ABSTRACT

A framework is developed with what we call technology capital. A country is a measure of locations. Absent policy constraints, a firm owning a unit of technology capital can produce the composite output good using the unit of technology capital at as many locations as it chooses. But it can operate only one operation at a given location, so the number of locations is what constrains the number of units it operates using this unit of technology capital. If it has two units of technology capital, it can operate twice as many operations at every location. In this paper, aggregation is carried out and the aggregate production functions for the countries are derived. Our framework interacts well with the national accounts in the same way as does the neoclassical growth model. It also interacts well with the international accounts. There are constant returns to scale, and therefore no monopoly rents. Yet there are gains to being economically integrated. In the framework, a country’s openness is measured by the effect of its policies on the productivity of foreign operations. Our analysis indicates that there are large gains to this openness.

*McGrattan, Federal Reserve Bank of Minneapolis and University of Minnesota; Prescott, Arizona State University and Federal Reserve Bank of Minneapolis. We thank the National Science Foundation for providing financial support for this research under grant SES-0422539. We thank Loris Rubini, Johanna Wallenius and particularly Simona Cociuba for exceptional research assistance. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.
1. Introduction

There is general agreement that the gains to openness are large. There is empirical support for this view, but the theoretical support is lacking. One avenue for gains from openness is increased trade. However, the estimated gains from the increased trade resulting from greater openness are smaller than what the empirical observations indicate. In this paper we examine another avenue for gains from openness. With this avenue, openness results in foreign know-how or technology capital being brought into a country and used there by foreign multinationals. This avenue shows great promise of providing theoretical underpinnings for large gains from openness.

We begin with a theoretical framework with plant production technologies and derive the aggregate production function of a country. This aggregate production function has three inputs and a composite output good. The first two inputs are the usual, namely the services of labor and tangible capital. The third input is technology capital. Absent technology capital, the aggregate technology is the same as the one in the neoclassical growth model that is so heavily used by macroeconomists. In our framework, the aggregate production function of each country displays constant returns to scale. But, summing identical countries’ aggregate production sets results in a bigger aggregate production set. It is as if there were increasing returns, when in fact there are none.

A unit of technology capital permits the firm owning this capital to set up one production unit at as many of the locations as this firm chooses. This technology capital is

---

2 In McGrattan and Prescott (2006b) addressing current account issues, we introduce two varieties of type one capital, namely tangible and production-unit specific intangible capital. Differences in tax treatments and reporting necessitated this feature. For the issues addressed in this paper, however, we need not and do not make the distinction between tangible and intangible capital. The extension is straightforward, but complicates the notation. All that needs to be done is to introduce an aggregator of tangible and firm-specific intangible capital at the production level.
what gives rise to foreign direct investment and to gains from being open and losses from being closed. Technology capital does not result in increasing returns because a production operation at a given location using this unit of technology capital has an optimal scale of operation given the rental price of capital and the wage rate in the country of the location. A country is totally open in our sense of the word if it treats foreign multinationals and domestic firms the same.

As we want countries to differ in size, a country is a measure of locations. As the measure of locations and people are equal, we refer to the size of a country by this measure. Labor services are country specific, not location specific, which means labor is mobile within a country but not between countries. Bigger countries have more locations and people. A country’s technology set is a convex cone, as is the sum of the technology sets of any subset of countries. Thus, price taking is assumed and there are no monopoly rents.

Our abstraction focuses on the role of economic integration making possible the use of foreign know-how or technology capital in production in a country and a country’s know-how in other countries. We emphasize that this is not the only reason for gains from states becoming economically integrated. The integration of economically sovereign states almost surely leads to fewer barriers to efficient production and higher productivities through competition. But this is not the topic of this paper, which focuses on the gains associated with exploiting both domestic technology capital and foreign technology capital in production.

In Section 2 we present some empirical evidence that becoming economically integrated with the advanced industrial economies—in particular becoming open to foreign direct investment—results in that country catching up to the industrial leader in terms of GDP per worker. In Section 3 we develop the aggregation theory leading to our aggregate

\[\text{See, for example, Holmes and Schmitz (1995 and 2001) and Burstein and Monge-Naranjo (2007).}\]
production functions. In Section 4 we develop formulas for the steady state. We use these formulas to assess the importance of openness for development. In Section 5 we examine the equilibrium adjustment path to the balanced growth path. We associate this with the transition from a set of closed countries to a set of open countries. We also examine the adjustment path for a closed economy becoming open relative to a set of already open countries. Here we are determining implications for countries joining a European Union type organization. We limit international borrowing and lending. Without this constraint, the convergence to the new balanced growth rate would be almost instantaneous, which is inconsistent with the experiences of countries that became more open by joining the European Union. In Section 6 we examine the adjustment path when countries differ in terms of their total factor productivity parameter and have this parameter adjust over time to the level of the countries making the direct foreign investment as happened in the European Union. Section 7 contains some concluding comments.

