Sharpening the Arithmetic of Active Management

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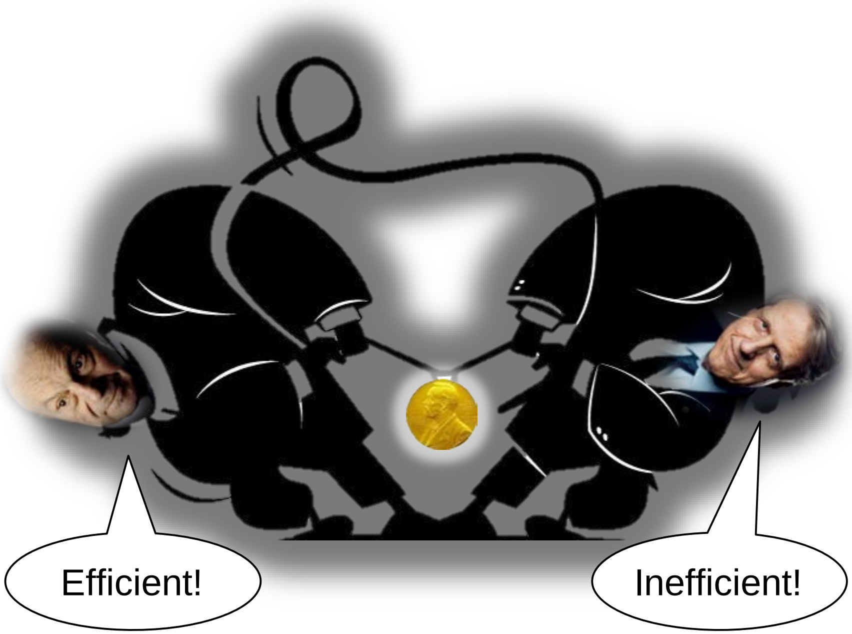
Active vs. passive: efficient vs. inefficient

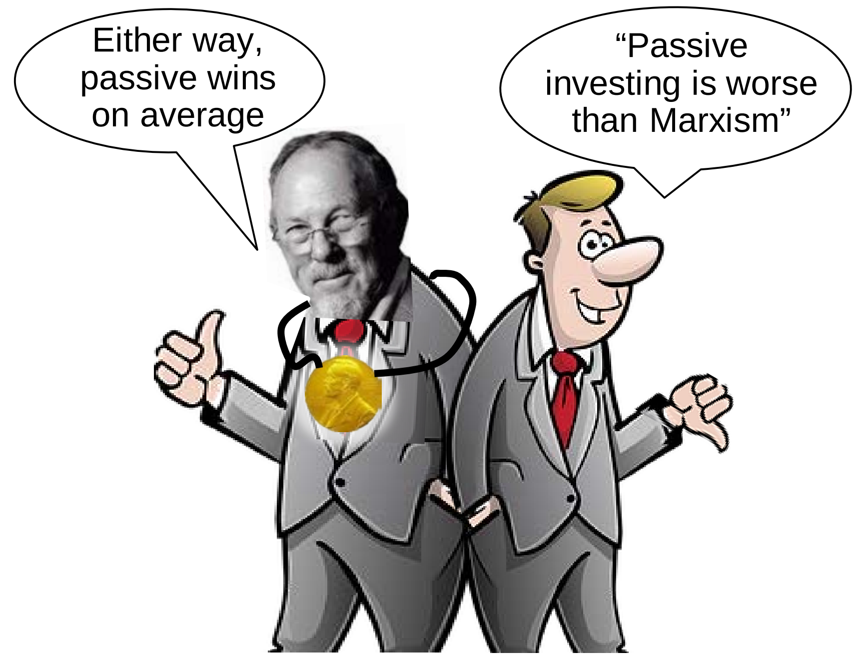
Academics vs. academics



Eugene Fama
Nobel Prize 2013

Robert Shiller
Nobel Prize 2013

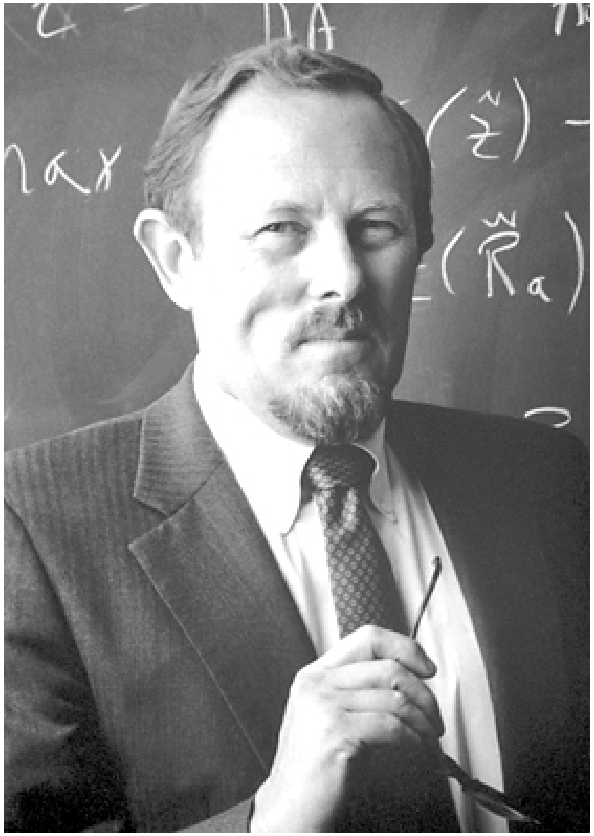
Academics vs. practitioners



William Sharpe
Nobel Prize 1990

Bernstein, L.P.
2016

Sharpe's "Arithmetic of Active Management"



William Sharpe
Nobel Prize 1990

"
it *must* be the case that
(1) before costs: average active return = passive return
(2) after costs: average active return < passive return
"

"
These assertions ...
depend only on the laws of addition,
subtraction, multiplication and division.
Nothing else is required.
"

