Universal versus Targeted Preschools:
A Mechanism-Design Perspective*

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Abstract

We study optimal fee schedules for public preschools in the presence of private alternatives and endogenous parental labor supply. Our model includes rich heterogeneity in parental ability, preferences and home environments, all of which are private information. The optimal sliding fee schedule trades-off public enrollment, crowding-out of private enrollment and the labor supply incentives of parents. We estimate our model of parents’ preschool and labor supply choices using data from the Early Childhood Program Participation Survey, the Current Population Survey and the Head Start Impact Study. Our model replicates several targeted behavioral observations based on (quasi-) experimental variation, as well as the distributions of enrollment and parental preschool expenditures. We find that overall preschool enrollment could increase by up to 12 percentage points, without raising any new tax revenue, by using the optimal fee schedule to target current subsidies more efficiently. Optimal targeting implies negative fees for the very poor, lower than current fees for parents just above the poverty line, and less crowding out of private preschool enrollment. We show that uncertainty with respect to the optimal policy derives from uncertainty about the mechanisms through which policy affects parenting decisions.

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1 Introduction

“One solution to these problems is to make the programs universal but to offer a sliding fee schedule based on family income.” Heckman (2013, p. 36)

The importance of early human capital investments has recently received substantial attention from economists and policy makers. While much of the recent economics literature has focused on evaluations of preschool and other early childhood programs, policy debates have focused on how these programs ought to be implemented, with particular attention given to the relative merits of universal and targeted pre-K programs. Targeted programs – which are usually means tested by family income – are considered a cost effective way to provide early education programs to disadvantaged children, but depend on arbitrary cut-off rules for participation. Such cutoff rules may exclude some children for whom benefits and economic returns would justify their participation, and might also create a disincentive for parents to earn income. Universal programs, such as those recently introduced in Georgia and Oklahoma, ensure access to early education for all children and do not reduce parental labor supply incentives, but do crowd out private enrollment in preschool leading to inefficient use of public funds (Cascio and Schanzenbach 2013). The crucial question is: What would be the design of a program that trades off these enrollment, crowding-out and parental earnings margins in an optimal way?

We address this problem by applying Mirrlees’ (1971) optimal policy approach to the problem of designing an optimal nonlinear fee schedule for public preschools. We primarily analyze the case of a social objective that is to maximize the number of children going to preschool, but also consider cases where larger policy weights might be placed on children whose baseline home environments are weaker. Whether a child is enrolled in a private or a public preschool does not matter for this objective, only that they attend a preschool program. Such objectives are deliberately different than utilitarian welfare criterions: we do not want to confuse the goal of efficiently promoting preschool enrollment with equalization or redistributive goals. Indeed, the optimal policy we derive implies very progressive public preschool fees, to the extent that certain parents even receive financial benefits for enrolling their child. Such a fee schedule is not a veiled mechanism for redistributing from rich to poor, but rather is a mechanism for creating parental incentives that lead to as much preschool enrollment as possible. Clarity in this distinction is only possible because we consider non-welfarist social objectives.

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1See Currie and Almond (2011) and Duncan and Magnuson (2013).
4These greater weights would be justified by the relatively larger returns to public preschool investments in children whose baseline (home care only) environment and outcomes are relatively weaker (Elango, García, Heckman, and Hojman 2015).
For related reasons, we treat the existing system of taxes and transfers as fixed, and focus on revenue neutral policy reforms. We would not want to consider altering the larger tax system because public preschool is only a small part of the overall function of government, and these other programs are not accounted for by the objective we consider. We consider our overall approach to be balanced: the social objective only considers preschool outcomes, and only the preschool fee schedule can be altered to achieve this objective, with all else remaining fixed. Public preschool spending is nevertheless endogenous. This is partly because changes in the fee schedule can affect parental labor supply and therefore income tax revenue. A change in tax revenue translates one to one in a change in the preschool budget because we consider budget-neutral reforms. In addition, the planner also raises funds for the public preschool program by charging fees, the revenue from which is directly added to program spending.

We first analyze the problem from a theoretical perspective. We determine optimal incentive-compatible allocations, and the implied optimal fee schedule, using the random participation approach pioneered by Rochet and Stole (2002). By using a fee perturbation approach\(^5\) in these derivations, we are able to provide a great deal of intuition for how the optimal fee schedule relates to the tradeoffs faced by the planner.\(^6\) Because the fee perturbation approach isolates effects working through the government budget from effects working through the social objective, we are able to better understand how variations of the policy weights would affect the optimal fee schedule.

The family decision process does not need to be fully parameterized in order to carry out our theoretical analysis. However, before we can apply the optimal policy formulas we derive in our quantitative analysis we must estimate a fully specified model of households. We apply a method of moments estimator, utilizing data from the Early Childhood Program Participation Survey, the Current Population Survey and the Head Start Impact Study. We use bootstrapping to not only estimate the uncertainty associated with the parameters of the model, but also to estimate the sampling distributions of the optimal fee schedule, the benefits of the optimal policy reform, and the relationships between these and certain ‘sufficient’ statistics.

We estimate that the optimal policy reform would lead to an increase in overall preschool enrollment of 12 percentage points (\(s.e. = 1.09\)). These gains in preschool enrollment are attained without any additional taxation being imposed by government, i.e. the reform is tax-revenue ‘neutral’. We also estimate that 39\% (\(s.e. = 13\%\)), or 4.7 percentage points, of this effect is purely due providing public preschool to those who previously did not have access, with the remaining 7.3 percentage points of the increase being due to improved targeting of subsidies under the optimal fee schedule. Importantly, this decomposition helps us understand

\(^5\)In the optimal taxation literature, perturbation methods have become popular complements to the mechanism design approach since Piketty (1997) and Saez (2001).

\(^6\)We demonstrate equivalence with the traditional mechanism design approach in an appendix.
the relatively large uncertainty regarding the effectiveness of the optimal policy: whenever better targeting composes a larger fraction of the enrollment gains the overall enrollment effect is larger. Thus, our analysis suggests that a better understanding of the sources of enrollment changes after policy reforms is an important future research agenda.

Recent literature has identified the effects of some recent policy changes on the distribution of preschool enrollment. Such data is important and we make use of it when estimating our model of parental behavior. One statistic of interest is the effect of implementing high quality free universal public preschool on public preschool enrollment. In the baseline specification of Cascio and Schanzenbach (2013) this is estimated to have been between 16.9 and 19.6 percentage points in Georgia and Oklahoma, depending on maternal education. Another issue of importance is the extent to which introducing universal public preschool crowds out private investments in preschool. The estimates of Cascio and Schanzebach indicate that this was of minimal importance for less educated families in Georgia and Oklahoma, but for families where the mother has at least some college education, 4-5 out of every 10 new public preschool enrollees would otherwise have been in a private program. Kline and Walters (2015) also provide evidence on preschool enrollment behavior using data from the Head Start Impact Study. They find that about one-third of those who are randomly not offered a Head Start slot find an alternative public program to attend. After this ‘control group contamination’ is accounted for the return from running Head Start programs is estimated to be positive. This is one of many studies showing the positive benefits of early investments in children (see Heckman and Kautz (2013) for a detailed summary).

In addition to the elasticity of preschool enrollment, the literature discussed in the previous paragraph also shows that there is rich heterogeneity in preschool attendance decisions. Firstly, even when preschool is free and of high quality, as it now is in Georgia and Oklahoma, a large fraction of families still do not enroll their children. For these families, the utility gained from preschool is not enough to induce enrollment of their children even though there is no direct cost. Many dimensions of heterogeneity could generate such behavior, including, but not limited to, variation in altruism, home environment or costs of travelling to a center. Secondly, although many families switched from private preschool to the high quality free public option in Georgia and Oklahoma, many others did not. Heterogeneity in relative net valuations of public and private options explains this.

About the quality of preschool programs: Our focus in this work is on the implementation of a high quality program, similar to those introduced in Georgia and Oklahoma. Evidence of positive social returns to high quality programs exists, most famously the High/Scope Perry Preschool Program analysed by Heckman et al. (2010b). Evidence from other programs exists as well, as surveyed by Heckman and Mosso (2014) and Elango et al. (2015). However, when programs are at the lower end of the quality spectrum, evidence suggests that negative effects
on children’s outcomes may occur (Baker, Gruber, and Milligan 2015a). We assume that after considering this evidence the social planner would choose to focus on implementing a high quality program only.

This paper is also related to the optimal taxation literature following Mirrlees (1971). In contrast to most of that literature we do not treat redistribution as the social objective. Another example of a non-welfarist objective can be found in Kanbur, Keen, and Tuomala (1994), who consider poverty reduction as a social objective. Kanbur, Pirttilä, and Tuomala (2006) provide a survey of other related papers. Whereas that strand of literature shares the non-welfarist approach with our paper, our paper differs in that we are not studying income taxation, but rather use the mechanism design approach to study optimal preschool pricing.

Our paper is also related to a literature in public finance that has asked whether the public provision of private goods can be welfare enhancing. Besley and Coate (1991) were the first to argue that universal public provision of private goods can facilitate the redistribution of income from rich to poor. Blomquist and Christiansen (1995) were the first to emphasize that this argument could be applied to education and day-care. See Currie and Gahvari (2008) for a comprehensive review of this literature. To the best of our knowledge, our study provides the first serious quantitative application of these arguments.

Some recent papers in the ‘New Dynamic Public Finance’ tradition study implementations of second-best efficient allocations, where government makes use of both income taxes and income-contingent repayment of student loans (Findeisen and Sachs 2015, Stantcheva 2015, Koeniger and Prat 2015). As in these papers, there are two different policy instruments that influence labor supply: Labor income taxes and income-contingent repayment of student loans in their case and payment of preschool fees in our case. Note, however, that our paper is conceptually different in that we take the labor income tax as given and focus on the design of one instrument in isolation. Whereas such an optimization of one policy instrument in isolation is naturally less ambitious in terms of the social objective, it may be of more immediate policy relevance if it is easier for governments to reform one policy instrument (preschool policies in our case) instead of changing many at the same time (which would be preschool policies and income taxes in our case). In line with our approach is the paper by Ho and Pavoni (2016) who study efficient child care subsidies in the tradition of Mirrlees (1971). Their focus, however, is on setting efficient incentives for maternal labor supply.

Finally, this paper is related to Blundell and Shephard (2012) in the sense that we study optimal policies using a structural empirical model. Like them, we connect the statistical uncertainty of our estimated model to uncertainty about the optimal policies and what they can achieve.

