Variance Risk Premium Dynamics in Equity and Option Markets*

Laurent Barras† Aytek Malkhozov‡

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Abstract

We analyze the quarterly dynamics of the Variance Risk Premium (VRP) in both the equity and option markets. For each market, we provide consistent estimators of the path followed by the VRP and examine the drivers of its time variation. Whereas the VRPs in the two markets follow similar patterns, they also exhibit large, but temporary differences. We find that such differences are largely explained by changes in the risk-bearing capacity of financial intermediaries, which only affect the option VRP. These results point to a degree of market segmentation between the two markets and suggest that the option VRP does not directly capture changes in the risk attitude of equity investors.

Keywords: Variance Risk Premium, Cross-Section of Stock Returns, Broker-Dealer Leverage

JEL Classification: G12, G13, C23, C51, C52, C58

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†McGill, Finance, laurent.barras@mcgill.ca
‡McGill, Finance, aytek.malkhozov@mcgill.ca
1 Introduction

The Variance Risk Premium (VRP) is the compensation required by investors for holding assets that perform poorly when stock market volatility rises. Because a version of the VRP can be easily calculated from option prices as the difference between the expected realized variance and the squared VIX index, it is often considered to be the most readily available proxy for fluctuations in investors’ risk aversion, or more colloquially "fear". As a result, the option VRP is used not only to assess the expected returns on option strategies, but also as a much broader measure of fluctuations in aggregate discount rates.\footnote{See Bollerslev, Gibson, and Zhou (2011), Drechsler and Yaron (2011), and Bekaert and Hoerova (2014).}

Whereas the wide use of the option VRP relies on the assumption that risk is priced consistently across markets, previous studies provide evidence of potential mispricing between equity and option markets and stress the key role played by broker-dealers in determining option prices.\footnote{See Constantinides, Jackwerth, and Perrakis (2009) and Bates (2003), Garleanu, Pedersen, and Poteshman (2009) respectively.} Therefore, the sources of variation as well as the informational content of the VRP in the option market can be quite different from those of the VRP required by equity market investors.

In this paper, we conduct an in-depth analysis of the quarterly dynamics of the VRP in both the option and equity markets. Our contributions to the existing literature are fourfold. First, we develop a new methodology to obtain consistent estimators of the entire path followed by the VRP in each market. Second, we study the sources of the time-variation of the VRP using a rich set of macro-finance predictors. Third, we compare the prices of variance risk in both markets to identify periods when they differ significantly. Finally, we determine to which extent this difference can be explained by changes in the risk-bearing capacity of broker-dealers active in the option market.

To preview our main findings, we plot in Figure 1 the paths followed by the equity and option VRPs (each datapoint provides a conditional estimate of the premium for the next quarter). First, we see that both series take negative values in most quarters,...
consistent with the idea that investors are willing to pay a premium for portfolios that perform well in times of high volatility. Second, the magnitude of both premia is higher when the market volatility is high (e.g., economic shocks in 1973-74, 1987 crash, recent financial crisis) and when the economy is in recession. Third, the price of variance risk is, on average, the same in both markets and approximately equal to -1.50% per year; however, Figure 1 reveals that in 16 quarters, the gap between the two premia is above 3.00% per year (two times larger than the average premium itself). Therefore, a simple analysis of the unconditional VRP is not sufficient to uncover the large, but temporary differences across the equity and option markets. Fourth, we find that these differences are largely explained by changes in the risk-bearing capacity of broker-dealers. When the latter become risk-constrained, option prices become dislocated relative to equity prices and imply a relatively higher VRP (in absolute value). The opposite phenomenon occurs when broker-dealers actively expand the size of their balance sheets. These findings are statistically significant and robust to a wide range of specification changes (set of macro-finance predictors, proxies for the risk-bearing capacity, and sample period). In short, we find that the ability of financial institutions to intermediate variance risk mostly affects the option VRP. These results point to a degree of market segmentation that limits risk sharing between equity investors and these institutions. In addition, they suggest that the option VRP does not directly reflect the attitude towards risk of equity investors.

We model the VRP consistently across both markets as the difference between the conditional expectations of the realized variance under the physical and risk-neutral measures. Each of these conditional expectations is specified as a linear function of lagged predictive variables that capture both business cycle conditions and the risk-bearing capacity of financial intermediaries. To estimate these linear coefficients, we develop a unified econometric framework that yields estimators that are both consistent and asymptotically normally distributed. For the physical expectation, we obtain the coefficients
by regressing the quarterly realized variance on the lagged predictors following the recent studies by Paye (2012) and Campbell, Giglio, Polk, and Turley (2013). For the risk-neutral expectation, we estimate the coefficients separately using either equity or option data to allow for potential differences between the two markets. In the equity market, we extract the risk neutral coefficients from a conditional two-factor model that includes the market return and the realized variance. After forming a set of 25 variance portfolios, we estimate this model using a conditional version of the two-pass cross-sectional regression developed by Gagliardini, Ossola, and Scaillet (2013). In the option market, the procedure is easier because the squared VIX index provides us with a model-free approximation for the risk neutral expectation of realized variance (e.g., Britten-Jones and Neuberger (2000)). Therefore, we can estimate the coefficients by regressing the squared VIX index on the lagged predictors. Because the VIX index is only observed during the second half of our sample, we apply the Generalized Method of Moments (GMM) for samples of unequal lengths proposed by Lynch and Wachter (2013) to improve the precision of the estimated coefficients.

We apply our methodology to estimate the risk premia associated with the quarterly realized variance of SP500 returns. To track business cycle conditions, we use a set of macro-finance variables that contains the lagged realized variance, the Price/Earnings (PE) ratio, the default spread, and the quarterly employment and inflation rates. In addition, we include two predictors that proxy for the risk-bearing capacity of financial intermediaries. The first one is the leverage ratio of broker-dealers available quarterly from the Fed Flow of Funds. This is motivated by previous work by Adrian and Shin (2010), and Adrian and Shin (2013) who provide supporting evidence that financial intermediaries actively manage their leverage in response to the tightness of their Value-at-Risk constraints. The second one is the quarterly return of the Prime Broker Index (PBI) used by Boyson, Stahel, and Stulz (2010), among others, to measure the financial standing of the major players in the brokerage sector, including Citigroup, Goldman Sachs, and UBS.

Examining the drivers of the time-variation of the equity VRP yields several new in-
sights. First, we find that the negative spikes documented in Figure 1 are mainly driven by the lagged realized variance. In volatile periods, investors revise their expectations about the future realized variance (physical expectation); concurrently, assets that provide insurance against a rise in future volatility become extremely valuable (risk-neutral expectation). Because the second effect dominates the first one, the estimated coefficient for the equity VRP is negative and significant. Second, these two effects offset each other for the default spread and the PE ratio and produce coefficients that are not statistically different from zero. Whereas both variables are important predictors of the future realized variance (as shown by Campbell, Giglio, Polk, and Turley (2013)), they do not significantly affect the equity VRP. Finally, both the employment and inflation rates yield positive estimated coefficients, which possibly explains why the magnitude of the VRP is higher during recessions (as shown in Figure 1).

In the option market, the relationship between the price of variance risk and the macro-finance variables is similar to the one observed in the equity market and explains the co-movement between the two VRP series in Figure 1. The major difference comes from the strong exposure of the option VRP to changes in the risk bearing capacity of financial intermediaries. When the latter deleverage or suffer from short-term losses, the magnitude of the VRP increases dramatically in the option market, but stays largely unchanged in the equity market. For example, the price of variance risk is very high in the option market in 1998 and in 2009-10 as a consequence of the LTCM collapse and the recent financial crisis. On the contrary, we observe the opposite phenomenon when broker-dealers take on leverage, as evidenced by the 2001-2003 monetary easing period. In both cases, the impact of such changes is economically large: a one-standard deviation variation in the leverage ratio changes the difference (in absolute value) between the option and equity VRPs by 1.12% per year—a change nearly as large as the average premium itself.

Our empirical evidence is consistent with the role played by financial intermediaries in risk sharing in the option market. Previous studies provide evidence that option prices
are determined by market makers who supply options to other market participants and require a premium in exchange for holding residual risk. Therefore, their ability to take on risk should have a direct impact on the VRP in the option market. The fact that we do not observe the same effect in the equity market and the resulting difference between the two VRPs constitute an apparent violation of no-arbitrage. From a theoretical point of view, this result can persist in equilibrium if investors face portfolio constraints that limit their ability to take advantage of mispricing and induce segmentation between the two markets (Basak and Croitoru (2000)). Alternatively, identical assets can be priced differently in equilibrium if investors face funding constraints and different margin requirements across markets (Garleanu and Pedersen (2011)). Whereas both explanations are likely to play a role, market segmentation seems more consistent with our main findings. First, we find that more direct measures of funding constraints, such as the default spread and the TED spread, cannot explain the difference between the two VRPs. Second, a margin-based explanation cannot easily account for the positive and negative VRP differences because the margin gap between the equity and option markets is unlikely to change signs.

This paper also sheds additional light on the effect of monetary policy on investors’ risk-taking behavior and asset prices, which is a major concern for policymakers (e.g., Bernanke and Kuttner (2005), Rajan (2006)). We note that during the investigated period, the leverage of broker-dealers is considerably higher in times when the Federal Reserve pursues accommodative monetary policy. These episodes are associated with more risk-taking by financial intermediaries and lower risk premia in the option market where these institutions trade. This finding resonates with the model recently proposed by Drechsler, Savov, and Schnabl (2014) in which lower nominal rates result in increased

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4Similar mechanisms are documented in other derivative markets. Cheng, Kirilenko, and Xiong (2012) report that, in normal times, financial traders in commodity futures markets accommodate the demand of commercial hedgers, but reduce the amount of risk sharing in times of distress. Etula (2009) shows that the risk-bearing capacity of broker-dealers, which is proxied by their leverage, drives risk premia in commodity derivatives markets.
bank leverage and lower risk premia.

There is an extensive literature on the role of market volatility risk in pricing the cross-section of stock returns. Ang, Hodrick, Xing, and Zhang (2006) estimate the unconditional VRP from a two-factor model that includes the market return and the VIX index, whereas Bali and Zhou (2013) use the option VRP as a risk factor that proxies for fluctuations in aggregate stock market uncertainty. Campbell, Giglio, Polk, and Turley (2013) and Bansal, Kiku, Shaliastovich, and Yaron (2013) derive an intertemporal CAPM with stochastic volatility where the realized variance of stock market returns is an important factor that helps the model to explain the cross-section of average stock returns. Relative to these papers, we specify and estimate the entire path followed by the equity VRP over time. Second, several studies examine the time-variation of the VRP computed from the prices of index options (e.g., Todorov (2010), Bollerslev, Gibson, and Zhou (2011)). By estimating the path of the VRP using equity instead of option data, our paper provides new evidence about the informational content of the VRP. Third, Constantinides, Jackwerth, and Perrakis (2009) who document violations of stochastic dominance bounds derived the prices of call and put options written on the SP500 index. We provide one possible reason for this mispricing, namely the difference in the pricing of variance risk. Fourth, our paper is related to the study by Bekaert, Hoerova, and Lo Duca (2013) which documents a strong co-movement between the VIX index and measures of monetary policy stance. Our results link this co-movement with changes in the risk-taking behaviour of financial intermediaries. Finally, Bates (2008), Adrian and Shin (2010), and Chen, Joslin, and Ni (2013) show that the risk-bearing capacity of financial intermediaries is an important driver of option prices. Relative to these papers, we find that financial intermediaries affect the price of variance risk very differently in the equity and option markets.

The rest of the paper is organized as follows. Section 2 presents the methodology used to estimate the VRP dynamics in the equity and option markets. Section 3 describes the data, whereas Section 4 contains the main empirical results. The appendix contains
additional details on the estimation procedure.

2 Empirical Framework

2.1 Specification of the Variance Risk Premium

We denote the conditional VRP in the equity and option markets by $\lambda^e_{v,t}$ and $\lambda^o_{v,t}$, respectively. Following the formulation used in the option literature (e.g., Bollerslev, Tauchen, and Zhou (2009), and Carr and Wu (2009)), we define each VRP as

\[
\lambda^e_{v,t} = E_t (f_{v,t+1}) - E_t^{Q^e} (f_{v,t+1}), \tag{1}
\]

\[
\lambda^o_{v,t} = E_t (f_{v,t+1}) - E_t^{Q^o} (f_{v,t+1}), \tag{2}
\]

where $E_t (f_{v,t+1})$ is the conditional expectation of the realized variance $f_{v,t+1}$ under the physical measure between time $t$ and $t + 1$, and $E_t^{Q^e} (f_{v,t+1})$, $E_t^{Q^o} (f_{v,t+1})$ are the conditional expectations of $f_{v,t+1}$ under the risk-neutral measures in the equity and option markets, respectively.

