Inspecting the Mechanism:
Leverage and the Great Recession in the Eurozone*

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Abstract
We provide a first comprehensive account of the dynamics of Eurozone countries from the creation of the Euro to the Great recession. We model each country as an open economy within a monetary union and analyze the dynamics of private leverage, fiscal policy and spreads. A parsimonious model can replicate the time-series of nominal GDP, employment, and net exports of Eurozone countries between 2000 and 2012. We then ask how periphery countries would have fared with: (i) more conservative fiscal policies; (ii) macro-prudential tools to control private leverage; (iii) a central bank acting earlier to limit financial segmentation; and (iv) effective fiscal devaluation. To perform these counterfactual experiments, we use U.S. states as a control group that did not suffer from a sudden stop. We find that periphery countries could have stabilized their employment if they had followed more conservative fiscal policies during the boom. This is especially true in Greece. For Ireland, however, given the size of the private leverage boom, such a policy would have required buying back almost all of the public debt. Macro-prudential policy would have been especially helpful in Ireland and Spain. However, in presence of a spending bias in fiscal rules, macro-prudential policies would have led to less prudent fiscal policies in the boom. If spreads had not spiked, employment would have been stabilized in all countries because they would not have been constrained into fiscal austerity. Finally, a fall in export prices - through a fiscal devaluation - would have enabled countries to attenuate the employment bust and to reduce their public debt.

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The lesson to be learned from the crisis is that a currency union needs ironclad budget discipline to avert a boom-and-bust cycle in the first place. Hans Werner Sinn (2010)

On the eve of the crisis (Spain) had low debt and a budget surplus. Unfortunately, it also had an enormous housing bubble, a bubble made possible in large part by huge loans from German banks to their Spanish counterparts. Paul Krugman (2012)

The situation of Spain is reminiscent of the situation of emerging economies that have to borrow in a foreign currency...they can suddenly be confronted with a “sudden stop” when capital inflows suddenly stop leading to a liquidity crisis. Paul de Grauwe (2012)

Countries which lost competitiveness prior to the crisis experienced the lowest growth after the crisis. Lorenzo Bini Smaghi (2013)

These quotes illustrate the wide disagreement about the nature of the eurozone crisis. Some see the crisis as driven by fiscal indiscipline, some emphasize excessive private leverage, while others focus on sudden stops or competitiveness divergence due to fixed exchange rates. Most observers understand that all these “usual suspects” have played a role, but do not offer a way to quantify their respective importance. In this context it is difficult to frame policy prescriptions on macroeconomic policies and on reforms of the eurozone. Moreover, given the scale of the crisis, understanding the dynamics of the Eurozone is one of the major challenge for macroeconomics today. This requires a quantitative framework to identify the various mechanisms at play.

The ultimate goal of this paper is to perform counterfactual experiments. For instance, we want to understand what would have happened to a particular country if it had run a different fiscal policy during the boom years, or if the eurozone had figured out a way to prevent sudden stops. Our contribution is to propose a model and an identification strategy to answer these questions. Needless to say, this is a difficult task that requires several steps: (i) specify a model and collect the data; (ii) find an identification strategy; (iii) run counterfactual experiments. This is what we do.

One feature of our analysis needs to be explained immediately to avoid confusion: we do not attempt to explain the average dynamics of the currency union. Instead, we study the relative dynamics of each country within the eurozone. Our model explains the impact of a sudden stop, say, on employment in Spain versus employment in Germany. In our control group, we focus on the dynamics of each state within the United States. This is how we identify our model and how we can make progress, but it is also obviously a limitation of our analysis. We do not claim to have a fully structural analysis of the eurozone crisis, but we claim that all the steps we take in this paper are necessary for such a structural analysis.
Our model focuses on three variables: private debt, fiscal policy, and funding costs. We analyze a collection of small open economies in a monetary union. Each economy has an independent fiscal policy and is populated by patient and impatient agents. Impatient agents borrow from patient agents but are subject to a time-varying borrowing limit. Governments tax, spend, and borrow. Funding costs are linked to private and public debt sustainability. Nominal wage are rigid so that changes in nominal expenditures affect employment. Our first contribution is to show that this parsimonious model does a fairly good job at replicating the dynamics of each individual eurozone country over the 13 years for which we have data. More precisely, given the paths of private debt, government spending and interest rates from 2000 to 2012, the model predicts the correct paths for GDP, employment, inflation, net exports, etc.

It is clear, however, that private debt, fiscal policy and funding costs are jointly endogenous. A key challenge is then to identify structural shocks that give rise to the observed dynamics. For instance, we would like to identify “sudden stop” shocks, and “private lending” shocks. But both shocks are going to affect interest rates, private debt, and they will also affect fiscal policy and public debt via general equilibrium effects and policy responses. Our key idea is to use the United States as a control group to help us identify shocks within the eurozone. The U.S. experience is of great interest for us because of both its similarities and its differences with the eurozone experience. A salient feature of the great recession in both the US and the eurozone is that regions that have experienced the largest swings in household borrowing have also experienced the largest declines in employment and output. Figure 1 illustrates this feature of the data, by plotting the change in employment during the credit crunch (2007-2010) against the change in household debt-to-income ratios during the preceding boom (2003-2007) for the largest US states and Eurozone countries.\footnote{State level household debt for the US comes from the Federal Reserve Bank of New York, see Midrigan and Philippon (2010). Nominal (wage) rigidities play an important role in our model. As noted in Midrigan and Philippon (2010), the pattern of figure 1 is at odds with the predictions of standard models of financing frictions with flexible wages. Such models predict that a tightening of borrowing constraints at the household level leads to a decline in consumption but, due to wealth effects, to an increase in the supply of labor.}

Until 2010, the American and European experiences look strikingly similar. This suggests both similar shocks and similar structural parameters governing the endogenous propagation mechanism. A significant difference between the two regions appears only after 2010 when several eurozone countries experience sudden stops and sovereign debt crises.\footnote{Sudden stops have been frequent in the 19th and 20th centuries but we do not know of any other historical example of a sudden stop among countries or states inside a monetary union. See Accominotti and Eichengreen (2013).} Consider for example Arizona and Ireland. Both had large increases in household debt during the boom years. Figure 2 shows the evolutions of their employment rates relative to 2005. The boom-bust cycle is almost identical up to 2010 but diverges afterwards. A similar pattern emerges when we compare Spain and Florida. Again, divergence is clear after 2010.
Many states within the U.S. experienced large private leveraging/deleveraging cycle. This allows us to identify debt dynamics that are not due to interest rate spreads, i.e., the private debt dynamics that would have prevailed across the eurozone if it had not experienced a sudden stop. This is our most important and novel identification strategy. Our other identifying assumptions are more standard. We assume a fiscal rule that stabilizes employment and reacts to funding costs, and we introduce a country-specific spending bias, which we called the “political economy” factor. Together with the fiscal rule, it predicts government spending as a function of the state of the economy. We estimate a small (essentially zero) political economy bias in some countries, such as Germany and Portugal, and a large one in some other countries, such as Greece for instance. Finally, we think of “sudden stops” as a latent risk that grows after 2008 and we show that it materializes in countries with high public and private debts, including implicit liabilities via bank recapitalization needs. We use instrumental variables to estimate the impact of public and private debts on the economy’s cost of fund.

Our “structural” model therefore features endogenous private debt, endogenous fiscal policy and endogenous spreads. We show that this structural model fits the data fairly well. The critical advantage of the structural model – compared to the model that takes as given the paths of private debt, government spending and interest rates, as explained above – is that we can use the structural model to perform counterfactual
experiments. We perform four such experiments.

We first ask how countries would have fared if they had followed more conservative fiscal policies during the boom. To do so, we shut down the “political economy spending bias” of the structural model. We find that such policies lowered spreads and the need for fiscal austerity during the bust. We find that periphery countries would then have stabilized their employment. This is especially true for Greece, and to a lesser extent for Ireland and Spain. For Ireland, however, this more conservative policy would have entailed buying back the entire stock of public debt, which seems implausible. This suggests that fiscal policy alone cannot act as a stabilization tool in presence of a massive private credit boom. Most of these results are consistent with many policy makers’ beliefs about the crisis, but we are the first to formalize and quantify them.

We then ask how these countries would have fared if they had conducted macro-prudential policies to limit private leverage during the boom. This would have successfully stabilized employment, in part because this would have decreased the need for bank recapitalization, leading to lower spreads and more room for countercyclical fiscal policy. We also highlight a new interaction between macro-prudential and fiscal policies. For a given political economy bias, a government would substitute public debt to private debt in response to restrictive macro-prudential policy. This suggests a complementarity between fiscal rules and macro-prudential rules.

In a third counterfactual, we find that if the ECB words and actions (Mario Draghi’s declaration “Whatever it takes” and the OMT program) had come in 2008 rather than 2012 and had been successful in reducing the spreads, the four countries would have been able to avoid the latest part of the employment slump. Ireland’s employment, for instance, would have looked very much like Arizona’s in Figure (2). The
improvement comes from lower funding costs for the private sector and from less fiscal austerity. In Greece, however, an unconditional OMT would have led to a return of high and unsustainable government spending. This highlights the need for conditionality when central banks intervene.

In our last counterfactual, we let countries engineer a 10% fiscal devaluation in 2009 that generates a boom in exports. We find that they would have experienced a shorter and milder bust and a smaller buildup in public debt.

Relation to the literature

Our paper is related to three lines of research: (i) macroeconomic models with credit frictions, (ii) monetary economics, (iii) sudden stops and sovereign defaults. We discuss the connections of our paper to each topic. Following Bernanke and Gertler (1989), many macroeconomic papers introduce credit constraints at the entrepreneur level (Kiyotaki and Moore (1997), Bernanke et al. (1999), or Cooley et al. (2004)). In all these models, the availability of credit limits corporate investment. As a result, credit constraints affect the economy by affecting the size of the capital stock. Curdia and Woodford (2009) analyze the implication for monetary policy of imperfect intermediation between borrowers and lenders. Gertler and Kiyotaki (2010) study a model where shocks that hit the financial intermediation sector lead to tighter borrowing constraints for entrepreneurs. We model shocks in a similar way. The difference is that our borrowers are households, not entrepreneurs, and, we argue, this makes a difference for the model’s cross-sectional implications. Models that emphasize firm-level frictions cannot reproduce the strong correlation between household-leverage and employment at the micro-level, unless the banking sector is island-specific, as in the small open economy “Sudden Stop” literature (Chari et al. (2005), Mendoza (2010)). This “local lending channel” does not appear to be operative across U.S. states, however, presumably because business lending is not very localized\(^3\). Our framework is also related to heterogeneous-agent macroeconomic models such as Krusell and Smith (1998), and models in the tradition of Campbell and Mankiw (1989), that feature impatient and patient consumers. This type of models has been used by Gali et al. (2007) to analyze the impact of fiscal policy on consumption and by Eggertsson and Krugman (2012) to analyze macroeconomic dynamics during the Great Recession.

Papers in the sudden stop literature have aimed at reproducing the stylized facts of these crises in emerging markets. According to Korinek and Mendoza (2013) the key characteristics of a sudden stop are 1) a sharp, sudden reversal in international capital flows, which is typically measured as a sudden increase in the current account 2) a deep recessions and 3) sharp changes in relative prices, including exchange rate depreciations.

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\(^3\)For instance, Mian and Sufi (2010) find that the predictive power of household borrowing remains the same in counties dominated by national banks. It is also well known that businesses entered the recession with historically strong balanced sheets and were able to draw on existing credit lines Ivashina and Scharfstein (2008).
The eurozone crisis shares the two first characteristics even if the pace of current account adjustment in the euro area is slower than for non-euro area countries (such as Bulgaria, Latvia and Lithuania) and past experiences of emerging markets crises (see Merler and Pisani-Ferry (2012) for a discussion). Substitution of private-capital inflows by public inflows, especially Eurosystem financing, partly explains this difference. The third characteristic of an emerging market sudden stop has been absent in the eurozone crisis: there has been (so far) no currency depreciation and no sudden and large change in goods relative prices between countries hit at different degrees by a sudden stop (Greece, Spain, Ireland, Portugal and Italy) and the rest of the eurozone. That these countries belong to a monetary union means the eurozone sudden stop stands apart. These differences are important for the choice of modeling approach. The sudden stop literature on emerging markets (see Mendoza (2010) and Korinek and Mendoza (2013) for example) has focused on a Fisherian amplification mechanism where debts are denominated in different units than incomes and collateral. This is not the case in our model as we study countries that belong to a monetary union. Another difference is that the sudden stop literature in emerging markets has focused on the sudden imposition of an external credit constraint (see Mendoza and Smith (2006) and Christiano and Roldos (2004) for example) or on transaction costs on international financial markets with multiple equilibria, as in Martin and Rey (2006). Our model integrates, for the first time to our knowledge, both a domestic credit crunch and a sudden stop produced by a spike in interest rate so that we can compare the impact of both on macroeconomic aggregates. The role of interest rates in our model relates our work to the paper of Neumeyer and Perri (2005). In their paper, as in ours, the economy is subject to interest rate shocks that generate a sudden stop in the form of a current account reversal. However, the mechanism is very different. In Neumeyer and Perri (2005), real interest rates movements either exogenous or induced by productivity shocks amplify the effect of the latter on production because they induce a working capital shortage. In our model, the increase in interest rate generates a demand shock through a fall in consumption.

