Participation and unbiased pricing in CDS settlement mechanisms*

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April 2015, first draft: September 2013

Abstract

The centralized market for the settlement of credit default swaps (CDS), which governs more than $12 trillion's worth of outstanding CDS contracts, has been criticized for mispricing the defaulted bonds that underlie the contracts. I take a mechanism design approach to the market for the settlement of CDS contracts and characterize robust settlement mechanisms that deliver unbiased prices for the underlying assets. All robust settlement mechanisms are the payoff equivalent of a posted price mechanism. Because forced participation in the settlement mechanism is not possible, my approach requires the development of a new notion of the core of games of incomplete information. This new notion can be applied to mechanism design environments in which side trades are allowed or when joining the mechanism is a cooperative decision.

*I am indebted to Jeffrey Ely and Rakesh Vohra for our valuable conversations and their encouragement to write this paper. I have also received helpful comments from Eddie Dekel and other participants in the CET student seminars.

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1 Introduction

A credit default swap (CDS) is regarded as an insurance contract against the risk of default. After the default of a bond issuer, the corresponding CDS contracts are settled through a centralized mechanism. This mechanism produces a price for the defaulted bond that is used to measure the amount of loss due to the default. The mechanism underprices the asset in most cases, which results in efficiency loss due to a lack of insurance (see Gupta and Sundaram (2013)). I characterize the set of robust mechanisms that deliver unbiased prices in expectation. In addition, I model the participation to this mechanism and, thus, contribute to the literature on the core of games of incomplete information. In this new notion of the core of such games, the decision to join the blocking mechanism precedes participation in the mechanism. Therefore, this notion can be used to understand centralized markets in which side trades or side matches prior to or concurrent with the centralized mechanism are allowed. Examples include dark pools[1] and some centralized job markets, such as the National Residency Matching Program (NRMP).

Each CDS contract corresponds to a reference entity's bond. The corresponding CDS contracts can be settled via *physical settlement* or *cash settlement*. In the case of cash settlement, the protection seller pays the face value minus the value of the defaulted bond to the protection buyer. In the case of physical settlement, the protection buyer hands the defaulted bond to the protection seller and receives the face value of the bond. While physical settlement has the advantages of not requiring a price and the protection buyer's full insurance, it is often impossible to physically settle all contracts. First, in most cases, the number of outstanding CDS contracts is more than the number of bonds.[2] Second, even if protection buyers could purchase the defaulted bonds for phys-

[1]Dark pools are private platforms to trade securities.

[2]As stated in Summe and Mengle (2011), at the time of Delphi Corporation's bankruptcy, it was estimated that

ical settlement, doing so would artificially raise the price of the defaulted bond. For these reasons, an alternative way of settling the contracts by cash transfer has emerged. The challenge for cash settlement is to identify a value for the defaulted bond. To determine the quantity of contracts to be settled physically or by cash transfer, as well as a price for cash settlement, the ISDA introduced a two-stage mechanism. In the first stage of the mechanism, only agents with CDS contracts participate. In this first stage, the mechanism determines the number of defaulted bonds to be bought or sold in the second stage of the mechanism and a price cap or floor. In the second stage, a uniform price auction determines a price for the defaulted bond. As of 2009, all CDS contracts are pegged to the value of the defaulted bond determined by this mechanism, unless both the protection buyer and protection seller choose to opt out.[3] Also, agents may sell their CDS contracts to other agents prior to participation in the settlement mechanism. The mechanism used by the ISDA has been the subject of criticism. Chernove et al. (2013) have observed that the defaulted bonds in this mechanism are underpriced in the vast majority of auctions. Due to the winner's curse in the second stage of the current mechanism, mispricing is inevitable. This mispricing implies that the protection buyer cannot fully insure against the risk of default by the issuer of the bond and that there is uncertainty in regard to the future payoff of a defaulted bond.

The goal of any design should be to settle contracts with unbiased prices. A settlement mechanism is unbiased if the cash settlement price is equal to the value of the defaulted bond or if an agent's payoff is equal to the payoff from physical settlement of all contracts. In this paper, I take a

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there were $28 billion in CDSs outstanding but only $2 billion in defaulted bonds. If short selling were facilitated in this market, in a physical settlement, the protection buyer would short sell, rather than hand, the defaulted bond to the protection seller. Because defaulted bonds are traded over the counter, short selling the defaulted bonds is difficult or even impossible.

[3] The ISDA argues that requiring all parties to a CDS to be bound by the results of the mechanism ensures certainty, consistency, enhanced transparency, and liquidity; see http://www.isda.org/press/press031209.html

mechanism design approach and look for a settlement mechanism that is unbiased. Moreover, the mechanism must satisfy four important properties, which I enumerate:

1. Ex-post incentive compatible, which means that the mechanism is incentive compatible for all possible agents' beliefs.

2. Weakly budget balanced, which means that the designer does not have to incur a cost to execute the settlement mechanism.

3. Robustness with respect to network, which means that, in the settlement, only the net number of CDS contracts matter.

4. Robust with respect to agents' participation decisions.

The revelation principle implies that, without loss of generality, one can restrict attention to direct settlement mechanisms that are incentive compatible (Myerson (1981)). The first property is a robustness property against agents' beliefs; see Bergemann and Morris (2005). Because there is cash transfer in the mechanism, the second property is also standard. In the third property, a mechanism is robust with respect to the network if the price and quantities are the same in two networks where agents have a net equal number of contracts. The current mechanism satisfies this property. The rationale for this property is that it lowers the systemic risk and transaction costs.

The fourth property also is important, as this contract does not compel agents to participate in the ISDA settlement mechanism. If both parties of a CDS contract agree, they can choose to settle some of their contracts outside of the settlement mechanism. I define participation-choice to describe the agents' decision about how they participate in the central mechanism as well as how they settle contracts outside of the mechanism. Participation-choices should satisfy two important criteria. First, when a pair of agents make a decision about their participation, it should benefit both of them. Second, these participation decisions should be self-confirming. This means that when a pair of agents engage in side settlement, each agent updates his beliefs about other agents' types, and given the updated beliefs, agents choose to exit when there is a benefit from doing so. The robustness property listed above is a strong one and requires the mechanism to be unbiased in all possible agent participation-choices. The main theoretical innovation of this paper is to model how agents choose their level of participation in the mechanism. For more discussion, see Sections 2 and 5.2.1.

A mechanism that sets the cash settlement price equal to the expected value of the defaulted bond conditional on the **designer** information and sets a constant cash settlement quantity is called a posted price mechanism. I show that all mechanisms with prope rties 1-4 listed above are a payoff equivalent to a posted price mechanism. The difficulty in designing a settlement mechanism, when participation is voluntary, is that agents may manipulate the outcome of the mechanism through side settlement. Because the designer faces the budget constraint, she cannot pay the agents to participate in the mechanism. When a posted price mechanism is employed, agents can no longer manipulate the settlement procedure through strategic choice of participation. Posted price mechanisms are simple and widely used in practice, see Einav et al. (2012).

As discussed, participation in this mechanism is voluntary.[4] Since CDS contracts are bilateral contracts between pairs of agents, no settlement mechanism can enforce participation. Choosing not to fully participate harms the transparency of the ISDA settlement procedure and drives away liquidity from the auction. This is important because liquidity and transparency were among the

[4]Some CDS contracts are between agents in different countries, this makes enforcing participation even more difficult.

main reasons offered for a central mechanism to settle CDSs in the first place. I show that the posted price mechanism is almost surely the unique mechanism that ensures that no pair of agents would side settle their contracts.

I extend the characterization result in two ways. First, I consider the case in which agents can sell their CDS contracts to other agents prior to the settlement mechanism but after they learn their private signals. Therefore, they can choose their level of participation by either selling their CDS contracts or settling their contracts outside of the mechanism. I provide a richer model for agents' participation. I show that the only mechanism that satisfies ex-post incentive compatibility and robustness with respect to agents' decisions about participation is a posted price mechanism. Second, I generalize the notion of biasedness. I consider settlement mechanisms for which, from an ex-ante point of view, the payoff of each agent is equal to his payoff from cash settlement with some known price. This property ensures that an ex-ante payoff of agents is proportional to their net number of contracts. This is relevant here because CDS contracts are homogeneous. I show that the characterization results hold if one replaces biasedness with this property.

The results of this paper mean that there is no robust and unbiased mechanism to settle CDS contracts whose outcome depends on the private information of agents. Therefore, all robust settlements should only depend on the publicly available information. Hence, the designer must employ other ways to generate a price for the defaulted bonds to be used for the settlement of the CDS contracts. One possible way is using the price of a comparable asset to find a valuation for the defaulted bond.

2 Related literature

My study contributes more broadly to the market design literature and, more specifically, to the nascent literature on the valuation and settlement of CDS contracts. The studies that examine the CDS contract settlement mechanism focus exclusively on the properties and modifications of the current mechanism in use. Gupta and Sundaram (2013) observe that there is a price bias for auctions held in the 2008-2012 period. Similarly, Helwege et al. (2009) compare the mechanism price to the pre- and post-auction prices of the defaulted bond in a sample of early ten auctions and find no mispricing in their sample. Coudert and Gex (2010) study the settlement procedure for a number of cases. Their empirical study also reveals a price bias in the auction. Du and Zhu (2015) develop a theoretical model to explain why the current auction misprices the defaulted bond and proposes a double auction to achieve efficiency. They consider the case in which a continuum of agents could have different valuations for the defaulted bond. Therefore, in their model, allocative efficiency becomes relevant. Motivated by the observation that the number of participants in CDS auctions rarely exceeds 15[5], I consider a framework in which there is small number of participants with private signals about the common value of the defaulted bond. Consequently, strategic behavior plays a much more crucial role in my analysis. Chernove et al. (2013) document the same price bias as do Gupta and Sundaram (2013). Taking into account multiple financial frictions in the market, they solve for equilibria of the two-stage auction, assuming that agents have no private information about the value of the defaulted bond. Assuming no private information about the value of the defaulted bond, they use a CDS auction to discover a price of the defaulted bond. My paper is the only one that takes a mechanism design approach. I complement the existing literature

[5] See http://www.creditfixings.com/CreditEventAuctions/fixings.jsp

by adopting an axiomatic mechanism design approach to characterize the settlement mechanisms that satisfy the key robustness properties described earlier.

A second contribution of my paper is a new notion of the core of a game of incomplete information. The notion of unravel-proofness under incomplete information can be interpreted as a stability condition. The set of mechanisms that satisfy the property can be interpreted as the core of the underlying game of incomplete information. The notion of core and stability has been generalized to games of incomplete information in Wilson (1978), Dutta and Vohra (2005), Myerson (2007), Serrano and Vohra (2007), Yenmez (2013), Liu et al. (2014), and Pomatto (2015). These notions differ by the way that agents communicate their private signals. Wilson (1978) considers two extreme cases: (i) all agents in a block share their private information completely (fine core) (ii) agents share no private information. Dutta and Vohra (2005) and Myerson (2007) consider the blocks for which the decision to join the block comes from a Bayesian Nash Equilibrium. Liu et al. (2014) study the implications of common knowledge of stability of a two-sided match when one side of the market has incomplete information about the other side.

In my notion a block exist if the exit game has an equilibrium in which a subset of agents with a positive measure subset of types participate in the blocking mechanism. The exist game, described in the paper, resembles the voting game described in Holmström and Myerson (1983). In their setup, all agents participate in the voting game and they ask if a mechanism can be pareto improved through reallocation by an unanimously elected alternative mechanism.

My notion of unravel-proofness differs in two important ways from notions of the core introduced in the literature. First, in the prior notions of the core, the decision to join the blocking mechanism comes after the realization of the grand mechanism's allocation. Therefore, the blocking designer or members of the blocking coalition take the allocation of the grand mechanism as exogenous. Because, in my framework, agents have quasilinear preferences and heterogeneous beliefs about a common value, the no-trade theorem implies that no subset of agents should agree to a reallocation once the contracts are settled; see Milgrom and Stokey (1982). Therefore, if one applies the notion of stability in the literature to the environment considered in this paper, all settlements would be stable and also durable, as defined in Holmström and Myerson (1983). In my notion of the block, agents simultaneously choose whether they want to participate in the blocking mechanism. The second difference is that, in my setup, the blocking mechanism and the settlement mechanism can coexist. This is because an agent may choose to partially participate in the settlement mechanism. None of the models of stability in the literature accommodates this possibility.

My paper is also related to a body of literature that studies the incentives of agents to participate in some centralized clearinghouses. Ashlagi and Roth (2013) studies the incentives of hospitals to partially enroll their patient-donor pairs in Kidney Exchange. Ekmekci and Yenmez (2015) studies the incentives of schools to participate in the centralized school choice clearinghouse. Sonmez and Unver (2015) proposes an incentive scheme in the Kidney Exchange. There are two main differences between my paper and this these papers, first, unlike kidney exchange, monetary transfer is possible in the CDS settlement clearinghouse, and second participation to the CDS settlement mechanism is the decision of (at least) two agents.

3 Leading Example

I illustrate how the CDS contracts work and the main theoretical contribution of the paper with the following example. The reader may omit reading this example and start from Section 4.

