AFFIRMATIVE ACTION AND STEREOTYPES IN HIGHER EDUCATION ADMISSIONS*

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This version: July 21, 2014

Abstract

We analyze the use of admission policy to affect stereotypes against students from disadvantaged groups. Many critics of affirmative action in admissions argue that lower standards result in stereotypes (statistical discrimination). We show that when stereotypes are a result of social inequality, they can easily persist under group-blind admissions. Perversely, eliminating stereotypes requires a higher admission standard for disadvantaged students. Even if schools seek both to treat students equally and counteract stereotypes, the optimal admission policy would still impose a higher standard on disadvantaged students. It requires a third goal, such as equal representation, to justify group-blind admissions. Even in this case, group-blind admissions are optimal only when the conflicting goals of equal representation and limiting stereotypes exactly balance. This is an implausible justification for group-blind admission because it implies that some schools desire higher standards for disadvantaged students. If a school values all three of these goals, some amount of affirmative action will be optimal.

*Preliminary and incomplete. Please do not cite without the authors’ permission. We thank Omri Ben-Shahar, Richard Brooks, Bob Cooter, Dhammika Dharmapala, David Gamage, Mark Gergen, Bert Huang, Will Hubbard, Justin McCrary, David Oppenheimer, Vicki Plaut, Ariel Porat, Eric Posner, Kevin Quinn, Russell Robinson, Arden Rowell, Talha Syed, Eric Talley, David Weisbach, and Glenn Weyl for helpful comments.
“When blacks take positions in the highest places of government, industry, or academia, it is an open question today whether their skin color played a part in their advancement. The question itself is the stigma—because either racial discrimination did play a role, in which case the person may be deemed ‘otherwise unqualified,’ or it did not, in which case asking the question itself unfairly marks those who would succeed without discrimination.”

Justice Clarence Thomas

“I hear the stigma argument all the time. ‘But affirmative action causes stigma.’ Well, yes, affirmative action causes stigma. That’s one of the costs of affirmative action, I acknowledge it. It’s a cost worth paying.”

Christopher Edley

Most critics of affirmative action argue that it is discriminatory. Many also argue that affirmative action harms its intended beneficiaries. As Justice Clarence Thomas states in his concurring opinion in Fisher v. Texas, “There can be no doubt that the University’s discrimination injures white and Asian applicants who are denied admission because of their race. But I believe the injury to those admitted under the University’s discriminatory admissions program is even more harmful.” For Justice Thomas, affirmative action is harmful because it results in the stereotype that its beneficiaries are less qualified or able than others.¹

In this paper, we analyze the use of admission policy to affect stereotypes or statistical discrimination against students from disadvantaged groups. Contrary to what some critics of affirmative action suggest, a concern for the effects of such stereotypes does not imply that a school should adopt group-blind admissions. We show that when stereotypes are a result of social disadvantage, they can easily persist even if schools adopt group-blind admissions. Eliminating these stereotypes requires a school to adopt higher admissions standards for students from disadvantaged groups. Such a perverse double standard is clearly unacceptable. The appropriate question to ask is how much weight should be accorded to combating stereotypes relative to other goals of admissions.

We present a simple model to illustrate the relationship between admission policies and stereotypes. A school chooses to admit students from an advantaged and a disadvantaged group on the basis of an academic score. The disadvantaged group has worse scores, in a

¹In response to this view, a large literature in social science attempts to measure whether affirmative action programs have an effect on the perceived competence of beneficiaries (Heilman et al (1992 and 1997), Nye (1998), Evans (2003)). A related literature examines whether the presence of stereotypes affects student performance (Steele (1997), Steele and Aronson (1998), Fischer and Massey (2006)).
statistical sense, than the advantaged group.\(^2\) We define a negative stereotype as the presence of statistical discrimination (Arrow (1972), Phelps (1972), Coate and Loury (1993)) against the disadvantaged group.

We first show that negative stereotypes can persist even under group-blind admissions. For example, if all students above a given score admitted, few disadvantaged students will have very high scores and many will have scores that are just above the admissions cutoff. As a result, admitted students from the disadvantaged group may still have a lower average score than other admitted students (Bowen and Bok (2000)). We provide sufficient conditions under which a negative stereotype will exist under any group-blind admission policy. To illustrate their plausibility, we show that they hold for the ACT scores of high school students who self identify as “black” and “white.”\(^3\) Therefore, racial stereotypes would continue to exist under any race-blind admissions policies based on the ACT or similar tests. If, as Justice Thomas suggests, admitted students from a disadvantaged group are viewed as less qualified under affirmative action, then this is likely to continue even in its absence.

Perversely, eliminating a negative stereotype requires actively discriminating against disadvantaged students. The admissions policy must make it more difficult for disadvantaged students to gain admission than for advantaged students. As a result, the percentage of admitted students who are disadvantaged will be even lower than the percentage under group-blind admissions. The greater the inequality between the two groups, the greater the admissions penalty that must be imposed on disadvantaged students to eliminate any stereotype. Using ACT data we show that eliminating the racial stereotype through admissions would require a substantial penalty on black students and greatly reduce their representation. An admission policy that would eliminate stereotypes is, admittedly, extreme. We ask how considering other goals in admissions changes this extreme result.

We show that if a school is willing to consider tradeoffs between equal treatment and combating stereotypes, it still optimal to set a higher admissions standard for disadvantaged students. We define equal treatment as the desire to treat all applicants the same, regardless of group status.\(^4\) We then consider the choice of an admission policy by a school with preferences over both equal treatment and limiting stereotypes. When both these ends are valued with diminishing margins, it is never optimal to choose a group-blind admissions

\(^2\)This definition reflects the view that deeper social forces that give rise to the inequality between the two groups. A disadvantaged group will have fewer opportunities to acquire many of the characteristics valued by the school in admissions.

\(^3\)The ACT does not entirely determine any school’s admission policy, but tests like the SAT and ACT do play a substantial role in admissions for many selective colleges (Krueger, Rothstein, and Turner (2006)).

\(^4\)This procedural definition of equality is at odds with other, substantive definitions of equality. In our view, this formal or procedural definition closely tracks the way equality is described by critics of affirmative action.
In fact, it requires a third goal, such as equal representation, to justify group blind admissions. We define equal representation as the desire for the demographics of admitted students to reflect those of the broader population. We then consider the choice of an admission policy by a school with preferences over equal treatment, combating stereotypes, and equal representation. We show that group-blind admissions are optimal only when concerns over reducing stereotypes and equal representation exactly balance. In our view, this knife edge case is an implausible justification for group-blind admission. If some schools are in perfect balance, this implies that other schools should actually desire higher standards for disadvantaged students. No school, however, would support such a discriminatory double standard. Therefore, some amount of affirmative action is optimal if a school values all three of these goals with diminishing margins.