In a frictionless, no-arbitrage environment, the equity and option VRPs have to be equal and any difference between them constitutes an arbitrage opportunity that can be exploited by market participants. However, mispricing between the two markets can possibly exist in equilibrium in a variety of cases. First, it can be caused by portfolio constraints that limit the ability of investors to take advantage of arbitrage opportunities. For instance, Basak and Croitoru (2000) describe a setting in which traders with heterogeneous preferences face portfolio constraints that limit the size of their positions in different markets, whereas Gromb and Vayanos (2002) demonstrate that when arbitrageurs are capital constrained, they cannot fully correct mispricing between two segmented markets. Second, deviations from the law of one price can also be explained by different margin requirements, as discussed by Garleanu and Pedersen (2011). In all
these cases, the difference between the two premia reflects the shadow cost of portfolio or margin constraints. Such frictions are potentially important when considering the equity and option markets. For example, stock market investors may face information costs or regulatory constraints that prevent them from buying or writing options. In addition, margin requirements generally differ between the underlying and derivative securities. Motivated by these considerations, we model the risk-neutral expectations in the two markets ($E^Q_e(f_{v,t+1})$ and $E^Q_o(f_{v,t+1})$) separately, thus allowing for a time-varying difference between the equity and option VRPs.

To determine the path followed by the VRP in each market, we need to specify the dynamics of the variance factor. Similar to the previous literature on variance predictability (e.g., Campbell, Giglio, Polk, and Turley (2013), Paye (2012)), we specify the conditional expectation of $f_{v,t+1}$ as a linear function of lagged instruments, i.e.,

$$E_t(f_{v,t+1}) = F_v' z_t,$$

where $z_t$ is a $J$-vector that includes a constant and $J - 1$ centered predictive variables, and $F_v$ is the $J$-vector of associated coefficients. This framework is not restrictive since non-linearities can be accounted for by including powers of the predictive variables. Similarly, we set the risk-neutral expectations of $f_{v,t+1}$ equal to

$$E^Q_e(f_{v,t+1}) = V^e_z z_t, \quad E^Q_o(f_{v,t+1}) = V^o_z z_t,$$

where $V^e_v$ and $V^o_v$ are the $J$-vectors that drive the risk-neutral conditional expectations in the equity and option markets, respectively. Combining Equations (1)-(4), we can write each VRP as a linear function of the lagged instruments:

$$\lambda_{v,t}^e = (F_v - V^e_v)' z_t,$$

$$\lambda_{v,t}^o = (F_v - V^o_v)' z_t.$$
The procedure for estimating these two risk premia can be summarized in three steps. First, we simply run a time-series regression of the realized variance on the predictors to estimate $F_v$. Second, we estimate the values taken by $V^e_v$ and $V^o_v$ using data from the equity and option markets, respectively. Finally, we plug the three estimated vectors, $\hat{F}_v$, $\hat{V}^e_v$ and $\hat{V}^o_v$, into Equations (5) and (6). In the next two sections, we describe the methodology used to compute the two vectors $\hat{V}^e_v$ and $\hat{V}^o_v$.

2.2 Estimating the Equity Variance Risk Premium

2.2.1 A Two-Factor Model for Equity Returns

We model equity returns using a two-factor representation that includes the market return $f_{m,t+1}$, and the variance factor $f_{v,t+1}$. This specification is used by Ang, Hodrick, Xing, and Zhang (2006), among others, to estimate the unconditional VRP in the equity market. To extend their analysis to a conditional setting, we assume the following return dynamics for each stock $j$:

$$r_{jt+1} = a_{jt} + b_{jm,t} \cdot f_{m,t+1} + b_{jv,t} \cdot f_{v,t+1} + e_{j,t+1},$$

(7)

where $b_{jm,t}$ and $b_{jv,t}$ are the time-varying risk loadings on the market and variance factors, and $e_{j,t+1}$ is the residual term. Following the large literature on return predictability (e.g., Keim and Stambaugh (1986), Fama and French (1986), Ferson and Harvey (1991)), we also write the conditional expectations of the market factor as linear functions of the predictors, i.e., $E_t(f_{m,t+1}) = F'_m z_t$ and $E_t^Q(f_{m,t+1}) = V'_m z_t$. The two-factor model implies the following restriction on the intercept:

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\[ a_{j,t} = b_{jm,t} (\lambda_{m,t}^e - E_t(f_{m,t+1})) + b_{jv,t} (\lambda_{v,t}^e - E_t(f_{v,t+1})) \]  
\[ = -(b_{jm,t} E_t^{Q_m}(f_{m,t+1}) + b_{jv,t} E_t^{Q_v}(f_{v,t+1})) \]  
\[ = -(b_{jm,t} \cdot V_m^e + b_{jv,t} \cdot V_v^e) z_t, \]

where the market risk premium, \( \lambda_{m,t}^e \), is defined as \( E_t(f_{m,t+1}) - E_t^{Q_m}(f_{m,t+1}) \). This equation is equivalent to the standard equilibrium condition applied to the conditional expected return: \( E_t(r_{j,t+1}) = b_{jm,t} \lambda_{m,t}^e + b_{jv,t} \lambda_{v,t}^e \).

If one fully specifies the dynamics of the market and variance betas of individual stocks, it is possible to estimate the equity VRP directly from Equations (7) and (8). However, this estimation procedure is challenging for several reasons. First, modeling the time-variation in individual stock betas largely increases the number of parameters, especially when common and stock-specific instruments are considered. Second, Ghysels (1998) shows that a wrong specification of time-varying betas may result in large pricing errors, possibly greater than those produced by a constant beta model.

To address this issue, we borrow from Ang, Hodrick, Xing, and Zhang (2006) and estimate the two-factor model using return data on 25 investable portfolios sorted on market and variance risk betas. These portfolios, referred to as variance portfolios, are rebalanced each month to maintain stable exposures to the market and variance risk factors.

### 2.2.2 Construction of the Variance Portfolios

To form the variance portfolios, we proceed as follows. First, we estimate each month the market and variance betas of individual stocks using daily returns over the previous month. As discussed by Lewellen and Nagel (2006), using a daily frequency allows us to

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5Because the market factor is traded, its average excess return is equal to its risk premium (i.e., \( \lambda_{m,t}^e = E_t(f_{m,t+1}) \)). Therefore, if the two-factor model is correctly specified, a testable restriction is that \( E_t^{Q_m}(f_{m,t+1}) \) must be equal to zero.
pin down the conditional risk loadings without specifying the conditioning information. Specifically, we regress the return of each stock on the market return and the innovation of the variance factor. Computing a model-free variance innovation based on intraday return observations is not feasible because this data is only available in the latter part of the sample. To address this issue, we model the daily conditional variance of the market return, \( \sigma_d^2 \), using a standard GARCH (1,1): \( \sigma_d^2 = \gamma + \alpha \sigma_{d-1}^2 + \beta \varepsilon_d^2 \), where \( \varepsilon_d^2 \) is the daily squared market return. Estimating the parameters \( \gamma, \alpha, \) and \( \beta \) using daily returns over a one-year rolling window, we compute the daily variance innovation as \( \varepsilon_d^2 - \hat{\sigma}_{d-1}^2 \), where \( \hat{\sigma}_{d-1}^2 \) is the estimated variance on the previous day.\(^6\)

Second, we sort stocks according to their exposures to the market and variance factors. Since short-window regressions can produce large estimation errors, we use the beta \( t \)-statistics, \( \hat{t}_{jm,t} = \hat{b}_{jm,t} / \hat{\sigma}_{jm,t} \) and \( \hat{t}_{jv,t} = \hat{b}_{jv,t} / \hat{\sigma}_{jv,t} \), where \( \hat{\sigma}_{jm,t}, \hat{\sigma}_{jv,t} \) denote the estimated standard deviation of \( \hat{b}_{jm,t} \) and \( \hat{b}_{jv,t} \), respectively.\(^7\) Stocks are ranked first into quintiles based on the variance \( t \)-statistic, and then into quintiles based on the market \( t \)-statistic.

Third, we compute the value-weighted return of all stocks in each of the 25 groups. Repeating these three steps each month over the entire sample period, we obtain return time-series for 25 investable variance portfolios.

### 2.2.3 Estimation Procedure

The estimation procedure builds on a recent paper by Gagliardini, Ossola, and Scaillet (2013) that extends the traditional two-pass regression to the conditional setting examined here. In the first step, we run a time-series regression of the excess return of each variance portfolio \( p \) \((p = 1, \ldots, 25)\) on the \( J \)-vector of predictors and the two risk factors:

\[
r_{p,t+1} = \beta_{p1} t_{z,t} + b_{pm} \cdot f_{m,t+1} + b_{pv} \cdot f_{v,t+1} + e_{p,t+1}. \tag{9}
\]

\(^6\)Alternatively, Ang, Hodrick, Xing, and Zhang (2006) use the daily change in the VIX index. We follow a different approach for two reasons. First, as noted by these authors, the VIX is a noisy proxy of the innovation of the variance factor because it also captures changes in the risk premium itself. Second, data on the VIX is only available in 1990, while our sample begins in 1970.

\(^7\)Previous papers (e.g., Kosowski, Timmerman, Wermers, and White (2006)) show that the \( t \)-statistic allows for an improved ranking because it controls for the precision of the estimated coefficient.
This formulation is a special case of Equation (7) with constant conditional betas. In the second step, we exploit the parameter restrictions implied by the equilibrium condition in Equation (8), which can be rewritten as

\[
\beta_{p1} = -(b_{pm} \cdot V^e_{m} + b_{pv} \cdot V^e_{v}) = -\beta_{p2} V^e,
\]

where \( V^e \) is a 2\( J \)-vector equal to \([V^e_m, V^e_v]'\), \( \beta_{p2} \) is a \( J \times 2 \cdot J \) matrix equal to \([b_{pm} \cdot I_J, b_{pv} \cdot I_J]\), and \( I_J \) is the \( J \times J \) identity matrix. Therefore, if we run a cross-sectional regression of the estimated \( J \)-vector \( \hat{\beta}_{p1} \) on the estimated coefficient matrix \( -\hat{\beta}_{p2} = [\hat{b}_{pm} \cdot I_J, \hat{b}_{pv} \cdot I_J] \), we obtain an estimator of \( V^e \) denoted by \( \hat{V}^e = [\hat{V}^e_m, \hat{V}^e_v]' \).

Inserting \( \hat{V}^e_v \) in Equation (5), we compute the estimated equity VRP as \( \hat{\lambda}^e_{v,t} = (\hat{F}_v - \hat{V}^e_v)' z_t \). As a by-product of the estimation procedure, we also obtain an estimate of time-varying market risk premium \( \hat{\lambda}^e_{m,t} \) equal to \( (\hat{F}_m - \hat{V}^e_m)' z_t \), where \( \hat{F}_m \) is the \( J \)-vector of estimated coefficients from a time-series regression of the market excess return on the predictors. The appendix contains additional details on the estimation procedure and on the asymptotic properties of the different estimators.

### 2.3 Estimating the Option Variance Risk Premium

#### 2.3.1 Extracting Information from Option Prices

The option VRP is easier to estimate than the equity VRP because we can use the information contained in the VIX index computed from the prices of index options. Specifically, Britten-Jones and Neuberger (2000), Jiang and Tian (2005), and Carr and Wu (2009) show that the squared VIX index, or implied variance, provides a model-free approximation of the risk-neutral expectation of the realized variance:

\[
\[Equations (9) and (10) are simply the conditional counterparts to the widely used two-pass regression used in an unconditional setting: (i) the time-series regression becomes \( r_{p,t+1} = a + b_{pm} \cdot f_{m,t+1} + b_{pv} \cdot f_{v,t+1} + e_{p,t+1} \); (ii) the cross-sectional regression becomes \( E(r_{p,t+1}) = b_{pm} \cdot \lambda_m + b_{pv} \cdot \lambda_v \), where \( E(r_{p,t+1}) \) is the average portfolio return, and \( \lambda_m, \lambda_v \) are the unconditional risk premia.\]
Zhou (2011)) to estimate a "raw" version of the VRP in which the risk-neutral expectation of the realized variance is conditioned on all information available to option market participants:

$$\lambda_{r,t} = E_t (f_{t,t+1}) - iv_t = F_v^t z_t - iv_t,$$

where $iv_t$ denotes the implied variance computed from index option prices observed at time $t$.

This "raw" version of the option VRP is easy to compute and extracts as much information as possible from option prices. However, it does not allow us to determine to which extent the predictors $z_t$ drive the dynamics of the option VRP. For this purpose, we rely on the formulation in Equation (6) to estimate the predictor-based option VRP defined as $(F_v - V_v^o)' z_t$. The only required input is an estimate of the $J$-vector $V_v^o$ that we obtain by regressing the implied variance on the predictors using the procedure described below.

### 2.3.2 Estimation Procedure

The main difficulty in estimating the vector $V_v^o$ comes from data limitation. While the realized variance and predictors are observed over a long period starting in 1970 (i.e., the long sample), data on the implied variance is only available in the early 90’s (i.e., the short sample). To address this issue, we use the Generalized Method of Moments (GMM) extended by Lynch and Wachter (2013) for samples of unequal lengths. The basic idea behind this approach is to exploit information contained in the long sample to obtain an estimator of $V_v^o$ that exhibits lower volatility than the one based on the short sample only.

The estimation procedure can be described as follows. First, we run a time-series regression of the implied variance on the predictors over the short sample to obtain an initial estimate denoted by $\hat{V}_v^{o,S}$. Second, we compute an adjustment factor, $\hat{A}$, that depends on the estimated vector $\hat{F}_v$ computed over the long sample. Third, we obtain the final estimate $\hat{V}_v^o$ by taking the difference between $\hat{V}_v^{o,S}$ and $\hat{A}$.

