Macro Risks and Term Structure of Interest Rates

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39th Annual NBER Summer Institute
Forecasting and Empirical Methods
July 13, 2016
Aggregate Demand and Supply Shocks

Define aggregate demand/supply shocks using minimal theoretical restrictions (Blanchard, 1989):

- Aggregate demand (AD): moves GDP growth and inflation in the same direction
- Aggregate supply (AS): moves GDP growth and inflation in opposite directions

Macro risks=second and higher order moments of AD/AS shocks

Identification of aggregate demand (AD) and aggregate supply (AS) shocks is important in many areas of economics
Macroeconomics

- Which shocks drive recessions?
- Which shocks drive long-term GDP growth?
- Our contribution:
  - A novel method to extract AD/AS shocks exploiting non-Gaussian properties of data
  - Characterizing US business cycles as AD/AS (e.g., Great Recession)
  - Supply shocks have permanent impact on real GDP, while demand shocks don’t
Explaining bond risk and term premia:

- Most of the literature uses financial factors (e.g., Campbell and Shiller, 1991)

- Most of the literature which deals with macro factors relies on level factors (e.g., Ludvigson and Ng, 2009): exceptions are Wright (2011) and Bansal and Shaliastovich (2013)

Economic insight: bond risk and term premia should be higher (lower) in aggregate supply (aggregate demand) environment

Our contribution:

- Non-Gaussian AD/AS macro risk factors drive substantial variation in bond risk-premia

- AD/AS macro risks factors affect bond risk and term premia differently
Macroeconomic Shocks

- Shocks to real GDP growth and inflation:

\[ g_{t+1} = E_t[g_{t+1}] + \epsilon^g_{t+1}, \]
\[ \pi_{t+1} = E_t[\pi_{t+1}] + \epsilon^\pi_{t+1}. \]

- Modeling using demand and supply shocks:

\[ \epsilon^g_{t+1} = \sigma^d_g u^d_{t+1} + \sigma^s_g u^s_{t+1}, \]
\[ \epsilon^\pi_{t+1} = \sigma^d_\pi u^d_{t+1} - \sigma^s_\pi u^s_{t+1}, \]

\[ \text{Cov}(u^d_{t+1}, u^s_{t+1}) = 0, \text{Var}(u^d_{t+1}) = \text{Var}(u^s_{t+1}) = 1. \]
If supply and demand shocks are heteroskedastic, $\text{Cov}_t(\epsilon_{g,t+1}, \epsilon_{\pi,t+1})$ will vary over time:

$$\text{Cov}_t(\epsilon_{g,t+1}, \epsilon_{\pi,t+1}) = \sigma_g \sigma_{\pi} \text{Var}_t(u_{d,t+1}) - \sigma_g \sigma_{\pi} \text{Var}_t(u_{s,t+1})$$

- Demand shock environment: large $\text{Cov}_t(\epsilon_{g,t+1}, \epsilon_{\pi,t+1}) \Rightarrow$ nominal bonds hedge well

- Supply shock environment: small $\text{Cov}_t(\epsilon_{g,t+1}, \epsilon_{\pi,t+1}) \Rightarrow$ nominal bonds hedge poorly
Demand and supply shocks are not identified with Gaussian shocks: 4 coefficients ($\sigma_g^d$, $\sigma_{\pi}^d$, $\sigma_g^s$, $\sigma_{\pi}^s$) to identify but only 3 moments to match (2 variances and covariance)

Approach: use non-Gaussian data aspects for the identification:

- Is macroeconomic data non-Gaussian?
- How to model non-Gaussian features?
Modeling Demand and Supply Shocks

- Demand (and supply) shocks modeled using Bad Environment-Good Environment (BEGE) structure (Bekaert and Engstrom, JPE 2016):

\[ u^d_{t+1} = \sigma^d_p \omega^d_{p,t+1} - \sigma^d_n \omega^d_{n,t+1} \]

- Shocks follow demeaned gamma distributions:

\[ \omega^d_{p,t+1} \sim \Gamma(p^d_t, 1) - p^d_t, \]
\[ \omega^d_{n,t+1} \sim \Gamma(n^d_t, 1) - n^d_t, \]

\( \Gamma(x, y) \)—shape parameter \( x \) and scale parameter \( y \)
Bad Environment-Good Environment

probability density function
Time-varying variances: Probability density functions

- $p_t$ can be interpreted as good variance and $n_t$ as bad variance.