2. Some Motivation

The rich industrial countries are all open and economically integrated with the other advanced industrial countries. One set of major advanced industrial countries is virtually all the Western European countries plus a few small nearby countries economically integrated with these countries. Another set is Australia, Canada, New Zealand, and the United States. The last set are trade oriented Eastern Asian countries, namely Japan, Hong-Kong, South Korea and Singapore. All these countries are open in the sense that multinationals can locate in these countries with little fear of their operations being expropriated. All these economies are exporting industrial goods and services and have a vested interest in staying open.

Our measure of development is GDP per hour, which is what we mean when we say
productivity. We use this measure rather than output per capita because our paper is about the consequence of openness for productivity and not why incentives to work differ so much across the advanced industrial countries, and over time for the Western European countries.

In 1957 six countries signed the Treaty of Rome to form what became the European Union. These six countries are Belgium, France, Italy, Luxembourg, Netherlands, and West Germany. Figure 1 plots productivity of this set of countries, which we will call the EU-6, relative to U.S. productivity. Productivity rose to the U.S. level for the EU-6 over the subsequent 30 years.

The blip in the ratio in the late 1990s is almost surely the result of unmeasured U.S. output being abnormally high in this period as found in McGrattan and Prescott (2006a). The technology boom gave rise to exceptionally large expensed investments in starting of new businesses, R&D, and developing new products and processes. This leads us to conclude
the EU-6’s productivity was near 100 percent of U.S. productivity subsequent to 1990.

Figure 2 plots the productivity of the 1973-EU joiners, Denmark, Ireland, and the United Kingdom. This group is dominated by the experience of the United Kingdom, as its population is far larger than the populations of either Denmark or Ireland. What can be seen is that the productivity of this set of countries fell dramatically relative to the EU-6 prior to joining the European Union and, with a lag, increased subsequent to joining.

The relative productivity performance of the United Kingdom subsequent to joining is almost surely higher than Figure 2 indicates. We say this because the EU-6 instituted policies that screen out of the market a disproportionate numbers of low human capital people. Bourlès and Cette (2005) makes a human capital adjustment for France and estimates that French 2002 productivity is 8 percent lower than U.S. productivity, and not 7 percent higher, when corrections are made for workforce composition. This suggests that the productivities
of the plotted countries subsequent to 1985 could well be 5 to 10 percent higher than the plotted values.

Figure 3: 1995 Joiners’ Labor Productivity as Percent of EU-6 (1950-2006)

We turn now to the set of countries that joined the European Union in 1995, namely, Austria, Finland, and Sweden. Figure 3 shows that these countries’ productivities lost ground relative to the EU-6 prior to joining the European Union and gained ground subsequent to joining. A Western European country that is not richly endowed with oil and that never joined is Switzerland. Figure 4 shows that it, like the 1995 joiners, lost ground prior to 1995 but did not recover subsequent to 1995 as did the 1995 joiners.

What about those that entered in the 1980s? Greece, 15 years after joining in 1981, began to catch up. Portugal did some catching up, but then faltered over the last 10 years. The third country is Spain, a populous country by EU standards. Productivity measurement is difficult for Spain. It reformed its labor market policies and flattened its tax rates in the late
Figure 4: Switzerland’s Labor Productivity as Percent of EU-6 (1950-2006)

Figure 5: CE-8 Labor Productivity as Percent of EU-6 (1989-2006)
1990s. As a result its market hours per working age person increased by 30 percent between 1996 and 2006, which led to a reduction in the average human capital of its workforce. As a result, the composition effect is large. In addition to having new entrants into the workforce there are huge on-the-job investments in human capital. See Imai and Keane (2004), Parente and Prescott (2000), and Heckman, Lochner and Taber (1998). This is part of unmeasured output, and failure to include this factor would depress Spanish measured productivity relative to the EU-6.

To summarize, a strong regularity is that new members of the EU caught up to the industrial leader in terms of productivity. The deviations from this pattern, with the exception of Portugal, could well be due to errors in measuring productivity.

We turn now to the 2004 joiners and, in particular, to the eight Central European joiners. We call these the CE-8. Figure 5 plots their post 1989 performance. The plot shows some impressive productivity catching up associated with becoming open to the EU. The fact that EU membership fosters openness suggests that the CE-8 will be catching up to the EU-6 in the coming years.

