Winning by Default: Why is There So Little Competition in Government Procurement?

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Abstract

In government procurement auctions, eligibility requirements are often imposed and, perhaps not surprisingly, contracts generally have a small number of participating bidders. To understand the effects of the restrictions of competition on the total cost of government procurement, we develop, identify, and estimate a principal-agent model in which the government selects a contractor to undertake a project. We consider three reasons why restricting entry could be beneficial to the government: by decreasing bid processing and solicitation costs, by increasing the chance of selecting a favored contractor and consequently reaping benefits from the favored contractor, and by decreasing the expected amount of price to the winning contractor. When the participation is costly and bidders are heterogeneous, the expected amount of price to the winning contractor may decrease by excluding ex-ante less efficient contractors. Using our estimates, we quantify the effects of the eligibility restrictions on the total cost of procurement.

1 Introduction

In recent ten years, the market for the United States federal government procurement is worth over $460 billion annually, which constitutes about 18% of the annual federal government spending. Despite its vast size, the extent of competition for a procurement contract is not very intense. Among the procurement contracts awarded during 2004–2012, about 43% of them were awarded under either limited or no competition. Even when a contract was open to full competition, attracting only one bidder for the contract was not uncommon. In this paper, we develop, identify, and estimate a procurement model to better understand the extent of competition observed in the data.

There are two important institutional features of federal government procurement that have received relatively little attention from the literature. First, for each procurement contract, the extent and method by which the contract will be competed is chosen by contracting officers who are hired by the government. The regulations allow them to eliminate bidders from consideration, although full and open competition is encouraged. As a result, the number of bids is relatively small; in fact, winning a contract by default is not uncommon in federal government procurement.

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Second, a sealed-bid auction is not always a dominant procedure to choose a contractor, depending on the nature of the products or services to be procured.\(^1\) An alternative solicitation procedure is *competitive proposal evaluation*, through which the proposals submitted by contractors are evaluated, negotiated, and selected. After the request for proposals is posted, the qualified contractors can submit their proposals, which will be reviewed in detail to determine which proposals are within a competitive range. Discussions and negotiations may then be carried out with the contractors within the competitive range, and the contractor will be selected whose proposal is found to be most advantageous to the procuring agency. During the discussions and negotiations, the contract terms are prices are considered together, which determines the winner.\(^2\)

In this paper, we construct a principal-agent framework that incorporates these features. In our model, the principal (the procurer) chooses the extent of competition, i.e., the eligibility conditions and the expected number of bidders, and offers a menu of contracts to the participating contractors (the agents) with a hidden type (cost). A contract chosen by a contractor is interpreted as a bid, which reflects the competitive proposal evaluation procedure. This procedure can be more profitable to the procurer than a sealed-bid auction when the number of bidders is very small.

The procurer may choose to decrease the extent of competition for three reasons. First, bid processing and solicitation costs may increase as the number of bidders increases. These costs include the time cost of waiting to receive more bids and the cost of reading the proposals and assessing various attributes of the contractors. Second, decreasing competition may increase the chance that a favored contractor will win the contract. If the procurer receives positive benefits from hiring a contractor favored for reasons that cannot be formally measured or verified, such as better quality or long-term relationship that could potentially be related to corruption, the procurer may want to decrease the extent of competition. Third, an unrestricted competition does not necessarily guarantee a lower expected amount of price to the winning contractor. By excluding ex-ante less efficient contractors, the procurer may incentivize the remaining bidders to bid more aggressively.

Using the federal procurement contracts awarded during 2004–2012, we nonparametrically estimate the model following our identification argument. Using the estimates, we conduct counter-factual analyses to decompose the effects of the three sources of entry restrictions. We also quantify the effects of the eligibility restrictions on the total cost of procurement.

The rest of the paper is organized as follows. We describe the model in Section 2, and then introduce our data and present descriptive analyses in Section 3. The identification of the model follows in Section 4, which we closely follow to nonparametrically estimate our model. We show the estimation results in Section 5. Section 6 concludes.

\(^{1}\)According to the Federal Acquisition Regulation, a sealed-bid auction is not considered appropriate to use if the award will be made on the basis of factors other than price; if it is necessary to conduct discussions with the responding contractors about their bids; or if there is not a reasonable expectation of receiving more than one sealed bid.

\(^{2}\)Contracting officers typically decide on the contract type prior to issuing a solicitation. However, particularly in negotiated procurements, selection of the contract terms can be a matter for negotiation between the procuring activity and the contractor. (48 C.F.R. 16.103(a)).
1.1 Related Literature

Our paper is related to the large literature on procurement and auctions. One strand of the literature explains why less competition does not necessarily lower the payoff of the auctioneer in independent private value auctions. Li and Zheng (2009) show that when the number of bidders is endogenously determined, the equilibrium bidding behavior can become less aggressive as the number of potential bidders increases. Krasnokutskaya and Seim (2011) study a bid preference program, and Athey, Coey, and Levin (2013) compare a set-asides program and the bid subsidy program. Both papers show the importance of allowing endogenous entry when assessing restrictive competition policies.

An important contribution of our paper is that we build and estimate a model where the procurer is assumed to optimally choose the extent of competition, considering favoritism and bid processing and solicitation costs. Using the estimates, we quantify the effects of favoritism and bid processing and solicitation costs on the procurement outcomes. In this regard, Bandiera, Prat, and Valletti (2009) and Coviello, Guglielmo, and Spagnolo (2014) are closely related to our paper. Bandiera, Prat, and Valletti (2009) develop a formal framework for distinguishing active waste and passive waste in the total government cost of procurement, and separately estimate them exploiting a policy experiment in Italy’s public procurement system. Active waste entails utility for the public decision makers, part of which is related to favoritism in our paper, while passive waste does not, such as bid processing and solicitation costs. Coviello, Guglielmo, and Spagnolo (2014) study government discretion on public goods provision in terms of whether or not to impose entry restrictions, and document the casual effect of increasing such discretion on procurement outcomes using a database for public procurement in Italy.

Another strand of the literature studies nonstandard contractor selection procedures, such as scoring auctions (Asker and Cantillon (2010)) or multi-attribute auctions (Krasnokutskaya, Song, and Tang (2013)), where the price is not the only factor in selecting a contractor. We consider an optimal direct revelation mechanism in a competitive environment, studied by Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987). We extend their models by allowing the procurer to choose the optimal extent of competition.

Lewis and Bajari (2014) and Bajari, Houghton, and Tadelis (2014) are related to our paper in that they study the price adjustments after the winning contractor is chosen and the project initiates. We distinguish the price adjustments into two categories: those incurring due to the cost changes related to the unknown type of the contractors and the rest. Lewis and Bajari (2014) study the former type of the price adjustments, and Bajari, Houghton, and Tadelis (2014) study the latter. We consider both types of the price adjustment, and this is possible because we observe the reasons for contract modifications. Furthermore, although they take the contract type as exogenously given, we allow that the contract type and the winning contractor are endogenously determined.

Our paper also belongs to a literature on the identification of a principal-contractor model, for example, Perrigne and Vuong (2011). In their paper, an implicit identifying assumption is that the optimal contracts are linear in costs. Their theoretical model provides the payoffs of the principal and the contractor, but it does not provide a particular contractual form that an optimal contract must follow. In our identification result, we do not impose any functional form assumptions on the optimal contracts.
2 Model

This section lays out a procurement model in which a procurer selects a contractor from multiple bidders with hidden efficiency type to undertake a project. In doing so, the procurer chooses the extent of competition by restricting the group of contractors who are eligible to participate and by deciding the amount of search efforts to receive enough bids. We characterize the equilibrium extent of competition and the selection mechanism in this model. Figure 1 presents the timeline of this model.

![Figure 1](attachment:image.png)

Timeline of the Procurement Process in the Model

2.1 Setup

A procurement project is realized, and the total cost of completing the project depends on the minimum expected cost \( c \), the extent of inefficiency of the contractor \( \beta \), and the ex-post cost changes due to stochastic realizations of demand or supply shocks \( \epsilon \):

\[
c + \beta + \epsilon.
\]

The realization of \( c \) and \( \epsilon \) is observed by both the procurer and the contractors at the same time: \( c \) is observed before the project is let and \( \epsilon \) is observed after the project is initiated. We assume that \( c \) and \( \epsilon \) are distributed independently of \( \beta \).

In completing the project, a signal, denoted by \( s \), is revealed to both the procurer and the contractor. The procurer cannot directly observe the cost of completing the project or \( \beta \), but does observe \( s \). We assume that \( \beta \) can take two values: \( \underline{\beta} \) and \( \overline{\beta} \). We normalize \( \underline{\beta} = 0 \) and \( \overline{\beta} > 0 \) without loss of generality. The signal is drawn from cumulative density functions \( F \) and \( \overline{F} \), conditional on \( \beta = \beta, \overline{\beta} \), with common support. We assume that the signal is informative in the following sense.

**Assumption 1** (Informative Signal) There exists a set of signals, \( S \), such that \( \Pr(s \in S | \beta) \neq \Pr(s \in S | \overline{\beta}) \).

Some contractors are favored by the procurer, while others are not. By hiring a favored contractor, the procurer receives a private benefit, \( \delta > 0 \). The subjective belief of the procurer that a contractor is efficient varies by the favor type: \( \pi_f \in (0, 1) \) for favored contractors and
\[ \pi_u \in (0, 1) \] for the rest. The proportion of favored contractors is \( \rho \in (0, 1) \), which is a common knowledge.

Upon a realization of a project, the procurer determines the extent of competition, i.e., whether to exclude non-favored contractors from competition and how many bids to receive on average. The realized number of bidders follows a shifted Poisson distribution with the average arrival rate of choice, \( \lambda \), with support \( 1, 2, \ldots, \infty \).\(^3\) We assume that this guaranteed one participant is favored.

Assumption 2 (At Least One Favored Participant) For all procurement auctions, at least one favored contractor participates.