We conclude that harm from stereotypes contributes little to the case for group-blind admissions. Many critics of affirmative action advocate group-blind admissions in all circumstances. This view is best described by a lexicographic preference for equal treatment. Critics of affirmative action undoubtedly care about equal treatment, and many suggest that negative stereotypes are harmful to affirmative action recipients. However, if only equal treatment and combating stereotypes are of concern, then schools should actively discriminate against disadvantaged groups, and no one advocates this position. We show that a desire for equal representation (or some related value) must exactly balance the desire to combat stereotypes if group-blind admissions are optimal. Yet few critics of affirmative action concede that equal representation is an appropriate goal for admission policy. In his concurring opinion in *Fisher v. Texas*, Justice Thomas states that “for constitutional purposes” it does not matter whether the effects of affirmative action are “insidious” or “benign.” In our view, this statement by Justice Thomas also captures the case for group-blind admissions outside the realm of constitution interpretation.

The effect of admission standards on group stereotypes is one of many concerns in formulating an admission policy. Affirmative action will not limit such stereotypes—in the way narrow we define them—to the same extent group-blind admissions. As Christopher Edley suggests, stereotypes may be cost of affirmative action. However, when stereotypes result from disadvantage, they are a cost of any admission policy. The issue for admission policy is how much should it seek to affect stereotypes relative to pursuing other goals.

This paper contributes to the literature analyzing affirmative action and statistical discrimination (Fang and Moro (2011)). It is closely related to Loury and Coate (1993), who

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5 Throughout, we make the distinction between (i) a preference for equal treatment in admissions that may exist alongside other preferences, and (ii) the choice of a group-blind admission policy. We try to be clear in the text when we are referring to a preference or a choice.
explain how racial stereotypes can be self-fulfilling as a result of the strategic interaction between workers and firms. Unlike Loury and Coate (1993), we focus on the decision problem of the school and ignore the effort choice of students so as to consider different values in admissions. It is also related to Eyster and Chan (2003), who analyze optimal admissions policies when schools value racial diversity and student quality, but are constrained from explicitly using race. Other papers analyzing optimal admissions under similar constraints include Fryer, Loury, and Yuret (2008), Eyster and Chan (2009), and Ray and Sethi (2010).

Section I describes the relationship between admissions policies and stereotypes. Section II describes admissions policies when schools also value equal treatment and equal representation. Section III discusses how our definition of stereotype is related to other common uses of stereotype and stigma.

I Admission Policy and Stereotypes

A A Simple Model of Admissions

Consider a single school that accepts students under competitive admission. The admission pool consists of advantaged (A) and disadvantaged (D) groups of applicants, where $\theta_A$ and $\theta_D$ represent the fraction of each group in the pool $(\theta_A + \theta_D = 1)$. The scores for each group are random variables $s_A$ and $s_D$ distributed on $[\underline{s}, \overline{s}]$ according to $F_A(s)$ and $F_D(s)$ with associated probability density functions $f_A(s)$ and $f_D(s)$. We first consider admission policies that take the form of a cutoff score ($c_A$ and $c_D$) for each group so that students with scores above the cutoff gain admission. We then consider more general admission policies.

The distribution of scores for $D$ is worse, in a statistical sense, than the distribution for $A$. In particular, we assume that the likelihood ratio $\frac{f_D(s)}{f_A(s)}$ is weakly decreasing in $s$. This property implies that the distribution $F_A(s)$ first order stochastically dominates the distribution $F_D(s)$ ($F_A(s) \leq F_D(s) \forall s$). Under first order stochastic dominance, any rational decision maker who prefers higher scores would choose a student from $A$ over $D$ if they knew only the student’s group. It also implies that the mean score for $D$ is lower than $A$ ($\mu_D < \mu_A$). This property motivates our definition of a negative stereotype.

Definition 1 A group $D$ experiences a negative stereotype in relation to $A$ if $F_A(s) \leq F_D(s) \forall s$.

Group-blind admissions will not eliminate the negative stereotype experienced by $D$. Suppose that the school wishes to accept students with the highest possible test scores subject to the constraint that it can only accept a fixed proportion ($K$) of applicants. The admissions problem can be formulated as:
max \left[ \frac{1}{K} \cdot [\theta_D \int_{c_D}^{c} s f_D(s) ds + \theta_A \int_{c_A}^{c} s f_A(s) ds] \right] \text{ subject to } \\
\theta_D \cdot (1 - F_D(c_D)) + \theta_A \cdot (1 - F_A(c_A)) = K

The optimal admission policy that solves this problem is group blind. The solution takes the form of a common cutoff score \( c_D = c_A = c^* \) where \( c^* \) is determined by:

\[ \theta_D \cdot (1 - F_D(c^*)) + \theta_A \cdot (1 - F_A(c^*)) = K \]

This policy does not eliminate the negative stereotype against admitted students from D. The likelihood ratio among admitted students is:

\[ \frac{f_D(s)}{f_A(s)} \frac{1 - F_A(c^*)}{1 - F_D(c^*)} = \frac{f_D(s)}{f_A(s)} \frac{1 - F_A(c^*)}{1 - F_D(c^*)} \]

By assumption, this ratio is weakly decreasing in \( s \). Therefore, the distribution of scores for admitted students from \( A \) continues to first-order stochastically dominate the distribution for admitted students from \( D \). The mean score of admitted students from \( A \) is also higher than the mean score of admitted students from \( D \).

In fact, admitted students from \( D \) will experience a negative stereotype under any group-blind admission policy. Suppose that an admissions policy consists of (i) a list of student characteristics, (ii) a score for each student based on these characteristics, and (iii) a probability of admission for each student based on this score. Such a policy is group blind if the student characteristics, score, and probability of admission do not vary by group.

**Definition 2** An admission policy is a vector \((X, s(x), q(s))\) where: (i) \( X \) denotes a random vector of student characteristics, (ii) \( s(x) : X \to \mathbb{R} \) assigns a score to each student, and (iii) \( q(s) : \mathbb{R} \to [0,1] \) assigns a probability of admission for each score.

**Definition 3** An admission policy is group blind with respect to \( D \) and \( A \) if \( (X_D, s_D(x), q_D(s)) = (X_A, s_A(x), q_A(s)) \)

It follows that group-blind admission policies will not eliminate negative stereotypes.

