A Quantitative Model of “Too Big to Fail,” House Prices, and the Financial Crisis

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Abstract

This paper develops a quantitative model that can rationally explain a sizeable part of the dramatic rise and fall of house prices relative to rents in the 2000-2009 period. The model provides a rationale for limits on loan-to-value ratios as a welfare-improving response to a government’s inability to pre-commit not to bail out financial institutions with large losses. A relaxation of those limits results in opportunistic behavior and, for a plausible range of parameters, house price increases of fifteen percent or more relative to fundamentals, along with increases in price/rent ratios and leverage. This “boom” is followed by a collapse with high rates of default. Maintaining tight loan-to-value limits, or otherwise forcing large financial institutions to internalize their risk-taking, can prevent the crisis equilibrium.

The housing boom and bust cycle of the 2000s continues to attract a wide range of explanations. As is well known, inflation-adjusted house prices rose, depending on the the price index, some 60 to 100 percent between 1998 and 2007, and then by 2010 in the wake of the financial crisis and recession fell back nearly to 1998 levels. (See Figure 1.\footnote{The figure is based on the S&P Shiller index that shows a nearly 100 percent increase. Other measures, such as the FHFA House Price Indexes, indicate a roughly 60 percent real increase over the period.}) The ratio of prices to rents behaved similarly, with increases of 40 to 60 percent. (See Figure 2.) While some portion of the rise can be attributed to macroeconomic factors such as income growth and low interest rates, the magnitude of the boom appears to have gone far beyond what standard fundamentals can explain.

Much of the attention of researchers has focused on credit markets as the source of volatility. The literature suggests that obtaining transmission from financial frictions into house prices or requires some nonstandard assumptions. As examples, Fostel and Geanakoplos (2008), Burnside, Eichenbaum, and Rebelo (2011) incorporate heterogeneous beliefs. Other authors, e.g. Favilukis, Ludvigson, and Nieuwerburgh (2010), obtain price effects by alternately impose or relaxing exogenous changes in credit limits for which there is
no clear rationale to begin with. Still others, e.g. Jeske, Krueger, and Mitman (2014), Corbae and Quintin (2013), limit their analysis to credit outcomes and treat house prices as exogenous.

The heterogeneous beliefs models face the question of testability, since they rely on unobservable variation in beliefs. Models that incorporate exogenous reductions borrowing limits to explain the boom shares with other work along these lines (such as Guerrieri and Lorenzoni (2011), Kocherlakota (2009)) the peculiar feature that the “bubble” or boom is welfare-improving. If a borrowing limit is just exogenously imposed (as opposed to being motivated by some other market failure), eliminating it tends to make agents better off by increasing their options.²

By contrast, we provide a quantitative model in which house prices are endogenously distorted above fundamentals as the result of a costly combination of relaxed of borrowing limits and “Too Big to Fail,” i.e. the inability of the government to let large financial institutions incur large losses or fail. And while the relaxation of the limits is, strictly speaking, exogenous in our model as well, the presence of the limits in the first place is motivated by a market failure that makes them socially beneficial. Moreover, there is a divergence between private and social interests regarding borrowing limits that makes their relaxation a natural outcome, though we do not model that in any detail.

In addition, the model maintains standard assumptions that beliefs are rational and homogeneous. In our setting, house prices are bid up as a consequence of increased leverage coupled with a system of guarantees or implicit promises of bailouts. That system, the intent of which is to support home ownership by subsidizing borrowers, gives rise to an ever-present incentive toward excessive leverage, as borrowers and lenders do not face the full consequences of higher default risk. Normally that incentive is blunted by strict limits on leverage as well as scrutiny of borrowers to weed out bad risks, and the result is a system that indeed supports expansive borrowing with little impact on either defaults or house prices.

With that benign outcome as a baseline, we then examine the impact of relaxing the limits on borrowing. Again, why this regulatory forbearance occurred is something we do not model explicitly. There is substantial evidence that it did occur, however. There are documented increases in loan-to-value (LTV) ratios, as well as the apparently increased disregard for other characteristics of borrowers (see Demyanyk and Hemert (2011), for example). In fact, a large number of mortgages in the period leading up to the crisis had combined LTVs (including second mortgages, home equity loans, etc.) of 100 percent.

While aspects of this story are not new, this is the first effort we are aware of to quantify the impact on house prices, leverage, and default rates in a calibrated general equilibrium model. To do this we have

²Moreover, there remains a question (see Kiyotaki, Michaelides, and Nikolov (2011)) whether the transmission of time-varying frictions into house prices is robust to the presence of a viable rental market. Provided credit is not a major constraint on potential landlords, and renting is not subject to the same frictions, it seems likely that credit constraints would affect ownership rates more directly than prices.
to make certain concessions for feasibility: We have a perfect foresight model in which the only actual risk is idiosyncratic. Nonetheless we are able to realistically make the government’s intervention contingent on aggregate defaults. In this regard the paper differs from Jeske, Krueger, and Mitman (2014), who only compare across steady states. They also effectively fix the price of housing (by having a linear transformation between housing and non-housing consumption) and focus on default risk. We endogenize house prices by fixing the stock but, like Jeske, Krueger, and Mitman (2014), impose discipline by having realistic default rates and default costs in our baseline.

1 Background: Government-Sponsored Enterprises and Mortgage Lending

A complete history of government involvement in mortgage lending is beyond the scope of this paper. Suffice it to say that since the Great Depression, the government has had a major role in making mortgages more widely available and affordable to borrowers, and more liquid for lenders. The primary mechanisms were purchasing, insuring, and securitizing mortgages. These efforts were successful in greatly expanding mortgage loans and, arguably, home ownership. For most of this period, through the mid-1980s, government agencies such as the Federal Housing Administration (FHA), and government-sponsored enterprises (GSEs) such as Fannie Mae, confined their involvement to loans that met relatively strict and objective standards for quality. The FHA, which insured private mortgages, was in principle self-financing, i.e. the insurance premiums were set to price default risk accurately.

Beginning with the Fair Housing Act of 1968, policy began to focus on expanding the availability of credit to those who had previously found it difficult to obtain, first by outlawing discrimination, but then by encouraging the extension of credit to riskier pools of borrowers. During this same period, Fannie Mae was privatized (though was widely perceived to have implicit government backing), and was allowed to purchase private non-insured mortgages (as opposed to those insured by the FHA or other government agencies). By the 1990s, the GSEs were required to meet “affordable housing” goals, meaning targets for mortgages of low-income homeowners. These goals became more ambitious by the late 1990s, with private lenders also getting into the act with “subprime” and other loans that did not conform to GSE standards. Ultimately these markets grew enormously, and lenders, both government and private, took on more risk and became highly vulnerable to an economic downturn. It is widely believed that the implicit government backing of the GSEs, as well as the “too big to fail” nature of some of the largest private financial institutions, contributed to this process and ultimately to the magnitude of the crisis that developed in 2008.

The story that implicit government guarantees (most notably of Fannie Mae and Freddie Mac) and other credit market interventions, coupled with lax oversight, resulted in low standards and artificially high prices
is not a new one, yet is not without controversy. Some have argued, for example, that Fannie Mae and Freddie Mac “were victims, not culprits,” pointing out that these agencies did not originate subprime loans, and in fact their share of mortgage originations dropped in 2003-2006. Krugman\textsuperscript{4} writes, moreover, that “Fannie and Freddie didn’t do any subprime lending, because they can’t: the definition of a subprime loan is precisely a loan that doesn’t meet the requirement[s]” imposed on the agencies.