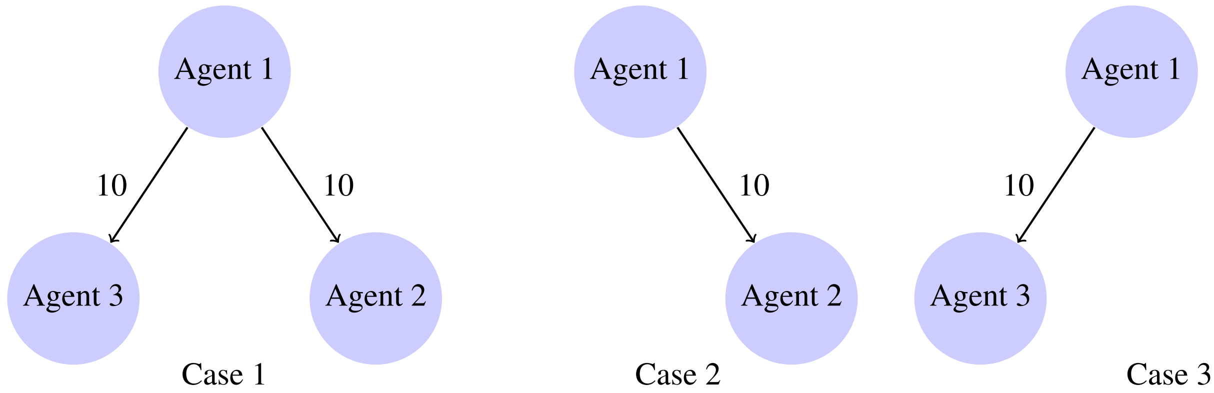

Figure 1: an arrow from agent i to j means that they have contracts and that i is a protection seller.

Leading Example: There are three agents, 1, 2, and 3. There is a bond with a face value of 100. Assume that the issuer of the bond has defaulted and that the value of the defaulted bond is $v(s)$, where $s = (s_1, s_2, s_3)$ is the agents' signal profile. Agent 1 is a protection seller and Agents 2 and 3 are protection buyers. Agents 2 and 3 each may have 10 CDS contracts with the protection seller. These homogeneous CDS contracts are on the bond. There are three possible cases (see Figure 1):

1. Agents 2 and 3 have 10 CDS contracts with Agent 1.
2. Agent 2 has 10 CDS contracts with Agent 1, and Agent 3 has no CDS contracts.
3. Agent 3 has 10 CDS contracts with Agent 1, and Agent 2 has no CDS contracts.

Denote the number of CDS contracts that agent i has in case j by n_i^j. Assume $n_i^j > 0$ if agent i is a protection buyer in case j, $n_i^j < 0$ if he is a protection seller, and $n_i^j = 0$ if he does not have any CDS contract. For example, $n_1^1 = -20$ and $n_2^1 = n_3^1 = 10$.

These contracts are settled by either physical settlement or cash settlement. In the case of physical settlement, the protection buyer hands the defaulted bond to the protection seller and, in return, receives \$100. Therefore, the protection buyer's payoff from the physical settlement of one contract is $100-v(s)$, and the protection seller's payoff from the physical settlement is $-(100-v(s))$. In the case of cash settlement, the protection seller pays the loss to the protection buyer(s) in the form of monetary transfer. Therefore, if p is a price for the defaulted bond, the protection seller pays $100 - p$ to the protection buyer to settle one CDS contract. If q_i^j is the number of agent i's contracts that are settled through cash settlement, and p_i^j is the cash settlement price, agent i's payoff is as follows:

$$u_i((n_i^j - q_i^j)(100 - v(s)) + q_i^j(100 - p_i^j)).$$

where $u_i : \mathbb{R} \to \mathbb{R}$ is agent i's utility function. Agents' signals about the value of the defaulted bond is either 0 or 1, $s_i \in \{0, 1\}$ for $i \in \{1, 2, 3\}$. Signals are independently distributed, and $s_i = 1$ with probability $\frac{1}{2}$. The value of the defaulted bond conditional on signals is as follows:

$$v(s) = 21(2s_1 + s_2 + s_3).$$

Agents 1 and 2 each possess nine defaulted bonds. Therefore, some of the contracts must be settled through cash settlement.

I describe a direct settlement mechanism. A description of a mechanism is a price and a quantity function for each agent in each network. The quantity is the number of CDS contracts that are settled by cash settlement, and the price is the cash settlement price. Let q_i^j and p_i^j denote the quantity of cash settlement and cash settlement price for agent i in network j, respectively. Consider the following settlement mechanism:

$$
\begin{aligned}
&q_1^1(s) = -6 + 4s_1 - s_2 - s_3, p_1^1(s) = 28 - 28s_1 + 8s_2 + 8s_3 + 20s_1s_2 + 20s_1s_3 + \frac{13}{4}s_2s_3 - \frac{27}{4}s_1s_2s_3 \\
&q_2^1(s) = 3 - 2s_1 + s_2, \qquad p_2^1(s) = 28 - 28s_1 - \frac{7}{4}s_2 + \frac{133}{4}s_1s_2 + 21s_3, \\
&q_3^1(s) = 3 - 2s_1 + s_3, \qquad p_3^1(s) = 28 - 28s_1 - \frac{7}{4}s_3 + \frac{133}{4}s_1s_3 + 21s_2, \\
&q_1^2(s) = -4, \qquad q_2^2(s) = 4,\ q_3^2(s) = 0,\ p_1^2(s) = p_2^2(s) = 42, \\
&q_1^3(s) = -3.5, \qquad q_3^3(s) = 3.5,\ q_2^3(s) = 42,\ p_1^3(s) = p_3^3(s) = 42.
\end{aligned}
$$

These prices and quantities guarantee ex-post incentive compatibility. Moreover, the following holds:

$$\forall \quad s \in \{0,1\}^3 \text{ and } \forall j \in \{1,2,3\} : \sum_{i=1}^{3} q_i^j(s) = 0, \tag{1}$$

$$\forall \quad s \in \{0,1\}^3 \text{ and } \forall j \in \{1,2,3\} : \sum_{j=1}^{3} q_i^j(s)(100 - p_i^j(s)) = 0. \tag{2}$$

Equation (1) is a market-clearing condition. Note that $q_i^j(100 - p_i^j)$ is the cash transfer that agent i receives in network j; therefore, equation (2) is the budget balanced condition. In addition to these properties, if $U_i^j(s)$ is agent i's payoff from the settlement mechanism in case j, the following holds:

$$E_s[q_i^j(s)(100 - p_i^j(s))] = E_s[n_i^j(100 - v(s))].$$

This condition is called unbiased pricing. This means, from an ex-ante point of view, that all contracts are settled by physical settlement or by cash settlement, with the price equal to the value of the defaulted bond.

I study agents' incentives to participate in the settlement mechanism when an arbitrary group of agents can form coalitions and settle some of their contracts with an arbitrary blocking mechanism. As an illustration, consider the settlement mechanism that I described above. I consider a block by Agents 1 and 3 (see Figure 2). In this blocking mechanism, seven contracts are settled by physical settlement, and three contracts are settled by cash settlement. The cash settlement prices for Agents 1 and 3 are 30 and 38.5, respectively. The following inequalities hold (see the appendix for the calculations.):

$$\begin{aligned} &E_{s_2}[U_1^1(0, s_2, 0)] \leq E_{s_2}[U_1^e(0, s_2, 0)], E_{s_2}[U_1^1(1, s_2, 0)] \geq E_{s_3}[U_1^e(1, s_2, 0)], \\ &E_{s_2}[U_3^1(0, s_2, 0)] \leq E_{s_2}[U_3^e(0, s_2, 0)], E_{s_2}[U_3^1(0, s_2, 1)] \geq E_{s_2}[U_3^e(0, s_2, 1)]. \end{aligned} \tag{3}$$

Therefore, there exists a Bayesian Nash Equilibrium in which Agents 1 and 3 choose the exit option when their signals are 0. In this blocking, when Agents 1 and 3 visit the blocking mechanism, i.e., when $(s_1, s_3) = (0, 0)$, the blocking designer's payoff is $3(38.5 - 30)$, which is positive. Given this model of agents' participation, in this paper, I answer the following two questions. First, which settlement mechanism ensures that all agents will participate with all of their contracts and is unbiased and budget balanced? Second, if we allow agents to settle a number of their contracts with some blocking mechanisms and take into account agents' payoff from blocking mechanisms, which settlement mechanism is unbiased and budget balanced? As I will show, the answer to both questions is a mechanism where the designer sets constant price and quantity.

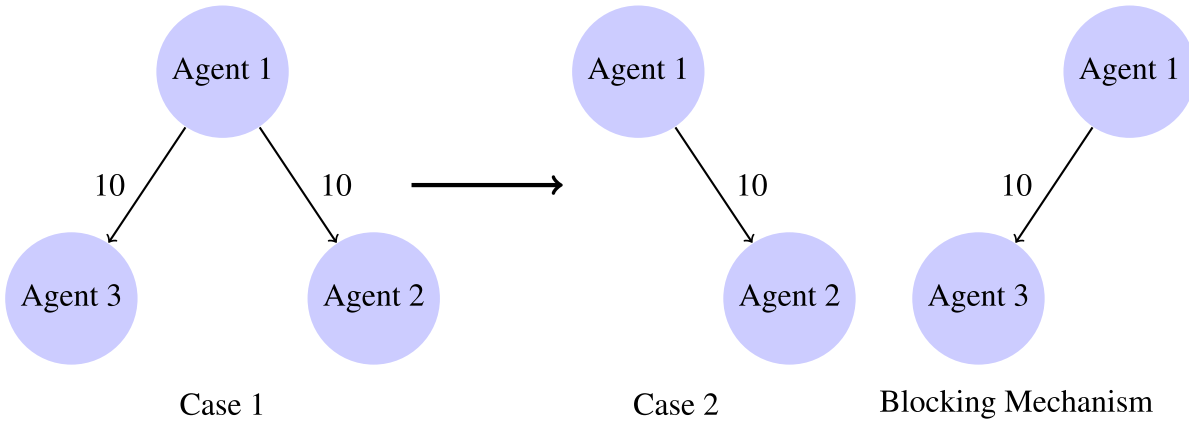

Figure 2

4 Model

Without loss of generality, I assume the face value of the **defaulted bond** is 100. Each **CDS contract** has a **protection buyer** and a **protection seller**. In the case of a default, the protection buyer should be compensated for the loss on the reference asset (bond) by the protection seller. These CDS contracts are homogeneous, and each corresponds to one bond. I assume that the default has happened, and I consider the contract settlement problem. Let K be the set of all agents. These agents may have CDS contracts on the bond between each other. A **contract matrix** specifies the number of contracts of a pair of agents. In a contract matrix $N = [n_{i,j}]$, agents $i, j \in K$ have net $n_{i,j}$ contracts. Assume $n_{i,j} > 0$ if j is a protection seller and i is a protection buyer, $n_{i,j} = 0$ if they do not have any CDS contracts, and $n_{i,j} < 0$ if i is the protection buyer.[6] Throughout this paper, I use the words **network** and **contract matrix** interchangeably. Each agent has a number of defaulted bonds; assume agent i has $b_i \geq 0$ defaulted bonds. Each agent has a private signal about the value of the defaulted bond. Agent i's signal is drawn from $S_i = [0, 1]$. Given $s \in [0, 1]^k$, a

[6]Note that $n_{i,j} + n_{j,i} = 0$.

profile of agents' signals, the expected value of the defaulted bond is $v(s)$. I assume $v(s)$ is non-decreasing and continuous in agents' signals. If $A \subseteq K$ is a subset of agents, set $S_A = \prod_{i \in A} S_i$. Given $B \subseteq A \subseteq K$ and $s \in S_A$, let $\pi_B(s) \in S_B$ be the projection of s on its B elements. For economy of exposition, I use the notation s_{-i} and $s_{-i,j}$ for $\pi_{K \setminus \{i\}}(s)$ and $\pi_{K \setminus \{i,j\}}(s)$ respectively.

CDS contracts are settled by either **physical settlement** or by **cash settlement**. In the case of physical settlement, the protection buyer hands in the defaulted bond to the protection seller and, in return, receives 100. Therefore, the protection buyer's payoff from the physical settlement of one contract is $100 - v(s)$, and the protection seller's payoff from the physical settlement is $-(100 - v(s)$. In the case of cash settlement, the protection seller pays the loss to the protection buyers. Therefore, if p_i is the price of the defaulted bond, and q_i^j is the number of agent i's contracts that are settled through cash settlement, agent's payoff at signal profile $s \in S_K$ is as follows:[7]

$$u_i(b_i v(s) + (n_i - q_i)(100 - v(s)) + q_i(100 - p_i)).$$

where $u_i : \mathbb{R} \to \mathbb{R}$ is agent i's utility function. The utility function, $u_i(.)$, is strictly increasing and is normalized such that $u_i(0) = 0$. Note that one can rewrite the payoff of agent i as follows:

$$u_i(b_i v(s) + n_i(100 - v(s)) + q_i(v(s) - p_i)).$$

where the first term inside $u_i(.)$ is his payoff if all of the contracts are physically settled or if the price is equal to the value of the defaulted bond. The second term can be thought of as the bias. In general, a settlement mechanism is a reallocation of the defaulted bonds and monetary transfer.

[7] Agent i gives/receives $n_i - q_i$ of his defaulted bonds and receives/pays the face value of the bond; in addition, he receives/pays his loss for the rest of the contracts.

Note that combinations of physical settlement and cash settlement can generate any allocation of the defaulted bonds and monetary transfer.

