Divided Majority and Information
Aggregation: Theory and Experiment*

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June 19, 2014

Abstract

This paper studies theoretically and experimentally the properties of plurality and approval voting when a majority gets divided by information imperfections. The majority faces two challenges: aggregating information to select the best majority candidate and coordinating to defeat the minority candidate. Under plurality, the majority cannot achieve both goals at once. Under approval voting, it can: welfare is strictly higher because some voters approve of both majority alternatives. In the laboratory, we find (i) strong evidence of strategic voting, and (ii) superiority of approval voting over plurality. Finally, subject behavior suggests the need to study equilibria in asymmetric strategies.

JEL Classification: C72, C92, D70
Keywords: Multicandidate Elections, Plurality, Approval Voting, Experiments

*We thank participants to numerous conferences and workshops, and seminars at Bonn, Boston University, Caltech, Chicago, Columbia, Copenhagen, CREED, ESSEX, EUI, Georgetown, IMT Lucca, Köln, Konstanz, LSE, Mannheim, Maryland, McGill, MIT, NYU, NYU-AD, Oxford, Pittsburgh, Princeton, PSE, Queen Mary, Royal Holloway, Saint Louis Belgium, Southampton, Tilburg and Warwick. We particularly thank David Ahn, Alessandra Casella, Eric Van Damme, Christoph Engel, Olga Gorelkina, Dura-Georg Granić, Kristoffel Grechenig, Alessandro Lizzeri, Roger Myerson, Santiago Oliveros, Tom Palfrey, and Jean-Benoit Pilet. We would also like to thank Erika Gross and Nicolas Meier for excellent assistance at running the experiments. We gratefully acknowledge financial support from the Max Planck Society. Micael Castanheira is a senior research fellow of the Fonds National de la Recherche Scientifique and is grateful for their support.
1 Introduction

Elections serve two main purposes: aggregating preferences and information.\(^1\) The literature proposes a variety of electoral systems to achieve these purposes, each of them featuring its own potential strengths and weaknesses. Yet, only few of them are actually used (Golder 2005). Not that there is either a proof or even a common belief that the currently used systems are better. To the contrary, their flaws have repeatedly been emphasized. Still, there is insufficient evidence that alternative systems would perform sufficiently better, thus stalling reform.\(^2\)

We therefore need to enhance our knowledge about the capacity of alternative electoral systems to outperform those currently in use. Filling this gap requires a combination of theory and empirics. Indeed, purely theoretical analyses suffer from the lack of consensus about actual voter behavior (e.g. are they strategic or sincere?). Similarly, purely empirical studies face an insuperable challenge: the lack of observational data available, since only a handful of electoral systems are used in practice.\(^3\)

With these issues in mind, we adopt a two-pronged approach, theoretical and experimental, in order to jointly study \((i)\) the behavior of voters under different electoral systems, and \((ii)\) the welfare properties of those systems. The theory produces crisp predictions about voters’ behavior and the performances of each electoral system when voters are either rational and strategic, or sincere. The experiments produce data that can be used to test those predictions, controlling for voters’ preferences and information.

Relying on experiments forces us to focus on a limited number of systems. As a benchmark, we study one of the most widely-used systems, plurality (Golder 2005), and one of its most oft-proposed, but little used, alternatives: approval voting (AV).\(^4\) Of course, the


\(^2\)These limitations resonate in civil society, where there is growing frustration with existing electoral systems. A large number of activists lobby in favor of reforming the electoral system (e.g. the Electoral Reform Society (www.electoral-reform.org.uk) and the Fair Vote Reforms initiative (www.fairvote.org)), and many official proposals have been introduced. A recent example comes from the UK, which held a national referendum in 2011 on whether to replace plurality voting with alternative voting.

\(^3\)Even when data are available, empirical work faces the constraint of having to back out voter preferences and information from voting behavior. Yet, this requires an accepted theoretical model of voting, which brings us back to the initial problem faced by theory.

\(^4\)Under AV, voters can “approve of” as many candidates as they want. Each approval counts as one vote and the candidate that obtains the largest number of votes wins. For the strengths and weaknesses of AV,
approach used in this paper could (and should) also be applied to other electoral systems.

We consider a world of aggregate uncertainty in which a divided majority has to defeat a Condorcet loser who is only backed by a minority. Voters are strategic and imperfectly informed about which is their best alternative. Beyond capturing important features of real-world elections, such as the voters’ lack of knowledge about each candidate’s policy and/or competence, our setup has three advantages: first, majority voters face the double incentive of aggregating information and coordinating their ballots against the minority. Since fulfilling each goal requires a different strategy, varying their relative importance generates clear testable implications. Second, studying multicandidate rather than two-candidate elections widens the set of electoral systems (and voter incentives) that can be analyzed. In our case, predicted behavior is substantially different under plurality and AV. Third, in contrast with the classical private value setup, our setup builds on the Condorcet Jury literature and allows for clearer welfare comparisons. As far as we know, our paper offers the first experiment with multiple candidates and common value voters.\footnote{Surprisingly, the experimental literature on multicandidate elections with private value voters is also quite slim. The seminal papers of Forsythe et al (1993, 1996) are closest to our paper. See also Rietz (2008) or Palfrey (2012) for detailed reviews of that literature. Van der Straeten et al. (2010) also study AV experimentally, although in substantially different settings. For experiments with common-value voters in a two-candidate setting, see e.g. Guarnaschelli, McKelvey and Palfrey (2000), Battaglini et al. (2008, 2012), Goeree and Yariv (2010), and Bhattacharya, Duffy and Kim (2013).}

Our theoretical analysis shows that two types of equilibria coexist under plurality. Along with the classical Duverger’s Law equilibria (in which all majority voters cast their ballots on a single majority candidate), the presence of aggregate uncertainty implies that there also exists an informative equilibrium (in which voters use their information and all candidates receive strictly positive vote shares). Each equilibrium has its strength and weakness: Duverger’s Law equilibria feature perfect coordination on a single alternative and thus ensure a defeat of the Condorcet loser. Yet, they also prevent information aggregation. Conversely, the informative equilibrium produces information aggregation but, when minority size is large, it lets the Condorcet loser win with (very) high probability. Importantly, this equilibrium is “stable” even when majority voters would benefit from collectively deviating towards a Duverger’s Law equilibrium.

Next, we show that switching from plurality to AV allows a strict increase in the expected welfare of majority voters. Having the possibility to approve of all majority alternatives at once both increases the odds of electing the best alternative and reduces the odds that the Condorcet loser wins. We can show that, under mild restrictions, voters must mix between

approving of the candidate they deem best and approving of both majority candidates. We can also formulate the substantiated conjecture that there exists only one symmetric equilibrium (e.g. it is unique for the parameter values of the experiments, and for all the parameter values that we have checked). This produces clear testable implications regarding comparative statics.

In practice, the capacity of such an “electoral reform” to improve welfare will depend on the voters’ ability to actually exploit the opportunities offered by AV. On the one hand, the fact that AV produces a single symmetric equilibrium should simplify coordination. On the other hand, the voters’ challenge is to collectively adopt a strategy that is sufficiently close to the prescribed optimum, despite the larger set of voting choices available in AV. We thus need the experiments to test whether the potential benefits of AV materialize.

In our controlled laboratory experiments, subjects play a voting game that reproduces the setup of our model. By playing the same game many times, subjects familiarize themselves with the trade-offs of the environment. Experimental results confirm our predictions. In particular, information aggregation becomes so efficient with AV that payoffs come very close to what a Social Planner who observes all signals could achieve. In fact, the welfare superiority of AV is stronger in the laboratory than theory predicts, for two reasons: first, subjects make fewer strategic mistakes under AV than under plurality. Second, when the minority is large, subjects need more time to reach equilibrium play in plurality than in AV. This suggests that voters more easily handle the larger set of voting choices offered by AV than the need to select an equilibrium under plurality.

Regarding equilibrium selection under plurality, we observe the emergence of both Duverger’s Law and informative equilibria: when the minority is sufficiently small, all groups stick to playing the informative equilibrium. By contrast, when the minority is large, in the sense that the informative equilibrium leads to the Condorcet loser winning with a high probability, all groups end up coordinating their ballots on a single alternative, as predicted by Duverger’s Law. Under AV, we observe that some subjects double-vote to increase

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6In contrast, Bouton and Castanheira (2012) prove uniqueness when electorate size is arbitrarily large. The equilibrium implies full information and coordination equivalence. That is, the full information Condorcet winner always has the largest expected vote share. Goertz and Maniquet (2011) instead provide an example in which aggregate information does not occur if sufficiently many voters assign a probability zero to some states of nature.

7With three alternatives, plurality offers four possible actions: abstain, and vote for either one of the three alternatives. AV adds another four possible actions: three double approvals, and approving of all alternatives. Saari and Newenhizen (1988) argue that this may produce indeterminate outcomes, and Niemi (1984) argues that AV “begs voters to behave strategically”, in a highly elaborate manner. In contrast, Brams and Fishburn (1983, p28) show that the number of undominated strategies can be smaller under AV than under plurality.
the vote shares of both majority candidates. As predicted, the amount of double-voting increases with the size of the minority. However, the absolute level of double-voting is lower than predicted. Varying the quality of information across states, we also observe that subjects adjust their behavior in line with theoretical predictions to improve information aggregation. This holds both in AV and in plurality.

In contrast with our theoretical results (which focus on symmetric equilibria), individual behavior in AV displays substantial heterogeneity among subjects: some subjects always double-vote, whereas others always single-vote their signal. This pattern points to the need to extend the theory and consider equilibria in asymmetric strategies.\(^8\) Extending the model in this direction, we find that this type of behavior is indeed an equilibrium, which performs well in explaining the level of double-voting observed in the laboratory.

Last but not least, we use our experimental data to study a central issue in the literature (see e.g. Guarnascheli et al, 2000, Feddersen, 2004, Kawai and Watanabe, 2013, Spenkuch, 2013, Esponda and Vespa, 2013, and references therein): whether voters adopt a “rational” behavior, or vote “sincerely”. We find that a substantial fraction of the subjects behave rationally: we obtain a lower and an upper bound on this fraction of, respectively, 27.78% and 72.23% across treatments.

## 2 The Model

We consider a voting game with an electorate of fixed and finite size who must elect one policy \(P\) out of three possible alternatives, \(A, B\) and \(C\). The electorate is split in two groups: \(n\) active voters who constitute a majority, and \(n_c\) passive voters who constitute a minority. There are two states of nature: \(\omega = \{a, b\}\), which materialize with probabilities \(q(\omega) > 0\). The actual state of nature is not observable before the election.

A majority voter’s utility depends on the policy outcome and on the state of nature. It is high if the elected alternative matches the state, intermediate if there is a mismatch between the two, and lowest if alternative \(C\) is elected:

\[
U(P, \omega) = \begin{cases} 
V > 0 & \text{if } (P, \omega) = (A, a) \text{ or } (B, b) \\
v \in (0, V) & \text{if } (P, \omega) = (A, b) \text{ or } (B, a) \\
0 & \text{if } P = C.
\end{cases}
\]

\(^8\)In two-candidate elections, Ladha et al. (1996) have identified situations in which there exists an asymmetric equilibrium in which voters who receive the same signal behave differently.
For the sake of simplicity, minority voters are assumed passive in the sense that they cast their $n_C$ votes on $C$. Hence, to beat $C$, either $A$ or $B$ must receive at least $n_C$ ballots. We focus on the interesting case in which $C$-voters represent a large minority: $n - 1 > n_C > n/2$. Thus, $C$ is a Condorcet loser (it would lose both against $A$ and $B$ in a one-on-one contest), but it can win the election if active voters split their votes between $A$ and $B$.

**Timing.** Before the election (at time 0), nature chooses whether the state is $a$ or $b$. At time 1, each voter receives a signal $s \in S \equiv \{s_A, s_B\}$, with conditional probabilities $r(s|\omega) > 0$ and $r(s_{A}|\omega) + r(s_{B}|\omega) = 1$. Probabilities are common knowledge but signals are private. Signals are informative: $r(s_A|a) > r(s_B|b)$, and therefore $r(s_B|a) < r(s_B|b)$. We say that the distribution of signals is unbiased if $r(s_A|a) = r(s_B|b)$. The distribution of signals is biased if $r(s_A|a) \neq r(s_B|b)$ and, by convention, we will focus on the case in which the “more abundant” signal is $s_A$: $r(s_A|a) + r(s_A|b) \geq 1$.