A region of the world that was not open in the last half of the 20th Century is South America. In the 45 years between 1961 and 2006 its productivity fell from 35 percent of the U.S. level to 26 percent as shown in Figure 6. This is to be contrasted with the Asian countries whose productivity increased from 8.4 percent to 18 percent of the most advanced industrial countries. This rate of increase has increased over the last decade. This is shown in Figure 7. Openness in Asia increased in this period and Asia has become more integrated with the advanced industrial countries.
Figure 6: **Labor Productivity of South America relative to US (1960-2006)**

Figure 7: **Labor Productivity of Asia relative to US (1960-2006)**
3. Aggregation

Technology

The aggregate of capital services $k_i$ and labor services $l_i$ of country $i$ is

\[ z_i = k_i^\alpha l_i^{1-\alpha}, \text{ where } \alpha \in (0, 1) \]

One unit of domestic technology capital and $z_i$ units of this aggregate input at a given domestic location produces

\[ y_i = z_i^\phi, \text{ where } \phi \in (0, 1) \]

where $y_i$ is county $i$ final output.

The owner of this unit of technology capital “owns” the location production function specified by (2) for every domestic location. A given unit of the aggregate input $z_i$ can be used at one and only one location in country $i$.

This unit of technology capital can also be used to set up operations in a foreign location. One unit of $j \neq i$ technology capital operated at a location in country $i$ produces

\[ y = \sigma_i(k_i^\alpha l_i^{1-\alpha})^\phi, \text{ where } \sigma_i \in [0, 1]. \]

Our measure of openness of country $i$ is $\sigma_i$. The degree of openness of a country affects the relative productivity of foreign operations within its borders. If $\sigma_i = 1$, country $i$ is totally open to the use of foreign technology capital within its borders. If $\sigma_i = 0$, country $i$ is totally closed to the use of foreign technology capital within its borders.

The country specific parameter $A_i$ specifies the number of units of labor services provided per hour worked. It reflects differences in legal and regulatory environments. Thus,
implies \( l_i = A_i h_i \), where \( h_i \) is hours.

**A country's aggregate production function**

The set of countries is \( I = \{1, 2, \ldots, I\} \), where \( I \) is used to denote both the set and its cardinality. The measure of people is \( N_i \) and the measure of locations in country \( i \) is \( A_i N_i > 0 \). A country \( i \) firm in our analysis is a stock \( m_i \) of technology capital. The know-how is embodied within the firm. If a firm operates in foreign countries it is a multinational. Country \( i \)'s has technology capital stock \( M_i \). It is the sum of the technology capital stocks of all firms in country \( i \).

The aggregate production function is the maximum output that can be produced given the quantity of the factor inputs. It is

\[
\begin{align*}
\max_{z_1, z_2} \{ M_i N_i A_i z_1^\phi + \sigma_i \sum_{j \neq i} M_j N_i A_i z_2^\phi \} \\
\text{subject to } M_i N_i z_1 + \sum_{j \neq i} M_j N_i z_2 \leq Z_i.
\end{align*}
\]

(4)

The amount of the composite input used at each plant operated by a domestic firm is \( z_1 \), while the amount used at each plant operated by a foreign firm is \( z_2 \). The resulting country \( i \) production function upon substituting for \( Z \) is

\[
F_i(M_1, \ldots, M_I, K_i, L_i) = Y_i = (A_i N_i)^{1-\phi} \left( M_i + \omega_i \sum_{j \neq i} M_j \right)^{1-\phi} K_i^\alpha L_i^{(1-\alpha)\phi}
\]

(5)

where \( \omega_i = \sigma_i^{1/(1-\phi)} \).

An alternative and equivalent framework that leads to aggregate production functions (5) is to think of \( \omega_i \) as the fraction of foreign technology capital that is permitted to be used in country \( i \). A generalization of this is to have \( \omega_{ij} \) be the fraction of country \( j \) technology
capital that can be used in country $i$. This generalization is straightforward and is needed to deal with organizations such as the European Union.

We rewrite (5) as

$$F_i(M_1, \ldots, M_I, K_i, H_i) = (A_i N_i)^{1-\phi} \left( M_i + \omega_i \sum_{j \neq i} M_j \right)^{1-\phi} K_i^{\alpha \phi} (A_i H_i)^{(1-\alpha) \phi}$$

as $L_i = A_i H_i$. Functions $F_i$ display constant returns to scale in the inputs $\{K_i, H_i, \{M_j\}_{j=1}^I\}$. If two countries are totally open and have the same productivity parameter $A_i$, the stock of technology capital used in these countries is the world aggregate stock of technology capital. In this case the stocks of technology capital used are the same, there is no economic advantage or disadvantage in terms of living standards to being large.