**Proposition 1** If the likelihood ratio \( \frac{f_D(s)}{f_A(s)} \) is weakly decreasing in \( s \) then, under any group-blind admission policy, admitted students from \( D \) experience a negative stereotype relative to admitted students from \( A \).
Proof. The likelihood ratio among admitted students is given by:

$$\frac{q(s) \cdot f_D(s)}{\int q(u) \cdot f_D(u) du} = \frac{\int q(u) \cdot f_A(u) du}{\int q(u) \cdot f_D(u) du} \cdot \frac{f_D(s)}{f_A(s)}$$

which is strictly decreasing in $s$. ■

We conclude that group-blind admission policies will not necessarily eliminate negative stereotypes. If negative stereotypes are a cost of affirmative action, they are also likely to be a cost of group-blind admissions.

A.1 An ACT Illustration

We illustrate the persistence of negative stereotypes under group-blind admission using data on ACT scores. We limit our data to students who self-identify as black or white on the ACT. We choose this example because the use of race in higher-education admissions is the most debated issue in affirmative action in the United States, and because tests like the ACT are an important, though by no means exclusive, component of admissions (Krueger, Rothstein, and Turner (2006)). We take no position on whether, as an empirical matter, (i) such stereotypes are widely held or acted upon, or (ii) admitted students are harmed by it. Our aim is to demonstrate the persistence of a racial stereotype, as we have defined it, under a simple model of race-blind admissions.

The unconditional distributions of scores for both groups is depicted below.  

![ACT Distribution by Race](image)

In our data, black students comprise 17% of test takers and white students comprise the remaining 83%. White students have a mean score of 21.8, and black students have a mean score of 17.1.
score of 17.1, implying a difference in means of 4.7. In terms of percentiles the mean black score is at the 18th percentile of the white distribution, while the mean white score is at the 92nd percentile of the black distribution. The two distributions satisfy the monotone likelihood property. Therefore, the assumptions under which a negative stereotype exists under any race-blind admission policy are satisfied for this data. They are likely to hold in other cases where affirmative action policies are used.

We consider a race-blind admission policy in which the school admits the top 60% of students on the ACT. This implies admitting all students with ACT scores greater than or equal to 20. This policy maximizes the expected ACT score of admittees subject to constraint of admitting 60% of applicants. Under this policy, admitted white students have a higher average ACT score than black students. This difference in means is 1.9, which is about 40% of the pre-existing difference (4.7). In terms of percentiles, the mean score for admitted black students is at the 37th percentile of the distribution for admitted white students. The mean for admitted white students is at the 88th percentile for admitted black students. A race-blind policy substantially reduces the percentage of black admitted students relative to the population of test takers. Black students would make up only 5.3% of admitted students, while white students would comprise the remaining 94.6%.
Black students would continue to suffer from a negative stereotype, even though admissions are race blind. The distribution of ACT scores for white admitted students first order stochastically dominates the distribution for black students. Visually this can be seen from that fact that the likelihood ratio ($D/A$) among admitted students is decreasing. Affirmative action may policies may leave in place negative stereotypes, but ending affirmative action would not eliminate these stereotypes. Critics of affirmative action who emphasize the harm of such stereotypes should be also be concerned with their effects under group-blind admissions.

B Eliminating Negative Stereotypes

If negative stereotypes are harmful to disadvantaged students, what would it take to eliminate them? We show that eliminating such stereotypes would require making admission standards higher for disadvantaged students than for advantaged students. This, in turn, would reduce the representation of disadavantaged students among admits to an even greater extent than a group-blind policy.

B.1 Equalizing Mean Scores

Consider again a simple admission policy that takes the form of a cutoff score and maximizes the expected score of admitted students. A policy that combats stereotype by equalizing the mean score of admitted students from $D$ and $A$ solves:
The optimal policy \((c_D^*, c_A^*)\) is determined entirely by the two constraints. From our previous analysis, we know that if \(c_D^* \leq c_A^*\) the mean score for admitted students from \(A\) is strictly higher than admits from \(D\). It follows that \(c_D^* > c_A^*\). Disadvantaged students must clear a higher threshold in order to gain admission.

There is a fundamental tension between a commitment to group-blind admissions and a commitment to eliminating stereotypes. Perversely, eliminating the negative stereotype against admitted students from \(D\) requires not equality, but making it more difficult for these students to gain admission. It would reduce the representation of admitted students even more than a group-blind policy. If negative stereotypes are indeed harmful, then an “equal-mean” policy would help admitted students from \(D\), but it would do so by preventing other disadvantaged students from gaining admission.\(^7\)

We can again illustrate this conclusion using data from the ACT. Below, we depict the optimal, (approximately) equal-mean policy \(\{c_D^*, c_W^*\}\) that admits 60% of students.

Black students are required to score above 22 to gain admission whereas white students are required to score 20. Under this equal-mean policy, black students comprise only 2.8% of admitted students. This is a 54% reduction in the representation of black students.

\(^7\)An equal-mean policy would also result in a lower expected score for admitted students, relative to a group-blind policy.
relative to a race-blind admission policy, for which black students comprise 5.3% of admitted students. This example illustrates the substantial effect of trying to eliminate stereotypes on the admission prospects of disadvantaged students.

B.2 Equalizing Distributions

Even if an admission policy equalizes the mean scores across admits from $A$ and $D$, there will still be a larger fraction of students from $A$ with high scores. This can be seen by examining the distributions in our ACT example. Admission policies that assign a probability of admission to a score can completely eliminate stigma by equalizing the distribution of scores across $A$ and $D$. Under these policies, eliminating stereotypes still requires making it more difficult for disadvantaged students to gain admission.

Consider an admission policy that maximizes the expected score of admitted students, subject to the constraints that a constant proportion $K$ of students are admitted and that the distribution of scores across admitted students from $D$ and $A$ are identical. Let $\phi_D$ and $\phi_A$ denote the probability of admission for each group, so that:

$$\phi_D = \int q_D(s)f_D(s)ds, \phi_A = \int q_A(s)f_A(s)ds$$

In contrast to contractual screening, in which a principal wishes to separate types, this admission policy can be thought of as contractual “shrouding.” The principal wishes to maximize an objective while preventing third parties or agents themselves from making inferences about each other’s type. An optimal admission policy solves:

$$\max_{\{q_D(s), q_A(s)\}} \frac{1}{K} \cdot [\theta_D \int s \cdot q_D(s)f_D(s)ds + \theta_A \int s \cdot q_A(s)f_A(s)ds] \text{ subject to }$$

$$\phi_D \cdot \theta_D + \phi_A \cdot \theta_A = K, \quad (i)$$

$$\frac{q_D(s)f_D(s)}{\phi_D} = \frac{q_A(s)f_A(s)}{\phi_A} \forall s, \quad (ii)$$

