Comments on "The constraint on public debt when $r < g$ but $g < m$" by Ricardo Reis.

John H. Cochrane*

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1 Introduction

This is an important paper. The US is embarked on a historically unprecedented peacetime fiscal expansion. The debt to GDP ratio is already higher than it has ever been. And current deficits, spending plans, and looming entitlements mean we are only halfway done. Whether this will work out or not is the single most important macroeconomic question of our time. It's a fiscal 1968. And $r < g$ question is squarely at the center of academic analysis of this question. (See the appropriately influential Blanchard (2019).)

The debt-to-GDP ratio evolves as

$$\frac{d}{dt}\left(\frac{b_t}{y_t}\right) = (r_t - g_t)\frac{b_t}{y_t} - \frac{s_t}{y_t}. \tag{1}$$

with b = real value of debt, y = GDP, r = rate of return, g = GDP growth rate, s = real primary surplus. $r < g$ seems to offer a delicious scenario: Run up the debt with a string of big deficits. Then, just keep rolling over the debt without raising surpluses. Debt grows at r, but GDP grows at g, so the debt-to-GDP ratio slowly declines at rate $r-g$. Apparently debt never has to be repaid by higher surpluses, debt has "no fiscal cost."

If we solve this differential equation forward,

$$\frac{b_t}{y_t} = \int_{\tau=0}^{T} e^{-(r-g)\tau}\ \frac{s_{t+\tau}}{y_{t+\tau}} d\tau + e^{-(r-g)T}\ \frac{b_{t+T}}{y_{t+T}}$$

$r < g$ seems to imply that government debt is infinitely valuable, or that it contains a

*Hoover Institution, Stanford University and NBER. https://www.johnhcochrane.com. Comments on Reis (2021) delivered at the February 19, 2021 NBER Economic Fluctuations and Growth conference.

"bubble" terminal condition that can be "mined."

But this analysis suggests two ridiculous conclusions. First, it seems there are no fiscal limits at all. If our government can borrow, and never worry about paying back debts, why should any of us pay back debts? Why should the government not borrow, and repay our student debts, mortgage debts, business debts; bail out state and local pension promises, and more. Why should we pay taxes? Why should we work? Let the government just send us money and we can order stuff from Amazon.

Washington understands these logical implications of the proposition that debt has no fiscal cost better than economists who write about sober public investments, and Washington is acting on it as we speak.

Well, obviously not.

Second, it seems that a theoretical wall separates $r > g$ from $r < g$. If r is one basis point (0.01%) above g, we solve the differential equation forward to a present value, debts must be repaid, the government must return to fiscal "austerity" to ward off the "bond vigilantes" who might trigger hyperinflation or sovereign default. If r is one basis point below g, we should really solve the integral backward, debts never need to be repaid, the government may borrow and spend, or just give away money to voters, as it pleases, with no repercussions.

Well, obviously not.

So why not?

The conventional limitation is the fact that $r < g$ eventually cannot scale. Sooner or later more debt raises r. *Marginal* $r - g$ is what counts.

Therefore *there is a maximum debt/GDP ratio out there somewhere. The fiscal expansion cannot be unlimited or go on forever.*

This consideration still suggests a fiscal expansion up to the debt/GDP ratio where $r = g$, however.

But that limit may be a long way away. For example, standard investment crowding out is one mechanism that raises r if we overdo it. But crowding out, real interest rates that rise because there isn't enough savings to finance capital formation so the marginal product of capital rises, seems a long way away, and something we would easily see approaching by a slow rise in real interest rates.

Now, this paper is deeply about how the marginal product of capital is not measured by the real interest rate, but rather by the equity premium. And thus it's deeply about the flaw in using this sort of perfect foresight logic. But we'll get back to that.

If $r < g$ is driven by low r due to a liquidity premium, or money-like demand for government debt, that demand declines more swiftly than crowding-out as debt increases, suggesting a much lower limit.

Most salient to me, high debts leave us open to doom loop run dynamics. If markets sniff a crisis coming, they charge higher rates as a default premium. Higher rates mean higher debt service which explodes the debt faster, and then the default happens. Greece on steroids with no Germany to bail us out. Leaving ample unused fiscal space stops doom loops, and might also come in handy in the next unforeseen crisis. It's a good thing that WWII did not *start* with 100% debt to GDP already on the books.

2 Beside the point

But today I want to emphasize a more radical view: *The* = $r - g$ *debate is irrelevant to current US fiscal policy issues.* I think economists have to some extent chased a theoretically interesting rabbit down a hole, while the classic and important issues fester.