This sanguine view of the GSEs disregards several important facts—facts acknowledged but minimized by these writers. First, as Krugman acknowledges, the GSEs were undercapitalized. The impact of that is to make them more profitable, but with greater risk, much the same effect as holding a mortgage with a higher loan-to-value (LTV) ratio. Second, Pressman concedes that “Fannie and Freddie purchased billions of dollars of subprime-backed securities for their own investment portfolios.” He fails to recognize that this is tantamount to holding subprime mortgages, and given the size of these institutions, helping to support that market. In addition, while the GSEs were constrained to limit their loans to 80 percent LTV and a maximum dollar amount (in 2006 the limit was $417,000, and $625,500 in designated “high-cost” areas), they did expand lending to riskier pools of buyers. According to Doris Dungey\textsuperscript{5} of the “Calculated Risk” blog:

Fannie and Freddie had about as much to with the “explosion of high-risk lending” as they could get away with...[T]hey pushed the envelope on credit quality as far as they could inside the constraints of their charter: they got into “near prime” programs (Fannie’s “Expanded Approval,” Freddie’s “A Minus”) that, at the bottom tier, were hard to distinguish from regular old “subprime” except—again—that they were overwhelmingly fixed-rate “non-toxic” loan structures. They got into “documentation relief” in a big way through their automated underwriting systems, offering “low doc” loans that had a few key differences from the really wretched “stated” and “NINA” crap of the last several years, but occasionally the line between the two was rather thin.

In fact, this effort on the part of Fannie Mae to expand credit to previously ineligible borrowers dates back to the late 1990s.\textsuperscript{6}

Acharya et al. (2011) support this, pinpointing the origin of the problem to the ironically-named Federal Housing Enterprises Financial Safety and Soundness Act (FHEFSSA), somewhat reluctantly signed into law by President George H.W. Bush in 1992. The intent of the legislation had been to restrain the GSEs, but political compromises led to its containing a major Trojan horse: “mission goals” to support housing and mortgages for “underserved areas.” In addition, the newly created regulator, the Office of Federal Housing Enterprise Oversight (OFHEO), was placed in the Department of Housing and Urban Development

\textsuperscript{3}See Pressman, “Fannie Mae and Freddie Mac were victims, not culprits” Business Week, September 26, 2008.
(HUD) rather than a more politically independent entity such as the Federal Reserve. The presence of these goals facilitated massive growth of low-quality mortgages, both through the increased ability of the GSEs to repurchase them as well as the participation of arguably too-big-to-fail so-called “large complex financial institutions” (LCFIs).\footnote{The 14 LCFIs were considered to be, according to Acharya et al. (2011), Citigroup, Bank of America, JP Morgan Chase, Morgan Stanley, Merrill Lynch, AIG, Goldman Sachs, Fannie Mae, Freddie Mac, Wachovia, Lehman Brothers, and Wells Fargo. Arguably 11 of the 14 were at risk of failure at some point in 2008, and all but one of those eleven were either bailed out by the government or folded into one of the three relatively healthy institutions (Bank of America, Wells Fargo, and JP Morgan). Lehman, of course, was the unique case of an LCFI that was allowed to fail.}

Finally, the GSEs facilitated the growth of major subprime lenders. Again from Doris Dungey:

GSEs were major culprits in the growth of the mega-lenders. Over the years they were struggling so hard to maintain market share, they were allowing themselves to experience huge concentration risks. As they catered more and more to their “major partners”–Countrywide, Wells Fargo, WaMu, the usual suspects–they helped sustain and worsen the “aggregator” model in which smaller lenders sold loans not to the GSEs but to CFC or WFC, who then sold the loans to the GSEs.

The GSEs, armed with what was widely viewed as government government backing, thus likely played a role in the expansion of credit that was much larger than their direct role in subprime lending, which was officially negligible at least until the last few years of the boom.\footnote{In 2007 Fannie Mae increased its direct involvement in subprime. See “Fannie’s Perilous Pursuit of Subprime Loans,” The Washington Post, August 19, 2008.} Many private lenders either saw themselves as too big to fail (i.e. subject to government bailouts) or as being able to sell low quality mortgages to investors up a food chain that was ultimately being supported by the government.

There are of course many other aspects to the financial crisis, notably the errors of rating agencies, and, related, apparent misperceptions of the risk of aggregate declines in house prices. Our focus on the role of government is not intended to belittle the role these other factors played. The goal is simply to quantify what we believe to be an important contributor to the boom and bust.

\section{A Two-Period Model with Ex Ante Identical Agents}

Before proceeding to a fully dynamic, heterogeneous agent model, we can illustrate the mechanism at work more transparently in a simple two-period setup where agents are ex ante identical. Since all agents will be making identical choices, some quantitative features of the model will be unrealistic, but the impact of the market distortions on asset prices will carry over reasonably well into the more realistic dynamic heterogeneous agent model.
2.1 Endowments and Constraints

There are two time periods that we call dates 0 and 1. The economy consists of a continuum of ex ante identical agents of unit measure, each endowed at date 0 with a tangible asset \( \bar{h} \) (a “house”), plus “income” \( y \) in the form of certain claims on date 1 consumption. Each has identical concave utility \( u(c) \). Agents can trade any portion \( \bar{h} - h \) of their housing endowment for additional claims on date 1 consumption, and, as described below, borrow against \( h \). Without loss of generality, we can set \( \bar{h} = 1 \). At date 1, consumers’ holdings of \( h \) will turn into consumption, but first will be hit with an idiosyncratic shock \( x \) with unit mean and distribution function \( G \). We assume \( x \) has compact support on \([x, \bar{x}] \equiv \mathcal{X}\). Thus in autarky agents would have uncertain consumption \( c = y + x \).

Markets are incomplete. There are only two trades available. One is to sell housing to other agents at some market price \( P \) in exchange for claims on \( c \). Since agents are identical, in equilibrium such trades will not occur. The other is to issue “secured debt” \( b \) on \( h \) up to \( Ph \), via a financial intermediary that is able to pool risks. Specifically, each agent can trade his \( h \) for an asset that pays

\[
\max \{xh - b, 0\} + b - \rho(\cdot)b.
\]

Here the debt “principal” \( b \) is secured by \( h \). \( \rho(\cdot) \geq 0 \) is the “interest rate” on \( b \), which will depend on leverage, default costs, and “policy” as described in detail below. The agent repays \( \rho(\cdot)b + \min \{b, xh\} \) after \( x \) is revealed. There is also a default cost \( \gamma xh \) incurred by the lender in the event the borrower “defaults” (that is, repays \( hx < b \)).\(^9\) We assume that \( b/h \) does not exceed \( \bar{x} \).\(^{10}\) We treat the repayment of \( \rho(\cdot)b \) as enforceable, as though it is paid continuously, analogously to an interest-only mortgage, so that only the principal is at risk of default.

