

INTEGRATING SCHOOLS FOR CENTRALIZED ADMISSIONS

MEHMET EKMEKCI AND M. BUMIN YENMEZ

ABSTRACT. As school districts integrate charter schools for centralized admissions in Denver, New Orleans, Newark and Washington D.C., some schools have stayed out. We provide a new framework to study the incentives of a school to join a clearinghouse and we show that each school prefers to remain out of the system when others join it. Therefore, our analysis provides an explanation of why some charter schools have evaded the clearinghouse. To overcome this issue, we propose two schemes that can be used by policymakers to incentivize schools to join the system, which achieves the desired integration of schools.

1. Introduction

Market design has been particularly successful in environments for which either monetary transfers are not available due to legal constraints or transfers of certain goods among agents is not feasible. In such environments, the use of centralized clearinghouses together with a proper allocation or assignment procedure may replicate what the missing markets could have achieved.¹ However, the assumption that all agents in the market would voluntarily choose to join the centralized clearinghouse may be violated, since an agent may deem it more advantageous to evade the centralized system.

Date: June 2, 2015.

Keywords: School choice, charter schools, centralized admissions.

Ekmecki is affiliated with Department of Economics, Boston College, 140 Commonwealth Ave, Chestnut Hill, MA, 02467; Yenmez is with Tepper School of Business, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213. Emails: ekosistem@gmail.com, byenmez@andrew.cmu.edu. Yenmez gratefully acknowledges financial support from National Science Foundation grant SES-1326584.

¹See Gale and Shapley (1962) for pioneering work in market design, Roth (1984a) for the residency matching, Abdulkadiroğlu and Sönmez (2003) for school choice, Roth, Sönmez, and Ünver (2004) for kidney exchange, and Sönmez and Switzer (2013) for cadet-branch matching.

A specific example of this issue appears in school choice. While some school districts have centralized clearinghouses to assign students to district-run schools, almost all charter schools run their admissions systems separately.² As a result, students may be admitted to many public schools at the same time. Then these students have to decide which school they would like to attend, after which the empty seats in other schools can be offered to other students who would like to take them. Of course, when this is done in a decentralized way the inefficiencies cascade in the system delaying the admissions procedure of schools to the school year with families scrambling to get their children into preferred schools. Indeed, the D.C. Public Charter School Board reports that 1,141 students withdrew from a charter school within the first month of classes in fall 2011 while another 2,671 entered within that same time frame.³ Likewise, Gabriela Fighetti, the executive director of enrollment for the Louisiana Recovery School District, explains that before New Orleans started their centralized clearinghouse called OneApp, parents had to keep track of dozens of applications and deadlines, and

“...at the end of that process, you could’ve gotten into more than one school, or you could’ve gotten into no schools.”

Recently, there has been a surging interest in a small but growing number of districts for having a centralized clearinghouse that assigns students to all public schools to overcome the reshuffling problems that arise in a decentralized system. Thus far, in four different school districts, charter schools and district-run schools have started to participate in the same clearinghouse: Denver launched its system in 2010, New Orleans debuted a system in February 2012,⁴

²Charter schools are publicly funded but they have more freedom than district-run public schools. In particular, they can run their own admissions or they can structure their curriculum. In 2012-2013, there were 6004 charter schools nationwide, which is 6.4% of all schools (National Alliance for Public Charter Schools, 2014).

³Approximately 35,000 students, or roughly %43 of public school population in Washington D.C., attend charter schools.

⁴In New Orleans, there are two school districts: Orleans Parish School Board and Recovery School District. As of July 2014, all public schools in New Orleans Recovery School District are charters.

and the Newark and District of Columbia have started to implement their systems for 2014-15 school year admissions.⁵

When the system is integrated, schools will no longer find their students leaving in the fall as they get into other schools off waiting list, or simply decide they would be better off at another school where they also secured a spot. Even though the benefits of having a unified clearinghouse seems to be clear for charters, which are public schools that are generally granted greater autonomy and flexibility than typical district-run schools, none of the cities that are currently using a universal enrollment system - with the exception of Denver - have %100 participation from all the charters in their districts. In particular, around 16 of the charter schools in District of Columbia have opted out of the centralized clearinghouse.⁶

If a charter school anticipates that by not joining a centralized system, it may gain an advantage in the student selection process relative to other schools, then it may stay out of the system, making the desire for having a centralized system unfulfilled. Therefore, it is of utmost importance to study the incentives of charter schools to join the system. To this end, we propose a framework to study the incentives of a charter school (likewise, a magnet or private school) for joining a centralized clearinghouse and consider different schemes to promote schools to join the system. Put it differently, we explore the ability of clearinghouses that use various matching algorithms to give the necessary incentives to all schools for joining the system rather than evading it.

In our model, each school decides whether to join the clearinghouse or evade it. If a school joins the clearinghouse, then it participates in the matching process where its students are determined. On the other hand, if it evades the clearinghouse, it runs its own admissions program. In Theorem 1, we show that if the centralized clearinghouse adopts the most commonly used mechanism in the reformed school choice systems, the deferred acceptance algorithm (Gale and Shapley, 1962), then every school would prefer to evade the clearinghouse,

⁵For Denver's system see <http://schoolchoice.dpsk12.org/>, for New Orleans' <http://enrollnola.org/>, for Newark's <http://www.newarkenrolls.org/> and for Washington D.C.'s <http://www.myschooldc.org/>.

⁶More information can be found at <http://www.myschooldc.org/faq/#other-4>.

if all other schools have joined it. Therefore, it is impossible to sustain the integration of all schools for centralized admissions as part of some equilibrium. To show this result, we assume that the left-over school runs its admissions program after students learn the outcome of the clearinghouse. In practice, the left-over school can potentially run its admissions program at other dates to gain advantage over schools in the system. What we show is that it will always prefer to stay out of the system even when the school runs its program at the predetermined period (after the clearinghouse announces the outcome).

The intuition for Theorem 1 that a school always prefers to evade the clearinghouse is as follows. When a school evades the clearinghouse all students get weakly worse schools in the system because there is more competition for each seat. In particular, students who would have matched with this school get strictly worse schools. As a result, all of these and potentially more students apply to the left-over school. Consequently, the school prefers to evade the clearinghouse. The same result holds even when the Boston (or immediate acceptance) mechanism is used in the centralized clearinghouse (6).

Since it is impossible to integrate schools for centralized admissions in general, we consider specific student preferences where integration is possible for the student-proposing deferred acceptance algorithm. We show that it is an equilibrium for schools to join the centralized clearinghouse for all school choice rules if and only if students have the same preferences over schools (Theorem 2). More explicitly, if students do not have the same preferences then there exist school choice rules for which at least one school is strictly better off by evading the clearinghouse.

A potential remedy for establishing a clearinghouse in which all schools participate is imposing a ‘binding commitment’ policy that a student matched in the clearinghouse must attend this school. A variant of this policy is used in the National Resident Matching Program.⁷ However, as we show in Theorem 3, binding commitment does not always alleviate the problem: There exists an integration problem where a school is better off by evading the clearinghouse when the clearinghouse uses either version of the deferred acceptance algorithm. However, if a (charter) school only cares about the number of students

⁷See <http://www.nrmp.org/policies/the-match-commitment/> for more details.

that it gets, then it is an equilibrium for all schools to join the system for the aforementioned mechanisms (Theorem 5).

