Information, Misallocation and Aggregate Productivity

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This paper

“Misallocation,” i.e., dispersion in MP’s ⇒ large losses in TFP and output

- But sources of distortions still unclear...
- Role of imperfect information? Informational role of financial markets?

1. What we do

- Heterogeneous firms choose inputs under imperfect info
- Firms learn from internal/private sources and noisy asset prices
- Quantify frictions using stock market/production data in US, China, India

2. What we find

- Significant micro-level uncertainty, esp. in China and India
  → accounts for 20-50% (+...) of MRPK dispersion
- Sizable aggregate impact
  → TFP losses: 7-10% in China and India, 4% in US; can be much larger...
- Only limited learning from markets; firm internal sources are key
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Simplified model

Homogeneous good, only capital, no agg. risk

- Continuum of producers: \( Y_{it} = A_{it} K_{it}^\alpha, \quad a_{it} \sim iid, \quad \mathcal{N} (0, \sigma_\mu^2) \)

Input choice under incomplete info:

- Choice of \( K_{it} \) conditional on info \( I_{it} \), \( a_{it} | I_{it} \sim \mathcal{N} (\mathbb{E}_{it} a_{it}, \mathbb{V}) \)

\( \mathbb{V} \) is key object:

- Misallocation: \( \sigma_{mpk}^2 = \mathbb{V} \)

- \( TFP : \quad a = a^* - \frac{1}{2} \frac{\alpha}{1 - \alpha} \sigma_{mpk}^2 = a^* - \frac{1}{2} \frac{\alpha}{1 - \alpha} \mathbb{V} \)

\( \Rightarrow TFP \downarrow \) in \( \mathbb{V} \); effect of poor info \( \uparrow \) in \( \alpha \)
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\( \Rightarrow TFP \downarrow \) in \( \mathbb{V} \); effect of poor info \( \uparrow \) in \( \alpha \)
Characterizing $\nabla$

The firm’s information set $I_{it}$

1. Private signal: $s_{it} = a_{it} + e_{it}$, $e_{it} \sim \mathcal{N} (0, \sigma_e^2)$

2. Stock price: $p_{it}$
   - Equivalent to signal $a_{it} + \eta_{it}$, $\eta_{it} \sim \mathcal{N} (0, \sigma_\eta^2)$

3. For now: $(a_{it}, e_{it}, \eta_{it})$ mutually independent

$\Rightarrow$ Sharp characterization of $\nabla$:

$$\nabla = \frac{1}{\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_\eta^2}}$$
Identifying info frictions - simplified model

1. General strategy:
   - Measure $\sigma_\mu^2$ directly: $(a_{it} = y_{it} - \alpha k_{it})$
   - Use $(\rho_{pk}, \rho_{pa})$ to infer $(\sigma_e^2, \sigma_\eta^2)$ or equiv $(\sqrt{V}, \sigma_\eta^2)$

   $$\rho_{pa} = \frac{1}{\sqrt{1 + \frac{\sigma_\eta^2}{\sigma_\mu^2}}} \quad \frac{\sqrt{V}}{\sigma_\mu^2} = 1 - \left(\frac{\rho_{pa}}{\rho_{pk}}\right)^2$$

2. Some appealing properties:
   - Unaffected by correlations in firm and market signals
   - Unaffected by ‘correlated’ distortions
   - Conservative estimate if ‘uncorrelated’ distortions
Identifying info frictions - simplified model

1. General strategy:
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   \[
   \rho_{pa} = \frac{1}{\sqrt{1 + \frac{\sigma_\eta^2}{\sigma_{\mu}^2}}} \quad \frac{\mathbb{V}}{\sigma_{\mu}^2} = 1 - \left(\frac{\rho_{pa}}{\rho_{pk}}\right)^2
   \]

2. Some appealing properties:
   - Unaffected by correlations in firm and market signals
   - Unaffected by ‘correlated’ distortions
   - Conservative estimate if ‘uncorrelated’ distortions
Quantitative model