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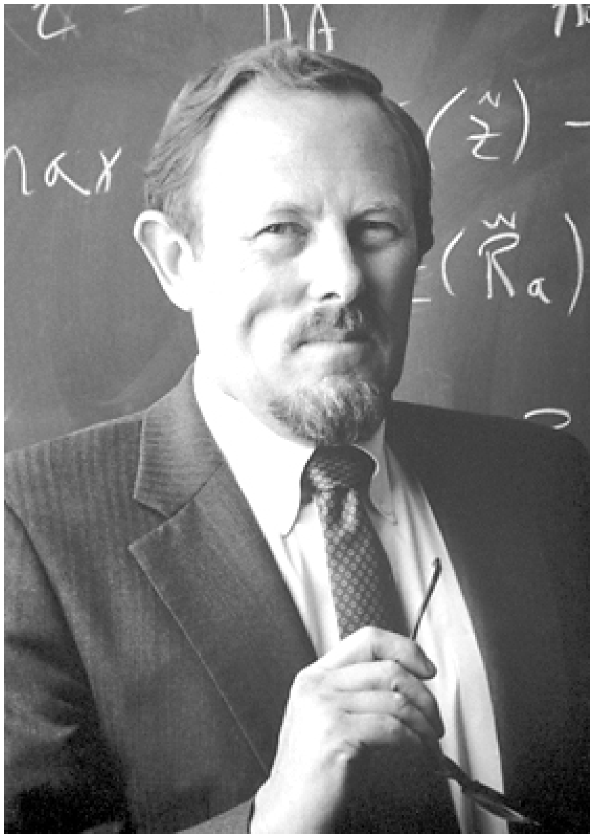
Sharpe's "Arithmetic of Active Management"



William Sharpe
Nobel Prize 1990

Focus first on returns before fees

Results for net returns follow from higher fees for active

Sharpe's starting point:

market = passive investors + active investors

market return = average (passive return, active return)

Passive investing defined as holding market-cap weights

market return = passive return

Conclusion:

market return = passive return = average active return

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Sharpening the Arithmetic of Active Management

Sharpe's arithmetic does not hold in the real world for several reasons:

First Objection:

- Informed (i.e. good) vs. uninformed (i.e., bad) managers
- Informed managers can outperform even if the average doesn't

Broader Objection:

- Can you be passive by being inactive?

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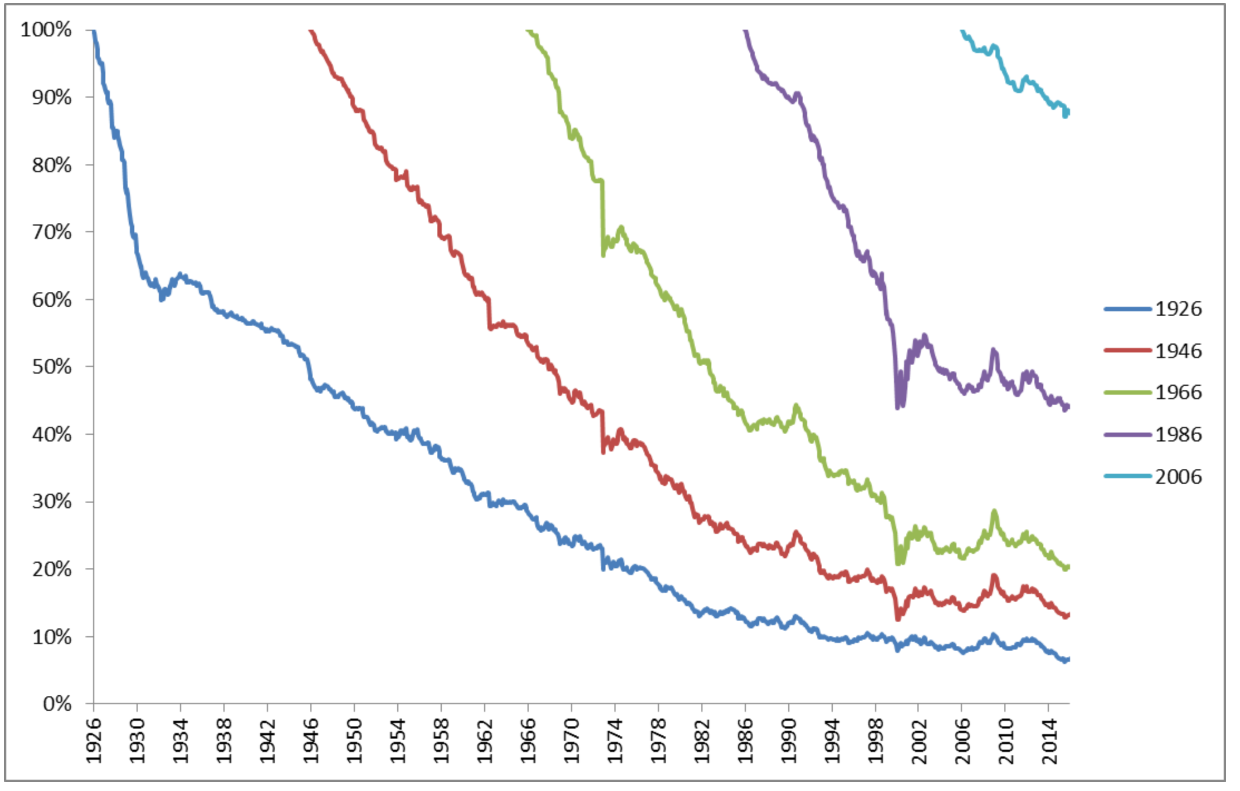
Even a “passive” investor must trade



The fraction of the market owned by an investor who starts off with the market portfolio but never trades after that (i.e., no participation in IPOs, SEOs, or share repurchases). Each line is a different starting date.

Source: Sharpening the Arithmetic of Active Management (Pedersen 2016). Shows path of an investor starting in a given year (1926, 1946, 1966, 1986, 2006) with the market portfolio and not trading thereafter. Market portfolio is all stocks included in the Center for Research in Security Prices (CRSP) database. For illustrative purposes only. Past performance is not a guarantee of future performance. Please read important disclosures in the Appendix.