This rest of this paper goes like this: In Section 2 we present the general structure of our model and use this to derive formulas describing the optimal policies. In Section 3 we parameterize
2 Theoretical Optimal Policy Analysis

2.1 Model Basics: Choices, Heterogeneity and Preferences

There is a continuum of heterogeneous parents. Parents make two decisions: how much income \( y \) to earn and where to enroll their children. Preschool enrollment decisions are denoted by \( ps \in \{no, pu, pr\} \), where the elements of the choice set are no preschool, public preschool and private preschool, respectively. The model allows for arbitrarily rich and complicated heterogeneity of families and preschool choices, but as we will show only three ‘summary’ dimensions of this heterogeneity need to be identified in order to analyze optimal policies. This result will be presented in Lemma 1 at the end of the subsection.

Heterogeneity. Parents are heterogeneous in many dimensions capture by a large-dimensional vector \( \alpha = (\alpha_1, \alpha_2, \omega) \), whose elements are arbitrarily correlated. The \( \alpha_1 \) part of this heterogeneity is a vector of \( N_1 \) characteristics that influence the preschool choices of parents, but are not directly relevant for the social objective of a non-welfarist planner. For example, \( \alpha_1 \) would include the idiosyncratic tastes of parents, e.g. how paternalistic they are. In contrast, \( \alpha_2 \) is a vector of \( N_2 \) characteristics that influence both the preschool choices of parents and are directly relevant for the social objective of a non-welfarist planner. One element of \( \alpha_2 \) might be the raw home environment, i.e. how well nurtured the child would be in the absence of preschool. This affects the social return to preschool enrollment (Elango, García, Heckman, and Hojman 2015), and is something we will focus on in our empirical work. A family’s ability to earn income is indexed by the latent heterogeneity \( \omega \) (one-dimensional). In our empirical work below (section 3) we discuss how this heterogeneity relates to family structure, i.e. it might be more difficult for a single- versus two-parent households to earn a given level of income. For now we maintain a more general interpretation of \( \omega \).

Denote the joint density of this heterogeneity by \( m(\alpha) \) and the cdf by \( M(\alpha) \). We normalize the measure of parents to unity, i.e. \( \int_A dM(\alpha) = 1 \), where \( A = A_1 \times A_2 \times \Omega \) and \( A_1 \in \mathbb{R}^{N_1}, A_2 \in \mathbb{R}^{N_2} \) and \( \Omega \in \mathbb{R}_+ \). Further, we often make use of the unconditional distribution \( F(\omega) \) and the conditional distribution \( H(\alpha_2|\omega) \), and their densities \( f(\omega) \) and \( h(\alpha_2|\omega) \), respectively.

Preferences. We assume that preferences over consumption, income and preschool are given by

\[
   u(c, y, ps; \alpha) = U \left( c - v \left( \frac{y}{\omega} \right) \right) + \theta_{ps}(\alpha_1, \alpha_2, \omega).
\]
Utility is separable in the consumption-leisure and the preschool dimension. Thus, $\alpha_1$ and $\alpha_2$ do not influence the labor-leisure decision directly, but rather only indirectly through the preschool decision. This allows us to denote the optimal consumption and income choices, conditional on preschool choices, by $c_{ps}(\omega)$ and $y_{ps}(\omega)$, and the optimal preschool choice as $ps(\alpha)$. Furthermore, with loss of generality, we make one normalization of utility rankings: $\theta_{no}(\alpha) = 0 \forall \alpha \in A$.

2.2 Equilibrium given Policies

In our environment there are two policy instruments: (i) labor income taxes $T(y)$ and (ii) public preschool fees $F_{pu}(y)$. We treat the labor income tax schedule as exogenous and focus on the optimal design of the public preschool fee schedule. The problem of a parent given these schedules reads as:

$$\max_{y \in \mathbb{R}_+, ps \in \{no, pu, pr\}} U \left( c - v \left( \frac{y}{\omega} \right) \right) + \theta_{ps}(\alpha_1, \alpha_2, \omega)$$

subject to:

$$c \leq y - T(y) - F_{pr}1_{ps=pr} - F_{pu}(y)1_{ps=pu},$$

where $F_{pr}$ are the fees that parents would have to pay for a private preschool.

**Choice of Income.** The first-order condition for $y$ reads as:

$$1 - T'(y) - F'_{pu}(y)1_{ps=pu} - v' \left( \frac{y}{\omega} \right) \frac{1}{\omega} = 0. \quad (1)$$

This shows that the labor-leisure decision is only driven by $\omega$ and not by $(\alpha_1, \alpha_2)$, and therefore we can denote optimal consumption and income by $c_{ps}(\omega)$ and $y_{ps}(\omega)$, as mentioned previously. Because weak separability of consumption and labor effort preferences implies no income effects on labor supply, we even have $y_{pr}(\omega) = y_{no}(\omega)$. This is because marginal incentives to earn income are the same for these parents, conditional on $\omega$.

If the public preschool fee varies with income, i.e. $F'_{pu} \neq 0$, marginal incentives to earn income will also depend on the preschool choice. Denote by $\tau_{ps}(\omega)$ the labor wedge of the parents which is defined as the wedge between the marginal rates of transformation and substitution, i.e. by

$$(1 - \tau_{ps}(\omega)) = v' \left( \frac{y_{ps}(\omega)}{\omega} \right) \frac{1}{\omega}.$$

6
For parents that do not send their children to public preschool, the labor wedge is simply equal to the marginal tax rate. For parents that do send their children to public preschool, the total labor wedge is the sum of the marginal preschool fee and the marginal income tax rate:

\[ \tau_{pu}(\omega) = F'_{pu}(y_{pu}(\omega)) + T'(y_{pu}(\omega)). \]  

(2)

The distortion of the labor supply decision is convexly increasing in the labor wedge. Since we take the income tax schedule as given, the costs of having a steeply increasing preschool fee schedule will be increasing in the progressiveness of the (pre-existing) income tax schedule.

There is an implicit assumption in the above paragraphs that labor force participation is not affected by these policies. We return to this assumption, and provide empirical that it is reasonable, in section 3.1 below. In essence, the quasi-experimental literature has not found any evidence that public preschool programs affect maternal labor supply.

**Preschool Decision.** Parents can choose between private, public or no preschool. Given the discreteness of the decision, it can simply be written as:

\[ ps(\alpha_1, \alpha_2, \omega) = \arg \max_{ps \in \{ pr, pu, no \}} U_{ps}(\omega) + \theta_{ps}(\alpha_1, \alpha_2, \omega), \]

where

\[ U_{ps}(\omega) = U \left( y_{ps}(\omega) - T(y_{ps}(\omega)) - F_{pr} \mathbb{1}_{ps=pr} - F_{pu} y_{ps}(\omega) \mathbb{1}_{ps=pu} - B \left( \frac{y_{ps}(\omega)}{\omega} \right)\right), \]

and \( y_{ps}(\omega) \) is the optimal income choice conditional on preschool choice according to (1).

We have now introduced enough structure and notation to show that, conditional on \( \omega \), the preschool decision only depends on the aggregates \( \theta_{pr}(\alpha) \) and \( \theta_{pu}(\alpha) \). That is, variation across the arbitrarily many dimensions of heterogeneity we allow for only matters in so far as is leads to variation in \( \theta_{pr}(\alpha) \) or \( \theta_{pu}(\alpha) \). Put differently, two sets of parents with characteristics \((\alpha_1, \alpha_2, \omega)\) and \((\alpha'_1, \alpha'_2, \omega)\) are observationally equivalent if \( \theta_{ps}(\alpha_1, \alpha_2, \omega) = \theta_{ps}(\alpha'_1, \alpha'_2, \omega) \) for \( ps \in \{ pu, pr \} \).

Thus, we can simplify our notation and express a type as \((\theta_{pu}, \theta_{pr}, \omega)\).

Given this simplified notation, it is evident that an individual of type \((\theta_{pu}, \theta_{pr}, \omega)\) prefers public over private preschool if

\[ \theta_{pu} \geq U_{pr}(\omega) - U_{pu}(\omega) + \theta_{pr}. \]

\( \text{7} \)This analysis is modelled after Choné and Laroque (2011) who consider the labor force participation model, but our analysis is slightly more involved because we consider three choices and endogenous income.
This allows us to directly define a threshold function \( \tilde{\theta}_{pu}(\theta_{pr}, \omega) \) such that for all \( \theta_{pu} \) larger than the threshold (given \( \omega \) and \( \theta_{pr} \)) public preschool would be chosen over private:

\[
\tilde{\theta}_{pu}(\theta_{pr}, \omega) = U_{pr}(\omega) - U_{pu}(\omega) + \theta_{pr}.
\] (3)

Thus, individuals of type \((\theta_{pu}, \theta_{pr}, \omega)\) prefer to send their child to public (private) preschool if \( \theta_{pu} \) larger than \( \tilde{\theta}_{pu}(\theta_{pr}, \omega) \).

Next, we turn to the question of whether parents prefer to send their children to private/public preschool as compared to sending them to no preschool. We also define threshold functions to characterize these decisions, although they are only functions of \( \omega \). First, the level of \( \theta_{pu} \), where parents are indifferent between public preschool and no preschool is

\[
\hat{\theta}_{pu}(\omega) = U_{no}(\omega) - U_{pu}(\omega).
\] (4)

The level of \( \theta_{pr} \), where parents are indifferent between private preschool and no preschool

\[
\hat{\theta}_{pr}(\omega) = U_{no}(\omega) - U_{pr}(\omega).
\] (5)

Using these thresholds we have now simplified a problem involving \( N_1 + N_2 + 1 \) dimensional heterogeneity into a problem involving 3-dimensional heterogeneity. We summarize the optimal preschool decisions of the parents in these terms in the following lemma.