Finally, we propose a new scheme to solve the integration problem. Fix any stable mechanism such as either version of the deferred acceptance algorithm.⁸ If a school evades the system, we add a virtual copy of this school to the mechanism and run it as if the actual school was participating in the clearinghouse. The set of students who get matched with the virtual school is unassigned whereas any other student gets their assigned schools. Because the mechanism is stable only the set of unassigned students including the ones who get matched with the virtual school apply to the left-over school. Then the school admits the set of students assigned to the virtual school since the mechanism is stable. Therefore, any school is indifferent between joining the clearinghouse and evading it if all other schools have joined it. Hence, it is an equilibrium for all schools to join the clearinghouse (Theorem 4). The virtual school approach can be used in conjunction with any stable mechanism to assign students for all agent preferences. This approach, however, requires the knowledge of school preferences that leave the system, which is innocuous for public schools including the charter schools.

There is a big literature on school choice following Abdulkadiroğlu and Sönmez (2003). In this literature, it is assumed that all schools participate in a centralized clearinghouse. This assumption is innocuous for district-run schools but not for charter, magnet and private schools, which can choose not to join the clearinghouse. Our analysis complements theirs by investigating how we can integrate all schools to the clearinghouse.⁹

The integration problem is mostly related to the matching manipulation literature where a school and a student can match before the centralized clearinghouse is run, or a school can underreport its capacity to get better students.¹⁰ In contrast to this literature, we focus on the incentives of a school to join a centralized clearinghouse but we assume that once a school joins, it

⁸Stability is a desirable property for matching problems. In the context of school choice, it is viewed as a fairness notion. See the discussion in Section 2.1.

⁹Manjunath and Turhan (2014) consider an alternative setting for school districts with vouchers in which two clearinghouses run in parallel.

¹⁰See Sönmez (1997, 1999); Konishi and Ünver (2006) who introduce the manipulation notions and Kojima (2007); Kojima and Pathak (2009); Kesten (2012) for further studies.

does not engage in any manipulation. This assumption is justified because the centralized clearinghouse can sanction schools if they engage in such behavior. For example, the aforementioned school districts and the National Resident Matching Program have such policies.¹¹

2. Model

There are two sets of agents: the set of students and the set of schools.¹² Each student has a strict preference ordering over all schools and remaining unassigned, and each school has a choice rule (or function) that selects a subset of students from a given set of applicants. More formally, there exist

- a set of students $\mathcal{S} = \{s_1, \dots, s_n\}$,
- a set of schools $\mathcal{C} = \{c_1, \dots, c_m\}$,
- a list of strict student preferences $\succ_{\mathcal{S}} = (\succ_{s_1}, \dots, \succ_{s_n})$, and
- a list of school choice rules $Ch_{\mathcal{C}} = (Ch_{c_1}, \dots, Ch_{c_m})$.

For any student s , \succ_s is a strict preference relation over $\mathcal{C} \cup \{s\}$ where $c \succ_s s$ means that student s strictly prefers school c to being unassigned (or unmatched). Let \succeq_s be the “at least as good as” relation induced by \succ_s . For any school c , Ch_c is a choice rule over all sets of students where $Ch_c(S)$ is the chosen subset for any set of students S .¹³ An *integration problem* is a tuple $\langle \mathcal{S}, \mathcal{C}, \succ_{\mathcal{S}}, Ch_{\mathcal{C}} \rangle$.

The basic model so far is a slight generalization of the well-known college admissions problem of Gale and Shapley (1962). In their seminal paper, each school has a capacity and a strict preference ordering over individual students, so the choice rule of a school can be constructed by selecting the highest ranked students up to the capacity. Similarly, our model also generalizes the school choice problem of Abdulkadiroğlu and Sönmez (2003).

¹¹See <http://www.nrmp.org/policies/match-participation-agreements-and-policies/> for the Match policies.

¹²We do not distinguish between private schools, district-run public schools or charter schools. But whenever we consider a school that can choose to join the clearinghouse, then we implicitly assume that it is not a district-run public school.

¹³Taking school choice rules as primitives of the model rather than their preferences has many advantages, see Chambers and Yenmez (2013). For example, school diversity policies (Ehlers, Hafalir, Yenmez, and Yildirim, 2014; Echenique and Yenmez, 2012) or regional distributional constraints (Kamada and Kojima, 2013) can be implemented using choice rules.

For a school c , a set of students S is *revealed preferred* to another set of students S' if the school chooses all students in S when all students in both S and S' are available. More formally, there exists \tilde{S} such that $\tilde{S} \supseteq S \cup S'$ and $Ch_c(\tilde{S}) = S$.

The outcome for an integration problem is a *matching* (or an *assignment*) between students and schools. Formally, a matching μ is a function on the set of all agents such that

- for any student s , $\mu(s) \in \mathcal{C} \cup \{s\}$,
- for any school c , $\mu(c) \subseteq \mathcal{S}$, and
- for any student s and school c , $\mu(s) = c$ if and only if $s \in \mu(c)$.

The first two conditions require that a student is either matched with a school or left unmatched and that a school is matched with a set of students. The last condition means that if a school is matched with a student then the student must be in the set of students matched with the school. This condition ensures the feasibility of the matching.

In an integration problem, each school decides whether to join a centralized clearinghouse, or evade it and run its own admissions program. In other words, each school has two actions: ‘join’ or ‘evade’. When all schools join, the clearinghouse uses a *mechanism* (or an *algorithm*) to assign students to schools. Below, we describe three particular algorithms. However, if some of the schools evade, the clearinghouse assigns students to schools that have chosen to join, first. Next, this assignment is observed by students. Afterwards, students decide whether to apply to schools that have evaded the centralized clearinghouse. Finally, these schools decide which students to admit.

We analyze when it is possible to induce all schools to join the centralized clearinghouse, i.e., when it is an *equilibrium* for all schools to join the system. Since schools do not have complete preferences over sets of students, we cannot compare any two outcomes for schools in general. However, we show in Section 3 that this is not an issue in our setting: We can compare the set of students that a school gets by joining the system or evading it using the revealed preference relation that we have defined above for the standard student assignment mechanisms. We discuss this issue further in Section 3.

In our analysis, we assume that choice rules for schools represent their own preferences. Even though this holds for private (and possibly magnet) schools, it does not have to for charter schools. Indeed, some states in U.S. enforce charter schools to use lotteries when there is excess demand for them. However, charter schools get funding per student. Therefore, they prefer to accept more students to less. Even when charter schools only care about the number of students, our main results continue to hold. We provide a discussion of this issue in Section 7.

2.1. Stability. The following solution concept of stability for matchings is crucial for the success of centralized clearinghouses. Empirical evidence shows clearinghouses that produce stable matchings have been successful whereas those that produce unstable matchings have failed (Roth, 1991; Pathak and Sönmez, 2013).

Definition 1. *A matching μ is **stable** if*

- (1) (**individual rationality for students**) for every student s , $\mu(s) \succeq_s \{s\}$,
- (2) (**individual rationality for schools**) for every school c , $Ch_c(\mu(c)) = \mu(c)$, and
- (3) (**no blocking**) there exists no (c, s) such that $c \succ_s \mu(s)$ and $s \in Ch_c(\mu(c) \cup \{s\})$.