Remember, the first scenario is a "one-time" fiscal expansion, and then run a few decades of zero primary surplus while $r < g$ whittles down the debt/GDP ratio. The second scenario is that $r < g$ allows the government to run a steady primary deficit and keep a constant debt/GDP ratio. At our 100% debt/GDP, and 1% $r < g$, we can run a steady 1% of GDP primary deficit, $200 billion today, as long as $r < g$ lasts.

But, as I illustrate in Figure 1 and Figure 2, the US runs $1 trillion, 5% of GDP deficits in good times, and $5 trillion, 25% of GDP deficits in each decade's once-in-a-century crises. And then in about 10 years unfunded Social Security, Medicare, and other entitlements really kick in. Our debt-to-GDP ratio is on an explosive upward path already.

Zero primary surplus while $r < g$ whittles down the debt/GDP ratio down means *zero* surpluses, not perpetual 5% of GDP deficits. Zero primary surplus means taxes that *equal* spending, not taxes = 0. The promise never was no taxes, the promise was no *extra* taxes, on the assumption that taxes equal spending already! Zero primary surpluses would be a dramatic, conservatives' dream, fiscal tightening for the US.

The US has *exponentially growing* debt to GDP, not gently declining debt to GDP

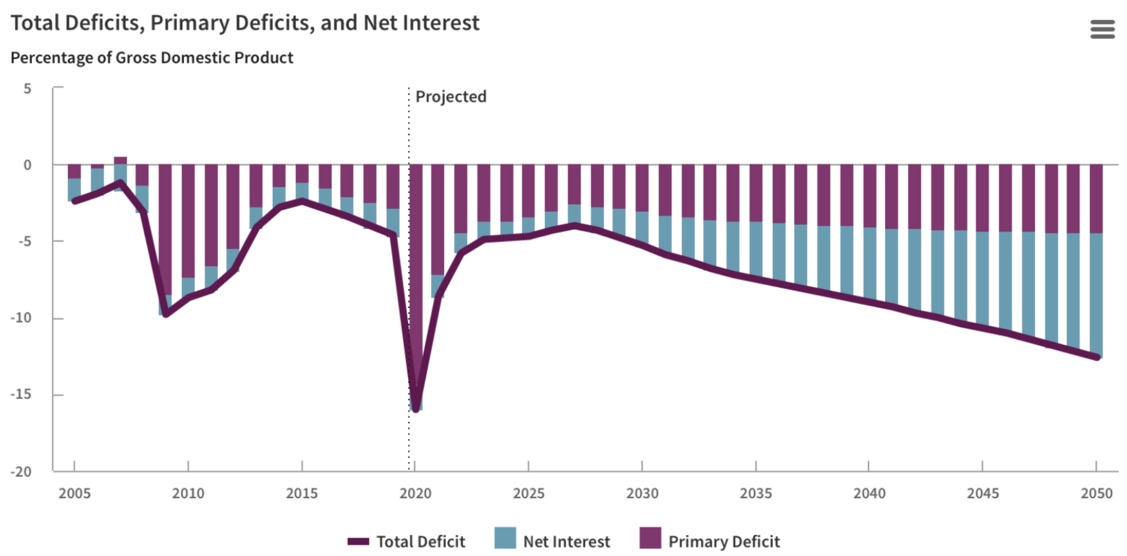

Figure 1: Deficits. Source: Congressional Budget Office https://www.cbo.gov/publication/56516