$$\phi_D = \int q_D(s)f_D(s)ds, \phi_A = \int q_A(s)f_A(s)ds, and \quad (iii)$$

$$0 \leq q_D(s) \leq 1, and 0 \leq q_A(s) \leq 1 \forall s \quad (iv)$$
Proposition 2  A solution \( q_D^*(s), q_A^*(s) \), to this program takes the form:

\[
q_D^*(s) = \begin{cases} 
0 & \text{if } s < s_L^* \\
\frac{f_A(s)}{f_D(s)} \cdot \frac{\phi_D^*}{\phi_A^*} & \text{if } s_L^* \leq s \leq s_H^* \\
1 & \text{if } s \geq s_H^* 
\end{cases}
\]

\[
q_A^*(s) = \begin{cases} 
0 & \text{if } s < s_L^* \\
1 & \text{if } s_L^* \leq s \leq s_H^* \\
\frac{f_D(s)}{f_A(s)} \cdot \frac{\phi_A^*}{\phi_D^*} & \text{if } s \geq s_H^* 
\end{cases}
\]

Proof. See Appendix □

Eliminating the negative stereotype requires making admission more difficult for applicants from \( D \). The probability of admission for applicants from \( D \) is positive and increasing over the range \([s_L^*, s_H^*]\), but it is less than the probability of admission for applicants from \( A \). It only equal to 1 for scores in the upper range \([s_H^*, \bar{s}]\). Because the threshold \( s_L^* \) is below the cutoff score under a group-blind policy, some students in both \( D \) and \( A \) are admitted under the no-stereotype policy that would not gain admission under a group-blind policy. The no stereotype policy also makes admission more difficult for high-scoring applicants from \( A \). Their probability of admission is less than 1 and actually decreasing over the range \([s_H^*, \bar{s}]\).

We again illustrate these conclusions using ACT data, where \( q_D^*(s) \) and \( q_W^*(s) \) represent the probability of admission for a black and white student, respectively, with score \( s \).

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\( ^8 \)If we constrain the probability of admission to be weakly increasing in \( s \), then all students from \( A \) above a threshold \( s^* \) are admitted with probability 1 and students from \( D \) with a score \( s \) above this threshold are admitted with probability \( q_D^*(s) = \frac{f_A(s)}{f_D(s)} \cdot \frac{\phi_D^*}{\phi_A^*} \), where \( \phi_D^* = \frac{K}{\theta_A + \theta_D} \) and \( \phi_A^* = \frac{K}{\theta_A + \theta_D + \frac{1}{f_D(s)}} \). If \( \frac{f_A(s)}{f_D(s)} \) is large, then the percentage of students from \( D \) is admitted is extremely small.
It is more difficult for black applicants to gain admission than white applicants under a no-stereotype policy. Black students with SAT scores between 19 and 34 have a lower probability of admission than white students who score in this range. Black students who score 35 or more have a higher probability of admission than white students, but overall only 2.9% of black students are accepted, while 71.7% of white students are accepted. The percentage of black students among admitted students is .8%, while the percentage of white students is 99.2%. The no-stereotype distribution of scores among admitted students, which is identical for black and white admits, is largely achieved by matching the black distribution to the white distribution. This can be seen by comparing the distribution of scores under a no-stereotype policy to that of white and black admits under a race-blind policy that admits all students who score over 20.

It is also more difficult for black students to gain admission under a no stereotype policy than a race-blind policy. Black students who score between 20 and 33 on the ACT have a strictly lower probability of admission, while the probability is unchanged for those who score above 33. The percentage of black students among admits (.8%) is lower than the percentage under a race-blind policy (5.3%) or even a policy that equalizes the mean score of black and white admits (2.8%).

We conclude that the conditions under which negative group stereotypes exist under group-blind admissions can be stated generally and are exhibited in real-world data. If admission policies sought to eliminate stigma, it would have dramatic consequences for students from disadvantaged groups. Disadvantaged students would be subject to even

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9Relative to a policy that equalizes the mean score of black and white admits, the probability of admission for black students under a no stereotype policy is higher for scores between 19 and 21, lower between 22 and 33, and equal when 34 and above.
more stringent admissions standards than those applied to advantaged students. As a
result, their representation among admitted students would fall substantially, even relative
to group-blind admissions.

Admittedly, admission policies are motivated by many values. In our view it is nev-
ertheless useful to understand the consequences of a particular value in isolation. Other
ethical values that inform higher-education admissions, when considered on their own, do
not imply outcomes that very few persons would support. For example, a commitment to
equal treatment implies that admission standards should be independent of group status. A
commitment to equal representation may require admission quotas or other steps to match
the characteristics of admitted students with those of the wider population. In comparison,
the desire to eliminate stereotypes does not stand well on its own.

II Other Values in Admissions

Eliminating stereotypes through admission policy would adversely affect disadvan-
taged students. We consider how this conclusion changes when admission policies also
embody other values. We have already shown there is a tension between a commitment to
equal treatment and concern over negative stereotypes. When both these ends are valued,
an optimal admission policy would still make it harder for disadvantaged students to gain
admission. It requires a third value, such as a commitment to equal representation, for
group-blind admission to be optimal. However, group-blind admissions are optimal only in
the knife edge case when marginal gains from equal representation exactly offset the mar-
ginal cost of negative stereotypes. Some affirmative action is optimal whenever the former
effect is larger.

A Stereotypes and Equal Treatment

We show that a commitment to both equal treatment and combating stereotypes is
inconsistent with a group-blind admission policy. Suppose that admission policies again
take the form of a cutoff score for each group. We assume that a school has a moral
ordering over different policies that can be represented by the function:

\[ S_A \left( 1 - \frac{\theta_A(1-F_A(c_A))}{\theta_A(1-F_A(c_A)) + \theta_D(1-F_D(c_D))} \cdot E_A[s|s > c_A] + \frac{\theta_D(1-F_D(c_D))}{\theta_A(1-F_A(c_A)) + \theta_D(1-F_D(c_D))} \cdot E_D[s|s > c_D] \right) \]

\[ S_D \left( 1 - \frac{\theta_A(1-F_A(c_A))}{\theta_A(1-F_A(c_A)) + \theta_D(1-F_D(c_D))} \cdot E_A[s|s > c_A] + \frac{\theta_D(1-F_D(c_D))}{\theta_A(1-F_A(c_A)) + \theta_D(1-F_D(c_D))} \cdot E_D[s|s > c_D] \right) \]

\[ 10 \text{In our previous examples, a school maximized the expected score of admitted students subject to}
\text{different constraints. None of our conclusions change if we also allow schools to value a higher expected}
\text{score among admitted students. In this case, the utility function would include a third argument:} \]