Individual rationality for a student requires that the student prefers her assigned school to being unassigned whereas for a school it requires that the school wants to keep all of its assigned students. In the context of college admissions problem, where each school has a strict preference relation over individual students and having an empty seat, individual rationality imposes that each assigned student is better than having an empty seat.

No blocking notion requires that there exists no student-school pair such that the student prefers the school to her assignment and the school wants to admit her. In the context of school choice, where each school has strict priority over individual students, it is viewed as a fairness notion (Balinski and Sönmez, 1999): There exists no student-school pair such that the student

wants to switch to the school and the school either has an empty seat or it has admitted another student with a lower priority.

2.2. Path Independence. It is well known that stable matchings need not exist without any assumptions on schools' choice rules (Kelso and Crawford, 1982; Roth, 1984b); nevertheless, when choice rules are path independent stable matchings exist.¹⁴

Definition 2. *School c 's choice rule is **path independent** if for every set of students S and S'*

$$Ch_c(S \cup S') = Ch_c(S \cup Ch_c(S')).$$

If a choice rule is path independent, then any set of students can be divided into (not necessarily disjoint) subsets and the choice rule can be applied to these subsets in any order without changing the final outcome. This axiom was first introduced informally by Arrow (1951), and formally by Plott (1973). Moulin (1985) provides an excellent survey of path-independent choice rules.

Path independence has strong implications on the choice behavior of schools. In particular, it rules out that students can be complements. That is, a student should not be chosen just because she is complementing another student. Chambers and Yenmez (2013) studies path-independent choice rules in the context of matching.

For what follows, we assume that choice rules are path-independent, which is satisfied in practical school choice settings. In particular, *responsive* choice rules are path independent:¹⁵ School c 's choice rule is **responsive** if there exist a capacity and a strict preference ordering over individual students and having an empty seat such that the choice from any given set of students is the set of best available students better than having an empty seat up to the capacity. A student is **acceptable** if it is ranked higher than having an empty seat. Formally, choice rule Ch_c is responsive if there exist a capacity $q_c \in \mathbf{Z}_+$

¹⁴When choice rules are primitives of a matching model, then path independence is the weakest condition known to guarantee the existence of a stable matching (Chambers and Yenmez, 2013). *Substitutability* alone does not guarantee the existence (Aygün and Sönmez, 2013).

¹⁵Echenique (2007) shows that the number of path-independent choice rules are exponentially more than the number of responsive choice rules.

and a strict preference ordering \succ_c over the set of students and having an empty seat such that

$$Ch_c(S) = \bigcup_{i=1}^{\min\{q_c, |S|\}} \{s_i^*\}$$

where s_i^* is defined inductively as $s_1^* = \max_{S \cup \{\emptyset\}} \succ_c$ and, for $i \geq 2$, $s_i^* = \max_{(S \setminus \{s_1^*, \dots, s_{i-1}^*\}) \cup \{\emptyset\}} \succ_c$.

2.3. Deferred Acceptance Algorithm. The student-proposing deferred acceptance algorithm (Gale and Shapley, 1962) is by far the most commonly adapted algorithm for assigning students to schools in the recent school choice reforms (Abdulkadiroğlu, Pathak, Roth, and Sönmez, 2005; Abdulkadiroğlu, Pathak, and Roth, 2005).¹⁶ It works as follows.

Step 1: Each student proposes to her most preferred school. Suppose that S_c^1 is the set of students who proposes to school c . School c tentatively accepts students in $Ch_c(S_c^1)$ and permanently rejects the rest. If there are no rejections, then stop.

Step k: Each student who was rejected in Step $k - 1$ proposes to her next preferred school. Suppose that S_c^k is the set of students who were tentatively accepted by school c in Step $k - 1$ and the set of student who just proposed to school c . School c tentatively accepts students in $Ch_c(S_c^k)$ and permanently rejects the rest. If there are no rejections, then stop.

This procedure ends in finite time since there can only be a finite number of proposals. When schools' choice rules are path independent, it produces a stable matching that is optimal for students: Each student prefers the outcome of this algorithm to any other stable matching (Roth, 1984b).

In the school-proposing version of the algorithm, schools make the proposals instead of students. See the descriptions in Abdulkadiroğlu and Sönmez (2003) when schools have responsive choice rules and Chambers and Yenmez (2013) when schools have path-independent choice rules.

¹⁶Pathak (2011) provides a detailed account of these reforms.

3. Integrating Schools

We are interested in establishing a centralized clearinghouse in which all schools want to join the system. To this end, we assume that all schools except one have joined the clearinghouse. Therefore, we analyze the situation when a school considers to join the clearinghouse if every other school has decided to join it. We show that the school always revealed prefers the set of students that it gets by evading the clearinghouse to the set of students that it gets by joining if either version of the deferred acceptance algorithm is used.

To study this question, we have to specify what happens when all schools except one join the clearinghouse more explicitly. In this case, students first participate in the centralized clearinghouse. Then they observe their assigned schools and decide whether or not to apply to the leftover school. For simplicity, we assume that only students who prefer the leftover school to their assigned schools apply.¹⁷ The leftover school admits students from the set of applicants. The admitted students then are permanently matched with the leftover school since they all prefer the leftover school to their assigned schools in the clearinghouse. The rest of the students are matched with their assigned schools in the centralized clearinghouse.

Theorem 1. *Suppose that all schools except school c have joined the centralized clearinghouse in which either version of the deferred acceptance algorithm is used. Then school c revealed prefers the set of students that it gets by evading the clearinghouse to the set of students that it gets by joining it.*

The proof is in the Appendix. Here, we provide some intuition. Removing school c in the deferred acceptance algorithm makes all students weakly worse off. In particular, students who previously got matched with school c are strictly worse off since student preferences are strict and the school is not in the clearinghouse anymore. Consequently, these students apply to school c . Therefore, school c considers a set of students including the ones that it would have gotten if it were participating in the centralized clearinghouse. The result follows.

¹⁷This assumption is justified, for example, if there is a small application fee for the leftover school or if the student has to exert costly effort to apply.

For any reasonable equilibrium concept, the profile in which all schools join the clearinghouse is an equilibrium if no school finds it strictly preferable to evade the system. On the other hand, Theorem 1 establishes that any school revealed prefers the set of students that it gets by evading the centralized clearinghouse to the set of students that it can get by joining it. Consequently, this profile is an equilibrium if every school is indifferent to joining the clearinghouse and evading it under the assumption that all remaining schools have joined the system. We state this result as a corollary.

Corollary 1. *Suppose that either the student-proposing or the school-proposing version of the deferred acceptance algorithm is used in the clearinghouse. Then it is an equilibrium for all schools to join the clearinghouse if and only if every school is indifferent between joining or evading the clearinghouse when the rest of the schools have joined it.*

The requirement that every school is indifferent between joining or evading the clearinghouse when others have joined is stringent. In the next section, we show that this is true for any stable mechanism if student preferences are identical. However, when students do not have the same preferences, there exists a school choice profile such that at least one school strictly prefers to remain out of the clearinghouse when the student-proposing deferred acceptance algorithm is used (Theorem 2).

In the Appendix, we provide an example which demonstrates that using the student-proposing deferred acceptance algorithm when all schools join but switching to the school-proposing version does not overcome the difficulty in integrating schools.