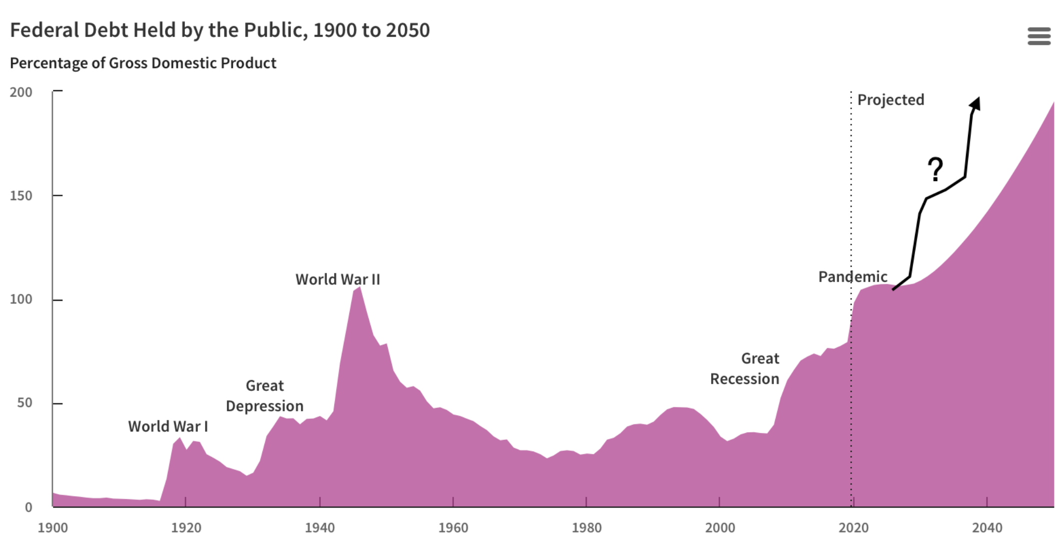

Figure 2: Debt to GDP ratio. Black line: An artistic guess that includes occasional crises. Source: Congressional Budget Office https://www.cbo.gov/publication/56516

that can be pushed to decline from a higher level.

The opportunity also has to last a long time. $r < g$ of 1% means that even with a return to zero primary surpluses, we bring down the debt to GDP ratio by one percent per year. Figure 3 illustrates that path. If the US raises debt from 100% of GDP to 150%, – less than we already have done since 2008 – and $r - g = -1\%$, we need 40 years of taxes actually equal to spending just to bring the debt / GDP ratio back to 100%, and 110 years to reduce debt/GDP to a historically more comfortable 50%. If we go up to 200%, those numbers are 70 years and 139 years. That's a long time to hope the bond vigilantes stay at bay, and we don't have a crisis that demands another "one time" fiscal expansion.

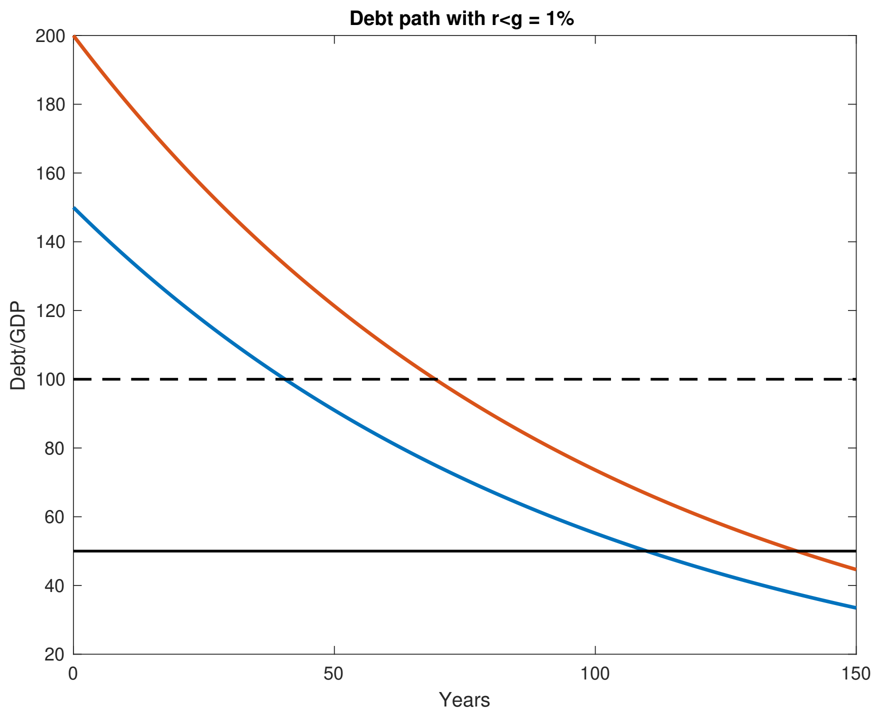

Figure 3: Debt paths with $r < g$ = 1%. Red starts at 200%, blue starts at 150%. Horizontal lines indicate 50% and 100% for reference.

To the scenario of a steady debt-to-GDP ratio with perpetual deficits, $r - g$ of 1% allows a 1% of GDP steady primary deficit, not 5% in good times, 25% in bad times, and then pay for Social Security and health care.

Looking at flows also makes sense of the apparent $r = g$ discontinuity. As we move from $r - g = 0.01\%$ (1 basis point) to $r - g = -0.01\%$ at 100% debt to GDP, we move from a steady 0.01% of GDP ($2 billion) surplus, to a steady 0.01% ($2 billion) of GDP deficit.