Sharpening the Arithmetic of Active Management

Sharpe’s hidden assumptions:

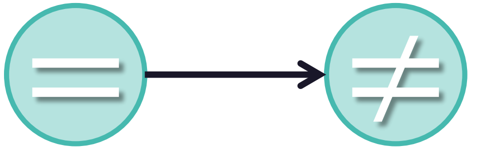
- Passive investors hold exactly the market
- The market never changes
- Passive investors trade to their market-cap weights for free

These assumptions do not hold in the real world:

- IPOs, SEOs, share repurchases, etc.
- Index inclusions, deletions

Relaxing these assumption breaks Sharpe’s equality

- When passive investors trade, they may get worse prices
- Passive investors deviate from “true market”



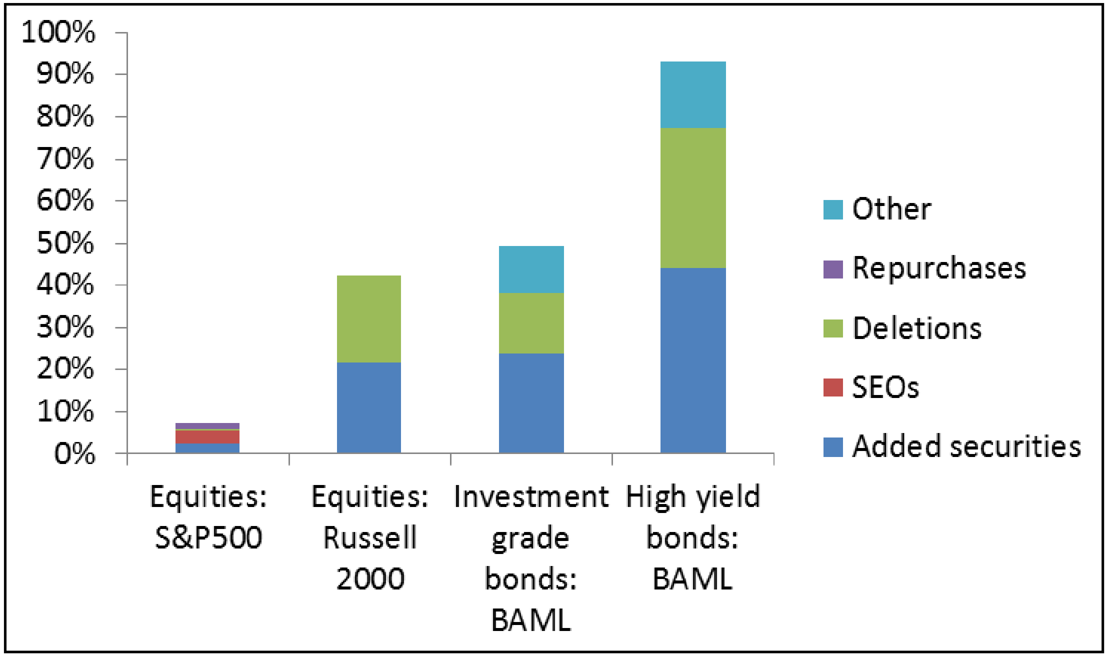
So active *can* be worth positive fees *in aggregate*

- Empirical questions:
  – Do they actually add value?
  – If so, how much? More than their fees?

<u>Fundamental economic issue</u>, not a small ”technical” issue

- Capital markets are about raising capital!
- The world is not a ”pure exchange economy”, the set of firms neither fixed nor ”given”

Trading by a “passive” investor: Indices



For S&P 500 and Russell 2000 (Petajisto, 2011)

- price impact from announcement to effective day has averaged
  - **+8.8% and +4.7%** for additions and **–15.1% and –4.6%** for deletions
- lower bound of the index turnover cost:
  - **21–28 bp** annually and **38–77 bp annually**

Source: Sharpening the Arithmetic of Active Management (Pedersen 2016). Turnover from 1926-2015 for equity indices (S&P500 and Russell 2000) and corporate bond indices (BAML investment grade and high yield indices), and turnover is computed as sum of absolute changes in shares outstanding as a percentage of total market value in the previous month. “Other” includes mergers that may not require trading. For illustrative purposes only. Past performance is not a guarantee of future performance. Please read important disclosures in Appendix.

Sharpening the Arithmetic: Model

- **Passive investors buy**
  - a fraction θ of each security i included in their definition of the "market"
  - zero of each non-included security n

- **Securities**
  - Non-included securities are added to the market ("switch up") with probability s^u
  - Included securities are deleted ("switch down") with probability s^d

- **Active investors**
  - solve standard portfolio problem

- **Equilibrium**
  - Active investors expect to outperform passive, before costs/fees

- **Calibration**
  - Outperformance of the order of institutional fees, smaller than typical retail fees

Conclusion: The future of asset management – doom?

Good For Me

Good for You

Implications of Sharpe's zero-sum arithmetic:

- Active loses to passive after fees
- Money flows passive → markets less efficient
- Surprisingly active still loses
- Eventually all money leaves active, sector is doomed

What happens if everyone is passive?

All IPOs successful regardless of price

- Everyone asks for their fraction of shares

Initial result: boom in IPOs

Eventual result: doom

- Opportunistic firms fail
- Equity market collapses
- People lose trust in financial system
- No firms can get funded
- Real economy falters

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Image Courtesy of http://dc.wikia.com/wiki/Wonder_Woman_Vol_1_601

Conclusion: The future of asset management – my arithmetic

My arithmetic:

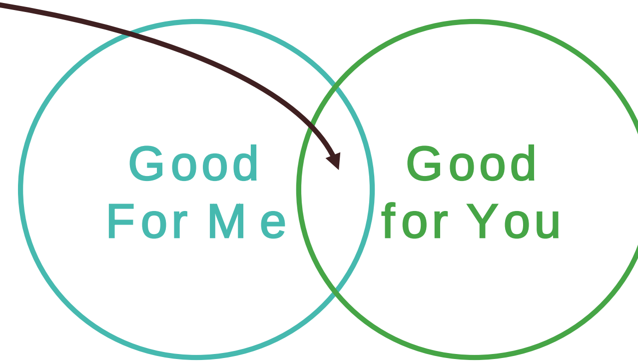
- Suppose active loses to passive after fees
- Money flows to passive → markets less efficient
- Active becomes more profitable → new equilibrium, no doom

The future of asset management

- Passive will continue to grow, but towards a level<100%
- Active management will survive, pressure on performance and fees
- Systematic investing and FinTech will continue to grow

Capital market is a positive-sum game

- Issuers can finance useful projects
- Passive investors get low-cost access to equity
- Active managers compensated for their information costs



