Minimally Altruistic Wages and Unemployment in a Matching Model

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Abstract:
This paper presents a model in which firms recruit both unemployed and employed workers by posting vacancies. Firms act monopsonistically and set wages to retain their existing workers as well as to attract new ones. The model differs from Burdett and Mortensen (1998) in that its assumptions ensure that there is an equilibrium where all firms pay the same wage. The paper analyzes the response of this wage to exogenous changes in the marginal revenue product of labor. The paper finds parameters for which the response of wages is modest relative to the response of employment, as appears to be the case in U.S. data and shows that the insistence by workers that firms act with a minimal level of altruism can be a source of dampened wage responses. The paper also considers a setting where this minimal level of altruism is subject to fluctuations and shows that, for certain parameters, the model can explain both the standard deviations of employment and wages and the correlation between these two series over time.

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Job vacancies drop considerably in recessions, suggesting that recruitment costs are quite procyclical. As emphasized by Rotemberg (2007b) in the context of the Mortensen-Pissarides (1994) model, the procyclicality of recruitment costs implies that real wages should be much more procyclical than they actually are. The model makes this prediction because the ease with which firms can recruit workers in recessions strengthens firms’ bargaining position so that Nash bargaining between firms and workers leads to substantially lower wages.

This paper departs from the Mortensen and Pissarides (1994) model in several respects. First, instead of assuming that wages are determined by a bargaining process, it assumes that firms set wages unilaterally. As in Burdett and Mortensen (1998), firms act somewhat monopsonistically in the model developed here. They realize, in particular, that reductions in their wages lead only some of their to employees to depart. In efficiency wage terms, the model of this paper is thus “turnover” based.¹

The result of this wage-setting assumption is that, unlike in the bargaining case, employee’s reservation wages no longer matter for wage determination. Instead, wages are greatly affected by the way employees weigh wage and nonwage aspects of a job when deciding among job opportunities. Nonwage aspects of jobs, and their role in creating job satisfaction, are stressed in the managerial literature on “voluntary turnover,” and they also play a role in the model of Nagypál (2005) that tries to explain the magnitude of this turnover.²

This paper also extends the canonical Mortensen and Pissarides (1994) model by considering not only a specification in which firms act selfishly but also one in which firms act somewhat altruistically. This specification is motivated by Rotemberg (2007a), which explains ultimatum and dictator experiments with a model where people react with anger

¹For a discussion of efficiency-wage models in general and their division into models based on reducing shirking, reducing turnover, improving selection, and increasing effort as a result of fairness considerations, see Katz (1986).
²For a discussion of the managerial literature, see Price (2001). Using exit interviews from a firm that experienced a great deal of turnover, Sutherland (2002) found that most people who left this firm for another job claimed that their reason for doing so was either higher wages or the opportunity to earn more overtime. Still, even in this case, 18 of the 48 people who left for another job reported doing so for other reasons. One difficulty with studying the sources of quits is that, as noted by Hinrich (1975), exit interviews do not provide the same answers as questionnaires on people’s intention to quit.
when they observe someone acting with insufficient altruism. The underlying idea behind this model is that people expect those with whom they interact to have a minimal level of altruism and that, while they initially assume at least a minimally acceptable level of altruism on the part of others, they get angry when people demonstrate a degree of altruism below this minimal level. Several previous models of fairness, most notably Fehr and Schmidt (1999) and Levine (1998), have been proposed to explain these experimental outcomes, but Rotemberg (2007a) argues that they can do so only with implausible preference parameters that are contradicted by other experiments. In contrast, when people’s reaction to insufficient altruism is suitably strong, these experiments can be explained with plausibly small degrees of altruism as in Rotemberg (2007a).3

One reason to be interested in this particular model of fairness in the labor context is that there are numerous instances in which workers who feel mistreated spend resources lashing out at their employer (or ex-employer in the case of wrongful termination lawsuits) (Rotemberg 2006). Bewley (1999) provides some evidence that firms are concerned about these potential reactions. In his survey asking firms why they did not cut wages in a recession, the most common answer was that “pay cuts hurt morale and demotivate workers” (Bewley 1999, p. 174). However, the relationship may be asymmetric: Rotemberg (2006) finds very little field evidence that workers increase their effort when they face better-than-normal conditions of employment.4

The model I propose should complement the literature that incorporates fair wage considerations into macroeconomic models, much of which is also inspired by Bewley (1999). This literature follows Akerlof’s (1982) gift-exchange model in which workers’ effort depends on the difference between the wage they receive and a reference wage. In the application

3There is, interestingly, also direct neurological evidence for this approach. Several investigators, including Morrison et al. (2004) show that pain-related areas of the brain become activated not only when a painful impulse is applied to a person but also when a person sees that someone else is being subjected to this impulse. This “mirror-neuron” response of observers turns out to depend on the altruism that the person being observed has evinced in the past. As shown by Singer et al. (2006), the neural response of subjects when they see someone receive an electric shock are smaller when that person has previously been observed making a low offer in a variant of a dictator game.

4The field experiments of Gneezy and List (2006) show that high wages motivate workers only for a very short period of time.
of this idea to macroeconomics, several distinct models of the reference wage have been proposed. Danthine and Donaldson (1990) and Chéron (2002) assume that the reference wage equals a geometric weighted average of the wage paid by other firms and the level of income received by self-employed individuals, with the latter having a weight equal to the unemployment rate. Collard and de la Croix (2000) and Danthine and Kurmann (2004) assume instead that past wages are also an important component of the reference wage. This obviously introduces additional wage stickiness. Lastly, Danthine and Kurmann (2006) lets effort depend on the relationship between the wage that the firm offers and the level of labor productivity, on the grounds that the latter represents the firm’s capacity to pay. This specification is somewhat related, though by no means identical, to Rabin (1993).

Aside from avoiding the difficult problem of specifying reference wages, the altruism-based model proposed here has the advantage of enabling straightforward consideration of the limit at which workers do not impose any fairness considerations on firms. It seems harder to consider this limit in models in which effort depends continuously on the wage offered by firms. A second advantage of focusing on outcomes in which firms are somewhat altruistic towards their workers is that this approach matches the content of at least some companies’ mission statements. In the compilation of such statements in Abrahams (1995), several companies promise to “care” for their employees. Gibson Greetings’ statement, for example, says “We are a team... We trust, respect and care for each other” (Abrahams 1995, p. 296.) Similarly, Tultex’s statement of values tells employees “We will create, through the contributions of each of us, a quality of worklife that is recognized by the caring, openness and understanding of each other” (Abrahams 1995, p. 549.) In a slightly different vein, but also consistent with some form of altruism, Johnson & Johnson’s corporate “credo” reads (in part) “We are responsible to our employees, the men and women who work with us throughout the world ... We must be mindful of ways to help our employees fulfill their family responsibilities.” These statements may be irrelevant, though some effort does appear to go into their creation and dissemination. They may also be seen as profit-maximizing

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5This point of view is adopted also by Kahneman, Knetch, and Thaler (1986).
strategies for recruiting good employees. Even in that case, the statements suggest that the analysis of firms that pretend to be altruistic may be valuable.

A final advantage of studying firms that are (or pretend to be) altruistic is that it allows one to consider a new source of wage and employment fluctuations. These variables respond, in particular, to changes in the level of altruism required by workers. When required altruism rises, wages rise and this leads firms to curtail their employment. This force thus induces a negative correlation between real wages and employment and thus serves to dampen the positive correlation induced by the technology shocks considered in Mortensen and Pissarides (1994) and Shimer (2005).

Hall (2006) employs a similar device in that he assumes that there are systematic variations in the bargaining power of workers. One difference between the two approaches is that Hall (2006) assumes that worker bargaining power is a deterministic function of the state of the labor market. By letting workers have more power in a slack labor markets model, he ensures that real wages fluctuate less in his model than they would if bargaining power were constant. I consider instead a situation where the fluctuations in required altruism (that induce countercyclical real wages) are not caused by changes in labor market tightness. This has the advantage of being consistent with the relatively low correlation between wages and employment.

The analysis proceeds as follows. Section 1 lays out the basic monopsony model without firm altruism. While inspired by Burdett and Mortensen (1998), this model differs in that it has an equilibrium in which all firms pay the same wage. The existence of such an equilibrium is due (in part) to the assumption that firms have multiple workers and that the marginal revenue product of labor decreases when firms hire more workers. Thus, a firm with systematically high wages keeps growing by attracting new workers and thereby finds itself with more workers than is optimal. In Burdett and Mortensen (1998) this effect is absent because each firm can accommodate at most a single worker (and must keep its wage high after attracting a worker because it is committed to a policy of constant wages).

Section 2 rationalizes a model with minimal firm altruism and derives its equilibrium
properties. Section 3 explains the numerical values assigned to the model’s parameters in the simulations whose results are reported in Section 4. Section 5 adds post-matching training costs. As Silva and Toledo (2006) have shown, such training costs tend to improve the performance of matching models when wages are set via bargaining. Training costs are conceptually just as important when firms act monopsonistically, since firms can reduce their training costs by raising wages so that employees remain at the firm. Unsurprisingly, the introduction of these costs affects the quantitative performance of this model as well. Section 6 concludes.

1 Basic Model

The utility of worker $j$ when employed by firm $i$ is assumed to be

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \rho_w^\tau u_i^j, \quad u_i = \frac{(C_i^j)^{1-\gamma}}{1-\gamma} + x_i^{ij}, \quad (1)$$

where the $\mathbb{E}_t$ operator takes expectations conditional on information available at $t$, $\rho_w$ is the discount rate of workers, $x_i^{ij}$ is the individual’s nonpecuniary utility from working at $i$ (set equal to $x^u$ if he were unemployed), while $C_i^j$ is an index of the individual’s purchases of a variety of different products.

Specifically, the index $C_i^j$ is given by the Dixit-Stiglitz aggregator

$$C_i^j = \left[ \sum_i (c_i^{ji}) \frac{\epsilon_t}{\epsilon_t - 1} \right]^{\frac{\epsilon_t}{\epsilon_t - 1}}, \quad (2)$$

where the parameter $\epsilon_t$ is allowed to vary over time and $c_i^{ji}$ is the quantity of good $i$ bought by $j$ at time $t$. If this individual’s nominal spending at $t$ is $S_i^j$ and the price of good $i$ at $t$ is $p_i^j$, his utility is maximized when his individual purchases satisfy

$$c_i^{ji} = \frac{S_i^j}{\bar{p}_t} \left( \frac{p_i^j}{\bar{p}_t} \right)^{-\epsilon_t^i}, \quad (3)$$

where

$$\bar{p}_t = \left[ \sum_i (p_i^t)^{-\epsilon_t} \right]^{\frac{1}{\sum_i \epsilon_t}}. \quad (4)$$
Using (2) and (3), it follows that the consumption index satisfies

$$C^j_t = S^j_t / \bar{p}_t.$$  

(5)

I assume that workers neither borrow or lend. This assumption becomes more plausible if one assumes that workers have discount rates that exceed equilibrium interest rates (so that they would like to borrow at current rates) while also assuming that bankruptcy provisions allow them to escape from debt obligations. The result is that no one lends to them in equilibrium. In practice, Kenickel, Starr-McCluer, and Sundén (1997) report that only about 10 percent of U.S. households had no financial assets in 1992. Over half, however, had no asset other than a “transactions account,” and some workers may exhaust these accounts right before they receive their paychecks.\footnote{While some borrowing, particularly “payday borrowing,” seems possible even for households without financial assets, its scope may well be limited.}

Since workers neither borrow nor lend, worker $j$’s real purchases $S^j_t / \bar{p}_t$ equal his real wage $w^j_t$ when he is employed. When the worker is unemployed, his real consumption equals the level of unemployment insurance $C_u$. The lack of borrowing and lending also implies that the discount rate $\rho^w$ is unimportant. The curvature parameter $\gamma$, by contrast, is shown to be important below. To further simplify the analysis, I assume that expenditures on unemployment insurance are financed by lump-sum taxes levied on the owners of firms.

For unemployed workers, the nonpecuniary compensation $x$ can be set to an arbitrary value $x_u$. In the case of employed workers, an important assumption for the analysis is that each worker expects his own $x$ to vary over time. This variability could capture changes in people’s preferred locations. Or it could be due more generally to changes in individual tastes for the particular amenities offered by any particular employer. As stressed by Nagypál (2005), this variability provides a rationale for worker mobility from job to job. This is particularly important in the current model, because I concentrate on equilibria with symmetric wages. When all firms pay the same wage, workers who care only about wages have no reason to change employers.
The owner of firm $i$ is assumed to choose her consumption path to maximize

$$E_t \sum_{\tau=0}^{\infty} \rho^\tau C^i_t,$$

where $C^i_t$ is the Dixit-Stiglitz aggregator introduced above and $\rho$ is the owner’s discount rate.

