International Prices and Demand for Value Added with Global Supply Chains∗

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Abstract

By linking domestic and foreign production processes, global supply chains alter how shocks are transmitted across borders. In this paper, we analyze the role of these input linkages in determining how demand for value added responds to changes in international relative prices. We emphasize that elasticities of substitution in production and final demand shift the balance between supply versus demand side transmission channels, and therefore affect both the magnitude and bilateral distribution of demand spillovers. Using global input-output data to parameterize the framework, we quantify these mechanics. When supply chains are inflexible, we find that the magnitude of multilateral spillovers is dampened, and bilateral spillovers are reallocated away from supply chain partners. We discuss how our results inform analysis of expenditure switching and price adjustment in macroeconomic models.

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Trade in intermediate inputs accounts for nearly two-thirds of international trade, and the import content of exports stands at 30 percent or more in many countries. Despite this, cross-border input linkages are largely absent in conventional macroeconomic models.\footnote{For example, see the International Real Business Cycle Model by Backus, Kehoe, and Kydland (1994) or various models in Obstfeld and Rogoff (1996).} To be precise, conventional frameworks treat exports as if they are composed entirely of domestic value added, while imports are entirely foreign value added. As a result, they focus on demand-side channels (e.g., expenditure switching) via which trade transmits shocks across countries. The growing importance of global supply chains poses a challenge to these frameworks, since they link countries together on the supply side as well. This therefore raises the question: do global supply chains alter the transmission of shocks across countries?  

To fix ideas, consider two ways in which these supply-side linkages might matter. First, global supply chains break the one-to-one correspondence between demand for gross exports and demand for domestic value added. This means that gross expenditure switching toward exported goods translates less than one-for-one into demand for domestic value added. Put differently, part of the benefit from increased exports is passed back through the production chain to countries that supply inputs. Furthermore, when those inputs come from third countries, this implies that gross bilateral trade may be a poor guide to bilateral spillovers.\footnote{For example, final goods imported by the United States from China contain value added from other Asian countries. Following a Chinese appreciation that induces US consumers to substitute away from Chinese goods, we need to trace the shock backwards through the supply chain to assess the bilateral repercussions of the shock.} On the flip side, low domestic content in exports means that increases in domestic production costs do not raise export prices one for one, and this dampens expenditure switching away from home goods when domestic production costs increase.  

Second, a common view is that supply chain relationships are inflexible in the short run. That is, producers may have limited possibilities for substitution across suppliers, meaning that elasticities of substitution in production may be lower than in final demand. This implies that countries heavily engaged in global supply chains would have lower effective elasticities of demand for their output and value added. Looking across bilateral partners, it would also mean that elasticities would vary across partners, such that relative price changes would lead to smaller changes in trade with global supply chain partners. Thus, global supply chains may have important implications for both the multilateral and bilateral strength of shock transmission.  

Given these observations, we believe supply chains ought to feature prominently in analyzing the transmission of shocks across countries. Recognizing growing importance of these channels, we put the macroeconomic role of supply chains under the microscope in this
paper. In doing so, we make two main contributions.

The first contribution is pedagogical. Starting from a framework with both final and intermediate input linkages across countries, we derive a reduced form expression that links changes in “demand for value added” from each country to changes in relative prices of value added and final expenditure levels. This expression boils down the complicated set of gross linkages across countries to describe how price changes induce expenditure switching between home and foreign value added. This allows us to succinctly characterize the strength of value-added expenditure switching, which in turn enables us to draw on standard intuition from conventional models (which abstract from input trade) to analyze shock transmission across countries.

We arrive at demand for value added in three steps. First, we write demand for gross output as a function of prices for gross output and final expenditure levels. This mapping involves the structure of final and intermediate linkages across countries, as well elasticities of substitution in production and final demand. Second, we convert demand for gross output into demand for value added by subtracting off input use, which is itself a function of relative gross output prices. Third, we then replace prices for gross output with prices of real value added, by mapping value-added prices to gross output prices through the global input-output structure.

Throughout our analysis, we highlight the role of various elasticities – among inputs, between inputs and value added in production, and among final goods – in governing the response of demand for value added to price changes. In studying the role of these elasticities, we develop two important auxiliary results.

First, when elasticities are all equal through the framework, we demonstrate that the derived demand for value added is isomorphic to that which one obtains from an alternative value-added CES-Armington representation of preferences. That is, it is as if consumers have CES-Armington preferences directly defined over real value added purchased from alternative source countries. This analysis anchors our discussion of demand for value added to conventional frameworks. Further, it identifies a set of assumptions under which one can justify writing down models with Armington-CES preferences over value-added.

Second, in general cases, we show that demand for value added depends on the interaction between gross substitution elasticities in production and consumption and the pattern of final and intermediate goods linkages across countries. To summarize the strength of expenditure switching over value added, we use the general formula to construct the effective elasticity of substitution between home and foreign value added. This “value-added elasticity” solves an aggregation problem: it takes gross elasticities of substitution, which can be estimated using conventional trade and production data, and aggregates them into a composite elasticity that
can be plugged into a value-added model to yield the same change in demand for value added as in the true gross model. We expect this result to prove useful in calibrating value-added models. We also use it to summarize aggregate expenditure switching in our framework.

The second contribution is that we use global input-output data to characterize how global supply chains modify shock transmission empirically. This global input-output data extends the domestic input-output accounts across borders, tracking final and intermediate shipments across both sectors and countries [Johnson and Noguera (2012), Timmer (2012), Koopman, Wang, and Wei (2014)]. These data, plus elasticity choices, completely parameterize the framework.

To illustrate the role of supply chains, we evaluate how changes in value-added prices spill over across borders. That is, we change the price of domestic value added in a particular country (e.g., Germany) and trace the impact of those price changes through supply chains to demand for value added originating in individual foreign partners. We compare results across two alternative elasticity parameterizations, one in which elasticities are equal (i.e., a value-added model) versus one in which elasticities are heterogeneous. We focus on a heterogeneous elasticity scenario in which possibilities for substitution in input sourcing are limited, as in Leontief production.

With limited input substitutability, we show that size of multilateral spillovers varies across countries, with smaller spillovers for countries that are integrated into global supply chains. At the bilateral level, we show spillovers are reallocated away from supply chain partners. Further, in some (rare) cases, supply chain partners can actually gain from a decline in foreign value-added prices (e.g., a decline in Chinese prices helps Vietnam). This stands in stark contrast to conventional frameworks. In conventional frameworks, all trade partners lose from an improvement in home competitiveness, and supply chain partners tend to lose most, since they have large bilateral gross trade volumes with home. More generally, even when the devaluation does not help individual trade partners, supply chain relationships substantially mitigate negative spillovers for supply chain partners.

We close the discussion of our empirical results by describing alternative ways in which the demand for value added framework we present can be embedded in general equilibrium models. The key choice concerns specifying how the supply of real value added (equivalently, primary factor inputs) is determined. For example, if we assume that real value added is an inelastically supplied endowment, then one can flip the prior analysis on its head. Where we previously manipulated prices and studied the resulting changes in demand for value added, shocks to the supply of real value added induce endogenous changes in prices. As such, the intuition we develop concerning the dependence of demand for value added on prices can inform analysis of price adjustment in equilibrium models.
Our paper makes contributions to several active areas in international economics. First, we obviously contribute to the broad literature that focuses on the role of expenditure switching and trade elasticities in governing shock transmission. Though elasticities play a central role in most empirical papers, we are not aware of prior work that analyzes the consequences of heterogeneity in elasticities in production versus demand as we do. Second, we also add to recent work on the role of cross-border vertical linkages in international macroeconomics [Burstein, Kurz, and Tesar (2008), Bems, Johnson, and Yi (2010), Di Giovanni and Levchenko (2010), Bussière, Callegari, Ghironi, Sestieri, and Yamano (2013), Bems (forthcoming), and Johnson (forthcoming)]. Third, our framework shares many features in common with recent efforts to incorporate input trade into quantitative trade models [Eaton, Kortum, Neiman, and Romalis (2011), Caliendo and Parro (2012), Levchenko and Zhang (2013), and Costinot and Rodriguez-Clare (forthcoming)]. While the basic structure of our framework parallels those trade models in a number of ways, our elasticity analysis and the macro-questions we ask are quite different.

1 Input Trade and Demand for Value Added

This section introduces some key ideas about how cross-border input linkages alter the response of demand for value added to price changes. To communicate these ideas, we analyze a highly stylized version of our general framework. This stylized framework provides a map for navigating the much richer framework that we introduce in Section 2 and use in quantitative analysis.

We consider an economy with three countries, and assume that each country produces an Armington differentiated good. Country 1 consumes some of its own output, and exports the remainder to 2. Country 2 also consumes some of its own output, and exports the remainder to country 3. In what follows, we assume exports from country 2 to country 3 are composed of final goods, which are consumed in country 3. Country 3 also consumes its own output, but does not export it.

As for trade between countries 1 and 2, we compare two alternative cases. In Case A, we assume that country 1 exports final goods, which are consumed by country 2. We refer to this as the ‘conventional case’, as it represents the default modeling approach in the international macroeconomics literature. In Case B, we assume that country 1 exports intermediate inputs, used by 2 to produce its gross output. In both cases, the pattern of gross trade is identical. However, the demand linkages implied by observed gross trade flows depends on whether country 1 ships final or intermediate goods to country 2.
1.1 Case A: Trade in Final Goods Only

As is standard, we assume that preferences take the constant elasticity of substitution form. Given the restricted trade patterns we have assumed, final expenditure in each country is:

\[ F_1 = f_{11}, \quad F_2 = \left( f_{12}^{(σ^{-1})/σ} + f_{22}^{(σ^{-1})/σ} \right)^{σ/(σ-1)}, \]  

and \[ F_3 = \left( f_{23}^{(σ^{-1})/σ} + f_{33}^{(σ^{-1})/σ} \right)^{σ/(σ-1)}, \]

where \( f_{ij} \) is the quantity of final goods from country \( i \) consumed in country \( j \). Denoting gross output in country \( i \) by \( Q_i \), market clearing requires:

\[ Q_1 = f_{11} + f_{12}, \quad Q_2 = f_{22} + f_{23}, \quad \text{and} \quad Q_3 = f_{33}. \]

Finally, we assume that gross output is produced directly from primary factors (i.e., there are no intermediate inputs in production). Therefore, gross output and real value added are equal: \( Q_i = V_i \) and \( p_i = p_i^v \), where \( p_i \) is the price of gross output, and \( V_i \) is real value added with price \( p_i^v \).

