bar{X} \), capture the difference in average growth rates, or the term

\[
\frac{1}{T-1} \sum_{s=0}^{T-1} (X_t - gs - \bar{X})^2,
\]

that is ignored by popular measures of risk sharing.\(^{18}\)

Figure 14 and Figure 15 depict the decomposition of our measure by showing the cross-country means of the first and the second terms of equation (6) over time. Lower values indicate better risk sharing. In Figure 14, we see that the high-frequency component is without trend for all country groups and it is quite noisy for MFIE and LFIE. This is probably the reason why the existing measures, whose focus is high frequency, cannot detect improved risk sharing. However, from Figure 15 we see that the low-frequency component is without trend over the full sample period for MFIE and LFIE, but shows an improvement more recently. For the Industrial countries, we see dramatic improvement early in the sample period. Indeed, the early improvement is so strong that there is little room for additional low-frequency improvement later on.

\(^{18}\) For example, the correlation and beta measures are usually derived from growth rate data or detrended data and are therefore high-frequency measures only.
IV. Conclusion

We propose and implement a natural and simple measure of risk sharing, finding that countries on average are sharing risk better during the era of financial globalization than previously. While this finding should not be surprising, it is different from the existing literature. The reason is that existing measures ignore growth rate differences and focus on whether per capita consumption across countries is synchronized at the business cycle frequency. Our measure considers both low-frequency and high-frequency elements.

The risk sharing we uncover is not short-term, brought about through insurance contracts or trading country-risk-specific securities. It is a long term phenomenon, driven perhaps by output-growth-rate convergence related to trade in ideas and technologies and to diffusion of institutions, that Kose, Prasad, Rogoff and Wei (2006) call the collateral benefits of globalization. Our new measure shows that risk sharing has improved over time because industrial countries consumption growth rates have converged dramatically since the 1960s and consumption growth rates for emerging markets may have started converging in the 1990s.
Figure 1
Figure 2

Rolling Volatility (mean) rw=20

- Industrial
- MFIE
- LFIE

rolling volatility

end year of rolling

Figure 3

Rolling Volatility (mean) \( w=15 \)

Thin lines indicate 90% Confidence intervals
Figure 4

RollingVolatility (mean) \( rw=15 \)

Thin lines indicate 90\% Confidence intervals

end year of rolling
Figure 5

Rolling Volatility (mean) \( r_w = 15 \)

Thin lines indicate 90% Confidence intervals
Figure 6
Figure 7
Figure 8
Figure 9
Figure 10
Figure 12
Figure 13
Figure 14
Figure 15: Rolling RVC_{G\alpha} (mean) rv=15
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In this appendix we explore an example in which we relate our measure of risk sharing to individual utility and social welfare. In this example we get a closed-form solution, but we must make an assumption about the statistical distribution of consumption.

Consumption Distribution

Suppose consumption in period $t$ is distributed log normally across $M$ individuals (countries):

$$C_{i,t} = \exp(grow_{i,t} + \nu_{i,t})$$

$grow_{i,t} = (\bar{g} + g_{i})t$

$\nu_{i,t}$ is $N(0, \sigma_{\nu}^2)$

$g_{i}$ is $N(0, \sigma_{g}^2)$.

In this example, individual $i$ draws an average growth rate at the outset, $\bar{g} + g_{i}$. Each period she draws also an idiosyncratic shock $\nu_{i,t}$. The shocks, $g_{i}$ and $\nu_{i,t}$, are mean zero and normally distributed - $\sigma_{\nu}^2$ is a measure of risk while $\sigma_{g}^2$ is a measure of cross-country consumption-growth heterogeneity.

Define $C_{w,t} = \frac{1}{M} \sum_{i=1}^{N} C_{i,t}$ - so $C_{w,t}$ is period-average level individual consumption. As $M$ goes to infinity $C_{w,t}$ converges to:

$$E(C_{w,t}) = \exp(\bar{g}t + \frac{t^2 \sigma_{g}^2 + \sigma_{\nu}^2}{2})$$

which is the unconditional expectation of time $t$ individual consumption.

Our measure is:

$$Var_{N}(\ln(C_{i,t} / C_{w,t}))$$

We use the notation $Var_{N}$ to denote the rolling $N$-period variance measure.
Since

\[ C_{i,t} = \exp((\bar{g} + g_i)t + v_{i,t}) \]

\[ \ln(C_{i,t}) = (\bar{g} + g_i)t + v_{i,t}. \]

This implies:

\[ \ln(C_{i,t} / C_{w,t}) = g_i t + v_{i,t} - \frac{t^2 \sigma_g^2 + \sigma_v^2}{2} \]  \hspace{1cm} (AI.1)

We apply the N-period variance operator to (AI.1) obtaining:

\[ \text{Var}_N(\ln(C_{i,t} / C_{w,t})) = \frac{1}{N}[(N^2 + (N-1)^2 + \ldots + 1)g_i^2 + \sum_{j=-N}^{i} v_{i,j}^2] \]  \hspace{1cm} (AI.2)

Now collapse the series in (AI.2) and define \( f(N) \):

\[ \frac{1}{N} \left( N^2 + (N-1)^2 + \ldots + 1 \right) = \frac{(N+1)(2N+1)}{6} = f(N). \]

We suppose that \( N \) is big enough so that:

\[ \frac{1}{N} \sum_{j=-N}^{i} v_{i,j}^2 = \sigma_v^2. \]

(This is not tight. In fact we can’t conceptually let \( N \to \infty \) or \( f(N) \) would blow up. We need some notation for the sample counterpart of the variance of \( v \)).