Let $K(N)$ be the set of agents who have some CDS contracts in network N and n_i be the net number of contracts of agent $i \in K$, formally,

$$K(N) = \{i \in K | n_{i,j} \neq 0 \text{ for some } j \in K\} \text{ and } n_i = \sum_{j \in K(N)} n_{i,j}.$$

In this environment, a direct **settlement mechanism** takes the network and the profile of reported signals as inputs and returns a cash settlement quantity and a cash settlement price for each agent. A direct settlement mechanism consists of functions $q_i^N : S_K \to \mathbb{R}$ and $p_i^N : S_K \to \mathbb{R}$ for all agents $i \in K$ and network N. The cash settlement quantity is q_i^N, and p_i^N is the cash settlement price for agent i in network N. Let $p^N = (p_i^N(.))_{i \in K}$ and $q^N = (q_i^N(.))_{i \in K}$ be the profile of price and quantity functions when the network is N and $P = (p^N)$ and $Q = (q^N)$ be the price and quantity profiles. Note that I allow agents to have different cash settlement prices; in other words, I am not restricting the case to $p_i^N = p_j^N$ for all $i, j \in K$. Therefore, any reallocation of money and the defaulted bonds can be generated by cash settlement and physical settlement. The number of defaulted bonds that are used for physical settlement must clear itself, formally, for all networks N and $s \in S_K$:

$$\sum_{i \in K} (n_i - q_i^N(s)) = 0.$$

This is equivalent to $\sum_{i \in K} q_i^N(s) = 0$. This mechanism is ex-post incentive compatible if, for all networks N, $i \in K$, and $s = (s_i, s_{-i})$ & $s' = (s'_i, s_{-i}) \in S_K$, the following holds:

$$u_i(b_i v(s) + (n_i - q_i^N(s))(100 - v(s)) + q_i^N(s)(100 - p_i^N(s))) \geq$$
$$u_i(b_i v(s) + (n_i - q_i^N(s))(100 - v(s)) + q_i^N(s')(100 - p_i^N(s'))). \quad (4)$$

This means that agent i with private information s_i should not find it profitable to misreport his signal as s'_i, when all other agents' signal profiles are s_{-i}. Since the utility function is increasing, inequality (5) is equivalent to

$$(n_i - q_i^N(s))(100 - v(s)) + q_i^N(s)(100 - p_i^N(s)) \geq$$
$$(n_i - q_i^N(s))(100 - v(s)) + q_i^N(s')(100 - p_i^N(s')).$$

I use the notation (P, Q, U) for a settlement mechanism with price, quantity, and payoff functions $p_i^N(.), q_i^N(.)$, and $U_i^N(.)$ for all networks N and agents $i \in K$. Agent i's payoff in the network N when all agents are reporting their signals truthfully is as follows:

$$U_i^N(s) = u_i(b_i v(s) + n_i(100 - v(s)) + q_i^N(s)(v(s)) - p_i^N(s))).$$

For economy of exposition, I define $\Lambda_i^N(s) = n_i(100 - v(s)) + q_i^N(s)(v(s)) - p_i^N(s))$ to be the **risk-neutral payoff** of agent i from settling the CDS contracts when the network is N; therefore, $U_i^N(s) = u_i(b_i v(s) + \Lambda_i^N(s))$.[8] Note that the cash settlement part of risk-neutral payoff, $q_i^N(s)(v(s)) - p_i^N(s))$, is a monetary transfer to the agent. The mechanism is ex-post **budget balanced** if, for all networks, N and $s \in S_K$, the equality $\sum_{i \in K} q_i^N(s)(100 - p_i^N(s)) = 0$ holds for all signal profiles s.

[8] I am not restricting attention to risk-neutral agents.

Because n_i's sum up to zero, this is equivalent to $\sum_{i\in K} \Lambda_i^N(s) = 0$ for all s. It is ex-post weakly budget balanced if, for all networks, N and $s \in S_K$, the inequality $\sum_{i\in K} \Lambda_i^N(s) \leq 0$ holds. It is ex-ante budget balanced if $E_\rho[\sum_{i\in K} \Lambda_i^N(s)] = 0$ for all networks N and signal profiles s. I define ex-ante weakly budget-balanced mechanisms similarly. I restrict attention to ex-post incentive compatible and ex-ante weakly budget-balanced settlement mechanisms. A mechanism has no short sell if $q_i(s) \leq n_i - b_i$ for all $i \in K$ and $s \in S_K$.

Agents' **prior** about the signals is denoted by μ; the probability of observing signal profile s is $\mu(s)$. The designer does not know μ, but she holds a prior ρ about μ. Let κ be the support of the designer's belief. Let ρ be the designer's prior about the agents' prior, that is, ρ is a probability distribution over agents' priors, μ. I use E_ρ to refer to the expected value symbol, given the designer's information. I maintain the following assumption throughout the paper.

Assumption 4.1. ***Full Rank Belief:*** *If function $x : S \to \mathbb{R}$ satisfies $E_\mu[x(s)] = 0$ for almost all μ in the support of the designer's belief, then $x(s) = 0$ for all $s \in S$.*

This assumption means that the designer is sufficiently unaware of the agents' prior. This assumption is violated if, for example, the designer knew the mean of the agents' signals. However, this assumption would be satisfied if the designer believes that the agents' prior is a small perturbation of some distribution.

An agent is short selling if he ends up with a net negative number of defaulted bonds after the settlement, that is: $b_i < n_i - q_i$. The problem is trivial if the market facilitates short selling, simply set $q_i^N(s) = 0$ and settle all contracts with physical settlement. However, short selling in this market is generally difficult or impossible. Therefore, any settlement should satisfy the **no short sell** constraint, which is $q_i \geq n_i - b_i$ for all $i \in K$.

5 Desired Properties

5.1 Participation

As of 2009 all CDS contracts are pegged to the result of the centralized CDS settlement mechanism. ISDA has argued that such policy ensures certainty, consistency, enhanced transparency, and liquidity. Even though CDS contracts are hardwired to the outcome of the settlement mechanism, if both parties of a CDS contract agree they are allowed to side settle their contracts by a concerted settlement procedure. Hence, we need a model to understand how agents participate in the settlement mechanism.[9]

An agent who does not have any CDS contracts is not obligated to participate in the settlement mechanism; he participates if there is a positive payoff. This motivates the following definition: A mechanism is **ex-post individually rational** for agents without contracts if, for all networks N, all signal profiles $s \in S_K$, all agents $i \in K$ that satisfy $n_{i,j} = 0 \; \forall j \in K$, the inequality $\Lambda_i^N(s) \geq 0$ holds. It is **interim individually rational** if the inequality $E_{s_{-i}}[U_i^N(s)] \geq E_{s_{-i}}[u_i^N(b_i v(s))]$ holds.

I formally model the participation decision of agents who have CDS contracts. In standard mechanism design, agents can choose whether to participate in the mechanism. They participate when they have a non-negative payoff from participating in the mechanism. Participation in this environment is different for an important reason. Agents with CDS contracts are required to participate by default; however, if both parties of a CDS contract agree, they can settle some of their contracts with another mechanism. In this environment, agents' outside options are no longer exogenous; rather, they depend on their signals as well as other agents' signals. In other words, if an agent agrees to settle a CDS contract though another mechanism, it reveals information about

[9] See http://www.isda.org/companies/auctionhardwiring/auctionhardwiring.html

his own private signal. I do not assume the number of contracts that a pair of agents has is private information; rather, agents are legally allowed to not bring a number of their contracts to the settlement mechanism if all parties of these contracts agree.

All the results of the paper are proven when I only consider side settlements by a pair of agents. However, to provide a more general model of participation, I consider side settlements by mulitple pairs of agents. Due to bilateral decisions that groups of agents may make prior to participating in the mechanism about the number of contracts, the designer may face contract matrices different from the original network of contracts. When the contract matrix is N, if a group of agents choose to settle some of their contracts outside of the settlement mechanism, the designer faces a new contract matrix, namely M. In this case, M is a **reduction** of N. Formally, $M = [m_{i,j}]$ is a reduction of $N = [n_{i,j}]$ if, for all $i, j \in K$, the inequality $|m_{i,j}| \leq |n_{i,j}|$ holds. I use the notation $M \prec N$, if M is a reduction of N. Let A be the set of all agents who choose to settle some of their contracts outside of the settlement mechanism. Note that $A = K(M - N)$ where $M - N$ is a contract matrix in which agents i and j have $m_{i,j} - n_{i.j}$ contracts.

A blocking mechanism can be viewed as a settlement mechanism when the set of agents is A and the network of contracts is $N - M$. The main differences are that it does not have to be budget balanced and does not have to clear the number of defaulted bonds used. However, the blocking mechanism must provide the designer with an expected positive surplus. A blocking mechanism has an important role; it is the process whereby all contracts that were not brought to the settlement mechanism are settled. I use the notation (P', Q', U') for the blocking mechanism. Let U_i^e be the payoff of an agent from joining the blocking mechanism, i.e.,

$$U_i^e(s) = u_i[b_i v_i(s) + \Lambda_i'(s) + \Lambda_i^M(s)] \tag{5}$$

where $\Lambda_i'(s) = (n_i - m_i - q_i'(s))v(s) + q_i'(s)(100 - p_i'(s))$. I present two models for the blocking: (i) complete information case and (ii) incomplete information case.

5.1.1 Complete Information Case

Agents in A for a subset of their types block the settlement mechanism and reduce the network from N to M if there exists a blocking mechanism (P', Q', U') and prescribed non-zero measure subset of types $S_i' \subseteq S_i$ for agents in A, such that the following holds:

1. For all $i \in A$ and $s_A \in \prod_{i \in A} S_i$, the following inequality holds:

$$E_{s_{-A}}[U_i^N(s_A, s_{-A}))] \leq E_{s_{-A}}[U_i^e(s_A, s_{-A})]. \tag{6}$$

   where U_i^e is agent i's payoff from joining the blocking mechanism; see equation (5). Agents in A join the coalition when their types are in the prescribed subset of types. Inequality (6) means that, if all signals of agents in A are in the prescribed sets, then the expected payoff of all agents $i \in A$ from the settlement mechanism with network N is not larger than an agent's total payoff from the blocking mechanism and the payoff from the settlement mechanism with network M. This gives agent i an incentive to join the coalition when all blocking agents' signals are in the prescribed sets.

2. For all $s_A \in S_K$ such that $\pi_{A\setminus\{i\}}(s_A) \in \prod_{j\in A\setminus\{i\}} S'_j$ and $\pi_{\{i\}}(s) \in S_i \setminus S'_i$, the following inequality holds:

$$E_{s_{-A}}[U_i^N(s_A, s_{-A}))] \geq E_{s_{-A}}[U_i^e(s_A, s_{-A})]. \tag{7}$$

   Inequality (7) means that if agent i's signal is not in S'_i, and the signal of all other agents in A are in the prescribed sets, then agent i's expected payoff from the settlement mechanism with network N is not smaller than the agent's total payoff from the blocking mechanism and his payoff from the settlement mechanism with network M.

3. The blocking designer has a positive payoff. Formally, the following inequality must hold:

$$E[-\sum_{i\in A} \Lambda'_i(s)|\pi_A(s) \in S_A] > 0. \tag{8}$$

Inequalities (6) and (7) mean that agents in A, for a subset of their private signals, may form a coalition and settle some of their contracts with the blocking mechanism. Agents in the coalition, A, choose between (P^N, Q^N, U^N) and (P^M, Q^M, U^M) plus the blocking mechanism. Consider a game in which agents in A choose whether to exit the mechanism. The block is formed only when all agents choose to exit. If agents choose the strategy of exiting only if the type is in the prescribed set, then these inequalities guarantee that agent $i \in A$, upon learning the types of all agents in $A \setminus \{i\}$, would not regret his participation decision. The mechanism is **ex-post unraveled** if blocking exists.

To understand inequality (8), think of the blocking designer as an agent. Note that, in general, the blocking mechanism does not have to balance the budget or clear the number of defaulted bonds that are used for physical settlement. Because there may be a surplus or a deficit in (i) monetary transfer and (ii) the number of defaulted bonds, the blocking designer's payoff may not be zero. Inequality (8) means that the blocking designer's expected payoff, conditional on the event that the block is formed, must be positive. The first term that appears in the summation is the blocking designer's payoff from defaulted bonds, and the second term is his payoff from the monetary transfer. The strict inequality (8) guarantees that no mechanism is blocked by the null mechanism.[10]

A settlement mechanism (P, Q, U) is **ex-post unravel-proof** if, for any pair of contract matrices M and N and subset of agents $A \subseteq K$, where agents in A reduced N to M, agents in A cannot form a block for a subset of their types. A settlement mechanism is **weakly ex-post unravel-proof** if there is no pair of agents who could form a block. One can strengthen the notion of unravel proof to consider blocks by more than two agents. However, the results of the paper are correct in either case.

The following proposition ensures that the set of unravel-proof mechanisms is not empty.

Proposition 5.1. *A settlement mechanism (P, Q, U) that settles all contracts with cash settlement with a constant price, i.e., $p_i^N = p_i$ and $q_i^N = n_i$ is ex-post unravel-proof. Moreover, if agents are risk neutral and the following inequality holds, then there is no ex-post unraveling in which agents in A reduce the network from N to M.*

$$\sum_{i \in A} E_{s_{-A}}[\Lambda_i^N(s_A, s_{-A})] \geq \sum_{i \in A} E_{s_{-A}}[\Lambda_i^M(s_A, s_{-A})] \text{ for almost all } s_A \in S_A \tag{9}$$

The inequality means that the sum of agents' risk-neutral expected payoffs that are in A is greater when the network is N compared to that of network M.

[10] A mechanism where all the quantities are equal to zero.