Since country 1 both consumes its own good and exports to country 2, demand for gross output from country 1 depends on the level of home final demand and demand for imports in country 2:

\[ \hat{Q}_1 = s_{11} \hat{f}_{11} + s_{12} \hat{f}_{12}, \]

where \( s_{ij} \) is the share of output shipped from \( i \) to \( j \) in country \( i \)'s total output, and \( \hat{x} = \Delta \log(x) \). Import demand in country 2 depends on relative prices and the level of final expenditure in country 2:

\[ \hat{f}_{12} = -σ \left( \hat{p}_1 - \hat{P}_2 \right) + \hat{F}_2, \]

with \( \hat{P}_2 = w_{12} \hat{p}_1 + w_{22} \hat{p}_2 \), where \( w_{ji} \) is the expenditure share on goods from \( j \) by \( i \). Combining market clearing and import demand, and recognizing that \( \hat{Q}_i = \hat{V}_i \) and \( \hat{p}_i = \hat{p}_i^v \) in this case, we get:

\[ \hat{V}_1 = -σ s_{12} w_{22} (\hat{p}_1^v - \hat{p}_2^v) + f(\hat{F}_1, \hat{F}_2), \]

where \( f(\hat{F}_1, \hat{F}_2) = s_{11} \hat{F}_1 + s_{12} \hat{F}_2 \) aggregates changes in final expenditure levels.

The economics of this conventional case are straightforward. First, a rise in the relative price of value added (equivalently, gross output) from country 1 lowers demand for value added from country 1, as it induces expenditure switching away from country 1 goods. Put differently, as country 1 loses competitiveness, demand for its value added falls. Second, demand for value added from country 1 is only a function of prices and expenditure levels in countries to which country 1 exports. Because country 1 does not export anything directly

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3Note that we do not specify a complete economic model here. Rather we omit consumer budget constraints, as well as assumptions on factor supply and factor market clearing. Our analysis (in both Cases A and B, as well as the full framework in Section 3) proceeds by analyzing the minimum set of equilibrium conditions necessary to characterize demand for value added. We return to discuss alternative options to close our framework and turn it into a complete model in Section 6.
to country 3, it does not compete head-to-head with either country 2 or country 3 in that
market.

Turning to country 3, we note that $\dot{Q}_3 = \dot{f}_{33}$, so $\dot{Q}_3 = -\sigma (\dot{p}_3 - \dot{P}_3) + \dot{F}_3$. Then, following similar steps to above, we can write demand for value added from country 3 as:

$$\hat{V}_3 = -\sigma w_{23} (\hat{p}_{2v} - \hat{p}_{3v}) + \hat{F}_3. \quad (4)$$

This expression is also intuitive, as increases in the price of country 3’s value added relative to country 2 induce expenditure switching toward imports by 3, reducing demand for its own value added.

Looking at countries 1 and 3 simultaneously, note that both suffer declines in demand for their value added when country 2 becomes more competitive. If we assume $\hat{p}_{2v} \neq 0$ and $\hat{p}_{1v} = \hat{p}_{3v} = 0$ and impose $\hat{F}_i = 0$ for all $i$ to focus on relative price effects, then $\hat{V}_1 = \sigma s_{12} w_{22} \hat{p}_{2}$ and $\hat{V}_3 = \sigma w_{23} \hat{p}_{2v}$. These expressions tell us two things. First, when country 2 becomes more competitive, demand for value added from countries 1 and 3 declines, since both compete with country 2 directly.

Second, the size of the decline in demand for value added depends on two things: (a) the elasticity ($\sigma$), and (b) bilateral trade linkages (via $s_{12} w_{22}$ and $w_{23}$). The absolute size of the decline in demand for value added, controlling for trade linkages, is governed by $\sigma$. Further, the bilateral distribution of declines in demand for value added is independent of the elasticity, and depends only on bilateral trade linkages. Put differently, the bilateral distribution of spillovers is independent of the absolute magnitude of the spillovers. This is a property of conventional CES-Armington models, which we will return to discuss further below.

### 1.2 Case B: Trade in Inputs and Final Goods

When country 1 ships inputs to country 2, instead of final goods, the economics of spillovers and linkages across markets change. Denoting inputs shipped from country $i$ to country $j$ as $x_{ij}$, the output market clearing condition for country 1 is now $Q_1 = f_{11} + x_{12}$. Accordingly, final expenditure in country 2 is now $F_2 = f_{22}$. On the production side, gross output and value added are no longer identical in country 2. We assume that gross output in country 2 is produced via CES production function: $Q_2 = \left(V_2^{(\gamma-1)/\gamma} + X_{12}^{(\gamma-1)/\gamma}\right)^{\gamma/(\gamma-1)}$.

We start again with demand for gross output from country 1, which (as in Case A)
depends on home final demand and demand for imports in country 2:

\[ \hat{Q}_1 = s_{11}\hat{F}_{11} + s_{12}\hat{x}_{12}. \]  
(5)

Import demand again is CES, but now it depends on relative prices and the level of gross output in country 2, not final expenditure as above. It is given by:

\[ \hat{x}_{12} = -\gamma (\hat{p}_1 - \hat{p}_2) + \hat{Q}_2. \]  
(6)

Plugging this into the previous equation, we get:

\[ \hat{Q}_1 = s_{11}\hat{F}_{11} - s_{12}\gamma (\hat{p}_1 - \hat{p}_2) + s_{12}\hat{Q}_2. \]  
(7)

Note that demand for gross output from country 2 depends positively on output in country 2.

Output in country 2 in turn depends on demand in countries 2 and 3: \( \hat{Q}_2 = s_{22}\hat{F}_{22} + s_{23}\hat{F}_{23} \). And demand for country 2 goods in country 3 is: \( \hat{f}_{23} = -\sigma (\hat{p}_2 - \hat{P}_3) + \hat{F}_3 \). Plugging these into Equation (7), we get:

\[ \hat{Q}_1 = -\gamma s_{12} (\hat{p}_1 - \hat{p}_2) - \sigma s_{12}s_{23} (\hat{p}_2 - \hat{P}_3) + f(\hat{F}_1, \hat{F}_2, \hat{F}_3), \]  
(8)

where \( f(\hat{F}_1, \hat{F}_2, \hat{F}_3) = s_{11}\hat{F}_1 + s_{12}s_{22}\hat{F}_2 + s_{12}s_{23}\hat{F}_3 \).

Finally, we note that \( \hat{V}_1 = \hat{Q}_1 \), and that \( \hat{p}_1 = \hat{p}_1^w \), \( \hat{p}_2 = (1 - s^w)\hat{p}_1^w + s^w\hat{p}_2^w \), and \( \hat{P}_3 = (1 - w)\hat{p}_2 + w\hat{p}_3 \) with \( \hat{p}_3 = \hat{p}_3^w \). Therefore, demand for value added from country 1 can be written in terms of primitive value-added prices alone:

\[ \hat{V}_1 = -s_{12} (\gamma s^w + \sigma s_{23}w(1-s^w)) \hat{p}_1^w + s_{12} (\gamma - \sigma s_{23}w) s^w\hat{p}_2^w + \sigma s_{12}s_{23}w\hat{p}_3^w + f(\hat{F}_1, \hat{F}_2, \hat{F}_3). \]  
(9)

This type of reduced form relationship linking final expenditure levels and value-added prices to demand for value added underpins all our subsequent analysis. Therefore, we remark on six features of this relationship, and related expressions for countries 2 and 3, which foreshadow key results from our full model.

**Remark 1** In terms of pedagogy, note what we did here. We first derived demand for gross output in terms of gross prices. We then linked demand for gross output to demand for value added, and finally substituted out for gross prices to write demand for value added
in terms of value-added prices.\footnote{The link between gross output and value added is trivial for country 1. Since country 2 uses imported inputs in production, it is more complicated for country 2: \( \hat{V}_2 = \hat{Q}_2 - \gamma (1-s^v)(\hat{p}_2^v - \hat{p}_1^v) \). We discuss this step further in the context of the full framework below.} We follow this same procedure in working through the full framework below.

**Remark 2** Demand for value added depends on a weighted average of final expenditure levels, via \( f(\hat{F}_1, \hat{F}_2, \hat{F}_3) \). This weighted average is identical for all possible values of the elasticities, both in this simple case and in our general framework. As we discuss further in the context of our general framework, these weights are value-added export shares – the share of value added from the source absorbed in each destination. We have explored these weights previously in Bems, Johnson, and Yi (2010), and so do not repeat that analysis here. Instead, our analysis in this paper focuses on the role of relative prices and elasticities.

**Remark 3** Demand for value added from country 1 depends on both the price of value added and the level of final expenditure in country 3, unlike Case A. One reason is that the level of final expenditure in country 3 determines demand for gross output from country 2. A second reason is that declines in the price of value added from country 3 lead consumers there to switch away from country 2 goods, symptomatic of a loss of competitiveness for country 2 in country 3’s market.

Importantly, country 1 and country 3 are linked together even though they do not directly trade with one another. This contrasts with Case A, where zero bilateral trade implies zero bilateral demand linkage. At a deep level, the linkage arises here because country 1 exports value added to country 3, embodied in country 2 goods. These value-added linkages shape bilateral spillovers.

**Remark 4** A decline in the price of country 1’s value-added (\( \hat{p}_1^v < 0 \)) leads to an increase in demand for its value added. This is not a surprising result, but it is instructive to note the mechanics. The first reason is that country 2 substitutes toward using country 1 inputs more intensively in production. The second reason is that country 2 gains in competitiveness in selling its final goods in country 3, since it sources inputs from country 1.

The change in demand for value added from country 1 following a change in its own price – the extent of expenditure switching toward country 1 value added, or ‘value-added expenditure switching’ for short – depends on the strength of gross expenditure switching for both inputs and final goods. Correspondingly, the overall impact of a price change in country 1 is governed by a mix of the production and final demand elasticities.
This means that input trade presents us with an aggregation problem. The reduced form
elasticity of demand for value added, which governs expenditure on home versus foreign
value added, is a composite of final expenditure and production elasticities. We use our
general framework below to provide guidance about how to aggregate gross elasticities into
equivalent value-added elasticities, which summarize the strength of value-added expenditure
switching and hence spillovers following relative price changes.

Remark 5 A decline in the price of value added in country 2 ($\hat{p}_2^v < 0$) has two offsetting
effects on country 1. First, it lowers demand for country 1 value added due to input ex-
penditure switching by country 2, who switches away from imported inputs toward using
its own value added in production. Second, it simultaneously raises demand for country 1
value added due to final goods expenditure switching by 3, which raises demand for country
2 output and hence country 1 inputs.

The values of the elasticities of substitution in production versus final expenditure ob-
viously play a crucial role in determining which effect dominates. One extreme would be
where country 1 supplies an essential input into country 2’s production process, so producers
cannot substitute at all in production ($\gamma = 0$). In this event, demand for value added from
country 1 unambiguously rises when the price of value added in country 2 falls. Put simply,
country 1 inherits country 2’s competitiveness gains, as it is able to sell more inputs when
country 2 can sell more output. While we think this assumption might be reasonable in the
short run, higher values of $\gamma$ are likely more appropriate at longer horizons. As substitution
in input expenditure becomes more important, it attenuates (and possibly overturns) any
increase in demand for value added from country 1.