Even if the bulk of the literature on sudden stops has put credit constraints at the center of the story, Gopinath (2004) and Aguiar and Gopinath (2007) have focused on an alternative explanation with TFP shocks taking center stage. Gopinath (2004) proposes a model with a search friction to generate asymmetric responses to symmetric shocks. A search friction in foreign investors entry decision into emerging markets creates an asymmetry in the adjustment process of the economy: An increase in traded sector productivity raises GDP on impact and it continues to grow to a higher long-run level. On the other hand, a decline in traded sector productivity causes GDP to contract in the short run by more than it does in the long-run. A related approach is the possibility of growth shocks as explored in Aguiar and Gopinath (2007). Because
of the income effect, a negative shock leads to a fall in consumption and an increase in the trade balance. Aguiar and Gopinath (2007) do not study the response of the labor market but it is well known that income effects tend move consumption and hours in opposite directions.

Shocks to trend TFP growth might be important in emerging markets, but they do not seem to explain the dynamics of euro area countries over the past five years. With the exception of Greece, countries that were hit by a sudden stop (Greece, Ireland, Italy, Spain, Portugal) are not those for which the reversal in TFP growth is the largest between the boom and the bust periods and no correlation appears between the differential in TFP growth (between the periods 2008-2012 and 2000-2007) and employment growth during the bust (2008-2012) as illustrated by figure 30 in Appendix B.

Most closely connected to our paper is the work of Midrigan and Philippon (2010), Guerrieri and Lorenzoni (2010) and Eggertsson and Krugman (2012) who also study the responses of an economy to a household-level credit crunch. Consistent with our results, Mian and Sufi (2012) show that differences in the debt overhang of households across U.S. counties partly explain why unemployment is higher in some regions than others. Schmitt-Grohe and Uribe (2012) emphasize the role of downward wage rigidity in the Eurozone recession. Our paper is also related to the literature on sovereign default (see Eaton and Gersovitz (1982), Arellano (2008) and Mendoza and Yue (2012)) that models default as a strategic decision with a tradeoff between gains from forgone repayment and the costs of exclusion from international credit markets. The objective of our paper however is to analyze how the sovereign default risk can affect the real economy through the impact it can have on liquidity available to households. The paper by Corsetti et al. (2013) considers a “sovereign risk channel,” through which sovereign default risk spills over to the rest of the economy, raising funding costs in the private sector. Finally the paper is related to the recent research on fiscal multipliers at the regional level (Nakamura and Steinsson (2014), Farhi and Werning (2013)).

In Section 1 we present the model and in section 2 we analyze its dynamic properties. 3 presents the calibration exercise and compares the reduced form model predictions to the data. In Section 4, we present the structural relations between private leverage, fiscal policy and sudden stops. These serve to conduct the final exercise on counterfactual policies presented in section 5. Section 6 concludes.

1 Model

We model a currency union with several regions. We follow Gali and Monacelli (2008) and study a small open economy that trades with other regions. Each region $j$ produces a tradable domestic good and is
populated by households who consume the domestic good and a basket of foreign goods. Following Mankiw (2000) and more recently Eggertsson and Krugman (2012), we assume that households are heterogeneous in their degree of time preference. More precisely, in region \( j \), there is a fraction \( \chi_j \) of impatient households, and \( 1 - \chi_j \) of patient ones. Patient households (indexed by \( i = s \) for savers) have a higher discount factor than borrowers (indexed by \( i = b \) for borrowers): \( \beta \equiv \beta_s > \beta_b \). Saving and borrowing are measured in units of the common currency (euros).

1.1 Within period trade and production.

Consider household \( i \) in region \( j \) at time \( t \). Within period, all households have the same log preferences over the consumption of home (\( h \)), foreign goods (\( f \)), and labor supply:

\[
\begin{align*}
\ u_{i,j,t} &= \alpha_j \log \left( \frac{C^h_{i,j,t}}{\alpha_j} \right) + (1 - \alpha_j) \log \left( \frac{C^f_{i,j,t}}{1 - \alpha_j} \right) - \nu (N_{i,j,t})
\end{align*}
\]

With these preferences, households of region \( j \) spend a fraction \( \alpha_j \) of their income on home goods, and \( 1 - \alpha_j \) on foreign goods. The parameter \( \alpha_j \) measures how closed the economy is, because of home bias in preferences or trade costs. The demand functions are then:

\[
\begin{align*}
p^h_{j,t} C^h_{i,j,t} &= \alpha_j X_{i,j,t}, \\
p^f_{t} C^f_{i,j,t} &= (1 - \alpha_j) X_{i,j,t},
\end{align*}
\]

where

\[
X_{i,j,t} \equiv p^h_{j,t} C^h_{i,j,t} + p^f_{t} C^f_{i,j,t}
\]

measures the spending of household \( i \) in region \( j \) in period \( t \), \( p^h_{j,t} \) is the price of home goods in country \( j \) and \( p^f_{t} \) is the price index of foreign goods. This gives the indirect utility

\[
U (X_{i,j,t}, P_{j,t}) = \log (X_{i,j,t}) - \log P_{j,t} - \nu (N_{i,j,t}),
\]

where the CPI of country \( j \) is \( \log P_{j,t} = \alpha_j \log p^h_{j,t} + (1 - \alpha_j) \log p^f_{j,t} \), the PPI is \( p^h_{j,t} \), and the terms of trade are \( \frac{p^f_{j,t}}{p^h_{j,t}} \). Foreign demand for the home good also has a unit elasticity with respect to export price \( p^h_{j,t} \). Production is linear in labor \( N_{j,t} \) and competitive, so \( p^f_{j,t} = w_{j,t} \). Market clearing in the goods market
requires
\[ N_{j,t} = \chi_j C_{b,j,t}^h + (1 - \chi_j) C_{s,j,t}^h + \frac{F_{j,t}^h}{p_{j,t}^h} + \frac{G_{j,t}}{p_{j,t}^h}, \]
where \( F_{j,t} \) is foreign demand and \( G_{j,t} \) are nominal government expenditures. Note that we assume that the government spends only on domestic goods. Define nominal domestic product as
\[ Y_{j,t} \equiv p_{j,t}^b N_{j,t} \]
and total private expenditures as
\[ X_{j,t} \equiv \chi_j X_{b,j,t} + (1 - \chi_j) X_{s,j,t}. \]
It is useful to write the market clearing condition in nominal terms (in euros)
\[ Y_{j,t} = \alpha_j X_{j,t} + F_{j,t} + G_{j,t}. \]

Each household supplies labor at the prevailing wage and receives wage income net of taxes \((1 - \tau_{j,t}) w_{j,t} N_{j,t}\). They also receive transfers from the government \( Z_{j,t} \). We assume that wages are sticky and we ration the labor market uniformly across households. This assumption simplifies the analysis because we do not need to keep track separately of the labor income of patient and impatient households within a country. Not much changes if we relax this assumption, except that we lose some tractability.\(^4\)

### 1.2 Inter-temporal budget constraints

Let \( B_{j,t} \) be the face value of the debt issued in period \( t - 1 \) by impatient households and due in period \( t \). It will be convenient to define disposable income (after tax and transfers but before interest payments) as
\[ \bar{Y}_{j,t} \equiv (1 - \tau_{j,t}) Y_{j,t} + Z_{j,t}. \]

The budget constraint of impatient households in country \( j \) is then
\[ \bar{Y}_{j,t} = (1 - \tau_{j,t}) Y_{j,t} + Z_{j,t}. \]

\(^4\)In response to a negative shock, impatient households would try to work more. The prediction that hours increase more for credit constrained households appears to be counter-factual however. One can fix this by assuming a low elasticity of labor supply, which essentially boils down to assuming that hours worked are rationed uniformly in response to slack in the labor market. Assuming that the elasticity of labor supply is small (near zero) also means that the natural rate does not depend on fiscal policy. In an extension we study the case where the natural rate is defined by the labor supply condition in the pseudo-steady state \( \nu'(n_i^e) = (1 - \tau_i) \frac{w}{\pi_i} \). We can then ration the labor market relative to their natural rate: \( n_{i,j,t} = \frac{n_i^e(\tau)}{\sum_i n_i^e(\tau)} n_{j,t} \) where \( n_i^e(\tau) \) is the natural rate for household \( i \) in country. This ensures consistency and convergence to the correct long run equilibrium. Steady state changes in the natural rate are quantitatively small, however, so the dynamics that we study are virtually unchanged. See Midrigan and Philippon (2010) for a discussion.
\[ \frac{B_{j,t+1}}{1 + r_{j,t}} + Y_{j,t} = X_{b,j,t} + B_{j,t}, \]

(2)

where \( r_{j,t} \) is the nominal cost of fund between \( t \) and \( t + 1 \). Notice that the budget constraint is written without the possibility of default by the borrower. In such a case, and without taking into account issues of market liquidity, the cost of fund is the same as the interest rate. When we discuss the model, we therefore refer to \( r_{j,t} \) as the interest rate. But when we turn to the data, it is obviously critical to remember that \( r_{j,t} \) is really meant to capture the cost of funds. We assume that interest rates are time-varying and potentially country-specific. Borrowing is subject to the exogenous limit \( B_{j,t}^{h} \):

\[ B_{j,t} \leq B_{j,t}^{h}. \]

(3)

The savers budget constraint is:

\[ S_{j,t} + Y_{j,t} = X_{s,j,t} + \frac{S_{j,t+1}}{1 + r_{j,t}}, \]

(4)

so their Euler equation is

\[ \frac{1}{X_{s,j,t}} = E_{t} \left[ \beta (1 + r_{j,t}) \frac{X_{s,j,t+1}}{X_{s,j,t+1}} \right]. \]

(5)

Note that financial markets clear in two ways in our model. For the impatient agents, given that they are quantity constrained, interest rates do not affect their borrowing. For the patient agents, their saving is determined by the interest rates through the Euler equation.

The government budget constraint is:

\[ \frac{B_{g,j,t+1}}{1 + r_{j,t}} + r_{j,t} Y_{j,t} = G_{j,t} + Z_{j,t} + B_{g,j,t}, \]

(6)

where \( B_{g,j,t} \) is public debt issued by government \( j \) at time \( t - 1 \).

### 1.3 Exports and foreign assets

Nominal exports are \( F_{j,t} \) and nominal imports are \( (1 - \alpha_{j}) X_{j,t} \) since the government does not buy imported goods while private agents spend a fraction \( 1 - \alpha_{j} \) on foreign goods. So net exports are:

\[ E_{j,t} = F_{j,t} - (1 - \alpha_{j}) X_{j,t}. \]

(7)
The net foreign asset position of the country at the end of period \( t \), measured in market value, is:

\[
A_{j,t} \equiv (1 - \chi_j) \frac{S_{j,t+1}}{1 + r_{j,t}} - \chi_j \frac{B_{j,t+1}^h}{1 + r_{j,t}} - \frac{B_{j,t+1}^g}{1 + r_{j,t}}.
\] (8)

Adding up the budget constraints, we have the spending equation

\[
X_{j,t} + G_{j,t} = Y_{j,t} + \chi_j \left( \frac{B_{j,t+1}^h}{1 + r_{j,t}} - B_{j,t}^h \right) - (1 - \chi_j) \left( \frac{S_{j,t+1}}{1 + r_{j,t}} - S_{j,t} \right) + \frac{B_{j,t+1}^g}{1 + r_{j,t}} - B_{j,t}^g.
\] (9)

Total spending (public and private) equals total income (nominal GDP) plus total net borrowing. If we combine with the market clearing condition (1), we get the current account condition

\[
CA_{j,t} \equiv A_{j,t} - A_{j,t-1} = E_{j,t} + r_{j,t-1} A_{j,t-1}.
\]

It will often be convenient to rewrite (9) with disposable income as

\[
(1 - \alpha_j) \tilde{Y}_{j,t} = \alpha_j \chi_j \left( \frac{B_{j,t+1}^h}{1 + r_{j,t}} - B_{j,t}^h \right) - \alpha_j (1 - \chi_j) \left( \frac{S_{j,t+1}}{1 + r_{j,t}} - S_{j,t} \right) + F_{j,t} + \frac{B_{j,t+1}^g}{1 + r_{j,t}} - B_{j,t}^g.
\] (10)

### 1.4 Employment and Inflation

The system above completely pins down the dynamics of nominal variables: \( Y_{j,t}, X_{i,j,t}, \) etc. Employment (real output) is given by

\[
N_{j,t} = \frac{Y_{j,t}}{p_{j,t}}.
\]

We need to specify the dynamics of inflation. Letting \( N^* \) denote the natural rate of unemployment, we assume the following Phillips curve:

\[
\frac{p_{j,t}^h - p_{j,t-1}^h}{p_{j,t-1}^h} = \kappa (N_{j,t} - N^*)
\] (11)

### 1.5 Discussion of the main modeling assumptions

Our modeling choices are motivated by just one goal: to be able to identify the sources of the Great Recession across eurozone countries. Any economic item that we feel is not strictly necessary has deliberately been left out. One such item deserves an explicit discussion: housing.\(^5\)

\(^5\)We also ignore corporate investment but this is a lesser concern. First, for all firms (SMEs) that are credit constrained, we can simply add their debts to our constrained households’ debts since what matters is only the implied budget constraint.
Our model does not incorporate housing. Given the obvious importance of housing in explaining the rise in household debt in some countries (namely Spain and Ireland), this might seem like a serious omission. We have thought rather carefully about this issue. A previous version of our paper had an explicit housing sector, but we decided to remove it to simplify our already rather lengthy paper. It is therefore important to explain this modeling choice. In a nutshell, given the structure of our model, the downside of not including housing explicitly is only that we fail to capture the dynamics of hours worked in construction relative to hours worked in the rest of the economy. To preview of results, this means that we underestimate the boom-bust cycle of employment in Spain. We argue that this is a small price to pay for a major simplification.