When we average across countries, as \( M \to \infty \):

\[ \frac{1}{M} \sum_{i=1}^{M} \frac{1}{N} [(N^2 + (N-1)^2 + \ldots + 1)g_i^2 = f(N)\sigma_g^2. \]

Therefore, our measure is:

\[ \text{Var}_N(\ln(C_{i,t} / C_{w,t})) = f(N)\sigma_g^2 + \sigma_v^2. \]
2. Risk, Heterogeneity and Welfare

We wish to study the relation of our measure to insurable risk, individual utility and world welfare. To study insurable risk and world welfare, each period we hold the per-capita world period endowment fixed at \( X_t / M_t = \exp(\bar{g} t + \frac{t \sigma^2_g + \sigma^2_r}{2}) \). Because the lognormal distribution is not symmetric in levels, we need to be careful how we think about mean, \( \bar{g} t \), and variance, \( \frac{t \sigma^2_g + \sigma^2_r}{2} \). When we hold \( \bar{g} t \) constant and increase \( \frac{t \sigma^2_g + \sigma^2_r}{2} \), we increase risk. We also, however, increase the endowment. In our thought experiments, we wish to study the effect of a mean-preserving spread (MPS), an increase in risk with no increase in the aggregate endowment, which ensures the risk we study is completely diversifiable.

To study the effect of variance changes in a MPS, we set \( K_t = \bar{g} t + \frac{t \sigma^2_g + \sigma^2_r}{2} \) where \( K_t = \ln(X_t / M_t) \) is held constant in the experiment.

With CRRA individual utility, unconditionally expected period utility for individual i utility is:

\[
EU(C_{i,t}) = \frac{1}{(1-\gamma)} \exp((1-\gamma)\bar{g} t + \frac{(1-\gamma)^2(t^2 \sigma^2_g + \sigma^2_r)}{2}), \quad \gamma > 0.
\]

Use our MPS condition to get:

\[
\bar{g} t = K(t) - \frac{t^2 \sigma^2_g + \sigma^2_r}{2}, \text{ which we substitute into expected utility giving:}
\]

\[
EU(C_{i,t}) = \frac{1}{(1-\gamma)} \exp((1-\gamma)[K(t) - \frac{\gamma(t^2 \sigma^2_g + \sigma^2_r)}{2}]) \quad (A1.3)
\]

We see from (A1.3) that expected utility declines when \( \frac{t \sigma^2_g + \sigma^2_r}{2} \) rises in a MPS.
Our Measure and Expected Utility

The relevant comparison is between:

\[ \text{Var}_h (\ln (C_{i,t} / C_{w,t})) = f(N) \sigma_g^2 + \sigma_r^2 \quad \text{FMM Measure} \]

vs.

\[ \frac{\gamma (r^2 \sigma_g^2 + \sigma_r^2)}{2} \quad \text{Utility-relevant measure.} \]

(Our measure matches the utility-relevant measure when our sample period matches the social welfare horizon.)
APPENDIX II

Industrial Countries

Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Germany (GER), Greece (GRC), Ireland (IRL), Italy (ITA), Japan (JPN), Netherlands (NLD), New Zealand (NZL), Norway (NOR), Portugal (PRT), Spain (ESP), Sweden (SWE), Switzerland (CHE), United Kingdom (GBR), United States (USA).

More Financially Integrated Countries:

Argentina (ARG), Brazil (BRA), Chile (CHL), China (CHN), Colombia (COL), Egypt (EGY), Hong Kong (HKG), India (IND), Indonesia (IDN), Israel (ISR), Korea, Republic of (KOR), Malaysia (MYS), Mexico (MEX), Morocco (MAR), Pakistan (PAK), Peru (PER), Philippines (PHL), Singapore (SGP), South Africa (ZAF), Thailand (THA), Turkey (TUR), Venezuela (VEN).

Less Financially Integrated Countries:

Algeria (DZA), Bangladesh (BDG), Benin (BEN), Bolivia (BOL), Botswana (BWA), Burkina Faso (BFA), Burundi (BDI), Cameroon (CMR), Costa Rica (CRI), Côte d’Ivoire (CIV), Dominican Republic (DOM), Ecuador (ECU), El Salvador (SLV), Gabon (GAB), Ghana (GHA), Guatemala (GTM), Haiti (HTI), Honduras (HND), Jamaica (JAM), Kenya (KEN), Mauritius (MUS), Nicaragua (NIC), Niger (NER), Nigeria (NGA), Panama (PAN), Papua New Guinea (PNG), Paraguay (PRY), Senegal (SEN), Sri Lanka (LKA), Syria (SYR), Togo (TGO), Tunisia (TUN), Uruguay (URY).
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