The overarching message is that, regardless of one’s view on the appropriate values of
input and final expenditure elasticities, input trade puts new structure to the pattern of
bilateral demand linkages. And this structure may be quite different than in conventional
frameworks. In particular, the result that a decline in country 2’s prices might actually help
country 1 runs counter to the beggar-thy-neighbor intuition that one obtains by interpreting
gross trade flows through the lens of conventional frameworks, as in Case A.

Remark 6 Consider a special elasticity parametrization, with $\epsilon \equiv \gamma = \sigma$. In this case, we
can pull the common elasticity out front in Equation (9) to yield:

$$\hat{V}_1 = -\epsilon [s_{12} (s^v + s_{23}w(1-s^v)) \hat{p}_1^v + s_{12} (1-s_{23}w) s^v \hat{p}_2^v + s_{12}s_{23}wp_3^v] + f(\hat{F}_1, \hat{F}_2, \hat{F}_3).$$ (10)
Further, noting that $\hat{Q}_3 = \hat{f}_{33}$ and $\hat{f}_{33} = -\epsilon (\hat{p}_3 - \hat{P}_3) + \hat{F}_3$ in this case, then we can write demand for value added from country 3 as:

$$\hat{V}_3 = \epsilon (1 - w)(1 - s^v)\hat{p}_1^v + \epsilon (1 - w)s^v\hat{p}_2^v - \epsilon (1 - w)\hat{p}_3^v + \hat{F}_3. \quad (11)$$

Demand for value added in country 3 depends on both prices of value added from countries 2 and 3, since they both provide value added into country 2’s final goods.

This special case shares two strong similarities with Case A. To see this, we again focus on price changes in country 2, setting $\hat{p}_2^v \neq 0$ and $\hat{p}_1^v = \hat{p}_3^v = 0$, and impose $\hat{F}_i = 0$ for all $i$ to focus on relative price effects. Then, the change in demand for value added in countries 1 and 3 in this special case is given by: $\hat{V}_1 = \epsilon s_{12} (1 - s_{23}w) s^v\hat{p}_2^v$ and $\hat{V}_3 = \epsilon (1 - w)s^v\hat{p}_2^v$. These expressions parallel the discussion of the size and distribution of spillovers in Case A.

The total size of changes in demand for value added is separable into two components: (a) the elasticity parameter $\epsilon$, and (b) trade and input-output linkages. This implies that the distribution of spillovers will be pinned down by trade and input-output linkages, invariant to the value of $\epsilon$. Both results clearly echo Case A. Further, a price decline in country 2 reduces demand for value added in both countries 2 and 3. This has a typical beggar-thy-neighbor intuition. Countries 1 and 2 both lose demand when country 2 gains competitiveness in this case, and the only relevant question is who loses more. This contrasts with the heterogeneous elasticity version of this framework, where there exist conditions under which demand for value added from country 1 can rise when value-added prices in country 2 fall.

These results serve as a useful benchmark, which we exploit to discipline comparisons across alternative parameterizations of the framework. First, in comparing Case A to Case B with homogeneous elasticities, we can match the total magnitude of value-added expenditure switching across models by choosing appropriate elasticities in each model. Given this, comparisons between the bilateral distribution of spillovers in Case A versus Case B have a natural interpretation. They tell us how an equal sized change in demand for value added from country 1 can rise when value-added prices in country 2 fall.

Second, these results also aid us in comparing across alternative elasticity parameterizations in Case B. The argument is essentially similar. We use the homogeneous elasticities case as a benchmark against which to compare spillover distributions with heterogeneous elasticities. Because the spillover distribution is invariant in the homogeneous elasticity case, we can implicitly match the total magnitude of value-added expenditure switching across these comparisons as well.

**Remark 7** Keeping our focus on the homogeneous elasticities case ($\epsilon \equiv \gamma = \sigma$), a second nice feature is that there a deep economic explanation as to why the results in Remark 6
look so similar to Case A. The reason is that the framework essentially collapses to a simpler CES framework with Armington preferences over value added.

Specifically, suppose that we were to define preferences directly over value added from particular source countries. Denoting \( v_{ji} \) as the quantity of real value added from country \( j \) consumed by \( i \), then final demand can be represented as:

\[
F_1 = v_{11}, \quad F_2 = \left( \frac{v_{12}^{(\epsilon-1)/\epsilon} + v_{22}^{(\epsilon-1)/\epsilon}}{\epsilon/(\epsilon-1)} \right) \\
F_3 = \left( \frac{v_{13}^{(\epsilon-1)/\epsilon} + v_{23}^{(\epsilon-1)/\epsilon} + v_{33}^{(\epsilon-1)/\epsilon}}{\epsilon/(\epsilon-1)} \right).
\]

These expressions capture the underlying pattern of trade in value added in our stylized example: country 1 sells value added to final purchasers in countries 2 and 3, in both cases embodied in country 2’s final goods.

Starting from these preferences, we can obtain an expression for demand for value added from country 1 that is identical to Equation (10). This explains why demand for value added in this case behaves like in Case A. The only real difference across the two cases is the pattern of trade. In Case A, we wrote down preferences taking gross trade as representing consumption patterns. In Case B, trade includes inputs, which implies that gross trade does not tell us where value added from a particular country is consumed. Nonetheless, with this homogeneous elasticity assumption, demand for value added behaves in conventional ways, once bilateral linkages are measured in value-added terms.

2 Framework

This section presents our general framework, extending the stylized framework above to accommodate general trade and input-output linkages across countries. As above, we continue to focus on partial equilibrium results, taking changes in the price of value added from each source country and real final expenditure in each destination as given. This approach requires us to specify only three basic components of the economic environment: (1) preferences over final goods, (2) production functions for gross output, and (3) market clearing conditions for gross output. Using this framework, we evaluate how changes in prices and real final expenditure in all countries are linked to changes in demand for value added from each source country. In Section 6, we discuss how this demand system can be embedded in general equilibrium.

2.1 Economic Environment

Suppose there are many countries indexed by \( i,j,k \in \{1, \ldots, N\} \). Each country is endowed with a production function for an aggregate Armington differentiated good, which is used

\[\text{The exact algebra is cumbersome, so we relegate the exercise to the appendix. We discuss this result in greater detail in the general version of the framework.}\]
both as a final good and intermediate input. Gross output in country $i$, denoted $Q_i$, is produced by combining domestic real value added, denoted $V_i$, with a composite intermediate input, denoted $X_i$. The composite input is a bundle of domestic and imported inputs, where inputs purchased by country $i$ from country $j$ are denoted $X_{ji}$.

We assume that the production structure takes the nested constant elasticity of substitution (CES) form:

$$Q_i = \left( \left( \omega_i^v \right)^{1/\gamma} V_i^{(\gamma-1)/\gamma} + \left( \omega_i^x \right)^{1/\gamma} X_i^{(\gamma-1)/\gamma} \right)^{\gamma/(\gamma-1)}$$

(12)

with

$$X_i = \left( \sum_j \left( \frac{\omega_{ji}^x}{\omega_i^x} \right)^{1/\rho} X_{ji}^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)}$$

(13)

where the $\omega$’s are aggregation weights, $\gamma$ is the elasticity of substitution between real value added and the composite input, and $\rho$ is the elasticity of substitution among inputs.\(^7\)

We assume that agents in each country have CES preferences defined of over final goods.\(^9\)

Denoting the quantity of final goods purchased by country $i$ from country $j$ as $F_{ji}$, preferences take the form:

$$F_i = \left( \sum_j \left( \omega_{ji}^f \right)^{1/\sigma} F_{ji}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

(14)

where $\omega$’s here denote preference weights and $\sigma$ is the elasticity of substitution among final goods.

Gross output can be used as both a final good and intermediate input, the market clearing condition for gross output is: $Q_j = \sum_{k=1}^{N} [F_{jk} + X_{jk}]$.

### 2.2 Linearization

The first order conditions for consumers and competitive firms are standard, as are the corresponding CES price indexes for gross output ($p_i$), the composite input ($p^x_i$), and the composite final good ($P_F$). To analyze these, we linearize and stack the first order conditions, price indexes, production functions, and market clearing conditions.

\(^7\)Domestic real value added can be thought of as a bundle of primary factor inputs. For example, we could model it as a Cobb-Douglas composite of capital and labor. Our discussion throughout the paper focuses on demand for the entire bundle of domestic inputs.

\(^8\)By assumption, only the elasticity between value added and inputs was relevant in stylized framework in Section [1]. Here we allow two separate elasticities in production.

\(^9\)We define final goods as in the national accounts, including consumption, investment, and government spending. Therefore, though we refer to ‘preferences’ throughout the paper, it might be more accurate to describe this as a final goods aggregator, which forms a composite final good used for consumption, investment, and by the government.
The final goods first order condition and final goods price index can be linearized as:
\[\hat{F}_{ji} = -\sigma(\hat{p}_j - \hat{P}_i) + \hat{F}_i, \quad \text{with} \quad \hat{P}_i = \sum_j \left(\frac{p_{ij}}{\hat{F}_{ij}}\right) \hat{p}_j.\]
We then define a vector \(\hat{F}\) to be a \(N^2\) dimensional vector that records final goods shipments: \(\hat{F} = [\hat{F}_{11}, \hat{F}_{12}, \ldots, \hat{F}_{1N}, \hat{F}_{21}, \hat{F}_{22}, \ldots]'\).

This allows us to rewrite the first order conditions and price index as:
\[\hat{F} = -\sigma M_1 \hat{p} + \sigma M_2 \hat{P} + M_2 \hat{F}, \quad \text{(15)}\]

where \(M_1 \equiv I_{N \times N} \otimes 1_{N \times 1}\) and \(M_2 \equiv 1_{N \times 1} \otimes I_{N \times N}\). The weighting matrix \(W_f\) is an \(N \times N\) matrix with \(ij\) elements equal to country \(i\)'s expenditure on final goods from country \(j\) as a share of total final goods expenditure in country \(i\).

Turning to production, the first order conditions for intermediates linearize as:
\[\hat{X}_i = -\gamma(\hat{p}_i^{x} - \hat{p}_i) + \hat{Q}_i, \quad \text{and} \quad \hat{X}_{ji} = -\rho(\hat{p}_j - \hat{p}_i) + \hat{X}_i. \quad \text{(10)}\]

These can be stacked in a similar way:
\[\hat{X} = -\gamma \hat{p}^x + \gamma \hat{p} + \hat{Q}, \quad \text{(17)}\]
\[\hat{X} = -\rho M_1 \hat{p} + \rho M_2 \hat{p}^x + M_2 \hat{X}, \quad \text{(18)}\]

where \(\hat{X} = [\hat{X}_{11}, \hat{X}_{12}, \ldots, \hat{X}_{1N}, \hat{X}_{21}, \hat{X}_{22}, \ldots]'\) is the \(N^2\) dimensional vector of intermediate goods shipments.