The economy wide resource constraint is

$$Y_i = C_i + X_{ik} + X_{im} + NX_i$$

which states that GDP equals consumption plus investment in $K_i$ plus investment in technology capital $M_i$ plus net exports $NX_i$. The laws of motion of the two capital stocks are the usual ones:

$$K_{i,t+1} = (1 - \delta_k) K_{it} + X_{ikt}$$

$$M_{i,t+1} = (1 - \delta_m) M_{it} + X_{imt}$$

where the depreciation rates are positive and less than one.
4. Steady State Analyses

In our steady state analyses we will assume that labor is supplied inelastically as the elasticity of labor supply quantitatively does not matter for steady state comparisons. In each country there is a stand-in household whose preferences are ordered by

\[ \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(c_t). \]

The function \( u \) is strictly increasing and strictly concave as well as being continuously differentiable. When we deal with balanced growth, further restrictions will be imposed that are sufficient for the existence of a balanced growth equilibrium. We deal with steady state with each country \( i \) owning its \( K_i \) and \( M_i \) and with no international borrowing and lending.

The world steady state interest rate is \( \rho \) given preferences. The rental price of \( K \) is therefore \( \rho + \delta_k \). Equating the marginal product of \( K \) to its rental prices yields the first steady state condition

\[(10) \quad (\rho + \delta_k)K_i = \alpha \phi Y_i.\]

A second equilibrium condition is

\[(11) \quad L_i = A_i N_i = A_i H_i \]

as each person is assumed to have one unit of time and to supply it to the market.

An equilibrium relation that we use in subsequent analysis is the one obtained by substituting equilibrium conditions (10) and (11) into the production function (6):

\[(12) \quad Y_i = \psi A_i N_i \left( M_i + \omega_i \sum_{j \neq i} M_j \right)^{(1-\phi)/(1-\alpha\phi)}.\]
where
\[ \psi = \left( \frac{\alpha \phi}{\rho + \delta_k} \right)^{\alpha \phi/(1 - \alpha \phi)} . \]

We turn now to developing a set of equilibrium conditions which the technology capital stocks \( M_i \) must satisfy.

The *domestic return* on country \( i \) technology capital \( M_i \) is

\[
(13) \quad \frac{\partial F_i}{\partial M_i} = (1 - \phi) N_i \left( M_i + \omega_i \sum_{j \neq i} M_j \right)^\theta
\]

where \( \theta = (\alpha - 1) \phi / (1 - \alpha \phi) \). This follows from differentiating country \( i \) production function (6) with respect to \( M_i \), then using (10) and (11) to eliminate \( K_i \) and \( L_i \), and finally using (12) to eliminate the \( Y_i \) that comes in when \( K_i \) is eliminated. The *foreign return* on country \( i \) technology capital in country \( j \neq i \) is

\[
(14) \quad \frac{\partial F_j}{\partial M_i} = \omega_j (1 - \phi) N_j \psi \left( M_j + \omega_j \sum_{k \neq j} M_k \right)^\theta .
\]

This is determined in essentially the same way as (13).

The total return on country \( i \) technology capital is

\[
r_i(M) = \sum_j \frac{\partial F_j}{\partial M_i}
\]

where \( M = \{M_1, \ldots, M_I\} \). Equilibrium conditions for \( M \), which do not depend upon the other inputs, are:

\[
(15) \quad r_i(M) \leq \rho + \delta_m
\]

with equality if \( M_i > 0 \).
Proposition 1. A non-zero steady state exists.

Proof. We develop a function \( f(M) \) whose fixed points are steady state \( \dot{M} \). We use the Kakutani fixed point theorem to establish existence of a fixed point.

We first define the functions \( g_i(M_{-i}) \) to be the solution to (15) given \( M_{-i} \), which denotes the \( I-1 \) dimensional vector of the \( M_j \) for \( j \neq i \). The function \( g_i(M_{-i}) \) is decreasing, and therefore

\[
g_i(M_{-i}) \leq g_i(0).
\]

The convex compact set over which the mapping \( f \) will be defined is

\[
\Delta = \{ M \in \mathbb{R}_+^I : M_i \leq g_i(0) \ \forall \ i \}.
\]

The function \( f : \Delta \rightarrow \Delta \) is defined as follows: function \( g_1 \) is used to compute \( f_1(M) = g_1(M_{-1}) \). The vector \( (f_1(M), M_2, \ldots, M_I) \) and \( g_2 \) are used to determine \( f_2(M) \), and so forth. This \( I \)-stage updating defines the function \( f \).