Having received her signal, the voter updates her beliefs through Bayes’ rule: $q(\omega|s) = \frac{q(\omega)r(s|\omega)}{q(a)r(s_{A}|a) + q(b)r(s_{B}|b)}$. Like Bouton and Castanheira (2012), we assume that signals are sufficiently strong to create a divided majority:

$$q(a|s_A) > 1/2 > q(a|s_B).$$

That is, conditional on receiving signal $s_A$, alternative $A$ yields strictly higher expected utility than alternative $B$, and conversely for a voter who receives signal $s_B$.

The election is held at time 2, when the actual state of nature is still unobserved, and payoffs realize at time 3: the winner of the election and the actual state of nature are revealed, and each voter receives utility $U(P, \omega)$.

**Strategy space and equilibrium concept.** The winner of the election is the alternative receiving the largest number of votes – ties are broken by a fair dice. Still, the action space, i.e. which ballots are feasible, depends on the electoral rule. We consider two such rules: plurality and approval voting.

In **plurality**, each voter can vote for one alternative or abstain. The action set is then:

$$\Psi_{Plu} = \{A, B, C, \emptyset\},$$

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A possible alternative interpretation of our setup is that voters vote on whether to reform a status quo policy ($C$). Two policies could replace this status quo ($A$ and $B$), and a qualified majority of $\frac{\frac{n_C}{2}}{n_C}$ is required for passing a reform (see e.g. Dewan and Myatt 2007).
where, by an abuse of notation, action \( A \) (respectively \( B, C \)) denotes a ballot in favor of \( A \) (resp. \( B, C \)) and \( \varnothing \) denotes abstention.\(^{10}\)

In approval voting, each voter can approve of as many alternatives as she wishes:

\[
\Psi_{AV} = \{ A, B, C, AB, AC, BC, ABC, \varnothing \},
\]

where, by an abuse of notation, action \( A \) denotes a ballot in favor of \( A \) only, action \( BC \) denotes a joint approval of \( B \) and \( C \), etc. Each approval counts as one vote: when a voter only approves of \( A \), then only alternative \( A \) is credited with a vote. If the voter approves of both \( A \) and \( B \), then \( A \) and \( B \) are credited with one vote each, and so on. Thus, the only difference between \( AV \) and plurality is that a voter can also cast a double or triple approval.

Let \( x_\psi \) denote the number of voters who played action \( \psi \in \Psi_R, R \in \{ PLU, AV \} \) at time 2. The total number of votes received by an alternative \( \psi \) is denoted by \( X_\psi \). Under plurality the total number of votes received by alternative \( A \), for instance, is simply: \( X_A = x_A \). Under \( AV \), it is: \( X_A = x_A + x_{AB} + x_{AC} + x_{ABC} \).

A symmetric strategy is a mapping \( \sigma : S \rightarrow \triangle (\Psi_R) \). We denote by \( \sigma_s (\psi) \) the probability that some randomly sampled active voter who received signal \( s \) plays \( \psi \). Given a strategy \( \sigma \), the expected share of active voters playing action \( \psi \) in state \( \omega \) is thus \( \tau_\psi^\omega (\sigma) = \sum_s \sigma_s (\psi) \times r (s|\omega) \). The expected number of ballots \( \psi \) cast by active voters is \( E [x (\psi) | \omega, \sigma] = \tau_\psi^\omega (\sigma) \times n \).

Let an action profile \( x \) be the vector that lists the realized number of ballots \( \psi \). Since we are focusing on symmetric strategies for the time being, and since the conditional probabilities of receiving a signal \( s \) are iid, the probability distribution over the possible action profiles is given by the multinomial probability distribution.

For this voting game, we analyze the properties of Bayesian Nash equilibria that satisfy what we call sincere stability. That is, the equilibrium must be robust to the case in which voters may tremble by voting sincerely (that is: \( \sigma_{s_A} (A) , \sigma_{s_B} (B) \geq \varepsilon > 0 \), and we look for sequences of equilibria with \( \varepsilon \rightarrow 0 \)). Sincere stability, by imposing that a small fraction of the voters votes for their preferred alternative, implies that at least some pivot probabilities remain strictly positive.\(^{11}\) As a consequence, sincere stability eliminates equilibria in weakly

\(^{10}\)Abstention will turn out to be a dominated action in both rules. Hence, removing abstention from the choice set would not affect the analysis.

\(^{11}\)Note that some equilibrium refinement is necessary to get rid of equilibria that would only be sustainable when all pivot probabilities are exactly zero, and voters are then indifferent between all actions. Imagine for instance that all active voters play \( A \). In that case, the number of votes for \( A \) is \( n \) and the number of votes for \( C \) is \( n_C \), with probability 1. Voters are then indifferent between all possible actions, since a ballot can never be pivotal.
dominated strategies.

The advantage of our sincere stability refinement is twofold: it captures the essence of properness in a very tractable way, and it is behaviorally relevant. Indeed, experimental data (both in our experiments and others) suggest that some voters vote for their ex ante most preferred alternative no matter what.

3 Plurality

This section analyzes the equilibrium properties of plurality voting. Below, we show that two types of equilibria coexist: in one, all majority voters play the same (pure) strategy independently of their signal: they all vote for either A or B. This type of equilibrium is known as a Duverger’s Law equilibrium, in which only two alternatives receive a strictly positive vote share. In the second type of equilibrium, a majority voter’s strategy does depend on her signal. Depending on parameter values, this equilibrium either features sincere voting, that is, voters with signal $s_A$ (resp. $s_B$) vote $A$ (resp. $B$) or a strictly mixed strategy in which voters with the most abundant signal ($s_A$ by convention) mix between $A$ and $B$. These three-party equilibria exist for any population size, are robust to signal biases, and do not feature any tie. They are thus not “knife edge” in the sense of Palfrey (1989).

3.1 Preliminaries

When deciding for which alternative to vote for, a voter must first assess the expected value of each possible action, which depends on pivot events: a voter’s ballot only affects her utility if it influences the outcome of the election. We denote by $piv_{QP}$ the pivot event that one voter’s ballot changes the outcome from a victory of $P$ towards a victory of $Q$. The probability of $piv_{QP}$ in state $\omega$ is denoted $p_{\omega}^{QP}$ (see Appendix A1 for details).

In our setup, comparing the value of different actions in the set $\Psi_{Plu}$ is simplified by two elements, without affecting generality: first, voting for $C$ is a dominated action. Hence, $\tau_C^\omega(\sigma)$ is equal to zero. Second, a vote for $A$ or for $B$ can only be pivotal against $C$, since we impose that $n_C > n/2$. This implies that abstention is also a dominated action.

It remains to check which of actions $A$ and $B$ each voter prefers to play. The payoff

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12We do not use more traditional refinement concepts such as perfection or properness because, in the voting context, the former does not have much bite, since weakly dominated strategies are typically excluded from the equilibrium analysis. The latter is quite intractable since it requires a sophisticated comparison of pivot probabilities for totally mixed strategies.
difference between the two is given by (see Appendix A1 for more detail):

\[ G(A|s) - G(B|s) = q(a|s) \left[ Vp^a_{AC} - Vp^a_{BC} \right] + q(b|s) \left[ Vp^b_{AC} - Vp^b_{BC} \right], \]  

(3)

where \( G(\psi|s) \) denote the expected gain of an action \( \psi \in \{A, B\} \) over abstention, \( \varnothing \).

### 3.2 Duverger’s Law Equilibria

The game theoretic version of Duverger’s Law (Duverger 1963, Riker 1982, Palfrey 1989, Myerson and Weber 1993, Cox 1997) states that, when voters play strategically, only two alternatives should obtain a strictly positive fraction of the votes in plurality elections. In our setup, these equilibria are as follows:

**Definition 1** A Duverger’s Law equilibrium **is such that all majority voters vote either for** \( A \) **or for** \( B \).

These Duverger’s Law equilibria feature pros and cons. On the one hand, they ensure that \( C \) cannot win the election, since either \( A \) or \( B \) must receive \( n > n_C \) votes. On the other hand, they prevent information aggregation. That is, the winner of the election is fully determined by voter coordination, and cannot vary with the state of nature. Our first proposition is that:

**Proposition 1** Under plurality, Duverger’s Law equilibria always exist.

The intuition for the proof (in Appendix A1) is straightforward: if an alternative, say \( A \), collects the ballots from all the other voters, then the pivot probability ratio \( p^A_{AC}/p^B_{BC} \) goes to infinity. From (3), this ensures that all majority voters value a vote for \( A \) strictly more than a vote for \( B \): they do not want to waste their ballot on an alternative that is very unlikely to win.

### 3.3 Informative Equilibria

In Duverger’s Law equilibria, voters discard the information in their possession: they coordinate their votes on a given party, independently of their signal. This type of equilibrium is typically considered the only reasonable one if voters are short-term instrumentally rational, in Cox’s (1997) terminology. Indeed, in a world without aggregate uncertainty, equilibria
with more than two alternatives obtaining votes are typically “knife edge” and “expectationally unstable” (Palfrey 1989, Fey 1997).\textsuperscript{13} Therefore, empirical research typically associates strategic voting with the voters’ propensity to abandon their preferred but non-viable candidates, and vote for more serious contenders (see Cox 1997, Alvarez and Nagler 2000, Blais et al 2005, Fujiwara 2011, Spenkuch 2013).\textsuperscript{14}

Instead, Propositions 2 and 3 show that even if all voters are short-term instrumentally rational, there exists an equilibrium in which no candidate is abandoned by her supporters. This thus breaks the link between instrumental voting and the observation that only relatively low fractions of the electorate switch to their second-best alternative. This discussion revolves around the existence and stability of what we call an informative equilibrium:

**Definition 2** An informative equilibrium is such that (i) all alternatives receive a strictly positive vote share, (ii) these vote shares are different across alternatives (no knife-edge equilibrium), and (iii) A is the strongest majority contender in state $a$, and B in state $b$.

To prove the existence of such an equilibrium, we first focus on the case in which information is close to being symmetric across states. Then, voters vote sincerely in an informative equilibrium: a voter who receives signal $s_A$ votes for $A$, whereas a voter who receives signal $s_B$ votes for $B$. That is, abandoning one’s preferred candidate is not a best response when one expects other voters to vote sincerely:

**Proposition 2** In the unbiased case $r(s_A|a) = r(s_B|b)$, the sincere voting equilibrium exists $\forall n, n_c$. Moreover, there exists a value $\delta(n, n_c) > 0$ such that sincere voting is an equilibrium for any distribution satisfying $r(s_A|a) - r(s_B|b) < \delta(n, n_c)$.

The intuition is that, in the unbiased case, sincere voting implies that the likelihood of being pivotal against $C$ is the same with an $A$-ballot in state $a$ as with a $B$-ballot in state $b$. Therefore, $s_A$-voters strictly prefer to vote for $A$ and $s_B$-voters strictly prefer to vote for $B$. The important feature of this equilibrium is that it is stable: it exists even if signals are slightly biased (we are coming back to this after Proposition 3). The pros and cons of

\textsuperscript{13}In contrast, Dewan and Myatt (2007) and Myatt (2007) emphasize the existence of three-candidate equilibria when there is aggregate uncertainty. In our setup as well, informative equilibria would still exist if $s_A$-voters always preferred $A$ and $s_B$-voters always preferred $B$, i.e. if they had private value preferences.

\textsuperscript{14}Kawai and Watanabe (2013) acknowledge the weakness of the private-value pivotal voter model in organizing observational data. For this reason, they relax the equilibrium restrictions on voter beliefs while admitting that their “way of modeling the mechanism may be somewhat unsatisfactory from a theoretical perspective”.
sincere voting are the exact flip-side of the ones identified for Duverger’s Law equilibria: as illustrated by the following example, it produces information, but does not guarantee a defeat of the Condorcet loser.