**Lemma 1.** The preschool choice of an individual of type \((\alpha_1, \alpha_2, \omega)\) in the presence of an income tax schedule \( T(y) \), a private preschool fee \( F_{pr} \) and an income-contingent public preschool fee schedule \( F_{pu}(y) \) can be summarized by

- \( ps(\alpha_1, \alpha_2, \omega) = pr \) if \( \theta_{pu}(\alpha_1, \alpha_2, \omega) < \tilde{\theta}_{pu}(\theta_{pr}(\alpha_1, \alpha_2, \omega), \omega) \) and \( \theta_{pr}(\alpha_1, \alpha_2, \omega) > \hat{\theta}_{pr}(\omega) \).
- \( ps(\alpha_1, \alpha_2, \omega) = pu \) if \( \theta_{pu}(\alpha_1, \alpha_2, \omega) > \tilde{\theta}_{pu}(\theta_{pr}(\alpha_1, \alpha_2, \omega), \omega) \) and \( \theta_{pr}(\alpha_1, \alpha_2, \omega) > \hat{\theta}_{pr}(\omega) \).
- \( ps(\alpha_1, \alpha_2, \omega) = no \) if \( \theta_{pu}(\alpha_1, \alpha_2, \omega) \leq \tilde{\theta}_{pu}(\omega) \) and \( \theta_{pr}(\alpha_1, \alpha_2, \omega) \leq \hat{\theta}_{pr}(\omega) \).

where \( \tilde{\theta}_{pu}(\theta_{pr}(\alpha_1, \alpha_2, \omega), \omega), \hat{\theta}_{pu}(\omega) \) and \( \hat{\theta}_{pr}(\omega) \) are as defined in (3), (4) and (5).

This lemma is very helpful in that it puts structure on the preschool decisions, which can be fully described by the threshold functions. When thinking about optimal preschool fees, one can take into account reactions along the preschool margin solely by the changes in the values of these thresholds. Despite the multidimensional heterogeneity of the problem, we can break it down to three dimensions: \((\theta_{pr}, \theta_{pu}, \omega)\).
**Distribution Functions.** Given that the preschool decision actually boils down to \( \theta_{pr} \) and \( \theta_{pu} \) it is useful to define the distribution functions of these reduced form variables, which will be helpful in deriving optimal preschool policies.

Firstly, we need notation for the conditional distribution of \((\theta_{pr}, \theta_{pu})\), where the conditioning variables are \(\omega\) and \(\alpha_2\). The reason \(\alpha_2\) must be conditioned on, as well as \(\omega\), is that these elements of heterogeneity may matter for the planner’s objective. We define this conditional joint density of \((\theta_{pr}, \theta_{pu})\) as:

\[
g(\theta_{pr}, \theta_{pu}|\alpha_2, \omega) = \int_{A_1} \mathbf{1}_{\theta_{pr}(\alpha_1, \alpha_2, \omega) = \theta_{pr}} \mathbf{1}_{\theta_{pu}(\alpha_1, \alpha_2, \omega) = \theta_{pu}} m(\alpha_1, \alpha_2, \omega) d\alpha_1
\]

and the respective cdf by

\[
G(\theta_{pr}, \theta_{pu}|\alpha_2, \omega) = \int_{\theta_{pr}}^{\theta_{pr}} \int_{\theta_{pu}}^{\theta_{pu}} g(x, y|\alpha_2, \omega) dy dx.
\]

Further, define the unconditional density for \(\theta_{pr}\):

\[
g_{pr}(\theta_{pr}|\alpha_2, \omega) = \int_{A_1} \mathbf{1}_{\theta_{pr}(\alpha_1, \alpha_2, \omega) = \theta_{pr}} m(\alpha_1, \alpha_2, \omega) d\alpha_1
\]

with cdf \(G_{pr}(\theta_{pr}|\alpha_2, \omega) = \int_{\theta_{pr}}^{\theta_{pr}} g_{pr}(x|\alpha_2, \omega) dx\). Application of Bayes’ Theorem yields

\[
g_{pu}(\theta_{pu}|\alpha_2, \omega, \theta_{pr}) = \frac{g(\theta_{pr}, \theta_{pu}|\alpha_2, \omega)}{g_{pr}(\theta_{pr}|\alpha_2, \omega)}
\]

with respective cdf \(G_{pu}(\theta_{pu}|\alpha_2, \omega, \theta_{pr}) = \int_{\theta_{pu}}^{\theta_{pu}} g_{pu}(x|\alpha_2, \omega, \theta_{pr}) dx\).

### 2.3 Optimal Policy Problem

The government optimally chooses the preschool fee schedule \(F_{pu}(\cdot)\) and we do not impose any ex-ante restriction on the functional form. Before we turn to the government’s budget constraint, we consider the government’s objective. Generally, we can consider the objective as

\[
\max_{F_{pu}(\cdot)} O(\mathcal{PS})
\]

where \(\mathcal{PS} = \{ps(\alpha)\}_{\alpha \in A}\) is the whole set of preschool decisions for each type \(\alpha = (\alpha_1, \alpha_2, \omega)\) and \(O\) is some functional. As discussed in the introduction, we deliberately restrict ourselves to a non-welfarist objective criterion so that redistributive effects are ignored and policies are evaluated solely on their capacity to promote preschool education. Extending our analysis to welfarist social objectives (such as Utilitarianism) would be straightforward.
In our quantitative analysis we will focus on a social objective of maximizing weighted preschool enrollment:

$$O(PS) = \int_\Omega \int_{A_2} b(\omega, \alpha_2) \int_{A_1} 1_{ps(\alpha) \neq no} dM(\alpha),$$  \hspace{1cm} (7)$$

Recall that only a subset $\alpha_2$ of the all heterogeneity affecting preschool decisions matters for a non-welfarist planner. Thus, the weights depend on $\alpha_2$ and $\omega$, but not $\alpha_1$. Clearly, if the return to investing in preschool for a particular type of child is larger than average the planner might make $b(\omega, \alpha_2)$ larger than average as well. In the case that the weight $b(\omega, \alpha_2)$ is the same for each type $(\omega, \alpha_2)$, then we have a special case where the objective is to maximize overall preschool enrollment.

As an aside, a non-welfarist social objective (6) could potentially capture many different concepts. For example, it could capture the idea to maximizing the average of children’s ex-post abilities $\omega_c$:

$$O(PS) = \int_A \omega_c(\alpha, ps(\alpha)) dM(\alpha)$$  \hspace{1cm} (8)$$

What one needs to do is construct equivalent weights such that those whose ability is most improved by preschool enrollment are given the most weight. For example, as has been shown by Elango et al. (2015), the returns to preschool depend on home environment and parenting style, so $\alpha_2$ should include at least these characteristics in order to maximize the ex-post skills of children. Another alternative would be an objective that captures aversion against inequalities in children’s abilities. This could be captured by

$$O(PS) = \int_A W(\omega_c(\alpha, ps(\alpha))) dM(\alpha),$$  \hspace{1cm} (9)$$

where $W(\cdot)$ is a concave transformation.  

\footnote{We can also easily capture peer effects, for that we would just have to replace $\omega_c$ in (8) or (9) by $\omega_c(i, ps_i, PS)$ so here, the ability of a child does not only depend on parent’s characteristics and the preschool decision, but also on where also on the preschool decision of all other children captured by $PS$.}
In the remainder of this section we derive the optimal fee schedule for the objective as defined in (7). To proceed we must be able to apply Lemma 1, thus we need to re-write (7) in terms of \((\theta_{pr}, \theta_{pu})\) and their distribution functions.

\[
\mathcal{O}(PS) = \int_{\Omega} \int_{A_2} b(\omega, \alpha_2) \left\{ \int_{\theta_{pr}^*(\omega)}^{\theta_{pr}^*(\omega)} G_{pu}(\tilde{\theta}_{pu}(\omega), \theta_{pr}) \left| \alpha_2, \omega, \theta_{pr} \right\} dG_{pr}(\theta_{pr} | \alpha_2, \omega) 
+ \int_{\theta_{pr}^*(\omega)}^{\theta_{pr}^*(\omega)} \left( 1 - G_{pu}(\tilde{\theta}_{pu}(\omega), \theta_{pr}) \right) \left| \alpha_2, \omega, \theta_{pr} \right\} dG_{pr}(\theta_{pr} | \alpha_2, \omega) 
+ \int_{\theta_{pr}^*(\omega)}^{\theta_{pr}^*(\omega)} \left( 1 - G_{pu}(\tilde{\theta}_{pu}(\omega), \theta_{pr}) \right) \left| \alpha_2, \omega, \theta_{pr} \right\} dG_{pr}(\theta_{pr} | \alpha_2, \omega) \right\} dH(\alpha_2 | \omega) dF(\omega).
\]

This expression is more cumbersome than (7) so we explain each line: The first line captures the measure of children in private preschool and the second and the third line captures the measure of children in public preschool. Writing the objective like this has the advantage that it is differentiable in preschool policies. Changing the preschool fee \(F_{pu}(y_{pu}(\omega))\) will result in a change of the thresholds \(\tilde{\theta}_{pu}(\omega)\) and \(\tilde{\theta}_{pu}(\omega)\), which can be captured formally in a simple way.

**Government’s constraints.** The government budget constraint reads:

\[
R \leq \int_{\Omega} \int_{A_2} h(\alpha_2 | \omega) f(\omega) \left( T(y(\omega)) \left[ \int_{\tilde{\theta}_{pr}^*(\omega)}^{\theta_{pr}^*(\omega)} G_{pu}(\tilde{\theta}_{pu}(\omega)) | \alpha_2, \omega, \theta_{pr} dG(\theta_{pr} | \alpha_2, \omega) \right] 
+ \left\{ T(y_{pu}(\omega)) \left[ \int_{\tilde{\theta}_{pr}^*(\omega)}^{\theta_{pr}^*(\omega)} \left( 1 - G_{pu}(\tilde{\theta}_{pu}(\omega), \theta_{pr}) \right) \left| \alpha_2, \omega, \theta_{pr} \right\} dG_{pr}(\theta_{pr} | \alpha_2, \omega) \right] 
+ \left\{ T(y_{pu}(\omega)) \left[ \int_{\tilde{\theta}_{pr}^*(\omega)}^{\theta_{pr}^*(\omega)} \left( 1 - G_{pu}(\tilde{\theta}_{pu}(\omega), \theta_{pr}) \right) \left| \alpha_2, \omega, \theta_{pr} \right\} dG_{pr}(\theta_{pr} | \alpha_2, \omega) \right] \right\} d\alpha_2 d\omega \right).
\]

where \(T(y_{pu}(\omega)) = T(y_{pu}(\omega)) + F_{pu}(y_{pu}(\omega)) - C\). The first line captures tax revenue from all individuals that do not send their children to preschool. The second line captures those families that send their child to a private preschool and the third line captures the public preschool parents. In addition to taxes, the government also obtains fees \(F_{pu}(y_{pu}(\omega))\) from them. However, each child in public preschool also implies resource costs \(C\) for the government; thus we assume constant marginal costs for public preschool. Extending the analysis to non-constant marginal costs would be straightforward.