The linearity of these preferences has two consequences. First, if owners can borrow and lend at the nominal rate of interest $i_t$, it implies

$$1 = E_t\rho \frac{\bar{p}_t(1 + i_t)}{\bar{p}_{t+1}}.$$  

This keeps owners indifferent between consuming a unit of consumption at $t$ and increasing consumption at $t+1$ by $(1+i_t)\bar{p}_t$ dollars. This indifference must hold at all $t$ if, in equilibrium, the consumption of owners is positive in each period. Since the wage bill is lower than the value of all goods produced in each period, owners’ $C^i_t$ is indeed positive for all $t$ and (7) must hold. An even more immediate implication of assuming that the owners maximize (6) is that firms that respond to their owner’s wishes use $\rho$ to discount future profits when they maximize the present discounted value of profits.

Firms generate revenues by giving workers productive tasks and selling the proceeds. The output of individual firm $i$ with $h^i_t$ workers at $t$ is $z_t f(h^i_t)$. When it charges a relative price of $p^i_t/\bar{p}_t$, its demand is $Y_t(p^i_t/\bar{p}_t)^{-\epsilon_t}$, where $Y_t$ are total final sales. Letting the firm set its price so that it sells the quantity that it has produced, its real revenues at $t$ are thus

$$R^i_t(h^i_t, Y_t, z_t, \epsilon_t) = Y_t^{1/\epsilon_t} [z_t f(h^i_t)]^{1-1/\epsilon_t}. $$

This implies that the marginal revenue product of labor is

$$\frac{dR^i_t}{dh^i_t} = Y_t^{1/\epsilon_t} z_t^{-1/\epsilon_t} \left(1 - \frac{1}{\epsilon_t}\right) f(h^i_t)^{-1/\epsilon_t} f'(h^i_t),$$

where primes denote derivatives. The marginal revenue product of labor depends positively on both technical progress $z_t$ and on $\epsilon_t$, the extent to which the product market is competitive at time $t$. These variables increase labor demand at $t$, either by making labor more productive or by reducing the monopolistic distortion that keeps labor demand low.

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Given the discussion above, the firm’s objective at time $t$ is to maximize $\Pi_i^t$

$$
\Pi_i^t = E_t \sum_{\tau=0}^{\infty} \rho^\tau \pi_i^{t+\tau},
$$

(10)

where $\pi_i^t$, its real profits at $t$, can be expressed as

$$
\pi_i^t = R_i^t - w_i^t h_i^t - \kappa(v_i^t).
$$

(11)

In this equation, $\kappa$ represents recruiting costs, while $w_i^t$ and $v_i^t$ are the real wage and the vacancies posted by firm $i$ at $t$, respectively. An important assumption in (11) is that the firm pays the same wage to all its workers. It is standard in turnover efficiency-wage models to assume that firms do not know the outside opportunities available to their employees, so it is optimal for them to pay the same wage to all their existing employees. On the other hand, firms may wish to treat new employees differently from existing ones. Firms with access to the relevant information may also wish to discriminate among new hires and let their wage depend on their previous employment status. As a first cut at the problem, I neglect these possibilities and imagine that each firm feels compelled to pay all its employees the same wage. In a more complete model, this result might be derived from informational imperfections and from firms’ desire to demonstrate their altruism.\footnote{Levine (1993) provides evidence that the structure of wages within job ladders (that is, across seniority for people doing similar tasks) is quite rigid. The compensation executives interviewed by Levine (1993) were unwilling to institute significant changes in relative wages within an occupation even when they were told that these relative wages had changed in the outside labor market. Quoting an executive, Levine (1993, p. 1256) says “If you pay new workers more than senior ones, ‘You will have an employee revolt on your hands.’” One cost borne by an existing employee when a new employee is brought in at a higher wage is that the employee regrets not having sought alternative employment at an earlier date. An altruistic firm might want to spare its employee this regret cost, and one way of doing so is to maintain a rigid wage structure. It is also worth noting that one of the issues that led to the unsafe tires discussed in Krueger and Mas (2004) was the attempt by Firestone to lower the wages of new employees by 30 percent.}

The dynamics of the labor market are similar to those in Mortensen and Pissarides (1994), with the proviso that firms that post vacancies attract both unemployed and currently employed individuals, as in the job-to-job transition models of Krause and Lubik (2005) and Nagypál (2005). Normalizing the labor force to equal one, let $h_t$ represent employment while $u_t$ equals the number of unemployed workers. Thus

$$
h_t + u_t = 1
$$

(12)
Unemployed people are assumed to meet open vacancies randomly. Letting $v_t$ denote the total vacancies posted by firms at $t$, the total number of meetings between the unemployed at $t - 1$ and firms at $t$ can be expressed as

$$m^u_t = u_{t-1} \left( \frac{v_t}{u_{t-1}} \right)^\eta.$$

(13)

This equation does not include a constant because the level of vacancies can be normalized, rendering this constant unnecessary. This is identical to the matching function in Mortensen and Pissarides (1994). I assume that the nonpecuniary utility of a particular individual at a job $x^{ij}_t$ is independently distributed over time and independent across individuals. This means that, as long as the equilibrium wage $w_t$ and the lowest possible level of $x$ on the job ($x_e$) satisfy

$$\frac{(w_t)^{1-\gamma}}{1-\gamma} + x_e > \frac{(C_u)^{1-\gamma}}{1-\gamma} + x_u,$$

any person who meets an open vacancy chooses to become employed. Assuming a constant rate $s$ at which workers leave jobs for unemployment, the meeting function (13) implies that employment evolves according to

$$h_t = (1 - s)h_{t-1} + u_{t-1} \left( \frac{v_t}{u_{t-1}} \right)^\eta.$$

(14)

Analogously to the meetings function for unemployed individuals, the number of meetings at $t$ between employers and people who were employed at $t - 1$ can be written as

$$m^h_t = \bar{m}h_{t-1} \left( \frac{v_t}{h_{t-1}} \right)^\ell,$$

(15)

where $\bar{m}$ and $\ell$ are constants.

The number of meetings that firm $i$ has with potential new employees is governed by the ratio of its own vacancies to the total number of vacancies. Firm $i$’s total number of

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8The use of a constant separation probability $s$ simplifies the analysis. In the United States, this separation rate does tend to rise somewhat in recessions. As a result, (14) overstates the extent to which vacancies need to rise in booms if $\eta$ is calibrated to match the relationship between the exit rate from unemployment and the $v/u$ ratio, as in Shimer (2005). As I discuss below, this leads me to use Mortensen and Nagypál’s (2005) value for $\eta$. 

9
meetings with unemployed individuals, $\tilde{m}_t^{ui}$ satisfy
\[
\tilde{m}_t^{ui} = \frac{v_t^i}{v_t} m_t^u = v_t^i \left( \frac{v_t}{u_t^{-1}} \right)^{\eta - 1},
\]
where the second equality uses (13). Let $\tilde{m}_t^{di}$ represent the total number of firm $i$ employees who meet new potential employers at $t$. The probability that a particular employee will encounter a potential employer ought not to depend on his original employer’s size. Therefore,
\[
\tilde{m}_t^{di} = \frac{h_t^i}{h_t^{-1}} m_t^h. \tag{17}
\]

By the same token, the number of meetings that firm $i$ has with people who were employed at other firms, $\tilde{m}_t^{ai}$ obeys
\[
\tilde{m}_t^{ai} = \frac{v_t^i}{v_t} \sum_{j \neq i} \frac{h_t^j}{h_t} m_t^h = \frac{v_t^i}{v_t} \left( 1 - \frac{h_t^i}{h_t} \right) m_t^h 
\approx \frac{v_t^i}{v_t} m_t^h. \tag{18}
\]
The approximation in the second line is valid when each firm represents only a small fraction of total employment, because this situation implies that the product of $v_t^i/v_t$ and $h_t^i/h_t$ is vanishingly small.

An individual $k$ who worked for firm $i$ at $t - 1$ and meets firm $j$ at $t$ can decide whether to stay at $i$ or join $j$. He stays if he expects
\[
E_t \sum_{\tau=0}^{\infty} \rho_{\tau}^s \frac{(w_t^i + \tau)^{1-\gamma}}{1-\gamma} + x_t^{ik} \geq E_t \sum_{\tau=0}^{\infty} \rho_{\tau}^s \frac{(w_t^j + \tau)^{1-\gamma}}{1-\gamma} + x_t^{jk}. \tag{19}
\]
I assume all firms face the same cost and demand conditions and seek a symmetric equilibrium where all firms offer the same wage. I thus assume that workers expect all firms to offer the same wage in the future, and I compute conditions under which firms also desire to do so today. Since $x_t^{ik}$ and $x_t^{jk}$ are independently and identically distributed over time, a worker $k$ at firm $i$ with a job prospect at firm $j$ stays at firm $i$ if
\[
\frac{(w_t^i)^{1-\gamma}}{1-\gamma} + x_t^{ik} \geq \frac{(w_t^j)^{1-\gamma}}{1-\gamma} + x_t^{jk}. \tag{20}
\]
Otherwise, he leaves. Let $F$ be the pdf for $[x_{ik}^j - x_{ik}^i]$. Since the $x$s are drawn from the same distribution, the resulting density must be symmetric, so that $F(y) = 1 - F(-y)$. The probability that a worker who can earn a wage $w_i^t$ at firm $i$ and a wage of $w_j^t$ at firm $j$ remains at firm $i$ is then

$$F\left(\frac{(w_i^t)^{1-\gamma} - (w_j^t)^{1-\gamma}}{1-\gamma}\right).$$

Assume for the moment that all firms other than $i$ pay the wage $\tilde{w}_t$ at $t$. The number of employees of firm $i$ at time $t$ is then

$$h_i^t = (1-s)h_{t-1}^i + \bar{m}_i^{ui} - \bar{m}_i^{di} + (\bar{m}_i^{di} + \bar{m}_i^{ui})F\left(\frac{(w_i^t)^{1-\gamma} - (\tilde{w}_t)^{1-\gamma}}{1-\gamma}\right).$$

The second equality uses (16), (18), and (17) to replace $m_i^{ui}$, $m_i^{di}$, and $m_i^{ui}$, respectively, and uses $F_t^i$ to denote $F([((w_i^t)^{1-\gamma} - (\tilde{w}_t)^{1-\gamma})/[1-\gamma])$.

I now study the profit-maximizing choices of wage and vacancy rates. To carry out this analysis, I use (10) and (11) to rewrite the expected present value of profits as

$$\Pi_i^t = R_i^t(h_i^t) - w_i^t h_i^t - \kappa(v_i^t) + E_t\rho(R_{t+1}^i - w_{t+1}^i h_{t+1}^i - \kappa(v_{t+1}^i)) + E_t\rho^2\Pi_{t+2}^i(h_{t+1}^i).$$

(22)

Since the firm is unable to influence workers’ beliefs concerning future wages, its choice of $w_{t+j}^i$ and $v_{t+j}^i$ affects profits only from $t + j$ onwards. It follows from (22) that, if the firm were able to choose the state-contingent levels of future wages and vacancies at $t$, it would choose the same levels as would be chosen at future dates. The time-consistency of the firm’s problem implies that an optimizing firm at $t$ should not be able to increase $\Pi_i^t$ by varying $w_i^t$, $v_i^t$, and $v_{t+1}^i$, while keeping constant its plan for future levels of the $h$’s. For the state contingent value of $h_{t+1}^i$ to stay the same in spite of changes in wages and vacancies at $t$, vacancies at $t + 1$ must respond. Equation (21) can be used to compute the requisite variation in $v_{t+1}^i$. This is given by

$$v_{t+1}^i = v_{t+1} \left(\frac{h_{t+1}^i - h_t^i(1-s) + (h_t^i/h_t)(1 - F_{t+1}^i)m_{t+1}^h}{F_{t+1}^i m_{t+1}^h + m_{t+1}^u}\right).$$

(23)

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Using this expression to substitute for \( v_{i+1}^{t} \) in (22), the firm’s objective function becomes

\[
\Pi_t^i = R_t^i(h_t^i) - w_t^i h_t^i - \kappa(v_t^i) + E_t \rho^2 \Pi_{t+1}^i (h_{t+1}^i) + E_t \rho \left( R_{t+1}^i - w_{t+1}^i h_{t+1}^i - \kappa \left( v_{t+1} - h_{t+1}^i h_t^i (1-s) + (h_t^i / h_t^i) (1 - F_{t+1}^i) m_{t+1}^h \right) \right). \tag{24}
\]