4. Conditions on Student Preferences

As we have shown in the previous section, when either version of the deferred acceptance algorithm is used, schools prefer to evade the centralized clearinghouse when the rest of the schools have joined it. Therefore, a natural question to ask is under what conditions on student preferences it is an equilibrium for all schools to join the clearinghouse. In the next theorem, we provide an answer to this question.

Theorem 2. *Suppose that students have the same preferences over schools and a stable matching mechanism is used in the clearinghouse. Then it is an equilibrium for all schools to join the clearinghouse. Conversely, if students find all schools acceptable but they do not have the same preferences over schools and the student-proposing deferred acceptance algorithm is used in the clearinghouse then there exist choice rules for schools such that it is not an equilibrium for all schools to join the clearinghouse.*

We relegate the proof to the Appendix. When all students have the same preferences over schools there is a unique stable matching. This stable matching can be produced by a serial dictatorship of schools in which schools choose the set of students that they like using the order in student preferences. Therefore, there is a clear hierarchy of schools and as a result there is no real competition between schools. Consequently, regardless of whether a school participates in the clearinghouse or not, a school chooses from the same set of students: The set of students who are unmatched after higher-ranked schools admit their students. Since each school is indifferent between joining the clearinghouse and evading it, it is an equilibrium for all schools to join the clearinghouse by Corollary 1.¹⁸

Theorem 2 also shows that common student preferences is not only sufficient for the strategy profile in which schools join to be an equilibrium but it is also necessary for the student-proposing deferred acceptance algorithm. To show this, for any student preference profile in which student preferences are not the same, we construct school choice rules for which at least one school strictly prefers to evade the clearinghouse.

Therefore, a centralized clearinghouse in which all schools participate is hard to establish whenever the student-proposing deferred acceptance algorithm is used. As a result, we consider other means that can help establish participation by all schools.

¹⁸In the case of responsive choice rules, we conjecture that a similar result holds when schools have the same priority ranking over students. If schools have common priorities over students, then it will be an equilibrium for all schools to join the clearinghouse. Otherwise, there will be a student preference profile for which at least one school will prefer to evade the clearinghouse. We leave this question for future research.

5. Binding Commitment

The National Resident Matching Program has a “binding commitment” policy:

All Match commitments are binding. The ranking of applicants by a program director and the ranking of programs by an applicant establishes a binding commitment to offer or to accept an appointment if a match results.

In other words, whenever a match results between a program and a doctor who have ranked each other, the residency program has to make an offer to the doctor and the doctor has to accept the offer. This policy is implemented to stop the programs from making offers to doctors after they learn the outcome of the Match.

In this section, we analyze the implications of such a policy for the integration problem. Intuitively, this policy prevents schools from evading the clearinghouse as a school that evades the clearinghouse can only choose from unassigned students. However, we show that a school may still strictly prefer to evade the clearinghouse.

Theorem 3. *Suppose that the centralized clearinghouse has a binding commitment policy and it uses either version of the deferred acceptance algorithm. Then there exists an integration problem in which a school revealed prefers the set of students that it gets by evading the clearinghouse to the set of students that it gets by joining it.*

In the Appendix, we provide examples to show these results.

Remark 1. In the context of resident matching, perhaps the National Resident Matching Program (NRMP) is able to enforce commitment because doctors may participate in the program more than once for residency, fellowships and specialties. Similarly, residency programs participate in the program every year. Doctors with a confirmed violation of NRMP policy are barred from starting a residency in institutions who participate in NRMP and also barred from participating in future NRMP Matches. Similarly, a residency program violating the policy is barred from participating in future NRMP matches.

Similarly, in the context of integration problem a district may be able to enforce such a rule by barring a school if they do not register a student assigned to them through the clearinghouse. Likewise a student who does not enroll to the assigned school can be barred from enrolling in other schools participating in the clearinghouse. Furthermore, if no state funding is provided for students who are assigned to schools in the centralized clearinghouse but enroll in schools outside of the system, then these students may not be able to attend charter schools outside of the clearinghouse.

Even though the binding commitment policy does not alleviate the problem when school choice rules represent schools' actual preferences, it does solve the problem for charter schools which only care about the number of students that they get. See Theorem 5 in Section 7 below.

6. Remedy: Virtual Schools

Suppose that when a school c evades the centralized clearinghouse, the clearinghouse implements a stable mechanism as if school c was participating in it, effectively adding a virtual school for school c in the algorithm used. The set of students matched with the virtual school remain unmatched in the clearinghouse. This implementation requires that students rank all schools including school c and it also requires that the clearinghouse knows the choice rule of school c even when school c evades the clearinghouse. Since charter and district-run public schools choice rules are determined by the school district or determined by law the second requirement is innocuous.¹⁹ The first requirement may not be innocuous in some environments but we show that this will only be required off the equilibrium path.

Theorem 4. *Suppose that all schools except school c have joined the centralized clearinghouse in which a stable mechanism with virtual schools is used. Then school c is indifferent to joining the clearinghouse or evading it. Therefore, it is an equilibrium for all schools to join the clearinghouse.*

¹⁹This assumption is satisfied, for example, if students take a standardized exam that determines their ranking for different schools as in Chicago (Pathak and Sönmez, 2013).

In the proof, we show that when a school evades the clearinghouse or joins it, the set of students who would like to be matched with that school remains the same. Since a stable mechanism is used in the clearinghouse, the set of students that the school gets remains the same under both scenarios. Consequently, it is an equilibrium for all schools to join the clearinghouse.

The intuition for this result is as follows. Consider either version of the deferred acceptance algorithm. When a school evades the clearinghouse, it avoids competition within the algorithm, which makes all students worse off. Furthermore, after the outcome of the clearinghouse is finalized, it can choose from the set of students who would like to go to this school. Since the remaining schools cannot make or receive additional offers at this point, the leftover school is better off. The virtual school mechanism takes away the advantage of the leftover school by creating competition within the clearinghouse by using the virtual school mechanism: When some students are matched with the virtual school, the remaining schools will still be able to make or receive offers. In other words, the virtual school mechanism *internalizes the external competition*.

7. Charter School Preferences

Even though charter schools are subject to fewer rules and regulations than district-run public schools, most of the states in the US have strict admission policies.²⁰ For example, in New York, charter schools have to give higher priorities in admissions to returning students, siblings of students already enrolled in the school and students who reside in the school district. Furthermore, these schools are also permitted to give higher priorities to lower achieving students.²¹ When there is excess demand, schools may have use lotteries to choose their students.²²

²⁰Magnet schools, which are public schools with specialized curricula, can choose its students based on exam scores, interview or audition. Therefore, it is safe to assume that choice rules present the actual preferences for public magnet schools as well as private schools.

²¹See <http://www.nyccharterschools.org/enrollment-faq> for various admissions policies.

²²A school that uses its own admission criteria may be subject to probation and closure: see, for example, <http://www.nytimes.com/2011/07/21/nyregion/bronx-charter-school-disciplined-over-admissions.html?pagewanted=all>.

On the other hand, charter schools are publicly funded and usually get funding per pupil. To this end, we can assume that they would like to admit as many students as possible without violating their capacity. We formalize this idea as a choice rule property.

Definition 3. *School c 's choice rule is **acceptant** if there exists a capacity q_c such that for every set of students S , $|Ch_c(S)| = \min\{q_c, |S|\}$.*

If a school has a quota filling choice rule it admits all students if the number of students is less than its capacity, and it fills its capacity when the number of students is weakly more than the capacity. For example, responsive choice rules satisfy this property.²³

Here, we discuss the implications of our results when charter schools prefer to have more students, since they usually receive per-pupil funding, but otherwise they do not have any actual preferences. To this end, we assume that choice rules are acceptant.