When non-favored contractors are allowed to participate, the probability that \( n \) contractors participate and \( n_f \geq 1 \) of whom are favored \( \lambda \) is:

\[
\Pr(n_f, n; \lambda) = \frac{(\lambda - 1)^{n-1} e^{-\lambda + 1}}{(n - 1)!} \left( \frac{n - 1}{n_f - 1} \right) \rho^{n_f - 1}(1 - \rho)^{n - n_f}. \tag{1}
\]

When determining the extent of competition, the procurer minimizes the total expected cost of procurement, consisting of (i) the expected transfer to the winning contractor, (ii) the expected bid processing cost, (iii) the expected private benefit of hiring a favored contractor, and (iv) the cost of holding an exclusive competition as opposed to an open competition. Here, we consider per-bidder bid processing cost, denoted by \( \kappa > 0 \), which includes the cost of reading the proposals, making sure that the language and terms of the proposals are unambiguous, and assessing various attributes of the contractors, as well as the time cost of delaying the initiation of the project. Furthermore, the last component of the cost, denoted by \( \eta \), represents the risk of bid protest or the administrative or political burdens to justify exclusion of sources.

Given the realized number of bidders by type, the procurer announces a menu of contracts, and the participating contractors simultaneously choose an item from the menu if the expected rent from doing so is nonnegative. When submitting their contract, the bidders do not know other bidders’ efficiency and have the same belief on the distribution as the procurer’s. However, they know whether or not each of other bidders is favored by the procurer. Given the submitted contracts, the procurer selects a contractor, who undertakes the project.

A typical contract in the menu has two components, a base price, \( p \), and a schedule of ex-post price adjustments, \( r(\cdot, \cdot) \), which are contingent on the realization of the signal and the cost shock \( \epsilon \). Given the realized value of \( s \) and \( \epsilon \), the payoff to the contractor under the contract is

\[ p - (c + \beta) + \psi(r(s, \epsilon) - \epsilon), \]

where \( \psi(\cdot) \) is a continuous function, with \( \psi(0) = 0, \psi'(0) = 1, \psi' > 0 \), and \( \psi'' < 0 \). Note that \( p - (c + \beta) \) is fixed while \( r(s, \epsilon) - \epsilon \) is variable. Due to liquidity concerns and potential adjustment costs, the variable part of the payoff is discounted, which is represented by \( \psi(\cdot) \).

Note that the procurer does not have liquidity concerns nor bear adjustment costs. Furthermore, the cost shock is independent of \( \beta \). Therefore, the procurer minimizes her expected

\(^3\)Because we do not observe contracts that received no bids in the data, we assume that the risk of drawing no bids is zero.
price by fully insuring the contractors against the cost shock. Therefore, we focus on the schedule \( r \) such that
\[
r(s, \epsilon) = q(s) + \epsilon,
\]
and solve for \( q(\cdot) \). In the following, we first characterize the optimal selection mechanism which induces a truth-telling Bayesian Nash equilibrium, given the number of bidders by favor type. Then we solve the optimal extent of competition to minimize the expected total cost of procurement. The proofs for theorems and corollaries are in Appendix.

### 2.2 Optimal Selection Mechanism

**Symmetric Bidders** Suppose \( n \geq 1 \) bidders of the same favor type participate, and the probability that a bidder is efficient is \( \pi \). We show that under Assumption 1, it is always optimal to offer a menu of two contracts, one fixed-price contract, \( p \), and one variable-price contract, \((\overline{p}, q(\cdot))\). Given this menu, contractors will reveal their efficiency type by their contract choices; an efficient contractor will choose the fixed-price contract while an inefficient one will choose the variable-price contract. Given this separation, procurer will select a participant that accepts the fixed-price contract, if there’s one. Otherwise, a random participant will be selected. Therefore, the probability that the fixed-price contract is selected is \( 1 - (1 - \pi)^n \), and the resulting expected transfer to a winning contractor is:

\[
[1 - (1 - \pi)^n] p + (1 - \pi)^n \left[ \overline{p} + \int q(s) f(s) ds \right].
\]

The contracts in the menu must satisfy the individual rationality (IR) and incentive constraints (IC) for each type \( \beta \). For example, the IR constraint for the inefficient contractors is:

\[
\overline{p} + \int \psi(q(s)) f(s) ds \geq c + \beta.
\]

The IC constraint for the efficient contractors is:

\[
\phi(n) \{ p - c \} \geq \overline{\phi}(n) \{ \overline{p} + \int \psi(q(s)) f(s) ds - c \},
\]

where \( \phi(n) \) and \( \overline{\phi}(n) \) denote the winning probabilities conditional on the choice of contract, \( p \) or the \((\overline{p}, q(\cdot))\), respectively. We show that these two constraints must hold in equality at the optimum, while the rest two constraints, i.e., the IR constraint for the efficient and the IC for the inefficient, may not be binding. This helps us characterize the optimal menu of contracts in the following theorem.

**Theorem 1** Suppose there are \( n \) symmetric bidders for a project. The optimal menu of contracts consists of two contracts: one fixed-price contract \( p \) and one variable-price contract \((\overline{p}, q)\). The price of the fixed-price contract is:

\[
p = c + \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left[ \overline{\beta} - \int \psi(q(s)) f(s) ds - f(s) ds \right].
\]
In the variable-price contract, the base price is:

\[ p = c + \beta - \int \psi(q(s))\overline{f}(s)ds, \quad (3) \]

and the ex-post price adjustment schedule depends on the realized value of the signal. The adjustment schedule is two-tiered. If \( f(s) / \overline{f}(s) < 1 / \pi \), then the ex-post price adjustment schedule satisfies the following equation:

\[ \psi'(q(s)) \left[ 1 - \pi \frac{f(s)}{\overline{f}(s)} \right] = 1 - \pi. \quad (4) \]

Otherwise, \( q(s) = M \), where \( M < 0 \) is the maximal penalty that the government can legally impose on contractors.

While the Appendix provides the formal proof, we discuss the outline of the proof here. We first prove that any equilibrium menu of contracts must include one fixed-price contract in Lemmas 1 and 2. Then we prove that if a menu includes more than one contract, it must consist of one fixed-price contract for efficient contractors and one variable-price contract for an inefficient contractors in Lemmas 3, 4, and 5. In Lemma 6, we show that under Assumption 1, a separating equilibrium, where two such contracts are offered, will occur.

The intuition for the characterization of the optimal menu of contracts is the following. Suppose that an inefficient contractor chose the fixed-price contract. Then the fixed-price would be at least \( c + \beta \), and the efficient contractor could extract all the rent of his efficiency, \( \beta \), by selecting the same contract. The procurer could reduce her expected price below the pooling equilibrium outcome if there is a set of informative signals, meaning \( f(s) \neq \overline{f}(s) \), that occurs with strictly positive probability. In that case the procurer could offer two contracts, a variable-price contract that is increasing in the likelihood ratio \( f(s) / \overline{f}(s) \) that leaves the inefficient contractor indifferent between undertaking the project versus not participating, as well as a fixed-price contract, with price between \( c \) and \( c + \beta \), which only the efficient contractor would take. Thus the procurer balances the losses associated with the liquidity premium paid to the inefficient contractor \( \mathbb{E}[\psi(q)\beta] \) against the constraint of offering a sufficiently high fixed-price contract to attract the efficient contractor.

Given the characterization of the menu of contracts, the following corollary summarizes the relationship between the optimal contracts and the number of bidders.

**Corollary 1** The expected transfer to a winning contractor decreases as the number of bidders increases. First, the price of the fixed-price contract, \( \overline{p} \), is non-increasing in the number of bidders. Second, the variable price contract is invariant to the number of bidders. Third, the fixed-price contract costs strictly less than the variable-price contract in expectation for any given number of bidders; i.e., \( \overline{p} + \mathbb{E}(q\beta) > \overline{p} \).

It can be seen that the expected transfer to a winning contractor decreases as the number of bidders increases through the following two channels. One channel is that the probability of hiring an efficient contractor, \( 1 - (1 - \pi)^n \), increases as the number of bidders increases. Because the fixed-price contract costs strictly less than the variable-price contract in expectation as in Corollary 1, the more likely an efficient contractor wins, the lower the expected...
price becomes. The other channel is through competition: the rent that an efficient contractor receives decreases as more contractors participate. Although the variable-price contract is invariant to the number of bidders, the fixed-price contract pays less as more bidders participate.

The following comparative statics with respect to $\pi$ is useful for the identification of the model, which is discussed in Section 4.

**Corollary 2** Suppose the conditional signal distributions are such that the probability of $f(s)/\overline{f}(s) < 1/\pi$ is zero in the neighborhood of $\pi$. As $\pi$ increases, the ex-post adjustment schedule of the variable-price contract, $q(\cdot)$, becomes more volatile and the price of the fixed-price contract, $p$, decreases.

**Asymmetric Bidders** Suppose $n_f$ favored contractors and $n_u$ non-favored ones participate. We assume that the procurer is not allowed to choose a favored, inefficient contractor if a non-favored, efficient contractor participates. Therefore, her selection rule is lexicographic: the first criterion is efficiency and the second criterion is favoritism. As a result, if at least one favored contractor participates, inefficient and non-favored contractors never win a project.

This leads to a stark result that non-favored contractors will receive a take-it-or-leave-it offer of the fixed-price contract of $c$. On the other hand, favored contractors will be offered a menu of two contracts, which induces the separation of efficiency types as a Bayesian Nash equilibrium.

**Theorem 2** Suppose there are $n_f \geq 1$ favored bidders and $n_u \geq 1$ unfavored ones. To the unfavored contractors, a fixed-price contract, $c$, is offered. To the favored contractors, a menu of two contracts, consisting of one fixed-price contract and one variable-price contract, is offered. The fixed-price contract in the menu is:

$$p = c + \frac{\pi_f (1 - \pi_f)^{n_f - 1} (1 - \pi_u)^{n_u}}{1 - (1 - \pi_f)^{n_f}} \left[ \beta - \int \psi(q(s)) [\overline{f}(s) - f(s)] ds \right]. \tag{5}$$

The variable-price contract is the same as the one when there are favored bidders only, as characterized in Theorem 1.