For ease of notation we can define \( z \equiv b/(Ph) \), the loan-to-value ratio (LTV). So the consumer can borrow \( zPh \) to finance \( h \) at price \( P \), where for enforceability \( z \leq 1 \), but regulatory policy may set an LTV limit requiring \( z \leq \zeta < 1 \). Essentially, the financial arrangements are made ex ante, when all agents are identical, prior to the realization of \( x \). Then \( x \) is realized, financial arrangements are concluded, and each agent consumes whatever resources remain (including \( hx \), which ex post has a “price” of one by assumption, meaning that it converts into consumption one-for-one). Letting \( a \) denote the non-housing (risk-free) asset

\(^9\) Similar results would obtain if the cost were incurred by the borrower.
\(^{10}\) More precisely, we will see that, except in extreme cases, \( P \) does not exceed \( \bar{x} \), and we impose \( b \leq hP \) on incentive-compatibility grounds—otherwise the borrower could just walk away at date 0.
held by the consumer, the resource constraints are

\[ \begin{align*}
Ph + a & \leq y + P + zPh \\
c & \leq a + h (\max \{ x - zP, 0 \} - \rho (\cdot) zP) \\
& = y + P (1 - h) + zPh + h (\max \{ x - zP, 0 \} - \rho (\cdot) zP)
\end{align*} \]

Absent any government intervention or renegotiation, default occurs if \( x < zP \). The first constraint is on portfolio allocation ex ante (before the realization of \( x \)); the second is ex post, where we know \( P = 1 \), and the consumer simply liquidates assets for consumption. Combining them, we get

\[ c \leq y + P (1 - h) + zPh + h (\max \{ x - zP, 0 \} - \rho (\cdot) zP), \tag{2} \]

where the choice variables are \( h \) and \( z \).

### 2.2 Financial Intermediaries

We suppose a finite number, say \( N \), of financial intermediaries operating under “Bertrand” assumptions: Any intermediary can capture the entire market by underpricing its competitors by an arbitrarily small amount. As a consequence, in equilibrium intermediaries participating in the market make zero profits. The actual size and number of intermediaries is not generally pinned down by the model in the absence of frictions or other technological factors. Moreover, these intermediaries are essentially just Walrasian auctioneers: They use up virtually no resources except in the case of foreclosure on mortgage contracts, and they have zero market value. Nonetheless, we need to give a name to these entities and describe their market behavior, for reasons that will become clear. Though it is not essential, we suppose that \( N \geq 2 \), and that participating firms divide the market equally, i.e. each gets a market share of 1/\( N \).

Below we examine the consequences of various policies involving bailouts and limits on loan-to-value ratios. In the absence of such interventions, the zero profit requirement means that \( \rho \) must satisfy (as a function of \( z \))

\[ \rho zP = zP - \mathbb{E} \left\{ \min \{ Pz, x \} + x \gamma 1 \{ zP > x \} \right\} \]

\[ = \int_{zP}^{x} (zP - x (1 - \gamma)) dG (x). \]

In this case the borrower will accurately internalize default risk and the associated costs, and the loan-to-value ratio will be largely irrelevant to the value of the house except for the negative impact of default
2.3 Bailouts and LTV Limits

We now alter the model to capture what we believe are key features of government’s role in the mortgage market, and more broadly, with respect to financial institutions: First, we posit that the government cannot credibly commit to let large financial institutions fail, or in the context of this model, incur huge losses—the "Too Big to Fail" (TBTF) phenomenon. Below we will define “large” in terms of market share $s$. Second, as a consequence, financial regulators take measures to limit large institutions’ risk-taking, which because of TBTF would tend to be excessive.$^{11}$ Indeed the key risk-reduction measures that we focus on, which in the model are distilled down to LTV limits, are those that historically applied particularly to large protected institutions such as Fannie Mae. The GSEs were long restricted to purchasing only “conforming” mortgages that were limited in size and LTV ratio.$^{12}$ Other institutions had more flexibility, though for the most part they historically could not sell non-conforming mortgages to the GSEs. Under our baseline assumptions, the equilibrium involves low default risk and house prices very close to what their values would be in the absence of TBTF. When the regulations are relaxed or circumvented, the equilibrium changes to one in which TBTF institutions take over the market, default risk increases, of course, but the main contribution of this paper is to show a substantial impact on asset prices.

In our simple setting, LTV regulation is sufficient to control risk, as we do not have ex ante heterogeneity in the borrower population.$^{13}$ One should bear in mind, however, that ultimately what matters is default risk, whether via higher LTVs or relaxed lending standards. We assume that “large” financial institutions anticipate that in the event that aggregate losses exceed some threshold, the government will bail them out by replenishing a share $\eta^* \in [0,1]$ of their losses. Since losses are a function of the default rate, we will simply specify the policy as thresholds $(f^*, s^*)$ such that any financial firm that has mortgage market share $s \geq s^*$ and that experiences a default rate $f \geq f^*$ will have $\eta^*$ of its losses covered by taxpayers. To combat the obvious moral hazard, for our baseline case we make the following assumption:

**Assumption 1:** Large institutions are required to restrict their loan-to-value ratios to be less than $\zeta^*$, where $\zeta^*$ is chosen so that the resulting default risk is less than $f^*$.

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$^{11}$This is not to suggest that small banks are not also regulated in their risk-taking, but that is largely because of deposit insurance, which is primarily in place to mitigate the problem of panics or “runs.” This aspect of regulation is largely tangential to the focus of our paper.

$^{12}$There are other requirements as well: Debt to income ratio, credit score, income documentation, etc.

$^{13}$Even in the dynamic heterogeneous agent model that we examine later, LTV limits will suffice for risk management, because agents differ only in their asset levels.
Consequently, even though the policy in place for firms with $s \geq s^*$ is

$$\eta = \begin{cases} 
0 & G(zP) < f^* \\
\eta^* & G(zP) \geq f^*
\end{cases},$$  

(3)

intermediaries cannot exploit this by adding risk, since $zP \leq \zeta^*$, and $\zeta^*$ is chosen so that $G(\zeta^*P) \leq f^*$. Consequently, there is no advantage to size, and possibly a disadvantage if optimal $z$ exceeds $\zeta^*$. In other words, if borrowers prefer $z > \zeta^*$, they will borrow only from small firms. Hence there are two possible scenarios: One in which the $Pz \leq \zeta^*$ constraint is binding, and one in which it is not. In the former, mortgage lending will be the exclusive province of small firms, as large firms would not be able to compete. In the latter, firm size is irrelevant. In either case the market outcome is the same: The house price and default rate will be the same as if the TBTF problem did not exist, as $\zeta^*$ is set appropriately. Essentially, the LTV limit suffices to prevent excessive risk-taking by TBTF intermediaries, and consequently has an important social benefit in a setting where the government cannot precommit to let large financial institutions fail or incur large losses.

2.4 Reduced Oversight

We next model the impact on equilibrium of reduced oversight, which takes the form of a relaxation of the LTV limit, of TBTF institutions. As we have seen, in reality financial institutions were allowed or even encouraged to make riskier loans in a variety of ways, but in our simple setting default risk is entirely related to the LTV ratio. In thinking about the actual events leading up to the 2008 crisis, however, the relaxed LTV limit in the model should be seen as a proxy for a wide variety of loopholes or leniencies that allowed large institutions to increase risk, perhaps even while officially adhering to the original LTV limit.

Clearly a modest increase in the LTV limit will have little impact other than potentially to allow TBTF institutions to gain a foothold in the market, which will be the case if the LTV limit rises to the point that it no longer binds. But once the limit is increased sufficiently, or even eliminated, any institution can, by the Bertrand assumption, gain an arbitrarily large market share by disregarding default risk and undercutting its competitors’ mortgage interest rate, and making high LTV loans. The resulting equilibrium is one in which effectively all mortgages are held by one or more TBTF institutions that underprice risk, taking advantage of the implicit subsidy from the government’s implicit guarantee. By becoming large relative to the market, these lenders can control the amount of aggregate risk and in effect bring about the circumstances that result in a bailout.