That's not going to finance anyone's federal spending wish list! *This* transition is clearly continuous.

The opportunity to grow out of debt with $r - g = -0.01\%$, means a 150% debt to GDP will, with zero primary surpluses, resolve back to 50% debt to GDP in $-\log(0.5/1.5)/0.0001 = 11,000$ years. This is not much different than the infinity, and beyond, required by $r > g$. A sensible understanding of how equations map to the economy is continuous as r passes g. If there is a "wealth effect," a transversality condition violation in debt to GDP that grows at 0.01%, rising from 150% by a factor of 3 to 450% in 11,000 years, then there is surely a "wealth effect" in a debt to GDP ratio that takes 11,000 years to decay by a factor of 3 from 150% to 50%.

This is a quantitative question. $r < g$ of 10% would solve our problems. But $r < g$ of 1% is a factor of 5 at least too small. $r < g$ of 1% would solve a 1% problem. Our problem is at least a factor of 5 larger.

So what does $r < g$ mean? $r < g$ may shift the *average* surplus to a slight perpetual deficit, just as seigniorage allows a slight perpetual deficit. But any substantial *variation* in deficits about that average – business cycles, wars, infrastructure programs – must be met by a substantial period of above average surpluses, to bring back debt to GDP in a reasonable time. The *variation about the average* remains well described by the standard forward-looking model even when r is a bit less than g.

3 Whch r?

But enough of the real world, how does $r < g$ matter in theory? Does $r < g$ represent a wall, on one side of which present values work, and on the other side of which some sort of magic occurs? No, and that is the main point as I see it of the paper. Present values converge, debts must be paid, even when r, as measured by the rate of return on government debt, is below g, the average growth rate of the economy.

Which r matters? The marginal product of capital or return on equity are comfortably above g, so if we use that discount rate everything looks normal. But is that right?

In a world of perfect certainty all interest rates are the same. That we have a choice tells us that the $r < g$ that we measure comes from a world with uncertainty and potentially liquidity premiums.

But it is misleading to pluck one measure, generated from our world, and use it in a perfect-foresight model. Our world can produce rates of return that, put in perfect foresight formulas, generate false infinities and false manna from heaven.

In a praiseworthy effort at intuition, the first half of the paper slips into this bad habit, adducing liquidity or uncertainty to drive a wedge between rates, but then using perfect foresight discounting. I think the message is, use present value relations that are appropriate to the return data we are using.

Indeed, we know the value of debt is finite. So, our job must be to *interpret* the observed finite value of debt in a sensible present value formula, not to decide if the value of debt should be infinite.

The second half of the paper builds a detailed model with both liquidity and uncertainty, which is the right way to go about it. But it's hard reading. So, as my discussant job, I will try to unpack what I think is the core messages of that model in a simpler discussion.

3.1 Liquidity

Start with liquidity. A liquidity value of government debt can drive down its rate of return, to produce $r < g$.

The simplest example is a government that finances itself entirely by non-interest-bearing money. This government can run slight deficits forever, printing money to satisfy economic growth and inflation. This is an $r = -\pi < g$. But it is obviously a limited opportunity. A big fiscal expansion from printing money quickly hits the revenue-maximizing inflation rate. Any significant deficit must still be repaid by surpluses.

We start with

$$\frac{dM_t}{dt} = -P_t s_t, \tag{2}$$

primary deficits are financed by printing money. There is a steady state with constant $M/(Py)$ at

$$\frac{M}{Py}(\pi + g) = -\frac{s}{y} \tag{3}$$

Massage (2) a bit, and we can integrate forward, discounting by the risk free rate, to write

$$\frac{M_t}{P_t y_t} = E_t \int_{\tau=t}^{T} e^{-\left(r^f - g\right)(\tau - t)} \left(\frac{s_\tau}{y_\tau} + i_\tau \frac{M_\tau}{P_\tau y_\tau} \right) d\tau + E_t e^{-(r^f - g)(T-t)} \frac{M_T}{P_T y_T}. \tag{4}$$

I assume $r^f > g$. The point is to generate a lower return on government debt $r = -\pi < g$ and to show how they differ I also write constant rates to keep the formulas simple. Write $e^{-\int_{s=t}^{\tau} - \left(r_s^f - g_s\right) ds}$ if you wish. Both terms converge as we take $T \to \infty$.