For any contingent plans concerning \( w_{i+j}^t \) and \( h_{t+j}^i \), an optimizing form must satisfy the following first-order conditions with respect to \( v_t^i \) and \( w_t^i \):

\[
\frac{d \Pi_t^i}{dh_t^i} - \kappa = 0, \quad \text{and} \quad \frac{d \Pi_t^i}{dw_t^i} - h_t^i = 0. \tag{25}
\]

In these equations, \( d \Pi_t^i / dh_t^i \) represents the derivative of the expression in (24) with respect to \( h_t^i \), so this derivative holds constant future plans for employment and wages. Equation (24) implies that

\[
\frac{d \Pi_t^i}{dh_t^i} = \frac{d R_t^i}{dh_t^i} - w_t^i + E_t \rho k_{t+1}^i \left( 1 - s - \bar{m} (1 - F_{t+1}^i) (v_{t+1} / h_t) \right) \left( v_{t+1} / h_t \right) \left( v_{t+1} / u_t \right)^\eta - 1,
\]

where (13) and (15) are used to substitute for \( m_{t+1}^u \) and \( m_{t+1}^h \), respectively. Meanwhile, differentiation of (21) implies that

\[
\frac{dh_t^i}{dv_t^i} = F_t^i m_t^h + m_t^u \left( v_t / h_t \right)^\eta - 1 + \left( v_t / u_t \right)^\eta - 1, \quad \text{and} \quad \frac{dh_t^i}{dw_t^i} = (w_t)^{-\gamma} F_t^i m_t^h \left( h_t / h_t \right)^\eta - 1 + \left( v_t / v_t \right)^\eta - 1. \tag{28}
\]

Because I consider the possibility that the vacancy cost function \( \kappa \) is concave, it is particularly important to check the second-order conditions with respect to vacancies. These conditions are necessary to prevent firms from preferring an oscillation between high and low vacancy levels, which lowers average recruitment costs when \( \kappa'' < 0 \). Since the left-hand side of (25) represents the derivative of profits with respect to vacancies, the second derivative of profits with respect to vacancies can be obtained by differentiating this expression:

\[
\frac{d^2 \Pi_t^i}{dv_t^i} = \frac{d^2 \Pi_t^i}{dh_t^i} \left( \frac{dh_t^i}{dv_t^i} \right)^2 + \frac{d \Pi_t^i}{dh_t^i} \frac{d^2 h_t^i}{dv_t^i} - \kappa''.
\]

\[
12
\]
This second derivative must be negative for the second-order condition to be satisfied. As shown in Appendix 2, this condition reduces to

\[
\frac{d^2 R_i}{(dh_i)^2} < \kappa'' \frac{1 + \rho[1 - s - (\bar{m}/2)(v_{t+1}/h_t)]^2}{[(\bar{m}/2)(v_i/h_{t-1})^\ell - 1 + (u_i/h_{t-1})^{\eta - 1}]^2}
\]

(31)
at a symmetric steady state.

Combining (25) and (26) to eliminate \(d\Pi_i/dh_i\), we have

\[
F_i \bar{m} \left( \frac{v_i}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_i}{u_{t-1}} \right)^{\eta-1} = \kappa'_i (w_i)^{-\gamma} F'_i \bar{m} \left( \frac{h_{t-1}}{h_{t-1}} + \frac{v_i}{u_i} \right) \frac{h_{t-1}}{h_t}.
\]

(32)

This equation admits of a solution where employment, wages, and vacancies at \(t\) are the same for all firms (equal to \(h_t, w_t,\) and \(v_t,\) respectively). At this symmetric solution, \(F'_i\) must be equal to \(F'(0),\) and, since \(F(y) = 1 - F(-y),\) \(F(0) = 1/2.\) Using \(\bar{F}'\) to denote the value of \(F'(0),\) (32) becomes

\[
\frac{\bar{m}}{2} \left( \frac{v_i}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_i}{u_{t-1}} \right)^{\eta-1} = 2\kappa'_i (w_t)^{-\gamma} \bar{F}' \bar{m} \frac{h_{t-1}}{h_t} \left( \frac{v_i}{h_{t-1}} \right)^{\ell}.
\]

(33)

Also, letting \(dR_i/dh_i\) denote the common value of \(dR_i/dh_i,\) (25) becomes

\[
\frac{dR_i}{dh_i} - w_i - \frac{\kappa'_i}{\frac{m}{2} \left( \frac{v_i}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_i}{u_{t-1}} \right)^{\eta-1}} + \frac{\rho k_{t+1}}{m} \left( 1 - s - \frac{m}{2} \left( \frac{v_{t+1}}{h_t} \right)^{\ell} + \left( \frac{v_{t+1}}{u_t} \right)^{\eta-1} \right) = 0.
\]

(34)

The symmetric equilibrium values of \(w_t, v_t, h_t,\) and \(u_t\) must thus satisfy the four equations (12), (14), (33), and (34). As long as the total number of firms is normalized to equal one, aggregate sales at this equilibrium \(Y_t\) equal the sales of each individual firm.

One interesting special case occurs when workers have a constant marginal utility of income, so \(\gamma = 0.\) In this case, (33) determines the level of vacancies at \(t\) for a given level of past employment \(h_{t-1}.\) Thus, employment is unaffected by \(dR/dh,\) the extent to which firms benefit from an additional employee. Rather, (34) implies that changes in \(dR/dh\) are simply reflected in wage changes.

Given that workers consume all of their income, the parameter \(\gamma\) does not affect intertemporal choices. It does, however, affect the relationship between the individual’s wage
(or income) and the number of dollars an individual is willing to give up for an additional unit of nonpecuniary consumption. When $\gamma = 0$, the wage has no effect on this number. From the point of view of firms, this means that the number of additional workers that firms retain by raising the wage by one dollar is independent of the equilibrium wage and depends only on the tightness of the labor market. At the same time, the number of extra workers that firms attracts by posting an additional vacancy also depends on the tightness of the labor market. Thus, the requirement that the firm be indifferent between attracting an additional worker through extra vacancies and attracting the worker by raising the wage determines the equilibrium level of labor market tightness and thus the level of employment.

2 Required Firm Altruism

One attractive feature of the monopsony model just described is that wages are clearly set by firms, so workers are able to their employer’s attitude from the wages she pays. Workers are, in particular, able to form an opinion about the altruism of their employer. This fits both with common parlance (where the term “good employer” is often used) and with the model of Rotemberg (2007a). In that model, the people who are affected by a decision use their information to assess the altruism of the decision-maker. If the affected party can reject the hypothesis that the decision-maker is minimally altruistic, he becomes angry. In other words, his utility function changes so that he now derives utility from harming the decision-maker. The opportunities available for workers to harm their employers are quite numerous, since workers have ready access to their employers’ assets. Indeed, workers have been observed to cause losses to their employers on several occasions. Krueger and Mas (2004), for instance, show that unhappy workers at a plant in Decatur were disproportionately to blame for the defective Firestone tires that were linked to Ford Explorer rollovers.

As discussed earlier, the idea that firms wish to be perceived as altruistic towards their workers fits with numerous corporate mission statements. One might imagine that people

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9 Typing “good employer” in Google returns about 250,000 hits.
10 See Rotemberg (2006) for further examples of harm caused by angry employees including wildcat strikes and increases in employee theft.
expect firms to be differentially altruistic towards different employees, with particularly high altruism reserved for employees who have been at the firm longer. While it is subject to alternate interpretations, the use of seniority to determine who should be laid off might be ascribable in part to a desire by firms to appear as if they cared more for employees with more seniority. Some evidence that is somewhat consistent with this interpretation is provided by Lee (2004). He points out that seniority rules for layoffs were introduced historically in the United States at the request of workers, who viewed these rules as being fairer than those that employers had used earlier.

I thus consider both the case where firms act as if they cared about all employees equally and the one where they act as if they were directly concerned only with more those that have already been at the firm in period \( t - 1 \). Since I impose the condition that wages have to be the same for old and new employees, new employees also benefit from firm altruism in this latter case. Let firm \( i \)'s actual altruism for all employees be denoted by \( \tilde{\lambda}^{Ai} \) while its actual altruism for its more senior employees is denoted by \( \tilde{\lambda}^{Ii} \). These altruism parameters are the firm’s “types.” At each point in time, these are drawn from discrete distributions so that, with \( j \) equal to either \( A \) or \( I \), the probability that a firm’s altruism \( \tilde{\lambda}^{ji} \) equals \( \bar{\lambda} \) at \( t \) equals \( d_{ij}(\bar{\lambda}) \).

An altruistic firm derives vicarious welfare from the welfare of its employees. To ensure that this vicarious welfare rises only if the employee is better off at \( i \) than he would be elsewhere, I let the utility of altruistic firms depend on the average difference between employees’ material welfare at \( i \) and the material welfare these employees would have elsewhere. For any particular employee, this difference depends on whether he has access to an alternate offer or not. If he does not, his instantaneous material payoffs would equal \( [(C_u)^{1-\gamma}/(1-\gamma) + x_u] \) if he lost his employment at \( i \). Since the \( x \)s are independent across employees, the expected difference for employees at \( i \) who do not have access to alternate employment is \( \psi^{1i} \), where

\[
\psi^{1i}_i = \frac{(w_i^{1i})^{1-\gamma} - (C_u)^{1-\gamma}}{1-\gamma} + \bar{x} - x_u,
\]

and \( \bar{x} \) denotes the unconditional mean of \( x \).
Now consider the welfare gains from having access to the job at \( i \) for employees who also have access to an alternate job with a wage of \( \hat{w}_t \). It is convenient to consider the expected gains of an employee before he knows either \( x^i \) or \( x^j \), the nonpecuniary benefits at the alternate job. At this point, the employee does know that he will remain at \( i \) only if (20) is satisfied. His expected gain from also having the job at \( i \) can therefore be written as

\[
\psi_{2i}^t = \int_{x^i}^\infty g(x^j) \int_{x^i + \hat{w}_t^{1-\gamma}(\hat{w}_t)^{1-\gamma}}^\infty \left[ \frac{(w^i)^{1-\gamma} - w_t^{1-\gamma}}{1-\gamma} + x^i - x^j \right] g(x^i) dx^i dx^j,
\]

where \( g \) is the density of \( x \). Since this individual decides to stay at firm \( i \) with probability \( F_{it} \) and derives no utility from the job at \( i \) if he decides to leave, his expected gain from having access to \( i \)'s job, conditional on staying, is \( \psi_{2i}^t / F_{it} \). This is also the average gain in material payoffs across the employees who stay and have alternate employment opportunities.

Depending on whether they have access to an alternate offer or not and whether they are incumbents at the firm or are new hires, employees fall into four categories. Their respective contribution to employment can be seen in equation (21), where the ones with outside offers are captured by the terms that include \( F_{jt} \), while the last full term represents new hires. Using (13) and (15), the expected material payoff gain of the incumbent employees at time \( t \) from being at firm \( i \) is given by \( \chi_{Ii}^t \), where

\[
\chi_{Ii}^t = \left[ 1 - s - \bar{m} \left( \frac{v_t}{h_t-1} \right)^{\ell} \right] h_{t-1}^{-1} \psi_{1i}^t + \bar{m} \left( \frac{v_t}{h_t-1} \right)^{\ell} h_{t-1}^{-1} \psi_{2i}^t.
\]

Adding in the gains of new employees, the expected gains of all employees are given by \( \chi_{Ai}^t \), where

\[
\begin{align*}
\chi_{Ai}^t = & \left[ 1 - s - \bar{m} \left( \frac{v_t}{h_t-1} \right)^{\ell} \right] h_{t-1}^{-1} \psi_{1i}^t + \bar{m} \left( \frac{v_t}{h_t-1} \right)^{\ell-1} \psi_{1i}^t \\
& \quad + \left[ \bar{m} \left( \frac{v_t}{h_t-1} \right)^{\ell} h_{t-1}^{-1} + \bar{m} \left( \frac{v_t}{h_t-1} \right)^{\ell-1} \right] \psi_{2i}^t.
\end{align*}
\]

(35)

It is worth noting for future reference that, while both \( \chi_{Ai}^t \) and \( \chi_{Ii}^t \) depend directly on the firm’s employment and wages, only the former also depends directly on the firm’s own
vacancy level. For future use, let \( \omega^i_t \) denote the derivative of \( \chi^{Ai}_t \) with respect to \( v^i_t \):

\[
\omega^i_t = \left( \frac{v^i_t}{w^i_{t-1}} \right)^{n-1} \psi^i_t + \bar{m} \left( \frac{v^i_t}{h^i_{t-1}} \right)^{t-1} \psi^{2i}_t.
\]

The sign of this derivative is positive. By increasing vacancies, firm \( i \) raises the welfare of both the unemployed people who thereby obtain jobs and the employed people at other firms who decide to move because firm \( i \) offers them a better package of wage and nonwage compensation.