Proof. I prove the second part of the proposition first. Assume that there is a blocking mechanism (P', Q', U'). Note that, when agents are risk neutral, inequality (6) is equivalent to:

$$\forall i \in A \text{ and } s_A \in \prod_{i \in A} S'_i,\ E_{s_{-A}}[\Lambda_i^N(s_A, s_{-A})] \leq E_{s_{-A}}[\Lambda_i^M(s_A, s_{-A}) + \Lambda'_i(s_A, s_{-A})]. \tag{10}$$

If one adds this for all $i \in A$ and takes expectations over all $s_A \in \prod S'_i$, then the following holds:

$$\sum_{i \in A} E[\Lambda_i^N(s) | \pi_A(s) \in \prod_{i \in A} S'_i] \leq E[\Lambda_i^M(s) + \Lambda'_i(s) | \pi_A(s) \in \prod_{i \in A} S'_i] \tag{11}$$

Inequalities (11) and (8) imply the following inequality:

$$\sum_{i \in A} E[\Lambda_i^N(s) | \pi_A(s) \in \prod_{i \in A} S'_i] < E[\Lambda_i^M(s) | \pi_A(s) \in \prod_{i \in A} S'_i] \tag{12}$$

Therefore, for some $s \in S_K$, $\sum_{i \in A} \Lambda_i^N(s) < \sum_{i \in A} \Lambda_i^M(s)$. This contradicts the assumption in the proposition.

I prove the first part of the proposition. When agents are not risk neutral, then inequality (6) does not in general imply (16). This is because agents may share the risk through the blocking mechanism. However, when the mechanism sets a constant price, the outcome of the mechanism becomes risk-free. Because agents are weakly risk averse, equation (6) implies equation (16). The rest of the argument follows as presented above. □

The contribution of this paper is that I consider the possibility that the outside option of an agent depends on the set of other agents who exercise their outside option. This is because the agents who choose to 'exit' may decide to band together and settle some of their contracts among themselves through a different mechanism. Such a possibility was first considered in the literature on cooperative games and culminated in the notion of the core. The notion of unraveling presented in this paper is related to the block in matching theory and the block in cooperative game theory. The difference between blocking in matching theory and the notion of unraveling is that, in my setup, the network is predetermined and only price and quantity of cash settlement are chosen through a mechanism. Unravel-proofness is a property of a mechanism that is defined over networks, but stability is defined over a possible match. Unlike similar concepts in corporate game theory, the notion of unravel-proofness can be naturally extended to environments with incomplete information. A generalization of the unravel-proofness notion to environments with incomplete information is presented in the following section.

5.1.2 Incomplete Information Case

I extend the blocking mechanism definition to environments in which agents who participate in a block do not know each other's signals but share a prior. When agents make decisions about whether to join the blocking mechanism, they update their belief upon observing other agents' decisions. An agent's choice to participate in a blocking mechanism reveals information about his private signal. Other agents take this into account when making their decisions. The notion of **interim blocking** is defined as follows:

For all $i \in A$, subsets $\emptyset \neq S'_i \subseteq S_i$ are called the prescribed sets. Let event E^i be defined as follows:

$$E^i = \{s \in S_K | \pi_{A\setminus\{i\}}(s) \in \prod_{j\in A\setminus\{i\}} S'_j \, , \pi_{\{i\}}(s) \in S_i \setminus S'_i\}.$$

Define event E as

$$E = \{s \in S_K | \pi_A(s) \in \prod_{j \in A} S'_j\}.$$

Note that E^i is the event that the private signals of all agents in A, except for agent i, are in the prescribed sets. Event E is the event that the private signals of all agents in A are in the prescribed sets. The inequalities in the blocking mechanism definition, inequalities (6), (7), and (8), change to the following inequalities:

$$E_{s_{-i}}[U_i^N(s)|E] \leq E_{s_{-i}}[U_i^e|E], \tag{13}$$

$$E_{s_{-i}}[U_i^N(s)|E^i] \geq E_{s_{-i}}[u_i^e|E^i]. \tag{14}$$

$$E[-\sum_{i \in A} \Lambda'_i(s))|E] > 0. \tag{15}$$

To interpret inequalities (13) and (14), imagine a game whose players are agents in A. These agents, after observing their private signals, choose whether to participate in the blocking mechanism. If all of these agents decide to participate in the blocking mechanism, their payoff is that of the blocking mechanism plus that of the settlement mechanism when the network is M. If some decide not to participate in the blocking mechanism, their payoff is only that of the settlement mechanism when the network is N. The mechanism is unraveled if this game has a Bayesian Nash Equilibrium in which agents in A, for a subset of their types, choose the blocking mechanism.

With the new definition of a block, unravel-proofness is naturally redefined. A mechanism is **interim unravel-proofness** if there is no pair of agents who could form an interim block. One can strengthen the notion of unravel-proofness to consider blocks by more than two agents. However, the results of the paper are correct in either case.

Proposition 5.2. *A settlement mechanism* (P, Q, U) *that settles all contracts with cash settlement with a constant price, i.e.,* $p_i^N = p_i$ *and* $q_i^N = n_i$ *is interim unravel-proof. Moreover, if agents are risk neutral and the following inequality holds, then there is no interim unraveling in which agents in* A *reduce the network from* N *to* M.

$$\sum_{i \in A} E_{s_{-A}}[\Lambda_i^N(s_A, s_{-A})] \geq \sum_{i \in A} E_{s_{-A}} \Lambda_i^M(s_A, s_{-A})] \textit{ for almost all } s_A \in S_A \tag{16}$$

Proof. The proof is an adaptation of the proof of proposition 5.1 and, hence, is omitted. □

5.2 Unbiased Mechanisms

As I mentioned in Section 2, several authors have criticized the current settlement mechanism for underpricing the underlying bond. The current mechanism in use sets a biased price, which results in a difference between physical settlement and cash settlement. Since at the time of contracting a CDS, it is not known if the CDS contract will be settled by cash settlement or physical settlement, the biased pricing results in uncertainty and hence efficiency loss. I look for mechanisms that overcome this issue. Due to the information rent, based on agents' private information about the value of the defaulted bond, ex-post correct pricing is not possible, unless all contracts are physically settled; I discussed in Section 1 that this is not possible. However, I consider a weaker condition; I look for mechanisms that are unbiased from the ex-ante perspective. I define unbiasedness in two ways.

5.2.1 Weakly-Unbiased

A **weakly-unbiased** mechanism is one that does not misprice the defaulted bond in expectation. Formally, mechanism (P, Q, U) is weakly unbiased if, for all networks N and agents $i \in K$:

$$E[\Lambda_i^N(s)] = [n_i(100 - v(s))]. \tag{17}$$

Equation (17) means that, from an ex-ante perspective, the agents' risk-neutral payoff from the settlement mechanism is the same as their payoff from physical settlement of all contracts or cash settlement with the price equal to the value of the defaulted bond. Note that, because both price and quantity may depend on the signal profile, this condition is not equivalent to $E[p_i^N(s)] = E[v]$.

Observation 5.1. *If* $E[\Lambda_i^N(s)] = n_i(100 - E[v])$*, then* $E_\rho[\sum_{i\in K} \Lambda_i^N(s)] = (\sum_{i\in K} n_i)(100 - E[v]) = 0$*. Therefore, a mechanism is weakly ex-ante budget balanced if it is weakly unbiased.*

5.2.2 Unbiased

If the mechanism is not strategy proof, some agents may settle some of their contracts outside of the settlement mechanism. Therefore, agents' total payoff is not only the payoff from the settlement mechanism; it should also include the payoff from the blocking mechanisms. A mechanism is **unbiased** if the agents' total payoff, including the payoff from the settlement mechanism and the blocking mechanisms, from an ax-ante perspective, is equal to the agents' payoff from physical settlement of all contracts or cash settlement with the correct price.

To formally define unbiased mechanisms, I first define the notion **participation-choice**. For some networks, a pair of agents may find it profitable to settle some of their contracts with a blocking mechanism. Using the participation model introduced in secion 5.1 , I allow agents to take these actions; a mechanism is unbiased if it is weakly unbiased, regardless of these actions. The results of the paper are valid if one considers only coalitions by pair of agents in the definiction of unbisedness. However, to provide a richer model we consider groups of agents.

Consider an ex-post incentive-compatible settlement mechanism, namely (P, Q, U), which may not be unravel-proof. Let Ω be the set of all possible networks,. We allow for several coalitions to coexist, let P_N be the set of coalitions (blockings) when the true network of contract is N. Hence, a participation-choice is a collection of sets $(P_N)_{N\in\Omega}$ whereby elements of P_N capture the sub-networks that join a coalition if the true network of contract is N. For all networks N, each element of $c \in P_N$ has a network $c_N \prec N$, a subset of agents $c_A \subseteq K(c_N)$, a set of type profiles $S_i^c \subseteq S_i$ for all $i \in c_A$, and a blocking mechanism for the c_N network, $(\hat{P}^c, \hat{Q}^c, \hat{U}^c)$. Let $c_t = \prod_{i\in c_A} S_i^c$ for all $i \in c_A$. This participation-choice should satisfy three conditions:

1. Let $A_N(s)$ be the set of all coalitions that are formed when the signal profile is $s \in S_K$. Formally,

$$A_N(s) = \{c \in A_N | \pi_{c_A}(s) \in c_t\}.$$

   It must be that $\bar{N}(s) = \sum_{c\in A_N(s)} c_N \prec N$. This means that the network that is left after coalitions are formed is a reduction of N.

2. Agent i's payoff from this participation-choice when the network is N is $u_i(\Lambda_i^{P_N}(s))$ where:

$$\Lambda_i^{P_N}(s) = \sum_{c\in A_N(s):i\in c_A} \hat{\Lambda}_i^c(s) + \Lambda_i^{N-\bar{N}(s)}(s).$$

Joining the coalitions for the prescribed types must be a Bayesian Nash Equilibrium. Formally, for all networks N and $c \in A_N$, let events E_c and E_c^i be defined as:

$$E_c = \{s \in S_K | \pi_{A_c}(s) \in \prod_{j \in K(c_N)} S_j^c\},$$

$$E_c^i = \{s \in S_K | \pi_{A_c}(s) \in \prod_{j \in K(c_N) \setminus \{i\}} S_j^c, \pi_{\{i\}}(s) \in S_i \setminus S_i^c\}.$$

For all $i \in c_A$, the following inequalities should hold:

$$E_{s_{-i}}[u_i^{P_N}(s) | E_c] \geq E_{s_{-i}}[u_i^{P_N \setminus \{c\}}(s) | E_c],$$

$$E_{s_{-i}}[u_i^{P_N}(s) | E_c^i] \leq E_{s_{-i}}[u_i^{P_N \setminus \{c\}}(s) | E_c^i].$$

3. Given a coalition $c \in A_N$, when the signal profile is $s \in S_K$, agent $i \in K(c_N)$ enters the coalition c if $\pi_{c_A}(s) \in S_i^{c_t}$. The blocking designer's expected payoff from the blocking mechanism should be positive, conditional on the event that all agents in c_A join the blocking mechanism. This is similar to inequality (15), which ensures that the blocking mechanisms are self-sustaining.

Consider a participation-choice for each agents' prior μ. The mechanism is **unbiased** if, for all networks N, all agents $i \in K$, and all participation-choices, the following holds:

$$E_\rho[\Lambda_i^{P_N}(s)] = E_\rho[n_i(100 - v(s))]. \tag{18}$$

This condition indicates that, from an ex-ante perspective, agent i's total payoff from the blocking mechanism and the settlement mechanism for all possible participation-choices is equal to the agent's payoff from physical settlement of all contracts. Note that, because I allow some contracts to be settled outside of the settlement mechanism, the notion of budget balancedness must be modified. The mechanism is *weakly budget balanced regardless of agents' participation-choice* if, for all networks N and all participation-choices, the following holds:

$$E_\rho[\sum_{i \in K(N)} \Lambda_i^{N-\bar{N}(s)}(s)] \leq 0.$$

5.3 Robustness With Respect to Network

The current mechanism that is in use takes only the net number of contracts as an input and not the details of the network of contracts. The rationale for this property is the lowering of the systemic risk and transaction costs. To elaborate, consider the following example: Agent 1 has sold a number of CDSs to Agent 2, and Agent 2 has sold the same amount of CDSs to Agent 3. If Agent 1 is unable to settle these contracts with Agent 2, it would harm all agents in the chain. To get around this issue, the clearinghouse treats Agent 2 as an agent without a CDS contract. This also results in fewer transactions and, hence, reduces the transaction cost. This motivates the property of **robust with respect to network**. Here is the formal definition: A mechanism is robust with respect to the network if, for all pairs of networks $M = [m_{i,j}]$ and $N = [n_{i,j}]$ that satisfy $\sum_{j \in K} m_{i,j} = \sum_{j \in K} n_{i,j}$ for all $i \in K$, the following equality holds: $\forall i \in K$, $q_i^N(s) = q_i^M(s)$ and $p_i^N(s) = p_j^M(s)$.

6 Results

I provided a model of participation when groups of agents can join the blocking mechanism, however, all the results of papers are proven when only a pair of agents engage in side settlement.

I look for mechanisms that satisfy a combination of properties that I have introduced in the previous section. Before presenting the characterization results, I introduce a class of mechanisms called **pooling-sepreable** mechanisms and **posted-price** mechanisms.