In this 2-period setting, we do not have explicit aggregate uncertainty, so this “bad” equilibrium occurs
with probability one if the LTV limits are relaxed. But it would be straightforward to tie the bad outcome to an aggregate shock, such that default rates are generally low, but with some probability the adverse shock hits and results in high default rates occur and trigger the bailout.

We need to demonstrate that with a sufficiently relaxed LTV limit, at least one financial institution will deviate from the low default, fair risk pricing equilibrium to one in which risk is underpriced and LTVs are as high as possible. We have the following intuitively obvious result, proved in the Appendix:

**Proposition 1** For $\zeta^*$ sufficiently close to one, a single lender can profit and capture the entire market by offering mortgages with LTV equal to $\zeta^*$ at an interest rate below the rate that fully prices in default risk.

It then follows any lender can capture the entire market by offering a mortgage with $z = \zeta^*$ and $\rho$ sufficiently small but positive. The only equilibrium with $\zeta$ sufficiently close to one will involve a small number of large firms making high LTV loans at the risk-free rate. Each firm will have market share $s \geq s^*$, the threshold for TBTF.

The above results are not terribly surprising. And while we are not aware of similar results in the literature, the idea that lax oversight coupled with TBTF resulted in excessively risky mortgages and ultimately high default rates is not a novel idea. The real innovation in this paper is to show quantitatively how these factors could have directly (and rationally) increased house prices. This is less obvious and requires some additional work.

In the event that the bailouts are triggered, their cost, assumed to be financed by lump-sum taxes $T$, will be (letting $1 \{ \cdot \}$ denote an indicator function equaling one if the condition in brackets is true, zero otherwise):

\[
T = \eta^* (zP - E \{ \min \{ Pz, x \} + x\gamma 1 \{ zP > x \} \}) \\
= \eta^* \int_{zP}^{\infty} (zP - x (1 - \gamma)) dG(x)
\]

Competitive financial intermediaries must break even on their transactions, so $\rho(\cdot)$ satisfies

\[
\rho zP = (1 - \eta^*) (zP - E \{ \min \{ Pz, x \} + x\gamma 1 \{ zP > x \} \}) \\
= (1 - \eta^*) \int_{zP}^{\infty} (zP - x (1 - \gamma)) dG(x)
\]

This shows that for given $\gamma$ and $G$, $\rho$ is a function of $zP$ and the policy rule for $\eta$. For a given $\eta^* < 1$, $\rho$ is increasing in $zP$, but clearly for the policy rule (3) $\rho$ will drop discretely if a common $zP$ reaches the point that $G(zP) = f^*$. 

10
The consumer solves the following problem: Again denoting the distribution function for \( x \) by \( G(\cdot) \),

\[
V(y, P; f^*, \gamma, \zeta^*) = \max_{h, z} \int_\mathbb{X} u(c) \, dG(x)
\]

where \( \mathbb{X} \) is the support of \( x \), subject to (2) and the credit limit \( z \leq \zeta^* \). We need \( P \) to be such that in equilibrium, \( h = 1 \), so that \( a = y + zP \). We solve this by first verifying that the higher LTV limit actually makes that limit binding (because of the change in the equilibrium to one in which bailouts of TBTF firms become operative). \(^{14}\) We can then solve for the equilibrium price by imposing \( z = \zeta^* \). It is straightforward (see the Appendix) to show that the \( P \) can be expressed as

\[
P = \frac{\mathbb{E}\{u'(c) \, dc\}}{\mathbb{E}\{u'(c)\}} \tag{4}
\]

where

\[
\frac{dc}{dh} = \max\{x, zP\} - \rho(zP) zP
\]

With this formulation we get the following result:

**Proposition 2** If \( \eta^* = \zeta^* = 1 \), then \( P \geq \bar{x} \).

**Proof.** From (4) we have

\[
G(zP) u'(y + zPh - h \rho(zP) zP - T) P z (1 - \rho(zP)) + \int_{\bar{x}}^{\mathbb{X}} u'(y + h(x - \rho(zP) zP) - T) (x - \rho(zP) zP) \, dG(x)
\]

\[
= P \left( G(zP) u'(y + zPh - h \rho(zP) zP - T) + \int_{\bar{x}}^{\mathbb{X}} u'(y + h(x - \rho(zP) zP) - T) \, dG(x) \right).
\]

Also, \( \eta^* = 1 \) implies \( \rho(zP) = 0 \), and \( z = 1 \). The terms involving \( G(zP) \) cancel out, so we are left with

\[
\int_{\bar{x}}^{\mathbb{X}} u'(y + hx - T) \, dG(x) = 0
\]

which rules out any \( P < \bar{x} \). \( \blacksquare \)

Thus when \( \eta^* = \zeta^* = 1 \), house prices can become arbitrarily high—in fact they are essentially indeterminate, since they no longer perform any allocative function. While this is an extreme case, it illustrates what

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\(^{14}\)The problem of finding optimal \( z \) is generally not a matter of applying a first-order condition, as utility need not be concave in \( z \), particularly when \( \gamma > 0 \). Optimal \( z \) is in fact frequently an extreme point, either zero or one (or \( \zeta^* \) if \( \zeta^* < 1 \)). In particular, as \( \eta^*/\gamma \) increases, optimal \( z \) (if unconstrained by \( \zeta^* \)) often jumps from zero to one.
happens as $\eta^*$ and $\zeta^*$ approach one, namely that $P$ approaches $\bar{x}$.

Example: Let $x$ have the Kumaraswamy distribution. This distribution has 4 parameters (lower bound $\underline{x}$, upper bound $\bar{x}$, and two shape parameters $a, b > 0$), making it effectively as flexible as Beta distribution, but with the advantage of having a closed-form density and c.d.f. In its standard form the c.d.f. $\hat{G}$ and density $\hat{g}$ are

$$
\hat{G}(x) = 1 - (1 - x^a)^b \\
\hat{g}(x) = abx^{a-1}(1-x^a)^{b-1}
$$

for $x \in [0,1]$. For our purposes we will consider the generalized distribution with a change of variables so that the support of the distribution is $[\underline{x}, \bar{x}]$, where $0 \leq \underline{x} < \bar{x} < \infty$.\footnote{This makes the distribution and density functions}

$$
G(x) = 1 - \left(1 - \left(\frac{x - \underline{x}}{\Delta}\right)^a\right)^b \\
g(x) = \frac{ab}{\Delta} \left(\frac{x - \underline{x}}{\Delta}\right)^{a-1} \left(1 - \left(\frac{x - \underline{x}}{\Delta}\right)^a\right)^{b-1}
$$

where $\Delta \equiv \bar{x} - \underline{x}$.

We let utility be

$$
u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}
$$

where $\sigma \geq 0$ is the coefficient of relative risk-aversion.

We choose parameters such that absent any government intervention, the optimal $z$ would be well below one because of foreclosure costs. In other words, the efficient (second best) equilibrium involves low default rates similar to what had occurred prior to 2006.\footnote{We cannot a priori rule out the possibility that because of improved risk-sharing, for some parameters the social optimum could involve LTVs near one and high default rates. Even then the implicit bailout promise would similarly distort house prices, but the social cost would likely be minimal, at least with a fixed housing stock.} We consider two sets of parameters. In one ($I$), the default risk is positive but low with no government intervention (that is, there is an interior optimum for $z$, in this case at 0.81). In the second case ($II$), the optimal $z$ is generally either zero (or any value up to $\underline{x}$ such that there is zero default risk), or one, depending on $\eta^*$. The parameters are shown in Table 1, and the distributions depicted in Figure 3. In both cases, $\sigma = 2$ and $y = 1.15$.