Now imagine that firm \( i \) maximizes

\[
\Pi^i_t = E_t \sum_{\tau=0}^{\infty} \rho^\tau \tilde{\pi}^i_{t+\tau},
\]

where \( \tilde{\pi}^i_t \) equals

\[
\tilde{\pi}^i_t = \pi^i_t + \lambda^A \chi^{Ai}_t + \lambda^f \chi^{fi}_t.
\]

In this equation, \( \lambda^A \) and \( \lambda^f \) are the altruism parameters that govern the firm’s behavior. As discussed below, they need not equal the firm’s actual altruism parameters. Using the logic that leads to (24), \( \Pi^i_t \) can be written as

\[
\Pi^i_t = R^i_t(h^i_t) - w^i_t h^i_t - \kappa(v^i_t) + E_t \rho^2 \Pi^i_{t+2}(h^i_{t+1}) + E_t \rho \left( R^i_{t+1} - w^i_{t+1} h^i_{t+1} - \kappa \left( v^i_{t+1} - h^i_{t+1} - h^i_t \right) \right) \left( \frac{h^i_{t+1}}{h^i_{t+1} + h^i_t} \right) + \lambda^A \chi^{Ai}_t + \lambda^f \chi^{fi}_t + E_t \rho (\lambda^A \chi^{Ai}_{t+1} + \lambda^f \chi^{fi}_{t+1}).
\]

The first-order conditions for the maximization of \( \Pi^i_t \) are

\[
\frac{d\Pi^i_t}{dh^i_t} \frac{dh^i_t}{dv^i_t} - \kappa^t + \lambda^A \omega^i_t = 0, \quad \text{and} \quad \frac{d\Pi^i_t}{dh^i_t} \frac{dh^i_t}{dw^i_t} - h^i_t (1 - \lambda^A (w^i_t)^{-\gamma}) + \lambda^f \left( (w^i_t)^{-\gamma} h^i_{t-1} \left( 1 - s + (F^i_t - 1) \frac{m^h_t}{h^i_{t-1}} \right) \right) = 0,
\]

where \( dh^i_t/dv^i_t \) and \( dh^i_t/dw^i_t \) are given by (28) and (29), respectively, while

\[
\frac{d\Pi^i_t}{dh^i_t} = \frac{dR^i_t}{dh^i_t} - w^i_t + E_t \rho (\kappa^i_{t+1} - \lambda^A \omega^i_{t+1}) \frac{1-s}{F^i_{t+1}} \frac{(v^i_t/h^i_t)^{t}}{m^h_t (v^i_{t+1}/h^i_t)^{t-1} + (v^i_{t+1}/h^i_t)^{t-1}} + E_t \rho (\lambda^A + \lambda^f) \left( 1-s \right) \psi^{fi}_{t+1} + \bar{m} \left( \frac{v^i_{t+1}}{h^i_t} \right)^{t} \left( \psi^{2i}_{t+1} - \psi^{fi}_{t+1} \right).
\]
In equation (40), the derivative of $\chi_{Ai}^t$ with respect to the wage has been set to the marginal utility of income $(w_i^t)^{-\gamma}$ multiplied by employment $h_i^t$, while the derivative of $\chi_{Ii}^t$ with respect to the wage has been set equal to the expression in curly brackets. The validity of these substitutions is demonstrated in Appendix 1.

Since the second line of (41) is independent of $v^t$ or $h^t$, the expression for the second derivative of $\hat{\Pi}$ with respect to $v$ is the same as that for the second derivative of $\Pi$ with respect to $v$. Thus, the discussion of second-order conditions for the nonaltruistic case remains valid here as well. Taking the ratio of (39) and (40), while using (15), (28), and (29), one obtains

$$\bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\eta-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} = \frac{(\kappa_i^t - \lambda^A \omega_i^t)^{-\gamma} \bar{F}_i^t h_{t-1} \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^t \left( \frac{h_{t-1}^i}{h_{t-1}} + \frac{v_t}{u_t} \right)}{h_i^t (1 - \lambda^A (w_i^t)^{-\gamma} - \lambda^A h_i^t (w_i^t)^{-\gamma}) 1 - s + (F_i^t - 1) \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^t}.$$  (42)

An increase in firm altruism $\lambda^I$ lowers the denominator of this expression so that, with $\gamma > 0$, $w_i^t$ must rise to maintain this equality. Similarly, higher values of $\lambda^A$ require higher wages as long as $\omega_i^t$ is small.\(^{11}\) This means that workers obtain information about the altruism parameter that governs firm behavior from the wage that the firm pays. Following Rotemberg (2007a), assume that workers care about this parameter and become angry if it is demonstrably too low. Simplifying Rotemberg (2007a) somewhat, assume that workers’ altruism towards firm $i$ at $t$ is given by the function

$$\xi(\lambda_i^t, \bar{\lambda}_t).$$

In this function, $\lambda_i^t$ represents all the information that workers have about firm $i$’s altruism parameters, while $\bar{\lambda}$ represents the worker’s altruism threshold. In general, $\lambda_i^t$ need not be a scalar and can include a wide range of data. Here, I specialize and give workers full information about the firm’s actions and environment.

If, using a test of size $\alpha$, the information in $\lambda_i^t$ allows workers to reject the hypothesis that $\lambda_i^A$ (or $\lambda_i^I$) is equal to at least $\bar{\lambda}$, $\xi$ is equal to a large negative number. If, instead, workers

\(^{11}\)For the parameters considered below, $\omega$ is indeed small enough that the right-hand side of (42) rises with $\lambda^A$.\)
are unable to reject the (statistical) hypothesis that their employer is minimally benevolent, \( \xi \) equals zero. Workers thus give employers the benefit of the doubt, and this means that it can be profitable for selfish firms to act as if they were altruistic.

A negative \( \xi \) implies that a worker is willing to incur a cost of \(|\xi|\) units in exchange for a reduction in employer utility of one unit. Given that workers have numerous opportunities for causing harm to their employers, a large negative \( \xi \) should prove costly to firms. While I do not model the nature of these costs explicitly, I assume that they equal \( \Xi \).

I now turn to the equilibrium determination of wages in the case where \( \tilde{\lambda}^{Ai} = 0 \) for all firms while \( \tilde{\lambda}^{Ii} \) has a nondegenerate distribution with \( d_I(\tilde{\lambda}) = \alpha \). The same arguments apply to the case where \( \tilde{\lambda}^I = 0 \) for all firms while \( \tilde{\lambda}^{Ai} \) has a nondegenerate distribution, as long as \( \omega_i^t \) is either negative or small in absolute value, so I do not deal with this case explicitly. Consider then an allocation where all firms with \( \tilde{\lambda}^{Ii} \leq \lambda^* \) set a wage given by (42) with \( \lambda^I = \lambda^* \), and all others set it with \( \lambda^I = \tilde{\lambda}^{Ii} \). It is immediately apparent that, for sufficiently large \( \Xi \), such an allocation cannot be an equilibrium unless \( \lambda^* \geq \bar{\lambda} \). If, instead, \( \lambda^* < \bar{\lambda} \), some firms would pay a wage that corresponds to an altruism level below \( \bar{\lambda} \), incurring the cost \( \Xi \) by identifying themselves as insufficiently altruistic.

In contrast, allocations where \( \lambda^* \geq \bar{\lambda} \) are equilibria as long as firms believe that lower wages will identify them as having \( \tilde{\lambda}^{Ii} \leq \bar{\lambda} \). The reason is that a test with size \( \alpha \) does not reject the hypothesis that firms paying the wage implied by \( \lambda^* \) have an altruism parameter of \( \bar{\lambda} \). Thus, \( \xi \) is zero for all firms paying this wage. Similarly, the hypothesis is not rejected for firms whose wage is even higher, because their \( \tilde{\lambda}^{Ii} \) exceeds \( \lambda^* \).

It should be noted that, in the case of \( \lambda^* > \bar{\lambda} \), the equilibrium beliefs above are not “reasonable,” in the sense that genuinely altruistic firms with \( \tilde{\lambda}^{Ii} = \bar{\lambda} \) would want to pay a lower wage. These equilibria with excessively high wages can be eliminated under some additional assumptions. Assume, in particular, that a fraction \( a \) of firms with \( \tilde{\lambda}^{Ii} = \bar{\lambda} \) believe that they have already demonstrated their true altruism parameter in other ways, so they can set any wage they wish without fear of reprisal. Assume, on the other hand, that workers do not see these additional signals even though they know that some altruistic
firms are naive in the manner just described. Now, consider a firm that deviates from a
proposed equilibrium with \( \lambda^* > \bar{\lambda} \) by paying the wage that corresponds to \( \hat{\lambda}^I = \hat{\lambda} \). Using a
test of size of \( a \alpha \), workers are unable to reject the hypothesis that this firm has an altruism
parameter of \( \bar{\lambda} \). Thus, at least for this significance level, workers do not punish such a firm.
This eliminates all equilibria with \( \lambda^* > \bar{\lambda} \) so the only equilibrium has \( \lambda^* = \bar{\lambda} \).

The analysis is simplified by assuming not only that naive altruistic firms are not punished
but by also assuming that no firm has an altruism parameter \( \tilde{\lambda}^I \) that strictly exceeds \( \bar{\lambda} \).
This ensures that the only equilibrium is symmetric, with all firms acting as if \( \lambda^I \)
in (42) were equal to \( \bar{\lambda} \). As discussed above, a similar analysis applies when \( \tilde{\lambda}^I = 0 \) for all firms
while \( \tilde{\lambda}^A \) varies. I focus on outcomes where only one of the \( \lambda^I \)s is zero, while the other is
equal to \( \lambda^j \), with \( j \) equal to \( A \) or \( I \). At symmetric equilibria of this type, (42) becomes

\[
m \frac{v_t}{h_{t-1}} (v_t)_{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1} = \frac{2(\kappa^I_t - \lambda^A_t \omega_t) w_t^{-\gamma} F' m \left( \frac{v_t}{h_{t-1}} \right)_{\ell}}{h_t (1 - \lambda^A_t w_t^{-\gamma}) - \lambda^I_t w_t^{-\gamma} \left( 1 - s - \frac{m}{2} \left( \frac{v_t}{h_{t-1}} \right)_{\ell} \right)}.
\]

(43)

This equation obviously reduces to (33) when both \( \lambda^A_t \) and \( \lambda^I_t \) equal zero. At a symmetric
equilibrium \( \psi^2 \) is constant, and (39) implies

\[
0 = \frac{dR_t}{dh_t} - w_t - \frac{\kappa^I_t - \lambda^A_t \omega_t}{\frac{m}{2} \left( \frac{v_t}{h_{t-1}} \right)_{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1}} + \frac{\rho(\kappa^I_{t+1} - \lambda^A_{t+1} \omega_{t+1}) \left( 1 - s - \frac{m}{2} \left( \frac{v_{t+1}}{h_t} \right)_{\ell} \right)}{\frac{m}{2} \left( \frac{v_{t+1}}{h_t} \right)_{\ell-1} + \left( \frac{v_{t+1}}{u_t} \right)^{\eta-1}} + \lambda^A \left( \psi^I_t + E_t \rho \hat{m} \left( \frac{v_{t+1}}{h_t} \right)_{\ell} \left( \psi^2 - \psi^1_{t+1} \right) \right) + E_t \rho \lambda^I \left( 1 - s \right) \psi^I_{t+1} + \hat{m} \left( \frac{v_{t+1}}{h_t} \right)_{\ell} \left( \psi^2 - \psi^1_{t+1} \right),
\]

(44)

which is the analogue of (34) in the case where firms act altruistically. In this equation,
increases in \( \lambda^A \) and \( \lambda^I \) reduce the marginal revenue product of labor for a given wage and
recruitment cost. Since an altruistic firm derives utility from its employment, it hires more
workers than a selfish firm and thereby reduces its marginal revenue product.