The economic intuition can be understood from the work of Midrigan and Philippon (2010). In their model, the debt constraint comes from the usual collateral constraint. In equation (3), they use \( B_{h,j,t} = \kappa Q_{j,t} H_{j,t} \) where \( Q_{j,t} \) and \( H_{j,t} \) are the price and quantity of housing, \( \kappa \) is a parameter (which can be time varying if needed but this is immaterial for our discussion). If the supply of housing is fixed, i.e. if in equilibrium \( H_{j,t} = \bar{H}_j \), then the dynamics of this economy are exactly the same as the dynamics of an economy without housing where we exogenously impose \( B_{h,j,t} = \kappa Q_{j,t} \bar{H}_j \). It is indeed easy to check that both the first order conditions and the market clearing conditions are identical in the two economies. If the quantity of housing is endogenous, then the equivalence obviously breaks down, but only in the labor market clearing condition. The dynamics of GDP in units of the common currency are unchanged.\(^6\)

2 Dynamic Properties of the Model

We now study the dynamics of a small open economy subject to four types of shocks: the borrowing limit of the impatient households \( B_{h,j,t} \), foreign demand \( F_{j,t} \), interest rates, and fiscal policy. We present some simple impulse response functions to build intuition about the mechanics of the model. The details of the assumptions and policy functions used to compute these impulse responses are in the Appendix A, and Martinez and Philippon (2014) provide a more theoretical discussion of the same framework, and in particular of the behavior of savers. Here we only mention one insight that is useful to interpret the impulse responses. Saver’s spending (in euros) reacts neither to \( B_{h,j,t} \), nor to \( G_{j,t} \) nor to \( Z_{j,t} \), because shocks to these variables affect the path of disposable income but not the net present value (in euros) of disposable income.

\(^6\)This equivalence holds in our model in particular because we have two fixed types: patient and impatient. It would not hold in the more advanced setup of Kaplan and Violante (2011). In that setup modeling housing would bring in new insights. In a model à la Campbell and Mankiw (1989) and Mankiw (2000), it does not.
As a result, these shocks affect the expenditures of impatient agents (that are effectively hand-to-mouth) but not those of patient agents. Shocks to foreign demand or to interest rates, on the other hand, affect the expenditures of patient agents. These results rely on the log preferences of Cole and Obstfeld (1991), but they are convenient because they allow us to solve the nominal side of the model (all variables in euros) independently of the Phillips curve, and we show later that they seem consistent with the data. Of course, even when expenditures remain constant, real consumption changes because prices (wages) react to changes in aggregate spending.

2.1 Scaling and Spreads

We assume that the variance of interest rates shocks is small and we linearize the Euler equation (5) as

\[ \mathbb{E}_t [X_{s,j,t+1}] \approx \beta (1 + r_{j,t}) X_{s,j,t}. \]

The equivalent equation for the monetary union as a whole is \( \mathbb{E}_t [X^*_{s,t+1}] \approx \beta (1 + r^*_t) X^*_{s,t} \), where \( r^*_t \) is the interest for the monetary union as a whole. We define the spread as:

\[ 1 + \rho_{j,t} \equiv \frac{1 + r_{j,t}}{1 + r^*_t}. \]

We show in the Appendix A that if we scale all our variables by \( X^*_{s,t} \):

\[ x_{s,j,t} \equiv \frac{X_{s,j,t}}{X^*_{s,t}} \]

Then we have

\[ \mathbb{E}_t [x_{s,j,t+1}] \approx (1 + \rho_{j,t}) x_{s,j,t} \]

From now on we work with scaled variables (in lower case). For example, the patient budget constraint becomes:

\[ x_{s,j,t} + \frac{\beta}{1 + \rho_{j,t}} s_{j,t+1} = s_{j,t} + \tilde{y}_{j,t}. \]

Finally, we maintain assumptions A1 throughout the paper

**Assumptions A1.**

- \( \mathbb{E}_t [f_{j,t+1}] = f_{j,t} \),
- \( \mathbb{E}_t [\rho_{j,t+1}] = 0 \);
Assumptions A1 say that the shocks on foreign demand are permanent and spreads are iid. These two conditions are assumed to hold throughout the paper. Note that we do not impose that the interest rate in the currency union is iid, only that deviations for a particular country are expected to last one year.  

2.2 Impulse responses to shocks

Figures (3), (4), (5) and (6) illustrate the impact of shocks to household debt \( b_{j,t} \), public spending \( g_{j,t} \), interest rates \( r_{j,t} \) and foreign demand \( f_{j,t} \).

In Figure 3, an increase in household debt generates a boom in spending by impatient households, employment and imports. Public debt falls but the net foreign asset position deteriorates. A fiscal expansion (Figure 4) has qualitatively similar effects except that public debt increases. The multiplier for household debt and government spending are increasing functions of \( \alpha_i \) and \( \chi_i \). A higher share of impatient agents in the economy implies that an increase in disposable income has a larger impact on aggregate expenditures. A higher share of spending on domestic goods reduces leakage through imports. Remember that patient agents expenditures do not react to private or public debt changes.

An increase in interest rates (Figure 5) is very different because it induces patient households to save more so it reduces their expenditures and generates a recession (fall in nominal GDP and in employment) that obliges impatient households to reduce their spending. Imports fall and the net foreign asset position improves. Because of lower tax revenues, the recession increases public debt. Finally, in Figure 6, an increase in foreign demand permanently increases nominal GDP which induces patient households to increase their saving. Spending of both patient and impatient households increase. The net foreign asset position improves. Public debt falls because of higher tax revenues.

To be clear, this means that savers in our model anticipate spreads to last for one year. When we see in the data that spreads remain high for 2 years in row, as in 2011 and 2012 for several countries, we interpret this as two negative shocks, which seems consistent with the narrative of the crisis. For instance, concerns with bank liquidity created spreads in 2010. The ECB reacted by providing liquidity. But the relief was temporary because soon after investors became worried about sovereign risk and exit. Again, it took about a year for policy makers to find an appropriate response. We have also performed robustness checks to make sure that our results are not biased by this assumption (assuming for instance that investors anticipate spreads to last for 2 years).

For these impulse response functions, we use the following parameters: \( \alpha = 0.75, \chi = 0.5, r = 0.05, \kappa = 0.2, \tau = 0.4 \). Prices, wages and employment are normalized to unity at time \( t = 0 \). The debt to income ratio is set at 60% for impatient households at time \( t = 0 \), so that the household debt to income ratio is 30%. The government debt to GDP ratio is set at 50% and the net foreign asset position over GDP is set at zero at time \( t = 0 \). The shock is a 20% increase of the variable at \( t = 1 \).
Figure 3: Private Credit Expansion

Figure 4: Fiscal Expansion
Figure 5: Interest rate shock

Figure 6: Foreign demand shock
### Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Discount Factor β</td>
<td>0.98</td>
</tr>
<tr>
<td>Domestic share of consumption α_j</td>
<td>country specific</td>
</tr>
<tr>
<td>Share of credit constrained households χ_j</td>
<td>country specific</td>
</tr>
<tr>
<td>Phillips curve parameter κ</td>
<td>0.3</td>
</tr>
</tbody>
</table>

## 3 Reduced Form Model

We simulate 11 Eurozone countries from 2000 to 2012: Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands and Portugal and calibrate the shocks on the observed data. The data sources are described in Appendix B.

### 3.1 Calibration

The parameters used in the simulations are presented in Table (1). The discount factor (of patient households) and the Philipps curve parameter are standard. The country-specific parameters – the share of credit constrained households (χ_j) and the domestic share of consumption (α_j) – are shown on Figure (7).

For the country specific domestic share of consumption, α_j, we rely on Bussiere et al. (2011) who compute the total import content of expenditure components, including the value of indirect imports. For consumption expenditures and for our sample our countries the average implied domestic share in 2005 (the latest date in their study) is 72.7%. The lowest is 66.4% for Belgium and the highest is 78.7% for Italy.

We also need to measure foreign demand F_{jt}. Given the absence of an intermediate goods sector in our model, we cannot use the value of gross exports. The trade linked to international production networks has been well documented (see for example Baldwin and Lopez-Gonzalez (2013)). In the context of our model, we need to measure the domestic value added that is associated with final consumption in the rest of the world, which corresponds to value added based exports. As detailed in appendix B, we use the data from the OECD-WTO Trade in Value-Added (TiVA) initiative to measure domestic value added embodied in gross exports. The normalized value-added based exports are shown in figure (35) in Appendix B. Finally, we take into account net EU transfers which are the difference between EU spending in the country and the country contribution to the EU. In our model, they play exactly the same role as foreign demand so we add EU net transfers to exports in the goods market equation.
Figure 7: Share of credit constrained households \( (\chi_j) \) and domestic share of consumption \( (\alpha_j) \)

Share of Constrained Households For the country specific share of credit constrained borrowers, \( \chi_j \), we use a measure by Mendicino (2014) based on the Eurosystem Household Finance and Consumption Survey (HFCS).\(^9\) For each country, Mendicino (2014) computes the fraction of household with liquid assets below two months of total households gross income to approximate the share of credit constrained households. The average for our set of countries is 48% with a maximum of 64.8% for Greece and a minimum of 34.7% for Austria. Ireland did not participate in the survey so for this country we use the average of the eurozone. Note that \( b_{j,t}^h \) in the model is debt per impatient household so the counterpart to the empirical measure of aggregate debt is \( \chi_j b_{j,t}^h \).

Cost of Fund The cost of fund \( \rho_{j,t} \) enters the Euler equation of unconstrained agents. It should represent at the same time the expected return of savers, the borrowing cost of unconstrained borrowers, and the funding cost of firms. The issue is that we only observe some interest rates, not expected returns or funding costs. We use several interest rates: (i) yields on 10-year government bonds; (ii) loans rates for SMEs; (iii) deposit rates; (iv) wholesale bank funding costs. In all cases we compute the difference between the rate in country \( j \) and the median of the Eurozone in year \( t \). Sovereign spreads are not expected returns, however, because they include expected credit losses. On the other hand, we know from a huge literature

\(^9\)The survey took place in 2010. In Greece and Spain, the data were collected in 2009 and 2008-09 respectively. This survey has been used recently by Kaplan et al. (2014) to quantify the share of hand-to-mouth households. They define these as consumers who spend all of their available resources in every pay-period, and hence do not carry any wealth across periods. They argue that measuring this behavior using data on net worth (as consistent with heterogeneous-agent macroeconomic models) is misleading because this misses what they call the wealthy hand-to-mouth households. These are households who hold sizable amounts of wealth in illiquid assets (such as housing or retirement accounts), but very little or no liquid wealth, and therefore consume all of their disposable income every period. They define hand-to-mouth consumers as those households in the survey whose average balances of liquid wealth are positive but equal to or less than half their earnings.
in finance that credit spreads create significant differences in funding costs. This is the basic point of all models with distress costs, agency costs, debt overhang, safety premia, etc. For bank-dependent borrowers, the funding costs of banks and the opportunity cost of lending are obviously critical. Both are tightly linked to sovereign spreads. Banks almost never borrow more cheaply than their own sovereign, and debt overhang in the banking sector makes it more attractive to invest in the debt of home sovereign.\footnote{In the limit of a model à la Myers (1977), the bank may end up treating the entire yield as an expected return because it only cares about the non-default state. See Philippon and Schnabl (2009) for a discussion of debt overhang.} This suggests that one could think of $\rho$ as the common component of rates (i)-(iv). But in practice we are constrained by data availability. The only rates consistently available for all years and all countries are the sovereign yield. We find that the following transformation of the sovereign yields makes them comparable to the other rates:

$$\rho = \Delta \mathbb{1}_{\Delta < 1\%} + \frac{\Delta - 1\%}{2} \mathbb{1}_{\Delta \in [1\%, 3\%]} + \frac{\Delta - 3\%}{4} \mathbb{1}_{\Delta \in [3\%, 5\%]} + \frac{\Delta - 5\%}{8} \mathbb{1}_{\Delta > 5\%},$$

where $\Delta$ is the deviation of the yield from the Eurozone median and $\rho$ is our measure of spreads in funding costs. What this means is that, for the first 100 basis points, we treat the spread as a funding cost. This is consistent with estimates of flight to quality towards German assets and liquidity risk premia. Then we divide the spread by two, etc. Above 500 basis points, we assume that only 1/8th of the spread represent funding costs. This filter creates funding costs that are comparable to the (limited) data we have on deposit rates, SME rates, and wholesale funding costs. Our results are robust as long as we trim the large spreads, otherwise the drop in consumption by savers is simply too large to be consistent with the data. Both the 10 year government bond spreads (the deviation of the yield from the Eurozone median) and $\rho$ as measured in the equation above are shown in figure (36) in appendix B.