Given the lexicographic selection rule and the revealed efficiency types of the favored contractors, the probability that a favored contractor, $P_f(n_f, n)$, is hired is:

$$P_f(n_f, n) = 1 - (1 - \pi_f)^{n_f} [1 - (1 - \pi_u)^{n - n_f}]. \tag{6}$$

### 2.3 Optimal Extent of Competition

The procurer chooses (i) whether to allow non-favored contractors to participate and (ii) the rate at which bidders arrive. The optimal bid arrival rate when eligibility restrictions are

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4In our model, the extent to which the favoritism negatively affects efficiency is fairly limited. Alternatively, Burguet and Che (2004) and Celentani and Ganguza (2002) allow the procurer to manipulate the quality assessment in exchange for a bribe.
imposed, $\lambda_e$, minimizes the expected total procurement cost:

$$V_e(\lambda) = \sum_{n=1}^{\infty} \left\{ [1 - (1 - \pi_f)^n] \pi_e,n + (1 - \pi_f)^n [\overline{p} + \mathbb{E}(q|\beta)] \right\} \frac{(\lambda - 1)^{n-1} e^{-\lambda+1}}{(n-1)!} + \kappa \lambda - \delta + \eta, \quad (7)$$

where $\pi_e,n$ denotes the fixed-price contract with $n$ favored bidders. Here, $\pi_e,n$ is defined in equation (2) in Theorem 1 where $\pi$ is replaced by $\pi_f$, and by writing $\pi_e,n$ for $\pi$ in the equation, we make the dependence of the fixed-price contract on the number of bidders explicit. Note that the variable-price contract, $\{p, q(\cdot)\}$ is invariant to the number of bidders, and is defined in equations (3) and (4) with $\pi = \pi_f$. As $\lambda$ increases, the expected transfer to a winning contractor decreases as shown in Corollary 1 while the total bid processing cost increases. Balancing this trade-off guarantees one unique $\lambda_e$ that minimize the expected total procurement cost under eligibility restrictions.

Without such restrictions, both favored and non-favored contractors are allowed to participate. Given Assumption 2, if a variable-price contract is selected in equilibrium, it must be by a favored contractor. Therefore, the variable-price contract is invariant to the eligibility restrictions, which is not the case for the fixed-price contracts. Let $\pi_o,n_f,n_u$ denote the fixed-price contract for the favored when $n_f$ favored and $n_u$ unfavored contractors participate, as defined in equation (5) in Theorem 2. Note that Assumption 2 has another important implication: if a non-favored contractor wins a contract, then the contractor is paid at $c$. Considering these, the expected total procurement cost without eligibility restrictions is:

$$V_o(\lambda) = \sum_{n=1}^{\infty} \sum_{n_f=1}^{n} \Pr(n_f, n; \lambda) \left\{ [1 - (1 - \pi_f)^n] \pi_o,n_f,n_u + (1 - \pi_f)^n \beta | 1 - (1 - \pi_u)^n - n_f | c \right\}$$

$$+ (1 - \pi_f)^n \beta | 1 - \pi_u | (1 - \pi_u)^n | \overline{p} + \mathbb{E}(q|\beta) | - \delta P_f(n_f, n) \right\} + \kappa \lambda, \quad (8)$$

where $\Pr(n_f, n; \lambda)$ and $P_f(n_f, n)$ are defined in equations (1) and (6), respectively. The optimal participant arrival rate, $\lambda_o$, minimizes the above expected total procurement cost.

In sum, the procurer chooses to impose eligibility restrictions to prevent non-favored contractors from participating if and only if the expected total procurement cost is lower with the restrictions than without them. Corollary 2 implies that even when the procurer receives no private benefit from hiring a favored contractor, i.e., $\delta = 0$, it can be optimal to impose eligibility restrictions.

**Corollary 3** Consider two procurement auctions with $n > 1$ bidders. The bidders in one auction are all favored, i.e., $n = n_f$, while the other auction has both favored and non-favored bidders. If $\pi_u \geq \pi_f$, the expected transfer to a winning contractor is always lower in the auction with asymmetric bidders. Otherwise, the auction with symmetric bidders may lead to a lower expected price.

If non-favored contractors are on average more efficient than favored ones, i.e., $\pi_u \geq \pi_f$, then holding a non-exclusive auction lowers the expected transfer. In this case, a large $\delta$ rationalizes the choice of having an exclusive auction. In the opposite scenario, on the other hand, allowing non-favored contractors to participate will increase the chance of getting a variable-contract, i.e., $(1 - \pi_f)^{n_f}(1 - \pi_u)^{n_u} > (1 - \pi_f)^{n_f+n_u}$, and it will also increase the payment if a favored contractor wins a fixed-price contract. Although there is still a merit
of holding a non-exclusive auction in the sense that non-favored, efficient contractors will receive a fixed-price contract of the lowest possible price, this can be outweighed by the aforementioned benefits of having only the ex-ante better agents compete. Therefore, even with no private benefit from favoritism, the procurer may prefer an exclusive competition to the alternative.

3 Data

The theory predicts the optimal extent of competition and contract terms. In the remainder of this paper, we analyze data from contracts let by the U.S. federal government. Our dataset provides a very detailed information on the extent of competition and the contract terms for each procurement contract.

The data is a selected set of contracts let by the U.S. federal government during the period of FY 2004–2012. It is sourced from Federal Procurement Data System - Next Generation, which collects information on contracts and their modifications. We restrict attention to contracts with specified terms and conditions of an expected size of $300,000 or more.\(^5\) This

\(^5\)There are contracts without specified terms and conditions, i.e., indefinite delivery, indefinite quantity contracts (IDIQ). These are used when the government cannot determine the precise quantities of supplies or services that the government will require during the contract period. They only comprise 6 percent of the total obligated amount by the federal government in FY 2010, $33 billion out of $540 billion. Compared to the contracts with definite delivery and quantity, the extent of competition is more or less similar.
size threshold is chosen because the contracts of an anticipated size greater than $300,000
are not normally expected to be reserved exclusively for small business concerns. We further
narrow down our sample that satisfy the following criteria: (i) available for competition but
commercially unavailable, (ii) competed under the competitive proposal evaluation procedure
or other similar procedures, (iii) not set-aside for small business concerns, (iv) initiated and
completed during the period of study, (v) performed in the U.S., (vi) expected to take longer
than two weeks for completion, and (vi) without any inconsistent records on the contract.
There are 45,743 contracts that satisfy all of the above criteria, totaling $290 billion. For
each contract, we construct the variables in Table 1.

The base price of a contract is defined as the total value of the contract plus all options
that have been exercised at the time of award. On the other hand, the final price is the total
amount of funds obligated to the government. The base prices and the final prices often differ
from each other, and the final prices are on average larger than the base prices. These
differences are due to the contract terms that allow the final price to vary with the observed
outcomes of the project. Some outcomes are correlated with the efficiency of the contractor,
and others are not. Those correlated with the efficiency are considered as $\epsilon$ in the model,
denoted as ‘shock’ in Table 1, and those uncorrelated with the efficiency are considered as
signal, $s$, in the model. We consider the work requirement changes are in the former category
and the rest are in the latter.

The contract types are grouped into two broad categories: fixed-price and variable-price.
The specific contract types range from firm-fixed-price, in which the contractor has full re-
sponsibility for the performance costs and resulting profit or loss, to cost-plus-fixed-fee, in
which the contractor has minimal responsibility for the performance costs and the negotiated
fee is fixed. In between are the various incentive contracts, we consider firm-fixed-price con-
tracts as fixed-price, and the rest as variable-price. In our sample, about half of the contracts
are fixed-price.

The extent of competition, which can be determined by the contracting officers with
discretion, is observed in two dimensions. One is whether or not there was a full and open
competition, under which about half of the contracts in the sample were let. The other
dimension is the number of bidders. The average number is 7, but the median is 1. In our
sample, 53% of the contracts were awarded to a single bidder, and this ratio is even larger,
73%, when the entry restrictions are imposed. Putting it differently, $145 billion, about half
of the total amount, was obligated to contractors that won a contract by default during the
period of the study. This trend is not limited to our sample. For example, about 44% of the
total obligated amount during in FY 2010, $238 billion, is associated with contracts with a
single bidder and the average number of bidders during the period is 3.8 with median 2.

4 Identification

We signify each contract by unobserved type $\pi \in [\overline{\pi}, \overline{\pi}]$, with $0 < \pi < \overline{\pi} < 1$. We assume
that the unobserved type affects the probability that a contractor is efficient, $\pi_f$ and $\pi_u$ as

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According to the FAR (see 13.003(b)(1)), “acquisitions of supplies or services that have an anticipated
dollar value exceeding $3,000 but not exceeding $150,000 are reserved exclusively for small business concerns
and shall be set aside.” The upper limit can be $300,000 for certain supplies or services (see part 2.101 and
look for the definition of “simplified acquisition threshold”).
well as \( \kappa \). In particular, \( \pi_f(\pi) = \pi \) and \( 1 - \pi_u(\pi) = \zeta(1 - \pi) \). The remaining components of the model are assumed not to vary with \( \pi \).

We identify the model conditional on unobserved type \( \pi \) using the observed variables of a random sample of procurement contracts. For each contract, we observe the number of bidders, the mode of competition (i.e., whether or not eligibility restrictions were imposed), the contract type (i.e., fixed-price or variable-price), the base and the final prices, and the realized signal. Notice that the identity of the bidders that are favored by the procurer is not observed.

It is important to recover the unobserved type \( \pi \) for each variable-price contract. In doing so, we exploit Corollary 2, which holds if the observe signal \( s \) is such that \( f(s)/\overline{f}(s) < 1/\pi \).