The real value of government debt equals the present value of surpluses, including the interest savings generated by the liquidity benefit of money, treated as a flow. This seigniorage revenue can finance a steady primary deficit $s < 0$ as given by (3). The combined surplus term remains positive,

$$\frac{M}{Py}(i) + \frac{s}{y} = \frac{M}{Py}(r^f - g).$$

But the equation makes clear that a substantial rise in deficits must be repaid by later surpluses. Those deficits would typically be financed by adding interest-bearing debt,

$$\frac{b_t}{y_t} + \frac{M_t + B_t}{P_t y_t} = E_t \int_{\tau=t}^{\infty} e^{-\left(r^f - g\right)(\tau - t)} \left(\frac{s_\tau}{y_\tau} + i_\tau \frac{M_\tau}{P_\tau y_\tau} \right) d\tau$$

Here I add both real b and nominal B debt. Again, the transversality condition means that the limiting term goes to zero. Large deficits would be paid for by issuing such interest bearing debt, which pays $r^f > g$. We have an example in which the marginal $r = r^f > g$, though the average $r = -\pi < g$.

We can also try to discount by the return on government debt $r = -\pi$. Now we get

$$\frac{M_t}{P_t y_t} = E_t \int_{\tau=t}^{T} e^{(\pi+g)(\tau-t)} \frac{s_\tau}{y_\tau} d\tau + e^{(\pi+g)(T-t)} \frac{M_T}{P_T y_T}. \tag{5}$$

Now the terminal condition explodes. Since the left hand side is finite, the present value condition also explodes negatively.

Now both (4) and (5) are correct.[1] The question is, which is more useful or insightful? Is it more useful to think of the liquidity services of money as providing a con-

[1] From (2) you get to either

$$\frac{d}{dt}\left(\frac{M_t}{P_t y_t}\right) + \frac{M_t}{P_t y_t}(g - r_t^f) = -\frac{s_t}{y_t} - i_t \frac{M_t}{P_t y_t}$$

venience yield flow, seignorage in the form of a lower interest cost of debt, which we discount at the real interest rate? Or is it more insightful to think of the liquidity services of money as lowering the discount rate, and then say that government debt is a "bubble" that can be "mined" for deficits?

I prefer the former. The latter can lead you mistakenly think the mine is infinite. The two elements explode in exactly offsetting directions. Though the integral explodes, surpluses themselves do not explode. You can miss the fact that substantial surpluses still need to be repaid.

The terminal condition converges in (4) but not necessarily in (5), because *The transversality condition holds discounting with the marginal rate of substitution,*

$$E_t\left[e^{-\rho(T-t)}\frac{u'(c_T)}{u'(c_t)}\frac{M_T}{P_T}\right] = E_t\left[e^{-r^f(T-t)}\frac{M_T}{P_T}\right] = 0$$

The "transversality condition" does not necessarily hold discounting with the ex-post return. The right hand sides of (5) may or may not converge, depending on parameter values.

A mathematician would also say that in the latter case we are simply solving the integral the wrong way. We should solve backward to express debt as an accumulation of past deficits, cumulated at the rate of return.

$$\frac{M_t}{P_t y_t} = E_t \int_{\tau=-T}^{t} e^{-(\pi+g)(t-\tau)}\frac{s_\tau}{y_\tau}d\tau + e^{-(\pi+g)(t-T)}\frac{M_T}{P_T y_T}. \tag{6}$$

This is also correct, but not very insightful.

3.2 Discount rates vs. rates of return

Here is the fundamental technical problem: *The transversality condition does not hold with all one-period discount factors.* One can always discount one-period payoffs with the ex-post rate of return, as with marginal utility or the stochastic discount factor. While

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or

$$\frac{d}{dt}\left(\frac{M_t}{P_t y_t}\right) + \frac{M_t}{P_t y_t}(\pi+g) = -\frac{s_t}{y_t}$$

With $i = r^f + \pi$, these are the same. Integrate one or the other forward.