1. Monopolistic competition: \( Y_t = \left( \int A_{it} Y_{it}^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}} \)

2. Production: \( Y_{it} = K_{it}^{\alpha_1} L_{it}^{\alpha_2} \)
   - Case 1: both factors chosen under imperfect info
   - Case 2: only \( K \) chosen under imperfect info, \( L \) adjusts ex-post

\( \Rightarrow \) Preserves \( \max_{K_{it}} \prod E_{it} [A_{it}] K_{it}^\alpha - RK_{it}; \) with \( \alpha \) in case 1 > \( \alpha \) in case 2

3. Persistence in \( A_{it} \): \( a_{it} = \rho a_{it-1} + \mu_{it}, \mu_{it} \sim N(0, \sigma_\mu^2) \)

4. Explicit model of stock market trading
   - Same info in \( p_{it} \)

\( \Rightarrow \) Preserves \( \forall = \frac{1}{\sigma_{\mu}^2} + \frac{1}{\sigma_{\mu}^2} + \frac{1}{\sigma_{\eta}^2} \)
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\[ \Rightarrow \text{Preserves } \max_{K_{it}} \prod E_{it} [A_{it}] K_{it}^{\alpha} - RK_{it}; \text{ with } \alpha \text{ in case } 1 > \alpha \text{ in case } 2 \]

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4. Explicit model of stock market trading
   - Same info in \( p_{it} \)

\[ \Rightarrow \text{Preserves } \mathbb{V} = \frac{1}{\frac{1}{\sigma_{\mu}^2} + \frac{1}{\sigma_{e}^2} + \frac{1}{\sigma_{\eta}^2}} \]
⇒ Same intuition as simple model:

- $\rho_{pa} \rightarrow$ noise in prices
- $\rho_{pi}$ relative to $\rho_{pa} \rightarrow \nabla$
## General parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time period</td>
<td>Time period</td>
<td>3 years</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.90</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Capital share</td>
<td>0.33</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Labor share</td>
<td>0.67</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution</td>
<td>6</td>
</tr>
</tbody>
</table>

- If $K$ and $L$ both chosen under imperfect information (case 1)
  \[ \alpha = \frac{\theta - 1}{\theta} = 0.83 \]

- If only $K$ chosen under imperfect information (case 2)
  \[ \alpha = 0.62 \]
### The impact of informational frictions

<table>
<thead>
<tr>
<th>Case 2 ($\alpha = 0.62$)</th>
<th>$\frac{\sum}{\sigma^2_\mu}$</th>
<th>$\frac{\sum}{\sigma^2_{mrp}}$</th>
<th>$a^* - a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.41</td>
<td>0.22</td>
<td>0.04</td>
</tr>
<tr>
<td>China</td>
<td>0.63</td>
<td>0.34</td>
<td>0.07</td>
</tr>
<tr>
<td>India</td>
<td>0.77</td>
<td>0.48</td>
<td>0.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 1 ($\alpha = 0.83$)</th>
<th>$\frac{\sum}{\sigma^2_\mu}$</th>
<th>$\frac{\sum}{\sigma^2_{mrp}}$</th>
<th>$a^* - a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.63</td>
<td>0.35</td>
<td>0.40</td>
</tr>
<tr>
<td>China</td>
<td>0.65</td>
<td>0.39</td>
<td>0.55</td>
</tr>
<tr>
<td>India</td>
<td>0.86</td>
<td>0.56</td>
<td>0.77</td>
</tr>
</tbody>
</table>

- Substantial posterior uncertainty (US firms best informed) => significant misallocation, losses in TFP and output
- Effects increase with $\alpha$
Case 1 vs. Case 2

Quantitative impact sensitive to this assumption

- Interpret our results as bounds
- But can we say anything more...?