Therefore, we make the following assumption on the distribution of \( s \):

**Assumption 3** The probability of \( f(s)/\overline{f}(s) < 1/\pi \) is zero.

The model is deterministic in the prediction of whether to exclude non-favored bidders given \( \pi \). To allow that for any \( \pi \) the probability that eligibility restrictions are imposed is not degenerate, we assume that the administrative cost of imposing eligibility restrictions, \( \eta \), is stochastic. We make the following assumption on the distribution of \( \eta \).

**Assumption 4** \( \eta \) is drawn from a distribution with full support, and is independent of \( \pi \).

Given that equilibrium is separating, the observed type of a contract informs us of the efficiency type of the contractor. Therefore, the distribution of the ex-post signal conditional on \( \beta \) can be directly identified from the observed distribution of the signal conditional on the contract type. As shown in Theorems 1 and 2, the efficient type chooses a fixed-price contract, and the inefficient type chooses a variable-price contract. Therefore, we identify \( f(\cdot) \) from the observed distribution of the signal for fixed-price contracts and \( \overline{f}(\cdot) \) from that for variable-price contracts.

Identification of the remaining components of the model proceeds as follows. We first focus on the exclusively competed contracts. For each of these contracts, we recover the unobserved type. Given the recovered unobserved type, we identify \( \psi(\cdot) \) off variable-price contracts. Then we identify the cost parameters, \( c(\pi) \) and \( \overline{\beta}(\pi) \), from the base price of the contracts. Using the identified parameters and the observed bidder arrival rate given \( \pi \), we identify \( \kappa(\pi) \). To identify the rest of the model, we use the information contained in the non-exclusively competed contracts as well. To do this, we first recover \( \pi \) for each variable-price, non-exclusively competed contract. By exploiting the variation in the number of bidders, we identify \( \rho \) and \( \zeta \) from the average base price of non-exclusively competed contracts. Lastly, we identify the distribution of a weighted sum of \( \delta(\pi) \) and \( \eta \) from the observed probability that a variable-price contract is exclusively competed conditional on \( \pi \). Note that we cannot separately identify \( \delta(\pi) \) and the location of the distribution of \( \eta \).

### 4.1 Recovering Unobserved Types for Exclusively Competed Contracts

First, we rank each fixed-price contract by the unobserved type \( \pi \), exploiting the monotone relationship between \( \pi \) and the price. The identified probability density function of the price of fixed-price contracts provides any percentile of type \( \pi \) conditional on the number of bidders for fixed-price contracts. Second, similarly we rank each variable-price contract by the unobserved type for any signal and number of bidders, exploiting the monotone relationship
between π and the dispersion of the price adjustment. The identified probability density function of the price adjustment provides any percentile of type π conditional the realized value of signal ad the number of bidders results for variable-price contracts. Third, we identify ϕₙ, defined as the odds ratio of a variable versus a fixed price outcome occurring for an exclusive auction with n bidders. Given these three identified objects, we recover π for each contract. We now explain each step in detail.

First, given n bidders, each exclusive fixed-price contract \( p_{e,n}(\pi) \) can be ranked by the probability that a contractor is efficient, with \( p'_{e,n}(\pi) < 0 \). Consider the price \( p \) as a random variable with conditional probability density function \( g_p(p|n) \) and distribution function \( G_p(p|n) \). Both are clearly identified and can be estimated using standard nonparametric methods. For future reference, let \( p_{n,k} \) denote the price matching the \((1 - k)\)th percentile, solving \( k = 1 - G_p(p_{n,k}|n) \). Note that each \( p_{n,k} \) is associated with a unique π.

For any given signal \( s \in S \) and n bidders, each variable-price contract can be ranked by type π, with \( |q'_{s,n}(\pi)| > 0 \), where \( \bar{q}_{s,n} \) denotes the absolute value of the ex-post price adjustment given signal s. Consider the absolute value of the price adjustment \( \bar{q} \) as a random variable with conditional probability density function \( g_{\bar{q}}(\bar{q}|s,n) \) and distribution function \( G_{\bar{q}}(\bar{q}|s,n) \). Clearly both distributions are identified and can be estimated nonparametrically. For future reference, let \( \bar{q}_{s,n,k} \) denote the \( k^{th} \) percentile, solving \( k = G_{\bar{q}}(\bar{q}_{s,n,k}|s,n) \). Note that each \( \bar{q}_{s,n,k} \) is associated with a unique π.

Given n, define \( \varphi_n \) as the quotient of two probabilities, the probability that an exclusively competed contract is fixed-price and the probability that it is variable-price. Clearly \( \varphi_n \) is identified and has a sample analogue. If π was constant for a given n, then it could be obtained from the equation:

\[
\pi_n = 1 - \left[ 1/(1 + \varphi_n) \right]^{\frac{1}{n}}.
\]

However πₙ is a mixture of probabilities in our model, not a primitive, so this equation is of no use.

Let \( r_{n,k} \) denote the probability that an exclusive procurement auction of type πₖ with n bidders results in a fixed-price contract. This can be defined as:

\[
r_{n,k} = \frac{\varphi_n g_p(p_{n,k}|n)}{\varphi_n g_p(p_{n,k}|n) + \int g_{\bar{q}}(\bar{q}_{s,n,k}|s,n) \bar{q}(s) ds}.
\] (9)

This can be seen by analyzing a discrete case where there are \( K \) unobserved types and then taking the limit as \( K \to \infty \). There are exactly \( K \) values of \( p_n \) for n bidders, and we define \( G_{\bar{q}}^{(K)}(k|n) \) as the distribution function:

\[
G_{\bar{q}}^{(K)}(k|n) \equiv \sum_{k'=1}^{k} \text{Pr} \left( p = p_{n,k'}|n \right).
\]

Similarly for each value of \( s \in S \), there are exactly \( K \) values of \( \bar{q} \in \{\bar{q}_1, \ldots, \bar{q}_K\} \) and an
associated probability distribution function conditional on the number of bidders:

\[ G_q^{(K)}(k | s, n) \equiv \sum_{k'=1}^{k} \Pr \left( \frac{q}{p} = \frac{q}{s, n, k'} | s, n \right). \]

Given that \( p'(\pi) < 0 \) and \( q'(\pi) > 0 \) for any \( s \) and \( n \), the \((1 - k)^{th}\) order statistic of \( p_n \) and the \(k^{th}\) order statistic of \( q_{s, n} \) for any \( s \) are both associated with the same \( \pi \) type, say \( \pi_k \). Note too that the unconditional probability of a fixed price contract given \( n \) bidders is \( \varphi_n / (1 + \varphi_n) \). Given these observations, the joint probability that an exclusive auction with \( n \) bidders is of type \( \pi_k \) and results in a fixed-price contract is:

\[ \frac{\varphi_n}{1 + \varphi_n} \left[ \frac{G_q^{(K)}(k | n) - G_q^{(K)}(k - 1 | n)}{1 - (1 - r_{n,k})^{1/2}} \right], \]

which converges to \( \varphi_n g(p_n, k | n) / (1 + \varphi_n) \) as \( K \to \infty \). Similarly, the joint probability that an exclusive auction with \( n \) bidders is of type \( \pi_k \) and results in a variable-price contract is:

\[ \frac{1}{1 + \varphi_n} \left[ \int \left\{ G_q^{(K)}(k | s, n) - G_q^{(K)}(k - 1 | s, n) \right\} \tilde{f}(s) ds \right], \]

which converges to \( \int \tilde{g}(n, k | s, n) d\tilde{F} / (1 + \varphi_n) \) as \( K \to \infty \). By noting that \( r_{n,k} \) is the conditional probability that an exclusive auction of type \( \pi_k \) with \( n \) bidders results in a fixed-price contract, we establish equation (9). Because the right hand side of this equation is all identified, we can solve for \( r_{n,k} \) for any \( k \) and \( n \).

We have identified \( r_{n,k} \) for any \( n \), and the following formula represents the one-to-one mapping between \( \pi_k \) and \( r_{n,k} \):

\[ \pi_k = 1 - (1 - r_{n,k})^{1/2}. \]

Noting that \( r_{n,k} \) is defined for each \( n \), we use a minimum distance estimator to obtain a more efficient estimator of \( \pi_k \). This ultimately identifies the unobserved type \( \pi \) for each exclusively competed contract in the data.

### 4.2 Liquidity Cost Function

Given the identified \( \pi \) for each exclusive variable-price contract, we identify the ex-post price adjustment, \( q(s, \pi) \) for any signal \( s \) and \( \pi \). Using equation (4) in Theorem 1, we identify \( \psi'(\cdot) \) for a given \( \pi \). To see this, we rewrite the equation here:

\[ \psi'(q(s, \pi)) \left[ 1 - \pi \frac{\tilde{f}(s)}{\tilde{F}(s)} \right] = 1 - \pi. \]

Note that \( \pi, f(s), \tilde{f}(s), \) and \( q_s(\pi) \) for any \( s \) have been identified. Given that the above equation holds for any \( \pi \), we use a minimum distance estimator to obtain a more efficient estimator of \( \psi'(\cdot) \). Then using the assumption that \( \psi(0) = 0 \), we identify \( \psi(\cdot) \).

### 4.3 Cost Parameters

The deterministic part of the cost for an inefficient contractor to completing a contract of type \( \pi \) is \( c + \beta \). Inefficient contractors select a variable-price contract, and the base price of
the contract is characterized in equation (3) in Theorem 1. Rewriting the equation:

\[ c + \beta = \bar{p}(\pi) + \int \psi[q(s, \pi)] f(s) ds. \] (10)

Note that \( \pi \) has been identified for each exclusively competed variable-price contract, and accordingly we observe \( \bar{p}(\pi) \) for any \( \pi \). The last term in the above equation consists of \( \psi(\cdot) \), \( q(s; \pi) \), and \( f(\cdot) \), all of which have been identified earlier. This allows us to identify \( c + \beta \).