In Theorem 1, we show that a school revealed prefers the set of students that it gets by evading the clearinghouse to the set of students that it gets by joining it for various mechanisms. Since the choice rule is acceptant, the school gets weakly more revenue by evading the clearinghouse. Therefore, the result that the school prefers to stay out of the system remains valid even when the school only cares about the number of students or, equivalently, revenue. In practice, not all charter schools can fill their capacities. Indeed, there were 368 vacant seats across five schools at different grade levels in Washington D.C.²⁴ Our results indicate that these charter schools could have filled more positions if they did not join the centralized clearinghouse.

The first statement in Theorem 2, which shows that it is an equilibrium for all schools to join the centralized clearinghouse when a stable mechanism is used if students have the same preferences over schools, remains true. In this case, each school is indifferent to joining or evading the clearinghouse, so a school admits the same number of students in both cases. In contrast, the

²³Kojima and Manea (2010) uses acceptance to characterize DA. In an earlier work, Alkan (2001) calls this notion *quota filling*.

²⁴<http://www.dcpsb.org/Parents/Available-Charter-School-Seats-and-Wait-Lists.aspx>

other direction of the proof does not hold anymore. It is easy to construct a counter example. Consider an integration problem in which there are more students than the number of seats, and students find all schools acceptable. In this case, a school can always fill its capacity by either joining the clearinghouse or evading it, regardless of student preferences. Therefore, the corresponding result when schools only care about the number of students will depend on both the capacity of schools and student preferences.²⁵

One major difference is that the results in Theorem 3, which state that there exists an integration problem where a school revealed prefers to evade the clearinghouse even when there is a binding commitment policy, are overturned. Indeed, if there is a binding commitment policy and if schools only care about the number of students that they get, then it is an equilibrium for all schools to join the clearinghouse. This is rather easy to see. Suppose that students find all schools to be acceptable. Consider the case when school c evades the clearinghouse when other schools have joined the system. Then school c can only admit the unassigned students. Any student is unassigned only when all the remaining schools fill their seats. But if school c joins the clearinghouse then it will be able to admit at least the same number of students if not more. This argument remains valid for either version of the deferred acceptance algorithm and the Boston mechanism; and it yields the following result.²⁶

Theorem 5. *Suppose that school choice rules are acceptant. Furthermore, suppose that schools only care about the number of students and they prefer to have more students to less (up to their quota). If the centralized clearinghouse has a binding commitment policy and either version of the deferred acceptance algorithm is used, then it is an equilibrium for all schools to join the centralized clearinghouse.*

The proof is immediate following the argument above, so we omit it. The same result holds for the Boston mechanism described below.

²⁵We leave this question open for future research.

²⁶In a subsequent work, Afacan (2014) shows that even when students may find some schools unacceptable the same results hold.

Finally, Theorem 4 shows that when the virtual school mechanism is used, it is an equilibrium for all schools to join the system. Since each school gets the same set of students by joining or evading the clearinghouse in this setting, the implication is still true when the school cares only about the number of students.

8. Conclusion

In the school choice literature, the set of schools participating in the clearinghouse is given exogenously. As more school districts integrate independent charter schools, which can run their own admissions programs, this assumption has become questionable. To this end, we have provided a framework to study the incentives for a school to join a centralized clearinghouse and we have shown that for the standard mechanisms used in practice, any charter always prefers to evade the clearinghouse. This poses some serious difficulty in the implementation of the existing matching algorithms proposed by market design research and, in fact, this difficulty has been observed in New Orleans, Newark and Washington D.C.

To overcome this problem, we have considered two possible remedies. In the first one, which is used in the National Resident Matching Program, the outcome of the clearinghouse is binding. We have shown that this policy solves the problem for charter schools but private schools may still find it optimal to opt out. The second remedy is the use of a virtual school in the clearinghouse whenever a school evades the system, as if the school was participating in it. We have shown that it is an equilibrium for all schools to participate in the clearinghouse if the virtual school mechanism is employed for stable mechanisms.

We have provided a general model to study integration problems. In our analysis, we have assumed that schools can only choose to join or evade the centralized clearinghouse and that students are not strategic agents. Using our framework, the analysis can be expanded to situations in which students are strategic agents and schools have more actions. We will investigate these issues in the future.

References

- ABDULKADIROĞLU, A., P. PATHAK, A. ROTH, AND T. SÖNMEZ (2005): “The Boston Public School Match,” *American Economic Review*, 95(2), 368–371.
- ABDULKADIROĞLU, A., P. A. PATHAK, AND A. E. ROTH (2005): “The New York City High School Match,” *American Economic Review*, 95(2), pp. 364–367.
- ABDULKADIROĞLU, A., AND T. SÖNMEZ (2003): “School choice: A mechanism design approach,” *American Economic Review*, 93(3), 729–747.
- AFACAN, M. O. (2014): “Enrollment Manipulations in School Choice,” Working Paper.
- ALKAN, A. (2001): “On preferences over subsets and the lattice structure of stable matchings,” *Review of Economic Design*, 6(1), 99–111.
- ARROW, K. J. (1951): *Social choice and individual values*. Yale Univ Pr.
- AYGÜN, O., AND T. SÖNMEZ (2013): “Matching with Contracts: Comment,” *American Economic Review*, 103(5), 2050–2051.
- BALINSKI, M., AND T. SÖNMEZ (1999): “A Tale of Two Mechanisms: Student Placement,” *Journal of Economic Theory*, 84(1), 73–94.
- CHAMBERS, C. P., AND M. B. YENMEZ (2013): “Choice and matching,” Working Paper.
- ECHENIQUE, F. (2007): “Counting combinatorial choice rules,” *Games and Economic Behavior*, 58(2), 231 – 245.
- ECHENIQUE, F., AND M. B. YENMEZ (2012): “How to Control Controlled School Choice,” Caltech SS Working Paper 1366.
- EHLERS, L., I. E. HAFALIR, M. B. YENMEZ, AND M. A. YILDIRIM (2014): “School choice with controlled choice constraints: Hard bounds versus soft bounds,” *Journal of Economic Theory*.
- GALE, D., AND L. S. SHAPLEY (1962): “College Admissions and the Stability of Marriage,” *The American Mathematical Monthly*, 69(1), 9–15.
- KAMADA, Y., AND F. KOJIMA (2013): “Efficient Matching Under Distributional Constraints: Theory and Applications,” Working paper.
- KELSO, A. S., AND V. P. CRAWFORD (1982): “Job Matching, Coalition Formation, and Gross Substitutes,” *Econometrica*, 50, 1483–1504.