**Scaling** In order to map the observed data into the model we scale the data in a manner consistent with equation (12). We construct the following benchmark level of nominal GDP for country $j$ at time $t$:

$$\hat{Y}_{j,t} = \frac{Y_{j,t_0} \bar{N}_{t_0} \bar{Y}_t}{N_{j,t_0} \bar{N}_t N_{j,t}},$$

where $t_0$ is the base year (2002 in our simulations), $Y$ is GDP, $N$ is population, and $\bar{Y}_t$ and $\bar{N}_t$ denote aggregate for the Eurozone. In words, the benchmark is the nominal GDP the country would have if it had the same per-capita growth rate as the eurozone together with its actual population growth. The key point is that the only country level time-varying variable that we take as exogenous is population growth. We
than scale all our variables in euros by the benchmark GDP. For GDP itself, we define

\[ y_{j,t} = \frac{Y_{j,t}}{Y_{j,t}} \]

which is one in the base year. For sovereign debt, we define

\[ b_{j,t} = \frac{B_{j,t}}{Y_{j,t}} \]

which is the actual debt to GDP ratio for the base year, but then tracks the level of debt, as in the model. This is important when we consider deleveraging. With large fiscal multipliers, a reduction in debt might leave the debt to GDP ratio unchanged in the short run. Ratios might give a very misleading view of deleveraging efforts. The normalized data for private and sovereign debt are shown in figure (33) in Appendix B. Normalized public spending and transfers are shown (34). Note also that government spending is adjusted for expenditures on bank recapitalization. For unit labor costs, we scale by the average unit labor cost in the eurozone. For employment we use employment per capita and we take the deviation from the base year.

### 3.2 Reduced Form Simulations

In our reduced form simulations, we take as given the observed series for private debt \((b_{j,t}^h)\), fiscal policy \((g_{j,t}, z_{j,t}, \tau_j)\) and interest rate spreads \((\rho_{j,t})\). The reduced form model \(\mathcal{R}\) is a mapping

\[
\mathcal{R}: (b_{j,t}^h, g_{j,t}, z_{j,t}, \rho_{j,t}) \rightarrow (b_{j,t}^g, y_{j,t}, n_{j,t}, p_{j,t}, e_{j,t}, \ldots)
\]

The scaled data on observed shocks that serve to feed the model for each country are shown in figures (33), (34), (36) and (35) in Appendix B. For each country, we simulate the path between 2001 and 2012 of nominal GDP \(y_{j,t}\), employment \(n_{j,t}\), wages \(w_{j,t}\), net exports \(e_{j,t}\) and public debt \(b_{j,t}^g\).

It is important to emphasize that there is no degree of freedom in our simulations. There is no parameter which is set to match any moment in the data. The model is entirely constrained by observable micro estimates and by equilibrium conditions. The only parameter that we can adjust is the slope of the Phillips curve \(\kappa\) but it does not affect the nominal GDP in euros, it only pins down the allocation of nominal GDP between prices (unit labor cost) and quantities (employment).

Figures (8), (9), (10) show the simulated and observed nominal GDP, net exports and employment. The reduced form model reproduces very well the cross sectional dynamics in the euro zone for nominal GDP.
and net exports. In particular, it replicates well the boom and bust dynamics on nominal GDP and the current account reversal for the crisis hit countries. For employment, the model does also well for the crisis countries and for countries that were hit less severely.

Figure 8: Reduced Form Model, Nominal GDP
Figure 9: Reduced Form Model, Net Exports

Figure 10: Reduced Form Model, Employment
4 Structural Model

In order to run counterfactuals we need to take into account that private leverage, spreads and fiscal policy may be interrelated. Hence, we need to identify the structural relations between the three variables. We think of country dynamics as being driven by three structural shocks. The first one is a boom/bust cycle in private debt, which we call credit bubble for short. The second one is a political economy bias in government spending that creates fiscal imbalances. The third is a sudden stop that threatens the stability of the eurozone. Formally, we think of the structural model $\mathcal{I}$ as a mapping

$$\mathcal{I} : \text{(Credit Bubble, Political Economy, Sudden Stop)} \rightarrow (b^h_{j,t}, g_{j,t}, z_{j,t}, \rho_{j,t}; b^g_{j,t}, y_{j,t}, n_{j,t}, p_{j,t}, e_{j,t}, \ldots) \quad (15)$$

The key point of the structural model is to explain the variables that we took as exogenous in the reduced form model (14). The challenge is to identify these shocks in the data.

We now present our identification strategy. This strategy is based on a mix of theoretical modeling and empirical identification using instrumental variables. When we talk about the “structural” model, we do not
mean that we have provided micro-foundations for every detail of the model. Given the range of data and economic forces that we need to capture, this is not even remotely possible. But we mean that, either there is an explicit theoretical equation, or there is an identified empirical equation that allows us to capture the influence of one variable on the others.

4.1 Using the U.S. to Identify Private Debt Dynamics

We use the US as a control group to estimate leverage dynamics without sudden stops. More precisely, we estimate the following model for deleveraging in a panel of U.S. states

$$b_{h,US}^{j,t} = \sum_{k=1}^{3} \alpha_{US}^{k} b_{h,US}^{j,t-k} + \epsilon_{j,t}$$

for $t = 2008, ..., 2012$, $j = 1, .., 52$, and $b_{h,US}^{j,t}$ is household debt detrended exactly as in the Eurozone. The idea is that these private leverage bubbles reflected various global and financial factors: low real rates, financial innovations, regulatory arbitrage of the Basel rules by banks, real estate bubbles, etc. To a large extent these forces were present both in Europe and in the US. The difference of course is that there was no sudden stops within the US. Hence, we interpret the US experience as representative of a deleveraging outcome in a monetary union without sudden stops.\(^{11}\)

We then take the estimated coefficients $\alpha_k^{US}$ and use them to construct predicted deleveraging in Eurozone countries:

$$\hat{b}_{h}^{j,t} = \sum_{k=1}^{K} \alpha_k^{US} b_{j,t-k}$$

for $t = 2008, ..., 2012$ and $j = 1..11$. Figure 12 illustrates the results for California and Ireland. The model predicts a somewhat slower deleveraging in Ireland than actually happened. This is also the case for other countries that experienced a sudden stop.

We now posit the following structural equations. Private leverage in euro zone countries is given by:

$$b_{h}^{j,t+1} = \lambda_{0}^{h} + \hat{b}_{h}^{j,t} + \lambda^{h,p} \rho_{j,t}. \quad (16)$$

Private leverage is equal to the prediction from the US experience plus the impact of the spread. The first element of the private leverage, $\hat{b}_{h}^{j,t}$, is interpreted as an exogenous shock. The second element, the impact of the spread, is endogenous and is meant to capture various transmission channels. An obvious one is

\(^{11}\)We discuss differences in local fiscal policy in the Appendix.
the funding costs of banks: an increase in $\rho_{j,t}$ would decrease bank lending. Another channel is that debt demand by impatient agents is not entirely interest inelastic as we have assumed in our simplified model. It will turn out, however, that $\lambda^{\rho_{h}}$ is not large and that these channels do not appear to be very important.

### 4.2 Funding Costs and Sudden Stops

The second equation of the structural model captures bond pricing:

$$\rho_{j,t} = \sigma_t \times \left[ \lambda^0 + \lambda^{\rho_{g,h}} b^g_{j,t} + \lambda^{\rho_{h,\text{high}}} b^h_{j,t} + \lambda^{\rho_{h,\text{high}}} b^h_{j,t} + \lambda^{\text{rec}} recap_{j,t} \right]$$

where $\sigma_t$ captures the size of the sudden stop. This equation says that funding costs start to diverge when there is a sudden stop in the eurozone and that the extent to which this happens in different countries depends on public debt $b^g_{j,t}$, private debt $b^h_{j,t}$, but also recapitalization of the financial sector ($recap_{j,t}$). This specification captures financial frictions associated with high leverage (debt overhang, risk shifting, adverse selection, runs, etc.). We allow the impact of public debt on spreads to be non-linear to take into account the case of Greece. More specifically, we allow the coefficient to be higher for levels of debt above 100% of GDP so we define $b^{g,\text{high}}_{j,t} = (b^{g}_{j,t} - 1)(b^{g}_{j,t} > 1)$. In our data, this non-linear effect is only significant for Greece. Finally, we have assumed so far that spreads are iid (even though the risk free rate can be persistent). Hence, we assume that $E_t[\sigma_{t+1}] = 0$. This means that agents in our economy anticipate that sudden stops, if any, last for a year.

We estimate the coefficients $\lambda$’s on the period 2008-2012 for the 11 eurozone countries. We do so by running instrumental variable regressions. For $\rho_{j,t}$ in equation (16) we use government debt lagged two
and three years as instruments. For government and private debt $b_{j,t}^g$ and $b_{j,t}^h$, in equation (17), we use as instruments government debt levels ($b_{j,t}^g$ and $b_{j,t}^{g,high}$) lagged three years, the exogenous component of private debt (and its lag) predicted by the US experience $b_{j,t}^h$. To capture the possibility and the size of a sudden stop in the eurozone, we measure the coefficient $\sigma_t$ as the divergence in spreads across the eurozone countries. More precisely, for each year, $\sigma_t$ equals the mean of the absolute value of spreads in the eurozone.

As expected, it is low and close to zero up to 2007 and starts increasing in 2008 with a maximum in 2012.

The estimated coefficients, which we will use in our simulations, are shown in Table (2):

<table>
<thead>
<tr>
<th>$\lambda_{0}^g$</th>
<th>$\lambda_{0}^{g,h}$</th>
<th>$\lambda_{0}^h$</th>
<th>$\lambda_{0}^{g,\rho}$</th>
<th>$\lambda_{0}^{n,\rho}$</th>
<th>$\lambda_{0}^{rec,\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.005$</td>
<td>$-1.2$</td>
<td>$-2.3$</td>
<td>$2$</td>
<td>$1.2$</td>
<td>$15$</td>
</tr>
<tr>
<td>$(0.002)$</td>
<td>$(0.184)$</td>
<td>$(0.16)$</td>
<td>$(0.4)$</td>
<td>$(0.3)$</td>
<td>$(3)$</td>
</tr>
</tbody>
</table>

Note: standard errors in parenthesis.

For the non linear effect of public debt on spreads, we set $\lambda_{0}^{g,high,\rho} = 1$ since it gives the best fit for Greece.

4.3 Fiscal Policy

Our last task is to specify a fiscal policy function for the different governments. We assume that the government seeks to stabilize employment but is constrained by its costs of funds. The funding constraint is measured by the (lagged) spread $\rho_{j,t-1}$ and is only binding when the spread is positive.\(^{12}\) Hence, the policy rule for government spending, with parameters $\gamma^n$ and $\gamma^{\rho}$, is given by:

$$
\begin{align*}
G_{j,t} &= \hat{G}_{j,t} + \gamma^n(n_{j,t} - \bar{n}) + \gamma^{\rho}\rho_{j,t-1} \text{ if } \rho_{j,t-1} > 0, \\
G_{j,t} &= \hat{G}_{j,t} + \gamma^n(n_{j,t} - \bar{n}) \text{ if } \rho_{j,t-1} < 0,
\end{align*}
$$

(18)

where $\hat{G}_{j,t}$ is a country specific drift which is linearly increasing until 2008 and decreasing afterwards:

$$
\hat{G}_{j,t} = G_{j,0} + \delta_j^g (\min(t, t_1) - t_0) - \delta_j^g \max(t - t_1, 0)
$$

(19)

with $t_0 = 2002$ and $t_1 = 2008$. Hence, $\delta_j^g$ represents the average “excess” annual spending growth rate during the boom years. We interpret this drift as a political bias in spending decisions that is reversed after

\(^{12}\) We use the t-1 spread simply because it fits better, which probably reflects implementation lags in fiscal policy. This is not related to the identification of the model and our results are not sensitive to this detail.
2008. The change in political bias might come from new fiscal rules agreed at the EU level, from explicit requirements for countries in a program, or more broadly from a shift in attitudes and beliefs about fiscal responsibility.\(^\text{13}\) What matters for us, however, is that countries displayed different levels of spending bias during the boom years, and we want to analyze to what extent this spending drift during the boom years may have contributed to the euro crisis. The same structural equations apply to transfers \(z_{j,t}\) with specific values for \(z_{j,0}\) and \(\delta^z_j\).