To separately identify \( c \) and \( \beta \), we use the price of exclusively competed fixed-price contracts. By taking the difference between equations (2) and (3) and solving for \( \beta \), we have the following equation:

\[ \beta = \int \psi[q(s, \pi)] dF + \frac{[1 - (1 - \pi)^n] \left[ \bar{p}(\pi) - p_{e,n}(\pi) \right] + \pi (1 - \pi)^{n-1} \int \psi[q(s, \pi)] dF}{1 - (1 - \pi)^n}, \] (11)

for \( n > 1 \) and any \( \pi \). This establishes a constructive proof for the identification of \( \beta \).

4.4 Per-bidder Bid Processing Cost

Recall that the optimal bid arrival rate when only favored contractors are allowed to participate given \( \pi \), \( \lambda_e(\pi) \), minimizes the expected total procurement cost as written in equation (7). By taking the first order condition:

\[ \kappa(\pi) = \sum_{n=1}^{\infty} \left\{ [1 - (1 - \pi)^n] p_{e,n}(\pi) + (1 - \pi)^n \bar{p}(\pi) + \mathbb{E}(q(\pi)|\beta) \right\} \] (12)

\[ \times \frac{(\lambda_e(\pi) - n)(\lambda_e(\pi) - 1)^{n-2} e^{-\lambda_e(\pi)+1}}{(n-1)!}. \] (13)

Note that \( \lambda_e(\pi) \) can be identified from the average number of bidders to an exclusive procurement auction given \( \pi \). Therefore, all remaining arguments in the right hand side are identified, and so is \( \kappa(\pi) \).

4.5 Ratio of Favored Contractors and Relative Likelihood that a Non-favored Contractor is Inefficient

We first recover \( \pi \) for non-exclusively competed variable-price contracts. Note that \( q(s; \pi) \) satisfies equation (4) for any variable-price contract under Assumption 3. Rewriting the equation, we can solve for \( \pi \) as follows:

\[ \pi = \frac{\psi'(q) f(s)/\bar{f}(s) - 1}{\psi'(q)}. \]

As all arguments in the right hand side of the above equation have been identified, \( \pi \) is identified for each variable-price under Assumption 3. Given this assumption, we identify the probability density function of \( \pi \) conditional on that a non-exclusively competed procurement auction with \( n \) bidders results in a variable-price contract for any \( n \) and \( \pi \). By introducing
two variables, \( d \) indicating that the contract is variable-price and \( e \) indicating that eligibility restrictions are imposed on the contract, this function can be denoted by \( g(\pi|d = 1, e = 0, n) \).

Note that the following equality holds for any \( n \) and \( \pi \):

\[
g(\pi|d = 1, e = 0, n) \Pr(d = 1|e = 0, n) = \Pr(d = 1|e = 0, n, \pi)g(\pi|e = 0, n).
\]

In the above equation, \( g(\pi|d = 1, e = 0, n) \) has been identified and \( \Pr(d = 1|e = 0, n) \) is directly identified from the data. \( \Pr(d = 1|e = 0, n, \pi) \) can be expressed as a function of \( \rho \) and \( \zeta \), as well as the parameters of the model that have already been identified as follows:

\[
\Pr(d = 1|e = 0, n, \pi) = \sum_{n_f=1}^{n} (1 - \pi)^n \zeta^{n-n_f} \left( \frac{n-1}{n_f-1} \right) \rho^{n_f-1} (1 - \rho)^{n-n_f}. \quad (14)
\]

Therefore, \( g(\pi|e = 0, n) \) can be written as:

\[
g(\pi|e = 0, n) = \frac{g(\pi|d = 1, e = 0, n) \Pr(d = 1|e = 0, n)}{\sum_{n_f=1}^{n} (1 - \pi)^n \zeta^{n-n_f} \left( \frac{n-1}{n_f-1} \right) \rho^{n_f-1} (1 - \rho)^{n-n_f}}. \quad (15)
\]

To identify \( \rho \) and \( \zeta \), recall that the model predicts the base prices of the equilibrium contracts for the favored, as in equation (3) and (5), as a function of \( \rho \) and \( \zeta \), as well as the parameters that have been identified, and that for the non-favored, which is the lowest possible price, \( c \). The expected base price of type \( \pi \) contracts that are competed without exclusion among \( n \) bidders when \( n_f \) of them are favored is a weighted average of these prices.

\[
E(p|e = 0, n, n_f, \pi) = c + \pi(1 - \pi)^{n-1} \zeta^{n-n_f} \left[ \frac{\beta}{\beta - \int \psi(g(s; \pi)) \bar{I}(s) - f(s)} ds \right] \nonumber \\
+ (1 - \pi)^n \zeta^{n-n_f} \left[ \frac{\beta}{\beta - \int \psi(g(s; \pi)) \bar{I}(s)} ds \right].
\]

In the data, we observe the expected base price for non-exclusively competed contracts with \( n \) bidders, \( E(p|e = 0, n) \), which can be written as:

\[
E(p|e = 0, n) = \int \sum_{n_f=1}^{n} \left( \frac{n-1}{n_f-1} \right) \rho^{n_f-1} (1 - \rho)^{n-n_f} E(p|e = 0, n, n_f, \pi)g(\pi|e = 0, n) d\pi. \quad (16)
\]

Note that we have the above equation for each number of bidders, \( n \), with two unknowns, \( \rho \) and \( \zeta \).

### 4.6 Benefit from Selecting a Favored Contractor

To identify \( \delta \), we identify the probability that a contract with type \( \pi \) is competed with eligibility restrictions, \( \Pr(e = 1|\pi) \), which can be written as follows:

\[
\Pr(e = 1|\pi) = \frac{g(\pi|e = 1) \Pr(e = 1)}{g(\pi|e = 1) \Pr(e = 1) + g(\pi|e = 0) \Pr(e = 0)}.
\]
We have identified $g(\pi|e = 1)$ and $g(\pi|e = 0)$, and $Pr(e = 1)$ is directly identified from the data.

Now we exploit the model prediction on $Pr(e = 1|\pi)$ for any $\pi$. The procurer chooses to impose eligibility restrictions for type $\pi$ procurement if the total expected procurement cost of doing so is cheaper than otherwise, i.e.,

$$
\min_{\lambda} V_e(\lambda, \pi) \leq \min_{\lambda} V_o(\lambda, \pi),
$$

(17)

where $V_e(\lambda, \pi)$ and $V_o(\lambda, \pi)$ are defined in equations (7) and (8). We identify $\lambda_e(\pi)$, which minimizes $V_e(\lambda, \pi)$, from the average number of bidders to an exclusively competed procurement of type $\pi$. For a non-exclusively competed procurement, $\lambda_o(\pi)$, which minimizes $V_o(\lambda, \pi)$, by deriving $Pr(n|e = 0, \pi)$ for all $n$ as follows. First, given that we have recovered $\pi$ for all non-exclusively competed variable-price contracts, we identify $Pr(n|d = 1, e = 0, \pi)$. This can be written as:

$$
Pr(n|d = 1, e = 0, \pi) = \frac{Pr(n, d = 1|e = 0, \pi)}{Pr(d = 1|e = 0)} = \frac{Pr(n|e = 0, \pi) Pr(d = 1|n, e = 0, \pi)}{Pr(\pi, d = 1|e = 0)/g(\pi|e = 0)}.
$$

The above two equalities are based on the Bayes rule, and from the last equality, we can solve for $Pr(n|e = 0, \pi)$:

$$
Pr(n|e = 0, \pi) = \frac{Pr(n|d = 1, e = 0, \pi) Pr(\pi, d = 1|e = 0)}{Pr(d = 1|n, e = 0, \pi) g(\pi|e = 0)}.
$$

Note that $Pr(\pi, d = 1|e = 0)$ is identified directly from the data, and $Pr(d = 1|n, e = 0, \pi)$ and $g(\pi|e = 0)$ have been identified as in equations (14) and (15). Given $Pr(n|e = 0, \pi)$ for any $n$, we can derive $\lambda_o(\pi)$. By plugging in $\lambda_e(\pi)$ and $\lambda_o(\pi)$ in inequality (17), eligibility restrictions are imposed if

$$
V_e[\lambda_e(\pi), \pi] \leq V_o[\lambda_o(\pi), \pi],
$$

where $V_e[\lambda_e(\pi), \pi]$ and $V_o[\lambda_o(\pi), \pi]$ can be written as follows:

$$
V_e[\lambda_e(\pi), \pi] = \sum_{n=1}^{\infty} \left\{ \left[ 1 - (1 - \pi)^n \right] P_{e,n}(\pi) + (1 - \pi)^n \left[ \bar{p}(\pi) + E(q(\pi)B) \right] \right\} Pr(n; \lambda_e(\pi)) + k(\pi)\lambda_e(\pi) - \delta + \eta,
$$

where $Pr(n; \lambda_e(\pi))$ is derived from the shifted Poisson distribution of $\lambda_e(\pi)$. As for $V_o[\lambda_o(\pi), \pi]$: 

$$
V_o[\lambda_o(\pi), \pi] = \kappa(\pi)\lambda_o(\pi) + \sum_{n=1}^{\infty} \sum_{n_f=1}^{n} Pr(n_f, n; \lambda_o(\pi) \{ (1 - \pi)^n [1 - ((1 - \pi)\zeta)^{n-n_f} - \delta P_f(n_f, n) \}
$$

where $Pr(n_f, n; \lambda)$ and $P_f(n_f, n)$ are defined in equations (1) and (6). Note that all arguments in $V_e[\lambda_e(\pi), \pi]$ and $V_o[\lambda_o(\pi), \pi]$ have been identified except $\delta$ and the distribution of $\eta$. We can recover the distribution of $\eta - \delta \left[ 1 - \sum_{n=1}^{\infty} \sum_{n_f=1}^{n} Pr(n_f, n; \lambda_o(\pi)) P_f(n_f, n) \right]$. 