A suggestive statistic:

- Case 2 $\rightarrow \frac{\sigma_{mrpl}^2}{\sigma_{mrpk}^2} = 0$; case 1 $\rightarrow \frac{\sigma_{mrpl}^2}{\sigma_{mrpk}^2} = 1$

- In US data: $\frac{\sigma_{mrpl}^2}{\sigma_{mrpk}^2} = 0.57$
Decomposing $\nabla$: the contribution of learning and its sources

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Delta a$</th>
<th>Share from source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Private</td>
</tr>
<tr>
<td>Case 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>5%</td>
<td>92%</td>
</tr>
<tr>
<td>China</td>
<td>4%</td>
<td>96%</td>
</tr>
<tr>
<td>India</td>
<td>3%</td>
<td>89%</td>
</tr>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>23%</td>
<td>91%</td>
</tr>
<tr>
<td>China</td>
<td>30%</td>
<td>96%</td>
</tr>
<tr>
<td>India</td>
<td>12%</td>
<td>96%</td>
</tr>
</tbody>
</table>

1. Significant learning $\Rightarrow$ significant aggregate gains
2. Learning is primarily from private sources
   Interpretation? Manager skill/incentives, info collection/processing...
3. Only small role for market-generated info $\Rightarrow$ just too much noise in prices
## Effect of US information structure

<table>
<thead>
<tr>
<th></th>
<th>Case 2</th>
<th>Case 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Market Information</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td>India</td>
<td>1%</td>
<td>4%</td>
</tr>
<tr>
<td><strong>Private Information</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>3%</td>
<td>6%</td>
</tr>
<tr>
<td>India</td>
<td>5%</td>
<td>26%</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>1%</td>
<td>10%</td>
</tr>
<tr>
<td>India</td>
<td>2%</td>
<td>20%</td>
</tr>
</tbody>
</table>

1. Gains from US private info > US market info

2. Differences in fundamentals → differential impact of friction
Conclusion

Theory linking micro uncertainty to misallocation and aggregates

- Substantial uncertainty and associated aggregate losses
- Limited informational role for stock markets
- Significant role for private learning \( \Rightarrow \) drives cross-country differences

Where next?

- Entry/exit
- Other frictions...
Related literature

Misallocation

• Hsieh and Klenow (09), Restuccia and Rogerson (08), ...
• Financial frictions: Buera, Kaboski and Shin (11), Midrigan and Xu (13), ...
• Adjustment costs: Asker, Collard-Wexler and De Loecker (13)
• Information frictions: Jovanovic (13)

Stock price informativeness

• Morck, Yeung and Yu (00), Durnev, Yeung and Zarowin (03), ...

The “feedback” effect (Bond, Edmans and Goldstein (12))

• Investment: Chen, Goldstein and Jiang (07), Bakke and Whited (10), Morck, Schleifer and Vishny (90)
• R&D spending: Bai, Philippon and Savov (13)
• Mergers and acquisitions: Luo (05)
Full-info TFP

Simplified model:

\[ a^* = \frac{1}{2} \frac{\sigma^2_\mu}{1 - \alpha} \]

General model:

\[ a^* = \frac{1}{2} \left( \frac{\theta}{\theta - 1} \right) \frac{\sigma^2_a}{1 - \alpha} \]
The stock market

Unit measure of firm equity traded by 2 type of agents
1. Investors: Can purchase up to single unit at price $p_{it}$
2. Noise traders: purchase random quantity $\Phi(z_{it})$, $z_{it} \sim \mathcal{N}(0, \sigma_z^2)$

Information of investors:
- History: $a_{it-1}$
- Private signal: $s_{ijt} = a_{it} + v_{ijt}$, $v_{ijt} \sim \mathcal{N}(0, \sigma_v^2)$
- Stock price: $p_{it}$

Trading: buy asset if $E_{ijt} \Pi_{it} \geq p_{it}$ or $s_{ijt} > \hat{s}_{it}$

Market clearing: $1 - \Phi\left(\frac{\hat{s}_{it} - a_{it}}{\sigma_v}\right) + \Phi\left(z_{it}\right) = 1$