the stochastic discount factor is

$$1 = E_t \left(\frac{\beta u'(c_{t+1})}{u'(c_t)} R_{t+1} \right)$$

It is trivially true that we can use the ex-post return as an alternative discount factor,

$$1 = E_t \left(R_{t+1}^{-1} R_{t+1} \right).$$

It does not follow that one can always discount infinite streams of payoffs with the ex-post return or other alternative discount factors. It can happen that the present value of cashflows, discounted by the stochastic discount factor, is finite and well-behaved, i.e. that

$$p_t = E_t \sum_{j=1}^{T} \frac{\beta^j u'(c_{t+j})}{u'(c_t)} d_{t+j} + E_t \frac{\beta^j u'(c_{t+j})}{u'(c_t)} p_{t+T} \rightarrow E_t \sum_{j=1}^{\infty} \frac{\beta^j u'(c_{t+j})}{u'(c_t)} d_{t+j},$$

yet if we attempt to discount using returns,

$$p_t = E_t \sum_{j=1}^{T} \prod_{k=1}^{j} \frac{1}{R_{t+k}} d_{t+j} + E_t \prod_{k=1}^{T} \frac{1}{R_{t+k}} p_{t+T}$$

the two terms explode in opposite directions. It doesn't *always* happen. But it can happen, depending on parameters. It's very useful to discount with ex-post returns, but convergence is a second, parameter-dependent issue.

Uncertainty is key to this possibility. In a world of certainty without frictions, the stochastic discount factor is the same as the risk free rate is the same as the ex-post return. To understand the bubble, then you must understand that it doesn't *always* explode. The combinations of high terminal value and low cumulative return that generate a bubble are states of nature with low marginal utility.

4 Bohn's example

To see this possibility in action for government debt, I adapt an example from Bohn (1995).

Suppose consumption growth is i.i.d., and there is a representative consumer with power utility. The value of the consumption stream is

$$p_t = c_t E_t \sum_{j=1}^{\infty} \beta^j \left(\frac{c_{t+j}}{c_t} \right)^{1-\gamma}$$

$$\frac{p_t}{c_t} = \sum_{j=1}^{\infty} \beta^j \left[E \left(\Delta c^{1-\gamma} \right) \right]^j = \frac{\beta \left[E \left(\Delta c^{1-\gamma} \right) \right]}{1 - \beta \left[E \left(\Delta c^{1-\gamma} \right) \right]} \tag{7}$$

where $\Delta c_{t+1} \equiv c_{t+1}/c_t$. Assume that $\beta \left[E \left(\Delta c^{1-\gamma} \right) \right] < 1$, with the result that expected utility is finite. The risk free rate is

$$\frac{1}{1+r^f} = E \left(\beta \Delta c_{t+1}^{-\gamma} \right).$$

We also need to assume that consumption growth is volatile enough to drive the risk free rate down below the growth rate,

$$1 + g = E(\Delta c_{t+1}).$$

Now, suppose the government keeps a constant debt/GDP ratio. At each date t it borrows an amount equal to GDP, c_t, and then repays it the next day, paying $(1+r^f)c_t$ at time $t+1$. (To be precise here, you should check that time-t contingent claim value of the promise to pay $(1+r^f)c_t$ indeed c_t, i.e. $E_t \left(\beta \Delta c_{t+1}^{-\gamma} (1+r^f) c_t \right) = c_t$.) The primary surplus is then

$$s_t = (1+r^f)c_{t-1} - c_t.$$

Now, the end-of-period value of government debt at time t, just after the government has borrowed c_t is obviously, $b_t = c_t$. Our job is to express that fact in terms of sensible present value relations.

If we construct a present value of surpluses, discounting properly with marginal utility, we obtain

$$b_t = E_t \sum_{j=1}^{T} \beta^j \left(\frac{c_{t+j}}{c_t} \right)^{-\gamma} s_{t+j} + E_t \beta^T \left(\frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T}$$

$$= E_t \sum_{j=1}^{T} \beta^j \left(\frac{c_{t+j}}{c_t} \right)^{-\gamma} \left[(1+r^f) c_{t+j-1} - c_{t+j} \right] + E_t \beta^T \left(\frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T}$$

It takes just a little work to boil all this back down to

$$b_t = \left[c_t - E_t \beta^T \left(\frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} \right] + E_t \beta^T \left(\frac{c_{t+T}}{c_t} \right)^{-\gamma} c_{t+T} = c_t. \tag{8}$$

The present value of borrowing c_{t+j} and repaying $(1+r^f)c_{t+j}$ the next period is zero, so only the first term $(1+r^f)c_t$ at time $t+1$ survives. The last term converges to zero, via the transversality condition. (If you want to be picky, you can take a few more steps and start with b_{t+T} on the right hand side.)