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5 Results

5.1 Nonparametric Estimator

We closely follow the identification arguments to construct the nonparametric estimator. We assume that all components of the model to be estimated depend on the observed characteristics of a procurement project. These characteristics include (i) the number of unique bidders that won any of the procurement contracts of the industry in the state during the period of study, (ii) the average base price of the contracts of the industry in the state during the same period, and (iii) the base duration of the project.

5.2 Findings

We provide the estimation results conditional on the median value of the observed characteristics of a procurement project. The median values of these characteristics are (i) 5 unique bidders that won any of the procurement contracts of the industry in the state during the period of study, (ii) $1.1 million of the average base prices of the contracts of the industry during the same period, and (iii) 364 days of base duration. The average size of the total payment of such contracts is $2.1 million. This can be divided into two parts: one is the sum of the base price and the ex-post price adjustment related to signal, and the other is the ex-post adjustment related to cost shocks. The average size of the former is $1.5 million and that of the latter is $0.5 million. Note that the findings here are confined to these contracts. The results are preliminary and the standard errors will be provided in the next version of this draft.

The distribution of the delay divided by the base duration varies by type, as shown in Figure 2. The estimated liquidity cost function, \( \psi(\cdot) \), can be found in Figure 3. The liquidity

![Figure 2: Signal Distributions by Type](image-url)

- Efficient Type
- Inefficient Type
cost function is estimated to be increasing and concave. The unconditional distribution of

Figure 3
Liquidity Cost Function, $\psi(·)$

the unobserved type $\pi$, or the ratio of efficient bidders among favored contractors is shown in figure 4. The average ratio is 0.59, and that among non-favored contractors is estimated to be slightly smaller, 0.58. This is an important finding because this implies that favored contractors are very similar to or slightly better than non-favored ones in terms of ex-ante efficiency. Our estimates indicate that absent cost shocks, it takes $0.8$ million for efficient contractors to complete a procurement project, while it takes $1.8$ million for inefficient contractors, more than twice of the cost of efficient contractors.

We find that the per-bidder bid processing and solicitation cost is $85,723$, and the resulting average bid processing cost per a contract is $196,195$. This is about $9\%$ of the average size of the total payment. The average number of bidders for exclusively competed contracts is 3, while that for non-exclusively competed ones is 7.5. This implies a relatively large administrative cost of holding a full and open competition.

These findings point to a conclusion that imposing eligibility restrictions can be cost-effective to the government. To see this, we consider a counter-factual scenario where only favored contractors are allowed to bid. We find that by imposing eligibility restrictions to the non-exclusively competed contracts, the government can reduce the expected payment to contractors by 22%. By limiting the entry of non-favored contractors, who are ex-ante slightly less likely to be efficient than the favored counterparts, the government can induce a lower price from the efficient, favored contractors than otherwise. At the same time, the government forgoes a lower price from the efficient, non-favored contractors. Which of these opposing effects dominates is partially determined by how likely it is for a non-favored contractor to win a full and open competition. We find that although the ratio of favored contractors is estimated to be 52%, the probability that a favored contractor wins a non-exclusively competed contract is 97%. Therefore, the potential cost reduction from hiring an efficient,
non-favored contractor does not realize very often, which further supports the use of entry restrictions.

Although the procurement cost can decrease by imposing entry restrictions, the government holds a full and open competition because of various administrative or political costs that are associated with limiting competition, which is captured by $\eta$ in the model. The nature of such costs is to be further investigated.

6 Conclusion

In this paper, we study the determinants of eligibility requirements and the number of participating bidders in government procurement auctions. To understand the effects of the restrictions of competition on the total cost of government procurement, we develop, identify, and estimate a principal-contractor model in which the government selects a contractor to undertake a project. We consider three reasons why restricting entry could be beneficial to the government: by decreasing bid processing and solicitation costs, by increasing the chance of selecting a favored contractor and consequently reaping benefits from the favored contractor, and by decreasing the expected amount of price to the winning contractor. Using our estimates, we decompose the effects of these three sources of entry restrictions, and quantify the effects of the eligibility restrictions on the total cost of procurement.
Appendix

A.1. Proof of Theorem 1

The following lemma shows that variable price contracts are only offered in conjunction with fixed price contracts, not by themselves.

**Lemma 1** The equilibrium contract menu includes a fixed price contract.

**Proof.** The proof is by a contradiction argument. Suppose to the contrary that every contract on the menu is a variable price contract. Denote by \( \{ p, q(s) \} \) one of the contracts on the menu. There are three cases to consider.

First, suppose \( E \{ \psi [ q(s) ] | \beta \} > E \{ \psi [ q(s) ] | \beta \} \). Then, the procurer can offer an additional, fixed price contract of \( p' = p + E \{ \psi [ q(s) ] | \beta \} \). The inefficient contractor would accept the contract, but the efficient contractor will not. By strict concavity of \( \psi(\cdot) \), we have \( E \{ \psi [ q(s) ] | \beta \} < E \{ q(s) | \beta \} \). Therefore, the expected payoff of the procurer increases when the inefficient contractor accepts the fixed price contract with any positive probability.

Second, suppose \( E \{ \psi [ q(s) ] | \beta \} > E \{ q(s) | \beta \} \). The procurer can offer an additional, fixed price contract of \( p' = p + E \{ \psi [ q(s) ] | \beta \} \). The efficient contractor would accept the contract, but the inefficient contractor will not. Since \( E \{ \psi [ q(s) ] | \beta \} < E \{ q(s) | \beta \} \), the expected payoff of the procurer increases when the efficient contractor accepts the new contract with any positive probability.

Lastly, suppose \( E \{ \psi [ q(s) ] | \beta \} = E \{ q(s) | \beta \} \). The procurer can offer instead a fixed price contract of \( p' = p + E \{ \psi [ q(s) ] | \beta \} \). Both types of contractor would accept the contract. Since \( E \{ \psi [ q(s) ] | \beta \} < E \{ q(s) | \beta \} \), the expected payoff of the procurer increases when either or both contractor types accept the new contract with any positive probability. ■

The next lemma shows that if there are multiple fixed price contracts in the menu, then the procurer can achieve the same expected payoff with only one fixed price contract in the menu. Furthermore, if there is only one contract in the menu, it must be a fixed-price contract.

**Lemma 2** There is exactly one fixed price contract on the equilibrium menu.

**Proof.** Suppose there are multiple contracts in the menu. Among them, if there are multiple, distinct fixed price contracts, then one of them will dominate the rest, and be the only one chosen by the contractors.

Suppose there is only one contract in the menu. If the contract is variable contract, then we have shown that in Lemma 1 either offering another fixed-price contract or replacing the variable contract with a fixed-price contract is more profitable to the principal. Therefore, if there is only one contract in the menu, it must be a fixed price contract. ■

The next lemma shows that if it is a best response for the inefficient contractor to accept a fixed price contract in equilibrium, then that is the only contract offered on the equilibrium menu.

**Lemma 3** If a fixed price contract is on the equilibrium menu, and the inefficient contractor would maximize his expected payoff by accepting it, then the only contract on the menu is the fixed price contract \( c + \beta \).
Proof. Let $p$ denote the fixed price contract in question. Suppose there is another contract in the menu. By Lemma 2, that contract is a variable contract, denoted by $\{p', q(\cdot)\}$. Suppose this variable contract is equally attractive to the inefficient contractor as the fixed price contract. Then, $p' + E\{\psi[q(s)]|\beta\} = p$. Since $E\{\psi[q(s)]|\beta\} < E[q(s)|\beta]$, the procurer earns less rent than what would be obtained from the inefficient contractor accepting the fixed price contract.

Now that the variable contract is not chosen by the inefficient contractor, it must be chosen by the efficient contractor with any positive probability. This requires that $p' + E\{\psi[q(s)]|\beta\} \geq p$. Since $E\{\psi[q(s)]|\beta\} < E[q(s)|\beta]$, the procurer earns less than the rent from the efficient contractor with the fixed price. This proves that if a fixed-price contract is acceptable to the inefficient contractor, the procurer does not offer a variable-price contract in equilibrium.

By Lemma 2, there is exactly one fixed-price contract on the equilibrium menu. If there is more than one contract on the equilibrium menu, by Lemma 3, the fixed price contracts are only accepted by the efficient contractor. The next result, Lemma 4, establishes that in equilibrium the efficient contractor never accepts a variable-price contract.

Lemma 4 The efficient contractor accepts the fixed price contract on the equilibrium menu with unit probability.

Proof. By Lemma 3, the efficient contractor would maximize his payoff by accepting the fixed-price contract, which we denote by $p$. So if a variable-price contract of the form $\{p', q(\cdot)\}$ is offered, incentive compatibility requires: $p \geq p' + E\{\psi[q(s)]|\beta\}$. If a strict inequality holds for all variable contracts on the menu, the result follows immediately. Supposing an equality holds for a variable-price contract, then the procurer garners $E\{q(s) - \psi[q(s)]|\beta\}$ less rent if the efficient contractor selects the variable-price contract than the fixed-price contract. Therefore, in equilibrium the principal does not offer a variable contract that an efficient contractor would accept with any positive probability.

We conclude that in equilibrium either there is one fixed price or there are several contracts in the menu, which includes one fixed price contract that the efficient contractors select into, and one or more variable contracts that the inefficient contractors select into. Using the strict convexity of the constraint set we show that only one variable price contract will be offered.

Lemma 5 The equilibrium menu of contracts is either a single fixed-price contract $c + \beta$ or a menu of two contracts comprising a fixed-price contract, denoted by $p$, that the efficient contractor selects, and a variable-price contract, denoted by $(p, q(s))$, that the inefficient contractor accepts.