The function \( f \) is continuous and maps convex compact set \( \Delta \) into itself. Therefore, it has a fixed point \( \dot{M} \). This fixed point is not zero for the following reason. If components \( M_1 \) to \( M_{I-1} \) were all zero, then \( M_I \) would be strictly positive. Thus, a non zero steady state equilibrium exists. ■

An Algorithm to Compute a Candidate Steady State

Consider the system
\[ (16) \quad \rho + \delta_k = \sum_{j \in I} \frac{\partial F_j}{\partial M_i} \quad i \in J \subseteq I \]
\[ (17) \quad M_i = 0 \quad \text{for } i \notin J. \]

This system can be solved uniquely for \( M = \{M_i\}_{i \in I} \). We will show this involves solving two systems of linear equations. The algorithm is:

- **Step 1.** Solve the system with \( J = I \). If \( M \geq 0 \) a steady state vector of \( M \) has been found. If not, go to step 2.

- **Step 2.** Remove the \( i \) from the set \( J \) for which \( M_i \) is most negative. Go to step 1.

With this algorithm, eventually a \( J \) will be found with solution \( M \geq 0 \) as \( I \) is a finite set.

If the vector obtained satisfies (15), it is a steady state. For the examples considered, the algorithm finds the unique steady state \( M \) vector. These examples have special structures on the \( \{\omega_i, A, N_i\} \) that ensures uniqueness.

**Four Examples**

We now carry out steady state analyses for four examples. The first example considers the advantage of size for a totally closed economy. The second example has two countries, a big country and a small country with common levels of openness. This example is motivated by Canada and the United States. The third example is designed to determine gains from expanding the size of an economic union where upon joining a country adopts the same openness policies as existing members. We determine gains to both the existing members and new members. This example is motivated by a country joining the European Union. The fourth example has a large set of closed economies. We show there are large gains to a country unilaterally becoming open. This suggests openness is a contributing factor in the
superior performance of the Chilean economy relative to other South American economies.

### Table 1. Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production parameters</td>
<td>$\alpha = .3$, $\phi = .94$</td>
</tr>
<tr>
<td>Depreciation rates</td>
<td>$\delta_k = .054$, $\delta_m = .08$</td>
</tr>
<tr>
<td>Interest rate</td>
<td>$\rho = .04$</td>
</tr>
</tbody>
</table>

The parameter values used are reported in Table 1. The parameters were selected to match a labor income share of 66 percent; a capital to output ratio of 3; a real interest rate of 4 percent; and a technology capital to output ratio of 0.5 for a large economic union such as the European Union or the United States if it were totally closed to the rest of the world and internally perfectly open. We picked a depreciation rate for technology capital of $\delta_m = 0.08$ which is lower than the Bureau of Economic Analysis estimate of 15 percent for the depreciation rate of R&D. We choose a lower number since technology capital also includes brand equity and organizational capital which depreciate more slowly.\(^4\)

**Example 1: Advantage of Size When Countries are Closed**

We first consider totally closed economies to focus on the advantage of country size per se when $\omega_i = 0$. The per capita output is proportional to the size $AN$; that is

$$y \propto (AN)^{(1-\phi)/[\phi(1-\alpha)]}.$$ 

The quantitative implication of being 10 times larger, as measured by $AN$ results in steady state output being 23.4 percent larger. This implies there are large gains to countries or

\(^4\)Although there is little direct evidence on depreciation rates for these capital stocks, there is some indirect evidence because we observe equity values.
states forming an economic union that are open with respect to each other. We turn now to varying the degree of openness.

**Example 2: Gains from Increasing the Degree of Openness**

This example assesses how productivity and welfare varies with openness for a two country world. One country has a much larger population that the other. The large population country might be thought of as the United States and the small population country as Canada. Given that U.S. population is almost ten times that of Canada, we set $N_1 = 10$ and $N_2 = 1$. The parameters that matter for this comparison are $\alpha$ and $\phi$ in the production functions (6). The openness measure is $\omega$ for both countries. Only the $N_i$ differ. We set $A_i = 1$ for $i = \{1, 2\}$ as this is just a normalization.

The equilibrium $M_1$ and $M_2$ can be found from equilibrium condition (15). If the solution to

$$\rho + \delta_m = r_i(M) \quad i = 1, 2$$  

(18) is non negative, this solution is the equilibrium $M$ vector. Otherwise, $M_2 = 0$ and $M_1$ is the solution to

$$\rho + \delta_m = r_1(M_1, 0).$$  

(19)

We turn now to a comparison of productivities. Figure 8 plots productivities of the two countries relative to the productivities of a totally closed Canada. Figure 9 plots the stocks of technology capital. The gains from being large are sizable unless the countries are nearly open. By moving from totally closed to perfectly open, Canadian productivity
increases by 24.4 percent.