**Large $p_t$ - Good environment:** positive unscaled skewness

**Large $n_t$ - Bad environment:** negative unscaled skewness

[Graphs showing the probability density functions for $p_t$ and $n_t$]
Advantages of BEGE Distribution

- Fit non-Gaussian features of macroeconomic (Bekaert and Engstrom, JPE 2016) and financial data (Bekaert, Engstrom, and Ermolov, JoE 2015) well

- Theoretically tractable: unscaled moments linear functions of $p_t$ and $n_t$
General Overview

- US quarterly observations 1959Q2-2015Q2
- Identify macro expectations and shocks using VARMA(1,1) on real activity and inflation data
- Filter demand and supply shocks from macro shocks using classical minimum distance (CMD)
- Estimate BEGE dynamics of demand and supply shocks using approximate MLE (Bates, 2006)
Identify Macro Expectations and Shocks

- VARMA(1,1) on 6 variables (based on AIC):
  - Real GDP growth
  - Core and aggregate inflation
  - Unemployment gap
  - 1 quarter and 10 year Treasury yields

- Extract:
  - Expectations of real GDP growth, inflation, core inflation + unemployment gap
  - Shocks to real GDP growth, inflation, core inflation and unemployment gap
Filter Demand and Supply Shocks

- Shock structure:

\[
\begin{bmatrix}
\epsilon^g_t \\
\epsilon^\pi_t \\
\epsilon^\text{core}_t \\
\epsilon^\text{unemp}_t \\
\end{bmatrix}
= \sum_{4\times2}
\begin{bmatrix}
u^d_t \\
u^s_t \\
\end{bmatrix}
+ \Omega_{4\times4}
\begin{bmatrix}
\xi^g_t \\
\xi^\pi_t \\
\xi^\text{core}_t \\
\xi^\text{unemp}_t \\
\end{bmatrix}
\]

- \(\Omega\) - diagonal with \(\xi^g_t, \xi^\pi_t, \xi^\text{core}_t, \xi^\text{unemp}_t\) \(\sim\) i.i.d. distribution with 1 variance and 0 skewness and excess kurtosis

- Percentage of variance attributed to \(\xi^g_t, \xi^\pi_t, \xi^\text{core}_t, \xi^\text{unemp}_t\) is the same across all 4 macro series

- Estimate \(\Sigma\) and \(\Omega\) via CMD: matching 36 unconditional second, third, and fourth order moments of macro shocks

- Filter \(u^d_t\) and \(u^s_t\) with a Kalman filter
12 out of 26 third and fourth order macro shock moments are individually statistically significant at least at the 10% level.

26 third and fourth order macro shock moments are jointly significant at the 1% level.
### Demand vs. Supply

<table>
<thead>
<tr>
<th></th>
<th>Demand</th>
<th>Supply</th>
</tr>
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<tbody>
<tr>
<td><strong>GDP growth</strong></td>
<td>0.43</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.11)</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td>0.26</td>
<td>-0.27</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.07)</td>
</tr>
<tr>
<td><strong>Core inflation</strong></td>
<td>0.19</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.04)</td>
</tr>
<tr>
<td><strong>Unemployment gap</strong></td>
<td>-0.16</td>
<td>-0.11</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

Other shocks account for 45% of macro shocks variance
Demand and Supply Shocks

Demand Shocks: Skewness = -1.07, Ex. kurtosis = 4.83, Jarque-Bera test p-value: <0.1%

Supply Shocks: Skewness=-0.35, Ex. kurtosis=1.64, Jarque-Bera test p-value: <0.1%
Estimate BEGE Dynamics

- BEGE variances $p_t^d$, $n_t^d$, $p_t^s$, and $n_t^s$ follow autoregressive square-root-type processes.


Macro risk processes
Demand Variance Decomposition
Supply Variance Decomposition

![Graph showing supply variance decomposition over time]

Legend:
- Green line: Good Variance
- Red line: Bad Variance

Year:
- 1960
- 1970
- 1980
- 1990
- 2000
- 2010

Variance values range from 0 to 4.5.