Proof. We consider the following sub-problem of the procurer. The procurer minimizes the expected cost of attracting the inefficient contractor subject to the constraint that the efficient contractor is deterred from taking the contract given an outside option that provides an exogenously determined payoff. The IR constraint for the inefficient contractor is:

$$p + E\{\psi[q(s)]|\beta\} - (c + \beta) \geq 0,$$
and the IC constraint for the efficient contractor is:

\[ p \geq \bar{p} + E \{ \psi [q(s)] | \bar{\beta} \} . \]

By inspection the IR constraint holds with equality. (If the IR constraint is not binding, then the value of objective function can be reduced by reducing \( \bar{p} \).) Solving for \( \bar{p} \) reduces the problem to choosing \( q(s) \) to minimize:

\[ c + \bar{\beta} + E [q(s) | \bar{\beta}] - E \{ \psi [q(s)] | \bar{\beta} \} \]

subject to the constraint:

\[ p \geq c + \bar{\beta} - E \{ \psi [q(s)] | \bar{\beta} \} + E \{ \psi [q(s)] | \bar{\beta} \} . \]

There are two cases to consider. First suppose the constraint is not binding. Since \( \psi(\cdot) \) is concave, the objective function is convex, so the unconstrained problem has a unique solution, characterized by the first order condition \( \psi'(q(s)) = 1 \) for all \( s \), implying \( q(s) = 0 \). Substituting the unconstrained solution \( \bar{p} = c + \bar{\beta} \) into the IC constraint for the efficient contractor it follows that \( p \geq c + \bar{\beta} \) in this case. Otherwise the constraint is binding.

Alternatively assume the constraint is met with equality. Then, the above constrained minimization problem can be written to be an unconstrained minimization problem with the objective function

\[ p + E [q(s) | \bar{\beta}] - E \{ \psi [q(s)] | \bar{\beta} \} \]

Since \( \psi(\cdot) \) is concave, the objective function is convex, so the above problem also has a unique solution characterized by its first order condition \( \psi'(q(s)) f(s | \bar{\beta}) = f(s | \bar{\beta}) \). Since the solution to the subproblem is unique, all the variable contracts solving the subproblems are identical, thus proving only one (distinct) variable contract is offered to the inefficient contractor.

In the following lemma, we show that under Assumption 1, offering a menu of two contracts is optimal for the procurer.

**Lemma 6** If Assumption 1 holds, the equilibrium is separating. Otherwise, the equilibrium is pooling.

**Proof.** Suppose such \( S \) does not exist. This implies that there exists no \( \eta > 0 \) such that \( S \equiv \{ s : f(s|\beta) - f(s|\beta) > \eta \} \). It is because by construction, \( \gamma_1 < \gamma_2 \). Therefore, \( \Pr \{ s : f(s|\beta) = f(s|\beta) \} = 1. \) For any ex-post price adjustment \( q(\cdot) \), \( E(q(\cdot)|\beta) = E(q(\cdot)|\bar{\beta}). \) Hence the individual rationality constraint for the inefficient contractor implies

\[ c + \bar{\beta} \leq \bar{p} + E(q(\cdot)|\bar{\beta}) = \bar{p} + E(q(\cdot)|\beta) \leq p. \]

The last inequality holds due to the incentive compatibility constraint for the efficient contractor. Therefore the efficient contractor extracts rent of at least \( \bar{\beta} \). The pooling equilibrium is more efficient than inducing the inefficient contractor to take a variable contract because the procurer does not pay a liquidity premium of \( E(q(\cdot)|\bar{\beta}) \) to the inefficient contractor in the pooling case.
Now suppose $S$ exists. In that case, we show by construction that it is less profitable to offer one fixed-price contract than a menu of two contracts, one fixed-price and the other variable-price. For any $\Delta > 0$ choose $\mu(\Delta)$ for a two-part variable contract in which $p = c + \beta$ and:

$$q(s) = \begin{cases} 
\Delta & \text{if } s \in S, \\
\mu(\Delta) & \text{if } s \notin S,
\end{cases}$$

where

$$\gamma_2 \psi(\Delta) + (1 - \gamma_2) \psi(\mu(\Delta)) = 0.$$  

Note that the above equation implies that $\mu(\Delta) < 0$. Because $\psi(\cdot)$ is strictly increasing, $\mu(\Delta)$ is uniquely defined by the equation:

$$\mu(\Delta) = \psi^{-1}\left[\frac{-\gamma_2}{1 - \gamma_2} \psi(\Delta)\right],$$

and is twice differentiable with:

$$\mu'(\Delta) = \frac{-\gamma_2}{1 - \gamma_2} \frac{\psi'(\Delta)}{\psi'(\mu(\Delta))},$$

where $\mu(0) = 0$. The fixed contract takes the form:

$$p = c + \beta + \gamma_1 \psi(\Delta) + (1 - \gamma_1) \psi(\mu(\Delta)) = c + \beta + \gamma_1 \psi(\Delta) - (1 - \kappa_1) \left(\frac{\gamma_2}{1 - \kappa_2}\right) \psi(\Delta).$$

Note that the incentive compatibility constraint is satisfied with equality by the efficient contractor and strict inequality by the inefficient contractor because $\gamma_1 < \gamma_2$. Similarly, the participation constraint is satisfied with equality by the inefficient contractor and strict inequality by the efficient contractor as long as $\Delta > 0$ is small enough. The expected price by the procurer is:

$$E(T|\Delta) = c + \beta + \pi \left[\gamma_1 \psi(\Delta) + (1 - \gamma_1) \psi(\mu(\Delta))\right] + (1 - \pi) \left[\gamma_2 \Delta + (1 - \gamma_2) \mu(\Delta)\right],$$

$$= c + \beta + \pi \left[\kappa_1 \psi(\Delta) - \frac{(1 - \gamma_1) \gamma_2}{1 - \gamma_2} \psi(\Delta)\right] + (1 - \pi) \left[\gamma_2 \Delta + (1 - \gamma_2) \mu(\Delta)\right].$$

We now show this expression is decreasing in the neighborhood of $\Delta = 0$. Differentiating with respect to $\Delta$ yields:

$$\frac{\partial E(T|\Delta)}{\partial \Delta} = \pi \left[\gamma_1 \psi'(\Delta) - \frac{(1 - \gamma_1) \gamma_2}{1 - \gamma_2} \psi'(\Delta)\right] + (1 - \pi) \left[\gamma_2 - \gamma_2 \frac{\psi'(\Delta)}{\psi'(\mu(\Delta))}\right].$$

Evaluating $\frac{\partial E(T|\Delta)}{\partial \Delta}$ at $\Delta = 0$ gives us:

$$\frac{\partial E(T|\Delta = 0)}{\partial \Delta} = \pi \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} < 0,$$

which shows that a fixed-price contract fails to meet a first order necessary condition. ■
Using the lemmas, we provide the proof for Theorem 1 below.

**Proof.** The principal minimize the expected price:

$$[1 - (1 - \pi)^n] \bar{p} + (1 - \pi)^n \left[ \bar{p} + E(q(s)|\beta) \right].$$

At optimum, the participation constraint for the inefficient contractors must hold in equality:

$$\bar{p} = c + \beta - \int \psi(q(s))\bar{f}(s)ds,$$

which is the same as equation (3) in Theorem 1. Given that the incentive constraint for the efficient contractors is binding at the optimum, we have the following equation.

$$\phi(n)(p - c) = \phi(n)\{\bar{p} + E(\psi(q)|\beta) - c\}.$$

In the above equation, $\phi(n)$ and $\phi(n)$ denote the winning probabilities conditional on the choice of contract. If a contractor chooses the fixed-price contract, the winning probability, $\phi(n)$, is

$$\phi(n) = \sum_{k=0}^{n-1} \frac{(n-1)^k(1 - \pi)^{n-1-k}}{k+1} = \frac{1 - (1 - \pi)^n}{n\pi},$$

assuming that other participating contractors follow the equilibrium strategy. If the contractor chooses the variable-price contract instead, the winning probability, $\phi(n)$, becomes:

$$\phi(n) = \frac{(1 - \pi)^{n-1}}{n}.$$

By substituting $\bar{p}$ using equation (3), $p$ can be rewritten as:

$$p = c + \phi(n) \frac{\bar{f}(s)}{\phi(n)} \left( \beta - \int \psi(q(s))\bar{f}(s) - f(s) ds \right)$$

$$= c + \pi(1 - \pi)^{n-1} \left( \beta - \int \psi(q(s))\bar{f}(s) - f(s) ds \right),$$

where the last equality is derived from using the definition of $\phi(n)$ and $\phi(n)$. We substitute the above equations for $p$ and $\bar{p}$ in the expected price of the procurer:

$$[1 - (1 - \pi)^n] \left[ c + \pi(1 - \pi)^{n-1} \left( \beta - \int \psi(q(s))\bar{f}(s) - f(s) ds \right) \right]$$

$$+ (1 - \pi)^n \left[ c + \beta + \int [q(s) - \psi(q(s))]\bar{f}(s) ds \right].$$

To solve for $q(s)$ for any $s$, we take the first order condition:

$$\pi(1 - \pi)^{n-1}\psi'(q(s))[-\bar{f}(s) + f(s)] + (1 - \pi)^n[1 - \psi'(q(s))]/\bar{f}(s) = 0.$$
Rearranging terms after diving both sides of the above equation by \((1 - \pi)^n f(s|\overline{\pi})\) gives us:

\[
\psi'(q(s)) \left[ 1 - \pi \frac{f(s)}{\overline{f}(s)} \right] = 1 - \pi.
\]

Notice that this is the same as equation (4) in Theorem 1. ■

**A.2. Proof of Theorem 2**

**Proof.** The procurer minimizes the expected price to a winning contractor given \(n_f\) favored bidders and \(n_u\) unfavored bidders. Given that unfavored contractor will receive a fixed-price contract, \(c\), the procurer’s problem is to minimize the expected price to favored contractors:

\[
[1 - (1 - \pi_f)^{n_f}] p + (1 - \pi_f)^{n_f} (1 - \pi_u)^{n_u} \{ \overline{p} + \mathbb{E}(q(s)|\overline{\pi}) \}.
\]  \hspace{1cm} (18)

Note that the probability that the winning contractor is inefficient and favored depends not only the average efficiency and the number of favored bidders, \((\pi_f, n_f)\), but also those of unfavored bidders, \((\pi_u, n_u)\).