Given $\underline{x}$, $\bar{x}$, and $\gamma$, the $a$ and $b$ parameters are determined by the requirement that $E(x) = 1$, the default risk for 80 percent LTV loans be approximately 0.5 percent, and a “default discount” (average value of homes in default relative to overall average) of 22 percent. We then consider two scenarios regarding policy: The government either strictly enforces an LTV limit on large firms of $\zeta^* = 0.8$, in which case lenders remain small and default rates are low; or it fails to enforce the limit, resulting in an equilibrium with large lenders
and high default rates.

The results are displayed in Table 2. Here $\Delta P$ refers to the distortion of $P$ relative to the equilibrium with a low LTV limit. If the government can enforce a sufficiently low LTV limit, the equilibrium is “good”: House prices are slightly below the ex post price of one, reflecting a risk premium in a world with incomplete risk-sharing; risky borrowing is self-limiting (at an interior optimum in Case I and non-existent in Case II); and the default rate is low.

Without enforcement of the LTV limit, however, the equilibrium exhibits inflated house prices at date 0. At date 1 house prices drop and there is a high default rate. In this “bad” equilibrium, lenders compete away profits and underprice risk in the expectation that with high default rates they will be bailed out. With underpriced risk, consumers take out the largest loans they can, and bid up the price of housing to the point that their gains are dissipated as well. In the aftermath, house prices fall back (on average), the high default rates trigger bailouts and inflict real costs, and taxpayers collectively foot the bill.

The social cost of all of this is the foreclosure costs needlessly incurred, though these are partly offset by the greater degree of risk-sharing. As the effective LTV limit approaches one, the impact on price can be large—nearly 30 percent in the case of the more realistic distribution of $x$ (Case II). Of course the foreclosure rate is implausibly high, as one might expect in a representative agent model with LTVs of 100 percent and large ex post price declines. This high default rate is an artifact, however, of not just the representative agent setting, but also the unrealistic assumption that all negative equity loans result in foreclosure. In reality, a majority of such loans are resolved in less drastic (and less costly) fashion—“short sales” (where the property is sold and the lender accepts a loss) or voluntary transfer (“deed in lieu of foreclosure”). In addition, the government’s intervention may involve relief either to borrowers or lenders (or to both) that allows them to avoid foreclosure. If we were to assume that only some fraction $\phi < 1$ of negative equity loans actually end up in foreclosure, the quantitative results would look very similar but with a lower actual foreclosure rate. We use this assumption extensively in the dynamic version of this model, therefore we defer a detailed discussion until the next section.

The two-period model effectively and transparently describes the mechanism that results in house price inflation. On the one hand, the government’s inability to pre-commit to allow financial institutions to incur large losses need not have adverse consequences if adequate LTV limits are imposed. But if those limits are relaxed, financial institutions and their customers attempt to exploit the government’s inability to tolerate large losses and high foreclosure rates. In so doing, they dissipate any gains by bidding up the price of houses, but incur the (largely socialized) cost of a high default rate.

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17See, e.g., Pennington-Cross (2010).
18The calibration would require a correspondingly higher value of $\gamma$, so that expected default costs are invariant to $\phi$. 

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It is also worth noting that as an alternative to LTV limits, restricting the size of financial institutions would also be an effective antidote to the pre-commitment problem. So long as any one institution is unable to grow large enough to push aggregate default rates over the threshold for government intervention, and institutions are prevented from colluding, then no single lender will have an incentive to deviate from the low-default equilibrium in which risk is fully priced.

One additional limitation of the two-period model is that there is no obvious analog to “rent,” so we cannot say what happens to price-rent ratios. In the main section of the paper we will be able to show that the house price boom and bust also applies to price-rent ratios.

3 A Dynamic Model with Heterogeneous Agents

The dynamic model shares many of the features of the static model. Time $t \in \{0, 1, \ldots \}$ is discrete. There is a continuum of infinitely-lived households and a continuum of financial intermediaries/banks, each with measure one. A competitive representative firm produces consumption and capital goods. The housing stock of the economy is in fixed supply, equal to 1, and there is no explicit rental market for housing. There is a government that taxes household labor income and uses the proceeds to make transfers to the banks. In what follows, we suppress individual household subscripts, but in general all quantities vary across households.

3.1 Households

Households derive utility from consumption $c_t$ and housing services $h_{t+1}$, discounting future at rate $\beta \in (0, 1)$. The preferences are represented by a CRRA utility function with the Cobb-Douglas aggregator $\tilde{c}_t = c_t^{1-\theta} h_t^\theta$, where $\theta \in (0, 1)$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \mathbb{E}_t \prod_{t=1}^{1-\sigma} \frac{1}{1-\sigma}. $$

Housing services at time $t$ are produced by a linear technology that uses the stock of housing the household owns. With some abuse of notation, we use $h_{t+1}$ to denote both. The price of consumption is normalized to 1, and the price of housing at date $t$ is $P_t$. It will be clear that income will not affect default decisions in our framework, therefore we choose to simplify the analysis by assuming identical labor income across households. In particular, households supply labor inelastically at a common post-tax wage rate $\bar{w}_t = (1-\tau_t)w_t$.

The households are subject to i.i.d. idiosyncratic “quality” shocks $x_t \geq 0$ to housing. We assume that there is no aggregate risk, therefore, this is the only source of uncertainty in the economy. These quality shocks have a cumulative distribution function $G(x)$ and density $g(x)$ with support $[\underline{x}, \bar{x}]$, and $\mathbb{E}(x) = 1$. 

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Unsecured borrowing is assumed to be unenforceable and households only borrow using housing as collateral. We use $b_{t+1}$ to denote mortgage debt and $a_{t+1}$ to denote the holdings of risk-free assets acquired at time $t$. Given the “no unsecured borrowing” assumption, we have $b_{t+1} \geq 0$ and $a_{t+1} \geq 0$ for all time periods. Specifically, households borrow through financial intermediaries, and this is modeled as a sequence of one-period mortgage contracts, similar to the treatment in Jeske, Krueger, and Mitman (2014). Asset markets are incomplete, since there are two assets and a continuum of states. We will use $r_t$ to denote the risk-free rate on $a_{t+1}$ and $\rho_t$ to denote the mortgage interest rate that depends on the characteristics of the loan. Just as we did for the static model, we assume that interest payments on the mortgage contracts are enforceable.$^{19}$

As in the standard costly state verification framework, households choose to “default” on their mortgages only when they have negative equity, i.e. in equilibrium, they cede the house to the bank in lieu of repayment of the principal when the value of the house is worth less than what they owe on the debt. This is built into the mortgage contract and not a matter of moral hazard on the borrower’s part.

Here is the timing within a period $t$:

1. Households make interest payments on their existing mortgage.
2. Households observe $x$ and make default decisions.
3. Given prices, households choose $c_t$, $a_{t+1}$, $b_{t+1}$, $h_{t+1}$. Housing can be used immediately for housing services at time $t$.