The government also has to take into account individual optimization behavior when choosing the fee schedule. First, the government is constrained by individually optimal preschool
enrollment decisions that are summarized in Lemma 1. Second, for those parents that choose to send their child to a public preschool, the government has to take into account that these parents choose their labor supply endogenously with respect to the fee schedule, i.e.

\[ y_{pu}(\omega) = \arg \max_y U \left( c - v \left( \frac{y}{\omega} \right) \right) + \theta_{pu}(\alpha_1, \alpha_2, \omega) \quad \text{s.t.} \quad c \leq y - T(y) - F_{pu}(y). \]

In Section 2.4 we solve this problem by optimizing the function \( F_{pu}(\cdot) \) using a perturbation approach. In Appendix A.1, we show that this problem can also be solved using a mechanism-design approach in the tradition of the optimal income taxation literature following Mirrlees (1971). The solutions are the same.

### 2.4 The Optimal Sliding Fee Schedule

In the main following, we use a more intuitive perturbation approach in the spirit of Piketty (1997) and Saez (2001). We show that it is equivalent to the mechanism-design solution in Appendix A.1. The value of using a fee perturbation approach is that it shows in a more intuitive way where the various components of the optimal fee schedule come from. In what follows we list these components one by one, and then combine them into a formula for the optimal fee schedule.

Assume that the black bold line in Figure 1 presents the optimal preschool fee schedule. Then, slightly lowering the fee below income \( y_{pu}(\omega^*) \), as illustrated, should have no first-order effect on the planner’s objective (weighted overall preschool enrollment). Note that the perturbation is such that the marginal preschool fee is slightly increased by \( dF'_{pu} \) in an interval around \( y_{pu}(\omega^*) \) with length \( dy \). We think of \( dy \) and \( dF'_{pu} \) being infinitesimal. This small reform will have direct effects on the planner’s objective, as well as indirect effects that work through the government budget constraint. If the initial fee schedule is optimal, the sum of these effects has to be zero.
**Mechanical Revenue Effect**  First of all, each *infra-marginal* family that enrolls their child in public preschool and who has \( \omega < \omega^* \), will now pay \( dF'_{pu} dy \) dollars less in preschool fees. We call these families infra-marginal because they would enroll their children in public preschool regardless of this small reform. Denote by \( \Lambda \) the marginal value of public funds. The impact on the governments objective from this effect is

\[
\Delta M(\omega^*) = -\Lambda dF'_{pu} dy \int_{\omega}^{\omega^*} s_{pu}(\alpha_2, \omega) dH(\alpha_2 | \omega) dF(\omega),
\]

where

\[
s_{pu}(\alpha_2, \omega) = \int_{\theta_{pr}}^{\theta_{pu}} \int_{\max(\theta_{pu}, \theta_{pr})}^{\theta_{pu}} dG_{pu}(\theta_{pu} | \alpha_2, \omega, \theta_{pr}) dG_{pr}(\theta_{pr} | \alpha_2, \omega).
\]

The double integral captures the mass of parents with ability below \( \omega^* \) that send their child to a public preschool, i.e. the mass of parents that now pay less fees due to the small reform.

**Labor Supply Effect**  All public preschool parents with income \( y_{pu}(\omega^*) \) now face a higher (implicit) marginal tax rate. We follow Piketty (1997) and Saez (2001) on how to formalize this: These parents will change their behaviour according to

\[
\frac{\partial y_{pu}(\omega^*)}{\partial \tau} dF'_{pu} = \varepsilon_{y, 1-\tau} \frac{y_{pu}(\omega^*)}{1 - \tau(\omega^*)} dF'_{pu}.
\]

The labor supply response of these parents has no first-order effect on the planner’s objective because of the envelope theorem. However, it does influence preschool enrollment indirectly through the implied marginal change in government funds. At the margin, of each additional dollar earned, the government obtains \( \tau_{pu}(\omega^*) = T'(y_{pu}(\omega^*)) + F'_{pu}(y(\omega^*)) \). Per individual, the effect on the government’s objective is given by:

\[
\tau(\omega^*) \varepsilon_{y, 1-\tau}(\omega^*) \frac{y_{pu}(\omega^*)}{1 - \tau(\omega^*)} dF'_{pu}.
\]

(11)

What is the mass of individuals that change their labor supply? It reads as

\[
\int_{A_2} s_{pu}(\alpha_2, \omega^*) dH(\alpha_2 | \omega^*) \times f(\omega^*) = \int_{A_2} s_{pu}(\alpha_2, \omega^*) dH(\alpha_2 | \omega^*) \times f(\omega^*) \frac{dy}{\varepsilon_{y, \omega}(\omega^*)} \frac{\omega^*}{y(\omega^*)} d\omega
\]

\( \varepsilon_{y, 1-\tau} \) is the elasticity of labor income w.r.t. ones minus the labor wedge. Relatedly \( \varepsilon_{y, \omega} \) is the elasticity of labor income w.r.t. to the wage.
Multiplying the effect per individual (11) with the mass of individuals and the marginal value of government funds yields the overall effect on preschool enrollment of this implied labor supply change:

$$\Delta_{LS}(\omega^*) = \Lambda \frac{\tau(\omega^*)}{1 - \tau(\omega^*)} \varepsilon_{y_1,\omega^*} \frac{1}{\varepsilon_{y,\omega}} \times \int_{A_2} s_{pu}(\alpha_2, \omega^*) dH(\alpha_2 | \omega) f(\omega^*) \times dydF'_{pu}.$$  

**New Enrollment Effect** Public preschool enrollment will rise because of the fee reduction. Some of the newly enrolled children will have otherwise not been enrolled in preschool at all. The measure of such children is what we term the ‘New Enrollment Effect.’ That is, some parents with $\omega < \omega^*$ have been indifferent between sending their children to public preschool or to no preschool at all. Namely, all families of type $\theta_{pu}, \theta_{pr}, \omega$ with $\theta_{pu} = \hat{\theta}_{pu}(\omega)$, $\theta_{pu} < \hat{\theta}_{pu}(\omega, \theta_{pr})$ and $\omega < \omega^*$. Because of the small decrease in preschool fees these families will now decide to send their child to public preschool. The mass of children that are affected in this way is given by (using Leibnitz's rule):

$$\forall \ (\alpha_2, \omega) \text{ with } \omega \leq \omega^* : \text{New-Enrollment}(\alpha_2, \omega) =  
\int_{\theta_{pr}}{\hat{\theta}_{pr}(\omega)} \frac{\partial}{\partial F_{pu}(\omega)} g_{pu}(\hat{\theta}_{pu}(\omega, \theta_{pr})|\alpha_2, \omega, \theta_{pr}) dG_{pr}(\theta_{pr}|\alpha_2, \omega) h(\alpha_2 | \omega) f(\omega).$$

This change in behavior has both an indirect effect on the government’s budget, and a direct effect on the objective function.

The effect on the government budget for each such child is

$$\Delta T(\omega) = T(y_{pu}(\omega)) - T(y_{no}(\omega)) + F_{pu}(y_{pu}(\omega)) - C.$$  

Thus, the overall effect on the government budget is:

$$\Delta_{NE-fiscal}(\omega^*) = \Lambda \int_{\omega}^{\omega^*} \Delta T(\omega) \text{New-Enrollment}(\omega) d\omega,$$

where New-Enrollment$(\omega) = \int_{A_2} \text{New-Enrollment}(\alpha_2, \omega) d\alpha_2$. In addition, this increase in enrollment has a direct effect on the objective:

$$\Delta_{NE}(\omega^*) = dydF'_{pu} \int_{\omega}^{\omega^*} \int_{A_2} b(\alpha_2, \omega) \text{New-Enrollment}(\alpha_2, \omega) d\alpha_2 d\omega.$$  

Interestingly, we now see an effect that is directly influenced by the policy weight. If raw home environment or other factors lead to the weights on marginal children being larger, i.e. $b(\alpha_2, \omega)$ begin relatively larger, then this new enrollment effect will become a more influential margin and the planner is more likely to reduce fees for them.
Crowding-out Effect  There is also an increase in public enrollment that results from some families switching away from private preschool. That is, some private preschool parents that have been indifferent between sending their children to public preschool or to private preschool. Namely, all parents with $\theta_{pu} = \tilde{\theta}_{pu}(\omega, \theta_{pr})$ and $\theta_{pu} > \hat{\theta}_{pu}(\omega)$. Because of the small decrease in preschool fees they will now decide to send their child to public preschool instead. The mass of parents whose private preschool investment is crowded-out is given by (using Leibnitz’s rule):

$$\forall (\alpha_2, \omega) \text{ with } \omega \leq \omega^*: \text{Crowd-out} (\alpha_2, \omega) = \frac{dy F_{pu}'}{d\theta_{pr}'} \int_{\tilde{\theta}_{pr}(\omega)}^{\hat{\theta}_{pr}(\omega)} g_{pu}(\tilde{\theta}_{pr}(\omega), \theta_{pr}|\alpha_2, \omega, \theta_{pr}) dG_{pr}(\theta_{pr}|\alpha_2, \omega) h(\alpha_2|\omega) f(\omega).$$

This effect changes individual contributions to the government’s budget. For each instance where public preschool crowds out private preschool investment there will be an impact $\Delta T(\omega)$ on the government’s budget. The overall crowd-out effect through the implied change in public funds is

$$\Delta_{CO-fiscal} = \Lambda \int_{\omega}^{\omega^*} \text{Crowd-out}(\omega) dF(\omega).$$

where Crowd-out$(\omega) = \int_{A_2} \text{Crowd-out}(\alpha_2, \omega) d\alpha_2$. Given that we consider an objective where the planner values a child in private and public preschool the same, this crowding-out does not have a direct impact on the government’s objective. If, for example, we were considering an environment where public preschool were not necessarily high quality but private were, different weights might be placed on these outcomes. As we have stated already, here the focus is on high quality programs only.