The market clearing conditions can be linearized as:
\[\hat{Q} = S_F \hat{F} + S_X \hat{X}, \quad \text{(20)}\]

The \(S_F\) and \(S_X\) matrices collect shares of final and intermediate goods sold to each destination as a share of total gross output in the source country:
\[S_F \equiv \begin{pmatrix} s_1^f & 0 & \cdots \\ 0 & s_2^f & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \quad \text{and} \quad S_X \equiv \begin{pmatrix} s_1^x & 0 & \cdots \\ 0 & s_2^x & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \]

with \(s_i^f = [s_{i1}^f, \ldots, s_{iN}^f]\), \(s_i^f = \frac{p_{ij} F_{ij}}{p_i Q_i}\), \(s_i^x = [s_{i1}^x, \ldots, s_{iN}^x]\), and \(s_i^x = \frac{p_{ij} X_{ij}}{p_i Q_i}\).

Finally, we linearize components of the production function and the gross output price

\[^{10}\text{We do not explicitly linearize the first order condition for real value added (V)}_i\text{ here because we do not use it in the derivation.}\]
index as:

\[
\hat{Q} = [\text{diag}(s^v_i)]\hat{V} + [\text{diag}(s^x_i)]\hat{X} \\
\hat{X} = W_X\hat{X} \\
\hat{p} = [\text{diag}(s^v_i)]\hat{p}^v + [\text{diag}(s^x_i)]\hat{p}^x,
\]

where \(s^v_i \equiv \frac{p_{i\text{V}}}{p_{iQ}}\) and \(s^x_i \equiv \frac{p_{iX}}{p_{iQ}}\) are the cost shares of real value added and the composite input in gross output. And \(W_X = [\text{diag}(w_1^x), \text{diag}(w_2^x), \ldots]\) with \(w_i^x = [w_{i1}^x, \ldots, w_{iN}^x]\) and \(w_{ij}^x \equiv \frac{p_{iX_{ij}}}{p_{jX_j}}\) are shares of individual intermediates in the composite intermediate.

### 2.3 Demand for Gross Output

Demand for gross output depends on demand for both final and intermediate goods. To begin, we take changes in final goods purchases (\(\hat{F}\)) as given, and study how price changes influence production techniques for final goods. We then add endogenous changes in demand for final goods in to arrive at demand for gross output.

#### 2.3.1 Substitution in Input Use

Using the first order conditions for intermediate input demand (Equations 17 and 18) and the aggregate input price index (Equation (19)), we substitute out for bilateral input shipments (\(X\)) in the gross output market clearing condition (Equation (20)) to yield:

\[
\hat{Q} = S_F\hat{F} + S_X \left[-\rho M_1\hat{p} + \rho M_2\hat{p}^x + M_2(-\gamma\hat{p}^x + \gamma\hat{p} + \hat{Q})\right]
\]

Note that there is an input-output loop in production here, as gross output appears on both sides of this expression. Pulling output to one side, and using \(\hat{p}^x = W_x\hat{p}\) to eliminate the composite input price, we get:

\[
\hat{Q} = [I - S_X M_2]^{-1}S_F\hat{F} + [I - S_X M_2]^{-1}S_X [-\rho M_1\hat{p} + \rho M_2 W_x\hat{p} - \gamma M_2 W_x\hat{p} + \gamma M_2 \hat{p}] .
\]

The first term maps changes in bilateral final goods shipments (\(\hat{F}\)) through the initial input-output structure into changes in gross output. The second term captures how input choices, and hence the input-output structure, respond to gross output price changes. This input re-optimization alters the mapping from final goods to gross output. Further, note that input choices obviously depend on the production elasticities (\(\gamma\) and \(\rho\)). We provide additional discussion of both terms in appendix.
2.3.2 Substitution across Final Goods

Changes in demand for final goods depend on relative prices as well. Substituting for $\hat{F}$ in Equation (25) using Equation (15), we get:

$$\hat{Q} = [I - S_X M_2]^{-1} S_F M_2 \hat{F} - \sigma [I - S_X M_2]^{-1} S_F (M_1 - M_2 W_f) \hat{p}$$

$$- \rho [I - S_X M_2]^{-1} S_X (M_1 - M_2 W_x) \hat{p} + \gamma [I - S_X M_2]^{-1} S_X M_2 (I - W_x) \hat{p}. \quad (26)$$

The first term captures the role of changes in real final expenditure levels in altering demand for output. The second term picks up substitution in final goods purchases, hence the presence of the final goods elasticity $\sigma$ there. As above, the third term picks up substitution within the input bundle, and the fourth term picks up substitution between real value added and inputs. In the end, how price changes feed through to demand for gross output depends on both supply side elasticities ($\gamma, \rho$) and the demand side elasticity ($\sigma$).

2.4 Demand for Value Added

We need to convert demand for gross output into demand for value added. To do this, observe that rearranging the production function (Equation (21)) yields:

$$\hat{V} = \left[ diag(s^v_i) \right]^{-1} \left[ \hat{Q} - \left[ diag(s^x_i) \right] X \right]. \quad (27)$$

This formula corresponds to the definition of double-deflated real value added. Double deflation strips out changes in input use from changes in gross output to recover changes in real value added.

Pushing further, we substitute for $\hat{X}$ using Equations (17) and (19):

$$\hat{V} = \hat{Q} - \gamma \left[ diag(s^x_i/s^v_i) \right] (I - W_x) \hat{p}. \quad (28)$$

An increase in country $i$’s gross output price lowers the price of inputs relative to the price of gross output ($\hat{p}^x_i - \hat{p}_i < 0$), so producers substitute towards produced inputs and real value added falls relative to gross output.

---

11 The price of source country goods relative to the destination price level is captured in $(M_1 - M_2 W_f) \hat{p}$. When $\sigma > 0$, a rise in the relative price of country $i$ goods lowers demand for that country’s gross output, as all countries substitute away in their consumption baskets.
Combining Equations (26) and (28), we arrive at:

\[
\hat{V} = [I - S_X M_2]^{-1} S_F M_2 \hat{F} \\
- [I - S_X M_2]^{-1} \left[ \sigma S_F (M_1 - M_2 W_f) + \rho S_X (M_1 - M_2 W_x) - \gamma S_X M_2 (I - W_x) \right] \hat{p} \\
- \gamma \left[ \text{diag} \left( s^\tau_i / s^v_i \right) \right] (I - W_x) \hat{p}.
\]  

(29)

This summarizes how demand for real value added depends on the level of real final expenditure in all countries (\( \hat{F} \)) and gross price changes (\( \hat{p} \)).

### 2.5 Linking Value-Added to Gross Output Prices

As a final step, we substitute for gross price changes to write demand for real value added in terms of value-added prices. To do this, we combine the price indexes for gross output (Equations (23)) and the composite input (Equation (19)) to write gross output price changes as function of value added price changes. The resulting formula takes the form:

\[
\hat{p} = [I - \Omega']^{-1} \text{diag}(s^v_i) \hat{p}^v,
\]  

(30)

where we note that \( \Omega' = \text{diag}(s^\tau_i) W_X \) is a global input-output matrix, with \( ij \) elements equal to the share of inputs from \( i \) purchased by \( j \) in total gross output of country \( j \). Two points about this formula are important to note.

First, gross output price changes are a weighted average of value-added price changes (\( \hat{p}^v \)), where the weights reflect total cost shares. These weights capture a basic fact: when supply chains cross borders, the price of gross output depends on both domestic and foreign factor prices. If the price of value added in country \( i \) rises, it raises the price of gross output in all downstream countries. Further, when the price of domestic value added rises, the price of gross output rises less than one for one.

Second, the link between value-added and gross output prices involves no elasticities, only production input shares. The reason is that substitution effects are not first-order in construction of price indexes, and Equation (30) combines linear approximations the gross output and composite input price indexes. Due to the absence of substitution effects, this value-added to gross output price mapping holds in all variants of the framework that we discuss below.

---

\[\text{The } ij \text{ elements of } [I - \Omega']^{-1} \text{ describe the amount of gross output from country } j \text{ used directly or indirectly in producing gross output in country } i, \text{ and then the value-added to output ratios } s^v_i \text{ rescale these to reflect how important value added from } j \text{ is in producing gross output in } j.\]
2.6 Summary

To recap, there were three steps in the analysis. First, we described how demand for gross output depends on gross output prices and final expenditure levels. Second, we applied a double deflation procedure to convert changes in demand for gross output into changes in demand for value added. Together, these steps yielded Equation (29), describing how demand for value added depends on gross prices and aggregate expenditure levels $\hat{F}$. Third, we mapped value-added price to gross output prices through the global input-output structure, summarized in Equation (30). This is the same procedure that we summarized in Remark 1 above.

Combining Equations (29) and (30), we have a complete description of demand for value added in terms of aggregate expenditure levels $\hat{F}$ and value-added prices $\hat{p}^v$. This linear system that takes the stylized form:

$$
\hat{V} = -[\sigma T_\sigma + \rho T_\rho + \gamma T_\gamma] \hat{p}^v,
$$

with $T_\sigma \equiv [I - S_X M_2]^{-1} S_F (M_1 - M_2 W_f)[I - \Omega']^{-1} [\text{diag}(s_{v_i}^v)],$ $T_\rho \equiv [I - S_X M_2]^{-1} S_X (M_1 - M_2 W_x)[I - \Omega']^{-1}[\text{diag}(s_{x_i}^v)],$ and $T_\gamma = \left[[\text{diag}(s_{x_i}^x/s_{v_i}^v)] - [I - S_X M_2]^{-1} S_X M_2 (I - W_x)[I - \Omega']^{-1}[\text{diag}(s_{v_i}^v)] \right].$ The first term captures substitution in final goods. The second captures substitution within the input bundle. The third captures changes in demand for the composite input and the impact of those changes on the relationship between gross output to real value added.

As we vary individual elasticities, we are put more or less weight on different forces in the model and thus alter both the size and distribution of changes in value added across countries. We now turn to discussing key aspects of this system in detail.

3 The Mechanics of Demand for Value Added

The framework in Section 2 maps value-added prices to demand for value added, and this mapping depends on both gross bilateral final and intermediate goods shipments, as well as supply and demand side elasticities. In this section, we discuss how different elasticity parametrizations alter the balance of forces driving changes in demand for value added following price shocks. Further, we highlight how the seven remarks enumerated in discussion of the stylized example translate into the general framework.

We begin by interpreting demand for value added in a benchmark case in which all the elasticities are equal in the model. With this restriction, we show that demand for value added takes a familiar CES form, as if consumers have preferences defined over value
added. This result is linked to Remarks 2, 3, 6, and 7 above. When production and demand elasticities are not equal, then the behavior of demand for value added deviates from this conventional case. We discuss aggregation of heterogeneous elasticities and bilateral spillovers, generalizing ideas introduced in Remarks 4 and 5.