These gains are even bigger if we use measured productivity which we define as gross domestic product (GDP) per person. Gross domestic product is output $Y_i$ less investment in technology capital because the latter is, for the most part, intangible investments of multinationals such as R&D and advertising which are not included in the national accounts. As Figure 9 shows, the big country (United States) does all of the investment in $M$ for $ω > .081$. Thus, the measured productivity of the small country eventually exceeds that of the big country. At $ω = 1$, the measured productivity of the big country is 96 percent of the small country’s.

The measure of welfare that we use is consumption. For steady state consumptions to be determined, asset ownership must be specified. We assume citizens of country $i$ own
\( K_i \) and \( M_i \) and that there is no international borrowing and lending. An implication of this is that the large country will have positive net foreign income and an equal size trade deficit.

With this asset ownership aggregate consumptions are

\[
C_1 = Y_1 - \delta_m M_1 - \delta_K K_1 + (1 - \phi)Y_2 \frac{\omega M_1}{M_2 + \omega M_1} - (1 - \phi)Y_1 \frac{\omega M_2}{M_1 + \omega M_2} \tag{20}
\]
\[
C_2 = Y_2 - \delta_m M_2 - \delta_K K_2 - (1 - \phi)Y_2 \frac{\omega M_1}{M_2 + \omega M_1} + (1 - \phi)Y_1 \frac{\omega M_2}{M_1 + \omega M_2}. \tag{21}
\]

Table 2 shows values of consumption per capita when \( \omega = 0 \) and when \( \omega \) approaches 1. The gain for the big country is only 1.2 percent while the gain for the small country is 21.3 percent—similar in magnitude to the gain in productivity.

The argument for two countries generalizes to \( I \) countries. First order the countries so that \( N_1 > N_2 > \cdots > N_I \). The value \( i^* \) must be found such that the solution to
\[ r_i(M) = \rho + \delta_m \quad i \leq i^* \]
\[ M_i = 0 \quad i > i^* \]
satisfies the equilibrium condition (15). They can be violated in two ways. The first way is if \( M_i < 0 \) for some \( i \). In this case, a smaller \( i^* \) is needed. The second way is if \( r_{i+1}(M) > \rho + \delta_m \). In this case, a larger \( i^* \) is needed.

<table>
<thead>
<tr>
<th></th>
<th>( \omega = 0 )</th>
<th>( \omega = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country 1</td>
<td>98.4</td>
<td>99.6</td>
</tr>
<tr>
<td>Country 2</td>
<td>79.8</td>
<td>96.8</td>
</tr>
</tbody>
</table>

**Example 3: Gains from Forming Larger Unions**

We define a union of states as a set of states with common openness policy \( \omega \) and totally closed with respect to the rest of the world. Let the number of states be \( I \). For these examples all members have equal size \( A_iN_i \).

Steady state per capita output as a function of the number of members is

\[ y(I) \propto (1 + (I - 1)\omega)^{(1-\phi)/(\phi(1-\alpha))} \]

Using estimates from Table 1, the ratio of \( y(I) \) for \( \omega = 1 \) and \( \omega = 0 \) is \( I^{0.91} \). Table 3 reports relative per capita incomes for several values of \( I \). As can be seen from Table 3, forming a unit of 20 members increases their per capita income and consumption by 27 percent if \( \omega = 2/3 \).
Table 3. Per Capita Output (with $y=1$ when $I=1$, $\omega=0$)

<table>
<thead>
<tr>
<th>$I$</th>
<th>$\omega = 0$</th>
<th>$\omega = 1/3$</th>
<th>$\omega = 2/3$</th>
<th>$\omega = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>1.199</td>
<td>1.269</td>
<td>1.314</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>1.204</td>
<td>1.275</td>
<td>1.320</td>
</tr>
</tbody>
</table>

Suppose a country joins a union of size $I = 20$ thereby making the union size $I = 21$. As can be seen from Table 3, this has a small effect upon existing members, with balanced growth per capita income increasing only 0.5 percent. But the increase of the joiner is large, being 27.5 percent.

Example 4: Consequences of Opening Unilaterally

All states are totally closed ($\omega_i = 0$) and of equal size, which is normalized to $AN = 1$. One state becomes totally open while the others remain totally closed to foreign technology capital. There are $I + 1$ countries in the world. We use equations (10) and (11) and the production function (6) to obtain

\[(23) \quad Y_c = \psi M_c^{(1-\phi)/(1-\alpha \phi)}\]

for a closed country $c$.

The country opening up exploits the world stock of technology capital $IM_c$. We assume $I$ is sufficiently large that $M_o = 0$ for the open country $o$. Thus,

\[\frac{Y_o}{Y_c} = I^{(1-\phi)/(1-\alpha \phi)} = I^{0.084}.\]
The important point is that unilaterally becoming open benefits the country that becomes open. In this case, if $I = 100$ the gains are 47 percent.