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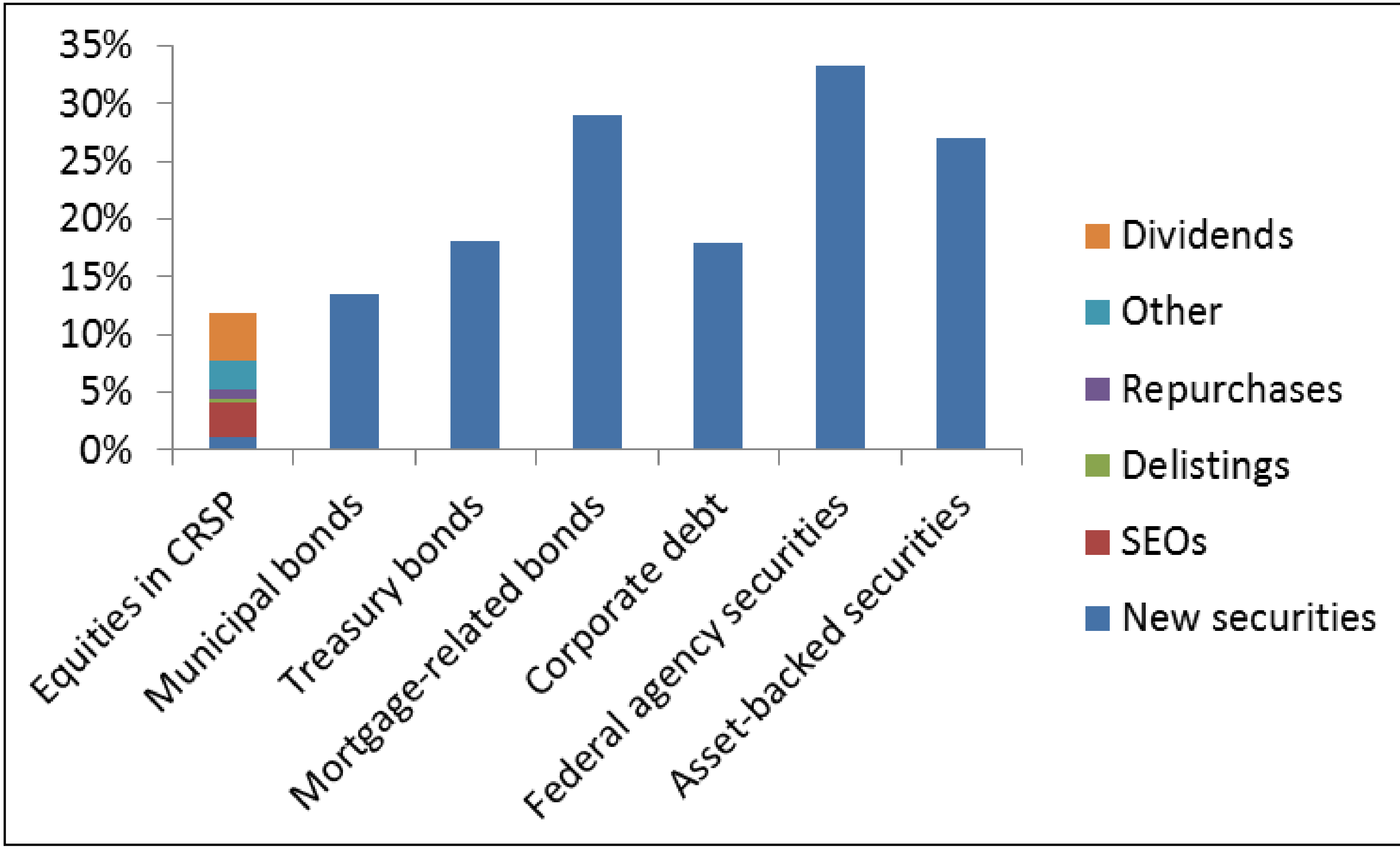
Appendix

Trading by a "passive" investor: Stocks and bonds



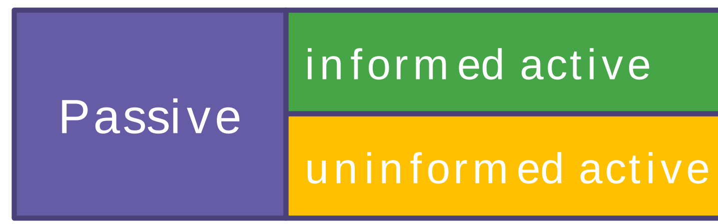
Source: Sharpening the Arithmetic of Active Management (Pedersen 2016). Turnover from 1926-2015 for all US listed stocks included in CRSP and the US municipal bonds, Treasury bonds, mortgage-related bonds, corporate debt, federal agency securities, and asset-backed securities, and turnover is computed as sum of absolute changes in shares outstanding as a percentage of total market value in the previous month. "Other" includes mergers that may not require trading. For illustrative purposes only. Past performance is not a guarantee of future performance. Please read important disclosures in Appendix.

Sharpening the arithmetic: Examples

Why can active managers outperform in aggregate?

Example 0: informed active managers win at the expense of non-informational investors

- Behavioral biases
- Leverage constrained investors
- Pension plans hedging liabilities
- Central banks intervening



Example 1: IPOs, SEOs, and repurchases

Example 2: Index additions and deletions

Example 3: Changes in the “market” and private assets

Example 4: Rebalancing

For illustrative purposes only.

Investing vs. running

If investing was like running a race

An above average investor would outperform the market, on average

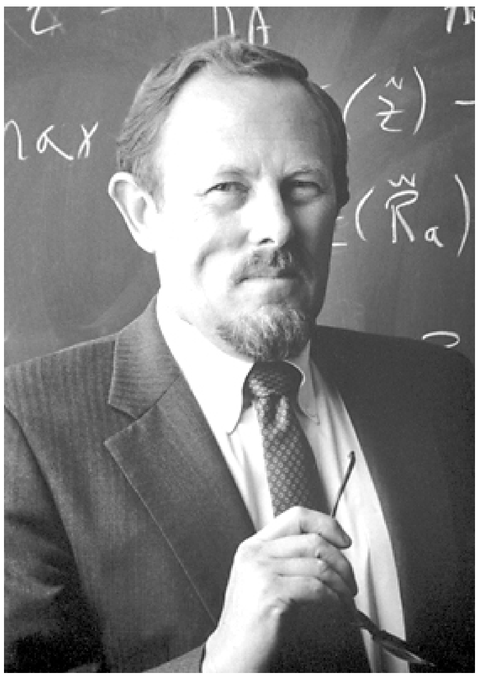
For illustrative purposes only. Past performance is not a guarantee of future performance.