3.1 Equal Elasticities: The Value-Added Armington-CES Model

Suppose that $\epsilon \equiv \gamma = \rho = \sigma$, so substitution elasticities are identical throughout the model. In this case, we can write demand for real value added as:

$$\hat{V} = -\epsilon \left( \hat{p}^v - \hat{p}^w \right) + \hat{F}^w$$

with $\hat{P}^w \equiv [I - SXM_2]^{-1}SFM_2W_f[I - \Omega']^{-1}[\text{diag}(s^w_i)]\hat{p}^v$

and $\hat{F}^w \equiv [I - SXM_2]^{-1}SFM_2\hat{F}$. (32)

Equation (32) tells us something intuitive: each country faces a single CES demand schedule for the value added it produces, as if each country sells value added to a single world market. The vectors $\hat{P}^w$ and $\hat{F}^w$ contain the aggregate price levels and final demand levels that each country faces in selling to the hypothetical world market. Demand for value added from country $i$ falls when the price of its own value added rises, relative to its perceived world price of value added (the $i^{th}$ element of $\hat{P}^w$), with an elasticity of $\epsilon$.

The perceived world prices of value added ($\hat{P}^w$) are weighted averages of price changes for value added originating from all countries. The weighting scheme for mapping from $\hat{p}^v$ to $\hat{P}^w$ has two components. The first component is: $W_f[I - \Omega']^{-1}[\text{diag}(s^w_i)]$, where again $\Omega$ is a global input-output matrix. This takes value-added prices, and aggregates them to form the price level for final demand in each destination market, where $[I - \Omega']^{-1}[\text{diag}(s^w_i)]$ converts value-added to gross output prices and $W_f$ aggregates these into final goods price indexes. The weighting scheme takes the form: $\hat{P}_j = \sum_k \left( \frac{p^V_{kj}}{p^w_{kj}} \right) \hat{p}_k$, where $V_{kj}$ is the amount of real value added from $k$ embodied in final goods absorbed in $j$.

The second component is: $[I - SXM_2]^{-1}SFM_2$. Each $ij$ element records the share of gross output from each source country $i$ used directly or indirectly to produce final goods absorbed in destination $j$. These weights are equal to the share of value added from source $i$ absorbed embodied in final goods in destination $j$: $\frac{p^V_{ij}}{p^w_{ij}}$. That is, they are export shares measured in value added terms. These shares measure the importance of destination $j$ in demand for value added from source $i$. The level of perceived demand ($\hat{F}^w$) is also computed.

---

13 We exploit several convenient cancellations to get from Equations (29) and (30) to Equation (32), which we discuss in the Appendix.

14 Note that $\sum_k \frac{p^V_{kj}}{p^w_{kj}} = 1$, since final goods are 100% value added attributable to some source country.
Using these value-added export shares\textsuperscript{15}

Combining these elements, we can re-write Equation (32) in summation notation as:

\[
\hat{V}_i = -\eta \left( \hat{p}_i^v - \hat{P}_i^w \right) + \hat{F}_i^w \\
\text{with } \hat{P}_i^w = \sum_j \left( \frac{p_i^v V_{ij}}{p_i^v V_i} \right) \hat{P}_j \quad \text{where } \hat{P}_j = \sum_k \frac{p_k^v V_{kj}}{\hat{F}_j F_{kj}} \hat{P}_k, \\
\text{and } \hat{F}_i^w = \sum_j \left( \frac{p_i^v V_{ij}}{p_i^v V_i} \right) \hat{F}_j. 
\]

(33)

Reflecting the discussion above, this formulation highlights that value-added trade patterns define which countries are important destination markets for a given source country, and which other countries provide competition in those destinations.

This CES-demand interpretation suggests an alternative way to characterize demand for value added for this case. Rather than specifying the entire gross production and trade framework, we could instead assume that countries produce and trade value added directly. This bypasses the intermediate step via which value added is aggregated into commodities prior to being sold to consumers, and instead connects consumers to producers of value added directly. Specifically, we could write preferences directly over value added coming from different countries, as in

\[
F_i = \left( \sum_j (\omega_{ji}^v)^{1/\eta} V_{ji}^{(\eta-1)}/\eta \right)^{\eta/(\eta-1)}, \quad \text{where } \omega_{ji}^v \text{ is now a value-added preference weight}. \textsuperscript{16}
\]

These preferences generate a CES demand system that can be combined with market clearing conditions for value added to yield Equation (33). Thus, one can in the end re-interpret the demand for value added formula as derived from an CES-Armington demand system for value added.

This CES-Armington interpretation is attractive because it connects our gross framework to conventional value-added macro-models, which abstract from production and trade in intermediate inputs. These models feature production functions for value added, so preferences in these models implicitly describe how consumers substitute across value added from alternative destinations. This means that value-added export data – which record how much value added from each source is sold in each destination – ought to be used in calibrating those preferences. This contrasts with current practice, which typically uses gross import

\textsuperscript{15} This weighting scheme is identical to the final demand weights in Bems, Johnson, and Yi (2010). In that paper, we assumed that technology and preferences were both Leontief ($\epsilon = 0$). Hence demand for value added depended on value-added exports weighted changes in final demand, but was independent of price changes in that paper. An alternative way to interpret Bems et al. is that we assumed that price changes were zero (i.e., $\hat{p} = 0$). Equation (32) generalizes this result by dropping this restrictive assumption.

\textsuperscript{16} One way to think about these preferences is that they combine two aggregation steps. They capture substitution across commodities along with substitution across value added in the production of commodities. So, the elasticity of substitution in preferences is an amalgam of elasticities in preferences with elasticities in production.
data, along with total final expenditure, to calibrate preference shares\(^{17}\).

These results and interpretation emphasize that – when elasticities are not too different – demand for value added behaves in standard ways. Increases in the price of value-added from country \(i\) will put downward pressure on demand for value-added, as agents substitute away from final and intermediate goods that embody value added from country \(i\) and country \(i\) itself substitutes toward using produced inputs more intensively. And demand for value added from foreign countries \(j\) will rise.

3.2 Heterogeneous Elasticities and Spillovers

Moving away from the case with equal elasticities, demand for value added no longer takes the constant elasticity form, and so new concerns emerge. Most importantly, the relative values of production versus final goods elasticities shift the balance of different forces in determining demand for value added. In this section, we describe how price changes in one country affect demand for value added at home and abroad. We emphasize that both input linkages and relative elasticities influence the aggregate size and bilateral distribution of these demand spillovers.

3.2.1 Aggregate and Bilateral Spillovers

For any elasticity triple \(\{\sigma, \rho, \gamma\}\) and vector of price changes, we can compute the response of demand for value added in all countries using Equation (31), holding final expenditure constant to isolate the expenditure switching impact of price changes. To emphasize the dependence of changes in demand for value added on elasticities, we now write the corresponding changes as \(\{V_i(\sigma, \rho, \gamma)\}\). In the case that \(\sigma = \rho = \gamma\), then we denote the change \(\{V_i(\epsilon)\}\).

To characterize spillovers from country \(i\) to foreign partners, we compute changes in demand for value added that follow from a change in value-added prices in country \(i\) \((\hat{p}_i \neq 0)\), holding prices in all other countries fixed \((\hat{p}_j = 0 \ \forall \ j \neq i)\). We define the aggregate foreign spillover of a change in value-added prices in country \(i\) as the change in demand for country \(i\)’s real value added: \(\hat{V}_i(\sigma, \rho, \gamma; i)\), where the letter after the semi-colon indicates the country whose prices change. We then describe relative spillovers across bilateral partners by examining the change in demand for value added in each foreign country relative to country \(i\).

\(^{17}\)Under the assumption that \(\sigma = \rho = \gamma\), the elasticity of substitution in the value-added formulation of preferences is the same as the gross trade elasticity. Therefore, gross elasticities can be used directly to parameterize value-added preferences in this special case.
We first characterize spillovers in the equal elasticity case, and then discuss spillovers with heterogeneous elasticities with reference to this case.

**Equal Elasticities** With equal elasticities, a rise in the price of value added in country \(i\) yields decreased demand for value added from country \(i\) and increased demand for value added in foreign countries \(j\). The changes in demand are given by: \(\hat{V}_i(\sigma; i) = -\epsilon T^{ii} \hat{p}_i^v\) and \(\hat{V}_j(\epsilon; i) = \epsilon T^{ji} \hat{p}_i^v\), where \(T^{kl} \equiv T^{kl}_\sigma + T^{kl}_\rho + T^{kl}_\gamma\) and \(\{T^{kl}_\sigma, T^{kl}_\rho, T^{kl}_\gamma\}\) are the \(kl\) elements of \(T_\sigma, T_\rho, \) and \(T_\gamma\) respectively. Writing out these formulas explicitly using Equation \(33\) gives us:

\[
\hat{V}_i(\epsilon; i) = -\epsilon \left[ 1 - \sum_k \left( \frac{p_i^V k}{p_i^V i} \right) \left( \frac{p_j^V k}{p_j^V j} \right) \right] \hat{p}_i^v \quad \text{and} \quad \hat{V}_j(\epsilon; i) = \epsilon \sum_k \left( \frac{p_i^V k}{p_j^V j} \right) \left( \frac{p_j^V k}{p_i^V i} \right) \hat{p}_i^v.
\]

There are two important points to take away from this calculation. First, the level of aggregate spillovers here is the product of the elasticity \(\epsilon\) and a summary measure of trade \(T^{ii}\), echoing Remark 6. The trade measure \(T^{ii}\) captures the impact of the country \(i\) price change on the price of country \(i\)'s value added relative to its hypothetical world price, and is constructed using value-added trade flows. Given trade patterns (fixing \(T^{ii}\)), aggregate spillovers change one-for-one with the elasticity \(\epsilon\), so changes in this elasticity summarize changes in the strength of aggregate spillovers. Second, the distribution of spillovers across bilateral partners is given by: \(\hat{V}_j(\epsilon; i)/\hat{V}_i(\epsilon; i) = T_{ji}/T_{ii}\). This distribution depends on the structure of value-added trade alone. Thus, for given trade flows, the distribution of spillovers across partners is invariant to changes in the overall strength of foreign spillovers.

**Heterogeneous Elasticities** When we allow the elasticity parameters to be heterogeneous, new results concerning the size and distribution of spillovers emerge. Following the discussion above, we can write changes in home and foreign demand for value added as:

\[
\hat{V}_i(\sigma, \rho, \gamma; i) = - \left[ \sigma T^{ii}_\sigma + \rho T^{ii}_\rho + \gamma T^{ii}_\gamma \right] \hat{p}_i \quad \text{and} \quad \hat{V}_j(\sigma, \rho, \gamma; i) = - \left[ \sigma T^{ji}_\sigma + \rho T^{ji}_\rho + \gamma T^{ji}_\gamma \right] \hat{p}_i.
\]

As an expositional device, it is helpful to re-write the change in demand for country \(i\) as:

\[
\hat{V}_i(\sigma, \rho, \gamma; i) = - \left[ \sigma T^{ii}_\sigma + \rho T^{ii}_\rho + \gamma T^{ii}_\gamma \right] T^{ii} \hat{p}_i \equiv \tilde{\epsilon}_i.
\]

By multiplying and dividing by \(T^{ii}\), we link this heterogeneous elasticity case to the homogeneous elasticity case.