The graph illustrates the decomposition of supply variance into good and bad variance components over several decades.
Impulse Responses

Demand shock - real GDP
Cumulative impact: 0.19% (se: 0.28%)

Supply shock - real GDP
Cumulative impact: 0.67% (se: 0.25%)

Demand shock - price level
Cumulative impact: 1.02% (se: 0.54%)

Supply shock - price level
Cumulative impact: -0.88% (se: 0.44%)
Time-varying Real-Nominal Covariance
State Variables

- Macro level factors (Ludvigson and Ng, 2009, - type):
  - Expected real GDP growth
  - Expected inflation
  - Expected core inflation
  - Unemployment gap

- Second/higher order moments = macro risks:
  - $p_t^d$ - good (positive skew) demand variance
  - $n_t^d$ - bad (negative skew) demand variance
  - $p_t^s$ - good (positive skew) supply variance
  - $n_t^s$ - good (negative skew) supply variance
Explanatory Power for Yield Levels

- Predictors: 4 macro level factors + macro risks
- Confidence interval is Bauer and Hamilton (2015) bootstrap confidence interval
Explanatory Power for Excess Returns

- Predictors: 4 macro level factors + macro risks
- Confidence interval is Bauer and Hamilton (2015) bootstrap confidence interval
## 1 Quarter Excess Return Regressions

<table>
<thead>
<tr>
<th>Macro level factors</th>
<th>1 year bond</th>
<th>5 year bond</th>
<th>10 year bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_d^t$</td>
<td>0.0006</td>
<td>0.0063</td>
<td>0.0211</td>
</tr>
<tr>
<td></td>
<td>(0.0020)</td>
<td>(0.0096)</td>
<td>(0.0208)</td>
</tr>
<tr>
<td>$n_d^t$</td>
<td>-9.8329</td>
<td>-44.0961</td>
<td>-69.0965</td>
</tr>
<tr>
<td></td>
<td>(2.7119)</td>
<td>(10.5535)</td>
<td>(24.7824)</td>
</tr>
<tr>
<td>$p_s^t$</td>
<td>0.0062</td>
<td>0.0208</td>
<td>0.0168</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td>(0.0100)</td>
<td>(0.0154)</td>
</tr>
<tr>
<td>$n_s^t$</td>
<td>0.0457</td>
<td>0.2028</td>
<td>0.4436</td>
</tr>
<tr>
<td></td>
<td>(0.0926)</td>
<td>(0.3440)</td>
<td>(0.5772)</td>
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</tbody>
</table>
Explanatory Power for Excess Returns Over Yield Factors

- Predictors: 4 macro level factors + 3 yield curve factors + macro risks

- Confidence interval is Bauer and Hamilton (2015) bootstrap confidence interval

Similar results for Ang-Piazzesi (2003) factors
Term Premium

- Blue Chip forecasts based 10 year term-premium: semi-annually 1986Q2-2015Q2

<table>
<thead>
<tr>
<th>macro level factors</th>
<th>$p_t^d$</th>
<th>$n_t^d$</th>
<th>$p_t^s$</th>
<th>$n_t^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>6.84E-06</td>
<td>-1.4758</td>
<td>0.0480</td>
<td>0.1046</td>
</tr>
<tr>
<td>...</td>
<td>(2.65E-04)</td>
<td>(0.3672)</td>
<td>(0.0949)</td>
<td>(0.0925)</td>
</tr>
</tbody>
</table>

- Adjusted $R^2$ without macro level factors only: 0.6437 (95% confidence upper bound 0.6814)
- Adjusted $R^2$ with macro risks: 0.7072
Conclusions

- Novel method for extracting aggregate demand and supply shocks based on exploiting non-Gaussian features of data

- Characterizing macroeconomic dynamics via AD/AS shocks

- Demand-supply composition of macroeconomic shocks matters for bond and term premia

- Term-structure model with AD/AS macro risks (work in progress):
  - Economic intuition
  - Non-Gaussian features
  - Closed form solutions!
Appendix: BEGE Moments

\[ u_t \sim \sigma_p (\Gamma(p_t, 1) - p_t) - \sigma_n (\Gamma(n_t, 1) - n_t) \]