Investing vs. running: if anyone can be average

If the worst investors use index funds and Sharpe’s arithmetic holds

The investor who is just above average suddenly gets a below-average result

Sharpe’s Arithmetic



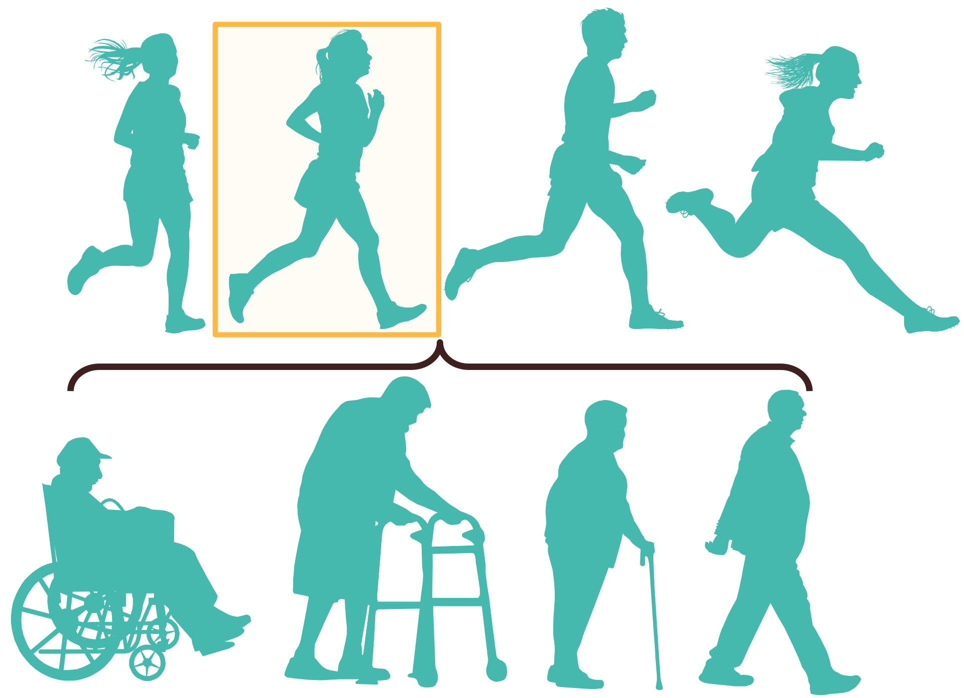


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Image on left courtesy of http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1990/sharpe-bio.html

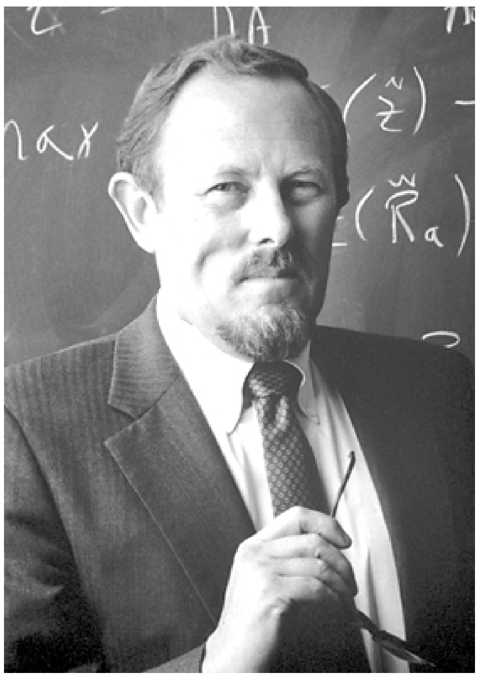
Investing vs. running: asset managers

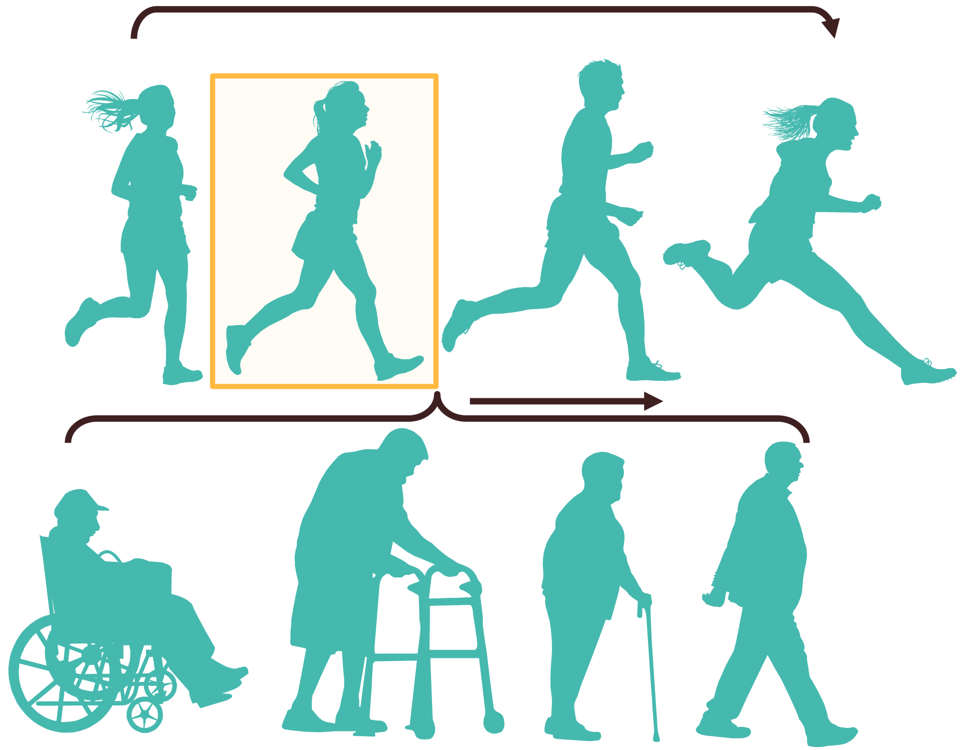
Active management

- Some investors benefit from the skills of managers
- But they pay a free

→ **These effects make it even harder to perform well**

Sharpe's Arithmetic





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Investing vs. running: my arithmetic