The individual rationality condition must be binding for inefficient contractors, which characterizes \(\overline{p}\):

\[
\overline{p} = c + \overline{\beta} - \mathbb{E}[\psi(q(s))|\overline{\pi}].
\]  \hspace{1cm} (19)

Note that this equation is the same as equation (3) in Theorem 1. The incentive compatibility constraint for efficient contractors must hold at equality as well:

\[
\phi(n_f)\{p - c\} = \phi(n_f, n_u)\{p + \mathbb{E}(\psi(q(s)|\overline{\beta}) - c\},
\]

where \(\phi(n_f)\) (or \(\phi(n_f, n_u)\)) denotes the probability of winning for a favored contractor if he selects the fixed-price (or variable-price) contract.

\[
\phi(n_f) = \sum_{k=0}^{n_f-1} \binom{n_f - 1}{k} \pi_f^k (1 - \pi_f)^{n_f - 1 - k} \frac{1 - (1 - \pi_f)^{n_f}}{n_f \pi_f},
\]

\[
\phi(n_f, n_u) = \frac{(1 - \pi_f)^{n_f-1} (1 - \pi_u)^{n_u}}{n_f}.
\]

Using these definitions of \(\overline{\phi}(n_f)\) and \(\phi(n_f, n_u)\), equation (19), and the above incentive compatibility constraint, the amount of the price of the fixed-price contract, \(p\), is characterized as follows:

\[
p = c + \frac{\pi_f (1 - \pi_f)^{n_f-1} (1 - \pi_u)^{n_u}}{1 - (1 - \pi_f)^{n_f}} \left[ \overline{\beta} - \int \psi(q(s))[\overline{f} - f(s)] ds \right].
\]

By taking the derivative of the objective function (18) with respect to \(q(s)\) for any \(s\), we have the following equation:

\[
\pi_f (1 - \pi_f)^{n_f-1} (1 - \pi_u)^{n_u} \psi'(q(s)) \left[ -\overline{f}(s) + f(s) \right] + (1 - \pi_f)^{n_f} (1 - \pi_u)^{n_u} [1 - \psi'(q(s))] \overline{f}(s) = 0.
\]
Rearranging terms after diving both sides of the above equation by \((1 - \pi_f)^n(1 - \pi_u)^n\) gives us:

\[
\psi'(q(s)) \left[ 1 - \pi f \frac{f(s)}{f(s)} \right] = 1 - \pi_f.
\]

Notice that this is the same as equation (4) in Theorem 1, where \(\pi\) is replaced with \(\pi_f\). ■

A.3. Proof of Corollaries

**Corollary 1 Proof.** We start with equation (2) which characterizes \(p\). Note that the last term in the equation,

\[
\bar{\beta} - \int \psi(q(s)) [\bar{f}(s) - f(s)] ds,
\]

must be nonnegative; otherwise, the individual rationality constraint for the efficient contractors will not be satisfied. Because \(q(\cdot)\) is invariant to the number of bidders, it suffices to show that the derivative \(\frac{\partial \bar{p}(n)}{\partial n} \varphi(n)\) with respect to \(n\) is always negative.

\[
\frac{\partial \bar{p}(n)}{\partial n} \varphi(n) = \log(1 - \pi) \frac{\pi (1 - \pi)^{n-1} (2 - (1 - \pi)^n)}{(1 - (1 - \pi)^n)^2}.
\]

The above expression is always negative as long as \(p \in (0, 1)\). This proves that \(p\) is non-increasing in the number of bidders. On the other hand, the number of bidders does not affect \(\bar{p}\) nor \(q(\cdot)\), as can be seen in equations (3) and (4) in Theorem 1.

To compare the expected transfer under the fixed-price contract and that under the variable-price contract, we consider the one-participant case. Because \(p\) is non-increasing in the number of bidders, if \(p < \bar{p} + E(q|\bar{\beta})\) when only one contractor participates, the proof is done. Let us start by considering the incentive constraint for the inefficient contractor. Given that the individual rationality constraint holds at equality, the incentive constraint can be written as:

\[
p - c - \bar{\beta} \leq 0.
\]

Plugging in equation (2), the above inequality can be written as:

\[
-\bar{\beta} + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left[ \bar{\beta} - \int \psi(q(s)) [\bar{f}(s) - f(s)] ds \right] \leq 0.
\]

Now, the difference in the expected prices, \(\bar{p} + \int q(s) \bar{f}(s) ds - p\), can be rearranged as:

\[
\left( \bar{\beta} - \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left[ \bar{\beta} - \int \psi(q(s)) [\bar{f}(s) - f(s)] ds \right] \right) + \int [q(s) - \psi(q(s))] \bar{f}(s) ds.
\]

The first term in parentheses are nonnegative and the second term is positive because \(q(s) > \psi(q(s))\) for all \(s \neq 0\). Therefore, \(\bar{p} + \int q(s) \bar{f}(s) ds > p\). ■
Corollary 2 Proof. (i) Dispersion of \( q(\cdot) \): For the purposes of this exercise we make explicit the dependence of \( q(s,\pi) \) on \( \pi \). Recall the first order condition is:

\[
\psi'[q(s,\pi)][1 - \pi l(s)] = 1 - \pi,
\]

where \( l(s) \equiv f(s)/\overline{f}(s) \). This holds if \( l(s) < 1/\pi \). Note that \( q(s,\pi) = 0 \) if \( l(s) = 1 \) and \( q(s,\pi) > 0 \) if \( l(s) < 1 \). Similarly \( q(s,\pi) < 0 \) if \( l(s) > 1 \). Totally differentiating the first order condition with respect to \( \pi \) yields:

\[
\psi''[q(s,\pi)] \frac{\partial q(s,\pi)}{\partial \pi} [1 - \pi l(s)] - \psi'[q(s,\pi)] l(s) = -1.
\]

Rearranging to make \( \frac{\partial q(s,\pi)}{\partial \pi} \) the subject of the equation gives:

\[
\frac{\partial q(s,\pi)}{\partial \pi} = \frac{l(s) - 1}{\psi''[q(s,\pi)] [1 - \pi l(s)]^2}.
\]

Noting \( \psi''(\cdot) < 0 \) it follows that \( \frac{\partial q(s,\pi)}{\partial \pi} > 0 \) when \( l(s) < 1 \) and \( \frac{\partial q(s,\pi)}{\partial \pi} < 0 \) when \( l(s) > 1 \). In other words, if \( l(s) < 1/\pi \), then

\[
\frac{\partial q(s,\pi)}{\partial \pi} > 0 \text{ if } q(s,\pi) > 0,
\]

\[
= 0 \text{ if } q(s,\pi) = 0,
\]

\[
< 0 \text{ if } q(s,\pi) < 0.
\]

as was to be proved.

(ii) Decline in \( p \): For the purpose of this exercise we make the dependence of the fixed price contract explicit by writing \( p(\pi) \) for \( p \).

\[
p = c + \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left[ \frac{1}{\beta} - \int \psi(q(s))|\overline{f}(s) - f(s)|ds \right].
\]

To show that \( p'(\pi) < 0 \) we consider the two expressions involving \( \pi \) separately. First:

\[
\frac{\partial}{\partial \pi} \ln \left[ \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \right] = \frac{1 - n\pi - (1 - \pi)^n}{\pi(1 - \pi)(1 - (1 - \pi)^n)}
\]

Note that the derivative is zero at \( n = 1 \) and that at \( n = 2 \) is \(-\pi^2\), which is negative. Now suppose it is negative for all \( n \in \{2, \ldots, n_0\} \). Then for \( n_0 + 1 \) the denominator is clearly positive and the numerator is:

\[
1 - (n_0 + 1)\pi - (1 - \pi)(1 - \pi)^{n_0} < \pi(1 - \pi)^{n_0} - \pi < 0.
\]

The first inequality follows from an induction hypothesis, and the second one from the inequalities \( 0 < \pi < 1 \). Therefore \( \pi(1 - \pi)^{n-1}/(1 - \pi)^{n-1} \) is decreasing in \( \pi \) for all \( n > 1 \).
Second, we note that:

$$\frac{\partial}{\partial \pi} \int \psi [q(s, \pi)] [1 - l(s)] \overline{f}(s) \, ds = \int \psi' [q(s, \pi)] \frac{\partial q(s, \pi)}{\partial \pi} [1 - l(s)] \overline{f}(s) \, ds$$

$$= \int (1 - \pi) \frac{\partial q(s, \pi)}{\partial \pi} \left[ \frac{1 - l(s)}{1 - \pi l(s)} \right] \overline{f}(s) \, ds = \int \frac{\pi - 1}{\psi'' [q(s, \pi)] [1 - \pi l(s)]^2} \overline{f}(s) \, ds > 0.$$ 

The second equality follows from using the first order condition to substitute out $\psi' [q(s, \pi)]$, and the third equality uses the expression we derived for $\frac{\partial q(s, \pi)}{\partial \pi}$. The last inequality appeals to the concavity of $\psi(\cdot)$.

Finally note that since the participation constraint is satisfied with an inequality for the efficient bidder:

$$\overline{\beta} - \int \psi [q(s, \pi)] [1 - l(s)] \overline{f}(s) \, ds > 0.$$ 

Hence:

$$\frac{\partial}{\partial \pi} p(\pi) = \frac{\partial}{\partial \pi} \left[ \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left\{ \overline{\beta} - \int \{\psi [q(s, \pi)]\} [1 - l(s)] \overline{f}(s) \, ds \right\} \right]$$

$$+ \frac{\pi (1 - \pi)^n}{1 - (1 - \pi)^n} \int \frac{[1 - l(s)]^2}{\psi'' [q(s, \pi)] [1 - \pi l(s)]^3} \overline{f}(s) \, ds < 0,$$

as claimed.  

References


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