Equations (43) and (44) go beyond the previous analysis in two ways. First, they add
time subscripts to the altruism parameters \( \lambda^A \) and \( \lambda^I \). For these parameters to be varying
for all firms at the same time, one would have to assume that workers form new judgments
about their employers in each period and that their required level of altruism (as well as
the altruism of the most altruistic firms) is time varying. The implications of these rather strong assumptions are considered further below.

Second, (44) assumes that the number of employees and the number of vacancies posted are also used as signals of a firm’s altruism. This is consistent with the full information assumptions I have made, though the dependence of current vacancies on the expected future altruism of the firm raises the question of whether workers at time $t$ insist that this altruism be equal to their current or their expected future $\bar{\lambda}$. A second complication with letting workers base their altruism judgments on the level of vacancies is that, while the wage in (42) depends only on overall labor market conditions and the firm’s share of employment and vacancies, the level of vacancies also depends on the productive opportunities available to the firm. Workers’ information about these opportunities is likely to be much poorer than that of employers, and workers may thus be unable to use vacancy data effectively to judge a firm’s altruism. While a full analysis of this informational difficulty is beyond the scope of this paper, I consider a simple alternative where firms set vacancies at the profit-maximizing level. In other words, all the $\lambda$ parameters in (44) are set equal to zero. This yields a good approximation to the equilibrium where firms choose their vacancies without being concerned with worker reactions, as long as most firms are indeed selfish (so that the number of truly altruistic firms is fairly small).

3 Choice of Parameters

A disadvantage of this model relative to the Mortensen-Pissarides model is that more of its parameters seem difficult to calibrate on the basis of microeconomic or steady-state observations. The parameters of the model are $\rho$, $\eta$, $\gamma$, $s$, $\bar{m}$, $\ell$, $\bar{F}'$, and $\psi^2$, while the variables are $u$, $h$, $v$, $w$, $dR/dh$, $\kappa'$, $\omega$, $\psi^1$, $\lambda^A$, and $\lambda^I$. To study the effect of shocks, one must calibrate the steady-state value of many of these variables. Both the baseline and some alternative values for the relevant parameters are displayed in Table 1.

One purpose of studying a model with minimal altruism is to ascertain whether relatively small degrees of altruism (or of altruism variation) can produce important consequences.
Thus, my baseline assumes that firms are selfish, so $\lambda^A = \lambda^I = 0$. I then compare the results of this baseline with the situation where, on average, either $\lambda^A$ or $\lambda^I$ is equal to 0.4. While this is a nontrivial level of altruism, it is important to stress that it would not lead an employer to be willing to donate money to employees without receiving something in return. Employers would, however, be willing to give a dollar to their employees if this led the employees to receive $2.50.

I treat the length of the period as being equal to one month and therefore choose $\rho = 0.996$, as in Shimer (2005). I also follow Shimer (2005) and set $s = 0.034$. According to Shimer (2005), the average job-finding rate for the unemployed is 0.45. This should equal the steady-state value of $(v/u)\eta$. Using (12) and (14), this implies that the steady-state rate of unemployment is $s/(s + (v/u)\eta)$. Knowing $\eta$ as well as the steady-state job-finding rate $(v/u)\eta$, this formula for steady-state unemployment allows one to compute the steady-state value of vacancies.

Shimer (2005) and Mortensen and Nagypál (2005) calibrate the parameter $\eta$ rather differently, with the former choosing a value of 0.28, and the latter a value of 0.54. In effect, Shimer (2005) uses the regression coefficient of the vacancy-unemployment ratio on the job-finding rate as his estimate of $1/\eta$, so he is treating the equality between the finding rate and $(v/u)\eta$ as an estimating equation. Mortensen and Nagypál (2005) calibrate this parameter instead by considering the regression of vacancies on the unemployment rate itself. This second regression is essentially equivalent to a regression of $v/u$ on the finding rate, where the finding rate is measured in such a way that its movements rationalize all unemployment fluctuations. Since this relationship would require that the finding rate be somewhat more procyclical than it actually is, the resulting estimate of $1/\eta$ is smaller, and the estimated value of $\eta$ larger. I adopt the value of 0.54 in this paper, because, by capturing the magnitude of the rise in vacancies resulting from a rise in employment, this parameter value is likely to reflect more accurately the extent to which the marginal cost of hiring rises in booms.

Because I am unaware of estimates of the matching function of the currently employed with new employers, I set $\ell = \eta$, so the elasticity of this matching function with respect
to vacancies is the same as the corresponding elasticity of matching between firms and unemployed workers. The constant $\bar{m}$ is based on Nagypál (2005), who shows that the total volume of job-to-job transitions is equal to about twice the number of people who become unemployed by separating from their employer. At a symmetric equilibrium, half the workers who find alternate employment accept it. This means that the total number of matches between employed workers and potential future employers, which equal $\bar{m}h(v/h)^{\ell}$, should equal $4s$. This pins down $\bar{m}$, because the calculations above provide the steady-state values of $h$ and $v$.

Silva and Toledo (2006) use micro evidence to obtain an estimate of the average cost of recruiting one worker. Their cost estimate is about 0.12 times the amount the individual is paid in one month, and I thus set $\kappa'/w(dh/dv)$ equal to this value. Given the lack of absolute measures of $\kappa'$, $\psi$, and $dR/dh$, observations of the steady-state wage $w$ do not help to determine the parameters in (34) or in (44). Similarly, because the absolute value of $\kappa'$ is unobservable, the observability of the steady-state $w$ does not help to determine any parameters that appear in (33) or in (43). Therefore, I normalize this average wage and set it equal to 1.

As in Rotemberg (2007b), I let marginal vacancy costs depend on the level of vacancies. I consider, in particular, the recruitment cost-function

$$ \kappa(v^t_i) = \kappa_0(v^t_i)^{\zeta_v}, $$

where the standard constant-cost case obtains when $\zeta_v = 1$. Rotemberg (2007b) argues that the assumption that the vacancy-posting function has increasing returns to scale (so $\zeta_v < 1$) makes it easier to rationalize the cyclical behavior of wages with the Mortensen-Pissarides (1994) model. In the United States, vacancies rise sharply in booms, increasing marginal recruitment costs considerably when $\zeta_v = 1$. With such a big increase in recruitment costs, workers are in a particularly strong bargaining position in booms, because their employer has more to lose from their departure. As a result, wages must rise strongly as well. With $\zeta_v$ smaller than 1, recruitment costs rise more modestly, and hence wage increases are more
muted. I thus consider both $\zeta_v = 1$ and $\zeta_v = 0.66$. This latter value is presented mainly for illustration.

When $\zeta_v < 1$, $\kappa'' < 0$ and it becomes important to check the second-order conditions because the concavity $R$ with respect to $h$ no longer suffices to guarantee this condition. As can be seen in (31), what is needed in this case is that $d^2R/dh^2$ be smaller than a negative number that depends on $k''$. I now discuss the parameters that are needed to satisfy this condition. With $\zeta_v = 0.66$ and the values of the parameters chosen so far, the right-hand side of (31) is equal to -0.71. To compute the left-hand side, one must make more specific assumptions about the production function and the demand conditions facing the typical firm.

Assume the firm’s production function $f(h)$ in (8) is given by $h^\alpha - \Phi$, where $\Phi$ is a fixed cost. Using (9), the marginal revenue product at a steady-state equals

$$\bar{A}[h^\alpha - \Phi]^{1/\epsilon} h^{\alpha-1},$$

where $\epsilon$ is the elasticity of demand at the steady state and the constant $\bar{A}$ must be chosen so the steady-state version of (44) is satisfied. When $\lambda^A = \lambda^I = 0$, $\epsilon = 3$, $\alpha = 0.66$, and $\Phi/h^\alpha = 1/3$, the left-hand side of inequality (31) equals about -0.76, so (31) is satisfied. The inequality is violated, however, if the elasticity of demand is made arbitrarily large and fixed costs are set to zero.\(^{12}\)

These production and demand parameters do not have a direct effect on the comovement of employment and wages in the case where these variables are affected only by exogenous changes in $dR/dg$. The reason is that, in this model, fluctuations in the marginal revenue product of labor have the same effect on $h$ and $w$ regardless of whether they are due to changes in technology $z$ or changes in the elasticity of demand $\epsilon$. On the other hand, the size of the changes in $z$ or $\epsilon$ that are needed to justify these changes in $dR/dh$ does depend on demand and production function parameters. Another reason to calibrate these parameters is that they are necessary to obtain implications regarding the movement of labor productivity.

\(^{12}\)Setting either $\lambda^A$ or $\lambda^I$ equal to 0.4 changes the $\bar{A}$ that satisfies (44). However, the effect of this on the left-hand side of (31) is negligible for the baseline parameters.
It must be pointed out, however, that the predicted movements in labor productivity depend also on whether the movements in \( dR/dh \) are due to changes in \( z \) or to changes in \( \epsilon \). Indeed, in the case where employment changes are due exclusively to changes in \( \epsilon \), setting \( \alpha = 0.66 \) and \( \Phi/h^\alpha = 0.4 \) leads the predicted comovement of labor productivity and employment to resemble — in certain respects — the observed comovement in the United States. As reported in Rotemberg (2007b), the regression coefficient of a detrended measure of U.S. output on a detrended measure of employment has a coefficient of about 1.1. Using the production function above, a 1 percent increase in \( h \) leads to a \( \alpha/(1 - \Phi/h^\alpha) \) percent change in output. Thus, the parameters just mentioned lead output to rise by the correct percentage in response to a typical increase in employment. Unfortunately, the correlation between labor productivity and employment is relatively low in the United States so changes in \( \epsilon \) cannot explain all the movements in labor productivity. A more promising approach is to let productivity be affected by a variety of shocks, including changes in \( \epsilon \) and \( z \). With a smaller value of \( \Phi/h^\alpha \), the former impulse would lead to movements in productivity that are more countercyclical than the average movements that are observed empirically, while the latter generally leads to movements that are more procyclical. This decomposition of productivity movements is beyond the scope of this paper, however.

The calibration of production and demand parameters is even more important if one considers changes in firms’ required levels of altruism. An increase in required altruism always raises real wages. However, the effect of this increase in real wages on the quantity of labor demanded depends on \( \alpha \), \( \epsilon \), and \( \Phi \). My baseline calibration for these parameters is \( \alpha = 0.75 \), \( \epsilon = 3 \) and \( \Phi = 0.2 \).

Survey evidence may someday be used to clarify how much typical workers value their jobs as opposed to the alternative uses of their time. This knowledge might then allow one to calibrate \( \psi^1 \) (the steady-state level of \( \psi^1_t \)) and \( \psi^2 \). So far, however, the existing evidence on them is fairly scant. In the case where \( \lambda^A = 0 \) and \( \lambda^I \geq 0 \), \( \omega_t \) does not affect (43), so this equation links the tightness of the labor market with real wages in a manner that is independent of \( \psi^1 \) and \( \psi^2 \). It then follows that these parameters do not affect the relative
movements of wages and employment when both are being driven by changes in $dR/dh$. On
the other hand, $\psi^1$ and $\psi^2$ have large effects on (44) and thus on the vacancy choices of
altruistic firms. To keep this influence small, I set these parameters to relatively low values.

Given the parameters chosen so far, one can compute the derivative of the pdf of non-
pecuniary benefits on the job $\bar{F}'$ by using the steady-state version of (43). Because the wage
has been normalized to equal 1, this parameter is independent of $\gamma$ and depends only on $\kappa'$,
on the $\lambda$s and $\psi$s, and on the parameters governing the matching of workers to firms. Using
the parameters just described, $\bar{F}'$ equals 28.5, 19.7, and 18.3, depending on, respectively,
whether both $\lambda$s are set to zero, $\lambda^A = 0.4$ (with $\lambda^I = 0$), or $\lambda^I = 0.4$ (with $\lambda^A = 0$). To gain
an idea of the implications of the value of $\bar{F}'$, it is worth considering only workers who have
an outside offer and computing the elasticity of their departure with respect to the wage.
For $w = 1$, this elasticity is given by $\bar{F}'/F(0)$. Thus, the above values of $\bar{F}'$ imply elasticities
greater than 36, which seem rather large.

The implied value of $\bar{F}'$ is large primarily because $\kappa'$ is relatively small. A low $\kappa'$ indicates
that wages are large relative to recruiting costs, and in this monopsony model this can be
rationalized only if employees are quite sensitive to wages when choosing whether to stay or
leave. This requires, in turn, that there be a large number of firms whose nonwage features
are comparable with those offered by any given employer, and this corresponds to a high $\bar{F}'$.