A pooling-seprable mechanism settles CDS contracts that are between pair of agents separately. The prices and quantities that a pooling-sepreable mechanism sets to settle CDS contracts between a pair of agents does not depend on the two agents' reported signals. Also, the risk neutral pay-off of agents must satisfy an additive property, that is, given a pair of networks N and M in which no pair of agents have contracts in both networks, $\Lambda_i^{M+N}(s) = \Lambda_i^N(s) + \Lambda_i^M(s)$, where $M + N$ is a network in which agents i and j have $n_{i,j} + m_{i,j}$ contracts. Formally, a settlement mechanism is pooling-seprable if, for each pair of agents i and j and number of contracts n, there exists a quantity and price functions $q_{i,j}^n : S_{K\setminus\{i,j\}} \to \mathbb{R}$ and $p_{i,j}^n : S_{K\setminus\{i,j\}} \to \mathbb{R}$ satisfying $q_{i,j}^n(s_{-i,j}) + q_{j,i}^{-n}(s_{-i,j}) = 0$ and $p_{i,j}^n(s_{-i,j}) = p_{j,i}^{-n}(s_{-i,j})$ for all $s_{-i,j} \in S_{K\setminus\{i,j\}}$, such that, for all $i \in K$, network N and $s \in \prod_{i\in K}(0, 1)$:

$$\Lambda_i^N(s) = \sum_{j\in K} n_{i,j}(100 - v(s)) + q_{i,j}^{n_{i,j}}(s_{-i,j})(v(s) - p_{i,j}^{n_{i,j}}(s_{-i,j})) \tag{19}$$

Note that the term inside the summation is the risk-neutral payoff of agent i in a network in which agent i has only $n_{i,j}$ contracts with agent j, and price and quantities are $p_{i,j}^{n_{i,j}}(s_{-i,j})$ and $q_{i,j}^{n_{i,j}}(s_{-i,j})$ respectively. The pooling-separable mechanisms can be characterized by the prices and quantities that the mechanism sets when only two agents have contracts with each other. Note that a pooling-seprable mechanism is weakly unbiased if and only if

$$E[q^n_{i,j}(s_{-i,j})(v(s) - p^n_{i,j}(s_{-i,j}))] = 0 \ \forall i, j \in K, \ n \in \mathbb{Z}. \tag{20}$$

Proposition 6.1. *Any settlement mechanism that is pooling-seprable and satisfies equation* (20) *is weakly unbiased and ex-post individually rational for agents without contracts. Moreover, if agents are risk neutral, the settlement mechanism is unravel-proof (both complete information and incomplete information).*

Proof. The only non-trivial property is unravel-proofness when agents are risk neutral. Note that mechanisms in this class satisfy the sufficient conditions in propositions 5.1 and 5.2. Therefore, they are unravel-proof. □

A pooling-seprable settlement mechanism (P, Q, U) is a posted price mechanism if the price and quantity functions $q^n_{i,j}(.)$ and $p^n_{i,j}(.)$ do not depend on the reported signals or agents i and j. That is, for all $s_{-i,j} \in S_{K\setminus\{i,j\}}$, $q^n_{i,j}(s_{-i,j}) = q^n$ and $p^n_{i,j}(s_{-i,j}) = p$ for some constants p and $q^n_{i,j}$. A posted price mechanism has a fair price if the price is equal to the ex-ante value of the defaulted bond, conditional on the designer's information $p = E_\rho[v]$.

Observation 6.1. *A posted-price mechanism that settles all contracts with cash settlement is unravel-proof, unbiased, and robust with respect to network.*

6.1 Characterizations

Theorem 6.2. *If (P, Q, U) is a settlement mechanism that is full information unravel-proof, weakly unbiased, and robust with respect to network, and ex-post individually rational for agents without* *contracts, then almost certainly it is a posted-price mechanism with a fair price.*

Proof. A sketch of the proof is presented; for the complete proof, see the appendix. Proof is by induction. First, I establish the case in which there are only two agents. To prove the inductive step, I use the following lemma.

Lemma 6.3. *Consider the assumptions and the setup in proposition 5.1, if the settlement mechanism is pooling-seprable when the network is M, equation* (19) *holds for network M, then almost certainly for all $s \in S_K$:*

$$\sum_{i \in A} \Lambda_i^N(s) = \sum_{i \in A} \Lambda_i^M(s).$$

This lemma connects the mechanism in a network to its reductions. Since we only consider side settlments by two agents, we use lemma 6.3 when $|A| = 2$. To explain how this lemma is applied in the proof, I provide an example.

Consider the case of three agents $\{i, j, k\}$, where k has one contract with i, and i has one contract with j (see **Figure 4**); call this network of contracts N. Let M_1 be a network where agent k has one contract with agent i and M_2 be a network where agent i has one contract with agent j (see **Figure 4**). Because the result is true for two agents, I can apply the lemma 6.3 for two pairs of networks N & M_1 and N & M_2. Also, the mechanism is budget balanced when the network is N, hence I have:

$$\Lambda_i^N(s) + \Lambda_j^N(s) = \Lambda_i^{M_1}(s), \quad \Lambda_i^N(s) + \Lambda_k^N(s) = \Lambda_i^{M_2}(s) \text{ and } \Lambda_i^N(s) + \Lambda_j^N(s) + \Lambda_k^N(s) = 0 \qquad (21)$$

Therefore,

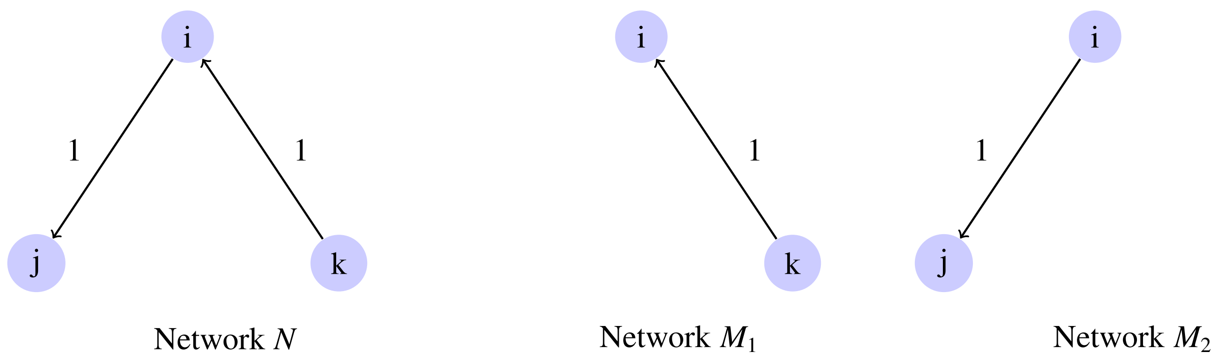

Figure 4

$$\Lambda_i^N(s) = \Lambda_i^{M_1}(s) + \Lambda_i^{M_2}(s), \quad \Lambda_j^N(s) = -\Lambda_i^{M_2}(s) \text{ and } \Lambda_k^N(s) = -\Lambda_i^{M_1}(s).$$

This proves the inductive step for this case.

I provide a sketch of the proof of lemma 6.3. Note that $\sum_{i\in A} n_i = \sum_{i\in A} m_i$, weakly-unbiasedness implies:

$$\sum_{i\in A} E_\rho[\Lambda_i^N(s)] = \sum_{i\in A} E_\rho[\Lambda_i^M(s)]$$

If the lemma does not not hold, then for some $s^* \in S$,

$$\sum_{i\in A} \Lambda_i^N(s^*) < \sum_{i\in A} \Lambda_i^M(s^*). \tag{22}$$

Inequality (22) implies that there exists $T_i \in \mathbb{R}$, such that

$$\sum_{i\in A} T_i < 0 \text{ and for all } i \in A\ \Lambda_i^N(s^*) = \Lambda_i^M(s^*) + T_i. \tag{23}$$

I construct a blocking for A. Because the mechanism is a posted price when the network is M,

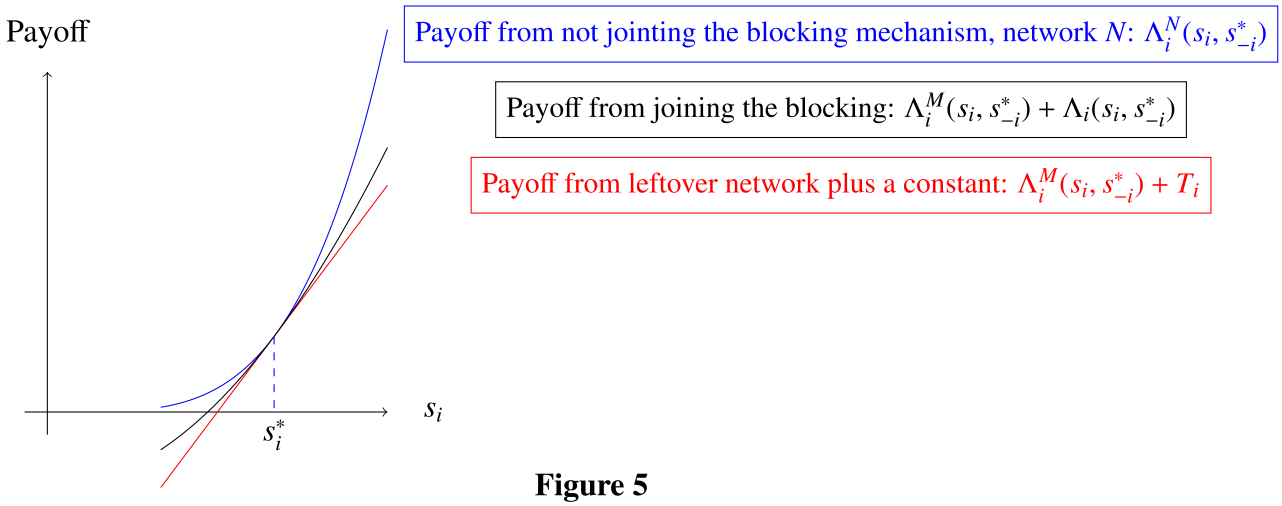

Figure 5

$\Lambda_i^N(s^*) = \Lambda_i^M(s^*) + T_i$ is linear. Convexity of $\Lambda_i^N(s_i, s_{-i}^*)$ follows from incentive compatibility of the mechanism. Because (23) holds, $\Lambda_i^N(., s_{-i}^*)$ lies above $\Lambda_i^M(., s_{-i}^*) + T_i$ (see **Figure 5**). In the candidate blocking, we want agent i to join the blocking mechanism only when his signal is s_i^*. (Here the set of type is a measure zero set; in the proof, I provide a blocking with a positive measure of types). Note that the candidate blocking mechanism must be ex-post incentive compatible. Therefore, all I need to find is a blocking mechanism with payoff Λ_i such that $\Lambda_i^M(., s_{-i}^*) + \Lambda_i(., s_{-i}^*)$ is convex and lies weakly below $\Lambda_i^N(., s_{-i}^*)$ and equal to $\Lambda_i^N(., s_{-i}^*)$ at s_i^*. Intuitively, this can be done by considering a convex function that lies between $\Lambda_i^N(., s_{-i}^*)$ and $\Lambda_i^M(., s_{-i}^*) + T_i$. The fact that $\sum_{i \in A} T_i < 0$ guarantees that the blocking mechanism satisfies equation (8). □

The unravel-proofness property in Theorem 1 is the full information notion. The same result still holds with the imperfect information model.

Theorem 6.4. *Under assumption 4.1, if (P, Q, U) is a settlement mechanism that is interim unravel-proof, robust with respect to network, weakly-unbiased, and interim individually rational for agents* *without contracts, then almost certainly it is a posted-price mechanism with a fair price.*

Proof. See the appendix for the proof. □

The motivation for unravel-proofness is that participation in the mechanism ensures liquidity and transparency of the settlement procedure. However, this property can be replaced with a stronger version of the weak-unbiasedness property, namely, the unbiasedness property.

Theorem 6.5. *Under assumption 4.1, if (P, Q, U) is a settlement mechanism that is robust with respect to the network, unbiased, weakly budget balanced regardless of agents' participation choice, and interim individually rational for agents without contracts, then almost certainly it is a posted-price mechanism with a fair price.*

Proof. See the appendix for the proof. □

7 Discussion

7.1 Relaxing Robustness with Respect to the Network

Note that, if agents are risk neutral, the class of pooling-seprable mechanisms satisfy all of the properties that are introduced in this paper, except for robustness with respect to network. I propose another example where the mechanism is not pooling.

Example 7.1. *There are four risk-neutral agents, 1, 2, 3, and 4. Agents 1 and 2 and Agents 3 and 4 have 10 CDS contracts, whereas Agents 1 and 3 are buyers. The expected value of the defaulted* *bond is the sum of the agents' signals. Price and payoffs are as follows:*[11]

$$
\begin{aligned}
q_1(s) &= s_1 + s_2 - s_3 - s_4 + 3, \qquad \Lambda_1(s) = \frac{s_1^2}{2} + s_1(3 + s_2 - s_3 - s_4) - \frac{s_2^2}{2} + s_2(3 - s_3 + s_4) \\
q_2(s) &= -s_1 + s_2 + s_3 - s_4 - 3, \qquad \Lambda_2(s) = \frac{s_2^2}{2} - s_2(3 + s_1 - s_3 + s_4) - \frac{s_1^2}{2} - s_1(3 - s_3 - s_4) \\
q_3(s) &= s_4 + s_3 - s_1 - s_2 + 3, \qquad \Lambda_3(s) = \frac{s_3^2}{2} + s_3(3 - s_4 + s_1 + s_2) - \frac{s_4^2}{2} + s_4(3 - s_1 + s_2) \\
q_4(s) &= s_4 - s_3 + s_1 - s_2 - 3, \qquad \Lambda_4(s) = \frac{s_4^2}{2} - s_4(3 + s_3 - s_1 + s_2) - \frac{s_3^2}{2} - s_3(3 - s_1 + s_2)
\end{aligned}
$$

This mechanism is ex-post incentive compatible, as $q_i(s) = \frac{\partial \Lambda_i(s)}{\partial s_i}$. Agents 1 and 2 as well as Agents 3 and 4 do not have an incentive for side settling, as their payoffs sum to zero; see Propositions 5.1 and 5.2. Consider a network where Agents 1 and 3 are both CDS buyers and have five CDS contracts with Agents 2 and 4. Agents in this new network have the same net number of contracts as the original network. If the mechanism is robust with respect to the network, then it should set the same price and quantity. However, my results show that, if we set the same price and quantity, then we lose unravel-proofness.