\[ \frac{\theta_A(1-F_A(c_A))}{\theta_A(1-F_A(c_A)) + \theta_D(1-F_D(c_D))} \cdot E_A[s|s > c_A] + \frac{\theta_D(1-F_D(c_D))}{\theta_A(1-F_A(c_A)) + \theta_D(1-F_D(c_D))} \cdot E_D[s|s > c_D] \]
All else equal, a school prefers a more equal policy to one that differentiates applicants according to group status. The school also prefers an admission policy in which there is a smaller difference in the average scores of admitted students from each group. We assume that there are diminishing marginal returns to achieving the most desired outcome of equal cutoffs and equal means. In particular, we assume that (i) $U(\cdot, \cdot)$ attains its global maximum at $U(0,0)$ and that $U(0,x)$ and $U(x,0)$ are local maxima for any $x$. Necessary and sufficient conditions for this to be true are that $U_1(0,0) = 0$, $U_2(\cdot, 0) = 0$, and $U(\cdot, \cdot)$ is strictly concave,

Under these assumptions, the admission policy would still make it harder for disadvantaged students to gain admission.

**Proposition 3** For these preferences, an optimal admissions policy implies a higher cutoff score for disadvantaged students ($c_D > c_A$).

**Proof.** Starting from any group-blind policy ($c_A = c_D = c$) consider effect of slightly slightly increasing $c_D$. This new policy is strictly preferred because:

$$
\frac{dU(0,+)}{dc_D} \bigg|_{c_A = c_D = c} = U_1(0,+) + U_2(0,+) \cdot \frac{-dE_D[s | s > c_D]}{dc_D}
$$

$$
= 0 + (-) \cdot (-) > 0
$$

The first term in this expression ($U_1(0, \cdot)$) is zero because, starting from a group-blind policy, the marginal loss from violating equal treatment is small. The second term ($U_2(0,+) \cdot \frac{-dE_D[s | s > c_D]}{dc_D}$) is positive because the increase in $c_D$ raises the mean score for the disadvantaged group and thereby lowers the stereotype experienced by this group.\footnote{Starting from any group-blind policy and slightly decreasing $c_A$ also results in a strictly preferred policy.} This reasoning also implies that it cannot be optimal to eliminate stereotypes ($E_A[s | s > c_A] \neq E_D[s | s > c_D^*]$). Therefore, an optimal admission policy entails a higher cutoff score for $D$ and a negative stereotype against $D$.\footnote{If the school also values a higher expected score among admitted students then, starting from the group-blind policy that maximizes the expected score subject to the constraint that fraction $K$ are admitted, the first derivative of the expected score with respect to changing the cutoff is zero. Therefore, a group-blind policy cannot be optimal, and an optimal policy will have a higher cutoff for $D$ than for $A$.}

These conclusions are closely related to those of the previous section. Admitted students from $D$ experience a negative stereotype under group-blind admissions. Therefore, a school that values both equal treatment and combating stereotypes would optimally increase
admission standards for disadvantaged groups.\textsuperscript{13}

A.1 Other Preferences Over Equal Treatment and Stereotypes

There are other ways to describe these preferences that do not imply this result. In our view, the best way to represent the preferences of advocates of group-blind admissions is to accord a lexicographic priority to equal treatment. Admission policies would first be ranked in terms of their proximity to a group-blind policy. If there are any ties on this dimension, then their effect on negative stereotypes would become relevant. Such preferences could reflect a concern for both equal treatment and combating stereotypes, but the optimal admission policy would be group blind. Because preferences are revealed through choices, these preferences are usually \textit{observationally equivalent} to preferences that completely ignore stereotypes. We conclude that combating negative stereotypes should not play an important role in the case for group-blind admissions.\textsuperscript{14}

Are there other preferences over equal treatment and stereotypes that would imply group-blind admissions? We cannot make claims about the entire space of preferences, but we can consider other plausible alternatives. The choice of group-blind admissions is a “corner solution” in a choice problem where the other corner is eliminating stereotypes. Corner solutions often arise for linear preferences where the marginal benefit of increasing equality or reducing stereotypes does not depend on the level of equality or stereotype.

Linear preferences do not adequately explain group-blind admissions. Suppose an opportunity arises in which a small change in the equality of treatment can result in a large change in the stereotype experienced by the disadvantaged group. For example, lowering the standard for the advantaged group could let in a disproportionately large number of students that would lower the average score for that group. Under linear preferences, a decision maker could then switch to a no-stereotype policy.\textsuperscript{15} Therefore, we do not believe that such preferences adequately rationalize the choice of group-blind admissions.

Nor are discontinuous preferences, which reflect a large, fixed cost of moving away from group-blind admissions, a plausible alternative. Consider a group of candidates (\textit{tier 1})

\textsuperscript{13}Note that advocating group-blind admissions is distinct from arguing that, starting from affirmative action, admission policy should move in the direction of equal treatment. The latter argument is consistent with the preferences we have described for equal treatment and reducing stigma, but the former is not. With these preferences, one should move not just toward equal treatment, but past it.

\textsuperscript{14}Group-blind admissions would also be optimal if a school had a lexicographic preference that disadvantaged groups be treated no worse than advantaged groups in the admission process. This can be described as an “antisubordination principle.” If a school had preferences over equal treatment and combating discrimination, the the antisubordination principle would then result in the choice of group-blind admission. We thank David Weisbach and Will Hubbard for bringing this point to our attention.

\textsuperscript{15}This logic also applies if there are increasing marginal returns to equalizing treatment and reducing stereotypes. The theory of revealed preference implies that observed choices cannot be used to distinguish preferences that exhibit increasing marginal benefits from preferences with constant marginal benefits.
that as school would strictly prefer to admit on the basis of their scores and a second group of candidates (tier 2) that the schools indifferent between admitting or not. Compare (i) the decision to favor candidates from D in tier 2 over candidates from A in tier 2 to (ii) the decision to favor a candidate in D from tier 2 over a candidate in A from tier 1. If there is a large fixed cost to violating equal treatment, then a decision maker would perceive a greater ethical violation in moving from a group-blind policy to (i), than in moving from (i) to (ii). We view this as implausible. Moreover, the existence of a large discontinuity would be observationally equivalent to advocating group-blind admissions in all circumstances.

We conclude that a lexicographic preference for equal treatment best explain the support for group-blind admissions. This conclusion also applies to the constitutional view articulated by Justice Thomas. In his concurring opinion in Fisher v. Texas, Justice Thomas states that “for constitutional purposes” it does not matter whether the effects of affirmative action are “insidious” or “benign.” In our view, Justice Thomas’s statement also describes arguments for group-blind admissions that do not explicitly rely on constitutional interpretation.