My Arithmetic

Sharpening the Arithmetic of Active Management

Lasse Heje Pedersen*

November 2016

Sharpe's (1991) famous "arithmetic of active management" states that

> "it *must* be the case that
> (1) before costs, the return on the average actively managed dollar will equal the return on the average passively managed dollar, and
> (2) after costs, the return on the average actively managed dollar will be less...
> These assertions will hold for *any* time period. Moreover, they depend *only* on the laws of addition, subtraction, multiplication and division. Nothing else is required." (emphasis in original)

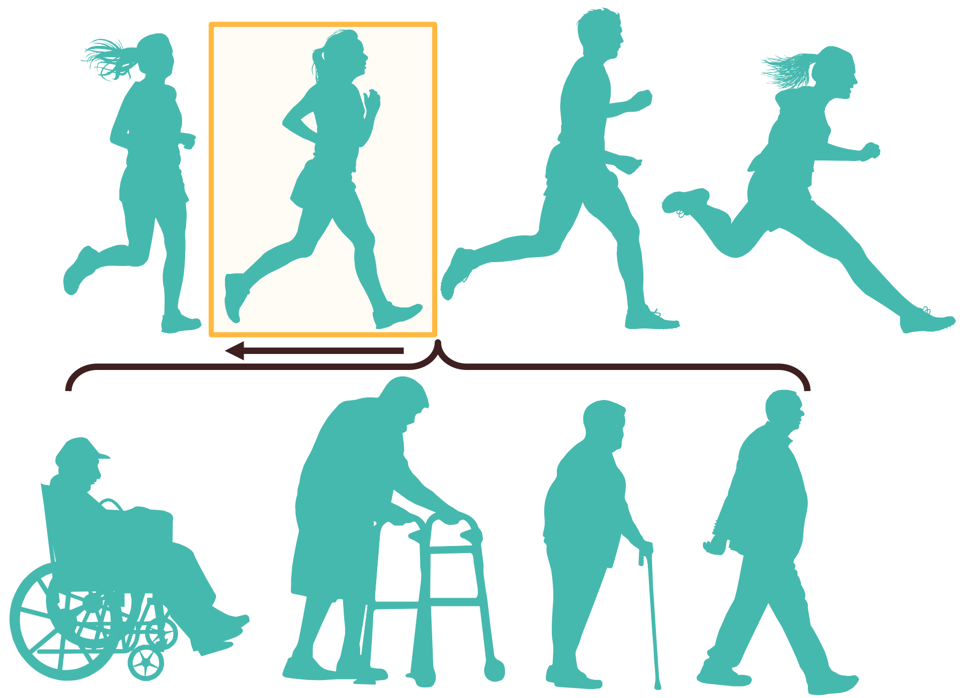
Sharpe's arithmetic is often stated as incontrovertible fact by speakers at conferences followed by a triumphant "QED!" and is cited as proof that active management is "doomed" in aggregate (French 2008).

If active management is doomed, then so is our market-based financial system because we need someone to make prices informative. However, I show that Sharpe's equality does not hold in general. His arithmetic is based on the implicit assumption that the market portfolio never changes. When we relax this assumption, which does not hold in the real world, Sharpe's arithmetic is no longer a mathematical identity.

Sharpe's argument ignores a key aspect of addition and subtraction; namely the addition of new firms and shares and the subtraction of disappearing ones. Although seemingly minor, the market portfolio changes importantly over time such that even "passive" investors must trade regularly, for instance to buy newly issued shares and sell those being repurchased. Whenever passive investors trade in order to maintain their market-weighted portfolios, they may trade at less favorable prices than active managers, which breaks Sharpe's equality.

This turnover of the market portfolio is important for two reasons. First, the changes of the market portfolio are large enough that active managers can potentially add noticeable returns relative to passive investors. Second, the issuance of securities is at the heart of a market-based economy. When we put these reasons together, we see that active management can be worth positive fees, which in turn allows active managers to provide an important, beneficial role in the economy — helping to allocate resources efficiently.

Sharpe (1991 and 2013) is fighting a good and important fight in pointing out the importance of fees and the flaws of many arguments in favor of active management. I think that low-cost index funds is one



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Sharpening the Arithmetic: Model

- **Securities**
  - Risk free rate r^f
  - A fraction I of all risky securities are included in passive investors' definition of the "market"
  - Non-included securities are added to the market ("switch up") with probability $\boldsymbol{s^u}$
  - Included securities are deleted ("switch down") with probability $\boldsymbol{s^d}$
  - No aggregate risk with changes in the market portfolio
  - Dividend payments, $D = E_t\,(D^i_{t+1})$

- **Passive investors buy**
  - a fraction $\boldsymbol{\theta}$ of each included security $\boldsymbol{i}$
  - zero of each non-included security $\boldsymbol{n}$

- **Active investors choose portfolio $\boldsymbol{\pi}$**

$$\max_\pi \pi'(\,E_t\big(D_{t+1}+P_{t+1}\big)-(1+r^f)P_t\ \,)-\frac{\gamma}{2}\pi'\pi$$

To understand the last term, note that we are looking for steady state equilibrium, $P_t = P$, so

$$\bar{\gamma}\,\boldsymbol{Var}\big(D_{t+1}+P_{t+1}\big)=\bar{\gamma}\,\boldsymbol{Var}\big(D_{t+1}\big)=\bar{\gamma}\boldsymbol{\sigma^2 Id}=:\frac{\gamma}{2}\boldsymbol{Id}$$

Equilibrium condition

- Active investor's optimal portfolio

$$\pi = \frac{1}{\gamma}\left(E_t\left(D_{t+1} + P_{t+1}\right) - (1 + r^f)P_t \right)$$

- In equilibrium, active investors must choose a position of
  - $\pi^i = 1 - \theta$ for included securities and
  - $\pi^n = 1$ for non-included securities