However, the value of this claim *cannot* be represented by the expected value of its cashflows discounted at its ex-post return when $r^f < g$. The one-period government debt portfolio return is r^f. The return on the government debt claim is also the risk free rate $(1+r^f)$. Attempting such a present value,

$$b_t = \sum_{j=1}^{T} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \right) s_{t+j} + \left(\prod_{k=1}^{T} \frac{1}{R_{t+k}} \right) b_{t+T} =$$

$$= \sum_{j=1}^{T} \frac{(1+r^f)c_{t+j-1} - c_{t+j}}{(1+r^f)^j} + \frac{1}{(1+r^f)^T} c_{t+T}$$

$$b_t = \left(c_t - \frac{c_{t+T}}{(1+r^f)^T} \right) + \frac{c_{t+T}}{(1+r^f)^T}.$$

Taking expected value,

$$b_t = c_t \left(1 - \frac{(1+g)^T}{(1+r^f)^T} \right) + c_t \frac{(1+g)^T}{(1+r^f)^T}. \tag{9}$$

If $r^f < g$ the present value of cashflows term builds to negative infinity, and the terminal value builds to positive infinity.

Now compare the present value discounted using marginal utility, (8) to the present value discounted using the ex-post return (9). Both equations are correct. Which is more useful? At a minimum, the latter invites mistakes. Seeing an exploding terminal condition, one is tempted to find bubbles of infinite value to mine. But don't forget that the present value condition explodes in the other direction. The present value explodes though all the elements in it are finite. And this government never does anything fancy, it just keeps a steady 100% debt to GDP ratio.

5 Ex post rather than present values

OK, you say, discount using marginal utility and present value formulas converge. But the government still can borrow at r^f and roll over debt forever, no? We sort of know the answer is no once we have a present value. But it's important to spell out what goes wrong.

The answer is no, because *growth is stochastic*. So though $r^f < g = E(\Delta c)$ means that the government will grow out of debt *on average*, but there now states of nature in which growth will persistently disappoint. Then the government will have to raise surpluses, and do so at the most painful time, because consumption is low and marginal utility is high.

Suppose the government borrows 100% of GDP once, and then tries to simply roll over the debt at $r^f < g$. Figures 4 and 5 plot what happens. (I use parameter values $g = E(\log \Delta c) = 3\%$, $\gamma = 2$, $\delta = 0$, $\sigma = 0.15$, which generate $r^f = exp(\delta + \gamma - 1/2\gamma^2\sigma^2) = 1.5\%$ I plot draws at the 1, 5, 50, 95 and 99 percentiles of terminal consumption.)

Since $r^f < g$, you see in the solid lines of Figures 4 and 5 that in a perfect certainty calculation growth outstrips the accumulating debt, and the debt to GDP ratio smoothly declines. But that doesn't always happen! The plots show two draws in which consumption growth disappoints, debt outstrips consumption, and the debt to GDP ratio rises spectacularly. Choose your favorite maximum debt to GDP ratio – 300, or 800 – and in these draws we discover the need to repay a massive debt with taxes, and just at the worst time because we have suffered an economic disaster, having missed what should have been 300% cumulative growth.

So, the one-time fiscal expansion, with "no fiscal cost" is no revealed for what it is: it is a bet, it is the classic strategy of writing an out-of-the-money put option that fails in bad times, and calling it arbitrage.

Though on average g beats r^f, it does not do so weighted by marginal utility, which is why the transversality condition fails in this example.

$$E_0\left[\beta^T \frac{u'(c_T)}{u'(c_0)} b_T\right] = E_0\left[\beta^T \frac{u'(c_T)}{u'(c_0)} b_0(1+r^f)^t\right] = E_0\left[\beta^T \frac{u'(c_T)}{u'(c_0)} b_0 \frac{1}{\beta^T \frac{u'(c_T)}{u'(c_0)}}\right] = b_0.$$