B Stereotypes, Equal Treatment, and Equal Representation

Finally we consider admissions when equal representation is valued in addition to equal treatment and combating stereotypes. We define equal representation as a desire for the demographics of admitted students to be similar to the general population. Equal representation can also be described as a concern for diversity. Other goals of affirmative action policies, such as remedying past or current discrimination, will usually be satisfied by moves toward more equal representation.

We show that it requires a third value such as equal representation for group-blind admissions to be optimal. However, group blind admissions are only optimal in the knife-edge case when preferences over equal representation and stereotypes exactly balance. We suggest that this perfect balance does not adequately capture the case for group-blind admission. When all three of these ends are valued, the most plausible conclusion is that some amount of affirmative action is optimal.

We again consider a school with preferences over equal treatment and combating stereotypes. The school also prefers more equal representation. All else equal, the school would like the relative proportion of admitted students from groups A and D be as close as possible
to their population proportion. These preferences can be represented by the function:\footnote{16} \footnote{17}

\[
U(c_A - c_D, E_A[s|s > c_A] - E_D[s|s > c_D], F_D(c_D) - F_A(c_A))
\]

We again assume that there are diminishing marginal returns to achieving the most desired outcome of equal cutoffs, equal means, and equal representation. In particular, we assume that (i) \(U(\cdot, \cdot, \cdot)\) attains its global maximum at \(U(0,0,0)\) and that \(U(0,x,0), U(x,0,0),\) and \(U(0,0,x)\) are local maxima for any \(x\). Necessary and sufficient conditions for this to be true are that \(U_1(0,\cdot,\cdot) = 0, U_2(\cdot,0,\cdot) = 0, U_3(\cdot,\cdot,0) = 0\), and \(U(\cdot,\cdot,\cdot)\) is strictly concave.

**Proposition 4** For these preferences, group-blind admissions are optimal only if the marginal benefit of reducing stereotype equals the marginal cost of lower representation.

**Proof.** Starting from a group blind policy \((c_A = c_D = c)\) consider effect of increasing \(c_D\). Whether the new policy is preferred is determined by the expression:

\[
\frac{dU(0,+,+)}{dc_D}|_{c_A=c_D=c} = U_1(0,+,+) + U_2(0,+,+) \cdot \frac{-dE_D[s|s > c]}{dc_D} + U_3(0,+,+) \cdot f_D(c)
\]

\[
= 0 + (-) \cdot (-) + (-) \cdot (+) = 0 + (+) + (-)
\]

The first term in this expression \((U_1(0,+,+))\) is 0 because, starting from group-blind admissions, the marginal cost of violating equal treatment is small. The second term \((U_2(0,+,+) \cdot \frac{-dE_D[s|s > c]}{dc_D})\) is positive because increasing \(c_D\) reduces the stereotype against admitted students from \(D\). The third term \((U_3(0,+,+) \cdot f_D(c))\) is negative because increasing \(c_D\) lowers the representation of students from \(D\).

Surprisingly, it requires a third value—such as equal representation—to rationalize a policy of group-blind admissions. We find it unlikely that such a delicate balancing act explains the advocacy of group-blind admissions. First, few critics of affirmative action, who emphasize the harm to recipients from stereotypes, voice an equal concern for promoting equal

\footnote{16}If the ratio of the two groups in the population is \(\frac{\theta_A}{\theta_D}\), then the ratio of the two groups among admitted students is:

\[
\frac{\theta_A(1 - F_A(c_A))}{\theta_A(1 - F_A(c_A) + \theta_A(1 - F_D(c_D)))} \cdot \frac{\theta_D(1 - F_D(c_D))}{\theta_D(1 - F_D(c_D))} = \frac{\theta_A}{\theta_D} \cdot \frac{(1 - F_A(c_A))}{(1 - F_D(c_D))}
\]

and the difference between the two ratios is

\[
\frac{\theta_A}{\theta_D} \cdot \frac{(1 - F_A(c_A))}{(1 - F_D(c_D))} - 1
\]

This difference is zero if and only if the probability of acceptance for both groups is equal.

\footnote{17}As in the previous section, our conclusions do not change if we also allow the school to prefer a higher expected score among admitted students.
representation through admission policy. Second, if many people found these tradeoffs to perfectly balance, then we should expect some to favor further reducing stereotypes. Yet no one takes seriously the position that admission standards should be higher for disadvantaged students.

Third, the balance between equal treatment and stereotypes should vary with other circumstances. For example, suppose scores are (truncated) normally distributed for students from $A$ and $D$ and group-blind admissions are optimal for a large, less selective school that admits all students above some $c$. This should inform the choice of admission policy for a highly selective school. For the highly selective school, the mean difference in scores between $A$ and $D$ will be lower, but the ratio of $A$ to $D$ will be higher. Relative to the less selective school, the marginal value of increasing $D$'s representation is higher and the marginal value of reducing $D$'s stereotype is lower. Therefore, some amount of affirmative action should be chosen at the more selective school. By the same logic, if group-blind admissions are optimal at the more selective school, then the admission standards should be higher for students from $D$ at the less selective school. Yet no advocates of group-blind admissions would vary their prescriptions in this way. We conclude that it is unlikely that group-blind admissions are explained by equally-balanced preferences over equal representation and stereotypes.

On the other hand, some affirmative action is optimal whenever, starting from group-blind admissions, the marginal benefit of more equal representation outweighs the marginal cost of greater stereotypes. In our view, this is almost always the case. Again, if it were not then some schools should advocate higher admission standards for disadvantaged students. Many would agree that equal treatment, equal representation, and combating stereotypes are worthy goals of admission policy. But no one would advocate such a perverse double-standard.

III Other Views of Stereotypes

We focus on statistical discrimination because we are interested in the argument that stereotypes harm beneficiaries of affirmative action. Models of statistical discrimination have been used since at least Arrow (1972, 1973) and Phelps (1972). They are conceptually distinct from “taste-based” models of discrimination (Becker (1957)), which assume that individuals from one social group have an aversion to individuals from another. In our view, this tast-based view is not closely related to the argument that affirmative action harms its beneficiaries.