These assumptions imply that the household will choose to default at time $t+1$ if and only if

$$x_{t+1}P_{t+1}h_{t+1} - b_{t+1} < 0$$

This defines a threshold value of shock, $z_{t+1} = \frac{b_{t+1}}{P_{t+1}h_{t+1}}$. If the household draws a value $x_{t+1} < z_{t+1}$ next period, which happens with probability $G(z_{t+1})$, default occurs. The dynamic nature of this model does not alter the way mortgages are priced. Just like the static model, in equilibrium, mortgages are priced exclusively based on $z_{t+1}$ along with the characteristics of the economy at the macro level.$^{20}$

Assuming that the economy is at a steady state, we drop the time subscripts from all prices, assuming $P_t = P$, $r_t = r$, $\bar{w}_t = w$ for all $t$. In addition, the discussion above suggests that the interest rates on mortgage contracts take the form $\rho(z_{t+1})$, where $z_{t+1}$ represents the loan-to-value (LTV) ratio.

$^{19}$One way to rationalize this assumption is to think that interest payments are made continuously but the shocks are realized in discrete intervals. The household would never choose to renege on interest payments, since that would trigger a foreclosure on a house worth more than the value of the debt.

$^{20}$In particular, household income is not factored into the interest rate since it does not affect the default decision.
Due to the structure of mortgage loans, we can formulate the budget constraint of a household using the LTV ratio $z_{t+1}$ rather than $b_{t+1}$ by substituting $b_{t+1} = P h_{t+1} z_{t+1}$.

\[ c_t + P h_{t+1} (1 - z_{t+1}) + a_{t+1} \leq \bar{w} + a_t (1 + r) + P h_t \left( \max \{ 0, x_t - z_t \} - \rho(z_t) z_t \right) \equiv I_t \]

A household enters period $t$ with post-tax wage $\bar{w}$, assets $a_t$ and housing $h_t$ net of the interest payment $P h_t \rho(z_t) z_t$. Having chosen an LTV ratio $z_t$ in period $t - 1$, upon realization of shock value $x_t$, the household receives a net return (or capital gain) of $P h_t \max \{ 0, x_t - z_t \}$ from housing. This term involves the optimal default decision. Including the wage level and return on assets, the total resources available to the household, after the default decision, is represented above by the term $I_t$. These resources are spent on consumption $c_t$, housing $h_{t+1}$, and assets $a_{t+1}$. Moreover, the household can get a loan of $P h_{t+1} z_{t+1}$ against the housing $h_{t+1}$ by writing a new mortgage contract.

We assume that $z_{t+1} \leq 1$, which is motivated from the observation that, given the opportunity, the household would simply default upon receipt of the loan and pocket the difference between the loan and the value of the house. Later, however, government intervention will motivate the imposition of tighter limits, which we will model as

\[ z_{t+1} \leq \zeta \]

where $\zeta$ may be strictly less than one.

The following Bellman equation represents the recursive formulation of a household's problem:

\[
V(I) = \max_{c, h', a', z'} u(c, h') + \beta \mathbb{E} V(I'(x, a', h', z'))
\]

subject to

\[ c + P h'[1 - z'] + a' \leq I \]

\[ c, z', a', h' \geq 0 \]

\[ z' \leq \zeta \]
where

\[ I'(x, a', h', z') = \bar{w} + a'(1 + r) + Ph'\left(\max\{0, x - z'\} - \rho(z')z'\right) \]

### 3.2 Production

There is a representative firm that uses capital \( K \) and labor \( N \), producing consumption and capital goods using a Cobb-Douglas production function. The output of the representative firm is

\[ Y = K^\alpha N^{1-\alpha} \]

For convenience, we also define \( F(K, N) \equiv K^\alpha N^{1-\alpha} - \delta K \), the output net of depreciation. The firm rents capital at rate \( r \) and labor at rate \( w \).

### 3.3 Financial Markets

Financial markets are perfectly competitive and banks are risk-neutral. These assumptions imply that profits are zero for each bank and for each mortgage contract, assuming a law of large numbers holds. These assumptions are very similar to those in Chatterjee et al. (2007).

Since interest payments are enforcable, a contract between the bank and the household yields interest payments to the bank with certainty. For a contract with LTV ratio \( z \), the household defaults with probability \( G(z) \). We assume that if the property is foreclosed after default, the bank loses a fraction \( \gamma \in [0, 1] \) of the value of the house. We model this cost as dead-weight loss measured in terms of consumption goods. It is clear that due to foreclosure costs, there are some gains from renegotiation ex-post.

The reasons why lenders want to avoid foreclosures are well-documented in the literature, and Ghent and Kudylak (2011) summarize them in detail. First, properties depreciate significantly (formalized by \( \gamma \) in the model) when the borrowers are in default, because the occupants have no incentive to maintain the property.\(^{21} \) Second, lenders may incur non-pecuniary costs, mainly due to negative publicity of forcibly removing a borrower from the property. Lenders can eliminate most, if not all, of these costs by taking alternative actions. For instance, the parties can negotiate on a short sale agreement in which borrower sells the property at a price lower than the purchase price, remitting the proceeds to the lender, and the lender waves the right to a deficiency. Another option is a voluntary conveyance where the borrower hands over the deed to the property to the lender, and the lender forgives the debt owed.

Motivated by the empirical evidence, we assume that there is a constant probability/fraction \( \phi \in [0, 1] \) of defaults ending up in costly foreclosure. The fraction \( \phi \in [0, 1] \) can be thought as the equilibrium outcome.

\(^{21}\)Consistent with this view, Ghent and Kudylak (2011) points out that the common view among foreclosure attorneys is that if the lenders decide to exercise the option of foreclosure, they have a strong interest in foreclosing quickly.
of a bargaining game between the borrower and lender, which is not modeled explicitly. A value of $\phi = 0$ represents the case of no costly foreclosures and $\phi = 1$ represents the case in which all defaults lead to foreclosures.

### 3.4 Government

The government's sole activity is to intervene in the housing market by means of (effectively) subsidizing mortgages. There could be an externality in home ownership or in housing consumption that could justify government intervention. There could be a credit market friction that inefficiently reduces home ownership. We do not model any of these reasons for intervention. The last one in particular would require an own versus rent decision that would make the model much more complex than it already is. We proceed on the basis that omitting these features does not have a substantial impact on our comparative steady state results.

The government bailout is triggered by a threshold rule. More specifically, we assume that when the aggregate foreclosure rate exceeds $f^* \in [0, 1]$, the government intervenes by guaranteeing a fraction $\eta^* \in [0, 1]$ of all outstanding debt. Government taxes labor income linearly at rate $\tau$ to finance these transfers. Since labor is supply is inelastic and the households are identical in terms of their labor endowment, this is effectively a lump-sum tax equal to $\tau w$. We assume that the government runs a balanced budget each period.

A side effect of the guarantees is that borrowers and lenders may have an incentive to agree to mortgages with excessive LTV rates, simply because taxpayers will bear a share of the foreclosure costs. As mentioned above, this motivates coupling the guarantees with LTV limits involving $\zeta^* < 1$. Specifically, we will suppose as our baseline that after a bailout, the government imposes an LTV limit $\zeta^*$ for all periods following an intervention. In our quantitative exercise, we will assume, realistically, that this limit will be tight enough to ensure that aggregate foreclosure rate never exceeds $f^*$. Observe that this implies, if $\zeta^*$ is tight enough, the effective subsidy is zero and the government subsidy has no effect on the economy. Although realistic choices for $\zeta^*$ will lead to no subsidy at a steady state, we will allow our definition of stationary recursive competitive equilibrium (RCE) to potentially involve a steady-state foreclosure rate exceeding $f^*$, which necessarily requires a relatively slack LTV limit. Putting all these pieces together, the government policy can be summarized by the tuple $(f^*, \zeta^*, \eta^*, \tau)$.