7.2 Selling CDS Contracts

Agents are legally allowed to sell some of their CDS contracts prior to participating in the settlement mechanism. In this section, I explore the possibility that agents trade their contracts before the settlement mechanism, but after they learn their signals. This motivates a change in the definition of unravel-proofness to ensure that agents do not have an incentive to take the following two actions: (i) settling some of their contracts with another mechanism and (ii) selling some of their

[11] Note that a settlement mechanism can be characterized by risk-neutral payoffs and quantities.

contracts. I change the definition of network reduction as follows:

Formally, M is a reduction of N if there exists a sequence of networks $(M^t = [m^t_{i,j}]_{i,j\in K})^{t=\tau}_{t=0}$ such that $M^0 = N$, M^1, M^2,...,$M^\tau = M$ and given M^t for $0 \le t \le \tau - 1$, M^{t+1} satisfies one of the following two cases:

1. Contract matrix $M^{t+1} \neq M^t$ is such that, for all $i, j \in K$, $|m^t_{i,j}| \ge |m^{t+1}_{i,j}|$ and if $m^t_{i,j} \neq 0$, then $m^t_{i,j}$ and $m^{t+1}_{i,j}$ have the same sign. In this case, the set of agents who took this action is all agents $i \in K$ such that $m^t_{i,j} \neq m^{t+1}_{i,j}$ for some agent $j \in K$.

2. Contract matrix M^{t+1} is constructed from M^t by removing some contracts that are between two agents $i \in K$ and $j_1 \in K$ and adding these contracts to contracts between i and another agent, $j_2 \in K$. It must be that

$$m^{t+1}_{i,j_1} + m^{t+1}_{i,j_2} = m^t_{i,j_1} + m^t_{i,j_2}.$$

   No other contract is removed or added. This is the case where agent j_1 buys some of the contracts that are between j_2 and i from agent j_2. In this case, agents i, j_1 and j_2 took an action.

The definition of unraveling is similar to the previous case. Let A be the set of agents who (potentially) take these actions. These agents block and reduce the network from N to M if there exists a blocking mechanism that satisfies the inequalities in the definition of blocking.[12] The blocking mechanism has two roles: (i) it is where all contracts that were not brought to the settlement mechanism are settled and (ii) it is where agents are compensated for selling their CDS contracts. With

[12]For the case of complete information, these inequalities are inequality (6), (7), and (8), and, for the case of incomplete information, they are (13), (14), and (15).

this modification, unbiasedness and unravel-proofness are naturally redefined.

Theorem 7.1. *Given the new definitions, the results of Theorems 6.2, 6.4, and 6.5 still remain true if one drops the property of robustness, with respect to the network, from the list of properties.*

Proof. See the appendix for the proof. □

7.2.1 Ex-ante Uniform Price

I propose a generalization for the unbiased and weakly-unbiased properties. A mechanism is weakly **ex-ante uniform price** if, from an ex-ante perspective, CDS contracts have the same payoff. Without this property, different contracts will be settled differently, even from an ex-ante perspective.

Here is a formal definition: A mechanism satisfies weak ex-ante uniform price property if, for some price function $p : S \to \mathbb{R}$, all networks N, and all agents $i \in K$:

$$E_\rho[u_i^N(s)] = E_\rho[n_i(100 - p(s))].$$

Ex-ante uniform price property is defined naturally. Given a participation-choice for agents' prior, agent i's ex-ante payoff is defined as $E[u_i^{P_N}(s)]$ for all $s \in S_K$. The mechanism is unbiased if, for all agents $i \in K$, all networks N and all participation plans and some price function $p : S \to \mathbb{R}$, the following holds:

$$E_\rho[u_i^{P_N}(s_{K(N)})] = E_\rho[n_i(100 - p(s))]. \tag{24}$$

Proposition 7.2. *Properties of unbiasedness and weakly-unbiasedness can be replaced with ex-ante uniform price and weakly ex-ante uniform price, respectively, in Theorems 6.2, 6.4, 6.5, and 7.1.*

Proof. See the appendix for the proof. □

8 Conclusion

I took a mechanism design approach to address the design of a CDS contract-settlement problem. The design would have been trivial if short selling were possible because one could settle all contracts physically. Physical settlement of all contracts would result in an ex-post unbiased-settlement procedure. Inability to short sell makes the design problem non-trivial. An important issue considered in this paper, neglected by other authors, is participation. Any settlement mechanism should take into account this issue when making predictions regarding the settlement price and agents' payoffs. The main result of the paper is that any settlement mechanism that is robust with respect to the network and, from a designer's ex-ante standpoint, sets an unbiased price is a posted-price mechanism. This mechanism sets a price equal to the expected value of the defaulted bond, conditional on designer's information. Moreover, this mechanism is almost surely the unique mechanism that satisfies enumerated properties in the introduction. In addition, it guarantees participation by all agents. The main result of the paper should be interpreted as an impossibility result, which means that there is no non-trivial settlement mechanism that can make this market efficient if naked CDSs are present.

The tool that is developed in this paper is a new approach to extending the notion of the core to the case of incomplete information. I considered the 'exit game' before joining the blocking mechanism. This model can be applied to mechanism design problems in which (i) agents are allowed to get together and use another mechanism for their purpose and (ii) the decision to leave the mechanism occurs before formation of the grand mechanism. An example of such environments is dark markets in the stock exchange.

9 Appendix

9.1 Proof of Inequalities in the Leading Example:

One can rewrite agent i's payoff that he receives when q_i contracts are settled physically with price p_i as $u_i(n_i(100 - v(s)) + q_i(v(s) - p_i))$. I have:

$$E[U_1^1(0, s_2, 0)] = \frac{1}{2}(E[U_1(0,0,0) + E[U_1(0,1,0)]) = -20(100 - E[v|s_1 = 0, s_3 = 0]) + \frac{1}{2}(168 + 105) =$$

$$-20(100 - E[v|s_1 = 0, s_3 = 0]) + 136.5$$

$$E[U_1^1(1, s_2, 0)] = \frac{1}{2}(E[u_1(1,0,0) + u_1(1,1,0)] = -20(100 - E[v|s_1 = 1, s_3 = 0]) + \frac{1}{2}(-84 - 105) =$$

$$-20(100 - E[v|s_1 = 1, s_3 = 0]) - 94.5$$

$$E[U_1^e(0, s_2, 0)] = -20(100 - E[v|s_1 = 0, s_3 = 0]) - 3.5(10.5 - 42) - 3(10.5 - 30) =$$

$$-20(100 - E[v|s_1 = 0, s_3 = 0]) + 168.75$$

$$E[U_1^e(1, s_2, 0)] = -20(100 - E[v|s_1 = 1, s_3 = 0]) - 3.5(52.5 - 42) - 3(52.5 - 30) =$$

$$-20(100 - E[v|s_1 = 1, s_3 = 0]) - 104.25$$

$$E[U_3^1(0, s_2, 0)] = 10(100 - E[v|s_1 = 0, s_3 = 0]) - 84$$

$$E[U_3^1(0, s_2, 1)] = 10(100 - E[v|s_1 = 0, s_3 = 1]) - 21$$

$$E[U_3^e(0, s_2, 0)] = 10(100 - E[v|s_1 = 0, s_3 = 0]) - 84$$

$$E[U_3^e(0, s_2, 1)] = 10(100 - E[v|s_1 = 0, s_3 = 1]) - 21$$

This proves the inequalities.

9.2 Proof of Theorem 6.2

I prove the following proposition:

Proposition 9.1. *Let M and N be a pair of contract matrices, where M is a reduction of N. Let A be defined as in the definition of blocking and (P, Q, U) be a settlement mechanism with no complete information unraveling, for which N is reduced to M by agents in A. If for all $i \in A$, p_i^M and q_i^M do not depend on s_i, then the following inequality must hold:*

$$\sum_{i \in A} E_{s_{-A}}[\Lambda_i^N(s_A, s_{-A})] \geq \sum_{i \in A} E_{s_{-A}}[\Lambda_i^M(s_A, s_{-A})] \textit{ for all } s_A \in S_A \tag{25}$$

The inequality means that sum of agents' risk-neutral expected payoffs that are in A is greater when the network is N compared to that of network M.

Proof. I prove that the following holds: For all intervals of types $S'_i = [\underline{s_i}, \overline{s_i}] \subseteq S_i$ $i \in A$ such that $\prod_{i \in A} S'_i$ has a positive measure,

$$\sum_{i \in A} E[\Lambda_i^N(s) | \pi_A(s) \in \prod_{i \in A} S'_i] \geq \sum_{i \in A} E[\Lambda_i^M(s_A, s_{-A}) | \pi_A(s) \in \prod_{i \in A} S'_i] \tag{26}$$

This is equivalent to (25). Let (P, Q, U) be a settlement mechanism that does not satisfy (26), i.e., for some interval of types $S'_i = [\underline{s_i}, \overline{s_i}] \subseteq S_i$ $i \in A$ such that $\prod_{i \in A} S'_i$ has a positive measure, the opposite of (26) holds. I construct a blocking mechanism (P', Q', U') in which agent $i \in A$ joins the blocking mechanism if $s_i \in S'_i$. For all $i \in A$ and $s \in S_K$, set the quantity as follows:

$q_i'(s) = q_i^N(s) - q_i^M(s)$

$$q_i'(s) = \begin{cases} q_i^N(s) - q_i^M(s) & \text{if } \underline{s_i} \leq s_i \leq \overline{s_i}, \\ q_i^N(\overline{s_i}, s_{-i}) - q_i^M(\overline{s_i}, s_{-i}) & \text{if } s_i > \overline{s_i}, \\ q_i^N(\underline{s_i}, s_{-i}) - q_i^M(\underline{s_i}, s_{-i}). & \text{if } s_i < \underline{s_i} \end{cases}$$

I set the price for the blocking mechanism as follows:

$$p_i'(s) = \begin{cases} \hat{p}_i(s) & \text{if } \underline{s_i} \leq s_i \leq \overline{s_i}, \\ \hat{p}_i(\overline{s_i}, s_{-i}), & \text{if } s_i > \overline{s_i}, \\ \hat{p}_i(\underline{s_i}, s_{-i}). & \text{if } s_i < \underline{s_i} \end{cases}$$

where $\hat{p}_i(s)$ is the unique solution to the following equation:

$$\Lambda_i^N(s) = \Lambda_i^M(s) + (n_i - m_i - q_i(s)))(100 - v(s)) + q_i'(s)(100 - \hat{p}_i(s)).$$

This equation has a unique solution, as the right-hand side is linear in $\hat{p}_i(s)$. Note that, by construction, if $\underline{s_i} \leq s_i \leq \overline{s_i}$ then $\Lambda_i'(s) = \Lambda_i^N(s) - \Lambda_i^M(s)$.

I prove that the mechanism (P', Q', U') is ex-post incentive compatible. Because the settlement mechanism is incentive compatible when the network is N and $q^M(., s_{-i})$ does not depend on s_i, for all $i \in A$ and $(s_i', s_{-i}), (s_i, s_{-i}) \in S_K$,

$$\begin{aligned}
\Lambda_i^M(s_i', s_{-i}) - \Lambda_i^M(s_i, s_{-i}) &= q_i^M(s_i, s_{-i})(E[v|s_i', s_{-i}] - E[v|s_i, s_{-i}]) \\
\Lambda_i^N(s_i', s_{-i}) - \Lambda_i^N(s_i, s_{-i}) &\geq q_i^N(s_i, s_{-i})(E[v|s_i', s_{-i}] - E[v|s_i, s_{-i}])
\end{aligned}$$

Therefore, if $\underline{s_i} \leq s_i, s_i' \leq \overline{s_i}$,

$$\Lambda_i'(s_i', s_{-i}) - \Lambda_i'(s_i, s_{-i}) \geq q_i'(s_i, s_{-i})(E[v|s_i', s_{-i}] - E[v|s_i, s_{-i}])$$

This proves that the blocking mechanism is incentive compatible in this case. Note that the settlement mechanism sets the same price and quantity if $s_i, s_i' > \overline{s_i}$ or if $s_i, s_i' < \underline{s_i}$. Therefore, the incentive compatibility is trivial for those cases. If $s_i' > \overline{s_i}$ and $s_i \in [\underline{s_i}, \overline{s_i}]$, then, by construction and incentive compatibility of the blocking mechanism in $[\underline{s_i}, \overline{s_i}]$, the following holds:[13]

$$\begin{aligned}
\Lambda_i'(s_i', s_{-i}) = \Lambda'(\overline{s_i}, s_{-i}) + q_i'(\overline{s_i}, s_{-i})(E[v|s', s_{-i}] - E[v|\overline{s_i}, s_{-i}]) &\geq \\
\Lambda'(s_i, s_{-i}) + q_i'(\overline{s_i}, s_{-i})(E[v|s', s_{-i}] - E[v|\overline{s_i}, s_{-i}]) + q_i'(s_i, s_{-i})(E[v|\overline{s_i}, s_{-i}] - E[v|s_i, s_{-i}]) &\geq \\
\Lambda'(s_i, s_{-i}) + q_i'(s_i, s_{-i})(E[v|s_i', s_{-i}] - E[v|s_i, s_{-i}])
\end{aligned}$$