**Example 1** Consider a case in which $n = 12$, $n_C = 7$, and $r(s_A|a) = r(s_B|b) = 2/3$. Sincere voting implies that the best alternative ($A$ in state $a$; $B$ in state $b$) wins with a probability of 73%. $C$ has the second largest expected vote share and wins with a probability of 23% in either state. The alternative with the lowest –but strictly positive– vote share is $B$ in state $a$ and $A$ in state $b$.

When $n_C$ is 9, the alternative with the largest expected vote share is $C$, who then wins with a probability larger than 71%, whereas the best alternative wins with a probability below 29%.

Based on Proposition 2 and Example 1, one may be misled into thinking that informative equilibria require signals to be (almost) unbiased. Instead, the fact that the signal structure becomes too biased to sustain sincere voting produces an equilibrium in which voters increase their support for the weakest candidate:

**Proposition 3** Let $r(s_A|a) - r(s_B|b) > \delta (n, n_c)$. Then, there exists an informative equilibrium in which voters with signal $s_A$ play a non-degenerate mixed strategy: $\sigma_{s_A} (A) \in (0, 1)$ and $\sigma_{s_B} (B) = 1$.

The intuition for the proof is best conveyed through a second numerical example:

**Example 2** Let electorate size be $n = 12$ and $n_C = 7$, and the signal structure be $r(s_A|a) = 8/9 > 2/3 = r(s_B|b)$. For these parameter values, an $s_A$-voter would strictly prefer to vote for $B$ if all the other voters were to vote sincerely. Indeed, with sincere voting, the probability of being pivotal in favour of $B$ in state $b$ is much larger than any other pivot probability. Hence, $G(A|s_A) - G(B|s_A) < 0$. The informative equilibrium is reached when $\sigma_{s_A} (A) = 0.915$ and $\sigma_{s_B} (B) = 1$: by reducing the expected vote share of $A$ and increasing that of $B$, the relative probability of being pivotal in favor of $A$ in state $a$ increases to the point in which $s_A$-voters are indifferent between voting $A$ and $B$, whereas $s_B$-voters

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15Each numerical example reproduces the parameters used in one of the treatments of our laboratory experiments (see Section 5). In all examples, the two states of nature are equally likely, and the payoffs are: $V = 200; v = 110$ and the value of $C$ is 20. Normalizing the latter to 0 would also reduce the other payoffs by 20.

16Note that, for a given bias $r(s_A|a) - r(s_B|b) > 0$, sincere voting is only an equilibrium if electorate size is sufficiently small: as electorate size increases to infinity, given the biased signal structure, the ratio of pivot probabilities would either converge to zero or infinity if voters kept voting sincerely.
still strictly prefer to vote B. Importantly, all vote shares are strictly positive and the full information Condorcet winner is the most likely winner in both states of nature (their winning probabilities are, respectively, 96% and 79% in states a and b):

\[ \tau_a^a = 0.81 > \tau_B^b = 0.69 > \frac{n_C}{n} = 0.58 > \tau_A^b = 0.31 > \tau_B^a = 0.19. \]

This informative equilibrium gives C a strictly positive probability of victory (3% in state a and 18% in state b) but expected utility is higher in this equilibrium than in a Duverger’s Law equilibrium.

The example illustrates that the existence of this equilibrium does not rely on some form of symmetry between vote shares.\textsuperscript{17} This stems from an underdog effect caused by aggregate uncertainty, as in Myatt (2007): ceteris paribus, in an informative equilibrium, the probability of being pivotal in favor of A in state a decreases when the proportion of A-supporters increases. This decreases the value of an A-ballot. In contrast, the value of a B-ballot remains intact. Therefore, in equilibrium, some A-supporters must lend support to the underdog, B.

The presence of common values provides another incentive for A-supporters to support B: with private values, they would benefit from a switch towards the all-vote-A Duverger’s Law equilibrium. With common values instead, B is their preferred alternative in state b, \textit{i.e.} exactly when their ballot is most likely to be pivotal against C. In contrast with the two-candidate Condorcet Jury literature, however, this informative equilibrium may not be advantageous to information-wary majority voters: since they are splitting their votes, they may pave the way to a victory of C, like in example 1 with \( n_C = 9 \). Plurality voting thus introduces a trade-off between information aggregation and coordination.

\section{Approval Voting}

\subsection{Payoffs and Dominated Strategies}

Under AV, voters have a larger choice set, which makes their voting decision potentially more complex. Single approvals (A, B, C) have the same effect as in plurality. Double or triple approvals instead allow voters to abstain selectively. For example, by giving one

\textsuperscript{17} As proved by Bouton and Castanheira (2009), such a mixed-strategy equilibrium also exists in arbitrarily large electorates, with the difference that the gap between \( \tau_A^A \) and \( \tau_B^B \) decreases to zero (\textit{i.e.} \( \lim_{n \to \infty} \tau_A^A = \lim_{n \to \infty} \tau_B^B \)).
vote to both A and B, an AB-ballot cannot be pivotal between these two alternatives, but increases the probability of being pivotal against C. The following lemma shows that the set of undominated strategies is more restricted:

**Lemma 1** Independently of a voter’s signal, the actions $\psi \in \{C, AC, BC, ABC, \varnothing\}$ are weakly dominated by some action in $\psi \in \{A, B, AB\}$. Hence, in equilibrium:

$$\sigma_s(A) + \sigma_s(B) + \sigma_s(AB) = 1, \forall s \in \{s_A, s_B\}.$$  \hspace{1cm} (4)

**Proof.** Straightforward. $\blacksquare$

The intuition is that abstaining or approving of C can only increase C’s probability of winning. In contrast, the actions in the undominated set can only reduce it. Next, how a voter wants to allocate her ballot across these undominated actions depends on the probability of each pivot event. Let $\pi_{QP}^s$ denote the probability that a single-Q ballot is pivotal in favor of Q against P in state $\omega \in \{a, b\}$ and the voting rule is AV. The derivation of these pivot probabilities and the values of each action are detailed in Appendix A2.

There are two important differences with plurality. First, the probability of being pivotal between A and B is no longer zero, since double-voting can increase the score of both A and B above that of C. Second, we need to determine the value of a double ballot. The payoff differential between actions A and AB is:

$$G^{AV}(A|s) - G^{AV}(AB|s) = q(a|s) \left[ \pi_{AB}^a (V - v) - \pi_{BC}^a v + \phi^a \right] + q(b|s) \left[ \pi_{AB}^b (v - V) - \pi_{BC}^b V + \phi^b \right],$$  \hspace{1cm} (5)

where $G^{AV}(\psi|s)$ is the expected value of a ballot $\psi \in \Psi_{AV}$ for a s-voter, and $\phi^a$ and $\phi^b$ are correcting terms for three-way ties (see Appendix A2). With straightforward, although tedious, manipulations, one finds that the first term in (5) may either be positive or negative, whereas the second is strictly negative. Similarly, the first term in (6) is strictly negative:

$$G^{AV}(B|s) - G^{AV}(AB|s) = q(a|s) \left[ \pi_{BA}^a (v - V) - \pi_{AC}^a V + \phi^a \right] + q(b|s) \left[ \pi_{BA}^b (V - v) - \pi_{AC}^b v + \phi^b \right].$$  \hspace{1cm} (6)

Equations (5) and (6) highlight the main trade-off faced by voters under AV. On the one hand, they want to double-vote more if C is a threat. On the other hand, they want
to double-vote less if \( C \) is not a threat. To see this, conjecture a strategy in which no other voter double-votes. In that case, a vote can never be pivotal between \( A \) and \( B \): 
\[
\pi^\omega_{AB} = \pi^\omega_{BA} = \phi^\omega = 0, 
\]
and both payoff differentials must be negative.\(^{18}\) Majority voters then prefer to maximize their probability of being pivotal against \( C \). It follows that:

**Lemma 2** The strategies that are an equilibrium in plurality cannot be an equilibrium in AV.

Conversely, imagine that all the other voters double-vote. In that case: 
\[
\pi^\omega_{AC} = \pi^\omega_{BC} = \phi^\omega = 0, 
\]
and either (5) or (6) must be strictly positive. In other words:

**Lemma 3** Pure double-voting is never an equilibrium in AV.

The reason is that, while double-voting reduces the risk that \( C \) wins the election, it also reduces the amount of information produced by the election. Lemma 3 implies that there can never be so much double-voting that information aggregation is impossible.\(^{19}\)

### 4.2 Equilibrium Analysis

The action set under AV is an extension of the action set under plurality. Therefore, in a pure common value setting, Ahn and Oliveros (2011, Proposition 1) show that there is an equilibrium in AV for which welfare is weakly higher than for any equilibrium in plurality.\(^{20}\) In contrast, our setup includes a minority with preferences opposite to that of the majority. As we observed in Section 3, the relatively large size of the minority implies that the probability of being pivotal between \( A \) and \( B \) is zero under plurality. Theorem 1 directly follows from that fact and from (5) – (6):

\(^{18}\)This is due to the fact that we focus on large minorities. If the size of the minority, \( n_C \), falls towards zero, then the propensity to double-vote may drop to zero as well (see Bouton and Castanheira, 2012).

\(^{19}\)Pure double-voting has been termed the *Burr dilemma* by Nagel (2007), who argues that approval voting is inherently biased towards such ties. He documents this with the “[approval] experiment [that] ended disastrously in 1800 with the infamous Electoral College tie between Jefferson and Burr.” Lemma 3 shows why such a “disaster” cannot be an equilibrium when voting behavior is not dictated by party discipline.

\(^{20}\)Ahn and Oliveros (2011) exploit McLennan (1998) to show that, in a common value setup, one can rank equilibrium outcomes under approval voting as opposed to plurality and negative voting. By revealed preferences, since the action set in the two other rules is a strict subset of the action set under AV, “the maximal equilibrium utility under approval voting is greater than or equal to the maximal equilibrium utility under plurality voting or under negative voting.” (p. 3).
Theorem 1 There always exists an equilibrium in AV for which expected welfare is strictly higher than for any equilibrium in plurality. In that equilibrium, some voters must double-vote, and \( s_A(A), s_B(B) > 0 \).

The intuition is as follows: when one compares the set of undominated actions in plurality and in AV, one sees that the only difference is the possibility to double-vote \( AB \). Following Ahn and Oliveros (2011), if one voter wants to double-vote, the other voters’ expected utility must also increase. From Lemma 2, a voter’s strict best response is precisely to double-vote when the other voters single-vote “excessively”. From Lemma 3, it is to single-vote sincerely if the other voters double-vote “excessively”. Hence the result.

In a large Poisson game setup, Bouton and Castanheira (2012) show that this pattern is monotonic, and that the relative value of the double and single-votes cross only once. In other words, AV displays a unique equilibrium. In contrast, we do not focus on arbitrarily large electorates, which means that one cannot establish a general proof of equilibrium uniqueness in our setup. Yet, our next theorem identifies unique voting patterns for any interior equilibrium:

Theorem 2 Whenever both \( s_A \)- and \( s_B \)-voters adopt a nondegenerate mixed strategy, then it must be that voters with signal \( s_A \) only mix between \( A \) and \( AB \), and voters with signal \( s_B \) only mix between \( B \) and \( AB \).

This theorem builds on the comparison between the preferences of \( s_A \) and \( s_B \) voters: conjecture a case in which the former play \( B \) with strictly positive probability. Since a voter with signal \( s_B \) values \( B \) even more, she must only play \( B \), which contradicts the nature of an interior equilibrium. To extend this result to equilibria in which (one of the two groups of) voters play pure strategies, we would have to focus on larger electorates, which is not our purpose. Yet, we can rely on numerical simulations: for all the parametric values we checked, the equilibrium was unique and such that voters with signal \( s_A \) only mix between \( A \) and \( AB \), while voters with signal \( s_B \) never play \( A \). This held both for interior equilibria and for equilibria in which (one of the two groups of) voters play a degenerate strategy.