Under perfect competition, the interest rates for contract $z$ must satisfy the following zero-profit condition derived from the expected present value of the returns for a financial intermediary, taking the degree of government intervention, $\eta$, into account:
\[
\rho(z; \eta)z = rz + (1 - \eta) \int_{\underline{x}}^{\bar{x}} [z - (1 - \gamma \phi)x]dG(x)
\] (9)

First, observe that, given the distribution \( G(x) \), LTV ratio \( z \) is a sufficient statistic to assess all risks in a contract from the perspective of a bank. Therefore, as conjectured in the previous section, the mortgage interest rate depends only on the LTV ratio \( z \). Second, \( \eta = 1 \), or full-subsidy, implies a risk-free borrowing rate, independent of the default probability, i.e. \( \rho(z; \eta) = r \) for all \( z \in [\underline{x}, \bar{x}] \).

We assume that \( G(.) \) is continuously differentiable everywhere in \((\underline{x}, \bar{x})\), and that \( g(\underline{x}) = G(\underline{x}) = 0 \). Using these assumptions, it is easy to show that

1. Function \( \rho(z; \eta) \) is continuously differentiable in \( z \in (\underline{x}, \bar{x}) \).
2. \( \lim_{z \to \underline{x}^+} \rho(z; \eta) = r \).
3. \( \rho'(z; \eta) > 0 \) and \( \rho'(\bar{z}; \eta) > r \) hold for all \( \eta \in (0, 1) \) and \( z \in (\underline{x}, \bar{x}) \).

For the rest of the exposition, unless the effect of a change in \( \eta \) is analyzed explicitly, dependence of the interest rate on \( \eta \) is suppressed for notational simplicity.

4 Equilibrium

To investigate the long-run effects of policy on the economy, we proceed with defining a stationary recursive competitive equilibrium for this environment.

For what is to follow, \( \mathcal{I} = [0, \infty) \) represents the space for resources \( I \), \( \Sigma \) represents the Borel \( \sigma \)-algebra on \( \mathcal{I} \), and \( \mathcal{P} \) represents all probability measures over the measurable space \( (\mathcal{I}, \Sigma) \).

**Definition 1** A stationary recursive competitive equilibrium with government policy rules \((\eta^*, \zeta^*, f^*)\) is a set of prices \( P, r, w \in \mathbb{R}^+ \), tax rate \( \tau \in \mathbb{R}^+ \), mortgage interest rates \( \rho : [0, 1] \to \mathbb{R}^+ \); policy functions \( c, h', z' : \mathcal{I} \to \mathbb{R}^+ \); steady-state distribution \( \mu \in \mathcal{P} \); foreclosure rate \( f \in [0, 1] \); and subsidy \( \eta \in [0, 1] \), such that

1. Given prices, tax rate and government policy rules, policy functions solve the households’ problem (7).
2. Given factor prices \((r, w)\), firms maximize profits, therefore

\[
F_K(K, N) = r
\]

\[
F_N(K, N) = w
\]

3. Intermediaries maximize profits, mortgage interest rates satisfy equation (9).
4. Equilibrium foreclosure rate and subsidy $\eta$ satisfies

$$f = \phi \int G(z(I))d\mu$$

$$\eta = 1[f \geq f^*] \eta^*$$

where $1[.]$ is an indicator function, taking value 1 if the condition in brackets is true and 0 otherwise.

5. Given policy functions, prices clear all markets:

(a) Labor market

$$N = 1$$

(b) House market

$$\int h'(I)d\mu = 1$$ \hfill (10)

(c) Capital market

$$K' = \int a'(I)d\mu - P \int z'(I)h'(I)d\mu$$ \hfill (11)

(d) Goods market

$$C + K' + DWL = Y + (1 - \delta)K$$

where aggregate dead-weight loss $DWL$ equals

$$DWL = \gamma \phi P \int h'(I) \left( \int_{\Xi}^{z'(I)} x dG \right) d\mu$$

6. Government runs a balanced budget and the tax rate $\tau$ satisfies

$$\tau wN = \eta P \int h'(I) \left[ \int_{\Xi}^{z'(I)} [z'(I) - (1 - \gamma \phi)x]dG(x) \right] d\mu$$

7. The stationary distribution of households $\mu$ is generated by policy functions.
\[
\mu(I_0) = \int \left[ \int 1[I'(x, a'(I), h'(I), z'(I)) \in I_0] dG(x) \right] d\mu \text{ for each } I_0 \in \Sigma
\]

5 Analysis of the Model

We first present the necessary conditions for the solution to the household’s problem, assuming that the value function is differentiable at the optimum. For what is to follow, we let \(\mu_z\) and \(\mu_a\) represent the Lagrange/Kuhn-Tucker multipliers associated with \(z' \leq \zeta\) and \(a' \geq 0\) respectively.

\[
z' : \quad Ph' \left[ u_1(c, h') - \beta \left( \int_{z'}^{x'} V'(I')dG(x) + \frac{d(z'\rho(z'))}{dz'} EV'(I') \right) \right] = \mu_z
\]

\[
h' : \quad u_1(c, h')P(1 - z') - u_2(c, h') - \beta P \left( \int_{z'}^{x'} V'(I')(x - z')dG(x) - EV'(I')\rho(z')z' \right) = 0
\]

\[
a' : \quad u_1(c, h') - \beta (1 + r) EV'(I') = \mu_a
\]

where equation (9) can be differentiated to show that

\[
\frac{d(\rho(z')z')}{dz'} = r + (1 - \eta)(G(z') + \gamma \phi z' g(z'))
\]

Using these conditions, we can derive some results on household decision rules. The first result points out that the solution to the portfolio allocation problem of the household is not unique if \(z' < x\), so that the optimal level of borrowing is done at risk-free rate \(r\).

**Proposition 3** Let \((z^*, a^*, h^*)\) be a solution to household’s problem for some \(I \in \mathcal{I}\) and suppose \(z^* < x\). Take any reallocation of \((a', z')\) with \(z^* \leq z' \leq x\) that satisfies

\[
a' - z' Ph^* = a^* - z^* Ph^*.
\]

Then \((z', a', h^*)\) also solve the household’s problem.

**Proof.** This result follows immediately from the fact that \(\rho(z) = r\) for all \(z \leq x\). Suppose the amount borrowed goes up from \(z^* Ph^*\) to some value \(z' Ph^* \in [z^* Ph^*, x Ph^*]\), and consider increasing \(a'\) by exactly the same amount, i.e. \(a' = a^* + \Delta a\) where \(\Delta a = (z' - z^*) Ph^*\). This change is budget-feasible, and it changes neither the probability of default, nor the resources carried over to the next period, \(I'\).

This indeterminacy has no effect on macroeconomic aggregates, but it is important in identifying the
borrowers in the economy. We call an agent a borrower if this agent optimally borrows at a cost, i.e. \( z^* > \bar{x} \)
so that \( \rho(z^*) > r \), or, if the agent borrows at risk-free rate but the net financial asset position is negative, i.e. \( z^* \leq \bar{x} \) and \( a^* - z^* Ph^* < 0 \).

The next proposition, characterizes the optimal LTV choice under specific cases. The proof is technical and it is included in the appendix section.

**Proposition 4** It is always optimal to choose \( z' = \zeta \) when (i) there is no cost of default, i.e. \( \gamma = 0 \), and when (ii) there is full subsidy, i.e. \( \eta = 1 \).