While the general expression for $\hat{A}$ is not particularly insightful, the intuition behind this adjustment can be easily illustrated with the following example. Suppose that we
just want to estimate the average of the realized and implied variances—in this case, $z_t$ equals 1 and $F_v$ and $V_o$ are scalars equal to the means of $f_{v,t+1}$ and $iv_t$, respectively. Now suppose that the estimated mean of $f_{v,t+1}$ over the short sample is above the more precise estimate computed over the long sample, i.e., $\hat{F}_{v,S} > \hat{F}_v$. If $f_{v,t+1}$ and $iv_t$ are positively correlated, the estimated mean of $iv_t$, $\hat{V}_{o,S}$, is also likely to be above average. Therefore, $\hat{A}$ takes a positive value to produce a final estimate $\hat{V}_v$ that is below $\hat{V}_{o,S}$. The appendix contains additional details on the estimation procedure and on the asymptotic properties of the different estimators.

3 Data Description

3.1 Risk Factors and Predictive Variables

We conduct the empirical analysis using quarterly data between April 1970 and December 2012 (the long sample). The market factor is proxied by the excess return of the CRSP value-weighted market index, while the variance factor is measured as the sum of the squared daily returns of the SP500 over each quarter. To compute the quarterly implied variance of the SP500, we use the squared VIX index computed from three-month option prices and available between January 1992 and December 2012 (the short sample).

We use a set of five macro-finance predictors to capture the time-variation of the VRP: the lagged realized variance, the Price/Earnings (PE) ratio, the quarterly inflation rate, the quarterly growth in aggregate employment, and the default spread (all of them expressed in log form). This choice is motivated by the recent studies of Bollerslev, Gibson, and Zhou (2011), Paye (2012), and Campbell, Giglio, Polk, and Turley (2013) in which these variables contain predictive information on the realized variance.\footnote{Our approach is different from Corsi (2009), and Bekaert and Hoerova (2014) who forecast the monthly realized variance using its lags calculated at different frequencies (monthly, weekly, daily). Here, we follow Paye (2012) and Campbell, Giglio, Polk, and Turley (2013) who show that over a quarterly horizon, including macro-finance variables in a multivariate regression improves forecast accuracy. In particular, the above-mentioned papers point out that a multivariate forecast can be useful for distinguishing between the short- and long-term variance fluctuations.} Whereas

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The lagged realized variance is informative about the persistent component of the variance process, the remaining variables help capture the fluctuations of aggregate volatility over the business cycle. The PE ratio is downloaded from Robert Shiller’s webpage and is defined as the price of the SP500 divided by the 10-year trailing moving average of aggregate earnings. Inflation data is computed from the Producer Price Index (PPI), aggregate employment is measured by the total number of employees in the nonfarm sector (seasonally-adjusted), and the default spread is defined as the yield differential between Moody’s BAA- and AAA-rated bonds. These three series are downloaded from the Federal Reserve Bank of St. Louis.

In addition to the macro-finance variables mentioned above, we consider two predictors used by previous studies as proxies for the risk-bearing capacity of financial intermediaries (both expressed in log form). The first one is the leverage ratio of broker-dealers, defined as their asset to equity values from the Federal Reserve Flow of Funds Accounts (Table L 128). Adrian and Shin (2010) provide supporting evidence that broker-dealers actively manage their leverage levels. In good times, they increase their leverage and expand their asset base, possibly because their Value-at-Risk constraints are less tight (see Adrian and Shin (2013)), whereas they deleverage in bad times. Second, we follow Boyson, Stahel, and Stulz (2010) and compute the value-weighted index of publicly-traded prime broker firms, including Goldman Sachs, Morgan Stanley, Bear Stearns, UBS, and Citigroup. The quarterly return of this index allows us to capture changes in the financial strength of the major players in the brokerage sector.

Table 1 provides summary statistics for the different predictors. To facilitate comparisons across the estimated coefficients presented in the empirical section, all the predictors are standardized. The comparison of the persistence levels for the two broker-dealer

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10 In the sensitivity analysis presented below, we consider an alternative set of macro-finance predictors that includes the dividend yield, the quarterly growth rate in industrial production, the business cycle indicator constructed by Aruoba, Diebold, and Scotti (2009), the 3-month Tbill rate, and the term spread. In all these cases, the main results of the paper remain unchanged.

11 The Federal Reserve defines broker-dealers as financial institutions who buy and sell securities for a fee, hold and inventory of securities for resale, or do both.

12 Lettau and VanNieuwerburgh (2008), among others, provide empirical evidence that the mean of
variables reveals that they contain information at different frequencies. The leverage ratio is a slow-moving predictor that proxies for long-term changes in the risk-bearing capacity of financial intermediaries, whereas the PBI return captures the short-term reaction of these intermediaries to aggregate losses. It is well known from the previous literature that persistent variables such as the leverage ratio can create inference biases in predictive regressions (e.g., Cavanagh, Elliott, and Stock (1995), Ferson, Sarkissian, and Simin (2003)). To mitigate this concern, we also run the estimation using the annual change in the leverage ratio—although this variable is a noisier measure of the leverage of broker-dealers, its first-order autocorrelation (0.74) is below the levels produced by all but one predictor (the PBI return).

Perhaps not surprisingly, the two broker-dealer variables also pick up some business cycle fluctuations—for instance, the correlation between the leverage and PE ratios equals 0.33. To explicitly distinguish between the two sets of predictors, we therefore regress the leverage ratio and the PBI return on the macro-finance variables and take the residual components of the two regressions.

[TABLE 1 HERE]

3.2 Variance Portfolios

To create the 25 variance portfolios, we consider all stocks traded on AMEX, NASDAQ and the NYSE, and impose two filters. First, we exclude tiny-cap stocks to avoid liquidity-related issues. Using the classification proposed by Fama and French (2008), we select all existing stocks with a size above the 20th percentile of the market capitalization for NYSE stocks. Second, we require each stock to have at least 15 daily return observations over the previous month in order to compute its market and variance betas.

To summarize the properties of the variance portfolios, we take an equally-weighted average of all portfolios in the same variance beta quintile (Low, 2, 3, 4, High). Overall, financial ratios exhibit large structural shifts after 1991. Therefore, we follow their recommendation and allow for the possibility that predictors have different means before and after 1991.
the results are similar to those reported by Ang, Hodrick, Xing, and Zhang (2006) for their VIX-based variance portfolios. Panel A of Table 2 shows that the low variance beta portfolio loads negatively on the variance factor (with a post-ranking beta of -0.30) and yields an average return of 7.55% per year. As we move toward the high variance portfolio, the post-ranking beta increases and the average return drops, consistent with the idea that investors are willing to pay a premium for portfolios that perform well in times of high volatility. Interestingly, unreported results show that during the five greatest volatility spikes (Oct. 1987, July 2002, July/Oct. 2008, July 2011), the market-hedged return of the high minus low variance portfolio is always positive (e.g., it equals 11.31% over the last quarter of 1987), whereas it turns negative during the five quarters with the lowest realized variance. These findings provide supportive evidence that the returns of the variance portfolios are exposed to variance risk and can be used to extract information about the VRP.

Whereas high volatility shocks are associated with stock market declines (i.e., the correlation between factor innovations equals -0.49), the two factors capture different dimensions of risk. Specifically, Panel B reveals that using the market factor alone produces CAPM alphas that exhibit the same pattern as the one observed for average returns. Next, we report the portfolio alphas obtained with the Fama-French model and two extensions that include the momentum and the Pastor-Stambaugh liquidity factors, respectively. The results show that the three models generally help reduce the magnitude of the alphas compared to the CAPM, but do not fully capture the cross-sectional variation in average returns.\(^\text{13}\)

\[^{13}\text{We reach a similar conclusion over the short sample (1992-2012). Unreported results show that the annual alphas range between -2.8\% and 4.1\% for the Fama-French (FF) model, between -2.1\% and 4.1\% for the FF-momentum model, and between -3.0\% and 3.2\% for the FF-liquidity model.}\]
4 Empirical Results

4.1 Realized Variance Predictability

We begin our empirical analysis by measuring the extent of variance predictability. We regress the quarterly realized variance on the set of predictors over the period 1970-2012 to estimate the $J$-vector of coefficients $F_v$. This procedure allows us to estimate the conditional expectation of the realized variance that we use as input to compute the equity and option VRPs.

Panel A of Table 3 reports the estimated vector $\hat{F}_v$ obtained with the (standardized) macro-finance variables. The first row shows the results when the lagged realized variance is used as a single predictor. We find that the estimated coefficient is positive and highly significant, i.e., a one-standard deviation increase in past variance increases future variance by 65% compared to its average level (0.49/0.75). In the second row, we condition on all macro-finance variables simultaneously. Consistent with Campbell, Giglio, Polk, and Turley (2013) and Paye (2012), we find a positive and statistically significant relationship between the default spread and future realized variance. The intuition for this result is that risky bonds are short the option to default. When expected future variance is above average, investors bid down the price of risky bonds, which in turn increases the default spread. Conditional on the other predictors, a high PE ratio also signals above-average future variance. As noted by Campbell, Giglio, Polk, and Turley (2013), the PE ratio helps capture episodes during which both stock prices and volatility are high, such as the dotcom bubble in the late 90’s.

In Figure 2, we plot the evolution of the realized variance and its conditional expectation based on the macro-finance variables. Overall, the fit is good and contrasts with the relatively low predictive $R^2$ reported in Panel A. This discrepancy is primarily driven by a few surprise spikes in realized variance, notably in 1987—when we only focus on the short sample (1992 – 2012), the predictive $R^2$ rises to 30.6% (unreported results).

Building on previous work by Brunnermeier and Pedersen (2009), Paye (2012) suggests
that financial intermediation possibly amplifies shocks to asset markets in times when financial intermediaries find themselves in deleveraging spirals. If this relationship holds, the leverage ratio of broker-dealers and the PBI return should capture information on future volatility beyond that already contained in the macro-finance variables. Contrary to this view, Panel B reveals that the incremental power of the broker-dealer variables is modest because none of the $t$-statistics is statistically different from zero.

4.2 Equity Variance Risk Premium

Next, we analyze the dynamics of the equity VRP inferred from the cross-section of variance portfolios. Specifically, we use the conditional two-pass regression approach described in Section 2 to estimate of the $J$-vector $V^e_v$ that drive the risk-neutral expectation of the realized variance. Then, we plug the two estimated vectors $\hat{F}_v$ and $\hat{V}^e_v$ into Equation (5) to compute the equity VRP as

$$b_{e,v,t} = b_{e,0} z_t,$$

where $b_{e,v} = b_{F,v} b_{V^e,v}$.

The estimated vector $\hat{\Lambda}^e_v$ obtained with the macro-finance variables is shown in Panel A of Table 4. In an multi-period setting, risk-averse investors want to hedge against increases in aggregate volatility because such changes represent a deterioration in investment opportunities. Therefore, stocks that do well in times of high volatility should command lower expected returns (e.g., Campbell, Giglio, Polk, and Turley (2013)). Consistent with this view, the average equity VRP equals $-1.48\%$ per year ($-0.38 \cdot 4$), and is comparable to the unconditional estimate of $-1.00\%$ per year found by Ang, Hodrick, Xing, and Zhang (2006).

The remaining coefficients in Panel A provide new insights into the dynamics of the equity VRP. First, the lagged realized variance has a significant impact on the VRP, both economically and statistically—a one-standard deviation increase in realized variance increases the magnitude of the VRP by $1.32\%$ per year ($0.33 \cdot 4$). The intuition for
this result is simple: in volatile periods, the price of assets that pay off when future volatility increases further becomes extremely valuable; this effect dominates the increase in expected future variance documented in Table 3 (i.e., $\hat{V}_{et} > \hat{F}_{et}$). Interestingly, the physical and risk-neutral expectation effects offset each other for both the PE ratio and the default spread because the estimated coefficients in Panel A are not statistically significant. Second, the coefficients associated with the inflation and employment rates are both positive. Since both predictors tend to be high in expansions, this result suggests that the VRP is countercyclical. However, only past inflation exhibits a statistically significant coefficient, possibly because its lower persistence level helps capture higher frequency business cycle fluctuations. Third, the conditional two-factor model does a good job at capturing the return dynamics of the 25 variance portfolios. Using the joint test of Kan, Robotti, and Shanken (2013), we find that the model is not rejected by the data at conventional significance thresholds.

In Figure 3, we plot the path of the equity VRP obtained with the macro-finance variables. The premium is negative most of the time and is characterized by transitory spikes during the 1973-74 economic crisis, the 1987 crash, the burst of the dotcom bubble, or the 2008 crisis. With an autocorrelation coefficient of 0.51, it also inherits some of the persistence exhibited by the predictors. Finally, there is only a partial overlap between episodes of highly negative VRP and the NBER recession periods. For instance, the magnitude of the premium is high throughout late 1990s and early 2000s.

Turning to the analysis of the broker-dealer variables, we observe in Panel B that none of the coefficients associated with the leverage ratio, the change in leverage and the PBI return is statistically different from zero. These results suggest that the risk-bearing capacity of financial intermediaries has little influence on the pricing of variance risk in the equity market—as shown in Figure 4 the equity VRP paths computed with and without the broker-dealer variables are nearly indistinguishable.

Bates (2000) argues that the probability of negative extreme events as perceived by investors has increased after the 1987 crash. This change could have potentially
contributed to increase the magnitude of the variance risk premium.\footnote{See Bollerslev and Todorov (2011) for how an increase in the risk neutral probability of jumps could contribute to increase the magnitude of the VRP.} Motivated by his findings, we examine whether the 1987 crash has triggered a structural change in the equity market by estimating the VRP over the short sample (1992-2012). Figure 5 reveals that the VRP paths computed over the long and short samples look very similar. In addition, the estimated coefficients associated with the different predictors do not change dramatically either (unreported results). Taken together, these empirical findings do not support the hypothesis of a structural break after 1987.