- KESTEN, O. (2012): “On two kinds of manipulation for school choice problems,” *Economic Theory*, 51(3), 677–693.
- KOJIMA, F. (2007): “When Can Manipulations be Avoided in Two-Sided Matching Markets?—Maximal Domain Results,” *The BE Journal of Theoretical Economics*, 7(1).
- KOJIMA, F., AND M. MANEA (2010): “Axioms for Deferred Acceptance,” *Econometrica*, 78(2), 633–653.
- KOJIMA, F., AND P. A. PATHAK (2009): “Incentives and stability in large two-sided matching markets,” *The American Economic Review*, pp. 608–627.
- KONISHI, H., AND M. U. ÜNVER (2006): “Games of Capacity Manipulation in Hospital-intern Markets,” *Social Choice and Welfare*, 27(1), 3–24.
- MANJUNATH, V., AND B. TURHAN (2014): “When two should be one but aren’t: Milwaukee Public Schools and the voucher system,” Working Paper.
- MCVITIE, D. G., AND L. B. WILSON (1970): “Stable marriage assignment for unequal sets,” *BIT*, 10(3), 295–309.
- MOULIN, H. (1985): “Choice functions over a finite set: A summary,” *Social Choice and Welfare*, 2(2), 147–160.
- NATIONAL ALLIANCE FOR PUBLIC CHARTER SCHOOLS (2014): “Public Charter School Dashboard,” <http://dashboard.publiccharters.org/dashboard/schools/page/overview/year/2013>, Accessed on 10/27/2014.
- PATHAK, P. A. (2011): “The Mechanism Design Approach to Student Assignment,” *Annual Review of Economics*, 3(1), 513–536.
- PATHAK, P. A., AND T. SÖNMEZ (2013): “School Admissions Reform in Chicago and England: Comparing Mechanisms by Their Vulnerability to Manipulation,” *American Economic Review*, 103(1), 80–106.
- PLOTT, C. R. (1973): “Path independence, rationality, and social choice,” *Econometrica*, 41(6), 1075–1091.
- ROTH, A., T. SÖNMEZ, AND U. ÜNVER (2004): “Kidney Exchange,” *Quarterly Journal of Economics*, 119(2), 457–488.
- ROTH, A. E. (1984a): “The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory,” *The Journal of Political*

- Economy*, 92(6), 991–1016.
- (1984b): “Stability and Polarization of Interests in Job Matching,” *Econometrica*, 52(1), 47–57.
- (1991): “A Natural Experiment in the Organization of Entry-Level Labor Markets: Regional Markets for New Physicians and Surgeons in the United Kingdom,” *American Economic Review*, 81(3), pp. 415–440.
- SÖNMEZ, T. (1997): “Manipulation via Capacities in Two-Sided Matching Markets,” *Journal of Economic Theory*, 77(1), 197 – 204.
- (1999): “Can Pre-arranged Matches Be Avoided in Two-Sided Matching Markets?,” *Journal of Economic Theory*, 86(1), 148 – 156.
- SÖNMEZ, T., AND T. B. SWITZER (2013): “Matching With (Branch-of-Choice) Contracts at the United States Military Academy,” *Econometrica*, 81(2), 451–488.

Appendix A. Omitted Proofs

First, we start with the following lemma.

Lemma 1. *Let c be a school. Suppose that μ_c and $\mu_{\mathcal{C} \setminus \{c\}}$ are the matchings produced by the centralized clearinghouse when \mathcal{C} and $\mathcal{C} \setminus \{c\}$ are the set of participating schools, respectively. If*

$$\{s : c \succ_s \mu_{\mathcal{C} \setminus \{c\}}(s)\} \supseteq \mu_c(c),$$

then school c revealed prefers the set of students that it gets by evading the clearinghouse to the set of students that it gets by joining it.

Proof. If school c evades the clearinghouse, then students who prefer school c to their assigned schools in $\mu_{\mathcal{C} \setminus \{c\}}$ apply to school c . Therefore, the set of students assigned to school c , when it evades the centralized clearinghouse, is $Ch_c(\{s : c \succ_s \mu_{\mathcal{C} \setminus \{c\}}(s)\}) \subseteq \{s : c \succ_s \mu_{\mathcal{C} \setminus \{c\}}(s)\}$. Let $\tilde{S} \equiv \{s : c \succ_s \mu_{\mathcal{C} \setminus \{c\}}(s)\}$, $S \equiv Ch_c(\{s : c \succ_s \mu_{\mathcal{C} \setminus \{c\}}(s)\})$ and $S' \equiv \mu_c(c)$. Then by construction $Ch_c(\tilde{S}) = S$, and, by assumption, $\tilde{S} \supseteq S' \cup S$. Therefore, S is revealed preferred to S' by school c . \square

Proof of Theorem 1. Consider the student-proposing DA first.

Let us consider the set of students who get matched with school c under two different scenarios depending on whether school c joins the clearinghouse or evades it. Let $\mu_{\mathcal{C}}^*$ be the matching produced by the student-proposing deferred acceptance algorithm when \mathcal{C} is the set of schools participating in the system.

First, suppose that school c joins the clearinghouse. Then school c is matched with $\mu_{\mathcal{C}}^*(c)$.

By Theorem C.1 of Echenique and Yenmez, for every student s , $\mu_{\mathcal{C}}^*(s) \succeq_s \mu_{\mathcal{C} \setminus \{c\}}^*(s)$. Therefore, for any student $s \in \mu_c(c)$, we have $c \succeq_s \mu_{\mathcal{C} \setminus \{c\}}^*(s)$. Since $\mu_{\mathcal{C} \setminus \{c\}}^*(s)$ cannot be school c and \succ_s is a strict preference, we get $c \succ_s \mu_{\mathcal{C} \setminus \{c\}}^*(s)$. Therefore, $s \in \mu_c(c)$ implies $s \in \{s' : c \succ_{s'} \mu_{\mathcal{C} \setminus \{c\}}^*(s')\}$ or equivalently $\{s' : c \succ_{s'} \mu_{\mathcal{C} \setminus \{c\}}(s')\} \supseteq \mu_c(c)$. By Lemma 1, we conclude that school c revealed prefers the set of students that it gets by evading the centralized clearinghouse to the set of students that it gets by joining it.

Consider the school-proposing DA next.

The proof is similar to the above and uses the property that for any student s , $\mu_C^*(s) \succeq_s \mu_{C \setminus \{c\}}^*(s)$ where μ_C^* is the matching produced by the school-proposing deferred acceptance algorithm when C is the set of participating schools. Here, we first establish this fact.

Claim: For every student s , $\mu_C^*(s) \succeq_s \mu_{C \setminus \{c\}}^*(s)$.

Suppose that we first run the school-proposing deferred acceptance algorithm with $C \setminus \{c\}$, i.e., when school c is absent in the clearinghouse. The matching outcome is $\mu_{C \setminus \{c\}}^*(s)$. Now, include school c and start the algorithm at $\mu_{C \setminus \{c\}}^*(s)$. Since the order of proposals does not change the outcome of the deferred acceptance algorithm (McVitie and Wilson, 1970), the matching outcome is $\mu_C^*(s)$. Since each student gets a weakly better school as the algorithm progresses, we conclude that $\mu_C^*(s) \succeq_s \mu_{C \setminus \{c\}}^*(s)$.

The rest of the proof is just an application of Lemma 1: We conclude that school c revealed prefers the set of students that it gets by evading the centralized clearinghouse to the set of students that it gets by joining it.

Proof of Theorem 2. We start with the first claim that when students have the same preferences over schools and a stable matching mechanism is used in the clearinghouse, it is an equilibrium for all schools to join the clearinghouse.

Since students have the same preferences over schools, there is a unique stable matching regardless of schools' choice rules. This stable matching can be produced by a serial dictatorship of schools in which schools choose their students one by one. The order is determined by the common student preferences.