This leaves the parameter $\gamma$, which governs both the substitutability of wage and nonwage
components of a job and the speed at which the marginal utility of income declines with
income. While it is standard to assume log utility $\gamma = 1$, the consumption commitments
model of Chetty and Szeidl (2007) can rationalize higher values. I therefore consider a
variety of values, with an eye towards understanding which values of $\gamma$ fit most easily with
the observed labor market dynamics.

4 Results of Simulations

I consider two different types of simulations. In the first, there are fluctuations in $dR/dh$
that induce fluctuations in $h$ and $w$. In the second, there are simultaneous fluctuations in
\(dR/dh\) and in either \(\lambda^4\) or \(\lambda'\). The first type is more similar to the exercise carried out in Shimer (2005), where there is a single exogenous variable (technology in his case), and the issue is how well the model reproduces certain features of the data. Shimer (2005) is particularly concerned with reproducing the cyclical movements in productivity, but, as I have just discussed, these hinge crucially on the underlying causes of the movements in \(dR/dh\). I therefore focus only on the relationship between real wages and employment.

To obtain some analogues of the moments predicted by the model, I detrend monthly data on the logarithms of total U.S. civilian employment and of the ratio of hourly earnings of production workers in manufacturing to the consumer price index (CPI). These series are detrended using the method of Rotemberg (1999), which is designed to keep the covariance of the detrended value of the series at \(t\) and \(t-k\) low, while also ensuring that the detrended value of a series at \(t\) is orthogonal to the difference between the trend at \(t\) and the average of the trend at \(t+v\) and \(t-v\). Because the series are monthly, \(k\) is set equal to 48, while \(v\) is set equal to 15.\(^{13}\) In practice, this method is essentially a band-pass filter that differs from Hodrick-Prescott in that only relatively low frequencies are allowed into the trend.

I use data from January 1948 to August 2006. Because the removal of a smooth, two-sided trend implies that detrended observations near the edges of the sample are inaccurate, I trim five years of data from the beginning and the end of the series. The resulting detrended real wage is fairly procyclical, and has a correlation of 0.41 with detrended employment. Abraham and Haltiwanger (1995) suggest that CPI-deflated hourly earnings are particularly procyclical relative to other aggregate real wage series. While these series may be somewhat atypical, the exercise of mimicking their joint movements with the present model should also be informative about the capacity of the model to match the movements in related series.

When I consider the effects of a single shock, I am mostly concerned with ascertaining the model’s capacity to match the “regression coefficient” of wages on employment. As can be seen in Abraham and Haltiwanger (1995), statistics of this sort are often used to summarize the extent to which real wages are procyclical. In the two series just discussed, this regression

\(^{13}\)These are three times larger than the parameters Rotemberg (1999) recommends for quarterly data.
coefficient equals 0.49. It is slightly larger than the correlation between the series because the standard deviation of detrended wages is slightly larger than the standard deviation of detrended employment. The former equals 0.017 while the latter equals 0.014.

Since the model seeks to explain the movements of employment by the movements of a single exogenous series, it is actually simpler to postulate a stochastic process for the detrended employment series and then derive the implied movements in $dR/dh$. Given that the first-order serial correlation of detrended employment equals 0.97, let the log of detrended employment follow

$$\tilde{h}_t = 0.97\tilde{h}_{t-1} + e^h_t,$$

(45)

where $\tilde{h}_t$ is the log difference between employment and steady-state employment, while $e^h_t$ is an i.i.d. random variable. In the interest of matching the behavior of actual series, let the standard deviation of $e^h_t$ be equal to 0.0036. This equals the standard error of the autoregression reported above.

The four equilibrium conditions — (12), (14), (43) and (44) — can then be solved using Dynare. The model’s predicted regression coefficients for several different values of the parameters are reported in Table 2. In addition to reporting regression coefficients of the log of $w$ on the log of $h$, Table 2 also reports coefficients from the regression of the log of $dR/dh$ on $h$. These give an idea of the extent to which the model needs “large impulses” to generate realistic movements in employment. It is important to note that, by themselves, increases in $h$ lower the marginal revenue product of labor, so the exogenous rise in $dR/dh$ that raises employment must be larger than the actual increase reported in Table 2.

With the baseline parameters of $\gamma = \zeta_v = 1$, the model implies extremely large procyclical movements in real wages and $dR/dh$. This shows that this monopsony-based model can also be subject to the difficulties uncovered by Shimer (2005). Even with selfish firms, it is possible to ameliorate these problems significantly by increasing $\gamma$ and lowering $\zeta_v$.

Equation (43) requires that the cost of attracting a worker by raising wages be the same as the cost of doing so by increasing vacancies. In booms, the cost of attracting a worker through vacancies rises. The higher is $\gamma$, the more the marginal utility of income falls for
workers when the wage rises. This implies that increases in wages are less effective as a recruiting tool when wages are already high, so the cost of increasing employment by raising wages rises with the wage rate. As a result, wages do not have to rise as much in booms for them to become as costly as increases in vacancies as a recruiting tool.

The effect of $\gamma$ on the extent to which wages are procyclical is opposite here to the effect in the standard market-clearing model of the labor market. In that model, a higher $\gamma$ means that the marginal utility of consumption falls more in booms, so the wage needs to rise more to keep people indifferent between their old hours of work and slight increases in their hours of work. Interestingly, the reduction in the marginal utility of income that occurs in booms plays a role here as well, but here the effect is to discourage firms from using wage increases to retain and recruit employees.

Table 2 also shows that the extent to which $dR/dh$ must rise to induce increases in employment is quite comparable to the rise in wages that accompanies this employment increase. Equation (43) determines wages from the degree of tightness in the labor market, without much regard for $dR/dh$. Equation (44), on the other hand, makes it clear that firms would not be willing to let the labor market become tight (which increases the cost of recruiting with vacancies) unless $dR/dh$ rose by essentially the same amount as wages.

The last eight lines of the table show that required firm altruism can dampen the needed changes in wages considerably. Indeed, for $\gamma = 8$, $\zeta_v = 0.66$, the model nearly reproduces the regression coefficient of wages on employment obtained when either $\lambda^A$ or $\lambda^I = 0$. Firm altruism has a number of effects that tend to make real wages less procyclical. First, increases in wages lower workers’ marginal utility of consumption $w^{-\gamma}$, and this decrease lowers an altruistic firm’s vicarious benefit from raising wages. This particular effect is larger when $\lambda^A = 0.4$ than when $\lambda^I = 0.4$ because, in the former case, the reduction in the marginal utility of income affects more people that the firm cares about.

Second, an increase in $h$ lowers the fraction of more senior employees because it is associated with a rise in $v/h$, which leads to more turnover. This means that a firm’s vicarious benefit from the utility of workers is smaller in booms, when $\lambda^A = 0$ and $\lambda^I > 0$. Such a
firm is thus less inclined to raise wages in economic expansions. However, this dampening of real wage increases is only valid when firms care disproportionately about more senior employees.

Third, increases in wages raise the utility of being employed relative to the utility of being unemployed, thus raising $\psi^1_t$ and thereby increasing $\omega_t$. For a firm with $\lambda^A > 0$, this increase in $\omega$ raises the attractiveness of increasing employment via vacancies rather than via wages. This dampens the incentive of such a firm to raise wages. This effect turns out to be quantitatively important and implies that wages are dampened more with $\lambda^A = 0.4$ than with $\lambda^I = 0.4$.

The dampening of wage movements suggests that a model with positive $\lambda$ fits with the interview evidence of Bewley (1999) in the sense that wages do not decline more in recessions in part because firms are worried about appearing sufficiently altruistic. Admittedly, one reason that altruism matters here is that workers’ marginal utility of income varies over time, and the extent of this variation may be controversial. There are, however, two reasons to imagine that this variation might well be substantial. First, as Chetty and Saez (2007) have emphasized, most workers have considerable consumption commitments that are difficult to unwind. This means that small reductions in disposable income can trigger large changes in the elements of consumption that can be freely varied. The result is that the marginal utility of consumption can fall dramatically even when income falls only by a small amount.\footnote{Formally, this would require consumption in (1) to be equal to the wage minus the level of consumption commitments $\bar{C}$. The expression $w_t^{1-\gamma}$ in equations such as (44) must then be replaced by $(w_t - \bar{C})^{1-\gamma}$. For this expression to rise by 1 percent requires a smaller percentage change in the wage than is required to increase $w_t^{1-\gamma}$ by 1 percent. The wage changes that are needed to balance changes in labor market tightness are thus smaller when $\gamma$ is larger.}

There is also a second reason for the marginal utility of income to vary substantially, which has not been widely considered in the macro literature. This is the existence of altruistic transfers within families and across friendship networks. In recessions, the unemployment rate rises and employed workers can be expected to give up more of their paycheck to people they know who have lost their jobs. This, presumably, ought to raise the marginal utility of income of employed workers by more than is implied by the reduction in their wage. If,
for example, a spouse loses his or her job, the marginal utility of income of the spouse who remains employed is presumably much larger.

The importance of variations in the marginal utility of income for the conclusion that firms smooth wages connects this model with models where observed wages are smooth because firms explicitly insure workers against income fluctuations. One difference between these two approaches is that I rule out binding long-term contracts in my analysis, in part because such contracts are quite rare among workers who occasionally experience unemployment. Contracts may, of course, be implicit as in the classic analysis of Azariadis (1975), although this formulation raises the question of how these contracts are enforced. One way of thinking about the model of this paper is that workers’ capacity for anger at insufficient altruism provides an enforcement mechanism that allows for a certain degree of insurance, though the details of this insurance are not identical to those of implicit-contracts models.

So far, I have treated the required altruism levels as fixed. One reason to study models where \( \lambda \), and thus \( \lambda^A \) or \( \lambda^I \), fluctuate is that the addition of such fluctuations to a model with variable \( dR/dh \) adds a force that can make real wages countercyclical, reducing the correlation between wages and employment. This potential source of countercyclical wage movements is worth contrasting with the more standard idea that there are movements in labor supply. Traditional labor supply shifts can be due either to preference shifts, as in Hall (1997), or to wealth effects. As an example of the latter, increases in government purchases reduce people’s wealth in standard models and increase their willingness to work at a given real wage. Similarly, the expectation of future technical progress makes people feel wealthier and reduces their willingness to work at a given real wage.

In matching models of the sort I have considered, these traditional movements in labor supply tend to have a counterfactual implication. When people increase their willingness to work at a given real wage, a matching model tends to predict an initial rise in unemployment as more people seek work. This increase in unemployment leads to eventual increases in employment (by increasing the number of workers who become matched to employers), even without an increase in vacancies. The result is that vacancies and unemployment are no
longer as negatively correlated as they would be if the only changes were changes in labor demand. Thus, matching models that include a labor supply channel, such as Merz (1995), do not have a Beveridge curve where, as in U.S. data, the negative correlation between vacancies and unemployment is nearly perfect.

In contrast, the model in this paper abstracts from labor supply variations by assuming that the labor force is fixed. The combination of (12) and (14) implies that all movements of employment lie on a Beveridge curve. This is not to say that changes in wealth cannot play any role. Perceived increases in firm wealth, for example, may lead people to expect a higher $\bar{\lambda}$ and thereby increase real wages for any given level of employment. Drawing out this connection is beyond the scope of this paper, however. Here, I simply consider exogenous variations in $\bar{\lambda}$. To simplify, I assume that $\bar{\lambda}$ follows the AR(1) process:

$$\bar{\lambda}_t = 0.96 \bar{\lambda}_{t-1} + e^\lambda_t,$$

where $e^\lambda$ is an i.i.d. random variable. The choice of the AR parameter is broadly dictated by a desire to match the serial correlation of employment, though the correspondence is not exact. One aim of the analysis is to study whether the standard deviation of $\bar{\lambda}$ needed to explain the broad features of employment and wage fluctuations is excessive. To make the results transparent, I report the standard deviation of $\bar{\lambda}$, which I denote by $\sigma_\lambda$, rather than reporting the standard deviation of $e^\lambda$.