To conclude this section, we examine the time-variation of the estimated market risk premium obtained from the two-factor model. As discussed in Section 2, this premium is defined as \( \hat{\Lambda}_t^m z_t \), where \( \hat{\Lambda}_t^m = \hat{F}_m - \hat{V}_m \), and \( \hat{F}_m \) is the \( J \)-vector of coefficients from the time-series regression of the quarterly market excess return on the predictors. Consistent with the previous literature, Figure (6) shows that the premium rises during recession periods. This countercyclicality is mostly driven by the PE ratio, which produces the most significant estimated coefficient across all predictors (unreported results).

Studying the time-series properties of the market risk premium provides a second test of the two-factor model. If the latter is correctly specified, the risk premium of the traded market factor must be equal to its conditional average return (e.g., Cochrane (2005)). To see if this restriction holds, we simply test whether the vector \( \hat{V}_m^e \) equals zero. We find that this equality is not rejected by the data because none of the estimated coefficients in \( \hat{V}_m^e \) is statistically significant at conventional levels (unreported results).

\textit{[FIGURE 6 HERE]}
4.3 Option Variance Risk Premium

Previous studies infer the dynamics of the VRP directly from the prices of index options. Specifically, the implied variance $i_v$ provides an option-based measure of the risk-neutral expectation of the realized variance. Exploiting this result, we can simply take the difference between $\tilde{F}_v^0 z_t$ and $i_v$ to estimate the "raw" option VRP (e.g., Bollerslev, Gibson, and Zhou (2011)). As shown in Figure 1, this premium is mostly negative, which is again consistent with the idea that investors are willing to accept lower returns to be protected against positive volatility shocks. In addition, it exhibits spikes that are much larger than those observed in the equity market. For instance, the magnitude of the premium is as large as 10% per year in the last quarter of 1998 (LTCM collapse), 2008 (height of the financial crisis), and 2011 (European debt crisis).

Whereas this simple approach provides a convenient way to determine the fluctuations of the option VRP, it is not informative about the role played by the different predictors in driving such fluctuations. To address this issue, we compute a version of the option VRP that is only conditioned on the predictors. As shown in Equation (6), it is defined as $\lambda_{o,t} = \tilde{\lambda}_v^o z_t$, where $\tilde{\lambda}_v^o = \tilde{F}_v^0 - \tilde{V}_v^o$, and the $J$-vector $\tilde{V}_v^o$ is obtained by regressing the implied variance on the predictors using the GMM approach described in Section 2.

In Panel A of Table 5, we report the estimated vector $\tilde{\lambda}_v^o$ associated with the macrofinance variables. Overall, the results are similar to those documented for the equity VRP, except for the positive and significant estimated coefficient associated with the PE ratio. More importantly, Panel B reveals a strong positive relationship between the two broker-dealer variables and the option VRP. When broker-dealers deleverage or suffer short-term losses, the magnitude of the option VRP increases (in absolute value), whereas the opposite holds when their leverage or their stock returns are above average. The estimated coefficient for the leverage ratio is not only highly significant, it is also economically large: a one-standard deviation decrease in leverage boosts the magnitude of the premium by 1.40% per year (-0.36). Because the two orthogonalized broker-dealer
variables are negatively correlated (-0.26), the predictive information contained in the PBI return is obscured when this predictor is used alone in the regression. Adding the leverage ratio cleans up the relationship between the PBI return and the option VRP and produces a positive and statistically significant coefficient (0.18). The rightmost columns of Panel B confirm that all these results remain unchanged when the leverage ratio is replaced with the annual change in leverage.

As a further check, Table 6 reports the estimated vector $\hat{V}_o$ for the predictive regression of the implied variance. Because the implied variance is a measure of option expensiveness, the coefficients in $\hat{V}_o$ indicate how option prices respond to changes in the predictor values. The empirical evidence in Panel B is the mirror image of its counterpart in Table 5, i.e., the coefficients are all negative and highly significant, which implies that options become more expensive when broker-dealer leverage and the PBI return are low.

The overall evidence is consistent with the role played by financial intermediaries in the risk sharing process. Garleanu, Pedersen, and Potoshman (2009) and Chen, Joslin, and Ni (2013) show empirically that public investors have a large long net position in SP500 index options, in particular in deep out of the money put options. By market clearing, option dealers satisfy this demand by writing options and are, therefore, structurally short variance risk. The risk-bearing capacity of these broker-dealers can change, possibly because they face more or less tight risk constraints (Etula (2009), Adrian and Shin (2010)); in particular, Adrian and Shin (2013) find that intermediaries actively manage their balance sheet in response to Value-at-Risk constraints. When the ability or willingness of broker-dealers to supply options goes down, the price of options increases and vice versa.\textsuperscript{15}

Finally, the reported link between broker-dealer variables and option VRP cannot be fully attributed to an alternative demand-based option pricing mechanism. First, we would expect demand pressure from hedgers to be at least partially captured by

\textsuperscript{15}For instance, Chen, Joslin, and Ni (2013) show that financial intermediaries became net buyers of deep out-of-the-money options during major negative events that followed the onset of the global financial crisis in 2009. These periods also correspond to spikes in option VRP.
aggregate macro-finance variables and be reflected in the price of variance risk in both
equity and option markets. Second, a shift in the demand for options would imply
a positive relationship between the magnitude of the option VRP and risk-taking by
broker-dealers. The data suggest the opposite pattern, which is consistent with a shift in
the supply of options.\footnote{A similar conclusion is reached in Chen, Joslin, and Ni (2013), who document a negative relationship
between the quantity of options exchanged and their expensiveness.}

4.4 Equity versus Option Variance Risk Premia

The comparison of the equity and the "raw" option VRPs in Figure 1 reveals that the two
series diverge significantly at times—for instance, the magnitude of the option VRP is
much larger during the 2008 and European debt crises, whereas the opposite relationship
is observed in the beginning of 2000. To examine whether the different predictors can
explain these discrepancies, we study the properties of the predictor-based VRP difference
\( \hat{d}_{v,t} \), defined as \( \hat{\lambda}_{v,t}^e - \hat{\lambda}_{v,t}^o = \hat{D}_v z_t \), where \( \hat{D}_v \) is equal to the difference between the \( J \)-vectors
\( \hat{\Lambda}_v^e \) and \( \hat{\Lambda}_v^o \) that drive the dynamics of the equity and option VRPs (reported in Tables 4
and 5, respectively).

The difference vector \( \hat{D}_v \) is shown in Table 7, along with the vector of \( t \)-statistics
computed with the bootstrap approach described in the appendix. There are several
points worth noting. First, the average difference between the equity and option VRPs
is not statistically different from zero. Therefore, a simple analysis of the unconditional
risk premia is not sufficient to uncover the large, but temporary mispricing across the
two markets. Second, there is significant evidence that the gap between the two premia
varies with the leverage of financial intermediaries. Specifically, the estimated coefficient
shown in Panel B is negative and implies that a one-standard deviation drop in leverage
increases the magnitude of the option VRP by 1.12% per year compared to the equity

\footnote{A similar conclusion is reached in Chen, Joslin, and Ni (2013), who document a negative relationship
between the quantity of options exchanged and their expensiveness.}
VRP (−0.28 · 4)—a change nearly as large as the average premium itself. Third, the discrepancy between the two VRPs is also negatively related to the PBI return (-0.20). As a result, aggregate losses experienced by broker-dealers have a much larger impact on the option VRP. Fourth, the only relevant macro-finance variable for explaining the VRP difference is the PE ratio (see Panel A). However, unreported results reveal that its impact becomes insignificant when we allow the broker-dealer leverage to compete with the PE ratio (i.e., when we do not orthogonalize leverage). Therefore, the predictive ability of the PE ratio may arise for its positive correlation with the leverage ratio, in particular during the late 90's when both predictors are above-average.

The coefficients estimated over the short sample (1992-2012) and documented in Figure 8 largely support the analysis above. The leverage ratio and the PBI return still produce coefficients that are negative and highly significant.\textsuperscript{17} Furthermore, none of the macro-finance variables including the PE ratio explains the difference between the two markets.

\textbf{[TABLE 7 HERE]}

\textbf{[TABLE 8 HERE]}

Whereas the leverage of broker-dealers helps explain the gap between the equity and option markets, it remains to be shown that it explains a large fraction of the "raw" VRP difference displayed in Figure 1. To examine this issue, we first plot both the leverage ratio and the "raw" difference in Figure 7, and find that the relationship between the two variables is striking. A high leverage ratio signals periods when the magnitude of the option VRP is lower than its equity counterpart (in absolute value). This is the case prior to the monetary policy tightening in 1994, during the 2001-2003 monetary policy easing, and between 2006 and 2008. On the contrary, episodes when the magnitude of the option VRP increases dramatically correspond to sharp contractions in broker-dealer

\textsuperscript{17}One difference with Table 7 is that the estimated coefficient associated with the change in leverage is not statistically significant. Whereas the magnitude of this coefficient is lower during the short sample (-0.11 versus -0.19), the reduction in the sample size (84 versus 171 observations) lowers the precision of the estimated coefficients.
leverage. Interestingly, Figure 7 reveals that the negative relationship between leverage and the "raw" difference is not only observed during the recent financial crisis.

Second, we measure to which extent the predictor-based VRP difference $\hat{d}_{vt}$ tracks the time-variation of the "raw" difference. As shown in Figure 8, using macro-finance variables produces an overly-smoothed path that leaves 53% of the variation of the "raw" difference unexplained. In contrast, adding the two broker-dealer variables allows us to explain up to 65% of the time-variation of the "raw" difference.

[FIGURE 7 HERE]

[FIGURE 8 HERE]

4.5 Interpreting the Evidence

Our analysis so far reveals that the two proxies for the risk-bearing capacity of financial intermediaries have a significant impact on the option VRP, but do not drive the price of variance risk in the equity market. As a result, we observe a time-varying difference between the two risk premia that constitutes an apparent arbitrage opportunity. Perhaps the most natural explanation for this discrepancy is the presence of informational or regulatory constraints that produce market segmentation and limit risk-sharing between marginal investors in the two markets.\textsuperscript{18} For instance, retail investors can lack the expertise required to monitor option positions, whereas mutual funds generally face limits on the amount of options they can hold in their portfolios. In addition, option trading desks of financial institutions may be constrained to trade exclusively in the underlying asset necessary to manage the delta of their positions, but not in other stocks (e.g., only in SP500 index futures for SP500 index options traders). As a result, the option VRP is determined by the capacity of broker-dealers to supply options—when they are constrained and the option VRP is high (in absolute value), equity investors are not able

\textsuperscript{18}Basak and Croitoru (2000) provide the theoretical foundations for this result. Specifically, they show that mispricing between two redundant securities can exist in equilibrium in presence of portfolio constraints that limit investors positions in the two markets.
to write options in sufficient amount to provide protection against spikes in aggregate volatility; conversely when broker-dealers are unconstrained, stock market investors do not fully take advantage of the cheap protection against volatility risk. This explanation is consistent with our key findings that changes in the risk bearing capacity of broker-dealers strongly affects the option VRP (but not its equity counterpart), and help explain both the positive and negative differences between the two VRPs observed in the data.

Alternatively, the gap in the pricing of variance risk may result from different funding constraints. Specifically, Garleanu and Pedersen (2011) provide a theoretical framework in which identical assets can exhibit different prices if they are traded in markets with different margin requirements. Applied to our setting, the theory predicts that the price of identical cash flows should be lower in the stock market because it exhibits higher margin requirements than the option market. In addition, this price discrepancy should increase with the tightness of funding constraints, leading to a time-varying and positive VRP difference between the equity and option markets. Given that prime brokers provide financing to institutions such as hedge funds, the two broker-dealer variables may simply capture the tightness of these funding constraints.\textsuperscript{19} This story is not supported by the data when we take more direct measures of funding constraints tightness. First, the default spread, which is included in the set of macro-finance variables, does not exhibit a significant relationship with the VRP difference (see Tables 7 and 8). Second, when we include the TED spread in the set of predictors (on its own or together with broker-dealer leverage), unreported results reveal that the coefficient has the wrong sign and is not statistically significant either ($t$-statistics of $-0.03$ (without leverage) and $-0.92$ (with leverage)). More importantly, a margin-based explanation cannot easily account for the possibility of both positive and negative VRP differences because margin requirements in the option market are unlikely to be greater than those in the equity market.

\textsuperscript{19}For instance, Adrian, Etula, and Muir (2012) interpret broker-dealer leverage as a proxy for the tightness of funding constraints.

Whereas market segmentation seems more consistent with our main findings, it is nat-
ural to consider the role of specialized arbitrageurs who have a strong incentive to correct any significant price discrepancy induced by segmented markets. For instance, when the magnitude of option VRP is sufficiently high, an arbitrageur could write expensive index options and invest in a portfolio of stocks with a positive beta to the market variance factor. Our evidence is not entirely consistent with arbitrageurs playing an active role in correcting the relative mispricing of variance risk, possibly because of high trading costs associated with rebalancing variance portfolios. If their ability to set up arbitrage trades is limited by funding constraints, we would expect to observe pronounced differences between the two VRPs only when broker-dealer leverage and prime broker index returns are low—that is, when the funding liquidity of arbitrageurs is limited. However, we find that discrepancies in the pricing of variance risk also arises when financial intermediaries’ leverage and their stock returns are high.