This proves the ex-post inventive compatibility for this case. All other cases admit a similar proof. Inequality (6) is satisfied by construction. To check inequality (7), let $i \in A$, $(s_i, s_{-i}) \in \prod_{j \in A} S_j'$ and $s_i' > \overline{s_i}$. For economy of exposition, let $s = (s_i, s_{-i})$, $s' = (s_i', s_{-i})$ and $\overline{s} = (\overline{s_i}, s_{-i})$. Incentive compatibility of the settlement mechanism when the network is N, construction of the blocking

[13]Because the mechanism is ex-post incentive compatible in S_i', $q_i'(\overline{s_i}, s_{-i}) \geq q_i'(s_i, s_{-i})$

mechanism, and that prices and quantities do not depend on the agent's signal when the signal is M imply the following:

$$\Lambda_i^N(s') \geq \Lambda_i^N(\overline{s}) + q_i^N(\overline{s})(E[v|s'] - E[v|\overline{s}]) = \Lambda_i^M(\overline{s}) + \Lambda_i'(\overline{s}) + q_i^N(\overline{s})(E[v|s'] - E[v|\overline{s}])$$
$$= \Lambda_i^M(s') + \Lambda_i'(s') + (q_i^N(\overline{s}) - q_i^M(\overline{s}) - q_i'(\overline{s}))(E[v|s'] - E[v|\overline{s}]) = \Lambda_i^M(s') + \Lambda_i'(s')$$

If $s_i' < \underline{s_i}$, then:

$$\Lambda_i^N(s') \geq \Lambda_i^N(\underline{s}) + q_i^N(\underline{s})(E[v|s'] - E[v|\underline{s}]) = \Lambda_i^M(\underline{s}) + \Lambda_i'(\underline{s}) + q_i^N(\underline{s})(E[v|s'] - E[v|\underline{s}])$$
$$= \Lambda_i^M(s') + \Lambda_i'(s') + (q_i^N(\underline{s}) - q_i^M(\underline{s}) - q_i'(\underline{s}))(E[v|s'] - E[v|\underline{s}]) = \Lambda_i^M(s') + \Lambda_i'(s')$$

These prove inequality (7). Because the opposite of (26) holds, the blocking mechanism designer has a positive expected payoff. □

I prove lemma 6.3.

Proof. Note that, if the mechanism is pooling-seprable with network M, then the assumption in the second part of Proposition 9.1 is satisfied. Taking expectation from the conclusion of Proposition 9.1 implies:

$$\sum_{i \in A} E[\Lambda_i^N(s)] \geq \sum_{i \in A} E[\Lambda_i^M(s)] \text{ for all } s \in S. \tag{27}$$

Because the mechanism is weakly-unbiased, the following holds:

$$\sum_{i \in A} E_\rho[\Lambda_i^N(s)] = \sum_{i \in A} E_\rho[\Lambda_i^M(s)]. \tag{28}$$

Equality (44) and Proposition 9.1 imply that, almost surely for all $s \in S_K$, the following holds:

$$\sum_{i \in A} \Lambda_i^N(s) = \sum_{i \in A} \Lambda_i^M(s). \tag{29}$$

$\square$

I prove the following lemmas.

Lemma 9.2. *Consider an agent $i \in K$ and two ex-post incentive compatible settlement mechanisms with price, quantity, and risk-neutral payoff functions p_i, q_i, Λ_i and p'_i, q'_i, Λ'_i. If $q'_i(.)$ and $p'_i(.)$ do not depend on s_i for all $s_i \in (0,1)$ and almost surely for all $s \in S_K$, $\Lambda_i(s) = \Lambda'_i(s)$, then almost surely $q_i(s) = q'_i(s)$ and $p_i(s) = p'_i(s)$.*

Proof. Let E be that set of signal profiles s_{-i} such that, for almost all $s_i \in [0,1]$, $\Lambda_i(s_i, s_{-i}) = \Lambda'_i(s_i, s_{-i})$. Note that, because $\Lambda_i(s) = \Lambda'_i(s)$ holds almost certainly, the complement of E is measure zero. Given $s_{-i} \in E$, let s_i, s'_i, s''_i be such that $\Lambda_i(s_i, s_{-i}) = \Lambda'_i(s_i, s_{-i})$, $\Lambda_i(s'_i, s_{-i}) = \Lambda'_i(s'_i, s_{-i})$, $\Lambda_i(s''_i, s_{-i}) = \Lambda'_i(s''_i, s_{-i})$ and $s'_i < s_i < s''_i$. Ex-post incentive compatibility implies that:

$$\Lambda_i(s'_i, s_{-i}) - \Lambda_i(s_i, s_{-i}) \geq q_i(s_i, s_{-i})(E[v|s'_i, s_{-i}] - E[v|s_i, s_{-i}]).$$

Note that

$$\Lambda_i(s'_i, s_{-i}) - \Lambda_i(s_i, s_{-i}) = \Lambda'_i(s'_i, s_{-i}) - \Lambda'_i(s_i, s_{-i}) = q'_i(s_i, s_{-i})(E[v|s_i, s_{-i}] - E[v|s'_i, s_{-i}])$$

Therefore, $q'_i(s_i, s_{-i}) \geq q_i(s_i, s_{-i})$. Considering ex-post incentive compatibility for types s_i and s''_i implies that $q'_i(s_i, s_{-i}) \leq q_i(s_i, s_{-i})$. Therefore, $q'_i(s_i, s_{-i}) = q_i(s_i, s_{-i})$ and $p'_i(s_i, s_{-i}) = p_i(s_i, s_{-i})$.

One can choose s_i' close enough to 0 and s_i'' close enough to 1. Therefore, almost certainly for all $s_i \in [0, 1]$, , $q_i'(s_i, s_{-i}) = q_i(s_i, s_{-i})$ and $p_i'(s_i, s_{-i}) = p_i(s_i, s_{-i})$. □

Lemma 9.3. *Assume N is a network where only two pairs of agents have contracts. If the mechanism (P, Q, U) is ex-post individually rational, full information unravel-proof, ex-ante weakly budget balanced, and weakly-unbiased, then almost certainly (i) $q_i^N(s) = 0$ for all signal profiles $s \in [0, 1]^{|K|}$ and agents $i \in K$ who do not have any CDS contracts in network N and (ii) the mechanism is almost certainly ex-post budget balanced.*

Proof. Consider the contract matrix in which no agent has a CDS contract, namely, the $\emptyset$ network. Because the mechanism is ex-post individually rational, $U_i^{\emptyset}(s) \geq 0$ for all $i \in K$; therefore, $\Lambda_i^{\emptyset}(s) \geq 0$ for all $i \in K$. Ex-ante weakly budget balanced condition implies that $\sum_{i \in K} E_\rho[\Lambda_i^{\emptyset}(s)] \leq 0$. Hence, almost certainly, $\Lambda_i^{\emptyset}(s) = 0$ for all $i \in K$. In network N, consider a block by the two agents who have CDS contracts. Lemma 6.3 implies that, for all $s \in S_K$, the following holds almost certainly:

$$\sum_{i \in K(N)} \Lambda_i^N(s) = 0. \tag{30}$$

Ex-post budget balancedness implies that, for all $i \in K \setminus K(N)$ and $s \in S_K$:

$$\Lambda_i^N(s) \geq 0. \tag{31}$$

Because the mechanism is ex-ante weakly budget balanced, the following inequality holds:

$$E_\rho[\sum_{i \in K} \Lambda_i^N(s)] \leq 0. \tag{32}$$

Inequalities (30), (31), and (32) imply that, almost certainly for all $i \in K \setminus K(N)$ and $s \in S_K$, $\Lambda_i^N(s) = 0$ and $\sum_{i \in K(N)} \Lambda_i^N(s) = 0$. Therefore, $\sum_{i \in K} \Lambda_i^N(s) = 0$ for all $i \in K$. Moreover, lemma 9.2 implies that for all $i \in K \setminus K(N)$ and almost all $s \in S_K$, $q_i^N(s) = 0$. □

I establish a generalization of the induction base as a lemma.

Lemma 9.4. *If in network N, agents i and j have CDS contracts only with each other and for almost all $s \in K$ $q_i^N(s) + q_j^N(s) = 0$, then, almost certainly, $q_i^N(s), p_i^N(s), q_j^N(s)$ and $p_j^N(s)$ do not depend on s_i and s_j and $p_i^N(s) = p_j^N(s)$.*

Proof. Consider a block where agents i and j choose to side settle all of their contracts. Lemma 6.3 implies that $\Lambda_i^N(s) + \Lambda_j^N(s) = 0$ for almost all $s \in S_K$. Set

$$\begin{aligned} A &= \{s_{-i,j} \in [0,1]^{|K|-2} | q_i^N(s_i, s_{-i,}) + q_j^N(s_i, s_{-i}) = 0 \text{ for almost all } s_i \in [0,1] \text{ almost all } s_j \in [0,1]\} \\ A_{s_{-i,j}} &= \{(s_i, s_j) \in (0,1)^2 | q_i^N(s_i, s_{-i,}) + q_j^N(s_i, s_{-i}) = 0\} \end{aligned}$$

The assumption in the lemma implies that A has probability 1. Given $s_{-i,j} \in A$ and $(s_i, s_j) \in A_{s_{-i,j}}$, let $(s_i', s_j) \in A_{s_{-i,j}}$ and $(s_i, s_j') \in A_{s_{-i,j}}$ be such that $E[v|s_i', s_j] = E[v|s_i, s_j']$. Note that, because $v(s)$ is increasing and continuous, by definition of A, I can always find s_i', s_j' such that $s_i' > s_i$. Ex-post incentive compatibility implies that:

$$q_i^N(s_i, s_j, s_{-i,j})(E[v|s_i', s_j, s_{-i,j}] - E[v|s_i, s_j, s_{-i,j}]) \leq \Lambda_i^N(s_i', s_j, s_{-i,j}) - \Lambda_i^N(s_i, s_j, s_{-i,j}) \tag{33}$$

$$q_j^N(s_i', s_j', s_{-i,j})(E[v|s_i', s_j, s_{-i,j}] - E[v|s_i', s_j', s_{-i,j}]) \leq \Lambda_j^N(s_i', s_j, s_{-i,j}) - \Lambda_j^N(s_i', s_j', s_{-i,j}) \tag{34}$$

$$q_i^N(s_i', s_j', s_{-i,j})(E[v|s_i, s_j', s_{-i,j}] - E[v|s_i', s_j', s_{-i,j}]) \leq \Lambda_i^N(s_i, s_j', s_{-i,j}) - \Lambda_i^N(s_i', s_j', s_{-i,j}) \tag{35}$$

$$q_j^N(s_i, s_j, s_{-i,j})(E[v|s_i, s_j', s_{-i,j}] - E[v|s_i, s_j, s_{-i,j}]) \leq \Lambda_j^N(s_i, s_j', s_{-i,j}) - \Lambda_j^N(s_i, s_j, s_{-i,j}) \tag{36}$$

Note that the left- and right-hand side of all these sum up to zero; therefore, all of these inequalities must hold with equality. Equality of inequality (33) means that agent i is indifferent about reporting s_i and s_i' when his signal is, in fact, s_i'. Let s_i'' be a signal for agent i such that $s_i < s_i'' < s_i'$, ex-post incentive compatibility implies that:

$$\begin{aligned} q_i^N(s_i'', s_j, s_{-i,j})(E[v|s_i', s_j, s_{-i,j}] - E[v|s_i'', s_j, s_{-i,j}]) &\leq \Lambda_i^N(s_i', s_j, s_{-i,j}) - \Lambda_i^N(s_i'', s_j, s_{-i,j}) \quad (37) \\ q_i^N(s_i, s_j, s_{-i,j})(E[v|s_i'', s_j, s_{-i,j}] - E[v|s_i, s_j, s_{-i,j}]) &\leq \Lambda_i^N(s_i'', s_j, s_{-i,j}) - \Lambda_i^N(s_i, s_j, s_{-i,j}) \quad (38) \\ q_i^N(s_i, s_j, s_{-i,j}) &\leq q_i^N(s_i'', s_j, s_{-i,j}) \quad (39) \end{aligned}$$

Adding inequalities (37) and (38) and applying inequality (39) implies inequality (33). Because inequality (33) holds with equality, inequalities (37), (38) and (39) hold with equality as well. This implies that, for all $s_i < s_i'' < s_i'$:

$$q_i^N(s_i, s_j, s_{-i,j}) = q_i^N(s_i'', s_j, s_{-i,j})$$

I can choose $s_i' < s_i$ as well. Therefore, I conclude that, in a neighborhood around s_i, the function $q_i(., s_i, s_{-i,j})$ is constant. Therefore, $q_i^N(., s_j, s_{-i,j})$ almost certainly does not depend on s_i.A similar argument holds for agent j. This proves the lemma. □

If only two agents, i and j, have CDS contracts, then lemma 9.3 implies that, almost certainly, the mechanism sets a quantity of zero for all other agents; therefore, the assumption of lemma 9.4 is valid. Lemma 9.4 implies that, when only two agents have CDS contracts with each other, then the price and the quantity do not depend on agents' signals. If agents i and j have n contracts, let $p_{i,j}^n(s_{-i,j})$ $(= p_{j,i}^n(s_{-i,j}))$ and $q_{i,j}^n(s_{-i,j})(= -q_{j,i}^n(s_{-i,j})$ be the price and quantity. Consider a network where three agents, i, j and k, have CDS contracts as follows: i has n CDS contracts with j, and j has n CDS contracts with k. Assume i is a CDS buyer and k is a CDS seller. Call this network M. Apply lemma 6.3 for $A = \{i, j\}$ and conclude:

$$\Lambda_i^M(s) = n(100 - v(s)) + q_{j,k}^n(s_{-j,k})(v(s) - p_{j,k}^n(s_{-j,k})) \tag{40}$$

Note that, in network M, agent j has net zero CDS contracts. Robustness with respect to network implies that

$$\Lambda_i^M(s) = n(100 - v(s)) + q_{i,k}^n(s_{-i,k})(v(s) - p_{i,k}^n(s_{-i,k})) \tag{41}$$

Equations (40) and (41) imply that

$$q_{j,k}^n(s_{-j,k})(v(s) - p_{j,k}^n(s_{-j,k})) = q_{i,k}^n(s_{-i,k})(v(s) - p_{i,k}^n(s_{-i,k})) \tag{42}$$

Note that the only term that depends on s_k in both sides of equation (42) is $v(s)$. Therefore, equation (42) implies that $q_{j,k}^n(s_{-j,k}) = q_{i,k}^n(s_{-i,k})$ and $q_{j,k}^n(s_{-j,k}) = q_{i,k}^n(s_{-i,k})$. Therefore, it must be that $q_{i,j}^n(s_{-i,j}) = q^n$ and $p_{i,j}^n(s_{i,j}) = p^n$ for all $i, j \in K$. Because the mechanism is unbiased, $p^n = E_\rho[v]$. This proves the induction base.