5. Adjustment Paths for Closed Economies Becoming Open

We turn now to the transition path of a country joining an economic union. For this analysis, we relax several assumptions made earlier in our steady state analysis. Here, we allow for elastically supplied labor and growth in population and technology. Utility flow in this case is given by

$$u(c, l) = \log c + \phi \log(1 - l),$$

with $\varphi = 2.5$ so that hours of work are consistent with observations. The growth rate in populations is $\gamma_N = .01$. The technology parameters evolve according to $A_t = (1 + \gamma_A)^t$, where $\gamma_A$ is set so that the interest rate ($= \rho$) is 4 percent as before. Therefore, total output in the economy grows at rate $\gamma_Y$ given by

$$\gamma_Y = [(1 + \gamma_A)(1 + \gamma_N)]^{1-\phi\alpha}/(\phi-\phi\alpha) - 1.$$ 

Along the transition paths, we constrain investments in both types of capital to be positive. These constraints ensure that the stocks are also positive.

Example 5. Catching-up After Joining a Larger Open Economy

We reconsider the case with two countries of size $N_1 = 10$ and $N_2 = 1$. We fix the technology parameters $A_i$, $i = 1, 2$, and set them equal to one across time and countries. The large country is a union of countries or states that are open to each other but not to the small country. In period 1, this country opens up to the small country. The small country
starts out closed and gradually opens to the large country. Here, we have in mind that a small European country joins the EU or countries like Canada and Mexico form an economic union with the United States.

The capital stocks in year 0 are the balanced growth values for $\sigma_1 = \sigma_2 = 0$, where $\sigma_i = \omega_i^{1-\phi}$. Figure 10 plots the paths of the openness parameter for the two countries. In year 1, the large country is effectively open and the small country is effectively closed. The openness parameter is equal to $\sigma_1 = .99$ which implies $\omega_1 = .85$. Subsequently, the openness parameter for the big country stays fixed near one, and the openness parameter for the small country increases to that of the big country. This choice of paths is motivated by the slow adjustments of GDP per hour following the formation of economic unions; the slow adjustment in $\sigma_2$ is due in part to political forces—that we treat exogenously here—
preventing an immediate opening to foreign multinational activity.\footnote{For the case that $\sigma_2 = \sigma_1$ in all periods of the transition, the results can be read off of our formulas in the last section.}

The paths of (aggregate) consumption relative to $(1 + \gamma)^t$ times the small country’s initial consumption level are shown in Figure 11.\footnote{The paths of per capita consumption are the same since both populations are proportional to $(1+\gamma_N)^t$.} Initially, virtually all of the small country gains are from returns on its technology being used in the big country. This is why the small country increases its technology capital initially as shown in Figure 12.

As the small country’s openness parameter increases, so does the amount of foreign technology used in the small country. The gains due to the increases in foreign technology capital used leads to further consumption growth. It also leads to a fall in the small country’s stock of technology capital beginning in year 17. In year 23 and subsequent years, no investment in technology capital is made by the small country. When both countries are
effectively open, their per-capita consumptions and labor inputs are equal but specialization in production persists with only the large country investing in technology capital.

Figure 12: Technology Capital Relative to $Y_{2,0}(1 + \gamma Y)^t$, Example 5

The increase in the small country’s actual productivity, but not the measured productivity, is similar in magnitude to the increase in its per-capita consumption. As Figure 12 makes clear, measured and actual productivity have very different paths. The large initial increases in the small country’s investment in technology capital implies that its measured productivity is initially low relative to the period before opening up to foreign technologies. For two years, the small country does no investment in nontechology capital $K$ while it builds $M$. During this time, measured productivity is roughly half of its initial level. After that, investment in technology capital slows but measured productivity does not recover to its initial level until after year 21. Thus, during the first two decades, it will seem as though opening has an adverse effect on the small country when in fact it has a very positive effect.
At the point that the small country’s technology capital starts to decline, there is rapid growth in measured productivity; it takes five years to catch up to the big country. Eventually, the measured productivity surpasses that of the big country because true productivities are the same but only the big country invests in technology capital. The example shows that measured productivity can give a distorted picture of actual economic performance.

Example 6. Leading in Openness and Lagging in Multinational Activity

As our earlier example of a country unilaterally opening demonstrated, opening up to other countries gives this country an advantage of using the world stock of technology. We turn now to an example motivated by the European Union opening to the United States and, subsequently, the United States opening to the European Union. The EU-6 countries in the period following World War II had a population similar to that of the United States but did little foreign direct investment in the United States relative to what the U.S. companies did in Europe.