We now focus on the four countries that were most harshly hit by the crisis, namely Spain, Greece, Ireland, and Portugal. We choose our parameters in the policy rule \(\gamma^n\) and \(\gamma^\rho\) and the spending and transfer drift coefficients \(\delta^g_j\) and \(\delta^z_j\) such that the model reproduces as best as possible the dynamics of observed nominal GDP, public debt, employment, and spreads.

This leads us to choose the parameters given in Table (3) and (4):

\begin{table}[h]
\centering
\begin{tabular}{lrr}
\hline
 & \(\gamma^n\) & \(\gamma^\rho\) \\
\hline
\(-0.5\) & \(-3\) \\
\hline
\end{tabular}
\caption{Fiscal policy coefficients}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{lrrrrrrr}
\hline
 & \(\delta^g_j\) & \(\delta^z_j\) \\
Spain & 1.5\% & 1.0\% \\
Greece & 1\% & 3.0\% \\
Ireland & 0.5\% & 2.0\% \\
Portugal & 0\% & 0.5\% \\
\hline
\end{tabular}
\caption{Fiscal policy drifts: government spending and transfers}
\end{table}

Note that the spending and transfers drift necessary to reproduce the debt dynamics is (not surprisingly) much larger in Greece than in the other periphery countries. It is intermediate and similar in Ireland and Spain and very small in Portugal.

We also need to take into account the specific case of Greece which benefited in 2012 from a debt relief that reduced its public debt by around 50\% of GDP and in 2011, from a reduction in interest rates and an extension in the repayment period for the EU and IMF rescue package. We do this by adding 10\% of GDP and 40\% of GDP to government revenues in 2011 and 2012. This allows to better replicate the Greek public debt in 2012.

\[^{13}\]The fact that it is reversed is not very important for our results. We could assume that \(\tilde{g}_{j,t}\) stays constant after \(t_1\) and our simulations would be similar. In fact, our counter-factual results would be stronger since the model would then choose a larger \(\gamma^\rho\) to fit the data. But this can create issues of debt sustainability if we simulate the model beyond 2012 and we assume that the spreads normalize. In practice we also see that governments are trying to reverse some of the spending decisions they made during the boom years.
4.4 Fit of the structural model

The structural model is a constrained version of the reduced form model presented earlier. We can now formally write equation (15) as

\[ \mathcal{M} : \left( \hat{b}^h_{j,t}, \tilde{g}_{j,t}, \tilde{z}_{j,t}; \sigma_t \right) \rightarrow \left( b^h_{j,t}, g_{j,t}, z_{j,t}, \rho_{j,t}; b^g_{j,t}, y_{j,t}, u_{j,t}, p_{j,t}, e_{j,t}, \ldots \right) \] (20)

subject to the equilibrium condition of the model and the structural equations (16), (17) and (18). There are three sets of exogenous shocks: the fiscal drifts (\( \delta^g_j \) and \( \delta^z_j \)) on government expenditures \( \tilde{g}_{j,t} \) and transfers \( \tilde{z}_{j,t} \), the private leverage that would have been predicted by the US experience \( \hat{b}^h_{j,t} \), the sudden stop shock \( \sigma_t \).

We show in figures (13), (14), (15), (16) and (17), how our structural model with these benchmark parameters performs in reproducing the main macro dynamics (nominal GDP, employment, net exports, public debt and spreads) of the four periphery countries. Overall, the structural model does very well in reproducing the boom and bust episodes for the different countries. The model and observed data depart in a few dimensions: 1) the Irish nominal GDP (but not employment) bust comes too late in the model; 2) the employment boom is under-predicted in Spain and the bust is over-predicted in Greece; 3) Spreads are under-predicted in Portugal.
Figure 13: Structural Model, Nominal GDP

Figure 14: Structural Model, Employment
Figure 15: Structural Model, Net Exports

![Graphs showing net exports for ESP, GRE, IRL, and PRT from 2001 to 2012.](image)

Figure 16: Structural Model, Government Debt

![Graphs showing government debt for ESP, GRE, IRL, and PRT from 2001 to 2012.](image)
5 Counterfactual experiments

We now present our main structural experiments. The goal is to provide counter-factual simulations of what would have happened to the four countries (Greece, Spain, Ireland and Portugal) that were most affected by the crisis had they followed a different set of policies. We consider four counterfactuals:

- fiscal policy: what would have happened if those countries had pursued a more conservative fiscal policy before 2008?
- macro-prudential policies: what would have happened if they had limited the growth of household debt?
- monetary policy: what would have happened if the “whatever it takes” commitment by the ECB had been announced in 2008 rather than in 2012?
- fiscal devaluation: what would have happened if they had been engineered a fiscal devaluation and reduce export prices?
For the counterfactual experiments, we use the structural equations (16) and (17) for private debt and spreads respectively and the fiscal policy rule (18). We use the same coefficient estimates from the instrumental variables regression (shown in Table 2) for the structural equations for private debt and spreads and the policy rule (18). For all counterfactual experiments, we compare on the same graph the data and the counterfactual experiment which we define as: structural model with counterfactual parameters + (data - structural model with benchmark parameters) so that we take into account that the structural model with benchmark parameters does not perfectly reproduce the data although we have seen that it does so very well. The simulation generates cross sectional time series for public debt, private debt, employment, nominal GDP, net exports and spreads on the period 2001-2012, using debt in 2000 as an initial point.

5.1 Counterfactual with a more conservative fiscal policy in the boom

How would countries have fared if they had followed more conservative fiscal policies during the boom? We answer this question by removing the fiscal drift bias in the fiscal rule (18). Hence, we set $\delta_g$ and $\delta_z$ equal to zero for the four periphery countries. For Spain, Ireland and Portugal all the other benchmark parameters are left unchanged. For Greece, we need to deal with the debt relief issue. Given that the counterfactual conservative fiscal policy generates debt to GDP ratios below 120%, we assume that debt relief would not have taken place. Hence, for Greece the counterfactual is the combination of a more conservative fiscal policy but also the elimination of a transfer of around 50% of nominal GDP in 2011-2012.

The elimination of the fiscal drift dramatically changes the public debt accumulation for Greece, Ireland and Spain but less so in Portugal which is not surprising given that our exercise suggests that the fiscal drift was very large in Greece, large in Ireland and Spain but small in Portugal. This can be seen in figure (18) where counterfactual public debt in Greece is stabilized in 2011 close to its level in 2001. Ireland would have eliminated all public debt just before the bust and in Spain it would have been reduced to around 26% of GDP. Hence, fiscal policy, once the fiscal drifts are removed, becomes more conservative but also more countercyclical. This large change in the public debt in turns reduces spreads during the sudden stop in Greece, Ireland and Spain but very little in Portugal as shown in figure (19). Lower spreads allow fiscal policy to be less constrained during the bust which explains part of the increase in debt in the periphery countries in the latest years. The counterfactual conservative fiscal policy in Greece is very successful in stabilizing employment as shown in figure (20). Remember that this more conservative fiscal policy in Greece also means that this country does not benefit from the debt relief at the end of the period. The counterfactual conservative fiscal policies in the boom - which allow for less fiscal austerity in the bust - in Spain and
Ireland do not eliminate the boom-bust cycle but allow for an earlier exit from the recession, especially in Ireland. The counterfactual employment dynamics in Ireland now look very similar to Arizona as illustrated in figure (2) in the introduction. In both Ireland and Spain, the boom-bust cycle is attenuated, but one should remember that this entails a large and probably implausible decrease in public debt during the boom, especially in Ireland. In Portugal, the conservative fiscal policy in the boom has little effect, except at the end of the cycle where the fall in spreads reduces fiscal austerity and improves moderately employment.

Figure 18: Government Debt under Conservative Fiscal Policy
Figure 19: Spreads under Conservative Fiscal Policy

Figure 20: Employment under Conservative Fiscal Policy
5.2 Counterfactual with macro-prudential policies in the boom

In this counterfactual we imagine that countries were able to implement policies that eliminated the household leverage boom. We assume that the growth rate of the exogenous part of the private leverage, the part predicted by the US experience, $b_{b_j,t}^h$, is set to zero in all four countries. This means that private debt in percentage of GDP is essentially flat during the whole cycle. We take into account the impact of such policy on the recapitalization of financial institutions during the bust. Using a cross-sectional regression, we estimate that a reduction of private debt by one euro during the boom reduces recapitalization needs by 0.25 euro during the bust. Figure (21) shows that this macro-prudential policy partially stabilizes employment in all four countries. This is especially so in Ireland: whereas in the data employment falls from peak to trough by 18%, in the counterfactual it falls by only 5%. Not surprisingly given that there was little private leverage boom in Portugal the impact of a counterfactual macro-prudential policy is small in this country.

The comparison of the conservative fiscal counterfactual and of the macro-prudential counterfactual shows that the private leverage boom was the key igniting element of the crisis in Ireland and Spain and that fiscal policies during the boom played a secondary role. The opposite is true for Greece. Given that the fiscal drifts are not affected in this counterfactual, the fiscal rule (that contains both a spending and transfer drift and a countercyclical component) induces a larger buildup of public debt than in a situation without macro-prudential policy. The public debt buildup in the boom can be seen in figure (22). It is especially stark in Spain, Greece and Ireland. With the spending and transfer drifts unchanged, larger public debt is substituted to private debt to achieve the employment target. There are two opposite effects on spreads: on the one hand the reduction in private leverage and in predicted recapitalization reduces spreads. On the other hand, the larger buildup of public debt pushes spreads upward. In Ireland, the first effect is larger than in the other three countries (see figure 24). This counterfactual suggests that macro-prudential policies that do not come with a more prudent fiscal rule may not have helped much to generate a fiscally sustainable stabilization of employment. In this sense, macro-prudential policies to contain private leverage and prudent fiscal policies to contain public debt appear to be complements not substitutes. Indeed, we checked that only a counterfactual that combines prudent fiscal policies and macro-prudential policies succeeds in stabilizing both total debt and employment.
Figure 21: Employment with Macro-Prudential Policies

Figure 22: Public debt with Macro-Prudential Policies
5.3 Counterfactual with “whatever it takes” in 2008

In this counterfactual we ask the following question: what would have happened if the announcements of July 2012 (Mario Draghi’s declaration “Whatever it takes”) and September 2012 (the OMT program) that were successful in reducing the risk of a euro breakup, the sudden stop and the spreads of the periphery countries had come earlier. The experiment is to imagine that these actions were implemented and successful in 2008 rather than 2012. We assume that, starting in 2008, $\sigma_t$, the mean of the absolute value of spreads in the eurozone, that enters the spread equation (17), is kept at its low 2007 level. This effectively eliminates the sudden stop and the dramatic rise in spreads as illustrated in figure (24). This allows to reduce the cost of borrowing constraint in the fiscal rule so that the countries can avoid fiscal austerity and stabilize employment (see figure 25). The employment dynamics of Ireland and Arizona (see figure 2 in the introduction) look very similar. Both Nevada and Ireland suffer a large private leverage induced boom and bust but then recover starting in 2011 because both belong to a monetary union that is successful in eliminating the risk of exiting the dollar and eurozone respectively. Finally, the dynamics of public debt are very similar to the data as illustrated in figure (24). The reason is that the relaxation of fiscal austerity that comes with the lowering of spreads is compensated by larger fiscal receipts thanks to better GDP growth.
There are three channels through which lower spreads affect aggregate spending in the economy: 1) savers increase spending through their Euler equation (13); 2) the leverage of borrowers increases through equation (16) which in turn affects their spending; 3) public spending and transfers increase because the borrowing costs are reduced in the fiscal rule (18). Transfers in turn affect the spending of the borrowers. We have checked that in all countries the fiscal channel is by far the most important channel through which lower spreads stabilize employment.