Optimality  If the fee schedule is optimal, the sum of the effects on the social objective has to be zero, thus the necessary conditions for an optimal public preschool fee schedule are:

$$\forall \omega \in \Omega : \Delta_M(\omega) + \Delta_{LS}(\omega) + \Delta_{New-NE}(\omega) + \Delta_{NE-fiscal}(\omega) + \Delta_{CO-fiscal}(\omega) = 0 \quad \text{(12)}$$

Rearranging (12) yields the following proposition:\textsuperscript{10}

**Proposition 1.** Optimal labor wedges for parents who send their children to public preschool are given by:

$$\frac{\tau_{pu}(\omega^*)}{1 - \tau_{pu}(\omega^*)} = \left(1 + \frac{1}{\varepsilon_{y,1-\tau}(\omega^*)}\right) \frac{\mu(\omega^*)}{\lambda \int_{A_2} s_{pu}(\alpha_2, \omega^*) dH(\alpha_2 | \omega^*) f(\omega^*) \omega^*}, \quad \text{(13)}$$

\textsuperscript{10}One also has to use that $\varepsilon_{y,\omega}(\omega) = \varepsilon_{y,1-\tau}(\omega)$.  

where
\[
\mu(\omega) = \int_{\omega}^{\omega^*} \left\{ \Lambda \left[ - \int_{\mathcal{A}_2} s_{pu}(\alpha_2, \omega) dH(\alpha_2 | \omega) f(\omega) + \Delta T(\omega) \left( \text{Crowd-out}(\omega) + \Delta \text{New-Enrollment}(\omega) \right) \right] \\
+ \int_{\mathcal{A}_2} b(\alpha_2, \omega) \text{New-Enrollment}(\alpha_2, \omega) d\alpha_2 \right\} d\omega.
\]

and $\Lambda$ is implicitly defined by $\mu(\omega) = 0$.

This condition for the optimal labor wedge trades-off the labor supply margin, the public/private preschool margin and the public/no preschool margin. Policy weights effect this directly by affecting the importance of the ‘New-Enrollment’ effect. It is formally similar to conditions for optimal redistributive taxes such as in the pioneering papers of Diamond (1998) and Saez (2001) and even closer to those papers that also studied an extensive margin in addition. The latter include Saez (2002) and Jacquet, Lehmann, and Van der Linden (2013) who consider the labor force participation margin, Lehmann, Simula, and Trannoy (2014) who consider migration and Scheuer (2014) who considers the occupational choice margin. The motives for labor supply distortions here, however, are very different. The usual redistribution motive coming from differences in marginal utility of income is shut down here as we deliberately choose this non-welfarist objective.

Note that Proposition 1 does not tell us explicitly what the optimal fee schedule is, but only about the optimal labor supply distortion (wedge). The way that this translates into the steepness of the preschool fee schedule depends on the pre-existing marginal tax rates:

**Corollary 1.** The optimal public preschool fee schedule is described by
\[
F'_{pu}(y_{pu}(\omega)) = \tau_{pu}(\omega) - T'(y_{pu}(\omega)).
\]

These theoretical derivations have shed light on the different forces at work and how they should optimally be traded-off. The interesting question is now what these results imply quantitatively. How steep should the optimal preschool fee schedule be? Or should it maybe even be decreasing? And how much better do optimal policies compare to current policies?

### 3 Empirical Model and Estimation

The theoretical work above provides a template for a quantitative implementation of the model. It shows us in Lemma 1 that all of the complex heterogeneity underlying both preschool decisions policy weights can be summarized by just three dimensions of heterogeneity $\theta_{pu}$, $\theta_{pr}$ and $\omega$. It also shows us in Proposition 1 that if the joint distribution of these variables can be
identified empirically then the optimal fee schedule can be recovered using equation (13). In what follows we introduce additional assumptions and notation that allow us to do just that.

3.1 Families: Preferences and Heterogeneity

Utility Functions: It is easiest to begin by parameterizing parental preferences, which include the functions $U$ and $v$ introduced above. The function $v$ determines the utility cost of providing effort in the labor market. We allow for the possibility of one or two parent families by parameterizing this function with an iso-elastic form:

$$v = \frac{\kappa_i^{1+\epsilon}}{1 + \frac{1}{\epsilon}} \left( \frac{y}{\omega} \right)^{1+\frac{1}{\epsilon}}.$$

The parameter $\kappa_i$ varies across one and two parent families, in the sense that a given level of labor market effort is more costly for one person to produce than two. Next we introduce a transformation of a family’s ability to produce income $\omega_i = \hat{\omega}_i/\kappa_i$. With this transformation we can write an alternative effort cost function that generally applies to any family:

$$v = \frac{1}{1 + \frac{1}{\epsilon}} \left( \frac{y}{\omega} \right)^{1+\frac{1}{\epsilon}}.$$

(14)

The elasticity of labor supply will be governed by $\epsilon$, which we exogenously set to $\epsilon = 1/3$. (Chetty, Guren, Manoli, and Weber 2011) but provide robustness analysis for other values of $\epsilon$.

We focus on the intensive margin of labor supply for two reasons. The first reason is the obvious fact that a sliding fee schedule will distort parental labor supply incentives, which must be accounted for in the policy analysis. The second reason is the empirical evidence suggesting no effect of universal preschool on parental labor supply (Fitzpatrick 2010, Cascio and Schanzenbach 2013). This evidence conflicts with alternative evidence that daycare subsidies do lead to increases in parental labor supply (Baker, Gruber, and Milligan 2015b, Guner, Kaygusuz, and Ventura 2013, Ho and Pavoni 2016). How can these be reconciled? We believe that recognizing and understanding the distinctions between preschool and daycare is crucial. Although they seem similar, preschool and daycare are not likely to be perfect substitutes. The purpose of preschool programs is to provide early education, while the purpose of daycare is to substitute for parental supervision. Not only do public preschool programs not seem to affect parental labor supply, they do not seem to affect parental spending on child care either. Cascio and Schanzenbach (2013) find that for less educated parents child care spending actually increased by about $35 dollars per month when universal preschool was introduced, even though there is a roughly 17% increases in public preschool enrollment and no effect on private
preschool enrollment. Preschool programs range from two to six hours per day. Perhaps these increased costs are associated with paying a daycare provider to take the child to and from preschool so that the parent can still work a full day? Perhaps there are no effects on parental labor force participation because a parent or other care giver must still be available to transport the child to and from preschool? Whatever the case may be, public preschool programs do not affect parental labor force participation in the same way daycare subsidies do, and we choose to study a model that shares this feature.

Parents willingness to trade consumption for the utility they get from their children’s preschool depends on the function $U$. We also parameterize this with an iso-elastic form $U(.) = (c - v(y/\omega))^{1-\gamma}/(1 - \gamma)$. The parameter $\gamma$ will determine the gradient of preschool enrollment on parental income, and will be estimated using exactly this type of variation.

**Preschool Preferences:** Next, we parameterize the reduced heterogeneity in preschool preferences $\theta_{pu}$ and $\theta_{pr}$, which were introduced in Section 2 above. To allow for an arbitrary correlation between these variables we write them as sums of the same uncorrelated components. That is, we specify

$$
\theta_i(ps_i) = \begin{cases} 
\bar{\theta}_{pr} - \xi_i + \phi_i & \text{if } ps = pr \\
\bar{\theta}_{pu} - \xi_i - \phi_i & \text{if } ps = pu
\end{cases},
$$

(15)

where $\bar{\theta}_{pr}$ and $\bar{\theta}_{pu}$ are constants, and $\xi_i$ and $\phi_i$ are random components. The $\bar{\theta}$ parameters are constants that reflect average utility from a given choice. We can interpret $\xi_i$ as a shock to preferences for preschool in general, and $\phi_i$ as a shock to the relative preference for public versus private preschool. Naturally, if the variance of $\xi_i$ is relatively large $\theta_{pu}$ and $\theta_{pr}$ will be positively correlated in the population, whereas if the variance of $\phi_i$ is relatively large the opposite will be true. The point is that this structure allows for arbitrary correlation between $\theta_{pu}$ and $\theta_{pr}$ as their definition in Section 2 requires.

We could also allow for arbitrary correlations between preschool preferences and parental income/ability by allowing the distributions of $\xi$ and $\phi$ to depend on $\omega$. For example, $\phi$ is modelled as normally distributed, and we could allow the mean or variance to be a function of $\omega$. However, we have not pursued this possibility yet.

The specific forms of the distributions of $\xi$ and $\phi$ that we currently estimate are as follows. We think of $\xi$ as reflecting a utility cost that is subtracted from $\bar{\theta}$, and hence restrict it to be positive. An exponential distribution with mean $\lambda$ is a simple way to capture this, where the parameter $\lambda$ will be estimated. Because of the way it enters equation 15, we must choose a distribution for $\phi$ that allows both positive and negative values. We adopt a normal distribution with variance $\sigma_{\phi}^2$, where this variance is to be estimated. Note that a mean of zero for this distribution imposes no restriction as $\bar{\theta}_{pu}$ and $\bar{\theta}_{pr}$ are estimated separately.
3.2 Income Distribution, Skill Distribution and Tax Policies

To recover the distribution of the latent variable $\omega$ we follow the approach of Saez (2001). For a given value of the labor supply elasticity (given above) and a parameterization of the labor income tax function, we can infer the $\omega$ that is associated to a certain income level. For the tax function we employ the Gouveia-Strauss specification estimated on married and unmarried households combined by Guner, Kaygusuz, and Ventura (2014). For the income distribution, we use CPS data from 2005 and focus on families with at least one preschool age child. Thus, we recover the distribution of $\omega$ for the population of families with preschool aged children.

3.3 Preschool Costs

A very important point is that the heterogeneity $\theta_{pu}$ and $\theta_{pr}$ are, by definition, orthogonal to the current policy environment. Thus, it is important to adequately control for the effect of current policies on preschool decisions when estimating the distributions of this heterogeneity. There are two aspects of current policies that are potentially important: the fees charged by current programs and their availability.

We model private preschool as always being available at cost $F_{pr}$, which will be estimated. The availability of public preschool depends to some extent on family income. For families below the poverty line Head Start is a possibility. However, given the prevalence of oversubscription and associated low enrollment rates, we choose to model the availability of Head Start as random with only a fraction of families $Pr(hs)$ having access. For those who have access the program is free. Families below and above the poverty line also potentially have access to other public preschool programs, mostly state run preschools. We also model access to these programs as possibly random, with a fraction of families $Pr(pu)$ having access. In modelling the fees for these public programs, $F_{pu}(y)$, we allow for progressiveness by estimating a fee $a_{low}$ that applies to families below the poverty line and another $a_{high}$ for families above the poverty line.