Consider the school whose ranking is k where $1 \leq k \leq n$, say c_k . When all schools join the clearinghouse, let S_i denote the set of students matched with school c_i , $1 \leq i \leq n$. It is clear that for every i , $S_i = \mathcal{C}_i(\mathcal{S} \setminus \bigcup_{j=1}^{i-1} S_j)$. In other words, school c_i accepts the students remaining after the previous schools. In particular, $S_k = \mathcal{C}_k(\mathcal{S} \setminus \bigcup_{j=1}^{k-1} S_j)$.

When all schools except school c_k join the clearinghouse, then again, these schools, excluding school c_k , choose the set of students one by one in the same order. Therefore, schools c_1, \dots, c_{k-1} get matched with the same set of students. The remaining students get matched with one of the remaining

schools that are ranked worse than school c_k . Therefore, the set of students that apply to school c_k is $\mathcal{S} \setminus \bigcup_{j=1}^{k-1} S_j$. Therefore, school c_k admits $\mathcal{C}_k(\mathcal{S} \setminus \bigcup_{j=1}^{k-1} S_j)$, which is S_k by construction.

Hence, school c_k is indifferent between joining the centralized clearinghouse or evading it for every k . By Corollary 1, it is an equilibrium for all schools to join the clearinghouse.

To show the second claim, suppose that students do not have the same preferences over schools. Then there exist s_1, \dots, s_k and c_1, \dots, c_k such that $c_i \succ_{s_i} c_{i+1}$ for $1 \leq i \leq k$ where $c_{k+1} = c_1$. We construct choice rules such that at least one school prefers to evade the clearinghouse.

Suppose that schools have responsive choice rules with capacity one. Consider the following preferences over students for each $1 \leq i \leq k$: $\succ_{c_i}: s_{i-1} \succ_{c_i} s_i \succ_{c_i} c_i$ where $s_0 = s_k$. Suppose that for the rest of the schools there are no acceptable students. Since the only acceptable students for school c_i are students s_i and s_{i+1} , we can assume that for student s_i , the only acceptable schools are c_i and c_{i+1} without changing the set of stable matchings for $i \leq k$. The rest of the students remain unmatched since no school finds them acceptable. Therefore, we focus on students s_1, \dots, s_k .

When all schools join the clearinghouse, the student-proposing deferred acceptance algorithm is used. At the first step, student s_i proposes to school c_i for each $1 \leq i \leq k$. Since student s_i is acceptable to school c_i , there are no rejections and the algorithm ends at this step.

Suppose now that c_k evades the clearinghouse. At the first step, student s_i proposes to school c_i for each $1 \leq i \leq k-1$ and s_k proposes to c_1 . Since school c_1 prefers student s_k over student s_1 , student s_1 is rejected. At the second step, student s_1 proposes to school c_2 . Since school c_2 prefers student s_1 over student s_2 , student s_2 is rejected and so on. The algorithm produces the following matching μ : $\mu(c_1) = s_k, \mu(c_2) = s_1, \dots, \mu(c_{k-1}) = s_{k-2}$. This is the assignment produced by the clearinghouse. There are only two students who are acceptable to school c_k : students s_{k-1} and s_k . Student s_{k-1} is unmatched at μ , so she applies to school c_k . Similarly, student s_k is matched with school c_1 , which she likes less than school c_k . Therefore, student s_k also applies to

school c_k . Since school c_k has a responsive choice rule with capacity one and student s_{k-1} is preferred over student s_k , it accepts student s_{k-1} .

When all schools join the clearinghouse school c_k is matched with student s_k whereas when all schools except school c_k join the clearinghouse school c_k is matched with student s_{k-1} . Therefore, it is not an equilibrium that all schools join the clearinghouse.

Proof of Theorem 3. Consider the student-proposing DA first.

We provide a simple example in which it is optimal for a school to evade the clearinghouse.²⁷

Suppose that there are two schools c_1, c_2 and two students s_1, s_2 . Schools have responsive choice rules with capacity one. Agents' preferences are as follows: $\succ_{c_1}: s_1 \succ s_2$, $\succ_{c_2}: s_2 \succ s_1$, $\succ_{s_1}: c_2 \succ c_1$ and $\succ_{s_2}: c_1 \succ c_2$. This information is summarized in Table 2 below.

Table 2				
	\succ_{c_1}	\succ_{c_2}	\succ_{s_1}	\succ_{s_2}
s_1	s_1	s_2	c_2	c_1
s_2	s_2	s_1	c_1	c_2
capacities	$q_{c_1} = 1$	$q_{c_2} = 1$		

Consider the case in which c_1 joins the clearinghouse. Then the student-proposing deferred acceptance algorithm is used to assign students to schools. At the first step of the algorithm, s_1 applies to c_2 and s_2 applies to c_1 . Both of them are tentatively accepted. Since there are no rejections, the algorithm ends after this step and the tentative assignment is made permanent. The final matching is $\{(c_1, s_2), (c_2, s_1)\}$.

Consider the case in which c_2 evades the clearinghouse and only c_1 joins the clearinghouse. In the clearinghouse, both students propose to c_1 . School c_1 accepts s_1 and rejects s_2 . Since there are no more schools to propose for s_2 , the algorithm ends. School c_1 is matched with student s_1 permanently.

²⁷Balinski and Sönmez (1999) use this example to show that *multi-category serial dictatorship* is not *Pareto efficient*.

Afterwards, student s_2 applies to school c_2 . School c_2 accepts student s_2 . The final matching is $\{(c_1, s_1), (c_2, s_2)\}$.

Since school c_2 prefers student s_2 over student s_1 , it prefers to evade the clearinghouse rather than joining it.

Consider the school-proposing DA next.

We provide a simple example in which it is optimal for a school to evade the clearinghouse. Recall the integration problem of Example 1.

There are two schools c_1, c_2 and three students s_1, s_2, s_3 . Schools have responsive choice rules with the following preferences: $\succ_{c_1}: s_1 \succ s_2 \succ s_3$, $\succ_{c_2}: s_2 \succ s_1 \succ s_3$. School c_1 has a capacity of one whereas school c_2 has a capacity of two. Students' preferences are as follows: $\succ_{s_1}: c_2 \succ c_1$, $\succ_{s_2}: c_1 \succ c_2$ and $\succ_{s_3}: c_1 \succ c_2$. This information is summarized in Table 1 above.

We have seen in Example 1 that when both schools participate in the clearinghouse and the school-proposing deferred acceptance algorithm is used the assignment is $\{(c_1, s_2), (c_2, \{s_1, s_3\})\}$.

Consider the case in which school c_1 remains out of the clearinghouse. Then the school-proposing deferred acceptance algorithm is used. School c_1 proposes to student s_1 . Student s_1 accepts school c_1 's offer. The matching is finalized after this step since there are no more offers. The unassigned students apply to school c_2 . School c_2 accepts both students. The final matching is $\{(c_1, s_1), (c_2, \{s_2, s_3\})\}$.

Since school c_2 prefers student s_2 over student s_1 , it prefers to evade the clearinghouse rather than joining it.

Proof of Theorem 4. Let μ be the matching produced by the mechanism when all schools participate in the clearinghouse. The outcome for school c is then $\mu(c)$.