We refer to the term \(\tilde{\epsilon}_i\) as the effective elasticity of substitution between home and foreign value added. The elasticity \(\tilde{\epsilon}_i\) yields the same change in demand for value added.
from country $i$ in a value-added Armington-CES framework as one obtains from the true heterogeneous elasticity framework. Mathematically, $\hat{V}_i^*(\sigma, \rho, \gamma; i) = \hat{V}_i^*(\tilde{\epsilon}_i; i)$. In this way, $\tilde{\epsilon}_i$ summarizes the strength of aggregate value-added expenditure switching under alternative heterogeneous elasticity triples.

This effective value-added elasticity is also a convenient way to summarize differences in the strength of spillovers across countries, since it strips out the role of value-added trade openness (embedded in $T_{ii}$) in explaining differences in realized spillovers across countries. The strength of aggregate spillovers differs across countries in our framework, for a given set of fundamental elasticities, because trade patterns differ. Countries where input trade is important will tend to put higher weight on input elasticities than countries where final goods are important. If the input elasticity is low relative to the final goods elasticities, then this will imply they would have relatively low effective value-added elasticities. With this motivation, we will document the spread in $\tilde{\epsilon}_i$ across countries in our results.

Turning to bilateral spillovers, we can write the relative spillover on country $j$ as:

$$\frac{\hat{V}_j^*}{\hat{V}_i^*} = \frac{\sigma T_{ji} + \rho T_{ji} + \gamma T_{ji}}{\sigma T_{i} + \rho T_{i} + \gamma T_{i}}.$$

(35)

This formula implies that changes in relative elasticities redistribute bilateral spillovers. One way to think about this is by rewriting Equation (35) as follows:

$$\frac{\hat{V}_j^*}{\hat{V}_i^*} = \frac{\tilde{\epsilon}_i T_{ji}}{T_{ji}} \text{ with } \tilde{\epsilon}_i \equiv \frac{\sigma T_{ji} + \rho T_{ji} + \gamma T_{ji}}{\sigma T_{i} + \rho T_{i} + \gamma T_{i}}.$$

(36)

Recall that $T_{ji}$ tells us the magnitude of bilateral spillovers in the homogeneous elasticity case. Then spillovers are scaled up relative to this benchmark for country $j$ when $\tilde{\epsilon}_ji > \tilde{\epsilon}_i$. This happens when trade linkages between $i$ and $j$ imply greater weight is attached to high elasticity components of trade than is true for country $i$ overall. For example, if input linkages are stronger for $i$ and $j$ than for $i$ with other partners, and the elasticity of substitution in inputs is high, then we expect stronger spillovers for shocks from $i$ to country $j$ than for $i$ to other partners.

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19 An equiproportional increase in all elasticities leaves the bilateral distribution of spillovers unchanged, as it simply scales up the aggregate spillover from country $i$ to the rest of the world.
4 Taking the Framework to Data

In this section, we introduce our data sources for parameterizing the framework, and discuss two important empirical components of the model. The first is the mapping from the primitive elasticities \( \{\sigma, \rho, \gamma\} \) to the aggregate, country-specific effective elasticity of substitution between home and foreign value added implied by these elasticities. The second is the mapping between changes in the price of domestic value added and the relative price of home to foreign gross output.

4.1 Global Input-Output Data

To parameterize the model, we populate matrices \( S_X, S_F, W_X, W_F, \) and \( \Omega \), along with measuring production shares \( s^v_i \) and \( s^x_i \). To do this, we need data on the value of gross output and value added by country, and the value of bilateral shipments of both final and intermediate goods.

We obtain these values from two alternative data sets. Our primary data source is the World Input-Output Database (WIOD) [Timmer (2012)], which covers 40 individual countries, plus a composite rest-of-the-world (ROW) region from 1995-2009. The 40 countries includes the 27 European Union countries and 13 other major developed and emerging markets, accounting for which account for 85-90% of world GDP.\(^{20}\) The underlying raw data on which the tables are based includes national accounts production data, country-specific supply and use tables from each country’s input-output accounts, and sector-level bilateral trade in final and intermediate goods.\(^{21}\) We aggregate across the 35 sectors available in the WIOD data to define the values needed for our one sector framework.\(^{22}\)

We switch to a secondary data source – the Global Trade Analysis Project Database (GTAP) for 2004 – for several figures in which we focus on Asia. The reason we switch is simply that WIOD only includes a few large Asian countries, and we need more detailed country coverage to document spillovers within the Asian production chain.

\(^{20}\)Croatia is the only current EU member not covered. The non-EU countries include Australia, Brazil, Canada, China, India, Indonesia, Japan, Mexico, Russia, South Korea, Taiwan, Turkey, and the United States.

\(^{21}\)For goods trade, these are measured by concording disaggregate trade commodity codes with end use classifications.

\(^{22}\)Our framework can be extended in a straightforward way to incorporate multiple sectors. Studying the role of intra-country relative price changes and sectoral reallocation in understanding the transmission of international relative price shocks is a topic for further work. See Bems (forthcoming) and Johnson (forthcoming) for discussion of related multi-sector models.
4.2 Mapping Gross to Effective Value-Added Elasticities

In Equation (34), we defined the effective elasticity of substitution between home and foreign value added as a country-specific weighted average of the primitive elasticities. In this section, we characterize how this spillover elasticity varies with changes in the underlying elasticities. This analysis summarizes broadly how sensitive aggregate spillovers are to changes in individual elasticities.

Recalling the definition of the effective elasticity, we can write: $\Delta \tilde{\epsilon}_i = \frac{T_{ii}}{\tilde{\sigma}} \Delta \sigma + \frac{T_{ii}}{\tilde{\rho}} \Delta \rho + \frac{T_{ii}}{\tilde{\gamma}} \Delta \gamma$. If we set $\Delta \sigma = \Delta \rho = \Delta \gamma = 1$, then we get $\Delta \tilde{\epsilon}_i = 1$ by construction, so a one unit change in the effective value-added elasticity. We can decompose this unit change in spillovers into contributions coming from each component by looking at the weights attached to changes in each primitive elasticity.

We plot these weights in Figure 1. As is evident, the weight attached to changes in $\rho$ is largest, at 50-60% for most countries. The weight attached to $\sigma$ is about 30%, while the weight attached to $\gamma$ accounts for the remainder. What this means is that the effective value-added elasticity will be most sensitive to the choice of $\rho$ – the elasticity of substitution among inputs. Further, if we are interested in thinking about whether changes in final goods elasticities or input elasticities are more important in explaining changes in the size of total spillovers, the answer is unequivocally input elasticities. Together, changes in $\rho$ and $\gamma$ induce large changes in the effective value-added elasticity.

4.3 Mapping Value-Added Prices to Gross Output Prices

Equation (30) states that the price of gross output in each country is a weighted average of the underlying prices of value added in all countries, where the weights reflect total cost shares. An immediate implication is that a change in the price of domestic value added will translate less than one-for-one into changes in the price of domestic gross output. Further, foreign output prices will move in the same direction as home output prices, since they put positive weight on domestic value-added. Therefore, if the price of home value added falls, the relative price of home versus foreign gross output will (i.e., the gross output terms of trade will depreciate less) will fall substantially less than one-for-one with the price of home value added. The empirical question is: how much less?

We can examine pass through simply by computing gross output price changes following idiosyncratic changes in value-added prices in each country. We then compute the gross output terms of trade as the change in the price of gross output less the median change in the foreign price of gross output in response to a change in the price of home’s value added.\footnote{The results are essentially identical if we take a imported weighted average of foreign gross output price}
The results are plotted in Figure 2 for selected countries. The gross output terms of trade falls by 0.6-0.8 percentage points for a one percentage point decline in the price of home value added. This indicates substantially muted gross output terms of trade responses. Movements in the terms of trade are substantially smaller for countries heavily integrated into global supply chains, such as Taiwan, Ireland, Hungary, and the Czech Republic.

An important further point is that the dampening effect of supply chains on gross output terms of trade movements is increasing over time. To illustrate this, we plot the median response across countries for the gross output terms of trade in response to a one percentage point change in the price of domestic value added in Figure 3. The median response has fallen from about 0.82 to 0.76 over the 1995-2008 period, consistent with the rise of vertical specialization over this period.

5 Relative Price Changes and Demand Spillovers

In Section 3.2 we articulated that both the level and distribution of foreign spillovers across bilateral partners depends on the relative elasticities of substitution in production and demand. In this section, we examine these spillovers in the parameterized framework.

To focus our discussion, we contrast one heterogeneous elasticity scenario with the homogeneous elasticity benchmark. The heterogeneous elasticity scenario is one in which substitution in production is limited, with Leontief production functions \((\rho = \gamma = 0)\) and \(\sigma = 4.5\). Our interest in this case is motivated by the commonly held view that production chains are ‘rigid’ or ‘inflexible’ in the short run, whereby producers find it difficult (if not impossible) to substitute across suppliers. This view is consistent with anecdotes regarding supply chain disruptions following the 2011 Japanese earthquake and floods in Thailand [Escaith, Teh, Keck, and Nee (2011), Fujita (2013)]. It is also consistent with emphasis on complementary in input sourcing in recent work on business cycle comovement and the TFP cost of misallocation [Burstein, Kurz, and Tesar (2008), DiGiovanni and Levchenko (2010), Jones (2011)].

In each scenario, we examine a uniform decrease in home relative to foreign prices, as in \(\hat{p}_i^h - \hat{p}_j^h = -1, \forall i \neq j\) for a given home country \(i\). Given these price changes, and assuming \(\hat{F} = 0\) to isolate relative price effects, we compute changes in demand for value added in all countries \(\hat{V}\) using Equations (29) and (30) (equivalently, Equation (31)). We repeat this exercise for price changes in different home countries. We report aggregate changes, as would be standard in computing the terms of trade.

\(24\) Given \(\rho = \gamma = 0\), we choose the value of \(\sigma\) here to yield an aggregate ‘trade elasticity’ near 1.5 in our framework.
spillovers – equivalently, the effective value-added elasticity – for 30+ countries, and then
discuss bilateral spillovers for price changes in Germany, China, and the United States as
representative cases.

5.1 Aggregate Spillovers

Building on Section 3.2, we summarize the size of aggregate spillovers by computing the
effective elasticity of substitution between home and foreign value added for each country
that results from setting $\rho = \gamma = 0$ and $\sigma = 4.5$. We present these effective value-added
elasticities by country in the left panel of Figure 4. Note that in a conventional macro-model
that ignores input trade, or a gross model with equal elasticities throughout the model,
these effective elasticities would be equal across all countries. In contrast, they vary across
countries in our framework, ranging from less than 1 for Indonesia (IDN) and Taiwan (TWN)
to over 1.6 in France (FRA) and Spain (ESP). That is, changes in relative prices in countries
like France and Spain will induce larger changes in demand for foreign value added than
equally sized price changes in Indonesia or Taiwan.