Two additional examples are useful to illustrate these results and better understand the features and comparative statics of voting equilibria in AV:

Example 3 Consider the same set of parameters as in Example 1: \( n = 12, n_C = 7 \) or \( 9 \), and \( r(s_A|a) = r(s_B|b) = 2/3 \). As just emphasized, the equilibrium is unique under AV.\(^{21}\)

\(^{21}\)In the strategy space \((\sigma_{s_A}(A), \sigma_{s_B}(B))\), there is a unique cutoff for which \( G(A|s_A) = G(AB|s_A) \), and
It is such that:

\[ \sigma_{s_A}(A) = \sigma_{s_B}(B) = 0.64 \text{ and } \sigma_{s_A}(AB) = \sigma_{s_B}(AB) = 0.36 \text{ when } n_C = 7, \]
\[ \sigma_{s_A}(A) = \sigma_{s_B}(B) = 0.30 \text{ and } \sigma_{s_A}(AB) = \sigma_{s_B}(AB) = 0.70 \text{ when } n_C = 9. \]

When \( n_C = 7 \), these equilibrium strategies imply that \( A \) wins with a probability of 82% in state \( a \) (as does \( B \) in state \( b \)), whereas \( C \)’s probability of winning is below 1%. When \( n_C = 9 \), \( A \) wins with a probability of 73% in state \( a \) (as does \( B \) in state \( b \)), whereas \( C \)’s probability of winning remains as low as 1.5%. These values should be contrasted with the sincere voting equilibrium in plurality (see example 1), in which the probability of selecting the best outcome was substantially lower, and the risk that \( C \) wins was substantially larger.

Comparing equilibrium behavior with \( n_C = 7 \) and \( n_C = 9 \) in Example 3 shows that the larger \( n_C \), the more double-voting in equilibrium. This pattern was found to be monotonic and consistent across numerical examples for any value of \( n \) and signal structures.

**Example 4** Consider the same set of parameters as in Example 3, except for \( r(s_A|a) = 8/9 \). This reproduces the biased signal setup of Example 2. As in the previous example, the equilibrium is unique: \( \sigma_{s_A}(A) = 0.26 < \sigma_{s_B}(B) = 0.52 \text{ and } \sigma_{s_A}(AB) = 0.74 > \sigma_{s_B}(AB) = 0.48 \). This equilibrium implies that \( A \) wins with a probability of 87% in state \( a \), whereas \( B \) wins with a probability of 90% in state \( b \). \( C \)’s winning probabilities are 0.5% in state \( a \) and 2.8% in state \( b \).

The equilibrium with biased information highlights a second feature of double-voting: when the signal structure is biased (here towards \( s_A \)), the voters with the most abundant signal double-vote more than the voters with the least abundant signal. In other words, like under plurality voting, they lend support to the underdog. But, in contrast with plurality, approval voting resolves the trade-off between coordination and information aggregation: by double-voting more, \( s_A \)-voters lend support to \( B \) without weakening \( A \), which allows them to maintain \( A \)’s high probability of winning in state \( a \) and increase \( B \)’s in state \( b \).

5 Experimental Design and Procedures

As explained in the Introduction, the best way to test our theoretical predictions is to run controlled laboratory experiments. To this end, we introduced subjects to a game the same holds for \( G(B|s_B) = G(AB|s_B) \). The equilibrium lies at the intersection between these cutoffs.
that had the very same structure as the one presented in the model of Section 2. All participants were given the role of an active voter, whereas passive voters were simulated by the computer.\footnote{Morton and Tyran (2012) show that preferences in one group are not affected by preferences of an opposite group. Therefore, having computerized rather than human subjects should not alter the behavior of majority voters in a significant way. Partisans (the equivalent to our passive voters) simulated by the computer has been used in previous studies – see Battaglini et al. (2008, 2010).} Following the experimental literature on the Condorcet Jury Theorem initiated by Guarnaschelli et al. (2000), the two states of the world were called blue jar and red jar, whereas the signals were called blue ball and red ball. One of the jars was selected randomly by the computer, with equal probability. The subjects were not told which jar had been selected, but were told how the probability of receiving a ball of each color depended on the selected jar. After seeing their ball, each subject could vote from a set of three candidates: blue, red or gray.\footnote{The colors that we used in the experiments were blau, rot and schwarz. Throughout the paper, however, we refer to blue, red and gray, respectively.} Blue and red were the two majority candidates and gray was the Condorcet loser. Subjects were told that the computer casts $n_C$ votes for gray in each election ($n_C$ was set equal to 7 or 9 depending on the treatments).

The subjects’ payoff depended on the color of the selected jar and on that of the election winner. If the color of the winner matched that of the jar, the payoff to all members of the group was 200 euro cents. If the winner was blue and the jar red or the other way around, their payoff was 110 cents. Finally, if gray won, their payoff was 20 cents.

We consider three treatment variables, which leads to six different treatments. The first variable is the voting mechanism: in PL treatments, the voting mechanism was plurality. In this case, subjects could vote for only one of the three candidates. In AV treatments, the voting mechanism was approval voting. In this case, subjects could vote for any number of candidates.\footnote{As in Guarnaschelli et al. (2000), abstention was not allowed (remember that abstention is a strictly dominated action in our setup). In a setting related to ours, Forsythe et al (1993) allowed for abstention and found that the abstention rate was as low as 0.65%.} With either mechanism, the candidate with the most votes wins, and ties were broken with equal probability. The second variable is the size of the minority, $n_C$, which was set to either 7 or 9. We will refer to them as small and large minority. The third variable is whether the signal structure is unbiased or biased. In unbiased treatments, signal precision was identical across states and set to $r(\text{blue ball} \mid \text{blue jar}) = r(\text{red ball} \mid \text{red jar}) = 2/3$. In biased treatments (which we indicate by $B$), $r(\text{blue ball} \mid \text{blue jar})$ was increased to $8/9$.

Table 1 provides an overview of the different treatments.

Experiments were conducted at the BonnEconLab of the University of Bonn between July 2011 and January 2012. We ran a total of 18 sessions with 24 subjects each. No subject...


<table>
<thead>
<tr>
<th>Treatment</th>
<th>Voting rule</th>
<th>Minority size ($n_c$)</th>
<th>Precision Blue State</th>
<th>Precision Red State</th>
<th>Sessions / Ind. Obs.</th>
<th>Group numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>PL7</td>
<td>Plurality</td>
<td>7</td>
<td>2/3</td>
<td>2/3</td>
<td>3 / 6</td>
<td>1-6</td>
</tr>
<tr>
<td>PL9</td>
<td>Plurality</td>
<td>9</td>
<td>2/3</td>
<td>2/3</td>
<td>3 / 6</td>
<td>7-12</td>
</tr>
<tr>
<td>AV7</td>
<td>Approval</td>
<td>7</td>
<td>2/3</td>
<td>2/3</td>
<td>3 / 6</td>
<td>13-18</td>
</tr>
<tr>
<td>AV9</td>
<td>Approval</td>
<td>9</td>
<td>2/3</td>
<td>2/3</td>
<td>3 / 6</td>
<td>19-24</td>
</tr>
<tr>
<td>PL7B</td>
<td>Plurality</td>
<td>7</td>
<td>8/9</td>
<td>2/3</td>
<td>3 / 6</td>
<td>25-30</td>
</tr>
<tr>
<td>AV7B</td>
<td>Approval</td>
<td>7</td>
<td>8/9</td>
<td>2/3</td>
<td>3 / 6</td>
<td>31-36</td>
</tr>
</tbody>
</table>

Table 1: Treatment overview. Note: ind. obs. stands for “individual observations”.

participated in more than one session. Students were recruited through the online recruitment system ORSEE (Greiner 2004) and the experiment was programmed and conducted with the software z-Tree (Fischbacher 2007).

All experimental sessions were organized along the same procedure: subjects received detailed written instructions, which an instructor read aloud (see supplementary appendix). Each session proceeded in two parts: in the first part, subjects played one of the treatments in fixed groups for 100 periods (repetition allows subjects to familiarize with the trade-offs of the environment). Before starting, subjects were asked to answer a questionnaire to check their full understanding of the experimental design. At the end of the experiment, subjects received new instructions, and made 10 choices in simple lotteries, as in Holt and Laury (2002). We ran this second part to elicit the subjects’ risk preferences.

To determine payment, the computer randomly selected four periods from the first part and one lottery from the second part. In total, subjects earned an average of €13.47, including a show-up fee of €3. Each experimental session lasted approximately one hour.

Our choice of fixed matching is not innocuous. The main advantage of fixed matching is that, for given costs, it delivers more independent units of observation, hence more power for non-parametric tests. A typical drawback of fixed matching is that it favours repeated game effects. One might thus fear that outcomes based on fixed matching could display more cooperative behavior than in the theory. However, this is not an issue in our setup: since voters have common values, there is no gap between a potential “cooperative equilibrium”

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25 Our choice of 100 periods is somewhat higher than usual. Yet we check the robustness of our results by also identifying the subjects’ behavior in the earlier periods.

26 In the first round of experiments (the seven sessions with the groups 1, 2, 7, 8, 9, 10, 13, 14, 15, 16, 19, 20, 21 and 22), we selected seven periods to determine payment. We reduced this to four periods after realizing that the experiment had taken much less time than expected. We find no difference in behavior between these two sets of sessions.
and the equilibrium in a one-shot game.\textsuperscript{27} Hence, the only relevant effect of fixed matching is that it may facilitate the subjects’ learning about the equilibrium selected by the group. For instance, Forsythe et al. (1993, 1996) observe that Duverger’s Law equilibria emerge more easily among voters with a common history. Since we are more interested in the equilibrium properties of the voting systems under consideration than out-of-equilibrium coordination failures, fixed matching emerges as a natural choice.\textsuperscript{28} Importantly, given equilibrium multiplicity under plurality and equilibrium uniqueness under approval voting, fixed matching essentially stacks the deck against the latter in welfare comparisons.

6 Experimental Results

Section 6.1 presents our experimental results when information is unbiased, and Section 6.2 when it is biased. Section 6.3 extends the model to asymmetric equilibria. Section 6.4 discusses voter rationality. Finally, Section 6.5 turns to welfare.

6.1 Unbiased Treatments

6.1.1 Plurality

As shown in Section 3, two types of equilibria coexist under plurality when information is unbiased: in Duverger’s Law equilibria, participants should disregard their signal and coordinate on always voting blue or always voting red. In sincere voting equilibria, participants should vote their signal. Table 2 shows the average frequencies with which subjects voted sincerely (we call this \textit{voting the signal}), for the color opposite to their signal (we will call this \textit{voting opposite}) or for gray.\textsuperscript{29}

In the presence of a small minority (PL7), the subjects’ voting behavior is consistent with sincere voting: taking an average across all groups and periods, 91.38\% of the ballots were sincere in PL7, with a lowest value of 86.42\% in one independent group. This behavior is quite stable over time: we regressed the frequency of “voting the signal” on the period number, and found that the coefficient was not significantly different from zero ($p = 0.648$).

\textsuperscript{27}In the setup of the Condorcet Jury Theorem, Ali et al. (2008) find no significant difference between random matching (or ad hoc committees) and fixed matching (or standing committees).

\textsuperscript{28}In any case, the first periods of the experiment provide information about possible coordination failures.

\textsuperscript{29}The figures with a ‘*’ report the predicted voting pattern for the last 50 periods, conditional on the color on which the group coordinated. For instance, if the group coordinated on blue, and if 42\% of the voters obtain a blue ball in a given draw, then 42\% should play “signal” and 58\% “opposite”.

19
Voting behavior is substantially different in the presence of a large minority (PL9). First, only 63.86% of the observations are consistent with sincere voting. Second, performing the same regression on the period number, we found a clear and significant ($p < 0.01$) trend: the frequency of voting the signal decreases over time. Participants begin the experiment by voting sincerely (94.44% of them voted their signal in the first period) but rapidly abandon that strategy (see below) and eventually adjust their behavior by frequently voting against their signal (only 53.70% voted sincerely in the last period). This pattern is fully consistent with the progressive shift from a sincere voting equilibrium to a Duverger’s Law equilibrium. Figure 1 illustrates this shift by plotting the observed frequency of voting blue, red and gray (irrespective of the signals subjects receive) for each group in the PL9 treatment. The horizontal dashed line displays the minimal vote share required to defeat gray (in case nobody plays the dominated strategy of voting gray). As one can see, all six groups converged to a Duverger’s Law equilibrium.