- Steady state equilibrium, $P_t = P$, given by

$$\left(1 + r^f\right)P^i = D + P^i - s^d\left(P^i - P^n\right) - \gamma(1 - \theta)$$
$$\left(1 + r^f\right)P^n = D + P^n + s^u\left(P^i - P^n\right) - \gamma$$

Equilibrium: Solution and comparative statics

- **Equilibrium price premium $\Delta P = P^i - P^n$ given by**

$$\Delta P = \frac{\gamma\theta}{r^f + s^d + s^u}$$

- **Comparative statics**
  - Price premium increases with γ and θ
  - Decreases with s^d and s^u
    - For *return difference*, there are additional effects – see below

- **Equilibrium prices**

$$P^i = \frac{D - \gamma(1-\theta) - s^d \Delta P}{r^f} = P^n + \Delta P$$
$$P^n = \frac{D - \gamma + s^u \Delta P}{r^f}$$

Return properties – Dollar returns

- Value change for included securities, in excess of risk free profit

$$\boldsymbol{R}_{t+1}^{i} = D_{t+1}^{i} + \left(1 - s^{d}\right)P_{t+1}^{i} + s^{d}P_{t+1}^{\boldsymbol{n}} - P_{t+1}^{i}(1 + r^{f})$$

- Expected value:

$$E_t\left(\boldsymbol{R}_{t+1}^{i}\right) = \boldsymbol{D} - s^{d}\Delta P - r^{f}P^{i}$$

- Value change for non-included securities

$$E_t(\boldsymbol{R}_{t+1}^{\boldsymbol{n}}) = \boldsymbol{D} + s^{\boldsymbol{u}}\Delta P - r^{f}P^{\boldsymbol{n}}$$

- Difference only depends on risk aversion and size of passive portfolio

$$E_t\left(\boldsymbol{R}_{t+1}^{\boldsymbol{n}} - \boldsymbol{R}_{t+1}^{i}\right) = \left(r^{f} + s^{d} + s^{u}\right)\Delta P = \gamma\theta$$

Return properties – percentage returns

- Return on included securities, with relative premium given by $x = \Delta P / P^n$

$$r_{t+1}^i = \frac{D_{t+1}^i + \left(1 - s^d\right) P_{t+1}^i + s^d P_{t+1}^i / (1 + x)}{P_t^i} - 1$$

- Expected return, given dividend yield $\delta_t = \frac{E_t(D_{t+1}^i)}{P_t^i}$ and price appreciation $\mu_t = \frac{E_t(P_{t+1}^i)}{P_t^i}$:

$$E_t(r_{t+1}^i) = \delta_t + (\mu_t - 1) - s^d \mu_t \frac{x}{1 + x}$$

- Return on non-included securities

$$E_t(r_{t+1}^n) = \frac{D_{t+1}^n + (1 - s^u) P_{t+1}^n + s^u P_{t+1}^n (1 + x)}{P_t^n} - 1 = (1 + x)\delta_t + (\mu_t - 1) + s^u x \mu_t$$

- Note that in steady state $\mu_t = 1$

Return differences – percentage returns

- **Return difference between non-included and included securities**

$$E_t\left(r_{t+1}^n - r_{t+1}^i\right) = x\left(\delta_t + \frac{s^d \mu_t}{1+x} + s^u \mu_t\right) = x\left(\delta_t + \frac{s^d}{1+x} + s^u\right)$$

- **Positive due to dividend-yield effect, additions, and deletions**
  - Comparative statics:
    - Increases in δ_t, s^d, s^u for given x
    - But x is endogenous and decreases in s^d, s^u (as discussed above) – see example below

- **Active investors hold**
  - all of the non-included stocks and $1-\theta$ of the non-included
  - the value-weighted fraction of non-included stocks in their portfolio is

$$\boldsymbol{f} = \frac{(1-I)P^n}{(1-I)P^n + I\theta P^i} = \frac{1-I}{1-I+I\theta(1+x)}$$

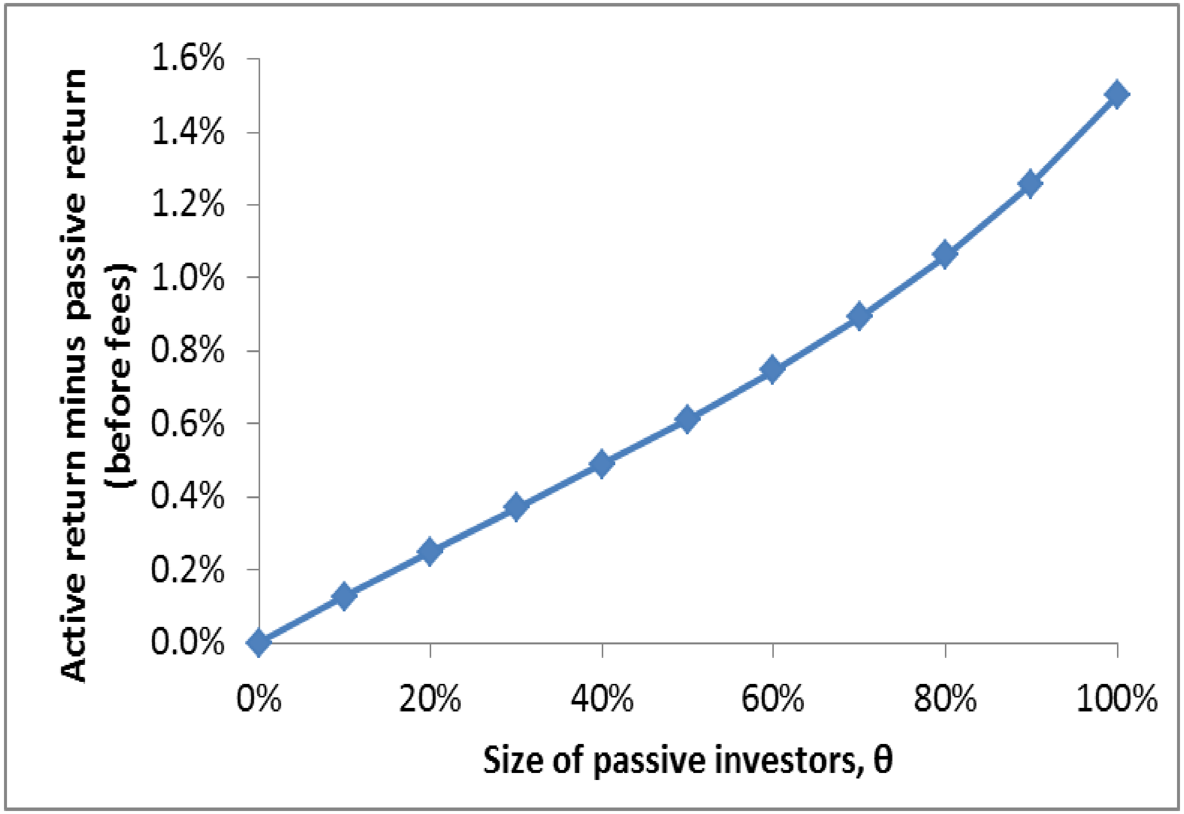
- **Return difference between active investor a and a passive (before fees)**