- Variance: \( \sigma_p^2 p_t + \sigma_n^2 n_t \)
- Unscaled skewness: \( 2\sigma_p^3 p_t - 2\sigma_n^3 n_t \)
- Unscaled excess kurtosis: \( 6\sigma_p^4 p_t + 6\sigma_n^4 n_t \)
Macro risks are persistent and driven by the realization shocks capturing volatility clustering (Gourieroux and Jasiak, 2006):

\[ p_{t+1}^d = \bar{p}^d + \rho_p^d (p_t^d - \bar{p}^d) + \sigma_{pp}^d \omega_{p,t+1}^d \]

Similar processes for \( n_t^d, p_t^s, \) and \( n_t^s \)

If \( \sigma_{pp} < \rho_p \), macro risks never hit a zero-lower bound
### Appendix: Unconditional Moment Values

#### 1/3

**Scaled skewness:**

<table>
<thead>
<tr>
<th></th>
<th>inflation</th>
<th>real growth</th>
<th>core inflation</th>
<th>unemployment gap</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>data</strong></td>
<td>-1.2632</td>
<td>0.1275</td>
<td>0.1866</td>
<td>0.6860</td>
</tr>
<tr>
<td><strong>standard error</strong></td>
<td>(0.9124)</td>
<td>(0.3064)</td>
<td>(0.4598)</td>
<td>(0.2372)</td>
</tr>
<tr>
<td><strong>fitted</strong></td>
<td>-0.2328</td>
<td>-0.3188</td>
<td>-0.2804</td>
<td>0.3576</td>
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</table>

**Excess kurtosis:**

<table>
<thead>
<tr>
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<th>inflation</th>
<th>real growth</th>
<th>core inflation</th>
<th>unemployment gap</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>data</strong></td>
<td>10.3646</td>
<td>1.6505</td>
<td>2.3854</td>
<td>1.9179</td>
</tr>
<tr>
<td><strong>standard error</strong></td>
<td>(5.1438)</td>
<td>(0.7314)</td>
<td>(1.1802)</td>
<td>(0.6808)</td>
</tr>
<tr>
<td><strong>fitted</strong></td>
<td>0.4552</td>
<td>0.8090</td>
<td>0.6070</td>
<td>0.9314</td>
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### Appendix: Unconditional Moment Values

#### 2/3

**Coskewness:**

<table>
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<tr>
<th></th>
<th>$infl^2 \times rgrw$</th>
<th>$infl^2 \times cinfl$</th>
<th>$infl^2 \times ugap$</th>
<th>$rgrw^2 \times infl$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>data</strong></td>
<td>-0.7728</td>
<td>-0.2339</td>
<td><strong>0.7322</strong></td>
<td>-0.2404</td>
</tr>
<tr>
<td><strong>standard error</strong></td>
<td>(0.4328)</td>
<td>(0.3097)</td>
<td>(0.4016)</td>
<td>(0.1780)</td>
</tr>
<tr>
<td><strong>fitted</strong></td>
<td>-0.2316</td>
<td>-0.2474</td>
<td>0.2416</td>
<td>-0.2982</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>$rgrw^2 \times cinfl$</th>
<th>$rgrw^2 \times ugap$</th>
<th>$cinfl^2 \times infl$</th>
<th>$cinfl^2 \times rgrw$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>data</strong></td>
<td>-0.1860</td>
<td>0.1877</td>
<td>0.0814</td>
<td>-0.1920</td>
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<tr>
<td><strong>standard error</strong></td>
<td>(0.1912)</td>
<td>(0.3358)</td>
<td>(0.3184)</td>
<td>(0.1459)</td>
</tr>
<tr>
<td><strong>fitted</strong></td>
<td>-0.3185</td>
<td>0.3314</td>
<td>-0.2632</td>
<td>-0.2721</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$cinfl^2 \times ugap$</th>
<th>$ugap^2 \times infl$</th>
<th>$ugap^2 \times rgrw$</th>
<th>$ugap^2 \times cinfl$</th>
</tr>
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<tbody>
<tr>
<td><strong>data</strong></td>
<td>0.0847</td>
<td>-0.1989</td>
<td>-0.4535</td>
<td>-0.0265</td>
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<tr>
<td><strong>standard error</strong></td>
<td>(0.2143)</td>
<td>(0.1384)</td>
<td>(0.3097)</td>
<td>(0.2290)</td>
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<tr>
<td><strong>fitted</strong></td>
<td>0.2833</td>
<td>-0.3172</td>
<td>-0.3443</td>
<td>-0.3392</td>
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## Co-excess Kurtosis:

<table>
<thead>
<tr>
<th></th>
<th>$infl^2 - rgrw^2$</th>
<th>$infl^2 - cinfl^2$</th>
<th>$infl^2 - ugap^2$</th>
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</thead>
<tbody>
<tr>
<td><strong>data</strong></td>
<td>1.7346</td>
<td>0.6331</td>
<td>1.2858</td>
</tr>
<tr>
<td><strong>standard error</strong></td>
<td>(1.0385)</td>
<td>(0.3624)</td>
<td>(0.7526)</td>
</tr>
<tr>
<td><strong>fitted</strong></td>
<td>0.6069</td>
<td>0.5257</td>
<td>0.6511</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>$rgrw^2 - cinfl^2$</th>
<th>$rgrw^2 - ugap^2$</th>
<th>$cinfl^2 - ugap^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>data</strong></td>
<td>0.7458</td>
<td>1.1438</td>
<td>0.7440</td>
</tr>
<tr>
<td><strong>standard error</strong></td>
<td>(0.3166)</td>
<td>(0.5808)</td>
<td>(0.2958)</td>
</tr>
<tr>
<td><strong>fitted</strong></td>
<td>0.7088</td>
<td>0.8680</td>
<td>0.7519</td>
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</table>
## Appendix: Yield Regression Coefficients

<table>
<thead>
<tr>
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<th>1 quarter</th>
<th>1 year</th>
<th>5 years</th>
<th>10 years</th>
</tr>
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<tbody>
<tr>
<td>Constant</td>
<td>1.2182</td>
<td>1.2616</td>
<td>1.4215</td>
<td>1.5314</td>
</tr>
<tr>
<td></td>
<td>(0.0665)</td>
<td>(0.0712)</td>
<td>(0.0733)</td>
<td>(0.0683)</td>
</tr>
<tr>
<td>$E_t c_{t+1}$</td>
<td>1.1418</td>
<td>1.1072</td>
<td>0.9680</td>
<td>0.8605</td>
</tr>
<tr>
<td></td>
<td>(0.0827)</td>
<td>(0.0928)</td>
<td>(0.1006)</td>
<td>(0.0920)</td>
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<tr>
<td>$E_t \pi_{t+1}$</td>
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<td>(0.2286)</td>
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<td>(0.2995)</td>
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<tr>
<td>$E_t g_{t+1}$</td>
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<td>0.5663</td>
<td>0.6054</td>
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<td>(0.1455)</td>
<td>(0.1279)</td>
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<tr>
<td>$u_t$</td>
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<tr>
<td></td>
<td>(0.0350)</td>
<td>(0.0357)</td>
<td>(0.0329)</td>
<td>(0.0254)</td>
</tr>
<tr>
<td>$p^d_t$</td>
<td>-8.10E-05</td>
<td>-3.39E-05</td>
<td>5.00E-05</td>
<td>4.25E-05</td>
</tr>
<tr>
<td></td>
<td>(9.53E-05)</td>
<td>(9.31E-05)</td>
<td>(8.60E-05)</td>
<td>(9.35E-05)</td>
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<tr>
<td>$n^d_t$</td>
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<td>0.4212</td>
<td>0.3177</td>
<td>0.2864</td>
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<td></td>
<td>(0.3179)</td>
<td>(0.3390)</td>
<td>(0.3111)</td>
<td>(0.2793)</td>
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<tr>
<td>$p^s_t$</td>
<td>0.0232</td>
<td>0.0240</td>
<td>0.0180</td>
<td>0.0121</td>
</tr>
<tr>
<td></td>
<td>(0.0076)</td>
<td>(0.0072)</td>
<td>(0.0055)</td>
<td>(0.0047)</td>
</tr>
<tr>
<td>$n^s_t$</td>
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<td>-0.1915</td>
<td>-0.1862</td>
<td>-0.1665</td>
</tr>
<tr>
<td></td>
<td>(0.0405)</td>
<td>(0.0479)</td>
<td>(0.0514)</td>
<td>(0.0483)</td>
</tr>
</tbody>
</table>
Appendix: Explanatory Power for Excess Returns over Ang-Piazzesi Factors

- Predictors: 4 macro level factors + Ang-Piazzesi (2003) factors + macro risks
- Confidence interval is Bauer and Hamilton (2015) bootstrap confidence interval