This raises two empirical questions regarding equilibrium selection. The first one is why all groups selected a Duverger’s Law equilibrium in the PL9 treatment, and the informative equilibrium in the PL7 treatment. The second question is how each PL9 group selected its Duverger’s Law equilibrium.

We can identify at least two reasons why Duverger’s Law equilibria are the most natural focal point in PL9: first, the expected utility in the informative equilibrium is 69.76 in PL9, instead of 152.76 in PL7. This compares with an expected utility of 155 in a Duverger’s Law equilibrium. The incentive to get away from sincere voting is thus substantial in PL9. Second, the range of strategy profiles for which sincere voting is a best response is quite narrow in the case of PL9. The phase diagrams in Figure 2 illustrate this graphically. Taking the standpoint of a given voter, the horizontal axis displays the “other” blue voters’
Figure 1: Frequency of voting blue, red and gray, irrespective of the signal in groups of treatment PL9U. The dashed line indicates the minimum frequency of vote share required to defeat gray (in case nobody from the majority votes for the Condorcet loser).

propensity to vote blue, and the vertical axis displays the “other” red voters’ propensity to vote red. The solid curve represents the locus of strategies for which our given voter is indifferent between playing blue and red if she gets a blue ball. To the left of that curve, her payoff of voting red is higher than that of voting blue, and conversely to the right of the curve. The dashed curve represents the equivalent locus for a voter who receives a red ball. Above the curve, she prefers red to blue, and conversely below the curve. The arrows display the attraction zones of each of the three equilibria mentioned: sincere voting in the top right corner, and the two Duverger’s Law equilibria in the bottom right and top left corners. One can readily see that the attraction zone of the sincere voting equilibrium is much larger in PL7 than in PL9. Therefore, even relatively small deviations from sincere voting make it optimal to vote for the leading majority candidate in PL9.

Turning to the second question, most groups coordinated on the first color that obtained strictly more than six of the majority votes. This is in line with the findings of Forsythe.

\[30\] Confronting the strategies actually played by the subjects to these theoretical predictions, we found that, even in early periods, the typical voting realization falls outside the sincere voting attraction zone in PL9, and inside that zone in PL7.

\[31\] It happened in period 1 for four groups and in period 2 for one group. The only exception is group 11,
Figure 2: Phase diagram of treatments PL7 and PL9. The horizontal axis displays the probability of sincere voting by blue voters while the vertical axis displays the probability of sincere voting by red voters. The solid line indicates the indifference curve for the blue voters, while the dashed line indicates the indifference curve for the red voters.

et al. (1993, p235): “a majority candidate who was ahead of the other in early elections tended to win the later elections, while the other majority candidate was driven out of subsequent races”. Yet, the transition from sincere voting to the selected Duverger’s Law equilibrium can take a substantial amount of time: the first period from which either blue or red consistently obtained enough votes to win was 50, 59, 83, 63, 21 and 26 for groups 7-12 respectively. This shows that experiments using shorter horizons may fail to capture equilibrium convergence (the welfare consequences of these coordination failures are analyzed in Section 6.5).

6.1.2 Approval Voting

Table 3 summarizes the subjects’ behavior in AV treatments. It displays the frequencies with which subjects single-vote their signal, double-vote red and blue, single-vote opposite to their signal, and vote gray (possibly in combination with another candidate).

These two treatments reproduce the parametric cases covered in Example 3, which we found to display a unique symmetric equilibrium. In that equilibrium a voter should only

where blue got 7 votes in the first period and then red received more votes from period 2 onwards.
Table 3: Aggregate voting behavior in approval voting treatments with unbiased information. Gray refers to voting for gray or a combination of gray and others.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Minority Size</th>
<th>Periods</th>
<th>Periods</th>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1-50</td>
<td>51-100</td>
<td></td>
</tr>
<tr>
<td>AV7</td>
<td>Small</td>
<td>Signal</td>
<td>70.92</td>
<td>71.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Double Vote</td>
<td>22.22</td>
<td>24.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Opposite</td>
<td>6.50</td>
<td>3.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gray</td>
<td>0.36</td>
<td>0.00</td>
</tr>
<tr>
<td>AV9</td>
<td>Large</td>
<td>Signal</td>
<td>47.08</td>
<td>43.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Double Vote</td>
<td>45.67</td>
<td>51.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Opposite</td>
<td>6.86</td>
<td>4.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gray</td>
<td>0.39</td>
<td>0.06</td>
</tr>
</tbody>
</table>

The second theoretical prediction drawn from Example 3 refers to the effect of minority size: it should increase the frequency of double-voting. Table 3 shows that this is indeed the way in which the subjects adapted their behavior: the percentage of double-voting was multiplied by more than two, from 23.29% in treatment AV7 to 48.66% in treatment AV9. This difference is significant at 1% (Mann-Whitney, $z = 2.722$, $p < 0.01$).

Although the comparative statics go in the direction predicted by theory, one should notice that the amount of double-voting was well below theoretical predictions. These differences are significant at 5% in both AV7 and AV9 (Mann-Whitney, $z = 2.201$, $p < 0.05$). One might think that risk aversion (or, more precisely, the lack of risk aversion) helps explain this discrepancy. However, we do not find any significant relation between subjects’ level of risk aversion and their propensity to double-vote. Another possibility for this discrepancy is that it is more costly for subjects to double-vote than to single-vote (they have to click twice instead of once). A third, and perhaps more subtle, possibility is that subjects actually coordinated on an asymmetric equilibrium. We discuss this in Section 6.3.

Finally, we note that aggregate behavior is very homogeneous across independent groups in AV7. In contrast, there is one group in AV9 (group 22) for which behavior differs substantially from the other five groups: in group 22, 17.33% of the votes were single-votes against the signal. According to our theoretical predictions, this should not happen in equilibrium. Looking closer at individual voting data, we observe that four subjects consistently single-voted red irrespective of their signal. This implies that red was consistently winning.
the elections, which prevented information aggregation.\textsuperscript{32}

6.2 The Effects of Biased Information

In PL7, we observed that all independent groups coordinated on the sincere voting equilibrium. One reason might be the symmetry between the blue and red signals, which made coordination challenging for the subjects. To test whether this is the case, treatment PL7B instead makes the signal structure strongly biased in favor of the blue signal by setting $r(\text{blue ball} | \text{blue jar}) = 8/9$. So, if the voters were to keep playing sincere, blue would win disproportionately more often than red. Propositions 1 and 3 show that voters may still coordinate on either the Duverger’s Law equilibrium or on the informative equilibrium in which blue voters should mix between voting blue with probability 91.53\% and voting red with probability 8.43\% (see Example 2).

In the experiment, we observe that one independent group (group 28) coordinated on the “blue” Duverger’s Law equilibrium. The other five adopted a strategy coherent with the informative equilibrium of Example 2. Let us analyze each in turn: in group 28, almost all subjects cast a blue ballot as of period 31. From that period onwards, blue consistently obtained enough votes to win. Table 4 summarizes the behavior of the other five independent groups. Looking at the last 50 periods, we observe that, in line with theoretical predictions, red subjects voted sincerely with a higher probability than blue subjects. The difference between these two behaviors is statistically significant (Mann-Whitney, $z = 2.023$, $p < 0.043$). Despite some heterogeneity across groups, the aggregate average is very close to the theoretical prediction: there is no statistically significant difference between the theoretical prediction and the observed frequency of voting blue when getting a blue ball (Mann-Whitney, $z = 0.405$, $p = 0.686$).

The model helps identify two reasons why the informative equilibrium is more likely to be selected. First, it yields a higher expected payoff than Duverger’s Law equilibria (178.37 instead of 155). Second, as identified by the phase diagram in Figure 3, when starting from sincere voting (the top-right corner), the local dynamics of individual best responses point towards the informative mixed strategy equilibrium (the black dot on the graph) rather than towards either Duverger’s Law equilibria.

This provides ample evidence that informative equilibria are empirically relevant when

\textsuperscript{32}Aggregate measures displayed in Table 3 are robust to the exclusion of group 22. In the last 50 periods, for example, the level of double voting would switch from 51.64 to 52.80 and the level single voting would switch from 43.33 to 44.63. In the same vein, the tests provided throughout the section are also robust to the exclusion of group 22.
Table 4: Aggregate voting behavior in treatment PL7B. Group 28 was excluded given that it converged to a Duverger’s Law equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>Periods 1-50</th>
<th>Periods 51-100</th>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal if blue</td>
<td>92.99</td>
<td>90.75</td>
<td>91.53</td>
</tr>
<tr>
<td>Opposite if blue</td>
<td>6.89</td>
<td>8.38</td>
<td>8.47</td>
</tr>
<tr>
<td>Signal if red</td>
<td>96.39</td>
<td>97.48</td>
<td>100</td>
</tr>
<tr>
<td>Opposite if red</td>
<td>3.13</td>
<td>1.74</td>
<td>0</td>
</tr>
<tr>
<td>Gray</td>
<td>0.27</td>
<td>0.83</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5: Aggregate Voting Behavior in treatment AV7B.

<table>
<thead>
<tr>
<th></th>
<th>Periods 1-50</th>
<th>Periods 51-100</th>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal if blue</td>
<td>66.95</td>
<td>61.16</td>
<td>50.1</td>
</tr>
<tr>
<td>Double-vote if blue</td>
<td>29.87</td>
<td>37.16</td>
<td>49.9</td>
</tr>
<tr>
<td>Signal if red</td>
<td>74.56</td>
<td>80.52</td>
<td>92.6</td>
</tr>
<tr>
<td>Double-vote if red</td>
<td>20.52</td>
<td>17.98</td>
<td>7.4</td>
</tr>
<tr>
<td>Opposite</td>
<td>2.94</td>
<td>1.56</td>
<td>0</td>
</tr>
<tr>
<td>Gray</td>
<td>0.89</td>
<td>0.06</td>
<td>0</td>
</tr>
</tbody>
</table>

voters face aggregate uncertainty. Note that subjects do not simply play “sincere”. They adjust their behavior to better aggregate information. To the best of our knowledge, our experimental results are the first to identify this pattern in a three-candidate setting.

Turning to approval voting, the only difference between treatments AV7B and PL7B is that subjects can exploit the possibility of double-voting instead of having to vote for or against their signal. In the AV treatment, voters face the more complex challenge of having to deal with a broader choice set but, as identified in Example 4, their task is simplified by the fact that the equilibrium is now unique. In light of Example 4, blue voters should double-vote more often than red voters, and no subject should single-vote against his or her signal. Table 5 shows that the subjects’ behavior was in line with this prediction. Actually, the difference between the blue and red voters is significant not only for the second half of the sample but also for the whole experiment (Mann-Whitney, $z = 2.201, p = 0.028$).

### 6.3 Asymmetric Equilibria with Approval Voting

We saw in Section 6.1.2 that the fraction of subjects who double-vote is lower than predicted by the theory in a symmetric equilibrium. To obtain a better understanding of this gap
between the theory and the observations, we can delve deeper into individual behavior. Figure 4 disaggregates voting behavior at the individual level in the last 50 periods of treatments AV7 (left panel) and AV9 (right panel). The horizontal axis plots the frequency with which each subject voted the signal and the vertical axis plots her frequency of double-voting. Each circle in the graph corresponds to the number of subjects who played that strategy: the larger this number, the bigger the circle.

According to Theorem 2, in a symmetric equilibrium, subjects should adopt the same strategy of mixing between voting their signal and double-voting. If all subjects voted in this way, all the circles should be located at the same point on the negative diagonal between (0,1) and (1,0). While most circles are indeed on this diagonal, we observe that very few subjects are in the vicinity of the orange triangle, which describes the predicted symmetric strategy. Instead, we observe two opposite subject clusters: one that plays the pure strategy of (almost) always double-voting and another one with subjects who (almost) always single-vote their signal. The treatment effect between AV7 and AV9 observed in Section 6.1.2 is mainly driven by a switch in the relative number of subjects in each cluster.