Given network N, let $f(N)$ be the number of pairs of agents who have contracts with each other and $g(N)$ be the number of pairs of agents who have contracts with each other and do not have contracts with other agents. I prove the result by induction on $h(N) = 2f(N) + g(N)$. So far, I have established the result for $h(N) \leq 3$. Given network N, where $h(N) > 3$ and $i \in K(N)$, I show that

Λ_i^N has the form described in the theorem. There are four cases:

Case 1: For some j and $k \in K$, i has contracts with j, j has some contracts with k, and k has contract with i.

Proof: construct contract matrix N^i, N^j and N^k as follows: N^i is constructed from N by removing the contracts between j and k, N^j is constructed by removing the contracts between i and k, and N^k is constructed from N by removing the contracts between i and j. Lemma 6.3 implies:

$$\Lambda_i^N(s) + \Lambda_j^N(s) = \Lambda_i^{N^k}(s) + \Lambda_j^{N^k}(s), \tag{43}$$

$$\Lambda_i^N(s) + \Lambda_k^N(s) = \Lambda_i^{N^j}(s) + \Lambda_k^{N^j}(s), \tag{44}$$

$$\Lambda_j^N(s) + \Lambda_k^N(s) = \Lambda_j^{N^i}(s) + \Lambda_k^{N^i}(s). \tag{45}$$

Equalities (43), (44) and (45) imply that

$$2\Lambda_i^N(s) = \Lambda_i^{N^k}(s) + \Lambda_j^{N^k}(s) + \Lambda_i^{N^j}(s) + \Lambda_k^{N^j}(s) - \Lambda_j^{N^i}(s) - \Lambda_k^{N^i}(s) \tag{46}$$

Since $h(N^i)$, $h(N^j)$ and $h(N^k) < h(N)$, the induction base implies that

$$\begin{aligned}
2\Lambda_i^N(s) &= \sum_{l\in K\setminus\{j\}} n_{i,l}(100-v(s)) + q^{n_{i,l}}(v(s)-p) + \sum_{l\in K\setminus\{i\}} n_{j,l}(100-v(s)) + q^{n_{j,l}}(v(s)-p) \\
&+ \sum_{l\in K\setminus\{k\}} n_{i,l}(100-v(s)) + q^{n_{i,l}}(v(s)-p) + \sum_{l\in K\setminus\{i\}} n_{k,l}(100-v(s)) + q^{n_{k,l}}(v(s)-p) \\
&- \sum_{l\in K\setminus\{k\}} n_{j,l}(100-v(s)) + q^{n_{j,l}}(v(s)-p) - \sum_{l\in K\setminus\{j\}} n_{k,l}(100-v(s)) + q^{n_{k,l}}(v(s)-p) \\
&= 2\sum_{l\in K} n_{i,l}(100-v(s)) + q^{n_{i},l}(v(s)-p) - n_{i,j}(100-v(s)) - q^{n_{i,j}}(v(s)-p) \\
&- n_{i,k}(100-v(s)) - q^{n_{i,k}}(v(s)-p) - n_{j,i}(100-v(s)) - q^{n_{j,i}}(v(s)-p) \\
&+ n_{j,k}(100-v(s)) - q^{n_{j,k}}(v(s)-p) - n_{k,i}(100-v(s)) - q^{n_{k,i}}(v(s)-p) \\
&+ n_{k,j}(100-v(s)) - q^{n_{k,j}}(v(s)-p) \qquad (47)
\end{aligned}$$

Note that for all $r, s \in K$, $n_{r,s} + n_{s,r} = 0$ and $q^n + q^{-n} = 0$ for all $n \in \mathbb{Z}$, hence, equality (47) implies that

$$\Lambda_i^N(s) = \sum_{l\in K} n_{i,l}(100-v(s)) + q^{n_{i},l}(v(s)-p)$$

Case 2: For some j and $k \in K$, i has contracts with j, j has some contracts with k, and k and i do not have contracts with each other.

Proof. Remove the contracts between i and j and call the network N^k. Also, remove the contracts between j and k and call the network N^i. Lemma 6.3 implies that for all $s \in S$:

$$\begin{aligned}
\Lambda_i^N(s) + \Lambda_j^N(s) &= \Lambda_i^{N^k}(s) + \Lambda_j^{N^k}(s) \\
\Lambda_j^N(s) + \Lambda_k^N(s) &= \Lambda_j^{N^i}(s) + \Lambda_k^{N^i}(s) \qquad (48)
\end{aligned}$$

I construct network M as follows:

$$m_{r,s} = \begin{cases} n_{i,j} & \text{if } (r,s) = (i,k) \\ n_{j,i} + n_{j,k} & \text{if } (r,s) = (j,k) \\ 0 & \text{if } (r,s) = (i,j) \\ -m_{s,r} & \text{if } (r,s) \in \{(k,i),(k,j),(j,i)\} \\ n_{r,s} & \text{otherwise} \end{cases} \tag{49}$$

Note that $h(M) = h(N)$. Robustness with respect to network implies $\Lambda_r^M(s) = \Lambda_r^N(s)$ for all $r \in K$ and $s \in S$. In network M, remove the contracts between j and k and call the network M^i. Note that $h(M^i) < h(N)$. Lemma 6.3 implies that for all $s \in S$:

$$\Lambda_j^N(s) + \Lambda_k^N(s) = \Lambda_j^M(s) + \Lambda_k^M(s) = \Lambda_j^{M^i}(s) + \Lambda_k^{M^i}(s). \tag{50}$$

Note that the inductive hypothesis applies to M^i, N^i, N^j and N^k. Equalities (49), (50) and similar argument as in the previous case proves the result for this case. □

Case 3: agent i has contracts with two agents j and k. This case admits a similar proof to previous cases.

Case 4: agent i only has contracts with agent $j \in K$ and j has no contract with other agents. Since $f(N) \geq 3$, there exists an agent $k \in K$ who has contracts with other agents. Construct network M from N as follows: remove the contracts between i and j, add contracts between i & k and j & k, such that $m_{i,k} = n_{i,j}$ and $m_{j,k} = n_{j,i}$. Note that all agents have the same net number of contracts in M and N, therefore, robustness with respect to network implies $\Lambda_i^N(s) = \Lambda_i^M(s)$. The rest of the proof is similar to case 2.

9.3 Proof of Theorem 6.4

I establish the result of lemma 6.3 for the case of incomplete information unravel-proof mechanisms. Let networks M, N satisfy $M \prec N$ and assume p_i^M and q_i^M do not depend on the reported messages. If, for some non-zero measure set of signals $\Pi_{i\in A} S'_i$, the inequality

$$E[\sum_{i\in A} \Lambda_i^N(s) | \pi_A(s) \in \Pi_{i\in A} S'_i] < E[\sum_{i\in A} \Lambda_i^M(s) | \pi_A(s) \in \Pi_{i\in A} S'_i]$$

holds, then one can design a blocking mechanism similar to proposition 9.1. This implies that, for all non-zero measure $\Pi_{i\in A} S'_i$, the following holds:

$$E[\sum_{i\in A} \Lambda_i^N(s) | \pi_A(s) \in \Pi_{i\in A} S'_i] \geq E[\sum_{i\in A} \Lambda_i^M(s) | \pi_A(s) \in \Pi_{i\in A} S'_i]. \tag{51}$$

Because the mechanism is unbiased,

$$E[\sum_{i\in A} \Lambda_i^N(s)] = E[\sum_{i\in A} \Lambda_i^M(s)]. \tag{52}$$

Equality (51) and (52) and assumption 4.1 imply that, almost certainly, for all $s \in S_K$:

$$\sum_{i\in A} \Lambda_i^N(s) = \sum_{i\in A} \Lambda_i^M(s). \tag{53}$$

A similar argument in the proof of lemma 6.3 implies the following: For almost all $i \in K \setminus K(N)$:

$E_\mu[\Lambda_i^N(s)] = 0$ holds. Assumption 4.1 implies $\Lambda_i^N(s) = 0$ for all networks N and agents $i \in K \setminus K(N)$. This proves the result of lemma 6.3 in this case. Proof of lemma 9.3 is followed from lemma 6.3. The rest of the proof is identical to the proof of Theorem 6.2.

9.4 Proof of Theorem 6.5

Let networks M, N satisfy $M \prec N$, and A is the set of agents who reduced the network from N to M. If the mechanism is payoff equivalent to a posted price mechanism when the network is M, then the following holds:

$$\sum_{i\in A} \Lambda_i^N(s) = \sum_{i\in A} \Lambda_i^M(s).$$

Proof: If for some positive measure set of types $\Pi_{i\in A} S'_i$, the following inequality holds:

$$E[\sum_{i\in A} \Lambda_i^N(s)|\pi_A(s) \in \Pi_{i\in A} S'_i] < E[\sum_{i\in A} \Lambda_i^M(s)|\pi_A(s) \in \Pi_{i\in A} S'_i]. \tag{54}$$

Consider a participation-choice in which agents in A block the mechanism and reduce the network from N to M (see proposition 5.2). Because the mechanism is unbiased, for all $i \in K$, the following holds:

$$E_\rho[\Lambda_i^M(s) I_{\{\pi_A(s)\in\Pi_{i\in A} S'_i\}} + \Lambda_i^N(s) I_{\{\pi_A(s)\notin\Pi_{i\in A} S'_i\}}] = E_\rho[n_i(100 - v(s))] \tag{55}$$

I sum up these equalities for all $i \in K(N - M)$ and apply equation (54) to conclude:

$$\begin{aligned} E_\rho[\sum_{i\in A} \Lambda_i^M(s)] &> E_\rho[\sum_{i\in A} n_i(100 - v(s))] \\ &= E_\rho[\sum_{i\in A} m_i(100 - v(s))] \end{aligned} \tag{56}$$

Consider a participation-choice in which the M network does not unravel; unbiasedness implies that (56) must hold with equality. This contradiction implies that (54) does not hold. I apply the unbiasedness for the participation choice with no unraveling to conclude: For all positive measure $\pi_A(s) \in \Pi_{i \in A} S'_i$

$$E[\sum_{i \in A} \Lambda_i^N(s) | \pi_A(s) \in \Pi_{i \in A} S'_i] = E[\sum_{i \in A} \Lambda_i^M(s) | \pi_A(s) \in \Pi_{i \in A} S'_i]. \tag{57}$$

Assumption 4.1 implies that, almost certainly,

$$\sum_{i \in A} \Lambda_i^N(s) = \sum_{i \in A} \Lambda_i^M(s). \tag{58}$$

The rest of the proof is the replicate of the proof of Theorem 6.2.

9.5 Proof of Theorem 7.1

Lemma 6.3 can be extended to this case. Because the new notions of unbiasedness and unravel-proofness are stronger, and cases 2, 3 and 4 of the proof of theorem 6.2 are the only parts that use the property of robustness with respect to the network of contracts, all I need to do is to replace these steps. Here is the replacement:

Case 2: For some j and $k \in K$, i has contracts with j, j has some contracts with k, and k and i do not have contracts with each other.

Proof. Construct network N_i by removing the contracts between j and k, and N_k by removing the contracts between i and j. If i sells all his contracts with j to agent k, network M is constructed. Lemma 6.3 implies:

$$\begin{aligned} \Lambda_i^N(s) + \Lambda_j^N(s) &= \Lambda_i^{N_k}(s) + \Lambda_j^{N_k}(s) \\ \Lambda_j^N(s) + \Lambda_k^N(s) &= \Lambda_j^{N_i}(s) + \Lambda_k^{N_i}(s) \\ \Lambda_i^N(s) + \Lambda_j^N(s) + \Lambda_k^N(s) &= \Lambda_i^M(s) + \Lambda_j^M(s) + \Lambda_k^M(s) \end{aligned} \tag{59}$$

Similar argument as in Case 2 of theorem 6.2 shows that equations (59) prove the result.

□

Case 3: agent i has contracts with two agents j and k. This case admits a similar proof to previous cases.

Proof. The proof of this case is similar to the previous case. □

Case 4: agent i only has contracts with agent $j \in K$ and j has no contract with other agents.

Proof. Let $k_1, k_2 \in K \setminus \{i, j\}$ be a pair of agents who have contracts with each other. □

9.6 Proof of Proposition 7.2

The unbiased and weakly-unbiased properties have been used in equations (44), (52), and (55). These equations still hold if I make the replacement. However, it has been argued that equation (53) contracts with the unbiasedness property. This equation also contracts with the ex-ante uniform price property.

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