---

18 We do not address the empirical question of the extent of statistical discrimination and its effect on admitted students. Instead, we take as given the claim that it exists and is harmful, and ask what are the logical implications for admissions policy.
Our use of stereotype as statistical discrimination is consistent with how test scores and other measures of academic performance are used in many discussions of affirmative action. For example, the admissions policy at issue in *Fisher v. Texas* is determined on the basis of a numerical Academic Index (AI) and a Personal Achievement Index (PAI). The AI is calculated from a student’s standardized test scores, class rank, and high school coursework. The PAI is based on a holistic review of the application and can take race into account. In his concurring opinion, Justice Thomas points out that among students admitted outside of the University of Texas’s Top Ten Percent Plan, “blacks scored at the 52d percentile of 2009 SAT takers nationwide, while Asians scored at the 93d percentile. Blacks had a mean GPA of 2.57 and a mean SAT score of 1524; Hispanics had a mean GPA of 2.83 and a mean SAT score of 1794; whites had a mean GPA of 3.04 and a mean SAT score of 1914; and Asians had a mean GPA of 3.07 and a mean SAT score of 1991.” In the context of our model, some minority groups would experience a negative stereotype with respect to their AI score or its components.\(^\text{19}\)

For these figures, Thomas relied on an amicus brief filed by Richard Sander, a law professor at UCLA and attorney Stuart Taylor. Sander is well known for his criticism of affirmative action programs in law schools (Sander (2004)). However, the use of differences in test scores has also been used to measure the extent of affirmative action and “stereotype threat” by scholars who are broadly supportive of affirmative action. For example, in their influential study, Fischer and Massey (2007) measure “affirmative action at the institutional level by taking the difference between the average SAT score earned by blacks or Hispanics and all students at a particular institution.” The authors use this measure to test for the effect of stereotypes on affirmative action recipients.

Finally, the logic of our argument is consistent with any “stereotype” (however defined) that (i) continues to exist under group-blind admissions, and (ii) decreases if admissions are made more difficult for the disadvantaged group. This includes stereotypes based on false priors about the academic performance of disadvantaged groups. It includes some accounts of “external stigma”–the perception of affirmative action recipients by others–in social psychology (Bowen (2010)). It can also include accounts of “internal stigma” such as “stereotype threat” (Steele and Aronson (1995), (Steele (1997)) if these effects decrease when admission standards are raised for disadvantaged groups.

Our conclusions do not follow for any “stereotype” that only exists in the presence of affirmative action. For example suppose negative stereotypes were only formed on the extensive margin of admissions. A negative stereotype could then be defined as the prob-

\(^{19}\)What Justice Thomas did not point out, but is consistent with our findings, is that these differences are substantially similar for candidates admitted under Texas’s Top Ten Percent program. When schools adopt race-blind admissions criteria that are intended to achieve diversity, stereotypes in the form of average test-score differences will likely still exist.
ability that a student was admitted because of affirmative action. This probability would increase with the extent of affirmative action. Ending affirmative action will eliminate this negative stereotype. Consistent with this view, there is evidence that negative inferences against affirmative action recipients are mediated by the perceived fairness of the admissions procedure (Heilman, McCullough, and Gilbert(1996), Evans (2003)). Ultimately, our claim is not that every form of stereotype or discrimination against affirmative action recipients would continue to exist under group-blind admissions. Our claim is that many of them would.

IV Conclusion

In this paper, we analyze the use of admission policy to affect stereotypes against students from disadvantaged groups. Many critics of affirmative action in admissions argue that lower standards cause such stereotypes. We show that when stereotypes are a result of social inequality, they can easily persist under group-blind admissions. Perversely, eliminating stereotypes requires making it harder for disadvantaged students to gain admission. Such an admission policy would increase social inequality and is clearly unacceptable.

This conclusion does not change if schools seek to both treat students equally and counteract stereotypes. Under these preferences, an optimal admission policy would still impose a higher standard on disadvantaged students. Group-blind admissions are optimal, however, if a school holds an ethically prior, lexicographic preference for equal treatment. We argue that the best case for group-blind admissions rests on such a preference. Consequently, stereotypes should play a negligible role in critiques of affirmative action that advocate group-blind admissions.

If a school is willing to actually consider tradeoffs between equal treatment and stereotypes, it requires a third goal such as equal representation to justify group-blind admissions. However, group blind admissions are optimal only when the conflicting goals of equal representation and limiting stereotypes exactly balance. In our view, this knife edge case is an implausible justification for group-blind admission. It would again imply a desire on the part of some schools to have higher standards for disadvantaged students. Consequently, some amount of affirmative action is always optimal if a school values all three of these goals.

References


V Appendix

Proof of Proposition 2

Claim 1 For optimal policies $q_D^*(s)$ and $q_A^*(s)$, the ratio $\frac{q_D^*(s)}{q_A^*(s)}$ is increasing in $s$.

Proof. The constraints imply that $\frac{f_A(s)}{f_D(s)} = \frac{q_D^*(s)}{q_A^*(s)} \cdot \frac{\phi_A^*}{\phi_D^*}$ so the conclusion follows from the monotone likelihood ratio. ■

We can substitute for $q_A^*(s)$ using the constraints and rewrite the program as:

$$\max_{\{q_D(s), \phi_D, \phi_A\}} \frac{1}{\phi_D} \cdot \int_s s \cdot q_D(s)f_D(s)ds subject to$$

$$\int_s q_D(s)f_D(s)ds = \phi_D$$

$$\phi_D \cdot \theta_D + \phi_A \cdot \theta_A = K,$$

$$0 \leq q_D(s) \leq \min \left[ \frac{f_A(s)}{f_D(s)} \frac{\phi_D}{\phi_A}, 1 \right]$$
Claim 2. A solution $q_D^*(s), \phi_D^*, \phi_A^*$ to this program takes the form:

$$q_D^*(s) = \begin{cases} 0 & \text{if } s < s_L^* \\ \min\left[ \frac{f_A(s)}{f_D(s)} \cdot \frac{\phi_D}{\phi_A}, 1 \right] & \text{if } s \geq s_L^* \end{cases}$$

where $s_L^*$ is defined by $\int_{s_L^*}^\infty \min\{1, \frac{f_A(s)}{f_D(s)} \cdot \frac{\phi_D}{\phi_A}\} f_D(s) ds = \phi_D^*$. The solution $q_D^*(s)$ is weakly increasing in $s$ and continuous almost everywhere.

**Proof.** For any choice of $\phi_D$, the program is linear and $q_D(s)$ is a “bang-bang” solution with respect to the constraint set. This is also true at an optimally chosen $\phi_D^*$. ■

Claim 3. The solution function $q_D^*(s) = 1$ on some interval $[s_H^*, \infty)$ where $s_L^* \leq s_H^* \leq s_H$.