Figure 24: Spreads with Early ECB Intervention
Figure 25: Employment with Early ECB Intervention

Figure 26: Public Debt with Early ECB Intervention
5.4 Counterfactual with fiscal devaluation

In this counterfactual we ask the following hypothetical question: what would have happened if periphery countries had been able to engineer a fiscal devaluation during the bust in order to recoup part of the competitiveness they had lost during the boom years. This is close to a “flexible” exchange rate counterfactual, but it is not identical because it does not impact the net foreign asset position.\(^{14}\)

More precisely, we assume in this counterfactual that the fiscal devaluation is the combination of a VAT tax on all domestic expenditures (private and public) and a payroll subsidy on wages. Appendix B.4 details how the equilibrium conditions are amended. The VAT is paid by firms and rebated to exporters. The payroll subsidy on labor is paid to domestic firms. To simplify further we assume that the VAT rate and the payroll subsidy are equal to \(\tau_{v,j,t}\). Hence, export prices \(p_{j,t}^*\) fall relative to domestic prices:

\[
p_{j,t}^* = (1 - \tau_{v,j,t}) p_{j,t}^h = (1 - \tau_{v,j,t}) w_{j,t}
\]

so foreign demand increases to \(\frac{f_{j,t}}{(1 - \tau_{v,j,t}) p_{j,t}^h}\) due to unit elasticity and domestic prices for domestic consumers remain unchanged.\(^{15}\) The fiscal devaluation is fiscally neutral because of a lump sum transfer \(\Gamma_{j,t}\) to households such that:

\[
\Gamma_{j,t} = \tau_{v,j,t} (\chi_j x_{b,j,t} + (1 - \chi_j) x_{s,j,t} + g_{j,t}) - \tau_{v,j,t} w_{j,t} n_{j,t}
\]

so that revenues of the VAT equal the cost of payroll subsidy and the transfer. However, the indirect effects on income tax revenues that arise from the stimulative effects of a fiscal devaluation on output remain as would be the case of an exchange rate devaluation. The fiscal devaluation is applied starting in 2009 and we set the VAT rate at 10% so that export prices fall relative to domestic prices by 10%. The increase in exports attenuates the fall in employment in all countries as shown in figure (27) especially in Ireland and a bit less so in Greece which is less open. Another effect of the fiscal devaluation is that the better employment figures induce governments to cut spending and transfers. Because of this and because of the stimulative effect of the fiscal devaluation on income taxes the trajectory in public debt is improved as shown in figure (26). This improvement in the debt dynamics is quite large in all countries: in 2012 public debt is lower by around 20 percentage points of GDP in Ireland and Portugal and 15 percentage points of GDP in Spain and

\(^{14}\)See Franco (2013) and Farhi et al. (2014) for conditions under which a fiscal devaluation is equivalent to an exchange rate adjustment.

\(^{15}\)Domestic prices and wages actually increase relative to data but only indirectly because of higher employment.
Greece. In all countries, because of the reduction in public debt spreads fall as shown in figure (28).

Figure 27: Employment with Fiscal Devaluation
Figure 28: Public Debt with Fiscal Devaluation

Figure 29: Spreads with Fiscal Devaluation
Table 5: Counter-Factual Employment Gains

<table>
<thead>
<tr>
<th>2008-2012</th>
<th>ESP</th>
<th>GRE</th>
<th>IRL</th>
<th>PRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Change in Employment</td>
<td>-17.3%</td>
<td>-20.1%</td>
<td>-16.1%</td>
<td>-10.5%</td>
</tr>
<tr>
<td>Counter-Factual Gains</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conservative Fiscal Policy during Boom</td>
<td>3.2%</td>
<td>7.8%</td>
<td>6.0%</td>
<td>1.6%</td>
</tr>
<tr>
<td>Macro-Prudential Policy during Boom</td>
<td>5.6%</td>
<td>5.1%</td>
<td>8.0%</td>
<td>3.1%</td>
</tr>
<tr>
<td>No Financial Segmentation After 2008</td>
<td>3.1%</td>
<td>12.3%</td>
<td>7.0%</td>
<td>4.7%</td>
</tr>
<tr>
<td>Fiscal Devaluation (10% in 2008)</td>
<td>2.8%</td>
<td>3.2%</td>
<td>3.4%</td>
<td>3.3%</td>
</tr>
</tbody>
</table>

Note: Spain lost 17.3 points of employment between 2008 and 2012. If Spain had run a successful macro-prudential policy during the boom (2002-2008), its employment would have been 5.6 points higher in 2012, i.e. the drop would have been only 17.3 - 5.6 = 11.7 points. Employment in our model is the same as real GDP.

6 Conclusion

Understanding the dynamics of the Eurozone is a major challenge for macroeconomics. Eurozone countries have experienced extraordinary levels of real and financial volatility. Unemployment rates have diverged to an extent that nobody had anticipated. While most observers recognize that private leverage, fiscal policy and sudden stops all played a role, it has proven challenging to analyze them jointly, and even more difficult to disentangle them.

Our paper makes three contributions. First, we present a model that accounts at the same time for domestic credit, fiscal policy, and current account dynamics. Second, we create a data set for 11 countries over 13 years that covers the variables of interest and deal with the various accounting issues. Third, and most importantly, we propose a new identification strategy that allows us to run counter-factual experiments on fiscal policy, macro-prudential policy, ECB policy averting the sudden stop episode and on regaining competitiveness. These counter-factual experiments are summarized in Table 5. More conservative fiscal policies during the boom would have helped but Ireland would have had to enter the crisis with no public debt. A macro-prudential policy to limit private leverage during the boom would have stabilized employment in all countries, and especially in Ireland. However, in the absence of a more prudent fiscal policy, this could have induced a larger buildup in public debt. One lesson we take from our exercise is that fiscal and macro-prudential policies are complements not substitutes in order to stabilize the economy. A prudent fiscal policy, although helpful, cannot by itself undo the consequences of a private leverage boom and the reverse is also true. Both prudent fiscal policies and macro-prudential policies are required to stabilize the economy. The sudden stop episode worsened the crisis by further constraining the fiscal reaction of governments during the bust. If the ECB words and actions (Mario Draghi’s declaration “Whatever it takes” and the OMT program) had come earlier and had been successful in reducing the spreads, the four countries would have been able
to avoid the latest part of the slump but would not have avoided the large buildup in public debt. Finally, if those countries had been able to recoup competitiveness through a fiscal devaluation in 2009, the fall in employment would have been stabilized and fiscal austerity would have been successful in reducing public debt.

Political economy issues and sovereign debt the pricing play an important role in our model, but we treat them in a rather empirical way. We estimate them and we study their consequences in a coherent framework, but they require a more in depth theoretical analysis.
References


Lee, D. and W. van der Klaauw (2012, November). An introduction to the frbny consumer credit panel. Federal Reserve Bank of New York Staff Reports.


Appendix

A  Model

A.1 Scaling

We have already defined the Euler equations for country $j$ and the monetary union (with an asterisk) as a whole:

$$E_t [X_{s,j,t+1}] \approx \beta (1 + r_{j,t}) X_{s,j,t} \quad ; \quad E_t [X_{s,t+1}^*] \approx \beta (1 + r_{s,t}^*) X_{s,t}^*.$$  

and the spread as

$$1 + \rho_{j,t} \equiv \frac{1 + r_{j,t}}{1 + r_{s,t}^*}$$

If we scale the budget constraint (assuming perfect foresight, or equivalently, neglecting the conditional variance of the aggregate shocks), we get

$$\frac{X_{s,j,t}}{X_{s,t}^*} + \frac{S_{j,t+1}}{X_{s,t+1}^*} \frac{X_{s,t+1}^*}{1 + r_{j,t}} = S_{j,t} \frac{X_{s,t}^*}{X_{s,t}^*} + \tilde{Y}_{j,t}$$

and up to the usual approximation we have

$$x_{s,j,t} + \frac{\beta}{1 + \rho_{j,t}} s_{j,t+1} = s_{j,t} + \tilde{y}_{j,t}$$

A.2 Phillips Curve

$$\left(\frac{p_{j,t}^h}{p_{j,t-1}^h}\right)^2 + (\kappa N^* - 1) p_{j,t}^h - \kappa Y_{j,t} = 0$$

Defining $\Delta \equiv (\kappa N^* - 1)^2 + 4\kappa^2 Y_{j,t}^2$, we find that

$$\frac{p_{j,t}^h}{p_{j,t-1}^h} = \frac{1 - \kappa N^* + \sqrt{\Delta}}{2}$$

Note that if $\frac{Y_{j,t}}{p_{j,t-1}^h} = n^*$, then $\Delta = (\kappa N^* + 1)^2$, and $\frac{p_{j,t}^h}{p_{j,t-1}^h} = 1$.

A.3 Budget constraints.

Let us first rewrite the budget constraints and market clearing conditions. Using the market clearing condition, and competition $p_{j,t}^h = w_{j,t}$, we get

$$y_{j,t} = \alpha_j (x_{b,j,t} + (1 - \chi_j) x_{s,j,t}) + f_{j,t} + g_{j,t}.$$
Nominal exports are \( f_{j,t} \), nominal imports are \( (1 - \chi_j) p^f_{j,t} c^f_{s,j,t} + \chi_j p^f_{j,t} c^f_{b,j,t} \) since the government does not buy imported goods. So net exports are \( e_{j,t} = f_{j,t} - (1 - \alpha_j) (\chi_j x_{b,j,t} + (1 - \chi_j) x_{s,j,t}) \). We define disposable (after-tax) income as \( \tilde{y}_{j,t} \equiv (1 - \tau_j) y_{j,t} + z_{j,t} \).

We can then write the system for nominal variables:

- \( x_{b,j,t} = \frac{b^h_{j,t+1}}{1 + \tau_{j,t}} + \tilde{y}_{j,t} - b^h_{j,t} \), budget constraint of impatient agents
- \( x_{s,j,t} = s_{j,t} + \tilde{y}_{j,t} - \frac{s_{j,t+1}}{1 + \tau_{j,t}} \), budget constraint of patient agents
- \( y_{j,t} = \alpha_j (\chi_j x_{b,j,t} + (1 - \chi_j) x_{s,j,t}) + f_{j,t} + g_{j,t} \), market clearing
- \( g_{j,t} + z_{j,t} - \tau_j y_{j,t} = \frac{b^g_{j,t+1}}{1 + \tau_{j,t}} - b^g_{j,t} \), budget constraint of the government
- \( e_{j,t} = \frac{1}{\alpha_j} (f_{j,t} - (1 - \alpha_j) (y_{j,t} - g_{j,t})) \), definition of net exports

Combining the first four equations, we get market clearing at time \( t \):

\[
(1 - \alpha_j) \tilde{y}_{j,t} = \alpha_j \chi_j \left( \frac{b^h_{j,t+1}}{1 + \tau_{j,t}} - b^h_{j,t} \right) + \alpha_j (1 - \chi_j) \left( s_{j,t} - \frac{s_{j,t+1}}{1 + \tau_{j,t}} \right) + \frac{b^g_{j,t+1}}{1 + \tau_{j,t}} - b^g_{j,t} + f_{j,t}.
\] (21)

### A.4 Pseudo-Steady State

We consider a steady state with constant interest rates equal to the rate of time preference of savers, i.e. \( \beta (1 + r) = 1 \) and the spread is zero: \( \rho = 0 \). The borrowing limit \( b^h_j \) is exogenous and we consider equilibria where the borrowing constraint (3) binds. Our notion of steady state is complicated by the fact that savings \( s_{j,t} \) are history-dependent. We define a steady state as the long run equilibrium of an economy with initial savings \( s_j \) and government debt \( b^f_j \), subject to no further shocks, constant government spending and constant government debt. All nominal quantities are constant and employment is at its natural rate \( n^* \).\(^{16}\) The long-run equilibrium conditions are

\[
p^h_j n^* = \chi_j \alpha_j x_{b,j} + (1 - \chi_j) \alpha_j x_{s,j} + f + g_j
\]

\[
x_{b,j} = \tilde{y}_j - \frac{r}{1 + r} b^h_j
\]

\[
x_{s,j} = \tilde{y}_j + \frac{r}{1 + r} s_j
\]

\[
\tau_j p^h_j n^* = g_j + z_j + \frac{r}{1 + r} b^g_j
\]

Nominal output (the price of home goods) is pinned down by

\[
p^h_j n^* = \alpha_j \left( (1 - \tau_j) p^h_j n^* + z_j \right) + \alpha_j \frac{r}{1 + r} \left( (1 - \chi_j) s_j - \chi_j \tilde{y}_j \right) + f_j + g_j.
\]

There are several ways to specify government policy. Here we assume that the policy is to keep government debt and nominal spending \( g_j \) constant. Long run nominal output is then given by

\[
p^h_j n^* = \alpha_j \frac{r}{1 - \alpha_j} a_j + \frac{f_j}{1 - \alpha_j} + g_j
\] (22)

\(^{16}\)We consider here the case where labor supply is inelastic, so \( n^* \) is effectively exogenous.
where recall that we have defined \( a \) as net foreign assets. This equation shows the determinants of the long run price level. The long run price level depends on the exogenous components of spending: net asset income, foreign demand, and government spending. All these are inflationary. For a given tax rate \( \tau_j \), transfers are then chosen to satisfy the government’s budget constraint:

\[
z_j = \tau_j p_j^b n^* - g_j - \frac{r}{1 + r} b_j^p.
\]

**A.5 Euler Equation and Expected Income**

The Euler equation of savers is

\[
\frac{1}{x_{s,j,t}} = (1 + \rho_{j,t}) E_t \left[ \frac{1}{x_{s,j,t+1}} \right].
\]

and we use the linear approximation

\[
E_t [x_{s,j,t+1}] = (1 + \rho_{j,t}) x_{s,j,t}.
\]

Consider the following experiment. Savers enter the period with a given level of savings. Then there is a shock to interest rates. For instance, starting from a steady state where \( \rho = 0 \), if the new rate is such that \( \rho > 0 \), savings jumps up, and if the new rate is such that \( \rho < 0 \), spending jumps up. The budget constraint at time \( t \) is

\[
x_{s,j,t} = s_{j,t} + y_{j,t} - \frac{s_{j,t+1}}{1 + \rho_{j,t}}
\]

and the expected budget constraint at time \( t + 1 \) is

\[
E_t \left[ \frac{s_{j,t+2}}{1 + \rho_{j,t+1}} \right] = s_{j,t+1} + E_t \left[ y_{j,t+1} - x_{s,j,t+1} \right].
\]

Combining the budget constraints and the linearized Euler equation, we get

\[
(1 + \beta) s_{j,t+1} + E_t \left[ y_{j,t+1} \right] - \beta E_t \left[ \frac{s_{j,t+2}}{1 + \rho_{j,t+1}} \right] = (1 + \rho_{j,t}) (s_{j,t} + y_{j,t}).
\]

since on average \( \beta (1 + r) = 1 \).