Overall, our evidence leads to a more nuanced view of the informational content of the VRP computed from option prices. Spikes in the option VRP can arise when financial intermediaries are in a deleveraging phase, whereas a low price of risk could be the consequence of an increased willingness of broker-dealers to take on risk. Because of market segmentation, such fluctuations might not reflect actual changes in investors’ risk aversion in the equity market. Interestingly, we also find that episodes when options seems cheap relative to the equity market valuation levels correspond to periods of monetary easing. Figure 7 plots the VRP difference and broker-dealer leverage series against shaded areas that correspond to periods where the Federal Reserve reduced its target for the federal funds rate by more than 100 basis points in total and subsequently kept it at this level. The correlation between the changes in the target federal funds rate and changes in leverage is equal to -0.29. This suggests a connection between monetary policy, risk-taking by financial intermediaries, and option prices.
4.6 Sensitivity Analysis

To check the robustness of our results, we examine whether the dynamics of the equity and option VRPs are sensitive to the choice of the macro-finance variables. First, we replace the PE ratio with the dividend yield computed from the CRSP index. Second, we replace the quarterly growth rate in employment with two alternative indicators of real activity: the (seasonally-adjusted) quarterly growth rate in industrial production, and the business cycle indicator constructed by Aruoba, Diebold, and Scotti (2009) that aggregates information about employment, industrial production, and interest rates. Third, we take the initial set of macro-finance variables and add two commonly-used interest rate variables: the 3-month Tbill rate and the term spread, defined as the difference between the 10- and 1-year Tbond yields. Finally, we add the quarterly variance of the inflation rate following recent work by Paye (2012) which shows that this variable helps forecast quarterly future volatility. The results in Table 9 reveal that the paths followed by the VRP is extremely stable across the different specifications—in each market, the correlation with the baseline VRP computed with the initial set of predictors is always above 0.9.

[TABLE 9 HERE]

The previous analysis reveals that the VRP difference between the equity and option markets is particularly large during volatile periods. Therefore, it raises the concern that the explanatory power of the broker-dealer variables is only driven by a few extreme observations for the volatility process. Unreported results show this is not the case, i.e., the coefficients that drive the VRP difference remain significant for both predictors when 1% and 2.5% of the most negative and positive variance observations are winsorized.
5 Conclusion

In this paper we infer the path of the VRP from the cross-section of stock returns and compare it to the VRP implied by option prices. Whereas the two premia are in line with each other on average, we identify episodes when they diverge and find that such differences are explained to a large extent by changes in the risk-taking behaviour of financial intermediaries who supply options. More precisely, proxies for broker-dealer risk-bearing capacity are significant explanatory variables for the option VRP, but come out as not significant for the equity VRP. Interestingly, we find that times when options are relatively cheap coincide with periods of monetary easing. The relationship between monetary policy, financial intermediation, and asset prices is an interesting topic for future research about which our paper provides novel empirical evidence. In sum, our findings can be exploited in future theoretical work that attempts to explain the aggregate pricing of variance risk and model local demand and supply factors in option market.
References


Appendix

A The Equity Variance Risk Premium

A.1 Two-pass Regression in a Conditional Setting

This section explains how to estimate the $J$-vectors of coefficients $F_v$ and $V_v^e$ that drive the evolution of the equity VRP. To this end, we use the two-pass regression approach developed by Gagliardini, Ossola, and Scaillet (2013). In the first step, we estimate, for each variance portfolio $p$ ($p = 1, ..., n$), the coefficients of the following time-series regression:

$$r_{p,t+1} = \beta_{p,1}'z_t + b_{p,m} \cdot f_{m,t+1} + b_{p,v} \cdot f_{v,t+1} + e_{p,t+1}, \quad (A1)$$

where $z_t$ is the $J$-vector of lagged predictors (including a constant), $f_{m,t+1}$ is the excess market return, and $f_{v,t+1}$ is the variance risk factor. The $(J+2)$-vector of coefficients, $\beta_p = (\beta_{p,1}', b_{p,m}, b_{v,m})'$, is equal to $Q_x^{-1}E[x_{t+1}r_{p,t+1}]$, where $Q_x$ is a $(J+2) \times (J+2)$ matrix equal to $E[x_{t+1}x_{t+1}']$, and $x_{t+1} = (z_t', f_{m,t+1}, f_{v,t+1})'$. The OLS estimator is computed as

$$\hat{\beta}_p = \hat{Q}_x^{-1} \frac{1}{T} \sum_{t=1}^{T} x_{t+1}r_{p,t+1}, \quad (A2)$$

where $T$ is the total number of return observations, and $\hat{Q}_x = \frac{1}{T} \sum_{t=1}^{T} x_{t+1}x_{t+1}'$.

In the second step, we compute the estimator of the $2J$-vector $V^e = [V_m^e, V_v^e]'$ that drives the risk-neutral expectations of the two risk factors. Specifically, we use a WLS approach to estimate the following cross-sectional regression:

$$\beta_{p1} = -\beta_{p2}V^e, \quad (A3)$$

where $\beta_{p2}$ a $J \times 2J$ matrix equal to $[b_{p,m}I_J, b_{p,v}I_J]$, and $I_J$ is the $J \times J$ identity matrix. For each portfolio $p$, we compute a $J \times J$ matrix of weights $w_p$ equal to $diag(v_p)^{-1}$, where $v_p$ is
the $J \times J$ covariance matrix of the $J$-vector of standardized errors $\epsilon_p = \sqrt{T} (\hat{\beta}_{p1} - \hat{\beta}'_{p2} V^e)$, i.e.,

\[ v_p = C_V' Q_x^{-1} S_{pp} Q_x^{-1} C_V', \quad (A4) \]

where $S_{pp}$ is a $(J + 2) \times (J + 2)$ matrix equal to $E[\epsilon_{p,t+1}^2 x_{t+1} x_{t+1}']$, and $C_V$ is a $(J + 2) \times J$ matrix defined as $[E'_1 - (I_J \otimes V^e) J_A E'_2]'$, with $E_1 = [I_J, 0_{J \times 2}]'$, $E_2 = [0_{2 \times J}, I_2]'$, $J_A = W_{J,J-2}(I_2 \otimes \text{vec}(I_J))$, $0_{J \times 2}$ is a $J \times 2$ matrix of zeros, and $W_{J,J-2}$ is a $(J, J \cdot 2)$-commutation matrix.\(^{20}\) The empirical counterpart of Equation (A4) is given by

\[ \hat{v}_p = C_{\hat{V}_1}' \hat{S}_{pp}^{-1} \hat{S}_x^{-1} C_{\hat{V}_1}, \quad (A5) \]

where $\hat{S}_{pp} = \frac{1}{T} \sum_{t=1}^{T} \hat{\epsilon}_{p,t+1}^2 x_{t+1} x_{t+1}'$, $\hat{\epsilon}_{p,t+1} = r_{p,t+1} - \hat{\beta}'_{p2} x_{t+1}$, $C_{\hat{V}_1} = [E'_1 - (I_J \otimes \hat{V}^e_1) J_A E'_2]'$, and $\hat{V}^e_1$ is the first-step OLS estimator of $V^e$ obtained using unit weights, i.e., $\hat{V}^e_1 = \left(\sum_{p=1}^{n} \hat{\beta}'_{p2} \hat{\beta}_{p2}\right)^{-1} \sum_{p=1}^{n} \hat{\beta}'_{p2} \hat{\beta}_{p1}$. Using the estimated matrix of weights $\hat{w}_p$ equal to $\text{diag}(\hat{v}_p)^{-1}$, we obtain the following estimator of $V^e$:

\[ \hat{V}^e = \hat{Q}^{-1}_{\hat{\beta}_2} \frac{1}{n} \sum_{p=1}^{n} \hat{\beta}'_{p2} \hat{w}_p \hat{\beta}_{p1}, \quad (A6) \]

where $\hat{Q}_{\hat{\beta}_2}$ is a $2J \times 2J$ matrix equal to $\frac{1}{n} \sum_{p=1}^{n} \hat{\beta}'_{p2} \hat{w}_p \hat{\beta}_{p2}$.

The conditional expectation of the risk factors, $F_{t+1} = (f_{m,t+1}, f_{v,t+1})'$, is given by $F' z_t$, where $F = [F_m, F_v]$ is a $J \times 2$ matrix of coefficients equal to $Q_z^{-1} E[z_t f_{t+1}']$, with $Q_z = E[z_t z_t']$. To estimate $F$, we run a time-series regression of the factors on the lagged predictors to obtain

\[ \hat{F} = \hat{Q}_z^{-1} \frac{1}{T} \sum_{t=1}^{T} z_t f_{t+1}', \quad (A7) \]

where $\hat{Q}_z = \frac{1}{T} \sum_{t=1}^{T} z_t z_t'$. Combining Equations (A6) and (A7), we can determine the

\(^{20}\)The commutation matrix $W_{n,m}$ of order $n \cdot m \times n \cdot m$ is defined such that $W_{n,m} \text{vec}(A) = \text{vec}(A')$ for any matrix $A \in \mathbb{R}^{n \times n}$. 38
path followed by the market and variance risk premia:
\[
\hat{\lambda}_{m,t} = \left( \hat{F}_m - \hat{V}_m \right)' z_t, \\
\hat{\lambda}_{v,t} = \left( \hat{F}_v - \hat{V}_v \right)' z_t.
\]  
(A8)

**A.2 Distribution of the Estimated Coefficients**

In an unconditional setting, Jagannathan and Wang (1998) show that the estimated vector of unconditional risk premia, \( \hat{\lambda} \), is consistent and asymptotically normally distributed. Its covariance matrix, \( \Sigma_\lambda \), is equal to \( \Sigma_f + \frac{1}{n} \Sigma_V \), where \( \Sigma_f \) is the covariance matrix of the factors, \( n \) is the number of test assets, and \( \Sigma_V \) is the covariance matrix of the vector of estimated expectations of the risk factors under the risk-neutral measure.

Gagliardini, Ossola, and Scaillet (2013) show that a similar result holds in a conditional setting. Specifically, the 2J-vector \( \hat{\Lambda}^c = vec(\hat{F}) - \hat{V}^c \) is consistent and asymptotically normally distributed as
\[
\sqrt{T} \left( \hat{\Lambda}^c - \Lambda^c \right) \Rightarrow N(0_{2J \times 1}, \Sigma_{\Lambda^c}).
\]  
(A9)

The 2J × 2J covariance matrix \( \Sigma_{\Lambda^c} \) is the sum of two terms, \( \Sigma_F + \frac{1}{n} \Sigma_V \), defined as
\[
\Sigma_F = (I_J \otimes Q_z^{-1}) \Sigma_u (I_J \otimes Q_z^{-1}),
\]  
(A10)
\[
\Sigma_V = \left( \frac{1}{n} B'_n W_n B_n \right)^{-1} \frac{1}{n} B'_n W_n V_n W_n B_n \left( \frac{1}{n} B'_n W_n B_n \right)^{-1},
\]  
(A11)
where \( \Sigma_u \) is a 2J × 2J matrix equal to \( E(u_{t+1} u_{t+1}' \otimes z_t z_t') \) with \( u_{t+1} = f_{t+1} - F' z_t \), \( B_n \) is a \( Jn \times 2J \) matrix defined as \([\beta_{12}', \ldots, \beta_{n2}']\), \( W_n \) is a \( Jn \times Jn \) block diagonal matrix with elements \([w_p]_{p=1,\ldots,n} \) on its diagonal, and \( V_n \) is a \( Jn \times Jn \) matrix composed of \( J \times J \) submatrices \([V_{pk}]_{p,k=1,\ldots,n} \) with \( V_{pk} = C'_V Q_x^{-1} S_{pk} Q_x^{-1} C'_V \), and \( S_{pk} = E[e_{p,t+1} x_{k,t+1} x_{t+1}' x_{t+1}'] \). A consistent estimator of \( \Sigma_{\Lambda^c} \) can be obtained by replacing \( \Sigma_u, Q_z^{-1}, B_n, W_n, \) and \( V_n \) with their empirical counterparts in Equation (A11).
A.3 Joint Test of Correct Specification

To test whether the two-factor model is correctly specified, we use the test statistic proposed by Kan, Robotti, and Shanken (2013). Under the null of correct specification, the sum of the pricing errors, \( \sum_{p=1}^{n} (\beta_{p1} - \beta_{p2} V^e)^t (\beta_{p1} - \beta_{p2} V^e) \), is equal to zero. The test is based on the weighted sum of squared residuals,

\[
\hat{Q}_e = \frac{1}{n} \sum_{p=1}^{n} \hat{\epsilon}_p^t \hat{w}_p \hat{\epsilon}_p, \tag{A12}
\]

where \( \hat{\epsilon}_p \) is the J-vector of estimated errors, \( \hat{\epsilon}_p = \hat{\beta}_{p1} - \hat{\beta}_{p2} \hat{V}^e \). Under the null, the test statistic, \( T \cdot \hat{Q}_e \), is asymptotically distributed as \( \frac{1}{n} \sum_{p=1}^{n} \epsilon_p^2 \), where \( \epsilon_p^2 \) are i.i.d. chi-square variables with one degree of freedom, and \( \epsilon_p \) are the non-zero eigenvalues of the matrix \( D \) equal to

\[
D = V_n^{1/2}(W_n - W_nB_n(B_n'W_nB_n)^{-1}B_n'W_n)V_n^{1/2}. \tag{A13}
\]

B The Option Variance Risk Premium

B.1 Treatment of Samples with Unequal Lengths

This section explains how to estimate the J-vectors of coefficients, \( F_v \) and \( V_v^o \), that drive the evolution of the option VRP. To estimate \( F_v \), we simply regress the realized variance \( f_{v,t+1} \) on the predictors \( z_t \). Therefore, the vector of estimated coefficients, \( \hat{F}_v \), is exactly the same as the one used for the equity VRP (see Equation (A7)).