**Proof.** Suppose not, so that $\frac{f_A(s)}{f_D(s)} \cdot \frac{\phi_D}{\phi_A} < 1$. This implies that (i) $q_D^*(s) = \frac{f_A(s)}{f_D(s)} \cdot \frac{\phi_D}{\phi_A} < 1 \ \forall s \in [s_L^*, s_H]$, (ii) $q_D^*(s) = 1$ in this interval, and (iii) $\phi_A = 1 - F_A(s_L^*)$. But then the objective function is $E_A[s: s > s_L^*]$, which is strictly increasing in $\phi_D$, so this cannot be an optimum. Therefore $\frac{f_A(s)}{f_D(s)} \cdot \frac{\phi_D}{\phi_A} \geq 1$ and $q_D^*(s) = 1$. ■

We can then rewrite the concentrated program as:

$$\max_{\left\{ \phi_A, \phi_D, s_L, s_H \right\}} \frac{1}{\phi_A} \cdot \int_{s_L}^{s_H} s \cdot f_A(s) ds + \frac{1}{\phi_D} \cdot \int_{s_H}^{\infty} s \cdot f_D(s) ds \text{ subject to}$$

$$\phi_D \cdot \theta_D + \phi_A \cdot \theta_A = K \quad \text{(i)}$$

$$\frac{f_A(s_H)}{f_D(s_H)} \cdot \frac{\phi_D}{\phi_A} = 1 \quad \text{(ii)}$$

$$\frac{F_A(s_H) - F_A(s_L)}{\phi_A} + \frac{1 - F_D(s_H)}{\phi_D} = 1 \quad \text{(iii)}$$

$$0 < \phi_A \leq 1, 0 < \phi_D \leq 1 \quad \text{(iv)}$$

For any choice of $\phi_D$, the constraints define implicit functions $\phi_A(\phi_D), s_L(\phi_D), s_H(\phi_D)$. By implicit differentiation we obtain:

$$\phi_A'(\phi_D) = \frac{-\theta_D}{\theta_A}$$

$$s_L'(\phi_D) = \frac{\phi_A}{f_A(s_L)} \cdot \left[ \frac{F_A(s_H) - F_A(s_L)}{\phi_A^2} \cdot \frac{\theta_D}{\theta_A} - \frac{1 - F_D(s_H)}{\phi_D^2} \right]$$

We can see that $s_L'(\phi_D) > 0$ when $s_H = \infty$ and $s_L'(\phi_D) < 0$ when $s_H = s_L$. After some algebraic manipulation, we can reexpress the first order condition with respect to $\phi_D$ (with $\phi_A, s_L, s_H$ defined implicitly) as:

\[
\frac{f_A(s) \cdot \phi_D}{f_D(s) \cdot \phi_A} = \min\{1, \frac{\phi_A}{\phi_D} \cdot \frac{\theta_D}{\theta_A} \}
\]
Claim 4 The solution function $q_D^*(s) = 1$ on some interval $[s_L^*, s_H^*]$ where $s_L^* < s_H^* < \bar{s}$.

Proof. Suppose that $s_H^* = \bar{s}$ so that $\frac{f_A(s_H^*)}{f_D(s_H^*)} \frac{\phi^*_D}{\phi^*_A} = 1$. This is a feasible solution only if

$$\phi^*_D = \frac{K}{\theta_A \left( \frac{f_A(s_H^*)}{f_D(s_H^*)} \right) + \theta_D}$$

and

$$\phi^*_A = \frac{K}{\theta_A + \theta_D \left( \frac{f_A(s_H^*)}{f_D(s_H^*)} \right)} < 1.$$  

The first order condition with respect to $\phi_D$ at $s_H^* = \bar{s}$ is:

$$(E_A[s|s_L^* < s < \bar{s}] - s_L) \cdot \left( \frac{1 - F_A(s_L^*)}{\phi^*_A} \cdot \frac{\theta_D}{\theta_A} \right) > 0$$

Therefore this cannot be an optimum and $s_L^* < \bar{s}$ (also $\phi_D^* > \frac{K}{\theta_A \left( \frac{f_A(s_H^*)}{f_D(s_H^*)} \right) + \theta_D}$). Suppose next that $s_L^* = s_H^* = s^*$. This is a feasible solution only if:

$$(1 - F_D(s^*)) \cdot \left( \theta_D + \theta_A \left( \frac{f_A(s^*)}{f_D(s^*)} \right) \right) = K$$

The first order condition is then:

$$-(E_D[s|s > s^*] - s^*) \cdot \left( \frac{1 - F_D(s^*)}{\phi^*_D} \right) < 0$$

Therefore, this cannot be an optimum and $s_L^* < s_H^*$. If this feasible set is nonempty, we can say $\phi_D^* < 1 - F_D(\max\{s^*\})$. If it is empty, then $\phi_D^* < \min\{\frac{K}{\theta_D}, 1\}$. 

Therefore, a solution takes the form:

$$q_D^*(s) = \begin{cases} 
0 & \text{if } s < s_L^* \\
\frac{f_A(s)}{f_D(s)} \cdot \frac{\phi^*_D}{\phi^*_A} & \text{if } s_L^* \leq s \leq s_H^* \\
1 & \text{if } s \geq s_H^* 
\end{cases}$$

$$q_A^*(s) = \begin{cases} 
0 & \text{if } s < s_L^* \\
\frac{f_D(s)}{f_A(s)} \cdot \frac{\phi^*_A}{\phi^*_D} & \text{if } s_L^* \leq s \leq s_H^* \\
1 & \text{if } s \geq s_H^* 
\end{cases}$$

The first-order condition and constraints for the concentrated program are:
These equations determine an interior solution \((\phi_D^*, \phi_A^*, s_L^*, s_H^*)\) if \(\frac{K}{\theta_A} < 1\) and \(\frac{K}{\theta_A} \geq 1\) and \(\frac{f_A(s_H)}{f_D(s_H)} \cdot \frac{\phi_D}{\phi_A} \leq 1\).

If otherwise, then the smallest \(\phi_D\) that satisfies the quota constraint \((\phi_D = \frac{K}{\theta_A}, \phi_A = 1)\) is not feasible because one cannot match the distributions across the entire support.

Let \(\underline{\phi}_D\) be the smallest solution to:

\[
\frac{1 - F_D(s_H)}{\phi_D} + \frac{F_A(s_H)}{\phi_A} = 1
\]

\[
\phi_D \cdot \theta_D + \phi_A \cdot \theta_A = K
\]

\[
\frac{f_A(s_H)}{f_D(s_H)} \cdot \frac{\phi_D}{\phi_A} = 1
\]

Then this system \((s_L = 2)\) determines an optimum if the first-order condition for the concentrated program is weakly less than 0 at \(\underline{\phi}_D\). If this first order condition is strictly positive, then the solution is interior.