**A.6 Equilibrium Conditions of the Model**

For the scaled variables, we have:

- \( \frac{1}{x_{s,j,t}} = (1 + \rho_{j,t}) E_t \left[ \frac{1}{x_{s,j,t+1}} \right] \), Euler equation
- \( x_{b,j,t} = \frac{b_{j,t+1}}{1 + r_{j,t}} + y_{j,t} - b_{j,t}^b \), budget constraint of impatient agents
- \( x_{s,j,t} = s_{j,t} + y_{j,t} - \frac{z_{j,t+1}}{1 + \rho_{j,t}} \), budget constraint of patient agents
- \( y_{j,t} = \alpha_j (x_j f_{b,j,t} + (1 - \chi_j) x_{s,j,t}) + f_{j,t} + g_{j,t} \), market clearing
- \( g_{j,t} + z_{j,t} - \tau_j y_{j,t} = \frac{b_{j,t+1}}{1 + \rho_{j,t}} - b_{j,t}^g \), budget constraint of the government
\begin{itemize}
  \item $e_{j,t} = \frac{1}{(1 - \alpha_j)} (f_{j,t} - (1 - \alpha_j) (y_{j,t} - g_{j,t}))$, net exports
  \item $\tilde{y}_{j,t} = (1 - \tau_j) y_{j,t} + z_{j,t}$, disposable income
  \item $(1 - \alpha_j) \tilde{y}_{j,t} = \alpha_j \chi_j \left( \frac{b_{j,t+1}^h}{1 + \rho_{j,t}} - b_{j,t}^h \right) + \alpha_j (1 - \chi_j) \left( s_{j,t} - \beta \frac{s_{j,t+1}}{1 + \rho_{j,t}} \right) + \beta \frac{b_{j,t+1}^g}{1 + \rho_{j,t}} - b_{j,t}^g + f_{j,t}$, market clearing
  \item $n_{j,t} = \frac{w_{j,t}}{p_{j,t}}$, labor market
  \item $\frac{p_{j,t}^h - p_{j,t-1}^h}{p_{j,t-1}^h} = \kappa (n_{j,t} - n^*)$, inflation
\end{itemize}

### A.7 Impulse Response Function

To compute the impulse response functions of the model we need to make an assumption about the path of household debt and fiscal policy

**Assumptions A2.**

- **Shocks on borrowing constraints are such that** $E_t \left[ b_{j,t+2}^h \right] = b_{j,t+1}^h$.
- **Fiscal policy is such that** $E_t \left[ b_{j,t+2}^g \right] = b_{j,t+1}^g$ and the tax rate $\tau_j$ is constant.
- **The variance of interest rates and foreign demand is small, and $\beta_h$ is small enough that** $b_{j,t} = b_{j,t}^h$ at all times.

**Assumptions A2** is only necessary to solve for the entire path of macroeconomic variables shown in the impulse response functions. The first condition says that shocks on borrowing are permanent. The second defines a class of fiscal policies. The last point is purely technical. It allows us to linearize Euler equations. We assume that the shocks are small enough that impatient households find it optimal to borrow up to the constraint (this is a joint restriction on the discount factor and the size of the shocks). Note that A2 is not necessary for the reduced form and structural simulations.

We now look for decision rules for the savers $\{s_{j,t}\}_{t=1,2,...}$ and the other variables of the model, $y_{j,t}, b_{j,t}^h, \ldots$ etc. We get the following Lemma:

**Lemma 1.** Under A1 and A2, savings dynamics satisfy $E_t \left[ s_{j,t+2} \right] = s_{j,t+1}$.

**Proof.** With assumptions A1 ($E_t \left[ f_{j,t+1} \right] = f_{j,t}$ and $E_t \left[ \rho_{j,t+1} \right] = 0$), we guess and verify that the Lemma is correct. Suppose the Lemma is true, then we obtain two important equations. Equation (23) becomes

\begin{equation}
  s_{j,t+1} = (1 + \rho_{j,t}) (s_{j,t} + \tilde{y}_{j,t}) - E_t \left[ \tilde{y}_{j,t+1} \right],
\end{equation}

and expected market clearing at $t + 1$ is

\begin{equation}
  (1 - \alpha_j) E_t \left[ \tilde{y}_{j,t+1} \right] = (1 - \beta) \left( \alpha_j (1 - \chi_j) s_{j,t+1} - \alpha_j \chi_j b_{j,t+1}^h - b_{j,t+1}^g \right) + f_{j,t}.
\end{equation}

Therefore

\begin{equation}
  (1 - \alpha_j) s_{j,t+1} = (1 - \alpha_j) (1 + \rho_{j,t}) (s_{j,t} + \tilde{y}_{j,t}) - (1 - \beta) \left( \alpha_j (1 - \chi_j) s_{j,t+1} - \alpha_j \chi_j b_{j,t+1}^h - b_{j,t+1}^g \right) - \frac{f_{j,t}}{1 - \alpha_j}.
\end{equation}

Using market clearing at time $t$ we get

\begin{equation}
  \frac{s_{j,t+1}}{1 + \rho_{j,t}} - s_{j,t} = \frac{\alpha_j \chi_j}{1 - \alpha_j \chi_j} \left( \frac{b_{j,t+1}^h}{1 + \rho_{j,t}} - b_{j,t}^h \right) + \frac{1}{1 - \alpha_j \chi_j} \left( \frac{b_{j,t+1}^g}{1 + \rho_{j,t}} - b_{j,t}^g \right) + \frac{\rho_{j,t}}{1 + \rho_{j,t} (1 - \alpha_j)} f_{j,t}.
\end{equation}
We are interested in simple rules that deliver the property that the public debt follows a random walk: $\mathbb{E}_t [b^g_{j,t+2}] = b^g_{j,t+1}$ and deliver some automatic stabilization, characteristic of modern fiscal systems in all OECD countries (see Fatas and Mihov (2012)). We assume that spending and transfers are predetermined. From the government budget constraint, this means that a recession at time $t$ automatically increases government debt at time $t$. To maintain fiscal stability, we specify that transfers adjust from $t$ to $t+1$ to keep public debt constant thereafter: $\mathbb{E}_t [b^g_{j,t+2}] = b^g_{j,t+1}$. More precisely, we specify the general policy rule as follows

1. Fiscal variables $g_{j,t}$ and $z_{j,t}$ are pre-determined. Government debt $b^g_{j,t}$ is determined in equilibrium at time $t$.

2. Set transfers $z_{j,t+1}$ for next period so that $\mathbb{E}_t [b^g_{j,t+2}] = b^g_{j,t+1}$ assuming a martingale for $g_{j,t}$.

   (a) The expected budget constraint is

   $$\tau_j \mathbb{E}_t [y_{j,t+1}] = g_{j,t} + z_{j,t+1} + \frac{r}{1+r} b^g_{j,t+1}$$

   (b) By definition we have

   $$\mathbb{E}_t [\tilde{y}_{j,t+1}] = (1 - \tau_j) \mathbb{E}_t [y_{j,t+1}] + z_{j,t+1}$$

   (c) Therefore (recall that $\mathbb{E}_t [\tilde{y}_{j,t+1}]$ is part of our solution at time $t$), $z_{j,t+1}$ is given by

   $$z_{j,t+1} = \tau_j \mathbb{E}_t [\tilde{y}_{j,t+1}] - (1 - \tau_j) g_{j,t} - (1 - \tau_j) \frac{r}{1+r} b^g_{j,t+1}$$

### B Data & Calibration

#### B.1 Data sources

**B.1.1 Europe**

All economic data for Eurozone countries (employment, population, GDP, consumption, government debt, expenditures...) comes from Eurostat. We use data for 11 eurozone countries from 2000 to 2012. Austria,
Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, Netherlands and Portugal. We excluded Luxembourg for which household debt data is available only starting in 2005 and other countries that joined in 2007 and later.

The data on household debt comes from the BIS which itself compiled the data from national central banks. This is debt of household and non-profit institutions serving households. Credit covers all loans and debt securities and comes from both domestic and foreign lenders. The series have quarterly frequency and capture the outstanding amount of credit at the end of the reference quarter.

We call government expenditures total government expenditures net of transfers, interest payments and bank recapitalization. The data on spending on bank recapitalization comes from Eurostat. It includes interest payable, capital injections recorded as deficit-increasing (capital transfer) and calls on guarantees and is net of revenues generated by bank recapitalization (guarantee fees, interest and dividends). Transfers is the addition of direct social benefits and of social transfers in kind.

The data on interest rates (10 year government bonds) come from the ECB. The spread is defined as the difference of the 10 year interest rate on government bonds with the median of the euro zone.

For exports we measure the domestic value added that is associated with final consumption in the rest of the world, which corresponds to value added based exports. We use the data from the OECD-WTO Trade in Value-Added (TiVA) initiative to measure domestic value added embodied in gross exports. Data is available only in 2000, 2005, 2008 and 2009. For missing years, we use the ratio of gross exports (from Eurostat) to value added gross exports of the nearest year and multiply this ratio by the gross exports of the missing year to obtain an approximation of value added exports of the missing years.

We use annual averages of 10 year government bond rates as long term rates. The source is OECD.

The source for the TFP figure (30) is the Conference Board:

B.1.2 United States

Data for the United States comes from the BEA, the Flow of Funds (FoF), and from the FRBNY Consumer Credit Panel. BEA and FoF data are standard and widely used so we do not discuss them.

The FRBNY Consumer Credit Panel is described in Lee and van der Klaauw (2012). It is a new longitudinal database with detailed information on consumer debt and credit. This panel is a random sample from consumer credit reports. It is available from 1999 onwards. Credit reporting agencies compile and maintain credit histories for all U.S. residents who have applied for or taken out a loan. Credit bureaus continuously collect information on individual consumers’ debt and credit from lenders and creditors. Most individuals begin building a credit history when they first obtain and use a credit or retail card or take out a student loan, usually when they are at least 18 years of age. New immigrants with little or no credit history from their home country are often older when a credit file is first created for them. The sample design implies that the target population consists of all US residents with a credit history. In addition to most individuals younger than 18, who had little need or opportunity for credit activity, the target population excludes
individuals who have never applied for or qualified for a loan.

The data at the State level is available in three data sets on the FRBNY web site:

- State level data for all States from 1999 to 2012, annual data for Q4 only.
- Selected states from 1999 to 2003, quarterly data.
- Selected states from 2003 to 2014, quarterly data.

Lee and van der Klaauw (2012) argue that household debt estimates based on the FRBNY Consumer Credit Panel are similar to estimates reported in the Board of Governors’ Flow of Funds Accounts. There are differences, however. First, the household debt measures in the Flow of Funds are not based on direct data but instead are derived as residual amounts. Total mortgage debt and non-mortgage debt in the second quarter of 2010 were respectively $9.4 and $2.3 trillion, the comparable amounts in the FoF for the same quarter were $10.2 and $2.4 trillion, respectively.

Second, the FoF measure of household mortgage debt includes some mortgage debt held by nonprofit organizations (churches, universities, etc.). On the other hand, FRBNY estimates exclude some debt held by individuals without social security numbers. There may also be differences in the speed at which changes in various types of debt are recorded, where new mortgage accounts usually appear on credit reports with some delay, making some direct comparisons difficult. The comparison is shown in figure (31).