$$E_t\left(r_{t+1}^a - r_{t+1}^i\right) = f E_t\left(r_{t+1}^n - r_{t+1}^i\right) = f x\left(\delta_t + \frac{s^d}{1+x} + s^u\right)$$

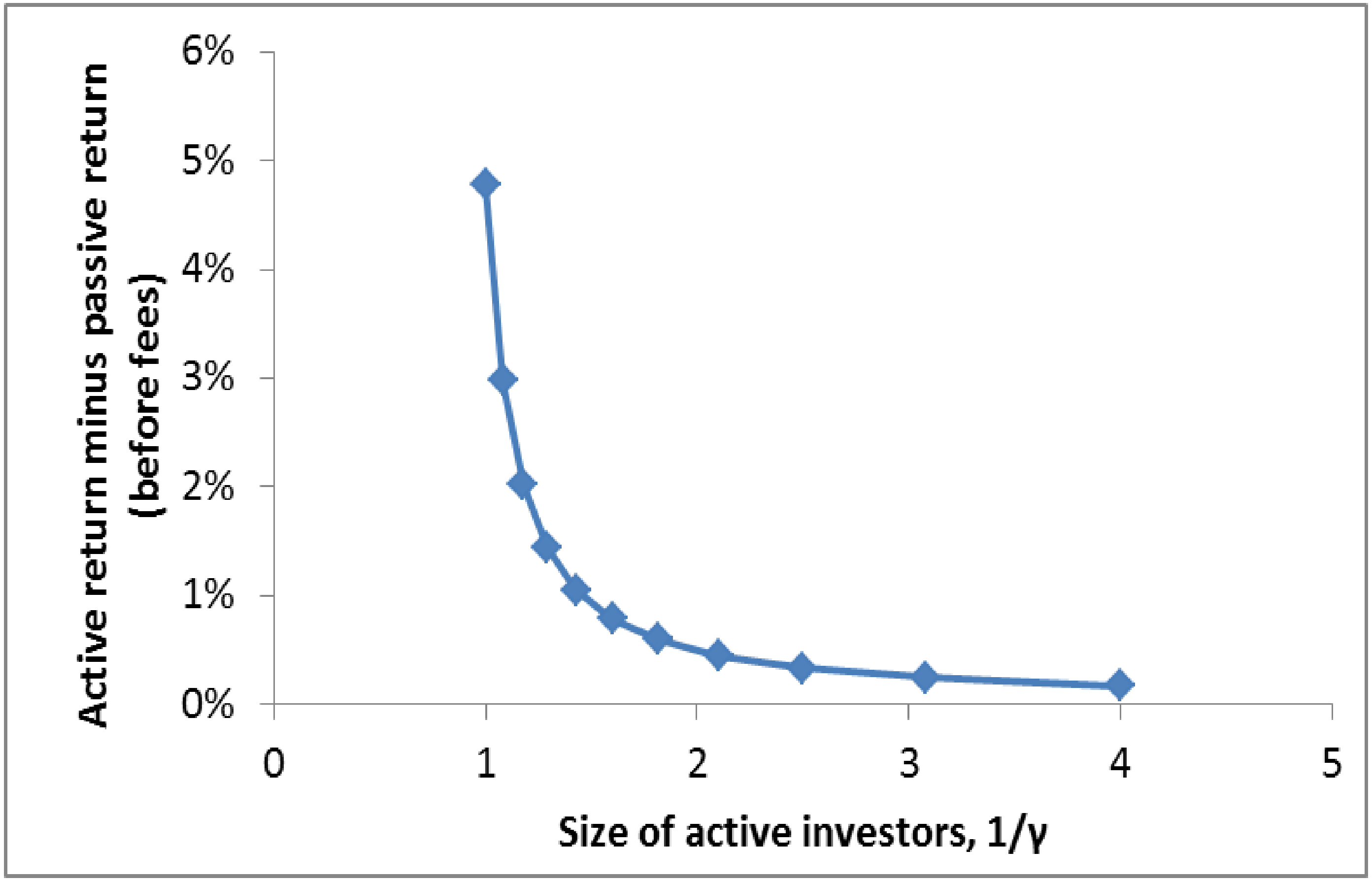
Numerical example

- **Securities**
  - Risk-free rate $r^f = 2\%$
  - Expected dividend $D = 1$
  - Half the securities are included $I = 50\%$
  - The fraction of deletions is $s^d = 2\%$, the fraction of non-included that are added is $s^u = 2\%$
- **Investors**
  - Passive investors buy $\theta = 40\%$ of the included shares
  - Active investors have a risk aversion corresponding to $\gamma = 0.5$
    - chosen to have a reasonable dividend yield of around 3%
- **Equilibrium**
  - Price of included securities $P^i = 31.7$
  - Price of non-included securities $P^n = 28.3$
  - Dividend yield of included securities is $\delta = 3.2\%$
  - Price premium is $x = 12\%$
  - The expected return difference for non-included vs. included stocks is $E_t\left(r_{t+1}^n - r_{t+1}^i\right) = 0.82\%$
  - Given that the active investors hold $f = 60\%$ of assets in non-included securities, the excess return of active relative to passive is $E_t\left(r_{t+1}^a - r_{t+1}^i\right) = 0.49\%$ (before fees).

Active minus passive return vs. the size of passive investing



Active minus passive return vs. the size of active investing



Active minus passive return vs. frequency of additions and deletions