3.4 Equilibrium

Given that the additional structure put on the model in this section specializes the general model described in Section 2, we can apply Lemma 1 and compute the model equilibrium for a candidate set of parameters. One caveat that should be explained relates to Head Start versus other public preschools. As we have modelled these programs the only difference is cost. Thus, whenever both programs are available to a family they will prefer Head Start. Thus, Head Start should be substituted for public preschool in Lemma 1 when available.
3.5 Model Estimation

We adopt a Method of Moments approach to estimate several parameters of our model. Parameters to be estimated are preference parameters $\lambda$, $\sigma^2$, $\theta_{pu}$, $\theta_{pr}$, and $\gamma$; the cost parameters $F_{pr}$, $a_{low}$ and $a_{high}$; and also the Head Start lottery offer probability $Pr(hs)$ and other public program offer probability $Pr(pu)$. Our main data sources for this estimation are the Early Childhood Program Participation (ECPP) portion of the 2005 National Health and Education Survey and the Current Population Survey (CPS). We supplement these primary data sources with some additional data sources. The effects of universal public preschool estimated from CPS data by Cascio and Schanzenbach (2013) are used in our estimation. We also use information on control group behavior in the Head Start Impact Study reported by Kline and Walters (2015), and national statistics on Head Start participation.

The identifying moments for our estimator group naturally into four blocks. The first block contains six moments that we find especially important to accurately replicate. The six moments are (1) The overall preschool enrollment rate of four year olds (60.36% in 2005 October CPS data), (2) The public preschool enrollment rate of four year olds (31.87% in 2005 October CPS data), (3) The Head Start enrollment rate of four year olds below the poverty line (54.9% in 2005 according to the National Head Start Association), (4) The estimated increase in overall preschool enrollment among four year olds in Georgia/Oklahoma when universal preschool was introduced (13.44% based on Cascio and Schanzenbach (2013)), (5) The estimated reduction in private preschool enrollment among four year olds in Georgia/Oklahoma when universal preschool was introduced (-5.46% based on Cascio and Schanzenbach (2013)), and finally (6) The percentage of four-year-old Head Start applicants who enroll in other public preschool programs upon being rejected from Head Start (41.0% based on Kline and Walters (2015)). We put greater weight on these moments than those that follow because we want to make sure our benchmark model closely replicates aggregate enrollment patterns, and, perhaps more importantly, we want the behavioral responses to policy changes in our model to replicate those observed in real world (quasi-)experimental settings.

One of the remaining blocks of moments includes valuable information on how much money parents actually spend on preschool. We separate parents into nine income groups and include average preschool expenditure within each group, conditional on preschool enrollment of their child. Income brackets, rather than actual income, are observed in the ECPP. The two remaining blocks of moments relate to how preschool decisions vary with income. One of the blocks includes overall preschool enrollment rates among the nine income groupings, and another includes private preschool enrollment rates among those same groups. Overall enrollment by income is taken directly from the ECPP data. For private preschool enrollment we use an

11The private preschool enrollment rate will be the difference between overall and public enrollment, thus it would be redundant to include as an additional moment.
imputation method in which (a) a probit regression of private enrollment on education level and employment status of parents is fit using CPS data, and then (b) this model is used to predict the probability that a given child in the ECPP is enrolled in private preschool. We average these predictions within income groups to arrive at estimate private enrollment rates within income groups.

3.5.1 Estimated Parameters and Goodness of fit

In Table 1 we present our estimates of the model parameters, as well as their bootstrapped standard errors. Most of the moments we target can be constructed based on bootstrap samples that we draw with replacement directly from the micro data. The exceptions are the responses of overall and public preschool enrollment to the Oklahoma/Georgia universal preschool introduction. For these we assume that the estimated effects are normally distributed, and so can re-sample these moments from normal distributions with standard deviations equal to the standard errors reported in Cascio and Shanzenbach (2013).

Several interesting aspects of the estimated parameters should be discussed. First, there is little evidence of greater utility earned from private versus public preschool. The point estimate of $\theta_{pr}$ is slightly higher than $\theta_{pu}$, but a bootstrapped 95% confidence interval for the difference crosses zero.\(^{12}\) The variance of the public-private preference parameter $\phi$ is relatively large. Being one standard deviations from the mean preference can increase or decrease utility by slightly more than 25%, depending on whether public or private preschool is chosen. The mean preference for children at home is small relative to mean utility from public preschool, but, given the low standard error of the estimate, this parameter should still be considered a significant determinant of preschool decisions. Our estimated public and private fees seem reasonable and in line with the data that identifies these parameters, and the public preschool rationing probabilities also seem reasonable and well identified.

Table 2 shows that the fitted model replicates the first block of targeted moments quite well. This is not hugely surprising given the larger weight on these moments in the estimation, but it is important to see that the parameterized model replicates current overall enrollment rates, and, perhaps more importantly, the behavioral moments from experimental data. We present the fit to the remaining blocks of moments in an appendix.

\(^{12}\)A bootstrapped 90% confidence interval has a lower bound just above zero, so one might be tempted to call the difference ‘marginaly’ significant.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>(s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{pu}$</td>
<td>6.003</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$\theta_{pr}$</td>
<td>6.304</td>
<td>(0.126)</td>
</tr>
<tr>
<td>$\sigma_{\phi}$</td>
<td>1.819</td>
<td>(0.026)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.210</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.806</td>
<td>(0.194)</td>
</tr>
<tr>
<td>$F_{pr}$</td>
<td>$6442$</td>
<td>($302$)</td>
</tr>
<tr>
<td>$a_{low}$</td>
<td>$475$</td>
<td>($139$)</td>
</tr>
<tr>
<td>$a_{high}$</td>
<td>$2225$</td>
<td>($492$)</td>
</tr>
<tr>
<td>$Pr(hs)$</td>
<td>0.706</td>
<td>(0.067)</td>
</tr>
<tr>
<td>$Pr(pu)$</td>
<td>0.796</td>
<td>(0.016)</td>
</tr>
</tbody>
</table>

Table 2: Primary Targeted moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enrollment overall</td>
<td>60.4%</td>
<td>60.3%</td>
</tr>
<tr>
<td>Enrollment public</td>
<td>31.9%</td>
<td>32.4%</td>
</tr>
<tr>
<td>Enrollment Head Start (below Poverty line)</td>
<td>54.9%</td>
<td>54.6%</td>
</tr>
<tr>
<td>Overall enrollment increase - okla/ga universal</td>
<td>+13.4%</td>
<td>+12.2%</td>
</tr>
<tr>
<td>Private enrollment decrease - okla/ga universal</td>
<td>-5.46%</td>
<td>-6.65%</td>
</tr>
<tr>
<td>HSIS - public enrollment rate of rejected appl.</td>
<td>41.0%</td>
<td>40.9%</td>
</tr>
</tbody>
</table>
4 Optimal Preschool Policies in the U.S.

Having now estimated the necessary features of the environment we can apply the formula in equation (13) to recover the optimal fee schedule. As a benchmark case we consider policy weights \( b(\alpha_2, \omega) = 1 \) for every \((\alpha_2, \omega)\). That is, we simply maximize the total measure of children in preschool. In an extension below we discuss a case where we weight children according to their raw (no preschool) home environment and outcomes.

4.1 Benchmark Case

Figure 2 shows the optimal fee schedule as a function of family income. As can be seen, the fee starts with negative values. Poor families receive up to $1,800 per year for sending their child to a public preschool. The reason is that even for a zero fee not all parents send their children, which might occur for two reasons: (i) they have a preference for not having their child in preschool or (ii) sending their child implies some costs such as transportation. Our reduced form model is silent about the exact mechanism and if the policy goal is only to maximize the number children going to preschool the exact reason does not matter. The elasticity of enrollment with respect to the fee is a sufficient statistic, and our quantitative model implies a value for this sufficient statistic at a zero fee for low incomes such that further decreasing the fee is optimal.

![Figure 2: Optimal Fee Schedule and Bootstrapped Confidence Bands](image)

The fee then concavely increases until parental income of around $100,000 where it flattens out. Interestingly, individuals with family income above $60,000 pay more than they would
pay for a private preschool. Some parents with income above $60,000 who nevertheless send their children to public preschool must therefore have a preference for sending their children to a public school, i.e. a negative value of φ. As we have tried to make clear, there are many dimensions of underlying heterogeneity that can influence φ. One of these is variation in proximity to private preschool. If a family lives in a small town where there is not enough demand, then this would show up as a strong preference for public preschool. But other factors like social ideology could also give rise to such realizations of φ. As discussed in more detail below, one reason for this feature of the optimal fee schedule is that it eliminates some of the crowding out of private investments inherent in the current policy environment by moving some higher income families towards private preschool.

The estimated empirical distribution is indicated in Figure 3 by including both 90% and 95% bootstrapped confidence intervals. The asymmetry of the uncertainty is interesting, especially for low income families. While the 95% confidence interval is about $3,000 wide for parental income around $15,000 range, much of this uncertainty is downward. It is more often the case that parameters re-estimated on bootstrap samples result in an optimal fee schedule that is more progressive than less progressive.

How does the optimal fee schedule perform in terms of increasing enrollment? Enrollment increases by 12 percentage points. This is a quantitatively meaningful increase as it is achieved without increasing taxes or decreasing any other spending. To put this into numbers, note that in the U.S. there are currently about 4,000,000 four-year-olds, thus this implies an increase in the number of four-year-olds in preschool of 480,000. Only part of the increase in preschool attendance can be attributed to better targeting of subsidies, as part of it also results from the fact that rationing is not part of the optimum. We decompose the gains into these two parts in Section 4.2 below.

Figure 3 shows preschool enrollment as a function of family income for both current and optimal policies. Bootstrapped 95% confidence bands are provided for both. Interestingly, in the optimal allocation, it is not the poorest children who are the least likely to go to preschool, but those with family income around $35,000. This happens for two reasons. One is that it is much easier to use subsidies to create incentives for very poor parents to enroll their child because their marginal utility from consumption is higher. Second, the number of families at a given income level is increasing with income over the lower range, thus it becomes increasingly expensive to offer such incentives to enroll children in preschool. As incomes rise above $35,000 parents become sufficiently rich to pay for preschool. As Figures 4(a) and 4(b) show, increasing enrollment above this income level results from rising private preschool enrollment.