⇒ Info in price: $\hat{s}_{it} = a_{it} + \sigma_v z_{it} \quad \left[\sigma_\eta^2 = \sigma_v^2 \sigma_z^2\right]$
Identification with iid shocks

\[ \rho_{pa} = \frac{1}{\sqrt{1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}}} \]

\[ \rho_{pk} = \frac{1}{\sqrt{\left(1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}\right) \left(1 - \frac{\nu}{\sigma_\mu^2}\right)}} \]

\[ \sigma_p^2 = \left(\frac{1 - \beta}{1 - \alpha}\right)^2 \left(\frac{\sigma_z^2 + 1}{\sigma_z^2 + \frac{1}{\rho_{pa}^2}}\right)^2 \frac{1}{\rho_{pa}^2} \sigma_\mu^2 \]

(ident)
Identification with permanent shocks

\[ \frac{\nabla}{\sigma_{\mu}^2} = \frac{\rho_{pk} - \rho_{pa}}{\eta} \quad \text{where} \quad \eta = \frac{1}{1 - \alpha} \frac{\sigma_{\mu}}{\sigma_p} \]

\[ \frac{\sigma_v^2 \sigma_z^2}{\sigma_{\mu}^2} = \frac{1 - \eta^2}{2 \rho_{pa}^2} \frac{\eta}{\rho_{pa}} - 1 \]

\[ \frac{\sigma_z^2 + 1}{\sigma_z^2 + 1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_{\mu}^2}} = \frac{1}{\eta} \]
Step 1. \( \text{cov} (p, k) = \text{cov}(p, a) \).

- follows from \( k = E (a|p, s_i) \)
- and since we can write \( a = E (a|p, s_i) + \varepsilon \)
- \( \text{cov} (a, p) = \text{cov} (E (a|p, s_i), p) + \text{cov} (\varepsilon, p) = \text{cov} (k, p) \).

Step 2. divide both sides by \( \sigma_a \sigma_p \) so we get

\[
\frac{[\text{cov} (p, k)]^2}{(\sigma_a \sigma_p)^2} = \rho (p, a)^2 \tag{1}
\]

Step 3. By the law of total covariance, \( \sigma_a^2 = \sigma_k^2 + V \) so

\[
\frac{\sigma_k^2}{\sigma_a^2} = 1 - \frac{V}{\sigma_a^2} \tag{2}
\]

Substituting (2) in (1) we get

\[
\left(1 - \frac{V}{\sigma_a^2}\right) = \left(\frac{\rho (p, a)}{\rho (p, k)}\right)^2
\]

identical to our identification equation.
Introduce alternative ‘distortions’ into capital choice:

\[ \tau_{it} = \gamma \mu_{it} + \varepsilon_{it}, \quad \varepsilon_{it} \sim \mathcal{N} \left( 0, \sigma^2_\varepsilon \right) \]

\[ \Rightarrow k_{it} = \frac{(1 + \gamma) \mathbb{E}[\mu_{it}] + \varepsilon_{it}}{1 - \alpha} \]

1. ‘Correlated’ distortion \( (\gamma \neq 0, \sigma^2_\varepsilon = 0) \)

\[ \Rightarrow \sigma^2_{mrpk} = \gamma^2 (\sigma^2_\mu - \mathbb{V}) + \mathbb{V} > \mathbb{V} \]

But, our measure \( 1 - \left( \frac{\rho_{pa}}{\rho_{pk}} \right)^2 = \frac{\mathbb{V}}{\sigma^2_\mu} \) still valid!