Now, suppose that, school c evades the clearinghouse. Then the same stable mechanism is run with virtual school c . The set of students matched with virtual school c are unmatched in the clearinghouse. Now, let us identify the set of students who would like to switch to school c : $\tilde{S} = \{s : c \succ_s \mu(s)\} \cup \mu(c)$. But this is equivalent to $\{s : c \succeq_s \mu(s)\}$. The set of students who get matched with school c is then $Ch_c(\tilde{S}) = Ch_c(\{s : c \succeq_s \mu(s)\})$. Since μ is a stable

matching we get that the set of students who are matched with school c is $\mu(c)$.

Therefore, if all schools except one join the centralized clearinghouse, the remaining school is indifferent to joining the system or evading it. Therefore, it is an equilibrium that all schools join the clearinghouse.

Appendix B. An Example

We have shown that, regardless of which version of the deferred acceptance algorithm is used, a school always weakly prefers to evade the clearinghouse. To overcome this difficulty of establishing a clearinghouse in which all schools participate, we investigate whether using different stable mechanisms depending on which set of schools are participating helps. Suppose that the clearinghouse uses the school-proposing deferred acceptance algorithm when all schools join the clearinghouse but otherwise uses the student-proposing version. Therefore, schools are rewarded when they all join the clearinghouse but otherwise they are punished. As a result, students have less incentives to apply to the school absent in the clearinghouse and schools have more incentives to join the clearinghouse. Even under this scenario, a school may still find it better to evade the clearinghouse. The following example demonstrates this point.

Example 1. Suppose that there are two schools c_1, c_2 and three students s_1, s_2, s_3 . School c_1 has capacity one whereas school c_2 has capacity two. Both schools have responsive choice rules with the following preferences: $\succ_{c_1}: s_1 \succ s_2 \succ s_3$, $\succ_{c_2}: s_2 \succ s_1 \succ s_3$. Students' preferences are as follows: $\succ_{s_1}: c_2 \succ c_1$, $\succ_{s_2}: c_1 \succ c_2$ and $\succ_{s_3}: c_1 \succ c_2$. This information is summarized in Table 1 below.

Table 1

	\succ_{c_1}	\succ_{c_2}	\succ_{s_1}	\succ_{s_2}	\succ_{s_3}
s_1	s_2	s_1	c_2	c_1	c_1
s_2	s_1	s_2	c_1	c_2	c_2
s_3	s_3	s_3			
capacities	$q_{c_1} = 1$	$q_{c_2} = 2$			

Consider the case in which both schools join the centralized clearinghouse. Then the school-proposing deferred acceptance algorithm is used to assign students to schools. At the first step of the algorithm, c_1 applies to s_1 and c_2 applies to s_1 and s_2 . Student s_1 rejects c_1 . At the second step, school c_1 proposes to s_2 . Student s_2 rejects c_2 . At the third step, school c_2 proposes to s_3 . Since there are no rejections, the algorithm ends after this step and the tentative assignment is made permanent. The final matching is $\{(c_1, s_2), (c_2, \{s_1, s_3\})\}$.

Consider the case in which c_2 evades the centralized clearinghouse. In the clearinghouse, the student-proposing deferred acceptance algorithm is used. All students apply to c_1 . School c_1 accepts s_1 and rejects s_2 and s_3 . The algorithm ends after the first stage. Since all students prefer c_2 to the outcome of the algorithm, they apply to c_2 . School c_2 accepts s_1 and s_2 . The final matching is $\{(c_1, \emptyset), (c_2, \{s_1, s_2\})\}$.

School c_2 revealed prefers $\{s_1, s_2\}$ over $\{s_1, s_3\}$, so it evades the clearinghouse rather than joining it. Therefore, it is not an equilibrium for all schools to join the clearinghouse.

Appendix C. Boston (or Immediate Acceptance) Mechanism

The Boston mechanism is a widely used algorithm in school choice (Abdulkadiroğlu and Sönmez, 2003). To apply it, we need to assume that each school has a capacity and a strict preference ordering over students and having an empty seat, that is, each school has a responsive choice rule.

Step 1: Each student proposes to her most preferred school. Each school permanently accepts its most-preferred acceptable students up to its capacity and rejects the rest. If there are no rejections, then stop.

Step k: Each student who was rejected in Step $k - 1$ proposes to her next preferred school. Each school accepts its most-preferred acceptable students up to its remaining capacity and rejects the rest. If there are no rejections, then stop.

The main difference between the Boston mechanism and the deferred-acceptance algorithm is that in Boston mechanism the acceptances at each step are permanent whereas in the deferred acceptance algorithm the acceptances are made permanent only after the last step of the algorithm.

Theorem 6. *Suppose that all schools except school c have joined the centralized clearinghouse in which the Boston mechanism is used. Then school c revealed prefers the set of students that it gets by evading the clearinghouse to the set of students that it gets by joining it.*

Proof. Let k be the last step at which school c admits a student in the Boston mechanism when all schools have joined the centralized clearinghouse.

Claim: For every $i \leq k$, 1) each student s who is matched at Step i of the Boston mechanism when all schools are present gets a weakly worse school when school c evades the clearinghouse, 2) the number of unfilled seats in the beginning of Step i for a school $c' \neq c$ is weakly smaller when school c evades the clearinghouse, and 3) there are weakly more applicants to a school $c' \neq c$ at Step i when school c evades the clearinghouse.

We prove this claim by mathematical induction on i . For $i = 1$, all students who get matched at Step 1 of the algorithm when all schools are present are matched with their top choice schools, so when school c evades the clearinghouse they get weakly worse schools. The second claim is trivial since all seats are unfilled in the beginning of Step 1. The third claim follows from the fact that each student whose top choice is c applies to another school at Step 1 when school c evades the clearinghouse.

Suppose that all of the claims hold for every $i < r$ where $r \leq k$. Let us prove the claim for $i = r$. For the first claim, note that any student s who gets matched in step r is not applying to school c before step r since school c has empty seats at least until step k . Moreover, by the second and third claims each school c' has fewer unfilled seats remaining at any previous step and more applicants when school c evades the clearinghouse. Therefore, student s cannot be matched before step r when school c evades the clearinghouse. In other words, student s gets a weakly worse school when school c evades the clearinghouse.

To prove the second claim, note that there are weakly smaller unfilled seats in the beginning of step $r - 1$ for a school $c' \neq c$ and also there are more applicants when school c evades the clearinghouse. As a result, school c' has weakly smaller number of unfilled seats in the beginning of step r when school c evades the clearinghouse.

For the third claim, note that any student s applying to school c' at Step r is not applying to school c before this step. Since each school gets more applications before Step $r - 1$ with fewer unfilled seats when school c evades the clearinghouse, student s cannot get into a better school when school c evades the clearinghouse. As a result, student s still applies to school c' when school c evades the clearinghouse.

One implication of these claims is that any student matched with school c gets a weakly worse school when school c evades the centralized clearinghouse. Therefore, $\{s : c \succ_s \mu_{C \setminus \{c\}}(s)\} \supseteq \mu_C(c)$ where μ_C is the matching produced by the Boston algorithm when C is the set of schools present. By Lemma 1, each school c revealed prefers the set of students that it gets by evading the clearinghouse to the set of students that it gets by joining it. \square

Like the deferred acceptance algorithms that we have considered above, a school always prefers to evade the clearinghouse under the Boston mechanism. So Boston mechanism does not mitigate the problem either.