This pattern points at the need to consider asymmetric strategies. Pushing the line of reasoning of McLennan (1998) and Ahn and Oliveros (2012) further, allowing for asymmetric strategies can be interpreted as an extension of the group’s choice set, which may
increase expected welfare. Allowing some voters to specialize in double or single-voting may produce significant advantages. The challenge is to identify potential equilibria by relaxing the assumption of symmetric strategies, ubiquitous as it is in the voting literature.\textsuperscript{33}

In this subsection, we extend our theoretical analysis and no longer impose that voters who receive the same signal play the same strategy. The following proposition proves, for a broad set of parameter values (including the ones used in the experiment), the existence of at least one asymmetric equilibrium, i.e. in which voters play asymmetric strategies. We also characterize this asymmetric equilibrium: voters specialize independently of their signal in either single-voting or double-voting. That is, some voters always single-vote and others always double-vote. If the signal structure is sufficiently unbiased, all “single-voters” vote their signal, i.e. $A$ if signal $s_A$ and $B$ if signal $s_B$. If the bias in the signal structure is stronger, then the voters receiving the less abundant signal vote sincerely whereas those who receive the more abundant signal mix between $A$ and $B$.

\textsuperscript{33}There are noticeable exceptions such as McLennan (1998), and Ladha, Miller and Oppenheimer (2000).
Proposition 4 Suppose that $q(a) = q(b)$, $r(s_A|a) \geq r(s_B|b)$ and $V \leq 2^v$. Any strategy profile satisfying the following conditions is an asymmetric equilibrium:

1. $2n_C - n + 1$ voters always double-vote;

2. The rest of the voters single-vote informatively with $\sigma_{s_B}^{1^v} = 1$ and

$$\sigma_{s_A}^{1^v}(A) = \begin{cases} \frac{\rho^{n-n_C-1}}{\rho^{n-n_C-1} - r(s_A|a) - r(s_B|b)} & \text{if } \rho^{n-n_C-1} > \frac{r(s_B|b)}{r(s_A|a)}, \text{ where } \rho = \frac{r(s_A|a)}{r(s_B|a)} \\ 1 & \text{if } \rho^{n-n_C-1} \leq \frac{r(s_B|b)}{r(s_A|a)} \end{cases}$$

where $\sigma_{s}^{1^v}(\psi)$ is the probability that a single-voter of type $s$ plays action $\psi$.

Proof. See supplementary appendix. □

Such an asymmetric behavior is an equilibrium because the voters who specialize in single-voting perceive the expected payoff of each ballot differently from voters specializing in double-voting. In particular, “single-voters” are pivotal only when $A$, $B$, and $C$ receive exactly the same number of votes, whereas “double-voters” are pivotal if either $A$ is trailing behind by one vote or if it is leading by one vote. The best responses of these two groups of voters are thus different. The following example illustrates this result in more detail.

Example 5 Assume (as in Example 3) $n = 12$, $n_C = 7$, and $q(s_A|a) = q(s_B|b) = 2/3$. In the asymmetric equilibrium, $2n_C - n + 1 = 3$ voters double-vote, and the other 9 single-vote their signal.

Compared with the symmetric equilibrium, the aggregate level of double-voting decreases from 36% to 25%, but this is enough to ensure that the Condorcet loser never wins the election. Indeed, with 3 double-votes and 9 single-votes, one of the two majority alternatives must receive at least 8 votes, i.e. strictly more than the Condorcet loser. Finally, the likelihood of choosing the best candidate increases from 82% in the symmetric equilibrium to 85.5%. Information aggregation is improved because the (expected) number of voters who reveal their information, i.e. the expected number of single-voters, is larger in this asymmetric equilibrium than in the symmetric one (9 vs. 7.68).\(^{34}\)

Such asymmetric equilibria appear to organize laboratory data better than the symmetric equilibrium. In treatment AV7, the predicted level of double-voting in the asymmetric

\(^{34}\)Note that, given the asymptotic efficiency of AV proven in Bouton and Castanheira (2012), the welfare gains of playing the asymmetric equilibrium vanish when $n$ grows large. We thank David Ahn for this point.
equilibrium is 25%, to be compared with the observed 24.46% in the laboratory. This difference is not significant (Wilcoxon, $z = -0.524, p = 0.60$). In the case of AV9, the predicted level of double-voting is 58.33% compared to the observed 51.64%. The difference is still significant (Wilcoxon, $z = 2.201, p = 0.028$), although the gap is much smaller than with the unique symmetric equilibrium.

The equilibrium described in Proposition 4 also makes an interesting prediction for the biased treatment AV7B. In the asymmetric equilibrium, the level of double-voting is independent of the signal structure. This is not what we observe in the data (see Table 5). Although it is beyond the scope of this paper, it might be useful to explore other types of asymmetric equilibria (if any) in this type of setting.

6.4 On Voter Rationality

An important question in the voting literature is whether we can take seriously the hypothesis that voters adopt a “rational” behavior, as opposed to voting “sincerely”. We proceed in three steps to explore this question. First, without distinguishing between sincere and rational behavior, we analyze whether subject behavior is random or follows some minimal criteria of self-interest. Second, we study the groups’ behavior across treatments to confront the sincere voting and strategic voting hypotheses. Third, we assess the fraction of subjects who adopted a rational voting behavior.

To assess random behavior, we first calculate how often each subject cast a ballot that is a dominated strategy according to our theory: less than 2.1% of the subjects chose a dominated action more than 5% of the times. A stricter test is to identify how often each voter cast ballots that were not in their equilibrium best-response set. Concretely, this means not voting sincerely in PL7, and not voting for the color on which the rest of the group coordinated in PL9 (i.e. one action out of three is in each voter’s best-response set). In AV, the best response set contains sincere single-voting and double-voting (i.e. two actions out of seven). Figure 4 illustrates the outcome: it depicts the cumulative distribution of the percentage of times a subject cast a ballot that is not in her best response set. The left (respectively, right) panel describes behavior under a minority of 7 (resp., 9). The dashed and solid curves respectively represent AV and plurality. One can draw two clear conclusions from this figure. First, subject behavior is far from random: more than 70% of the subjects played a non-equilibrium strategy less than 5% of the time. Second, subjects

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35 This percentage is similar across treatments: 2.78%, 1.39%, 1.39%, 1.39%, 1.39% and 4.17% in PL7, PL9, PL7B, AV7, AV9 and AV7B respectively.
make more mistakes under plurality than under AV: the AV curve clearly stands above the former for both minority sizes. This points to the difficulty in coordinating on the right equilibrium in plurality (see Section 6.5).

A problem with these measures is that they cannot discriminate between strategic and sincere behavior. Using the results described in Section 6, we can however clearly reject the hypothesis that all subjects behave sincerely. In contrast, the differences across treatments show that a substantial fraction of the subjects adapt their behavior in the direction predicted by our model. In particular, (1) subject behavior in PL9 implies that 40.67% of the votes (see Table 2) were incompatible with sincere voting. (2) Biased information treatments show that the subjects with the more abundant signal, blue, significantly increased their propensity to vote for red. This compensation mechanism goes against the sincere voting hypothesis, while it exactly matches the predictions of our model. (3) Theorem 2 highlights that, *ceteris paribus*, a switch from plurality to approval voting should produce a clear shift in voting behavior: comparing PL7 with AV7, their propensity to single-vote their signal should drop. Comparing PL9 with AV9, they should stop single-voting oppo-

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36 For instance, if 100% of people single voted their signal in AV9, then 100% of the data would be consistent both with equilibrium and with sincere voting according to either of the above criteria.

37 Importantly, the equilibrium behavior in PL7 *is* to vote sincerely. We can thus *not* use PL7 to discriminate between strategic and sincere behavior. This highlights an important weakness of much of the empirical research on strategic voting, which considers sincere voting as direct evidence against strategic behavior, in setups where the researcher is actually unable to identify the equilibrium selected by the electorate. Kawai and Watanabe (2013) also underline this issue.
site, and double-vote instead. This is exactly what we find in the laboratory: in the last 50 periods, with a minority size of 7, their propensity to single-vote their signal drops from 90.94% to 71.94%. With a minority size of 9, the subjects’ propensity to single-vote opposite drops from 40.67% to 4.97%. These differences are statistically significant (Mann-Whitney, $z = 2.882$, $p < 0.01$ in both cases). (4) One could try to rationalize these differences by adapting the sincere voting hypothesis and assume that some fraction of the voters are “sincere double-voters”.\textsuperscript{38} This modified hypothesis is however rejected by the comparison between AV7 and AV9, which shows that double-voting did increase significantly.

Note still that similar arguments can be made to exclude the reverse hypothesis that all subjects behaved as predicted by our pivotal voter model: in PL9, for instance, 15.28% of the subjects voted sincerely more than 90% of the times. An obvious question is thus to assess which fraction of subjects behaved “rationally” according to the model. Fully addressing this question is clearly beyond the achievable with our experiment, but we can easily obtain back-of-the-envelope lower and upper bounds on this fraction. We first observe that 72.23% of the subjects behaved rationally in PL9, in the sense that they voted for the majority candidate more than 90% of the times in the last 50 periods (63.89% of the subjects voted for the majority alternative 100% of the times). Second, we compare the fraction of subjects who double-voted more than 90% of the times under AV7 and AV9: this fraction was 18.06% in AV7 and 45.84% in AV9. The difference, 27.78%, provides another estimate for the fraction of subjects who are rational in the sense that they adapt voting behavior in the way predicted by theory. In summary, this discussion suggests that the fraction of rational subjects lies between a lower bound of 27.78% and an upper bound of 72.23% across the experimental treatments.\textsuperscript{39} This figure is in line with recent findings on observational data in the literature (Kawai and Watanabe 2013, Spenkuch 2013).

### 6.5 Welfare

Previous multicandidate election setups used in laboratory experiments were based on theories that are inconclusive when it comes to comparing welfare across voting systems (this is the case, for instance, with the theoretical predictions of Myerson and Weber (1993) used in Forsythe et al. (1996)). A valuable feature of our common value setup is that it allows for such direct comparisons: by Theorem 1, in equilibrium, the active voters’ payoff should be strictly higher with AV than with plurality.

\textsuperscript{38}Nuñez (2013) shows that different voting patterns are compatible with sincere voting in approval voting.

\textsuperscript{39}In a substantially different laboratory experiment setup, Esponda and Vespa (2013) find that between 50% and 80% of their subjects behaved non-strategically.
Table 6: Average payoff and theoretical predictions. * In the case of plurality, equilibrium predictions refer to the equilibrium where experimental groups converged to.

We perform welfare comparisons separately for the first and second 50 periods of the experiment. The first half provides a good idea of how each system performs when subjects have not been given much time to solve the coordination problem or fine-tune their (mixed) strategies. The second half instead provides a good idea about how the systems perform when the subjects’ play is closer to the equilibrium. Note that our results hold for many different subsets of periods, e.g. 1-20, 1-30, and 10-40.

Table 6 (in columns 2 and 3) displays the average payment obtained by the subjects in each treatment, respectively for the first and second 50 periods. Comparing PL and AV treatments two by two, one can see that realized payoffs are systematically higher in AV treatments. All these differences are significant at the 1% confidence level. Interestingly, realized payoffs under AV are remarkably close to the payoffs that a benevolent dictator would achieve if informed about all the signals in the group (191.00, 188.90 and 197.60 in the last 50 periods for AV7, AV9 and AV7B, respectively).

These results lead to several conclusions. First, plurality performs relatively poorly both when coordination problems are salient and when they have been resolved. Why AV performs better even in early periods can be due to several reasons. For instance, (i) subjects fine-tune their strategies much faster under AV than they can solve coordination problems under plurality. (ii) AV does not require full fine-tuning to perform better than plurality. (iii) Plurality performs very poorly unless coordination problems have been fully achieved.

Regarding point (i), we observed in Section 6.4 that subjects make fewer “mistakes” in AV than in plurality. This happens despite the fact that voters have to choose from a larger

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40In a different setup, Bassi (2008) finds that, while subjects play in a more sophisticated manner under plurality, the latter rule performs worse than AV and the Borda count in terms of Condorcet efficiency.