To organize this heterogeneity, we plot the effective value-added elasticities against the
share of final goods in total trade for each country in the right panel of Figure 4. As is evident,
there is sizable variation in the final goods share of trade across countries. And there is a
strong positive correlation between effective elasticity and final goods share of trade and the
effective value-added elasticity. Countries that are more involved in global supply chains,
and hence have larger shares of intermediate inputs in their trade, have lower effective value-
added elasticities. This is because they put larger weight on production elasticities, which
are set to zero here, than on the final goods elasticity. As a result, demand spillovers are
more muted for these countries.

As is well known, global supply chains have generically become more import in inter-
national trade over the last few decades. In a conventional macro-model that
ignores input trade, or a gross model with equal elasticities throughout the model, this has
no impact on value-added elasticities. In contrast, with heterogeneous elasticities, the rise
of global supply chains tends to push the effective value-added elasticity down over time. We
plot these changes for representative countries and the sample median in Figure 5 between
1995 and 2008. We see gradual decreases in effective value-added elasticities, especially from
2000 onwards. The effective elasticity decreased by 20% for China and 30% for Korea, who
have both deepened their involvement in Asian supply chains.

To sum up, if one believes that supply chains are relatively inflexible, then the sensitivity
of demand for value added to value-added price changes appears to be heterogeneous across
countries, lower for countries involved in supply chains, and falling over time.

5.2 Bilateral Spillovers

We now turn to describing bilateral spillovers. We start by tracing out the impact of a decline in the price of value added from Germany on its partners. We illustrate results for identical exercises for China and the United States. In each exercise, we normalize bilateral spillovers as in Section 3.2, taking the change in demand for value added in each foreign country and dividing it by the change in value added in home country. By doing this, we can compare the pattern of spillovers in our heterogeneous elasticities version of the model to the homogeneous elasticities version of the model. In effect this normalization tells in each scenario how a one percentage point change in demand for GDP in the home country, which implies an equivalently sized decrease in demand for foreign value added, is distributed across its partners.

We report results for a decline in the price of value added in Germany in Figure 6. In the left panel, we present percentage changes in demand for value added in the homogeneous and heterogeneous elasticity cases. As would be expected, spillovers are concentrated among Germany’s neighbors, where bilateral trade linkages are strong. In terms of magnitudes, a one percentage point increase in demand for German real value added induces a 0.25-0.3 percentage point fall in demand for real GDP from Austria, the Czech Republic, Hungary, and Slovakia. Turning to the Leontief production model, spillovers are significantly smaller in these neighboring countries, and off-set with larger negative spillovers in the EMU periphery (Greece, Ireland, Italy, and Spain) and other major markets (China, Japan, U.S.). Differences in the impact of the price change across models is summarized in the right panel of Figure 6, which reports the difference in spillovers in the conventional and Leontief models.

What explains these differences? With Leontief production, the impact from a fall in the relative price of German value added shifts away from countries that trade inputs with Germany and towards countries that trade final goods with Germany. That is, spillovers are mitigated for supply chain partners when supply chains are rigid, since demand for inputs is not price sensitive. To summarize this effect, Figure 7 plots differences in bilateral spillovers across the two models against the share of final goods in bilateral trade with Germany for each country. Similar to aggregate spillovers, depicted in the right panel of Figure 4, the share of final goods in trade is a strong predictor of the strength of spillovers.

Turning to China, the basic ideas developed in the German example hold as well. Results for China are presented in Figure 8. In the homogeneous elasticity version of the model, negative spillovers are concentrated in regional trade partners, such as Hong Kong, Malaysia,
South Korea, Taiwan, etc. Shifting to the Leontief case, spillovers are reallocated away from these regional supply chain partners, and these large adjustments push spillovers for these countries down substantially. Remarkably, demand for real value added from Vietnam actually increases following the fall in Chinese prices. This demonstrates that, in principle, supply chain linkages can actually flip the sign on bilateral spillovers relative to conventional models.

Finally, we consider spillovers for changes in the price of United States value added in Figure 9. For NAFTA partners Canada and Mexico, demand for value added falls dramatically – more than one-for-one in the homogeneous elasticity model. In the Leontief scenario, these declines are attenuated, as would be expected based on North American supply chain integration, but still remain large for Mexico and Canada. One reason for this is that bilateral trade within North America actually includes a lot of trade in final goods. For example, the bulk of Mexican exports to the US are finished products, which compete with US produced final goods in the US. This is somewhat unlike the China and Germany cases, where production sharing partners with China and Germany tend to supply inputs but not compete as intensely head-to-head with China in final goods markets.

Looking outside North America, we do see the same pattern of shifting bilateral spillovers away from countries most involved in input trade with the US (e.g., Ireland). For countries where this reallocation happens, the economic size of the effects tends to be large. The negative impact on demand for value added in Ireland is reduced from -0.75% to -0.35%. On the other end of the spectrum, the negative impact on demand for value added from China is increased from -0.5% to -0.7%.

6 Closing the Framework

The framework we presented in Section 2 links demand for value added to value-added prices and final expenditure levels, as in $\hat{V} = f(\hat{p}^v, \hat{F})$. To embed this demand framework in general equilibrium, we need additional assumptions about how final expenditure and the supply of value added (and hence equilibrium value-added prices) are determined. In this section, we sketch several simple ways to close the model, which reflect the types of closures adopted in different streams of research. Rather than pursuing one particular model in detail, our main objective is to illustrate how the results regarding the behavior of demand for value added can be applied in a variety of contexts.

We start by discussing the determination of final expenditure levels, with reference to the budget constraints of the final consumer. Consistent with the static framework we developed above, we assume the budget constraint takes the form: $p_i^e V_i + T_i = P_{Fi} F_i$, where $T_i$ is an
exogenous transfer received by country \(i\), with \(\sum_i T_i = 0\). When \(T_i = 0\) for all \(i\), we then have balanced trade in all countries. Setting trade balances to zero, or holding them constant, then these budget constraints imply changes in final expenditure are a function of value-added price changes and changes in real value added, so \(\hat{\mathcal{F}} = g(\hat{p}_v, \hat{V})\). Using this, we can re-write demand for value added, eliminating final expenditure levels: \(\hat{V} = f(\hat{p}_v, g(\hat{p}_v, \hat{V}))\). Since our framework is linear, we can then easily solve for \(\hat{V}\) as a function of prices alone.

To completely close the framework, we need to specify how the supply of real value added is determined. At a primitive level, real value added is produced using factor inputs, so one should think of this step as specifying how factor supplies are determined. To bracket the possible alternatives, we discuss both perfectly elastic and perfectly inelastic factor supplies, and then note that general environments with endogenous factor supply lie between these extremes.

**Perfectly Elastic Supply of Real Value Added** At one polar end of the spectrum, one could write down a model with a perfectly elastic supply of real value added. For example, if value added is produced using labor alone, then one could assume that labor supply is perfectly elastic. In this case, equilibrium value added would be entirely demand determined, and thus demand for value added would pin down the equilibrium level of value added. While this represents an extreme view, it is worth noting that sticky wage models have this flavor in the short run. Translating this to our context, imagine an environment in which labor is the only factor of production and wages are pre-set in the producer’s currency (i.e., the producer currency price of value-added is fixed). Relative prices across countries then depend on nominal exchange rates. Shocks to nominal exchange rates would then trigger relative price changes, which would induce expenditure switching and changes in equilibrium demand for value added, and hence equilibrium value added when output is demand determined.

**Perfectly Inelastic Supply of Real Value Added** The opposite extreme would be an environment in which the supply of real value added is perfectly inelastic, where real value added is effectively an endowment. With this assumption, the logic of the exercises we have conducted above is reversed. Whereas we previously analyzed the impact of price changes on demand for value added, now endowments pin down equilibrium real value added. As a result, we can solve for equilibrium value-added prices.

In this case, we can use the intuition we have built up about the role of supply chains in determining demand for value added to help us interpret the role of supply chains in

\(^{25}\)We include the transfer term here because trade is unbalanced in our raw global input-output data.
explaining how prices adjust to different types of shocks. One straightforward exercise is to compute relative price changes following endowment shocks. A second exercise would be to compute relative price changes in response to closing trade imbalances, as in Obstfeld and Rogoff (2005, 2007) or Dekle, Eaton, and Kortum (2008). In both cases, the responsiveness of demand for value added to relative prices governs how much value-added prices need to adjust to absorb the shocks.

As an illustration, we compute bilateral value-added price changes following a shock to the value added endowment of Germany using our framework. Generically, a rise in Germany’s value added endowment depresses the relative price of German value added. The question we focus on is how input linkages determine the size of relative price changes across Germany’s trading partners. To illustrate the role of input linkages, we focus on comparing price adjustments across alternative elasticity parameterizations (equal elasticities versus Leontief production), as in previous figures.

In Figure 10, we present the change in each country’s value-added price relative to the price of German value added following a 1% rise in the endowment of German value added. We present the raw percentage changes in the left panel, and the difference across parameterizations of the model in the right figure. Following the rise in the German endowment, the relative price of German value-added falls against all partners, as would be expected. When production is inflexible, the distribution of price changes across countries shifts, such that supply chain partners experience larger increases in their value-added price relative to Germany.

We can draw on our previous results regarding demand for value added to supply intuition for this result. What we saw previously is that supply chains make demand for value added from Germany’s supply chain partners relatively insensitive to changes in the relative price of their value added with respect to Germany. Put differently, it takes big relative price changes to induce agents to switch expenditure away from these countries value added toward German value added. This in turn implies that their relative value-added prices need to rise a lot in order to induce sufficient expenditure switching to absorb the increased supply of German real value added. This points to an important role for global supply chains in explaining price adjustment, as well as the usefulness of our framework in interpreting these changes.

The Middle Ground. The prior two cases represent two extremes: demand-determined versus supply-determined output. It is straightforward to imagine realistic models that lie somewhere in the middle. For example, again focusing on labor as the sole input into producing value added, we could add a labor-leisure trade off to generate an upward sloping supply of real value added. This would lead to a model in which part of the adjustment is
loaded onto changes in endogenous equilibrium quantities (as in the elastic case above) and part is loaded onto changes in prices (as in the inelastic case above). Again, demand for value added remains a central concept in this context for organizing discussion about the equilibrium response to shocks.

7 Conclusion

In this paper, we studied the macroeconomics of global supply chains. As an analytical device, we introduced the concept of “demand for value added.” In doing so, we linked the pattern of gross final and input linkages across countries and relative elasticities in consumption and production to the strength of expenditure switching across value added from alternative sources. This framework is useful to organize discussion of the role of global supply chains in modifying shock transmission, because it links the analysis to well-understood expenditure switching mechanics in conventional macro-models.

Using global input-output data, we parameterized the framework to illustrate how supply chains can modify the size and bilateral distribution of spillovers across countries. Following a decline in prices in one country, inflexible supply chains reallocate changes in demand for value added away from supply chain partners. This has implications for understanding bilateral price adjustment following shocks to the supply of real value added.