In addition to fluctuations in $\bar{\lambda}$, I continue to let $dR/dh$ be subject to cyclical fluctuations. Note that the existence of diminishing returns implies that $dR/dh$ falls whenever reductions in $\bar{\lambda}$ lead to rises in employment. I thus introduce a variable $\hat{dR}/dh_t$, which is the level of $dR_t/dh_t$ that would be induced by the current level of employment if $\epsilon$ and $z$ were at their steady-state levels. One can then write $dR_t/dh_t$ as

$$\frac{dR_t}{dh_t} = \frac{\hat{dR}_t}{dh_t} + \frac{dR}{dh} \frac{\hat{dR}_t}{dh_t},$$

where $dR/dh$ is the steady-state value of $dR/dh$ and $dR/dh$ represents the effects of exoge-
nous variations in $z$ and $\epsilon$. Assume further that the exogenous changes in $dR/dh$ satisfy

$$\frac{\tilde{dR}_t}{dh_t} = 0.96 \frac{dR_{t-1}}{dh_{t-1}} + \epsilon_t^R,$$

(47)

where $\epsilon_t^R$ is i.i.d. and independent of $\epsilon^\lambda$ at all leads and lags. I let $\sigma_R$ denote the standard deviation of $\tilde{dR}/dh_t$ induced by $\epsilon_t^R$.

The results of using Dynare to simulate this model for various parameters are reported in Table 3. The first line of this table reports the standard deviations of cyclical employment and wages as well as their correlation in the U.S. data discussed earlier. The issue considered here is whether combinations of the two parameters $\sigma_R$ and $\sigma_\lambda$ can explain these three moments. I show that this is not possible for certain model parameters, while it is possible for others. This is obviously not a full estimation exercise, since many combinations of model parameters fit these three moments equally well. Rather, Table 3 provides some guidance as to whether the parameters that are able to replicate these moments are plausible.

The table is constructed by varying $\sigma_\lambda$ and $\sigma_R$ so the correlation between detrended employment and detrended wages equals 0.41 for all the specifications. When $\gamma$ and $\zeta_v$ are set at their baseline values of 1, one cannot choose values of $\sigma_\lambda$ and $\sigma_R$ to account for the standard deviations of $h$ and $w$. This can be seen by noting that, for the parameters reported, one of these predicted standard deviations is larger than the observed one, while the other is smaller. If one reduces either $\sigma_\lambda$ or $\sigma_R$, the other must be lowered as well to maintain the correlation between $h$ and $w$. As a result, both the standard deviation of $w$ and that of $h$ fall, and one of these declines renders the model more counterfactual. The same argument applies if either $\sigma_\lambda$ or $\sigma_R$ is increased.

The root cause of this problem is that, as we saw before, the baseline parameters lead wages to change much more than employment in response to changes in $dR/dh$. This leads the standard deviation of wages to be too high relative to the standard deviation of employment. If one tried to increase the latter relative to the former by raising $\sigma_\lambda$, the correlation between employment and wages would be too low, so the performance of the model cannot be improved in this way.
Letting $\gamma = 8$ and $\zeta_v = 0.66$ comes closer to matching these moments, because, as we saw, it ensures that $dR/dh$ leads to less procyclical wages. Nonetheless, the moments cannot be matched exactly. Even more troubling, the fit is actually worsened if $\gamma$ is raised further, because variations in $\lambda^4$ around the steady-state value of 0.4 lead to procyclical movements in real wages when $\gamma \geq 8.15$. The reason for these procyclical movements is that (44) implies that a more altruistic firm wishes to hire more unemployed people and, all else being equal, this leads to an increase in vacancies when $\lambda^4$ rises. A countervailing effect, obviously, is that increases in $\lambda$ lead wages to rise, reducing labor demand. However, this effect is weak when $\gamma$ is large, so the vacancy-increasing effect dominates for sufficiently large $\gamma$.

As discussed earlier, one might prefer to assume that selfish firms do not increase their vacancies when $\bar{\lambda}$ rises, because, workers inability to observe $dR/dh$ prevent them from using a firm’s hiring level to determine its altruism. One crude way of capturing this informational imperfection is to set $\lambda^4 = \lambda^I = 0$ in (44), and the four last rows of Table 3 show the results of this approach. In this case, increases in $\bar{\lambda}$ still raise wages, but now they unambiguously lead firms to lower employment. The result is that a lower value of $\gamma$, namely, $\gamma = 6.5$, suffices to match all three moments, even when $\zeta_v$ is set to 1.

5 Adding Training Costs

There are several reasons to consider an extension of the model that incorporates post-recruitment training costs. First, as discussed by Silva and Toledo (2006), these costs appear to be significantly larger than the costs of recruitment. Second, Silva and Toledo (2006) argue that incorporating realistic costs of this type has an important impact on the cyclical properties of the Mortensen-Pissarides (1994) model. It therefore seems important to analyze whether the present model is equally affected by the incorporation of training costs.

There are also two reasons to incorporate such costs that relate to the model itself. The first is that, without training costs, the model has difficulty rationalizing the existence of wages that are high relative to recruitment costs. It can do so only if the elasticity of worker departures with respect to the firm’s wage is very high. With training costs, the firm has an
additional reason to fear worker departures, and this rationalizes the payment of relatively high wages. The second is that concavity in training costs with respect to the number of workers trained can contribute to satisfying the second-order conditions in the case where the \( \kappa \) function is convex.

To simplify the presentation of this section, I derive equilibrium conditions under the assumption that the firm is selfish and \( \lambda^A = \lambda^I = 0 \). Adding the effects of altruism to these equations is straightforward, because these effects are captured by the difference between the equations in Section 2 and those in Section 1. The equations that correspond to (43) and (44) for this combination, which are used in the simulations, are displayed in Appendix 3.

Consider a firm that incurs training costs \( \tau \) that depend on the number of individuals newly hired by the firm. For firm \( i \), the earlier analysis makes it clear that the number of people newly hired at \( t \) equals \( h^i_t - h^i_{t-1}(1 - s - (1 - F^i_t)\bar{m}(v_t/h_{t-1})^\ell) \).

Analogously to (24), the present discounted value of profits can thus be written as

\[
\Pi^i_t = R^i_t(h^i_t) - w^i_t h^i_t - \kappa(v^i_t) + E_t \rho^2 \Pi^i_{t+2}(h^i_{t+1}) - \tau \left( h^i_t - h^i_{t-1} \left[ 1 - s - (1 - F^i_t)\bar{m}(v_t/h_{t-1})^\ell \right] \right)
\]

\[
- \rho \tau \left( h^i_{t+1} - h^i_t \left[ 1 - s - (1 - F^i_{t+1})\bar{m}(v_{t+1}/h_t)^\ell \right] \right)
\]

\[
+ E_t \rho \left( R^i_{t+1} - w^i_{t+1} h^i_{t+1} - \kappa \left( v_{t+1} \frac{h^i_{t+1} - h^i_t(1 - s) - (h^i_t/h_t)(1 - F^i_{t+1})\bar{m}^h_{t+1}}{F^i_{t+1} m^h_{t+1} + m^w_{t+1}} \right) \right). \tag{48}
\]

As before, this formulation allows one to hold \( h^i_{t+k} \) and \( w^i_{t+k} \) for \( k \geq 1 \) constant as one varies \( v^i_t \) and \( w^i_t \). The resulting first-order conditions are now

\[
\frac{d\Pi^i_t}{dh^i_t} \frac{dh^i_t}{dv^i_t} - \kappa^t = 0, \tag{49}
\]

\[
\frac{d\Pi^i_t}{dh^i_t} \frac{dh^i_t}{dw^i_t} - h^i_t + \tau^t(w^i_t)^{-\gamma} F^i_t h^i_{t-1} \bar{m}(v_t/h_{t-1})^\ell = 0, \tag{50}
\]

35
while
\[
\frac{d\Pi_t^i}{dh_t^i} = \frac{dR_t^i}{dh_t^i} - w_t^i + E_t \rho \kappa_t^i \frac{1 - s - \tilde{m}(1 - F_t^{i+1})^{\ell}}{F_t^{i+1} \tilde{m}} \left( \frac{v_t^{i+1}}{h_t^i} \right)^{\ell-1} + \left( \frac{v_t^{i+1}}{u_t^i} \right)^{\eta-1} + E_t \rho \tau_t^{i+1} \left( 1 - s - \tilde{m}(1 - F_t^{i})^{\ell} \right). 
\]

(51)

Note that (49) is identical to (25), whereas (50) differs from (26) because raising wages now has an additional advantage over raising vacancies as a device to increase employment. This advantage is that the incumbent employees who stay at the firm do not require training and thus reduce costs by \(\tau\).

Because (49) is the same as (25), the second derivative of profits with respect to \(v_t^i\) can still be written as (30). For the second-order conditions to be satisfied, this second derivative has to be negative. As shown in Appendix 2, at a symmetric steady state this now requires that
\[
\frac{d^2 R_t^i}{(dh_t^i)^2} < \kappa'' \frac{1 + \rho[1 - s - (\tilde{m}/2)(v_{t+1}/h_t^i)^{\ell}]}{[(\tilde{m}/2)(v_t/h_{t-1})]^{\ell-1} + (u_t/h_{t-1})^{\eta-1}]} + \tau'' \left( 1 + \frac{\rho}{2} \left( 1 - s - \frac{\tilde{m}}{h_t^i} \right)^{\ell} \right)^2, 
\]

(52)

which is easier to satisfy if \(\tau'' > 0\).

Combining (49) and (50) to eliminate \(d\Pi_t^i/dh_t^i\), one obtains
\[
F_t^i \tilde{m} \left( \frac{v_t}{h_{t-1}^i} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}^i} \right)^{\eta-1} = \frac{\kappa_t^i (w_t^i)^{\gamma} F_t^i \tilde{m} h_{t-1} \left( \frac{v_t}{h_{t-1}^i} \right)^{\ell} \left( \frac{h_{t-1}^i}{h_t^i} + \frac{v_t}{u_t} \right)}{h_t^i - \tau'(w_t^i) - \gamma F_t^i h_{t-1}^i \tilde{m} \left( \frac{v_t}{h_{t-1}^i} \right)^{\ell}}. 
\]

(53)

At a symmetric equilibrium, this becomes
\[
\frac{\tilde{m}}{2} \left( \frac{v_t}{h_{t-1}^i} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}^i} \right)^{\eta-1} = \frac{2 \kappa_t^i (w_t^{i+1})^{\gamma} \tilde{m} h_{t-1} \left( \frac{v_t}{h_{t-1}^i} \right)^{\ell}}{h_t^i - \tau'(w_t^i) - \gamma \tilde{F} h_{t-1}^i \tilde{m} \left( \frac{v_t}{h_{t-1}^i} \right)^{\ell}}, 
\]

(54)

while symmetry allows (49) to be written as
\[
\frac{dR_t}{dh_t} = w_t - \frac{\tilde{m}}{2} \left( \frac{v_t}{h_{t-1}^i} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}^i} \right)^{\eta-1} + E_t \rho \kappa_t^{i+1} \left( 1 - s - \tilde{m} \left( \frac{v_t^{i+1}}{h_t^i} \right)^{\ell} \right) - \tau_t^{i+1} + E_t \rho \tau_t^{i+1} \left( 1 - s - \frac{\tilde{m}}{2} \left( \frac{v_t^{i+1}}{h_t^i} \right)^{\ell} \right) = 0. 
\]

(55)
The quantitative analysis of the effects of training costs is simplified by assuming that $\tau'$ is constant. Silva and Toledo (2006) suggest that the marginal cost of training one individual worker equals about 13 times the cost of recruiting an additional employee. This implies that $\tau' = 13\kappa'/(dh/dv)$; so I calibrate $\tau'$ in this manner.

Table 4 shows elasticities of wages and $dR/dh$ with respect to employment in a model where detrended employment is given by (45). Its last column displays $\bar{F}'$, the density of $F$ at zero. While it does not vary with $\gamma$, this density does depend on $\bar{\lambda}$. With realistic training costs, $\bar{F}'$ is substantially lower than it was in Section 3. Increases in wages now differ from vacancies as a method for increasing employment in that some of the employees obtained by increasing wages do not require training. This leads firms to raise wages (relative to recruitment costs) and therefore requires a smaller elasticity of employee departures to ensure that wages are high.

The entries in this table lead to two additional conclusions. The first is that the elasticities of wages and $dR/dh$ with respect to employment are reduced relative to those in Table 2, but the reductions are modest. The intuition for this effect of training costs appears to be somewhat different from that provided in Silva and Toledo (2006) for their bargaining model. Here, $\gamma > 0$ implies that wages become less effective as a recruiting device when wages are increased (because wage and nonwage aspects of jobs are not perfect substitutes). This means that, when wages are already high in booms, wages are also not very effective relative to vacancies in reducing a firm’s training costs for new employees. This further reduces the firm’s incentive to raise wages in booms.