Two groups of children would tend to benefit the most in terms of increased preschool enrollment from a shift from current to optimal policies: the very poor and those just above the poverty line. The former group has much stonger enrollment rates because their parents
receive subsidies in excess of 100% (i.e. they pay negative fees) and the latter group because they are negatively affected by the targeted nature of the current system. In our benchmark model two aspects of the environment change at the poverty line. One is that eligibility for Head Start ends at the poverty line, and the second is that fees for public preschool increase there. Although the Head Start cutoff is clearly part of the current policy environment, the latter feature is arguably an artificial consequence of the fact that we model current public preschool fees as a step function. We have also considered an alternative model of current policies where public preschool fees increase linearly with parental income. That version of the model does not perform as well as our preferred benchmark, but it does continue to feature strong improvements in enrollment just above the poverty line when an optimal fee schedule is adopted, indicating that most of this effect results from the Head Start cutoff. It is worth noting that the extent to which Head Start acceptance determines the preschool enrollment of low income children is exactly replicated by our model based on experimental evidence from the Head Start Impact Study, thus the importance of this program for families near the poverty line is not overstated here.

In Figures 4(a) and 4(b) we decompose enrollment into public and private enrollment (we repress standard error bands for clarity). Public Enrollment decreases in a convex manner and then slightly increases as the fee stops increasing with income. Private enrollment starts at zero then quickly increases at a point where affordability becomes less of an issue. Overall, private preschool enrollment increases by one-third of a percentage point under the optimal fee schedule. This is a direct result of the fact that an optimal fee schedule does a better job of limiting the crowding out of private preschool enrollment than existing policies do.
4.2 Better Targeting versus Absence of Rationing

The gain from implementing the optimal policies are quite large, and there are improvements in overall enrollment at every income level. Because enrollment gains are ubiquitous, we can rule out the possibility that all of the gains from adopting the optimal fee schedule arise from better targeting of subsidies. Otherwise, enrollment would fall or remain stable for at least some income groups. The other potential source of enrollment increases is the implied change from a rationing system to a price system, which raises extra revenue and thereby allows for expansion in the number of children enrolled. Recall from the estimates in Table 1 that in the benchmark only 79.6% of public preschool and 70.6% Head Start applicants actually receive and enrollment offer. As explained above, this lottery assignment captures two important features of the real world: (i) oversubscription and (ii) local availability issues. A natural question then is how much of the overall gains arise from better targeting as opposed to eliminating of such rationing?

How do we decompose enrollment gains into ‘better targeting’ and ‘absence of rationing’? Our approach here is to construct a set of public preschool fees such that current targeting is held constant, but no rationing occurs:

1. For each income level we compute the total dollars currently spent on Head Start and other public preschool programs.

2. We eliminate rationing lotteries for public programs. Of course, at current fees this would necessitate an increase in public spending.

3. We increase income-specific fees (merging Head Start and public for low income groups) up to the point that income-specific public spending equals what is was in the benchmark equilibrium.
What this procedure identifies is the increase in fees that would be required in order for no extra spending to be needed in order to eliminate over-subscription. Higher fees work to eliminate rationing partly by pushing some marginal families out based on cost, and partly by increasing fee revenue, which funds additional preschool slots. This is done separately for each income level grid point, thus holding targeting constant.

Figure 5 illustrates this idea by plotting three fee schedules: benchmark fees, optimal fees and the ‘no-rationing’ fees resulting from the above procedure. Notice that higher fees are required to eliminate rationing, and the amount gets bigger as incomes get larger. Also notice that the difference between ‘no-rationing’ fees and optimal fees is relatively big, indicating how much targeting changes when the optimal policy is implemented.

How much of the enrollment gains does the elimination of rationing account for? The answer is 39.2%. If the ‘no-rationing’ fee schedule were implemented overall enrollment would increase by 4.7 percentage points as compared to current policies, which is is 39.2% of the 12 percentage points that the optimal policy can achieve.

Figure 6(a) illustrates the enrollment implications along the parental income distribution. The curves somewhat mirror what we can expect from Figure 5: those income groups that pay a lower (higher) fee under the optimal fees than under the ‘no-rationing’ fees have a higher (lower) share of children in preschool. One interesting result is that overall enrollment among higher income families is slightly higher under ‘no-rationing’ than under the optimal fee schedule, which is of course because the ‘no-rationing’ public fee for these groups is lower than the optimal fee. However, that fact that high income enrollment is only slightly different illustrates the power of optimal fees. Under the optimal fee schedule most of the higher income families that opt out of public programs instead enter private programs. This is shown in Figures 6(b) and 6(c), which plot public and private preschool enrollment rates in all three
cases. As expected, public enrollment for higher income families is substantially lower under the optimal policy than the no-rationing policy because of higher optimal fees. Most of this difference is simply a shift towards private enrollment, which is much higher under the optimal policy than the ‘no-rationing’ policy. Indeed, the planner’s overall enrollment objective suffers because some of the high income families pushed out of public programs by the high fee do not find a private alternative. However, this is offset by value of the dollars saved on the larger fraction of families that simply switch to private preschool. The value of those dollars is, of course, that they can be spent on encouraging enrollment among low income families where the propensity to substitute between public and private programs is lower.

4.3 Labor Supply Incentives

Optimal preschool policies are quite progressive in a sense that fees increase a lot with parental income. Figure 7 illustrates how the increasing fee schedule (or the decreasing subsidy schedule) translates into distortions of the labor supply of the parents. The dashed-line shows the labor supply distortions coming from the income tax. The dashed-dotted line illustrates the additional distortions – the marginal preschool fee – for parents, who send their children to public preschool. For low incomes they are even higher than the income tax itself. The highest value is reached around an income of $17,000, where the marginal fee is 29%. The bold line shows the effective marginal tax rate (i.e. the sum of the two) or more technically speaking the overall labor wedge for public preschool parents.

The implied decrease in labor supply reduces tax revenue. Given that we only consider budget neutral reforms, this decrease in tax revenue has to translate one to one into a decrease of the preschool budget. This effect turns out quantitatively important: the size of the preschool budget actually shrinks by 8.7%. So better targeting comes at the expense of a lower budget but nevertheless leads to a large enrollment increase. This actually highlights the power of optimal targeting: despite the decrease in the size of the budget, enrollment could be increased drastically. In Section 4.5.1, we provide robustness with respect to the preexisting distortions of the income tax schedule and it turns out that it is quantitatively significant.

4.4 The Importance of (Unknown) Price Responsiveness

Most of the uncertainty regarding to the optimal fee schedule, and the enrollment benefits of implementing it, can be attributed to a statistic that we refer to as “price responsiveness”. This statistic summarizes how important prices are in generating observed preschool enrollment
Figure 6: Changes in Enrollment
patterns, as opposed to the rationing that is caused by oversubscription of public preschools and Head Start centers and local availability issues.

To formally define price responsiveness we consider a counterfactual in which universal preschool is partially implemented: current public preschool fees are held fixed, but all rationing is eliminated. In our full implementation of universal preschool rationing is also eliminated, but fees are reduced to zero as well. Price responsiveness is computed as a ratio where the numerator is the difference between overall preschool enrollment under a full universal program and that under the no-rationing partial implementation, and the denominator is the difference between overall preschool enrollment under a full universal program and that in the benchmark economy. More simply, it is the fraction of the enrollment effect of universal preschool implementation that is attributable to the associated price reduction, and not to rationing.

In Figure 8 we plot the covariation in our bootstrapped samples between the price responsiveness statistic and the effect of the optimal policy on overall preschool enrollment. The relationship between these statistics in the model makes it very clear that uncertainty about price responsiveness, which arises from uncertainty about the estimated model parameters, explains why there is such uncertainty about the effectiveness of the optimal fee schedule.

Why is price responsiveness so important? This arises from the fact that introducing the optimal preschool policy entails not only a change in the way subsidies are targeted, but also a more basic change from a rationing system to a price system. The introduction of price system is something of a free lunch for the planner to the extent that as prices rise extra preschool slots can be funded. The price responsiveness statistic measures how much the planner can gain via this channel, and thus is a direct indicator of how much is gained by implementing the optimal policy.
Figure 8: Bootstrapped covariation between effectiveness of the optimal policy and price responsiveness

Where is the source of uncertainty about price responsiveness? This is related to the fact that there are at least two ways to interpret the data on universal preschool introduction. One is that the new enrollment effect results from lowering preschool fees, thus making preschool affordable for many more families. Under the same interpretation, the measured crowding out of private investments occurs because the gap in price between public and private options has become larger. These interpretations imply a pure price effect, and price responsiveness statistic equal to one. An alternative interpretation of the data is that the new enrollment effect of universal preschool is due to an expansion of the number of slots available, thus allowing public preschool to be accessed by many more families. For some families this could induce a switch from private to public if that would have been their preferred choice under the old system, but slots were unavailable. For other families private preschool may be too expensive so increased public preschool slots might induce them to now enroll their child in preschool.

Both of the aforementioned interpretations factor into our model, but their relative importance (which the price responsiveness statistic captures) depends on the parameter estimates. The data displayed in Figure 8 show that a great deal of variation in price responsiveness is generated by varying the estimated parameters over their joint empirical distribution, thus implying considerable uncertainty about both this statistic and the effects of optimal policy implementation.

This discussion isolates important gaps in our empirical knowledge about preschool in the United States. We do not have any direct evidence on the extent of rationing in Head Start and
other public preschool programs, nor do we have much sense of the elasticity of enrollment in quality preschool programs with respect to the fees charged for those programs. The availability of such evidence would not only reduce the amount of policy uncertainty in our exercise, but would also provide guidance to those studying policy design within other research paradigms. In a sense, we believe that strong evidence on what we call price responsiveness would provide an important sufficient statistic for policy analysis.

4.5 Robustness

Certainly there are many ways to study robustness. Currently we are working on various cases where we assume different functional forms for the distributions of preferences. What we present here is robustness with respect to the modelling of current policies.

4.5.1 Difference in Calibration of Current Tax System

The pre-existing tax function is also important for our results as our theoretical analysis has shown. The higher marginal tax rates, the more distortive is an increasing preschool fee schedule. To quantitatively assess this channel, we assume the tax function to be more progressive. We therefore take a slightly different specification from Guner, Kaygusuz, and Ventura (2014): the one which includes all state taxes as well. Figure 9(a) compares the two different cases. As can be seen tax rates are up to 5 percentage points higher in this case.

![Figure 9(a) Difference in Tax Rates](image1)

![Figure 9(b) Optimal Fee Schedule for Higher Tax Rates](image2)

Figure 9: Comparative Statics w.r.t. to Tax Distortions

Figure 9(b) illustrates the optimal fee schedule for this case. Comparing it to the optimal schedule illustrated in Figure 2, we can see that the optimal schedule is less steep.