We believe the results we have developed regarding the role of global supply chains in shaping demand spillovers have implications for addressing many classic macroeconomic questions. The framework has implications for assessing international competitiveness and constructing real exchange rates, as we explored in Bems and Johnson (2012). We also see applications to studying spillovers following exchange rate shocks, price adjustment in rebalancing episodes, and spillovers from technology or factor supply shocks. We look forward to applying our framework in analysis of these topics.
References


Figure 1: Decomposition of Changes in Effective Elasticity of Substitution Between Home and Foreign Value Added, by Country

Shaded regions show the contribution of changes in individual primitive elasticities on the overall effective elasticity of substitution between home and foreign value added for each country.
Each bar depicts the change in the price home relative to foreign gross output following a one percentage point decline in the price of home value added. The price change for foreign gross output is computed as the median across all foreign countries.
Figure 3: Cross-Country Median Changes in the Gross Output Terms of Trade in Response to Changes in the Price of Domestic Value Added over Time

The line indicates the cross-country median change in the price home relative to foreign gross output following a one percentage point decline in the price of home value added in each year.
Figure 4: Effective Value-Added Elasticity with Inflexible Global Supply Chains

(a) Effective Value-Added Elasticity  
(b) Correlation with Final Goods Share of Trade

The left panel presents the effective elasticity of substitution between home and foreign value added for each country, given primitive elasticities $\rho = \gamma = 0$ and $\sigma = 4.5$. The right panel plots the effective value-added elasticities against the share of final goods in total trade for each country.
The lines indicate the effective elasticity of substitution between home and foreign value added for each country for each year, given constant primitive elasticities $\rho = \gamma = 0$ and $\sigma = 4.5$. 
Bilateral spillover is defined as the change in demand for value added from each country relative to a 1 percent increase in demand for value added from Germany, induced by a decrease in the price of German value added. The left figure depicts the level of spillovers for each elasticity triple, and the right figure depicts the difference across parameterizations.
Figure 7: Difference in Bilateral Spillovers for Germany in Leontief and Equal Elasticity Cases versus Bilateral Trade Composition

The x-axis is the difference between normalized bilateral spillovers in the Leontief and Equal Elasticity cases, corresponding to the right panel of Figure 6.
Bilateral spillover is defined as the change in demand for value added from each country relative to a 1 percent increase in demand for value added from China, induced by a decrease in the price of Chinese value added. The left figure depicts the level of spillovers for each elasticity triple, and the right figure depicts the difference across parameterizations.
Figure 9: Bilateral Spillovers for the United States

Bilateral spillover is defined as the change in demand for value added from each country relative to a 1 percent increase in demand for value added from the United States, induced by a decrease in the price of the U.S. value added. The left figure depicts the level of spillovers for each elasticity triple, and the right figure depicts the difference across parameterizations.
Relative prices are defined as the change in the price of value added in each country relative to Germany after a 1% increase in Germany’s endowment of real value added. The left panel displays percentage changes under alternative elasticity parameterizations, and the right panel plots the difference in changes across the two parameterizations.
A Framework Appendix

This appendix collects supplemental results for both the stylized and general frameworks.

A.1 Details on Remark 7

In this section, we demonstrate the claim made in Remark 7 that CES preferences over value added from different countries yield Equation (10). To start, note that market clearing for country 1’s value added implies: \( \hat{V}_1 = \left( \frac{V_{12}}{V_1} \right) \hat{V}_{11} + \left( \frac{V_{13}}{V_1} \right) \hat{V}_{12} + \left( \frac{V_{12}}{V_2} \right) \hat{V}_{13} \). Then, further note that the first order conditions for final expenditure imply: \( \hat{V}_{11} = \hat{F}_1, \hat{V}_{12} = -\epsilon (\hat{p}_1^v - \hat{P}_2) + \hat{F}_2, \) and \( \hat{V}_{13} = -\epsilon (\hat{p}_1^v - \hat{P}_3) + \hat{F}_3 \). Together, these imply that:

\[
\hat{V}_1 = -\epsilon \left( \frac{V_{12}}{V_1} \right) (\hat{p}_1^v - \hat{P}_2) - \epsilon \left( \frac{V_{13}}{V_1} \right) (\hat{p}_1^v - \hat{P}_3) + \left( \frac{V_{11}}{V_1} \right) \hat{F}_1 + \left( \frac{V_{12}}{V_1} \right) \hat{F}_2 + \left( \frac{V_{13}}{V_1} \right) \hat{F}_3, \tag{37}
\]

where \( \hat{P}_2 = w_{12}^v \hat{p}_1^v + w_{12}^v \hat{P}_2, \hat{P}_3 = w_{13}^v \hat{p}_1^v + w_{23}^v \hat{P}_2 + w_{23}^v \hat{P}_3, \) and \( w_{ij}^v \) is the share of value added from country \( i \) in total final expenditure in country \( j \).

Getting from Equation (37) to Equation (10) requires working out the pattern of value added trade in the gross model, and then replacing value-added export and final expenditure shares with their gross counterparts.

First, we note that value-added export shares for country 1 are given by: \( \left( \frac{V_{12}}{V_1} \right) = s_{11}, \left( \frac{V_{13}}{V_1} \right) = s_{12}s_{22}, \) and \( \left( \frac{V_{13}}{V_1} \right) = s_{12}s_{23}, \) where \( s_{ij} \) is the share of gross output from country \( i \) shipped to country \( j \). Country 1 provides value added equal to \( p_1X_{12} \) into country 2’s production, a fraction \( s_{22} \) of which stays in country 2 and a fraction \( s_{23} \) of which is re-exported to country 3. Then value added exported from country 1 to countries 2 and 3 is \( p_1X_{12}s_{22} \) and \( p_1X_{12}s_{23} \). Since gross output equals value added in country 1, then value-added exports as a share of total value added is \( s_{12}s_{22} \) and \( s_{12}s_{23} \). Given this, we can re-write Equation (37) as:

\[
\hat{V}_1 = -\epsilon s_{12} (\hat{p}_1^v - \hat{P}_2) - \epsilon s_{12}s_{23} (\hat{p}_1^v - \hat{P}_3) + f(\hat{F}_1, \hat{F}_2, \hat{F}_3), \tag{38}
\]

where the \( f(\cdot) \) function is defined as in the main text.

Second, we note that the value-added expenditure weights used in defining \( \hat{P}_2 \) and \( \hat{P}_3 \) also reflect value-added export flows. These can be redefined to match terms in Equation (10). We note first that \( w_{12}^v = \frac{p_{12}V_{12}}{p_2F_2} \) and \( p_2F_2 = p_1^vV_{12} + p_2^vV_{22} \). Since value-added flows in the stylized model can be written as \( p_1^vV_{12} = (1 - s^v)s_{22}p_2Q_2 \) and \( p_2^vV_{22} = s^v s_{22}p_2Q_2, \) then \( p_{12}V_{12} = (1 - s^v) \) and \( p_{13}V_{13} = \frac{p_{13}V_{13}}{p_3F_3} = (1 - s^v)(1 - w), \) since \( p_1^vV_{13} = (1 - s^v)p_2C_{23} \). Further, \( w_{23}^v = \frac{p_2^vV_{23}}{p_3F_3} = s^v(1 - w) \) and \( w_{23}^v = w. \) Plugging these into the price indexes, inserting them into Equation (38), and simplifying yields Equation (10).
A.2 The Link Between Final Goods and Gross Output

This section discusses details regarding the link between demand for final goods and demand for gross output, embodied in Equation (25).

As mentioned in the main text, the first term links changes in bilateral final goods shipments ($\hat{F}$) to gross output. To understand the operation, it is helpful to recognize that $[I - S_X M_2]^{-1} \equiv [diag(p_i Q_i)]^{-1}[I - \Omega]^{-1}[diag(p_i Q_i)]$, where $\Omega$ is a global input-output matrix with $ij$ elements equal to the share of inputs from country $i$ as a share of the value of gross output in country $j$. This allows us to re-write the first term as:

$[I - S_X M_2]^{-1} S_F \hat{F} = [diag(p_i Q_i)]^{-1}[I - \Omega]^{-1}[diag(p_i Q_i)] S_F \hat{F}$.

Breaking this down, $[diag(p_i Q_i)] S_F \hat{F}$ aggregates changes in bilateral shipments into the aggregate change in final goods shipments (in levels) from each country to all destinations. Then the ‘Leontief Inverse’ matrix $[I - \Omega]^{-1}$ computes the amount of gross output needed directly and indirectly to produce those final goods, including domestic output needed to produce both domestic and foreign final goods.

One important feature of this calculation is that demand for country $i$’s output depends not only on demand for country $i$’s own final goods, but also demand for final goods produced in downstream countries that use inputs from $i$. Note that these downstream countries $k$ may use country $i$ goods directly, as in they import final or intermediate goods directly from country $i$. Alternatively, they may use country $i$ goods indirectly, in that they import final goods or intermediate goods from countries $j$ that source inputs from country $i$. In this case, there may be no direct gross trade recorded between $k$ from $i$, but we would observe value-added exports from $i$ to $k$. In this sense, the global input-output table framework both direct and indirect demand linkages.

The second term in Equation (25) describes how input choices depend on gross output price changes. To illustrate how production elasticities govern input sourcing, consider two special cases.

First, suppose that $\gamma = 0$, so relative prices do not induce substitution between real value added and inputs. In this case, an increase in the price of country $i$’s good induces substitution away from country $i$ in all countries. This implies that demand for country $i$’s gross output falls in response, while demand for output from other countries rises.

Second, suppose instead that $\rho = 0$, so relative prices induce substitution between inputs and real value added, but not among inputs. In this case, an increase in country $i$’s price raises demand for the composite input in country $i$ ($\hat{X}_i > 0$), which induces additional demand for inputs from all countries ($\hat{X}_{ki} > 0$). Abroad, demand for the composite input falls in countries $j \neq i$ ($\hat{X}_j < 0$), leading to declines in demand for imported inputs in those countries ($\hat{X}_{kj} < 0$). These input sourcing changes have further knock on effects, since inputs themselves are produced using intermediate imports, which is picked up by the presence of $[I - S_X M_2]^{-1}$ in the input calculation. Whether demand for country $i$’s output rises or falls after it’s price increases depends on the precise configuration of trade flows.

To illustrate the possibilities, consider two special cases where we can make definitive statements. One special case is when country $i$ uses only its own goods as inputs, then demand for its output certainly falls, because $p_i = \hat{p}_i$ and $\hat{X}_i = 0$ in this case. A second
special case is where foreign countries do not use country $i$ goods in production ($w^x_{ij} = 0$ for $j \neq i$), but country $i$ uses both home and foreign goods in the input bundle (so $p_i \neq \hat{p}_i^x$), then demand for country $i$’s output definitely rises after its own price rises. This highlights the need to explicitly quantify these offsetting channels.