2. ‘Uncorrelated’ distortion \( (\gamma = 0, \sigma^2_\varepsilon \neq 0) \)

\[ \Rightarrow \sigma^2_{mrpk} = \mathbb{V} + \sigma^2_\varepsilon > \mathbb{V} \]

Our measure \( 1 - \left( \frac{\rho_{pa}}{\rho_{pk}} \right)^2 = \frac{\mathbb{V}}{\sigma^2_\mu} - \frac{\sigma^2_\varepsilon}{\sigma^2_\mu} \) is conservative...
Investment-Q regressions

Model has reduced-form representation:

\[ \Delta k_{it} = \lambda_1 (\Delta \mu_{it} + \Delta e_{it}) + \lambda_2 \Delta p_{it} \]

Use model to derive:

\[ \lambda_2 \propto \frac{\nabla}{\sigma^2_{\eta}} \]

Intuition: \( \lambda_2 \uparrow \) in \( \nabla \), \( \downarrow \) in \( \sigma^2_{\eta} \)

But, regression ID's \( \lambda_2 \) only if \( \Delta e_{it} \perp \Delta \mu_{it}, \Delta p_{it} \)

- Violated if correlated signals, correlated distortions...
### Data and parameter values

<table>
<thead>
<tr>
<th></th>
<th>Target moments</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_{pi}$</td>
<td>$\rho_{pa}$</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.23</td>
<td>0.18</td>
</tr>
<tr>
<td>China</td>
<td>0.16</td>
<td>0.06</td>
</tr>
<tr>
<td>India</td>
<td>0.25</td>
<td>0.08</td>
</tr>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.24</td>
<td>0.10</td>
</tr>
<tr>
<td>China</td>
<td>0.15</td>
<td>0.02</td>
</tr>
<tr>
<td>India</td>
<td>0.26</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Data source: Compustat NA and Compustat Global.

- Cross-country variation in moments $\Rightarrow$ variation in parameters
- US : less fundamental uncertainty, better private info, less noise in markets
Transitory vs. permanent MRPK deviations

- Information speaks to dispersion in transitory component
- In US data: transitory $\approx$ one-third of total
- US $\forall$ accounts for 60% in case 2; entirety in case 1
Robustness: adjustment costs

Are we simply labeling adj. costs as info frictions?

- Simulate moments from full-info (for firms) adj. cost model
- Do we estimate large $\mathbb{V}$ with these moments?

<table>
<thead>
<tr>
<th></th>
<th>Adj. Cost</th>
<th>$\mathbb{V}$</th>
<th>Baseline $\mathbb{V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.03</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>0.06</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>0.08</td>
<td>0.22</td>
<td></td>
</tr>
</tbody>
</table>

- $\mathbb{V}$ (and agg effects) about 1/3 of baseline estimates

$\Rightarrow$ Unlikely that we are reading adj. costs as info frictions!
Robustness: correlated information

How would correlation between firm and investors’ signals affect results?

- Correlation $\rightarrow \rho_{pk} \rightarrow \nabla$?

- Re-estimate assuming $s_{ijt} = s_{it} + v_{ijt} = a_{it} + e_{it} + v_{ijt}$

<table>
<thead>
<tr>
<th></th>
<th>(\frac{\nu}{\sigma^2_{\mu}}) w corr. info</th>
<th>(\frac{\nu}{\sigma^2_{\mu}}) baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2 ($\alpha = 0.62$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>China</td>
<td>0.58</td>
<td>0.63</td>
</tr>
<tr>
<td>India</td>
<td>0.68</td>
<td>0.77</td>
</tr>
</tbody>
</table>

⇒ Results quite close to baseline!
Full-information adjustment cost model

- Value function

\[ V \left( \tilde{A}_{it}, K_{it-1} \right) = \max_{K_{it}, N_{it}} G \tilde{A}_{it} \bar{K}_{it} - I_{it} - H \left( I_{it}, K_{it-1} \right) + \beta \mathbb{E} V \left( \tilde{A}_{it+1}, K_{it} \right) \]

where \( l_{it} = K_{it} - (1 - \delta) K_{it-1} \) and \( H \left( I_{it}, K_{it-1} \right) = \zeta K_{it-1} \left( \frac{l_{it}}{K_{it-1}} \right)^2 \)

- Solve numerically for joint distribution of \( \tilde{A}_{it}, K_{it} \) in GE

- Target \( \left( \rho_{pa}, \sigma_p^2, \sigma_k^2 \right) \)

- Simulate data to compute \( \rho_{pi} \) and relative correlation