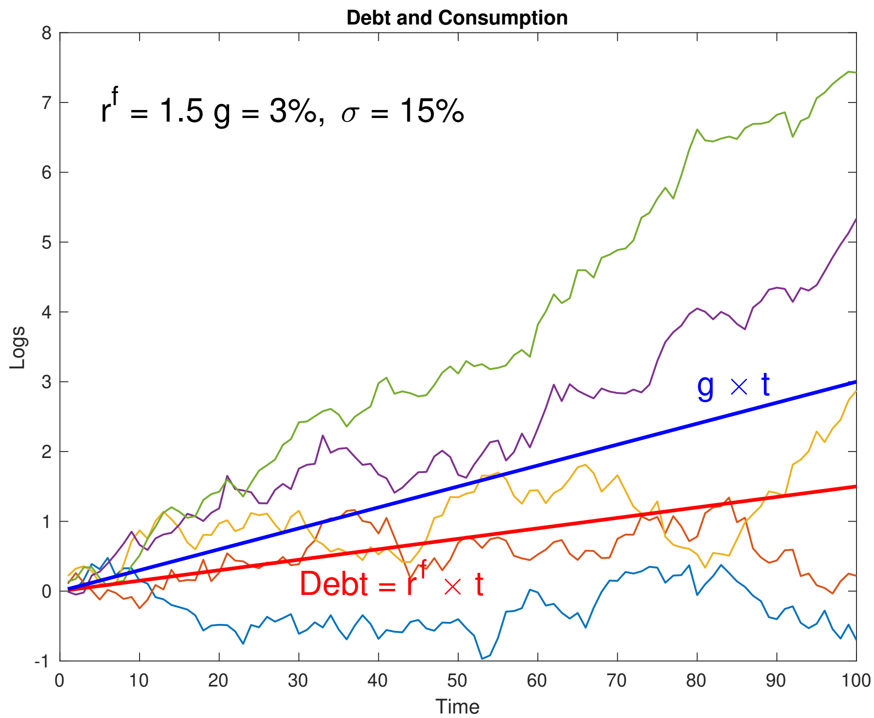

Figure 4: Path of perpetually rolled over debt, and consumption.

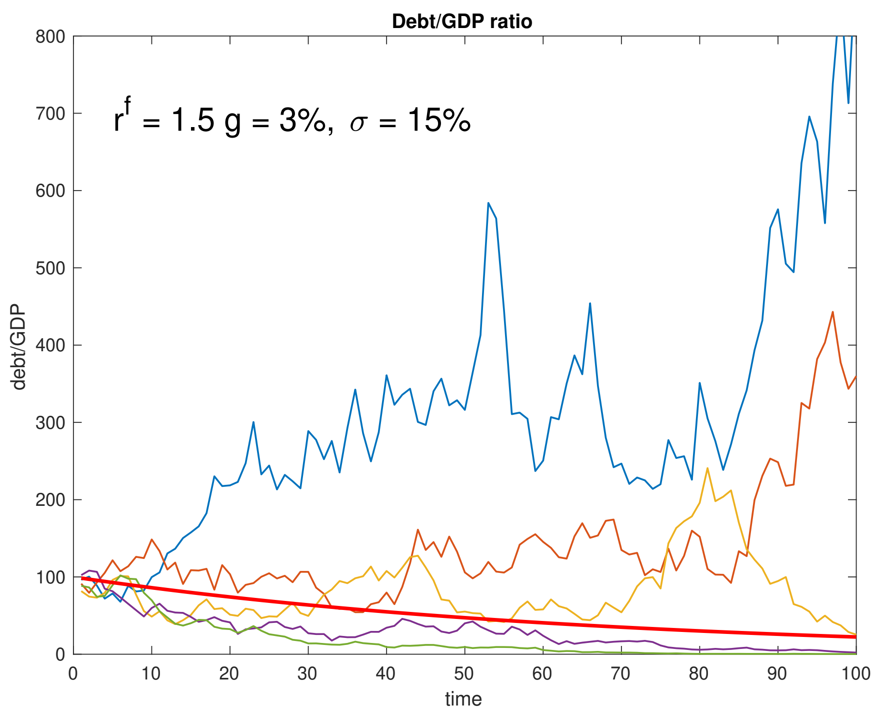

Figure 5: Paths of debt to GDP (consumption) ratio.

6 Paper and bottom line

The paper contains a detailed though rather complex model with uncertainty, production, and financial frictions generating a liquidity premium for government bonds. I think you now see why liquidity and uncertainty are key ingredients. The complexity is to some extent necessary to micro-found liquidity. It also helps to generate realistic parameter configurations for which r is low. To overcome

$$r = \delta + \gamma(g - n) - 1/2(\gamma)(\gamma - 1)\sigma^2$$

I had to assume an unrealistically large σ. One needs either different preferences or a more complex model to generate $r < g$ from uncertainty realistically.

But the basic point is much more general, and as usual microfounded detail and quantitative realism hide how important that basic point is.

The bottom line:

$r < g$ is like seignorage, allowing a small steady deficit. But $r < g$ is irrelevant for the big issues of US fiscal policy.

Despite $r < g$, *Large deficits still need to be repaid with primary surpluses*, at least in marginal utility weighted terms. The grow out of debt strategy is like writing out of the money put options and calling it arbitrage.

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