To estimate \( V_v^o \), we run a time-series regression of the implied variance \( iv_t \) on the predictors. The main issue is that the periods over which \( f_{v,t+1} \), \( iv_t \), and \( z_t \) are observed have unequal lengths–while \( f_{v,t+1} \) and \( z_t \) are available since 1970 (i.e., the long sample), \( iv_t \) is only observed since the early 90’s (i.e., the short sample). To exploit the information contained in the long sample, we use the extension of the Generalized Method of Moments (GMM) developed by Lynch and Wachter (2013).
To begin, we denote by \( g(V_{o,s}) \), \( g(F_{v,s}) \) the \( J \)-vectors of moments associated with \( V_{o} \) and \( F_{v} \) over the short sample, whereas \( g(F_{v}) \) is the \( J \)-vector of moments associated with \( F_{v} \) over the long sample:

\[
\begin{align*}
g(V_{o,s}) &= \frac{1}{\lambda T} \sum_{t=1}^{T} f_t(V_{o,s}) = \frac{1}{\lambda T} \sum_{t=1}^{T} z_t(u_{v,t+1} - V_{o,s}^\prime z_t), \\
g(F_{v,s}) &= \frac{1}{\lambda T} \sum_{t=1}^{T} f_t(F_{v,s}) = \frac{1}{\lambda T} \sum_{t=1}^{T} z_t(u_{v,t+1} - F_{v,s}^\prime z_t), \\
g(F_{v}) &= \frac{1}{T} \sum_{t=1}^{T} f_t(F_{v,s}) = \frac{1}{T} \sum_{t=1}^{T} z_t(f_{v,t+1} - F_{v,s}^\prime z_t),
\end{align*}
\]  

\( (B1) \)

where \( \lambda \) is the fraction of the period during which all variables are observed. The procedure proposed by Lynch and Wachter (2013) consists in using a new set of moments to estimate \( V_{o}^\prime \):

\[
g(V_{o}) = g(V_{o,s}) - B_{V_{o},F_{v}}(g(F_{v}) - g(F_{v,s})),
\]

\( (B2) \)

where each row of the \( J \times J \) matrix \( B_{V_{o},F_{v}} \) contains the coefficients of a regression of each element in \( g(V_{o,s}) \) on the \( J \)-vector \( g(F_{v,s}) \). To estimate this matrix, we compute the estimated vectors \( \hat{V}_{o,s} \) and \( \hat{F}_{v,s} \) over the short sample; then, we use Equations (B1) to compute the \( J \)-vectors \( f_t(\hat{V}_{o,s}) \) and \( f_t(\hat{F}_{v,s}) \) at each time \( t \), and run a time-series regression of each element in \( f_t(\hat{V}_{o,s}) \) on \( f_t(\hat{F}_{v,s}) \).

By construction, the estimated long sample moment, \( g(\hat{F}_{v}) \), equals zero because we use it to compute \( \hat{F}_{v} \), whereas the estimated short sample moment, \( g(\hat{F}_{v,s}) \), is given by

\[
\frac{1}{\lambda T} Z'(Y_{f_{v}} - Z\hat{F}_{v}),
\]

where \( Z = [z'_{(1-\lambda)T+1}, \ldots, z'_T]' \), and \( Y_{f_{v}} = [f_{v,(1-\lambda)T+2}, \ldots, f_{v,T+1}]' \). Plugging these estimates into Equation (B2), we have:

\[
\begin{align*}
g(V_{o}) &= \frac{1}{\lambda T} Z'(Y_{f_{v}} - ZV_{o}) - \hat{B}_{V_{o},F_{v}} \left( \frac{1}{\lambda T} Z'(Y_{f_{v}} - Z\hat{F}_{v}) \right) \\
&= \frac{1}{\lambda T} \left( Z'(Y_{f_{v}} - ZV_{o}) - \hat{B}_{V_{o},F_{v}} Z'(Y_{f_{v}} - Z\hat{F}_{v}) \right),
\end{align*}
\]  

\( (B3) \)
where \( Y_{iv} = [iv(1-\lambda)T+1, ..., ivT]' \). Therefore, the adjusted estimated vector given by
\[
\hat{V}_v^o = (Z'Z)^{-1} \left( Z'Y_{iv} - \hat{D} \right) = \hat{V}_{v,S}^o - \hat{A},
\] (B4)

where \( \hat{V}_{v,S}^o \) is the short sample estimate of \( V_v^o \) equal to \( (Z'Z)^{-1} Z'Y_{iv} \), and \( \hat{A} \) is the adjustment factor given by \( (Z'Z)^{-1} \hat{B}_{V_v,F_v} Z'(Y_{iv} - Z\hat{F}_v) \). Combining the two estimated vectors \( \hat{F}_v \) and \( \hat{V}_v^o \), we can compute the option VRP as \( (\hat{F}_v - \hat{V}_v^o)'z_t \).

### B.2 Distribution of the Estimated Coefficients

Using the results derived by Lynch and Wachter (2013), we can determine the properties of the \( 2J \)-vector of estimated coefficients, \( \hat{C}^o = [\hat{F}_v', \hat{V}_v^o]' \). Specifically, it is consistent and asymptotically normally distributed as
\[
\sqrt{NT} \left( \hat{C}^o - C^o \right) \Rightarrow N(0_{2J \times 1}, \Sigma_{C^o}).
\] (B5)

The \( 2J \times 2J \) covariance matrix \( \Sigma_{C^o} \) is equal to
\[
\Sigma_{C^o} = (I_2 \otimes E[z_t z_t']^{-1}) S^A (I_2 \otimes E[z_t z_t']^{-1}),
\] (B6)

where \( S^A \) is defined as
\[
S^A = \begin{bmatrix}
\lambda S_{F_v} & \lambda S_{F_v V_v^o} \\
\lambda S_{V_v^o, F_v} & S_{V_v^o} - (1 - \lambda) S_{V_v^o, F_v} S_{F_v}^{-1} S_{F_v, V_v^o}
\end{bmatrix},
\] (B7)

with \( S_{F_v} = \sum_{\tau = -\infty}^{\infty} E[f_t(F_v)f_{t-\tau}(F_v)'] \), \( S_{F_v, V_v^o} = \sum_{\tau = -\infty}^{\infty} E[f_t(F_v)f_{t-\tau}(V_v^o)'] \), and, finally, \( S_{V_v^o} = \sum_{\tau = -\infty}^{\infty} E[f_t(V_v^o)f_{t-\tau}(V_v^o)'] \). To estimate these elements, we build on the procedure described by Stambaugh (1997) and Lynch and Wachter (2013). First, we use the White estimator to compute \( \hat{S}_{F_v} \) over the full sample. Then, we use the estimated coefficient matrix \( \hat{B}_{V_v,F_v} \) and the estimated residual covariance matrix \( \hat{\Sigma} \) from the regression of \( f_t(\hat{V}_v^o) \) on \( f_t(\hat{F}_v) \) to compute the remaining terms:
\[ S_{F_v,V^o} = \hat{S}_{F_v,\hat{B}^o_{V^o,F_v}}, \quad \text{(B8)} \]
\[ \hat{S}_{V^o} = \hat{\Sigma} + \hat{B}^o_{V^o,F_v} \hat{S}_{F_v,\hat{B}^o_{V^o,F_v}}. \quad \text{(B9)} \]

This approach guarantees that the estimator of \( \hat{S}^A \) is positive-definite. Plugging this estimator in Equation (B7) and replacing \( E[z_t z'_{t}] \) with its estimated value over the long sample, \( \hat{Q}_z = \frac{1}{T} \sum_{t=1}^{T} z_t z'_t \), we get a consistent estimator of \( \Sigma_{C_v} \).

Building on Equation (B7), we can also derive the asymptotic distribution of the \( J \)-vector of estimated coefficients that drive the option VRP:

\[ \sqrt{T}(\hat{\Lambda}_v^o - \Lambda_v^o) \Rightarrow N(0_{J \times 1}, \Sigma_{\Lambda_v}), \quad \text{(B10)} \]

where \( \hat{\Lambda}_v^o \) is equal to the difference between \( \hat{F}_v \) and \( \hat{V}_v^o \). The \( J \times J \) covariance matrix \( \Sigma_{\Lambda_v} \) is given by

\[ \Sigma_{\Lambda_v} = \Sigma_{C_v}^1 + \Sigma_{C_v}^2 - 2\Sigma_{C_v}^{1,2}, \quad \text{(B11)} \]

where \( \Sigma_{C_v}^1 \) is the \( J \times J \) upper block of \( \Sigma_{C_v} \), \( \Sigma_{C_v}^2 \) is the \( J \times J \) lower block, and \( \Sigma_{C_v}^{1,2} \) is the off-diagonal block.

### C t-Statistics for the Difference in Estimated Coefficients

The time-variation of the equity and option VRPs is governed by the two estimated \( J \)-vectors \( \hat{\Lambda}_v^e \) and \( \hat{\Lambda}_v^o \) defined above. To determine whether these two vectors differ from each other, we compute the difference vector, \( \hat{D}_v = \hat{\Lambda}_v^e - \hat{\Lambda}_v^o \), and use a bootstrap approach to determine the \( t \)-statistics associated with \( \hat{D} \). Consistent with the specification chosen to estimate the equity and option VRPs, we assume that the dynamics of the excess return of each variance portfolio \( p \) (\( p = 1, \ldots, 25 \)), the market return, the realized variance, the
implied variance, and the vector of predictors is given by:

\[
\begin{align*}
    r_{p,t+1} & = -(b_{pm} \cdot V_m^{\epsilon} + b_{pv} \cdot V_v^{\epsilon})z_t + b_{pm} \cdot f_{m,t+1} + b_{pv} \cdot f_{v,t+1} + e_{p,t+1}, \\
    f_{m,t+1} & = F_m^{\epsilon}z_t + u_{m,t+1}, \\
    f_{v,t+1} & = F_v^{\epsilon}z_t + u_{v,t+1}, \\
    iv_{t+1} & = V_v^{\epsilon}z_t + u_{iv,t+1}, \\
    z_{t+1} & = \Phi z_t + u_{z,t+1}.
\end{align*}
\]  

(C1)

After estimating the different coefficients in the system of Equations (C1), we build a \(\lambda T \times N\) matrix of estimated residuals, \(\hat{\mathbf{R}} = [\hat{e}, \hat{u}_m, \hat{u}_v, \hat{u}_{iv}, \hat{u}_z]\), where \(\lambda T\) is the number of observations over the short sample, \(N = 25 + 3 + (J - 1)\), \(\hat{u}_m = [\hat{u}_{m,(1-\lambda)T+2}, \ldots, \hat{u}_{m,T+1}]'\), \(\hat{u}_v = [\hat{u}_{v,(1-\lambda)T+2}, \ldots, \hat{u}_{v,T+1}]'\), \(\hat{u}_{iv} = [\hat{u}_{iv,(1-\lambda)T+2}, \ldots, \hat{u}_{iv,T+1}]'\), \(\hat{u}_z = [\hat{u}_{z,(1-\lambda)T+2}, \ldots, \hat{u}_{z,T+1}]'\), and \(\hat{e} = [\hat{e}_{(1-\lambda)T+2}, \ldots, \hat{e}_{T+1}]'\), where each element \(\hat{e}_t\) is equal to \([\hat{e}_{1,t}, \ldots, \hat{e}_{25,t}]\).

For each bootstrap iteration \(b (b = 1, \ldots, 1,000)\), we first draw with replacement a set of \(T\) rows from the matrix \(\hat{\mathbf{R}}\). This procedure allows to preserve the cross-sectional correlation between the residuals. Second, we plug the estimated coefficients and the bootstrapped residuals into Equations (C1) to build, for each time \(t (t = 1, \ldots, T)\), the \(J\)-vector of predictors \(z_t^b\), the excess return of the variance portfolios \(r_{p,t+1}^b\), the market return \(f_{m,t+1}^b\), and the realized variance, \(f_{v,t+1}^b\). Third, we do the same for the implied variance \(iv_{t+1}^b\) using the bootstrapped residuals over the short sample. Fourth, we use all of these bootstrapped time-series and reestimate the \(J\)-vectors of coefficients \(\hat{F}_v(b)\), \(\hat{V}_v^\epsilon(b)\), and \(\hat{V}_v^{\epsilon}(b)\) using the approach proposed by Gagliardini, Ossola, and Scaillet (2013) and Lynch and Wachter (2013). Fifth, we compute \(\hat{A}_v^\epsilon(b)\) as \(\hat{F}_v(b) - \hat{V}_v^\epsilon(b)\), \(\hat{A}_v^\epsilon(b)\) as \(\hat{F}_v(b) - \hat{V}_v^{\epsilon}(b)\), and \(D_v(b)\) as \(\hat{A}_v^\epsilon(b) - \hat{A}_v^\epsilon(b)\). After repeating these five steps 1,000 times, we compute the \(t\)-statistic of each element, \(\hat{d}_{v,j} (j = 1, \ldots, J)\), as \(\frac{\hat{d}_{v,j}}{\hat{\sigma}_{d_{v,j}}}\), where \(\hat{\sigma}_{d_{v,j}}\) is the standard deviation of the 1,000 bootstrapped values.
Table 1: Summary Statistics for the Predictive Variables

Panel A reports the quarterly mean and standard deviation of the different variables used to explain the dynamics of the Variance Risk Premium (VRP), which are the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate of the producer price index, the quarterly growth rate of the number of employees in the nonfarm sector (seasonally-adjusted), the leverage ratio of broker-dealers, and the quarterly return of the prime broker index (all expressed in log form). The remaining columns of Panel A show the skewness, kurtosis, first-, and second-order partial autocorrelation coefficients of the standardized versions of the predictors. Panel B shows the correlation matrix of the standardized predictors. All statistics are computed using quarterly data between January 1970 and September 2012 (171 observations).