To complete the analysis, I now let both $\bar{\lambda}$ and $d\bar{R}/dh$ vary in the model with training costs. The stochastic processes for these variables are once again given by (46) and (47), and the results are displayed in Table 5. As before, the parameters $\sigma_\lambda$ and $\sigma_R$ are set so the correlation between the logarithms of $w$ and $h$ equals 0.41. When $\gamma$ and $\zeta_v$ equal their baseline values, it remains impossible to match the standard deviations of employment and wages by varying $\sigma_R$ and $\sigma_\lambda$. On the other hand it is almost possible to do so while letting $\zeta_v = 1$ if one raises $\gamma$ to equal 6. While this value is still substantial, it is lower than what
was needed in the absence of training costs.

The last four rows of Table 5 show the results of assuming that firms set vacancies selfishly, so that $\lambda^A$ and $\lambda^I$ equal zero in (55). With this modification, which makes real wages more countercyclical in response to changes in $\bar{\lambda}$, it is nearly possible to match the three moments with a value of $\gamma$ of only 4. The changes in $\lambda$ that are needed are not trivial, but they are still low relative to the steady-state value of 0.4. Interestingly, these variations in $\bar{\lambda}$ do not end up explaining a very large fraction of employment fluctuations, most of which remain accounted for by changes in “labor demand.”

6 Conclusion

This paper has shown that parameters can be found in a matching model with monopsonistic elements that mimic certain aspects of the joint behavior of real wages, employment, and vacancies in the United States. Interestingly, the performance of this model is enhanced by assuming that workers require that firms be minimally altruistic. This lends some credence to the idea that required firm altruism can capture some of the fairness considerations that employers and workers allude to when discussing wages. Nonetheless, it is important to note that the model requires nonstandard parameter values in order to explain the standard deviations of employment and wages as well as their correlation; so further work is needed to see whether these values are consistent with other observations.

The model is highly stylized, and numerous extensions could help to determine its applicability. A source of simplicity, but also an important shortcoming of the model, is that it considers homogeneous firms. Particularly because the business cycle is associated with differences in the rates at which different sectors expand and contract their employment, it would be useful to develop an analogous theory where wages differ across firms. Similarly, the model covers only homogeneous workers and therefore does not make predictions about wage dispersion within firms. Lastly, the model neglects variations in the extent to which firms lay off workers over the business cycle, and this, too, seems to be a promising area for further analysis.
7 References


Mortensen, Dale T. and Eva Nagypál, “More on unemployment and vacancy fluctua-


Table 1
Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Alts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$: Discount rate</td>
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<td></td>
</tr>
<tr>
<td>$s$: Steady-state separation rate into unemployment</td>
<td>.034</td>
<td></td>
</tr>
<tr>
<td>$(v/u)^\eta$: Steady-state finding rate for unemployed</td>
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<td></td>
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<tr>
<td>$\eta$: Elasticity of finding rate with respect to $v/u$</td>
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<td>$\ell$: Elasticity of finding rate with respect to $v/h$</td>
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<tr>
<td>$\bar{m}(v/h)^\ell$: Steady-state finding rate for employed</td>
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<td>$\psi^1$: Average welfare gain for unemployed</td>
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<tr>
<td>$2\psi^2$: Expected gains from second offer</td>
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<tr>
<td>$\kappa'/w(dh/dv)$: Steady-state recruitment cost in wage units</td>
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<td></td>
</tr>
<tr>
<td>$\zeta$: Elasticity of recruiting costs</td>
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<td></td>
</tr>
<tr>
<td>$\gamma$: Measure of $w - x$ substitutability</td>
<td>1 8</td>
<td></td>
</tr>
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<td>$\epsilon$: Steady-state elasticity of demand</td>
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<td></td>
</tr>
<tr>
<td>$\alpha$: Exponent on labor in the production function</td>
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</tr>
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<td>$\Phi/h^\alpha$: Index of returns to scale in production</td>
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<td></td>
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<td>Spec.</td>
<td>Parameters</td>
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<td>------------</td>
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Table 3
The effect of independent variations in $\lambda$ and $z$ (or $\epsilon$)

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<th>$\sigma_\lambda$</th>
<th>$\sigma_R$</th>
<th>S.D.($h$)</th>
<th>S.D.($w$)</th>
<th>Corr($h,w$)</th>
<th>Frac.($h$) due to $R$</th>
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<td>.41</td>
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<td></td>
<td></td>
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Table 4
Elasticities with respect to employment with nonzero training costs

<table>
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<th>Elasticity of $dR/dh$</th>
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<td>$\lambda^A = .4, \gamma = 1$, $\zeta_v = 1$</td>
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<td>3.43</td>
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<td>$\lambda^A = .4, \gamma = 8$, $\zeta_v = 1$</td>
<td>0.62</td>
<td>1.15</td>
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Table 5

The effect of independent variations in $\lambda$ and $z$ (or $\epsilon$) in the presence of training costs

<table>
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<th>Parameters</th>
<th>$\sigma_\lambda$</th>
<th>$\sigma_R$</th>
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<th>S.D.$(w)$</th>
<th>Corr$(h,w)$</th>
<th>Frac.$(h)$ due to $R$</th>
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<td>.017</td>
<td>.41</td>
<td></td>
<td></td>
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<td>Altruistic wages and hiring</td>
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<td></td>
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<td>.022</td>
<td>.007</td>
<td>.029</td>
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<td>.017</td>
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<td>.023</td>
<td>.007</td>
<td>.030</td>
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<tr>
<td>(4) $\lambda^I = .4, \gamma = 6$</td>
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<td>.023</td>
<td>.013</td>
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<td>.41</td>
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<tr>
<td>Altruistic wages — Selfish hiring</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(5) $\lambda^A = .4, \gamma = 1$</td>
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<td>.028</td>
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<td>.026</td>
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Appendix 1: The effect of wages on $\chi^1$ and $\chi^2$

Note first that the derivative of $\psi_i^{1i}$ with respect to $w^i_t$ is $(w^i_t)^{-\gamma}$. By Leibnitz’ rule the derivative of $\psi_i^{2i}$ with respect to $w^i_t$ is

$$(w^i_t)^{-\gamma} \left[ \int_{x^*}^{\infty} \int_{x^*}^{\infty} \frac{w_{i1}^{1-\gamma}}{y^j} g(y^i) g(y^j) dy^i dy^j \right]$$

$$= (w^i_t)^{-\gamma} F \left( \frac{(w^i_t)^{1-\gamma} - \bar{w}_i^{1-\gamma}}{1 - \gamma} \right).$$

It follows that the derivative of $\chi^{li}_t$ with respect to the firm’s wage is given by the expression in curly brackets in (40).

Using the definition of $\chi^{li}_t$ in (35), the derivative of $\chi^{li}_t$ with respect to $w^i_t$ is

$$\left\{ \left[ 1 - s - \bar{m} \left( \frac{v_t}{h_{l-1}} \right)^{\ell} \right] h^i_{l-1} + \left( \frac{v_t}{u_{l-1}} \right)^{\eta-1} \right\} (w^i_t)^{-\gamma} + \left[ \bar{m} \left( \frac{v_t}{h_{l-1}} \right)^{\ell} h^i_{l-1} + \bar{m} \left( \frac{v_t}{h_{l-1}} \right)^{\ell-1} \right] F_t^i (w^i_t)^{-\gamma}$$

$$= h^i_t (w^i_t)^{-\gamma},$$

where the equality follows from (21).
Appendix 2: Second-order conditions

Consider first the case without training costs. Differentiating (27) while noting that $\kappa'$ depends on $v^i$, which in turn is given by (23), we obtain

$$\frac{d^2 \Pi_i}{d(h_i^t)^2} = \frac{d^2 R_i^t}{(dh_i^t)^2} - E_t \rho \kappa''_{t+1} \left( \frac{1 - s - \bar{m}(1 - F_i^{t+1})(v_{t+1}/h_t)^\ell}{F_i^{t+1} \bar{m}(v_{t+1}/h_t)^{\ell-1} + (v_{t+1}/u_t)^{\eta-1}} \right)^2$$

Moreover, it follows from (28) that $d^2 h^t/dv^t$ is equal to zero, meaning that the number of vacancies a firm must post to hire an additional worker is independent of the number of people it hires. Using (30), we then have

$$\frac{d^2 \Pi_i}{dv^t} = \left\{ \frac{d^2 R_i^t}{(dh_i^t)^2} - E_t \rho \kappa''_{t+1} \left( \frac{1 - s - \bar{m}(1 - F_i^{t+1})(v_{t+1}/h_t)^\ell}{F_i^{t+1} \bar{m}(v_{t+1}/h_t)^{\ell-1} + (v_{t+1}/u_t)^{\eta-1}} \right)^2 \left( F_i^{t} \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{u_t}{h_{t-1}} \right)^{\eta-1} \right)^2 \right\}$$

This must be negative for the second-order condition to be satisfied. At a steady state, this requires that

$$\frac{d^2 R_i^t}{(dh_i^t)^2} < \kappa'' \left[ 1 + \rho \left[ 1 - s - \left( \bar{m}/2 \right) (v_{t+1}/h_t)^\ell \right] \right]$$

as in (31).

Now turn to the case with training costs. Differentiating (51), we obtain

$$\frac{d^2 \Pi_i}{d(h_i^t)^2} = \frac{d^2 R_i^t}{(dh_i^t)^2} - E_t \rho \kappa''_{t+1} \left( \frac{1 - s - \bar{m}(1 - F_i^{t+1})(v_{t+1}/h_t)^\ell}{F_i^{t+1} \bar{m}(v_{t+1}/h_t)^{\ell-1} + (v_{t+1}/u_t)^{\eta-1}} \right)^2 - \tau''_t$$

$$+ E_t \rho \tau''_{t+1} \left( 1 - s - \bar{m}(1 - F_i^{t+1}) \right) \left( \frac{v_{t+1}}{h_t} \right)^\ell \right)^2$$

Since (28) still holds, $d^2 h^t/dv^t$ remains equal to zero. Thus, (30) now implies

$$\frac{d^2 \Pi_i}{dv^t} = -\kappa''_t + \left( F_i^{t} \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{u_t}{h_{t-1}} \right)^{\eta-1} \right)^2$$

* For this to be negative at a symmetric equilibrium requires that

$$\frac{d^2 R_i^t}{(dh_i^t)^2} < \kappa'' \left[ 1 + \rho \left[ 1 - s - \left( \bar{m}/2 \right) (v_{t+1}/h_t)^\ell \right] \right]$$

as in (52).
Appendix 3: Equilibrium conditions with altruism and training costs

The equations that correspond to (43) and (44) (or (54) and (55)) are, respectively,

\[
\frac{m}{2} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \frac{v_t}{u_{t-1}}^{\eta-1} = \frac{2(\kappa'_t - \lambda_t^A \omega_t)(w_t)^{-\gamma} F_t \bar{m} h_{t-1} \left( \frac{v_t}{h_{t-1}} \right)^{\ell}}{h_t(1 - \lambda_t^A w_t^{-\gamma}) - (w_t)^{-\gamma} h_{t-1} \left[ \tau_t' \bar{m} \left( \frac{v_t}{h_{t-1}} \right)^{\ell} + \lambda^t \left( 1 - s - \frac{m}{2} \left( \frac{v_t}{h_{t-1}} \right)^{\ell} \right) \right]},
\]

and

\[
\frac{dR_t}{dh_t} = w_t - \frac{(\kappa'_t - \lambda_t^A \omega_t) \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1}}{\frac{m}{2} \left( \frac{v_t}{h_{t-1}} \right)^{\ell-1} + \left( \frac{v_t}{u_{t-1}} \right)^{\eta-1}} + E_t \frac{\rho(\kappa'_{t+1} - \lambda_{t+1}^A \omega_{t+1}) \left( 1 - s - \frac{m}{2} \left( \frac{v_{t+1}}{h_t} \right)^{\ell} \right)}{\bar{m} \left( \frac{v_{t+1}}{h_t} \right)^{\ell-1} + \left( \frac{v_{t+1}}{u_{t+1}} \right)^{\eta-1}}
\]

\[
+ \lambda^A \left( \psi^1_t + E_t \rho \bar{m} \left( \frac{v_{t+1}}{h_t} \right)^{\ell} \left( \psi^2_t - \frac{\psi^1_{t+1}}{2} \right) \right) + E_t \rho \lambda^t \left( 1 - s \right) \psi^1_{t+1} + \bar{m} \left( \frac{v_{t+1}}{h_t} \right)^{\ell} \left( \psi^2_t - \psi^1_{t+1} \right)
\]

\[-\tau_t' + E_t \rho \tau'_{t+1} \left( 1 - s - \frac{m}{2} \left( \frac{v_{t+1}}{h_t} \right)^{\ell} \right) = 0.
\]