Here, we show what can happen when two big countries or unions of similar size open at different times. We set $N_1 = 10$ and $N_2 = 10$ and assume that the initial capital stocks are equal for the two countries in year 0. The only difference between the countries is the timing of opening. The assumed paths of the openness parameters are plotted in Figure 13.

Figure 14 shows how seemingly similar economies, such as the United States and the European Union, can be very different in terms of their multinational activity. The country opening first exploits the more closed economy by using its technology capital. For this reason, it drops its investment in technology capital during the period when the relative σ’s are most different. Eventually, the country opening first does increase its investments in technology capital although the level of this investment is much lower than that done in the
Figure 13: Openness Parameters ($\sigma$), Example 6

Figure 14: Technology Capital Relative to $Y_{2,0}(1 + \gamma_Y)^t$, Example 6
country opening second.

If the countries were to open up further, the paths of technology capital would depend on the relative sizes of parameter $\sigma$. When fully open, nothing in the theory pins down the relative sizes of the technology capital stocks. Thus, it is possible for countries to have the same standard of living but very different levels of foreign direct investment.

6. Adjustment Path When TFP Rises

In the examples we have considered thus far, we have assumed that total factor productivities are fixed across time. One reason countries gain from opening up is that they improve their productive efficiency. For example, there is a lot of evidence that Central European countries gain in TFP as they open up. Their experience motivates our final example.

*Example 7. Gaining from Foreign Know-How and Efficiency*

We make a slight modification to Example 5 by assuming that the country opening also gains in economic efficiency as measured by $A_{2,t}/A_{1,t}$ increasing over time, with the rate of increase proportional to the rate of increase in $\sigma_{2,t}$ in Figure 10. When closed, the joining country has $A_2 = .9A_1$. During the transition, we assume that $A_{2,t}/A_{1,t} = .9 + .1\sigma_{2,t}$ while $A_{1,t} = (1 + \gamma_A)^t$.

In Figure 15 we plot the consumption paths for the small country joining the union. As before we plot consumption relative to $(1 + \gamma_Y)^tC_{2,0}$ and multiply by 100. We also plot consumption for the case with $A_{2,t}/A_{1,t} = 1$. This is the same series plotted in Figure 11.

Here, the gains from higher TFP are similar in magnitude to the gains from opening,
implying an overall gain to the smaller country joining the union of about 45 percent when compared to consumption levels for the countries when closed. This overall gain is far greater than any documented gains from trade.

7. Conclusions

In this paper we provide theoretical support for the view that gains from openness are large. The main avenue for gains in our theory is the foreign know-how brought into a country and used by multinationals in production there. We refer to this know-how as technology capital.

We see this paper as an extension of the neoclassical growth model. The extension is the concept of location which permits the introduction of know-how or technology capital without introducing nonconvexities. The theoretical structure matches well with the national
accounts and the international accounts. In this paper, we use the theory to assess the gains from openness, which we estimate to be large. In McGrattan and Prescott (2006b), we use the theory to successfully address a puzzle in the U.S. current account and net asset position. This extended theory leads us to view the world differently and some puzzling observations from the perspective of theory without our extension are not puzzling.
Appendix A: Data

Europe and U.S.

Labor productivity is defined as total gross domestic product (GDP) per annual hour worked. The labor productivity is calculated as:

\[
\frac{\sum_{i \in I} GDP_i}{\sum_{i \in I} \text{Hours Worked}_i}
\]

where \(i\) denotes a country in the group considered. For example, \(I\) is EU-6. Hence, the implicit assumption is that countries are weighted by their share of hours worked in total hours of the group.

South America and Asia

Data for GDP per annual hour worked in South America and Asia is scarce, so we measure productivity in these regions as GDP per person employed. This is calculated as:

\[
\frac{\sum_{i \in I} \text{GDP per person employed}_i \cdot \text{Population}_i}{\sum_{i \in I} \text{Population}_i}
\]

Units

The total GDP and GDP per person employed for a given country is measured in millions of US dollars (converted at Geary Khamis PPPs). Hence, labor productivity is expressed in 1990 Geary Khamis dollars.

Data Sources

• Groningen Growth and Development Center (GGDC), Total Economy Database. Go
to: http://www.ggdc.net/ then, Total Economy Database and download data. The
data used here are Total Economy Database, January 2007 (the database was last
updated in January).

– Data for Europe and the U.S. were downloaded on February 12, 2007.

– Data for South America and Asia were downloaded on March 21, 2007.

For all countries data prior to 1950 is from Maddison (1995). Table C-16a (page 249)
reports GDP in 1990 GK dollars and Table J-5 (pages 180-183) reports labor productivities
in 1990 GK dollars per hour.

For all countries, data for years 1950 - 2006 are from GGDC.
References