Figure 31: Comparison of Household Debt Measures

Local Fiscal Policy  Another issue is whether fiscal policy was not also active in the US. Perhaps private debt bubbles were associated with large fiscal revenues and large spending. This probably happened to some extent, but compared to the Eurozone, these effects are small (of course we are only talking about cross-sectional variation in government spending). Figure 32 shows this for two states and two countries. A regression for all the states and all the countries shows that the link between private debt and government spending was at least four times smaller in the US than in Europe.
We therefore argue that the US provides a benchmark for private deleveraging without sudden stops, and with relatively neutral (cross-sectional) fiscal policy.

B.2 Scaled data
The scaled data for the reduced form shocks for household and public debt, government expenditures and transfers, spreads and foreign demand are given in figures (33), (34), (36) and (35).
Figure 34: Government expenditures and transfers

Graphs by country

Figure 35: Value added based exports

Graphs by country
B.3 Simulation of the reduced form and structural models

To run the simulations, we first need to set the initial conditions in a particular year. We use 2002 as our base year.

1. Natural employment and prices (unit labor costs) are normalized to \( n^* = 1 \) and \( p^0_{j,t_0} = 1 \) (so nominal GDP is normalized in the base year to: \( y_{j,t_0} = 1 \))

2. Variables set to their observed values are: \( b^h_{j,t_0}, z_{j,t_0}, g_{j,t_0}, b^{g}_j, b^{g}_{j,t_0-1}, r_{j,t_0} \). We then get \( f_{j,t_0}, \tau_j \) and \( \tilde{y}_{j,t_0} \) from market clearing and budget constraints:
   
   (a) Foreign demand \( f_{j,t_0} \) is chosen to match net exports \( e_{j,t_0} = \frac{1}{\alpha_j} (f_{j,t_0} - (1 - \alpha_j)(y_{j,t_0} - g_{j,t_0})) \)
   
   (b) Get \( \tau_j \) from the government budget constraint \( g_{j,t_0} + z_{j,t_0} - \tau_j y_{j,t_0} = \frac{b^g_{j,t_0-1}}{1+r_{j,t_0}} - b^g_{j,t_0} \)
   
   (c) Disposable income at time \( t_0 \) is \( \tilde{y}_{j,t_0} = (1 - \tau_j) y_{j,t_0} + z_{j,t_0} \)

3. Savers’ assets \( s_{j,t_0} \) and \( s_{j,t_0-1} \) are chosen to solve the equilibrium conditions
   
   (a) \( s_{j,t} = (1 + \rho_{j,t}) (s_{j,t-1} + \tilde{y}_{j,t}) - E_t [\tilde{y}_{j,t+1}] \)
   
   (b) \( (1 - \alpha_j) E_t [\tilde{y}_{j,t+1}] = \frac{1}{1 + \tau_j} (\alpha_j (1 - \chi_j) s_{j,t+1} - \alpha_j \chi_j b^h_{j,t+1} - b^g_{j,t+1}) + f_{j,t} \)
   
   (c) \( (1 - \alpha_j) \tilde{y}_{j,t} = \alpha_j \chi_j \left( \frac{b^{h}_{j,t+1}}{1+r_{j,t}} - b^h_{j,t} \right) + \alpha_j (1 - \chi_j) \left( s_{j,t} - \frac{s_{j,t+1}}{1+r_{j,t}} \right) + b^g_{j,t+1} - b^g_{j,t} + f_{j,t} \).

We then feed exogenous processes for the different shocks (using scales values) for observed household debt \( (b^h_{j,t}) \), fiscal policy \( (\tau_j, z_{j,t}, g_{j,t}) \), and interest rate spreads \( \rho_{j,t} \). For the sake of completeness, we also feed exogenous foreign nominal demand shocks \( f_{j,t} \), even though they are not crucial for our analysis. For each
country, we simulate the path between 2001 and 2012 of nominal GDP $y_{j,t}$, employment $n_{j,t}$, wages $w_{j,t}$, net exports $e_{j,t}$ and public debt $b_{j,t}^p$.

The level of output, the price level and the level of net foreign assets is set to their 2002 levels in the data. Foreign demand is set using data on exports in value added terms for 2001-2012, normalized so that the level of foreign demand in 2002 satisfies goods market clearing in the model. Finally, taxes are set so that the path for government debt implied by the model coincides with the data.\footnote{Instead of using government debt directly from data, we construct a simulated government debt series in order to avoid including factors that affect government debt in the data but are not in the model, such as bank recapitalizations, default, revenues from privatizations, etc. The simulated debt series is constructed by adding to $t-1$ period debt government expenditures including interest payments and subtracting tax revenues.}

The structural model is simulated in the same way but the exogenous variables are replaced by structural equations (16) and (17) for private debt and spreads respectively and the fiscal policy rule (18). The tax rate is constant at its 2002 level and equal to total government revenues as a percentage of GDP. The structural shocks are now $b_{j,t}^p$, $\sigma_t$ and $g_{j,t}$.

### B.4 Fiscal Devaluation

We define a fiscal devaluation as the combination of a VAT tax on domestic expenditures (private and public) and a payroll subsidy. Let $p_{j,t}^h$ be the price of home goods for domestic consumers, and $p_{j,t}^f$ be the price of home goods for foreign consumers. $\tau_{v,j,t}$ is the VAT so that the government collects $\tau_{v,j,t}(x_jx_{s,j,t} + (1 - \chi_j)x_{s,j,t} + g_{j,t})$. The VAT is paid by firms and rebated to exporters. $\lambda_{j,t}$ is the payroll subsidy so the government pays $\lambda_{j,t}w_{j,t}n_{j,t}$ to firms. Profit maximization implies the following prices:

$$(1 - \tau_{v,j,t})p_{j,t}^h = p_{j,t}^f = (1 - \lambda_{j,t})w_{j,t}$$

So foreign demand becomes $\frac{f_{j,t}}{(1 - \tau_{v,j,t})p_{j,t}^f}$. Given that the VAT is imposed on imported goods, importers (assuming flexible prices for foreign firms as for domestic firms) increase the price of their imports to compensate for the VAT, so we have $p_{j,t}^f = \frac{p_{j,t}^f}{1 - \tau_{v,j,t}}$ where $p_{j,t}^f$ is the domestic price of foreign goods in country $j$ and $p_{j,t}^f$ is the foreign price of foreign goods. With log preferences, this leads to a one for one drop in the quantity of imported foreign goods, while the spending shares remain the same. For simplicity we further assume that the VAT rate and payroll subsidies are equal, $\tau_{v,j,t} = \lambda_{j,t}$ so that $p_{j,t}^h = w_{j,t}$, $y_{j,t} = w_{j,t}n_{j,t} = p_{j,t}n_{j,t}$ and domestic prices to domestic consumers are unchanged. The government budget constraint, is then:

$$\frac{b_{j,t+1}^g}{1 + r_{j,t}} + (\tau_{j,t} - \tau_{v,j,t})w_{j,t}n_{j,t} + \tau_{v,j,t}(x_jx_{b,j,t} + (1 - \chi_j)x_{s,j,t} + g_{j,t}) = g_{j,t} + z_{j,t} + \gamma_{j,t} + b_{j,t}^p,$$

where $\gamma_{j,t}$ is a lump sum transfer to households. We set this transfer so that the fiscal devaluation is neutral for the government budget constraint in the sense that the budget constraint remains:

$$\frac{b_{j,t+1}^g}{1 + r_{j,t}} + \tau_{j,t}y_{j,t} = g_{j,t} + z_{j,t} + b_{j,t}^g,$$

so the lump sum transfer is:

$$\gamma_{j,t} = \tau_{v,j,t}(x_jx_{b,j,t} + (1 - \chi_j)x_{s,j,t} + g_{j,t}) - \tau_{v,j,t}w_{j,t}n_{j,t}$$
Market clearing becomes:

\[ n_{j,t} = \frac{\chi_j x_{b,j,t} + (1 - \chi_j) x_{s,j,t}}{p_{j,t}^b} + \frac{f_{j,t}}{(1 - \tau_{v,j,t}) p_{j,t}^b} + \frac{g_{j,t}}{p_{j,t}^b}, \]

and budget constraints are

\[ x_{b,j,t} = \frac{b_{j,t+1}}{1 + r_{j,t}} + \tilde{y}_{j,t} - b_{j,t}^h \]

and

\[ x_{s,j,t} = s_{j,t} + \tilde{y}_{j,t} - s_{j,t+1} \frac{1}{1 + r_{j,t}} \]

where net income is now defined as:

\[ \tilde{y}_{j,t} = (1 - \tau_j) y_{j,t} + z_{j,t} + \Gamma_{j,t} \]

If we substitute the government budget constraint into market clearing, we get:

\[ y_{j,t} = \left( \alpha_j + \frac{\tau_{v,j,t}}{1 - \tau_{v,j,t}} \right) (\chi_j x_{b,j,t} + (1 - \chi_j) x_{s,j,t}) + \frac{f_{j,t}}{1 - \tau_{v,j,t}} + \frac{1}{1 - \tau_{v,j,t}} \left( \frac{b_{j,t+1}}{1 + r_{j,t}} - b_{j,t}^g - z_{j,t} - \Gamma_{j,t} \right) + \frac{(\tau_j - \tau_{v,j,t}) y_{j,t}}{1 - \tau_{v,j,t}} \]

Substitute for consumptions from the budget constraints to get:

\[ (1 - \alpha_j) \tilde{y}_{j,t} = \left( \alpha_j + \frac{\tau_{v,j,t}}{1 - \tau_{v,j,t}} \right) \left( \chi_j \left( \frac{b_{j,t+1}}{1 + r_{j,t}} - b_{j,t}^h \right) + (1 - \chi_j) \left( s_{j,t} - s_{j,t+1} \frac{1}{1 + r_{j,t}} \right) \right) + \frac{1}{1 - \tau_{v,j,t}} \left( f_{j,t} + \frac{b_{j,t+1}}{1 + r_{j,t}} - b_{j,t}^g \right) \]

Assuming permanent taxes and subsidies, permanent shocks on foreign demand, iid spreads and \( \mathbb{E}_t [b_{j,t+2}] = b_{j,t+1}^g \), expected market clearing at \( t + 1 \) is:

\[ (1 - \alpha_j) \mathbb{E}_t [\tilde{y}_{j,t+1}] = \frac{1}{1 - \tau_{v,j,t}} \left( \alpha_j + \frac{\tau_{v,j,t}}{1 - \tau_{v,j,t}} \right) \left( (1 - \chi_j) s_{j,t+1} - \chi_j b_{j,t+1}^h - \frac{1}{1 - \tau_{v,j,t}} b_{j,t+1}^g \right) + \frac{f_{j,t}}{1 - \tau_{v,j,t}} \]

So the equilibrium equations in the case of a fiscal devaluation (with VAT and payroll subsidy equal to \( \tau_{v,j,t} \)) and revenue neutrality become:

1. \( s_{j,t+1} = (1 + \rho_{j,t}) (s_{j,t} + \tilde{y}_{j,t}) - \mathbb{E}_t [\tilde{y}_{j,t+1}] \)
2. \( (1 - \alpha_j) \mathbb{E}_t [\tilde{y}_{j,t+1}] = \frac{1}{1 - \tau_{v,j,t}} \left( \alpha_j + \frac{\tau_{v,j,t}}{1 - \tau_{v,j,t}} \right) \left( (1 - \chi_j) s_{j,t+1} - \chi_j b_{j,t+1}^h - \frac{1}{1 - \tau_{v,j,t}} b_{j,t+1}^g \right) + \frac{f_{j,t}}{1 - \tau_{v,j,t}} \)
3. \( (1 - \alpha_j) \tilde{y}_{j,t} = \left( \alpha_j + \frac{\tau_{v,j,t}}{1 - \tau_{v,j,t}} \right) \left( \chi_j \left( \frac{b_{j,t+1}}{1 + r_{j,t}} - b_{j,t}^h \right) + (1 - \chi_j) \left( s_{j,t} - s_{j,t+1} \frac{1}{1 + r_{j,t}} \right) \right) + \frac{1}{1 - \tau_{v,j,t}} \left( f_{j,t} + \frac{b_{j,t+1}}{1 + r_{j,t}} - b_{j,t}^g \right) \]
4. \( \frac{b_{j,t+1}}{1 + r_{j,t}} - b_{j,t}^g = (1 - \tau_{v,j,t}) (g_{j,t} + z_{j,t} - \tau_j \tilde{y}_{j,t}) \)
5. \( \tilde{y}_{j,t} = (1 - \tau_j) y_{j,t} + z_{j,t} + \Gamma_{j,t} \)
6. \( \Gamma_{j,t} = \tau_{v,j,t} \left[ \frac{1 - \alpha_j}{\alpha_j} (y_{j,t} - g_{j,t}) + \frac{1}{\alpha_j} \frac{f_{j,t}}{1 - \tau_{v,j,t}} \right] \)