Interestingly, the increase in overall enrollment is even by .3 percentage points higher in this case. But note that to study this case we completely reestimated the model for this more
progressive tax function. As a consequence that price responsiveness as defined in Section 4.4 is now at 52% instead of 39%. Consequentially, even though the overall increase in enrollment is now higher, the increase due to better targeting is lower. It is 5.7% instead of 7.2%. Thus, slightly varying the progressivity of the tax schedule lowers the power of optimal targeting by a non-trivial amount. This highlights the importance of modelling other policies correctly which is generally true for any policy question.

4.5.2 Difference in Calibration of Current Preschool Policies

When estimating the benchmark model we considered a case in which current preschool fees follow a two tier system. We now consider an alternative case in which benchmark fees follow a linear function of parental income. Although we find that this version of the model does not fit the data as well, it is still a useful check on the robustness of our main results.

We have not bootstrapped this set of results yet. Parameter estimates with bootstrapped standard errors and a specification test against the benchmark model will be provided in this space in the final manuscript. However, we are still able to do the important exercise of comparing the estimated optimal fee schedule in this case against the benchmark case.

The diamond-marked line in Figure 10(a) illustrates the optimal fee schedule for this case. When compared to Figure 5 we see that there is very little difference in the optimal fee schedule. The overall enrollment increase in the linear case is even larger than the benchmark case: 14.9 percentage points versus 12.0 in the benchmark. This is certainly within the range of uncertainty of the effects of policy for the benchmark model, and shows that if anything our benchmark model underestimates the potential effectiveness of the optimal policy.

We also want to understand how important changes in the targeting of subsidies is in this case, and how this compares to the benchmark. In the linear case on 42% of the increase in enrollment is due to better targeting, as compared to 61% in the benchmark. Figure 10(b) illustrates how enrollment rates as a function of income differ in the two cases. For higher income families we see that both the ‘no-rationing’ enrollment rate and the optimal enrollment rate are quite a bit higher than the linear baseline enrollment rate, whereas under our benchmark specification all three were quite similar.

5 Conclusion

We have taken a mechanism-design perspective on the policy debate about public preschool provision. We have developed a theory of how a public preschool fee schedule should vary with family income. We thereby did not impose any restrictions on the potential shape of
such a schedule. Our formulas transparently highlight the trade-off between increasing public preschool enrollment, crowding-out private preschool enrollment and labor supply incentives.

We have estimated our model using data on existing U.S. preschool programs, enrollment patterns and quasi-experimental evidence. Our quantitative exploration revealed that preschool enrollment could be significantly increased by better targeting subsidies. Without increasing any taxes, the government could increase overall preschool enrollment by 12 percentage points. The necessary fee schedule would start around zero and steadily increase until a family income of roughly $90,000. The additional implicit marginal tax rate implied is up to 30%.

In future work it would be interesting the model the private sector with more sophistication. Private preschool providers could for example respond to public providers by increasing quality. Further, we neglected the question of the optimal level of quality. To address such questions, more empirical evidence is needed about returns to the intensive margin in terms of money spend per child at preschool. Lastly, we only asked how the government should target a given level of public funds to children along the parental income distribution, but did not ask how much subsidies the government should pay overall. It would also be interesting to look to what extent higher subsidies to preschool pay for themselves through higher tax revenue in the future in the spirit of Findeisen and Sachs (2016), who look at college education subsidies.

A Appendix

A.1 Derivation of Fee Schedule with Mechanism-Design Approach

Applying a variant of the revelation principle, we know that instead of optimally choosing the function $F_{pu}(-)$ (which would be an indirect mechanism), the social planner can also choose
directly the respective allocation variables \( \{y_{pu}(\omega), c_{pu}(\omega)\}_{\omega \in \Omega} \). This mechanism-design approach is non-standard because \( \{y_{pr}(\omega), c_{pr}(\omega)\}_{\omega \in \Omega} \) and \( \{y_{no}(\omega), c_{no}(\omega)\}_{\omega \in \Omega} \) are exogenously given. This is the ‘direct-mechanism equivalent’ of taking the income tax schedule \( T(\cdot) \) as given – note that the income of parents that do not send their children to public preschools only depends on the tax schedule.\(^{13}\) When optimizing over \( \{y_{pu}(\omega), c_{pu}(\omega)\}_{\omega \in \Omega} \), the planner has to obey the same government budget constraint as in (10), but with \( y_{pu}(\omega) - c_{pu}(\omega) \) substituted for \( F_{pu}(y_{pu}(\omega)) + T(y_{pu}(\omega)) \).

**Incentive Compatibility** When choosing \( \{y_{pu}(\omega), c_{pu}(\omega)\}_{\omega \in \Omega} \), the government has to take into account (i) that those parents who send their children to public preschool truthfully reveal their type \( \omega \) and (ii) how \( \tilde{\xi}(\omega, \phi) \) and \( \tilde{\phi}(\omega) \) depend on \( \{y_{pu}(\omega), c_{pu}(\omega)\}_{\omega \in \Omega} \). The former is equivalent to

\[
\forall \omega, \omega' \text{ with } \omega \neq \omega': U\left(c_{pu}(\omega) - \frac{y_{pu}(\omega)}{\omega}\right) \geq U\left(c_{pu}(\omega') - \frac{y_{pu}(\omega')}{\omega}\right).
\]

The definition of the thresholds are as in Lemma 1, which is why we refrain from denoting them again here. In Appendix A.1 we present a derivation of the optimal fee schedule using mechanism-design techniques.

Following the theory of optimal income taxation in the tradition of Mirrlees (1971), one can show that the incentive compatibility constraints (16) can be summarized by:

\[
\forall \omega : X'(\omega) = v'\left(\frac{y_{pu}(\omega)}{\omega}\right) \frac{y_{pu}(\omega)}{\omega^2}
\]

where \( X(\omega) = c_{pu}(\omega) - v\left(\frac{y_{pu}(\omega)}{\omega}\right) \) and a monotonicity constraint \( y'_{pu}(\omega) \geq 0 \). Following common practice, we ignore the monotonicity constraint in the analytical part and numerically check ex-post, whether it is fulfilled – and it always is.

The Lagrangian associated with the mechanism-design problem then reads as (where we use the negative of the objective function to write at as a maximization problem):

\(^{13}\)With income effects, the labor supply of the private preschool parents would also depend on \( F_{pr} \). It would be straightforward to take that into account.
and obtain the following necessary conditions:

After applying integration by parts \( \int_{\omega} \eta(\omega) X'(\omega) = \eta(\omega) X(\omega) - \int_{\omega} \eta'(\omega) X(\omega) \), we obtain the following necessary conditions:

\[
\frac{\partial \mathcal{L}}{\partial X(\omega)} = \int_{A_2} b(\alpha_2, \omega) \frac{\partial}{\partial \theta_{pr}(\omega)} \hat{\theta}_{pr}(\omega, \theta_{pr}) g_{pr}(\hat{\theta}_{pr}(\omega, \theta_{pr})|\alpha_2, \omega, \theta_{pr}) dG_{pr}(\theta_{pr}|\alpha_2, \omega) H(\alpha_2|\omega)f(\omega)
+ \Lambda (T_{pu}(\omega) - T(y(\omega))) \int_{A_2} \frac{\partial}{\partial \theta_{pr}(\omega)} \hat{\theta}_{pr}(\omega, \theta_{pr}) g_{pr}(\hat{\theta}_{pr}(\omega, \theta_{pr})|\alpha_2, \omega, \theta_{pr}) dG_{pr}(\theta_{pr}|\alpha_2, \omega) H(\alpha_2|\omega)f(\omega)
+ \Lambda (T_{pu}(\omega) - T(y(\omega))) \int_{A_2} \frac{\partial}{\partial F_{pu}(\omega)} \hat{\theta}_{pr}(\omega, \theta_{pr}) g_{pu}(\hat{\theta}_{pr}(\omega, \theta_{pr})|\alpha_2, \omega, \theta_{pr}) dG_{pr}(\theta_{pr}|\alpha_2, \omega) H(\alpha_2|\omega)f(\omega)
- \eta'(\omega) = 0.
\] (18)

and

\[
\frac{\partial \mathcal{L}}{\partial y_{pu}(\omega)} = \lambda \left( 1 - \frac{\partial}{\partial \theta_{pr}(\omega)} \hat{\theta}_{pr}(\omega, \theta_{pr}) \frac{1}{\omega} \right) f(\omega) \int_{A_2} s_{pu}(\alpha_2, \omega) dH(\alpha_2|\omega)
- \eta(\omega) \left( \frac{\partial}{\partial \theta_{pr}(\omega)} \frac{1}{\omega^2} + \frac{\partial}{\partial F_{pu}(\omega)} \frac{y(\omega)}{\omega} \right) = 0.
\] (19)
as well as the transversality conditions

\[ \eta(\omega) = \eta(\bar{\omega}) = 0. \]

One can show that (19) can be written as:

\[ \frac{\tau_{pu}(\omega)}{1 - \tau_{pu}(\omega)} = \left( 1 + \frac{1}{\varepsilon_{y,1-\tau}(\omega)} \right) \frac{\eta(\omega)}{\omega \lambda f(\omega) \int_{A_2} s_{pu}(\alpha_2, \omega) dH(\alpha_2 | \omega)} \]  

(20)

Then, integrating (18), solving for \( \eta(\omega) \) and inserting into (20) yields the result in Proposition 1. The optimal value for \( \lambda \) follows from the transversality conditions.

---

14First we use just the definition of the wedge \( \omega(1 - \tau_{pu}(\omega)) = v' \left( \frac{y_{pu}(\omega)}{\omega} \right) \). Second – using the implicit function theorem – one can show that the elasticity satisfies \( \varepsilon_{y,1-\tau}(\omega) = \frac{\omega(1 - \tau_{pu}(\omega))}{\frac{y'_{pu}(\omega)}{\omega} v'' \left( \frac{y_{pu}(\omega)}{\omega} \right)} \). Together this implies

\[ v' \left( \frac{y_{pu}(\omega)}{\omega} \right) + v'' \left( \frac{y_{pu}(\omega)}{\omega} \right) \frac{y_{pu}(\omega)}{\omega} = \omega(1 - \tau_{pu}(\omega)) \left( 1 + \frac{1}{\tau_{pu}(\omega)} \right) \].
References