### Panel A: Unconditional Moments

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std.</th>
<th>Skew.</th>
<th>Kurt.</th>
<th>AC1</th>
<th>AC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged Realized Variance (RV)</td>
<td>-5.32</td>
<td>0.80</td>
<td>0.82</td>
<td>4.43</td>
<td>0.65</td>
<td>0.13</td>
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<tr>
<td>Price/Earnings Ratio (PE)</td>
<td>1.24</td>
<td>0.20</td>
<td>0.14</td>
<td>2.18</td>
<td>0.93</td>
<td>-0.10</td>
</tr>
<tr>
<td>Default Spread (DEF)</td>
<td>1.03%</td>
<td>0.41%</td>
<td>2.16</td>
<td>10.81</td>
<td>0.81</td>
<td>-0.11</td>
</tr>
<tr>
<td>Producer Price Index (PPI)</td>
<td>0.94%</td>
<td>1.31%</td>
<td>0.00</td>
<td>5.69</td>
<td>0.34</td>
<td>0.18</td>
</tr>
<tr>
<td>Employment Growth (EMP)</td>
<td>0.37%</td>
<td>0.58%</td>
<td>-0.99</td>
<td>4.74</td>
<td>0.75</td>
<td>0.03</td>
</tr>
<tr>
<td>Broker-Dealer Leverage (LEV)</td>
<td>2.70</td>
<td>0.62</td>
<td>0.84</td>
<td>4.50</td>
<td>0.85</td>
<td>0.05</td>
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<tr>
<td>Prime Broker Index (PBI)</td>
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<td>16.6%</td>
<td>-0.54</td>
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</table>

### Panel B: Correlations

<table>
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<tr>
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<th>DEF</th>
<th>PPI</th>
<th>EMP</th>
<th>LEV</th>
<th>PBI</th>
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</thead>
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<tr>
<td>Lagged Realized Variance (RV)</td>
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<td>-0.05</td>
<td>-0.42</td>
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<td>-0.28</td>
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<tr>
<td>Price/Earnings Ratio (PE)</td>
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<td>-0.11</td>
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<td>0.33</td>
<td>0.07</td>
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<tr>
<td>Default Spread (DEF)</td>
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<td>-0.62</td>
<td>0.05</td>
<td>0.00</td>
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<tr>
<td>Producer Price Index (PPI)</td>
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<td>-0.11</td>
<td>-0.02</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Employment Growth (EMP)</td>
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<td></td>
<td></td>
<td>-0.12</td>
<td>-0.05</td>
<td></td>
</tr>
<tr>
<td>Broker-Dealer Leverage (LEV)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.14</td>
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</table>
Table 2: Summary Statistics for the Variance Portfolios

Panel A shows the annualized excess mean, standard deviation, and the pre-, post-rank variance betas of the quarterly returns of quintile portfolios formed by equally weighting all portfolios in the same variance beta quintile (Low, 2, 3, 4, High). For each portfolio, the pre-rank beta are defined as the mean of the variance beta across stocks on the portfolio formation date over the whole sample. The post-rank variance beta is computed from the time-series regression of the portfolio return on the market return, the realized variance, and the set of macro-finance variables. Panel B reports the annualized estimated alpha of each portfolio using the CAPM, the Fama-French model and two extensions that include the momentum and liquidity portfolios. The momentum portfolio is downloaded from Ken French’s website and the Pastor-Stambaugh liquidity portfolio is downloaded from Lubos Pastor’s website. The t-statistics of the different estimators are shown in parentheses and are robust to the presence of heterokedasticity. All statistics are computed using quarterly data between April 1970 and December 2012 (171 observations).

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Mean (% p.a.)</th>
<th>Std. Dev. (% p.a.)</th>
<th>Pre-rank beta</th>
<th>Post-rank beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>7.55</td>
<td>18.10</td>
<td>-0.71</td>
<td>-0.30</td>
</tr>
<tr>
<td>2</td>
<td>7.54</td>
<td>18.75</td>
<td>-0.34</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>6.40</td>
<td>17.91</td>
<td>-0.05</td>
<td>0.27</td>
</tr>
<tr>
<td>4</td>
<td>5.18</td>
<td>18.66</td>
<td>0.25</td>
<td>0.36</td>
</tr>
<tr>
<td>High</td>
<td>4.75</td>
<td>18.94</td>
<td>0.65</td>
<td>0.25</td>
</tr>
<tr>
<td>High-Low</td>
<td>-2.81</td>
<td>8.93</td>
<td>1.36</td>
<td>0.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quintile</th>
<th>CAPM (% p.a.)</th>
<th>Fama-French (FF) (% p.a.)</th>
<th>FF+Momentum (% p.a.)</th>
<th>FF+Liquidity (% p.a.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>1.84</td>
<td>(1.91)</td>
<td>1.46 (1.58)</td>
<td>1.22 (1.18)</td>
</tr>
<tr>
<td>2</td>
<td>1.47</td>
<td>(1.98)</td>
<td>1.79 (2.53)</td>
<td>1.89 (2.57)</td>
</tr>
<tr>
<td>3</td>
<td>0.56</td>
<td>(0.88)</td>
<td>0.98 (1.51)</td>
<td>1.33 (1.92)</td>
</tr>
<tr>
<td>4</td>
<td>-0.89</td>
<td>(-1.26)</td>
<td>-0.34 (-0.49)</td>
<td>0.11 (0.13)</td>
</tr>
<tr>
<td>High</td>
<td>-1.31</td>
<td>(-1.44)</td>
<td>-0.80 (-0.94)</td>
<td>-0.04 (-0.05)</td>
</tr>
<tr>
<td>High-Low</td>
<td>-3.16</td>
<td>(-2.11)</td>
<td>-2.26 (-1.72)</td>
<td>-1.27 (-0.88)</td>
</tr>
</tbody>
</table>
Table 3: Realized Variance Predictability

Panel A reports the estimated coefficients and the predictive $R^2$ of a time-series regression of the quarterly realized variance on the set of macro-finance predictors that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The figures in parentheses report the $t$-statistics of the estimated coefficients that are robust to the presence of heterokedasticity. Panel B examines the incremental predictive power of the orthogonalized broker-dealer variables. The leftmost columns report the estimated coefficients for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The rightmost columns repeat the analysis after replacing the leverage ratio with the annual change in the leverage ratio ($\Delta$LEV). ***, **, and * designate statistical significance at the 1%, 5%, and 10% level, respectively.

### Panel A: Macro-Finance Variables

<table>
<thead>
<tr>
<th>Mean RV Variance</th>
<th>R. Var. (RV)</th>
<th>PE ratio (PE)</th>
<th>Default (DEF)</th>
<th>Inflation (PPI)</th>
<th>Employ. (EMP)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R. Variance</td>
<td>0.75***</td>
<td>0.49***</td>
<td></td>
<td></td>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(8.77)</td>
<td>(3.49)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Variables</td>
<td>0.75***</td>
<td>0.39***</td>
<td>0.18*</td>
<td>0.25**</td>
<td>0.12</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(8.89)</td>
<td>(3.52)</td>
<td>(1.84)</td>
<td>(2.20)</td>
<td>(1.12)</td>
<td>(0.32)</td>
</tr>
</tbody>
</table>

### Panel B: Contribution of Broker-Dealer Variables

<table>
<thead>
<tr>
<th>Leverage (LEV)</th>
<th>PB Index (PBI)</th>
<th>$R^2$</th>
<th>$\Delta$Leverage (LEV)</th>
<th>PB Index (PBI)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Leverage</td>
<td>0.28</td>
<td>0.23</td>
<td>0.19</td>
<td>(0.99)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.18)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+Prime Broker</td>
<td></td>
<td>0.18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−0.07)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−1.09)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+Leverage &amp; Prime Broker</td>
<td>0.28</td>
<td>0.01</td>
<td>0.23</td>
<td>0.19</td>
<td>−0.05</td>
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<tr>
<td></td>
<td>(1.08)</td>
<td>(0.05)</td>
<td>(0.93)</td>
<td>(−0.61)</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Equity Variance Risk Premium

Panel A reports the estimated coefficients that drive the equity VRP for the set of macro-finance predictors that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). To compute the different coefficients, we apply the conditional two-pass regression approach of Gagliardini, Ossola, and Scaillet (2013) on the cross-section of variance portfolio returns. The figures in parentheses report the t-statistics of the estimated coefficients that are robust to the presence of heteroskedasticity. The J-stat. and associated p-values in parentheses are based on the joint test proposed by Kan, Robotti, and Shanken (2013). Panel B examines the incremental predictive power of the orthogonalized broker-dealer variables. The leftmost columns report the estimated coefficients for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The rightmost columns repeat the analysis after replacing the leverage ratio with the annual change in the leverage ratio (ΔLEV). ***, **, and * designate statistical significance at the 1%, 5%, and 10% level, respectively.

### Panel A: Macro-Finance Variables

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R. Variance</td>
<td>-0.37*</td>
<td>-0.27</td>
<td></td>
<td></td>
<td></td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td>(-1.92)</td>
<td>(-1.55)</td>
<td></td>
<td></td>
<td></td>
<td>(0.14)</td>
</tr>
<tr>
<td>All Variables</td>
<td>-0.37**</td>
<td>-0.33*</td>
<td>-0.06</td>
<td>0.12</td>
<td>0.27*</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(-2.06)</td>
<td>(-1.84)</td>
<td>(-0.25)</td>
<td>(0.44)</td>
<td>(1.77)</td>
<td>(0.22)</td>
</tr>
</tbody>
</table>

### Panel B: Contribution of Broker-Dealer Variables

<table>
<thead>
<tr>
<th></th>
<th>Leverage (LEV)</th>
<th>PB Index (PBI)</th>
<th>J-stat.</th>
<th>ΔLeverage (ΔLEV)</th>
<th>PB Index (PBI)</th>
<th>J-stat.</th>
</tr>
</thead>
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<tr>
<td>+Leverage</td>
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<td></td>
<td>(0.70)</td>
<td>(0.48)</td>
<td>(0.14)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+Prime Broker</td>
<td>-0.07</td>
<td>6.99</td>
<td>— —</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.37)</td>
<td>(0.23)</td>
<td>— —</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+Leverage &amp; Prime Broker</td>
<td>0.13</td>
<td>-0.02</td>
<td>7.48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.75)</td>
<td>(-0.11)</td>
<td>(0.33)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.07</td>
<td>-0.04</td>
<td>8.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.42)</td>
<td>(-0.25)</td>
<td>(0.19)</td>
</tr>
</tbody>
</table>
Table 5: Option Variance Risk Premium

Panel A reports the estimated coefficients that drive the option VRP for the set of macro-finance predictors that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). To compute the different coefficients, we run a time-series regression of the option-based implied variance on the set of predictors using the GMM approach developed by Lynch and Wachter (2013). The figures in parentheses report the $t$-statistics of the estimated coefficients that are robust to the presence of heteroskedasticity. Panel B examines the incremental predictive power of the orthogonalized broker-dealer variables. The leftmost columns report the estimated coefficients for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The rightmost columns repeat the analysis after replacing the leverage ratio with the annual change in the leverage ratio ($\Delta$LEV). ***, **, and * designate statistical significance at the 1%, 5%, and 10% level, respectively.

### Panel A: Macro-Finance Variables

<table>
<thead>
<tr>
<th></th>
<th>Mean (RV)</th>
<th>R. Var. (RV)</th>
<th>PE ratio (PE)</th>
<th>Default (DEF)</th>
<th>Inflation (PPI)</th>
<th>Employ. (EMP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R. Variance</td>
<td>$-0.48^{***}$</td>
<td>$-0.31^{***}$</td>
<td>$-0.70^{***}$</td>
<td>$-3.00$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Variables</td>
<td>$-0.48^{***}$</td>
<td>$-0.36^{***}$</td>
<td>$0.29^{***}$</td>
<td>$0.03$</td>
<td>$0.17^{*}$</td>
<td>$-0.08$</td>
</tr>
</tbody>
</table>

### Panel B: Contribution of Broker-Dealer Variables

<table>
<thead>
<tr>
<th></th>
<th>Leverage (LEV)</th>
<th>PB Index (PBI)</th>
<th>$\Delta$Leverage (LEV)</th>
<th>PB Index (PBI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Leverage</td>
<td>$0.35^{***}$</td>
<td>$0.07$</td>
<td>$0.27^{***}$</td>
<td>$0.14^{*}$</td>
</tr>
<tr>
<td>Prime Broker</td>
<td>$0.42^{***}$</td>
<td>$0.18^{**}$</td>
<td>$0.34^{***}$</td>
<td>$0.14^{*}$</td>
</tr>
</tbody>
</table>


Table 6: Implied Variance Predictability

Panel A reports the estimated coefficients and the predictive $R^2$ of a time-series regression of the quarterly implied variance on the set of macro-finance predictors that include the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). To compute the different coefficients, we use the GMM approach developed by Lynch and Wachter (2013). The figures in parentheses report the $t$-statistics of the estimated coefficients that are robust to the presence of heterokedasticity. Panel B examines the incremental predictive power of the orthogonalized broker-dealer variables. Panel B examines the incremental predictive power of the orthogonalized broker-dealer variables. The leftmost columns report the estimated coefficients for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The rightmost columns repeat the analysis after replacing the leverage ratio with the annual change in the leverage ratio ($\Delta$LEV). ***, **, and * designate statistical significance at the 1%, 5%, and 10% level, respectively.