41Mann-Whitney tests are: \( z = 2.882 \) and \( p\text{-value} = 0.0039 \) for AV7-PL7, \( z = 2.722 \) and \( p\text{-value} = 0.0065 \) for AV9-PL9, and \( z = 2.913 \) and \( p\text{-value} = 0.0036 \) for AV7B-PL7B.
choice set in AV than in plurality. This suggests that the “individual complexity” of having to choose from a larger choice set is actually much simpler to address than the “strategic complexity” of having to adapt behavior to the equilibrium selected by the other voters.

Regarding point (ii), one should realize that AV produces higher welfare even if subjects do not perfectly fine-tune their strategy to the equilibrium predicted by the theory. To show this, we calculate the predicted level of expected welfare under AV when voters adopt any given mixture between single and double-voting, and compare it with the case in which voters coordinate perfectly on the Duverger’s Law equilibrium under plurality, i.e. the payoff-maximizing equilibrium both in PL7 and PL9. This clearly stacks the deck against AV. The results shown in Figure 6 are striking: with symmetric strategies (remember from the previous section that asymmetric strategies perform even better), the welfare dominance of AV over plurality is robust to large mistakes in the mix between single and double-voting. For a small minority, virtually any positive amount of double-voting ($> 0.174\%$) makes AV welfare-superior to plurality. For a large minority, AV dominates plurality as long as voters double-vote with a probability higher than $45.1\%$. In contrast, in plurality treatments PL7 and PL9, respectively, at least $2/3$ and $83.3\%$ of the subjects must coordinate on a same color to achieve the maximum achievable payoff of 155. This shows that (a) AV does not require perfect fine-tuning to perform better than plurality, and (b) plurality performs poorly unless a very large fraction of the voters have solved the coordination problem of agreeing on which equilibrium to select.

Regarding point (iii), it is interesting to look at the effect of the size of the minority on welfare. In plurality, the expected payoff should be strictly decreasing in $n_C$ in an informative equilibrium. In a Duverger’s Law equilibrium instead, it should remain independent of $n_C$, at 155. Table 6 shows an interesting reversal: in the first 50 periods, the average payoff is higher in treatment PL7 than in treatment PL9, while the opposite is true for the second half. This is explained by the substantial costs of the coordination failures that we observe in the first 50 periods in PL9. In contrast, the expected payoff of the sincere voting equilibrium is as high as 152.8 in PL7. Across the entire experiment session, payoff is lower under PL9 than under PL7 (Mann-Whitney test, $z = 1.922$, $p$-value $= 0.0547$), while the opposite should hold if coordination was perfect.
Figure 6: Expected payoff under AV for different symmetric strategies. The light (dark) gray line represents the expected payoff under AV as a function of the frequency of double voting, when the minority is small (large). The dashed line is a benchmark that represents the expected payoff of the Duverger’s Law equilibrium strategy.

7 Conclusions

In this paper, we studied the properties of plurality and AV both theoretically and experimentally. We considered a case in which the majority is divided between two alternatives as a result of information imperfections, while the minority backs a third alternative, which the majority views as strictly inferior. The majority thus faced two problems: aggregating information and coordinating to defeat the minority candidate.

In plurality, two types of equilibria coexist: Duverger’s Law equilibria, which fulfill the coordination purpose at the expense of information aggregation, and informative equilibria, in which majority voters aggregate information but open the door to a victory of the Condorcet loser. Interestingly, this equilibrium is not “knife edge”. This theoretical finding helps rationalize some empirical regularities in the literature that are oft-considered as supporting evidence for the lack of a “rational-instrumental” voting behavior. In approval voting (AV), the structure of incentives is quite different. In equilibrium, some majority voters should double-vote. This allows for information aggregation and a significant reduction in the threat posed by the Condorcet loser. As a consequence, AV produces strictly higher expected welfare.

We then tested our predictions with laboratory experiments: under plurality, we ob-
served the emergence of both informative (when minority size was small) and Duverger's Law equilibria (when minority size was large). Under AV, double-voting significantly increased welfare: the subjects’ behavior allowed them to elect the full information Condorcet winner with a probability very close to what a Social Planner would have achieved after observing all available signals. Such behavior is statistically different from “sincere voting” and consistent with most theoretical predictions. However, in contrast with our theoretical prior, we also found that subjects used asymmetric strategies. This led us to extend the theoretical analysis in that direction.

We believe that this paper opens up many novel theoretical and experimental questions about multicandidate elections: how would other voting rules perform in such a setup? How would different voting rules perform when majority voters have a mix of private and common values? What is the importance of asymmetric equilibria both for these comparisons and for the results in the literature? Last but not least, how can we further use multicandidate setups in order to better understand voting behavior and voters’ strategic interactions?

References


Appendices

Appendix A1: Plurality

A ballot, say in favour of $A$ can only be pivotal if the number of other $A$-ballots ($x_A$) is either the same as or one less than the number of $C$-ballots ($n_C$). To assess the probability of such an event, a voter must identify the distribution of the other $n-1$ votes, given their strategy $\sigma$. Dropping $\sigma$ from the notation for the sake of readability, the pivot probabilities in favour of $A$ and $B$ are:

$$p_{AC}^\omega = \Pr(piv_{AC}|\omega, \text{Plurality}) = \frac{(n-1)!}{2} \left(\frac{x_A^{n-1}x_C^{-1}}{(n-C-1)!(n-n-C-1)!}\right) \left[\frac{\tau_A^n}{n_C} + \frac{\tau_B^n}{n - n_C}\right],$$

$$p_{BC}^\omega = \Pr(piv_{BC}|\omega, \text{Plurality}) = \frac{(n-1)!}{2} \left(\frac{x_A^{n-1}x_C^{-1}}{(n-C-1)!(n-n-C-1)!}\right) \left[\frac{\tau_B^n}{n_C} + \frac{\tau_A^n}{n - n_C}\right],$$

where the two terms between brackets represent the cases in which one vote respectively breaks and makes a tie. Note that pivot probabilities are continuous in $\tau_A^\omega$ and $\tau_B^\omega$.

The expected gains of action $A$ and $B$ over abstention are:

$$G(A|s) = q(a|s) p_{AC}^a + q(b|s) p_{AC}^b \quad (>0),$$

$$G(B|s) = q(a|s) p_{BC}^a + q(b|s) p_{BC}^b \quad (>0).$$

Since both actions yield higher payoffs than abstention, the latter is dominated.

Proof of Proposition 1

Consider e.g. $\sigma_{sA}(A) = \varepsilon$ and $\sigma_{sB}(B) = 1$. From (7) and (8), we have:

$$\frac{p_{AC}^a}{p_{BC}^a} = \left(\frac{\tau_A^\omega}{\tau_B^\omega}\right)^{2n_C-n} \frac{\tau_A^n}{\tau_A^n n_C + \tau_B^n (n - n_C)} \rightarrow 0.$$

Hence, from (3), we have that $G(A|s) - G(B|s) < 0$ for any $\varepsilon$ in the neighborhood of 0.

Proof of Proposition 2. We start with the unbiased case, i.e. $r(s_A|a) = r(s_B|b)$. Under sincere voting, $\sigma_{sA}(A) = 1 = \sigma_{sB}(B)$, (7) and (8) imply $p_{AC}^a = p_{BC}^b > p_{AC}^b = p_{BC}^b$. Then, from (3):

$$G(A|s) - G(B|s) = [Vp_{AC}^a - vp_{BC}^a] [q(a|s) - q(b|s)].$$

Hence $G(A|s) - G(B|s) > 0 > G(A|s_B) - G(B|s_B)$, since $q(a|s_A) - q(b|s_A) > 0 > q(a|s_B) - q(b|s_B).$ Sincere voting is thus an equilibrium strategy. By the continuity of pivot probabilities with respect to $\tau_A^\omega$ and $\tau_B^\omega$, it immediately follows that there must exist a value $\delta(n,n_C) > 0$ such that sincere voting is an equilibrium for any $|r(s_A|a) - r(s_B|b)| < \delta(n,n_c)$.

Proof of Proposition 3. Consider a distribution of signals such that $r(s_A|a) - r(s_B|b) > \delta(n,n_c)$, in which case sincere voting is not an equilibrium. That is, there exists a signal $\bar{s} \in \{s_A, s_B\}$ such that all the voters who received signal $\bar{s}$ strictly prefer to deviate from a strategy profile $\sigma_{sincere} = \{\sigma_{sA}(A), \sigma_{sB}(B)\} = \{1,1\}$. 

39
The pivotal event

Appendix A2: Pivot Probabilities and Correcting Terms in AV

Case 1: \( \tilde{s} = s_A \). In this case, \( i \sigma^\text{sincere} \Rightarrow G (A|s_A) - G (B|s_A) < 0 \). Now, consider a second strategy profile \( \sigma' \equiv \{ [r (s_A|a) + r (s_A|b)]^{-1}, 1 \} \). With this profile, we have: \( \tau^A_B = \tau^B_A \) and \( \tau^A_A = \tau^B_B \), and thus \( p^b_{BC} = p^a_{AC} > 0 \) and \( p^b_{AC} = p^a_{BC} > 0 \) and, from (3):

\[
G (A|s) - G (B|s) = [V p^a_{AC} - v p^a_{BC}] - [q (a|s) - q (b|s)] ,
\]

(11)

where (i) \([V p^a_{AC} - v p^a_{BC}]\) is positive, and (ii) \([q (a|s) - q (b|s)]\) is positive for \( s_A \) and negative for \( s_B \). In other words, all voters would strictly prefer to deviate from \( \sigma' \) by voting sincerely. This means that the value of \( G (A|s_A) - G (B|s_A) \) changes sign when \( \sigma_A (A) \) is increased from \([r (s_A|a) + r (s_A|b)]^{-1}\) to 1. Since all pivot probabilities are continuous in \( \sigma_A (A) \), there must exist a value \( \sigma^*_A (A) \) such that voters with signal \( s_A \) are indifferent between playing \( A \) and \( B \). It is easy to check that, for this strategy, a voter who received signal \( s_B \) strictly prefers to play \( B \), and hence that \( \{ \sigma_A (A), \sigma_B (B) \} = \{ \sigma^*_A (A), 1 \} \) is an equilibrium.

Case 2: \( \tilde{s} = s_B \). In this case, \( i \sigma^\text{sincere} \Rightarrow G (A|s) - G (B|s) > 0 \) for both signals. Now, consider another strategy profile \( \sigma'' = \{ \varepsilon, 1 \} \), with \( \varepsilon \to 0 \) (and hence \( \sigma_A (B) \to 1 \)). From Proposition 1, this strategy profile implies \( G (A|s) - G (B|s) < 0 \) for both signals. By the continuity of the payoffs with respect to \( \sigma_A (A) \), there must therefore exist a value \( \sigma^*_A (A) \) such that \( G (A|s_A) - G (B|s_A) = 0 \) and, by the same argument as above, \( G (A|s_B) - G (B|s_B) < 0 \). Hence, the strategy profile \( \{ \sigma_A (A), \sigma_B (B) \} = \{ \sigma^*_A (A), 1 \} \) is an equilibrium.