Panel A: Macro-Finance Variables

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>R. Var.</th>
<th>PE ratio</th>
<th>Default</th>
<th>Inflation</th>
<th>Employ.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R. Variance</td>
<td>1.22***</td>
<td>0.80***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>(23.20)</td>
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<tr>
<td>All Variables</td>
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<td>0.76***</td>
<td>−0.12</td>
<td>0.22***</td>
<td>−0.05</td>
<td>0.10</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>(26.46)</td>
<td>(9.46)</td>
<td>(−2.01)</td>
<td>(3.40)</td>
<td>(−1.11)</td>
<td>(1.54)</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Contribution of Broker-Dealer Variables

<table>
<thead>
<tr>
<th></th>
<th>Leverage (LEV)</th>
<th>PB Index (PBI)</th>
<th>$R^2$</th>
<th>$\Delta$Leverage (ΔLEV)</th>
<th>PB Index (PBI)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Leverage</td>
<td>−0.07*</td>
<td></td>
<td>0.77</td>
<td>−0.08</td>
<td></td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(−1.82)</td>
<td></td>
<td></td>
<td>(−2.13)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+Prime Broker</td>
<td>−0.14**</td>
<td></td>
<td>0.78</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>(−2.02)</td>
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</tr>
<tr>
<td>+Leverage &amp; Prime Broker</td>
<td>−0.14***</td>
<td>−0.18**</td>
<td>0.79</td>
<td>−0.15***</td>
<td>−0.19***</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>(−3.49)</td>
<td>(−2.42)</td>
<td></td>
<td>(−5.07)</td>
<td>(−2.84)</td>
<td></td>
</tr>
</tbody>
</table>

50
Table 7: Equity versus Option Variance Risk Premia

Panel A reports the difference between the estimated coefficients that drive the equity and option VRPs (reported in Tables 4 and 5, respectively) for the set of macro-finance variables that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The figures in parentheses report the t-statistics of the estimated coefficients computed using a bootstrap procedure described in the appendix. Panel B examines the incremental predictive power of the orthogonalized broker-dealer variables. The leftmost columns report the estimated coefficients for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The rightmost columns repeat the analysis after replacing the leverage ratio with the annual change in the leverage ratio (ΔLEV). ***, **, and * designate statistical significance (based on the bootstrap distributions) at the 1%, 5%, and 10% level, respectively.

### Panel A: Macro-Finance Variables

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>R. Var. (RV)</th>
<th>PE ratio (PE)</th>
<th>Default (DEF)</th>
<th>Inflation (PPI)</th>
<th>Employ. (EMP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R. Variance</td>
<td>0.10</td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.42)</td>
<td>(0.20)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Variables</td>
<td>0.10</td>
<td>0.04</td>
<td>-0.36**</td>
<td>0.10</td>
<td>0.11</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.36)</td>
<td>(2.06)</td>
<td>(0.39)</td>
<td>(0.67)</td>
<td>(0.75)</td>
</tr>
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</table>

### Panel B: Contribution of Broker-Dealer Variables

<table>
<thead>
<tr>
<th></th>
<th>Leverage (LEV)</th>
<th>PB Index (PBI)</th>
<th>ΔLeverage (ΔLEV)</th>
<th>PB Index (PBI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Leverage</td>
<td>-0.23**</td>
<td></td>
<td>-0.19*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.06)</td>
<td></td>
<td>(-2.55)</td>
<td></td>
</tr>
<tr>
<td>+Prime Broker</td>
<td>-0.14</td>
<td></td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.48)</td>
<td></td>
<td>(-1.48)</td>
<td></td>
</tr>
<tr>
<td>+Leverage &amp;</td>
<td>-0.28***</td>
<td>-0.20**</td>
<td>-0.27***</td>
<td>-0.19**</td>
</tr>
<tr>
<td>Prime Broker</td>
<td>(-2.55)</td>
<td>(-2.03)</td>
<td>(-2.42)</td>
<td>(-1.96)</td>
</tr>
</tbody>
</table>
Table 8: Equity versus Option Variance Risk Premia: Short Sample

Panel A reports the difference between the estimated coefficients that drive the equity and option VRPs over the short sample (1992-2012) for the set of macro-finance variables that includes the lagged realized variance (RV), the price/earnings ratio (PE), the default spread (DEF), the quarterly inflation rate (PPI), and the quarterly employment rate (EMP). The figures in parentheses report the t-statistics of the estimated coefficients computed using a bootstrap procedure described in the appendix. Panel B examines the incremental predictive power of the orthogonalized broker-dealer variables. The leftmost columns report the estimated coefficients for the broker-dealer leverage ratio (LEV) and the quarterly return of the prime broker index (PBI) in the presence of the macro-finance variables. The rightmost columns repeat the analysis after replacing the leverage ratio with the annual change in the leverage ratio (ΔLEV). ***, **, and * designate statistical significance (based on the bootstrap distributions) at the 1%, 5%, and 10% level, respectively. Note that we do not need to use the GMM approach developed by Lynch and Wachter (2013) to estimate the coefficients that drive the option VRP because the samples of data for realized and implied variances have equal length.

### Panel A: Macro-Finance Variables

<table>
<thead>
<tr>
<th>Mean</th>
<th>R. Var.</th>
<th>PE ratio</th>
<th>Default</th>
<th>Inflation</th>
<th>Employ.</th>
</tr>
</thead>
<tbody>
<tr>
<td>R. Variance</td>
<td>-0.04</td>
<td>-0.13</td>
<td>(−0.13)</td>
<td>(−0.62)</td>
<td></td>
</tr>
<tr>
<td>All Variables</td>
<td>0.04</td>
<td>-0.23</td>
<td>-0.08</td>
<td>0.11</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(−0.80)</td>
<td>(−0.37)</td>
<td>(0.34)</td>
<td>(0.34)</td>
</tr>
</tbody>
</table>

### Panel B: Contribution of Broker-Dealer Variables

<table>
<thead>
<tr>
<th>Leverage (LEV)</th>
<th>PB Index (PBI)</th>
<th>ΔLeverage (ΔLEV)</th>
<th>PB Index (PBI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Leverage</td>
<td>-0.30**</td>
<td>-0.11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−2.52)</td>
<td>(−0.65)</td>
<td></td>
</tr>
<tr>
<td>+Prime Broker</td>
<td></td>
<td>-0.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(−1.21)</td>
<td></td>
</tr>
<tr>
<td>+Leverage &amp; Prime Broker</td>
<td>-0.37***</td>
<td>-0.30***</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>(−2.95)</td>
<td>(−2.57)</td>
<td>(−0.82)</td>
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</tbody>
</table>
Table 9: Alternative Set of Macro-Finance Variables

This table examines the robustness of the equity and option VRPs to changes in the set of macro-finance variables. The first specification replaces the price/earnings ratio with the dividend yield. The second and third specifications replace the quarterly growth rate in employment with the quarterly growth rate in industrial production and the business cycle indicator proposed by Aruoba, Diebold, and Scotti (2009), respectively. The fourth, fifth and sixth specifications add two bond variables (the three-month TBill rate and the term spread) and the quarterly volatility of the inflation rate to the baseline set of macro-finance variables. For each specification and each market (equity and option), we report the estimated coefficient and t-statistic (in parentheses) associated with the new predictor, the correlation between the new VRP and the baseline VRP, and the quarterly mean and standard deviation of their difference.

<table>
<thead>
<tr>
<th></th>
<th>Equity Variance Risk Premium</th>
<th></th>
<th></th>
<th>Option Variance Risk Premium</th>
<th></th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Coeff.</td>
<td>Diff. with Baseline</td>
<td></td>
<td>Coeff.</td>
<td>Diff. with Baseline</td>
<td></td>
</tr>
<tr>
<td>Dividend Yield</td>
<td>0.23</td>
<td>0.02</td>
<td>0.16</td>
<td>0.94</td>
<td>−0.23</td>
<td>0.02</td>
</tr>
<tr>
<td>(0.98)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(−4.20)</td>
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<tr>
<td>Industrial Production</td>
<td>0.21</td>
<td>−0.02</td>
<td>0.17</td>
<td>0.93</td>
<td>−0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>(0.97)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(−0.37)</td>
<td></td>
</tr>
<tr>
<td>Business Cycle</td>
<td>0.15</td>
<td>0.00</td>
<td>0.12</td>
<td>0.96</td>
<td>−0.25</td>
<td>0.02</td>
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<tr>
<td>(0.78)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(−2.85)</td>
<td></td>
</tr>
<tr>
<td>Short Rate</td>
<td>0.15</td>
<td>0.00</td>
<td>0.12</td>
<td>0.95</td>
<td>−0.11</td>
<td>0.01</td>
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<tr>
<td>(0.63)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(−1.39)</td>
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<td>Term Spread</td>
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<td>0.03</td>
<td>0.10</td>
<td>0.97</td>
<td>0.18</td>
<td>0.01</td>
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<tr>
<td>(−0.54)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2.40)</td>
<td></td>
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<tr>
<td>Vol. Inflation</td>
<td>0.14</td>
<td>−0.01</td>
<td>0.11</td>
<td>0.97</td>
<td>0.13</td>
<td>0.01</td>
</tr>
<tr>
<td>(0.67)</td>
<td></td>
<td></td>
<td></td>
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<td>(1.32)</td>
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</table>
Figure 1: Equity and Option Variance Risk Premia

This figure reports the path of the quarterly equity VRP estimated from the cross-section of stock returns and the path of the quarterly option VRP inferred from prices of index options. The equity VRP is modeled as a linear function of a set of macro-finance variables that include the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, and the quarterly employment rate. The equity VRP is estimated between January 1970 and December 2012 using a conditional two-pass regression approach described in detail below. The option VRP is defined as the difference between the conditional expectation of the realized variance (based on the macro-finance predictors) and the squared VIX index computed from three-month options and available between January 1992 and December 2012. We refer to this specification of the option VRP as the "raw" option VRP because it uses all the information contained in the VIX index (an alternative formulation of the option VRP is discussed below). Shaded areas correspond to NBER recession periods.
This figure reports the time variation of the quarterly realized variance and its quarterly predicted value based on the set of macro-finance variables that includes the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, and the quarterly employment rate.
Figure 3: Equity Variance Risk Premium

This figure reports the path of the quarterly equity VRP obtained with the set of macro-finance predictors that includes the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, and the quarterly employment rate. Shaded areas correspond to NBER recession periods.
Figure 4: Equity Variance Risk Premium: Impact of the Broker-Dealer Variables

This figure compares the paths followed by two versions of quarterly equity VRP. The first version is based on the set of macro-finance variables that includes the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, and the quarterly employment rate. The second version is based on the macro-finance predictors and the two broker-dealer variables, which are the leverage ratio of broker-dealers and the quarterly return of the prime broker index.
Figure 5: Equity Variance Risk Premium: Short Sample

This figure compares the equity VRP paths computed over the long sample and short samples, respectively. In both cases, the equity VRP is based on the set of macro-finance variables that includes the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, and the quarterly employment rate.
Figure 6: Market Risk Premium

This figure reports the path of the quarterly market risk premium obtained with the set of macro-finance predictors that includes the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, and the quarterly employment rate. Shaded areas correspond to NBER recession periods.
Figure 7: VRP Difference and Broker-Dealer Leverage

This figure shows the relationship between the quarterly leverage ratio of broker-dealers (in log form) and the "raw" VRP difference, defined as the equity VRP minus the "raw" option VRP. The results are obtained with all predictors, including the leverage ratio. Shaded areas correspond to periods where the Federal Reserve reduced its target for the Federal funds rate by more than 100 basis points in total and subsequently kept it at this level.
Figure 8: Equity VRP and Option VRP Difference

This Figure examines the relationship between the quarterly "raw" VRP difference and the predictor-based VRP difference. The "raw" VRP difference is defined as the equity VRP minus the "raw" option VRP. The predictor-based VRP difference is defined as the equity VRP minus the predictor-based option VRP. Panel A shows the results obtained with the set of macro-finance variables that includes the lagged realized variance, the price/earnings ratio, the default spread, the quarterly inflation rate, and the quarterly employment rate. Panel B shows the results obtained with the macro-finance variables, the leverage ratio of broker-dealers, and the quarterly return of the prime broker index. In both panels, the $R^2$ denotes the explanatory power from a time-series regression of the "raw" VRP difference on the predictor-based VRP difference.