Note that sincere stability is not a binding restriction, since all voters vote for their preferred alternative with a probability strictly larger than 0. ■

Appendix A2: Pivot Probabilities and Correcting Terms in AV

The pivotal event \( piv^{AV} \) is defined as follows:

\[
\begin{align*}
&x_A > x_B - 1 \text{ and } x_A + x_{AB} \in \{ n_C - 1, n_C \}, \text{ or} \\
&x_A = x_B - 1 \text{ and } x_B + x_{AB} = n_C .
\end{align*}
\]

With the multinomial distribution: \( \text{Pr} (x|\omega) = n! \prod_{\psi \in \Psi_{AV}} ^{\psi} \frac{x_{\psi} (x_{\psi})^n}{x^n} \). Therefore, the probability of event \( piv_{AC} \) in state \( \omega \) under AV is:

\[
\pi^\omega_{AC} \equiv \text{Pr} (piv^{AV}_{AC}|\omega) = (n-1)! \sum_{i=0}^{2(n_C-i)-n} \sum_{x_{AB}=0}^{(\tau^A_{\omega})^{n_C-i-x_{AB}}(\tau^B_{\omega})^{x_{AB}}(\tau^B_{\omega})^{(n-1)-(n_C-i)}} \frac{(\tau^A_B)^{n_C-i-x_{AB}}(\tau^B_A)^{x_{AB}}(\tau^B_A)^{(n-1)-(n_C-i)}}{2(n_C-i-x_{AB})! x_{AB}!(n-1-n_C+i)!} + \frac{(n-1)!}{3} \frac{(\tau^A_{\omega})^{n_C-i} (\tau^B_{\omega})^{2n_C+i-1}}{[2(n_C-n)]!} + \frac{(n-1)!}{6} \frac{(\tau^A_{\omega})^{n_C-i} (\tau^B_{\omega})^{2n_C-n}}{[n-C-i]! (n-1-n_C)!}. 
\]

40
The value of a double ballot follows almost immediately from $G$ and $B$.

If there exists a signal $\alpha$ such that

\[ B \] trails behind $A$ and $C$ in three other cases: when $B$ trails behind $A$ or $C$ by one vote, when $A$ trails behind $B$ and $C$ by one vote, and when $A$, $B$, and $C$ have the same number of votes. We can express these overestimations in that and three other cases: when $A$ trails behind $B$ and $C$ by one vote, when $B$ trails behind $A$ and $C$ by one vote, and when $A$, $B$, and $C$ have the same number of votes. We directly see that $\tau_{AB} = 0$ when $\tau_{AB} \in \{0, 1\}$, or $\tau_{AB} = 0$, or $\tau_{AB} = 0$. These correcting terms become vanishingly small and can be omitted when the population size increases towards infinity.

Appendix A3: Approval Voting, Equilibrium Analysis

Lemma 4 If there exists a signal $s$ such that

\[ G^A (A|s_A) - G^A (AB|s_B) = 0 \text{ then } G^A (A|s_A) - G^A (AB|s_A) > G^A (A|s_B) - G^A (AB|s_B) \] (15)

\[ G^A (B|s_B) - G^A (AB|s_B) = 0 \text{ then } G^A (B|s_B) - G^A (AB|s_B) > G^A (B|s_A) - G^A (AB|s_A) \] , and

\[ G^A (A|s_A) - G^A (B|s_B) = 0 \text{ then } G^A (A|s_A) - G^A (B|s_A) > G^A (A|s_B) - G^A (B|s_B) . \]

Proof available upon request.
**Proof.** We detail the proof for (15). It is similar for the other two implications. Remember that the second term in (5) is necessarily negative. Thus $G^{AV} (A|s) - G^{AV} (AB|s) = 0$ implies that the first term must be strictly positive. It follows immediately that:

$$G^{AV} (A|s) - G^{AV} (AB|s) \geq 0 \text{ iff } \frac{q(a|s)}{q(b|s)} \geq \frac{\pi^b_{AB} (V - v) + \pi^b_{BC} V - \phi^b}{\pi^a_{AB} (V - v) - \pi^a_{BC} v + \phi^a}.$$ 

Thus, (15) follows from $\frac{q(a|sA)}{q(b|sA)} > \frac{q(a|sB)}{q(b|sB)}$. □

**Lemma 5** In any voting equilibrium under AV, neither $A$ nor $B$ can be approved by all voters.

**Proof.** We prove the proposition by contradiction and for the limit case in which $\varepsilon = 0$. By definition the results hold when $\varepsilon > 0$.

Policy $A$ is approved by all voters if and only if $\sigma_{sA} (A) + \sigma_{sA} (AB) = 1 = \sigma_{sB} (A) + \sigma_{sB} (AB)$. In this case, we have: $x_A + x_{AB} = n$ and hence $\pi_{AC}^a = 0 = \pi_{BC}^a$ and $\phi^w = 0$. The only possible pivot events are when $x_{AB} = n - 1$ or $n - 2$. Hence:

$$G (A|s) - G (AB|s) = [q(a|s) \pi^a_{AB} - q(b|s) \pi^b_{AB}] (V - v) \geq 0$$

$$G (B|s) - G (AB|s) = [q(b|s) \pi^b_{BA} - q(a|s) \pi^a_{BA}] (V - v) \geq 0.$$ 

with: $\pi_{AB}^w = \frac{(\tau_{AB}^w)^{n-1}}{2}$, and $\pi_{BA}^w = \frac{(\tau_{BA}^w)^{n-1}}{2} [(n - 1) + (2 - n) \tau_{AB}^w]$. Therefore,

$$\frac{\pi_{BA}^b}{\pi_{BA}^a} = \left( \frac{\tau_{BA}^b}{\tau_{BA}^a} \right) \frac{n-2}{(n-1) + (2 - n) \tau_{AB}^b},$$

$$\frac{\pi_{AB}^a}{\pi_{AB}^b} = \left( \frac{\tau_{AB}^a}{\tau_{AB}^b} \right)^{n-1}.$$ (16)

Now, we show that $\pi_{BA}^b$ is increasing in $\tau_{AB}^b$ (from (17), it is straightforward that $\frac{\pi_{BA}^b}{\pi_{BA}^a}$ is also increasing in $\tau_{AB}^b$). Taking logs, we have that the right-hand side of (16) is

$$(n - 2) \left[ \log \tau_{AB}^b - \log \tau_{AB}^a \right] + \log [(n - 1) + (2 - n) \tau_{AB}^b] - \log [(n - 1) + (2 - n) \tau_{AB}^a]$$

Differentiating with respect to $\tau_{AB}^b$ yields:

$$\frac{n - 2}{\tau_{AB}^b} - \frac{n - 2}{(n - 1) + (2 - n) \tau_{AB}^b}.$$ 

This is non-negative if and only if $\tau_{AB}^b \leq 1$. Therefore, we have that $\pi_{AB}^b > \pi_{AB}^a$ and $\pi_{BA}^b > \pi_{BA}^a$ when $\tau_{AB}^b > \tau_{AB}^a$, and conversely.

We now use this result to prove that $A$ cannot be approved by all voters. From Theorem 1, Lemma 3, and Lemma 4 (in this Appendix), there are 2 cases to check: (i) $\sigma_{sA} (A) = 1$ and $\sigma_{sB} (A) \in [0, 1)$, and (ii) $\sigma_{sB} (A) = 0$ and $\sigma_{sA} (A) \in (0, 1]$. If $\sigma_{sA} (A) = 1$ and $\sigma_{sB} (A) \in [0, 1)$, then $\tau_{AB}^b > \tau_{AB}^a$. Hence, we have that $\pi_{BA}^b > \pi_{BA}^a$, which implies $G (B|s_B) - G (AB|s_B) > 0$. Thus,
there cannot be any equilibrium in which $\sigma_{s_{A}}(A) = 1$ and $\sigma_{s_{B}}(A) \in [0,1)$. If $\sigma_{s_{A}}(A) \in (0,1]$ and $\sigma_{s_{B}}(A) = 0$, then either $\tau_{AB}^{a} > \tau_{AB}^{b}$ or $\tau_{AB}^{a} < \tau_{AB}^{b}$. If $\tau_{AB}^{a} > \tau_{AB}^{b}$, then $\pi_{AB}^{a} > \pi_{AB}^{b}$, and thus $G(AB|s_{A}) - G(AB|s_{B}) > 0$. If $\tau_{AB}^{a} < \tau_{AB}^{b}$, then $\pi_{AB}^{b} > \pi_{AB}^{a}$, and thus $G(B|s_{B}) - G(AB|s_{B}) > 0$. Therefore, there cannot be any equilibrium in which $\sigma_{s_{B}}(A) = 0$ and $\sigma_{s_{A}}(A) \in (0,1]$. ■

Proof of Theorem 1. From McLennan (1998), a strategy that maximizes expected utility must be an equilibrium of such a common value game (and any finite Bayesian game like ours must have an equilibrium). Now, conjecture some strategy profile $\sigma$ that can be played under plurality. That is, $\sigma_{s}(AB) = 0$ for $s = s_{A}, s_{B}$. In this case, $\pi_{AB}^{a} = \pi_{BA}^{a} = 0$, whereas $\pi_{BA}^{b} = \pi_{AB}^{b} = \phi^{5} = 0 < \pi_{AC}^{a}, \pi_{BC}^{a}, \omega = a, b$. Therefore, $G_{AV}(A|s) - G_{AV}(AB|s) < 0$ and $G_{AV}(B|s) - G_{AV}(AB|s) < 0, \forall s$. This means that $\tau_{AB}^{a} = 0$ cannot be part of an equilibrium under AV, and that the welfare-maximizing equilibrium under AV must produce strictly higher expected utility than plurality.

It remains to show that this equilibrium is sincerely stable. We actually show the stronger statement that, to maximize expected welfare, a strategy must satisfy $\sigma_{s_{A}}(A), \sigma_{s_{B}}(B) > 0$. We show this by contradiction: suppose that $\hat{\sigma}$ maximizes expected welfare and is such that $\hat{\sigma}_{s_{A}}(A) = 0$. By Lemma 5, we have $\tau_{AB}^{\hat{a}}, \tau_{AB}^{\hat{b}}, \tau_{AB}^{a} > 0$ and hence $\hat{\sigma}_{s_{B}}(A) > 0$. Then, compare $\hat{\sigma}$ with some other strategy $\hat{\sigma}'$ in which $s_{A}$-voters change some of their votes from $B$ towards $AB$, whereas $s_{B}$-voters adapt their voting strategy so as to maintain all vote shares unchanged in state $b$.43

As a result, the total vote share of $A$ in state $a$ must increase (i.e., $\tau_{A}^{\hat{a}}(\hat{\sigma}') + \tau_{AB}^{b}(\hat{\sigma}') > \tau_{A}^{\hat{a}}(\hat{\sigma}) + \tau_{AB}^{a}(\hat{\sigma})$), whereas the expected fraction of double-votes increases (the total vote share of $B$ remains unchanged). As a result, in state $a$, the probability that $A$ wins must increase, whereas the probability that $C$ wins decreases weakly. In state $b$, winning probabilities are unchanged. Hence, $\hat{\sigma}$ cannot maximize expected welfare: a contradiction. ■

Proof of Theorem 2. We prove the Theorem in two steps. First, we show that there is no interior equilibrium in which a strategy strictly mixes across the three actions $A$, $B$, and $AB$. Second, we show that $s_{A}$-voters never play $B$, nor $s_{B}$-voters play $A$ in an interior equilibrium. It follows that the only possible interior equilibrium is such that voters with signal $s_{A}$ mix between $A$ and $AB$, and voters with signal $s_{B}$ mix between $B$ and $AB$.

First, conjecture an equilibrium in which $\sigma_{s_{A}}(A), \sigma_{s_{A}}(B), \sigma_{s_{A}}(AB) > 0$. This requires $G(A|s_{A}) = G(B|s_{A}) = G(AB|s_{A})$. In this case, by Lemma 4 (in this Appendix), $s_{B}$-voters must be playing $B$ with probability 1, i.e. $\sigma_{s_{B}}(B) = 1$. The equilibrium is therefore not interior, a contradiction. Similarly, $s_{A}$-voters must play $A$ with probability 1 if $s_{B}$-voters strictly mix between $A$, $B$, and $AB$.

Second, imagine that $s_{B}$-voters play $A$ with strictly positive probability in equilibrium: $\sigma_{s_{B}}(A) \in (0,1)$. This requires either (i) $G(A|s_{B}) = G(AB|s_{B}) \geq G(B|s_{B})$ or (ii) $G(A|s_{B}) = G(B|s_{B}) \geq G(AB|s_{B})$. By Lemma 4, both (i) and (ii) imply that $G(A|s_{A}) > G(AB|s_{A}), G(B|s_{A})$, and hence that $A$’s strategy cannot be interior. By symmetry, $\sigma_{s_{A}}(B) \in (0,1)$ cannot be part of an interior equilibrium either. ■

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43If $\hat{\sigma}_{s_{A}}(AB) = 1$, then one must consider a transfer of $s_{A}$-votes from $AB$ towards $A$, and $s_{B}$-voters adapt their strategy to maintain all $\tau_{s_{B}}^{a}$ unchanged.