Zero-default-cost result is a natural implication of efficient risk-sharing between the lender and the borrower. When \( \gamma = 0 \), the risk-sharing comes essentially for free from the perspective of the borrower. The actuarially fair mortgage interest rate only reflects the probability of default, the rate at which a risk-neutral borrower would be indifferent between borrowing an extra dollar or not. Therefore, a risk-averse household uses the borrowing technology to its full extent. This result implies that the only reason households do not borrow up to the LTV limit is the probability of costly default which is reflected on the borrowing rate.

One consequence of this result is that when the LTV limits are eliminated, it is optimal for the households to borrow up to \( z' = \bar{x} \). This arrangement is identical to a rental contract, since every household buys housing and defaults with probability one next period. In the long run, this leads to perfect consumption smoothing for each household since all risk is borne by the lenders. An immediate implication is that this allocation cannot be part of an equilibrium, since there would not be any lenders in this environment.

The second result in proposition 3 should not come as a surprise, given the first result. When there is full subsidy, all default costs are covered by the government and hence households borrow up to the LTV limit. We would like to point out, however, that \( \eta = 1 \) is a sufficient condition for this outcome. In most cases, modest values for \( \eta \) also lead to borrowing up to the LTV limit.

6 Quantitative Analysis

Our ultimate goal is to investigate the dynamic impact of relaxed lending standards in the economy. To this end, we start with a steady state in which the LTV limits are tight and consider an exogenous relaxation of these limits. If these limits are sufficiently relaxed, the model potentially generates dynamics that lead to a bailout at some future date. Under the the assumption that the government can credibly commit to the

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22 This indeterminacy result implies that we should really speak about policy correspondences in the definition of recursive competitive equilibrium, not functions. To tackle this issue, we consider a selection from this correspondence. Without loss of generality, that choice of \( z \) is restricted to lie in the interval \( [\bar{x}, \zeta] \). Then, solution to household's problem is unique. Obviously, in light of the result above, this has no effect on aggregate variables.

23 Lemma 1 in the appendix section can provide us a tight lower bound for the values of \( \eta \) that lead to borrowing up to the LTV limit. However, this bound depends on other parameters in a non-trivial way.
policy of reinstating and keeping tight borrowing limits, following the bailout, the economy converges back to its initial steady state. The transition to the bailout period features a significant increase, followed by a large drop in house price following the bailout. These predictions are qualitatively very similar to the trend in home price index before and after the housing crisis in the U.S.

Next, we discuss the details of the calibration exercise.

6.1 Calibration

Our model period is one year. For the quantitative exercise, we exogenously set the coefficient of risk aversion $\sigma$, capital share in production $\alpha$, depreciation rate $\delta$, the baseline LTV limit $\zeta$, and the share of defaults going into foreclosures, $\phi$, based on previous studies on the U.S. economy. The rest of the parameters, discount rate $\beta$, labor income tax rate $\tau$, share of housing in utility function, $\theta$, the shock distribution parameters $a, b, \xi$, and the cost of default $\gamma$, are calibrated endogenously. Table 3 summarizes the results of the calibration exercise.

We choose a coefficient of risk aversion of $\sigma = 2$ for all quantitative results. Based on the U.S. data, we choose a capital share in production of $\alpha = 0.33$ and an annual capital depreciation rate of $\delta = 0.10$. For the baseline calibration, we choose a loan-to-value limit of $\zeta = 0.80$, reflecting the typical limit in the mortgage market for the “normal” times, before the U.S. market experienced a large increase in house prices.

To calibrate the foreclosure cost parameter of $\gamma = 0.035$, we rely on the estimates in the vast literature that aims to measure the consequences of foreclosures. Pennington-Cross (2004) estimated that at the national level, foreclosed property sells with a 22% discount relative to the market average. This figure is based on the average percent difference in sales revenue from foreclosed houses, to the market prices constructed by the OFHEO repeat sales index. Campbell, Giglio, and Pathak (2011) report a higher foreclosure cost of 27%, based on sales in Massachusetts state. Most of the estimates fall in between these two. (See Shilling, Benjamin, and Sirmans (1990), Forgey, Rutherford, and VanBuskirk (1994), Hardin and Wolverton (1996) among others.) None of these estimates satisfactorily control for selection. In particular, conditional on foreclosure, the low observed sale prices, with model equivalent measure $(1 - \gamma)zP_h$ reflects not only the actual cost to the lender $\gamma z P_h$, but also a low value of $z$ that triggered the default in the first place. The distribution of $z$, conditional on foreclosure, has a lower mean, compared to the unconditional distribution, biasing the estimates in the data upward. To deal with this problem, we employ the same empirical strategy on the model-generated data, omitting the selection bias, essentially taking an indirect inference approach. We choose a value of $\gamma = 0.03$ to match an average discount of 25% on the foreclosed property, not controlling for selection. The very low value of $\gamma$ we use should not come as a surprise: In an efficient real-estate
market, the observed price discount on foreclosed property reflects, to a large extent, selection bias from the fact that properties with diminished value are more likely to be delinquent or in a negative equity position. In this sense, the figures reported in the literature are unrealistically high. For instance, Carroll, Clauritie, and Neill (1997) find that the foreclosure discount is negligible once the condition of the foreclosed houses as well as neighborhood effects are properly accounted for. 24

We choose a discount rate of $\beta = 0.958$ to match an equilibrium risk-free rate of 4%. We use $\theta = 0.16$ for the utility share of housing to target an expenditure share of 14.1% based on the NIPA data. Since there is no rental market, we define an implicit rental rate for each household to measure housing expenditures. Household-specific expenditure shares are then aggregated to get an average expenditure share for housing. Given an LTV of $z$, the implicit rental rate below reflects the cost of buying a unit of housing net of the expected capital gain.

$$\tilde{\rho}(z) = (1 + r)(1 - z) + \rho(z)z - \int_z^\infty (x - z)dG.$$  

In the model, default occurs if and only if a borrower has negative equity, i.e. $x < z$. However, in the data, foreclosure proceedings can be initiated for a variety of other reasons (such as borrower illiquidity or carelessness). Most do not end up in costly foreclosures, and for our purposes we can ignore these cases. So instead of using the annual share of mortgages for which foreclosure proceedings are initiated, we match the share of mortgages that end up with negative equity at the resolution of foreclosure proceedings. Mortgage Banker Association (MBA) reports quarterly FHA foreclosure starts as a percentage of outstanding insured loans. This rate was fairly stable around 2% between 1990 and 2000. 25 Ambrose and Capone (1998) estimate that, conditional on starting foreclosure proceedings, there is a 25% probability that a borrower will have negative equity at resolution. These two findings together indicate that the probability of going into negative equity is around 0.5% annually for the decade immediately preceding the house market boom.

Due to its flexibility, we use a Kumaraswamy distribution for the quality shock $z$. For the baseline calibration, we fix $\bar{x} = 1.4$ and choose the shape parameters $(a, b, x)$ jointly to target $E(x) = 1$, the standard deviation $\sigma_x$ and the equilibrium annual default probability. The literature provides conflicting annual volatility estimates based on different data sets. OFHEO reported annualized volatility estimates quarterly for each state separately between 1996-2000. These estimates ranged from 0.08 to 0.12. We think that these estimates should be taken as a conservative lower bound since aggregate volatility should be higher than regional volatilities. Flavin and Yamashita (2002) estimate an annual volatility of around 0.15 based on data.

24 We neglect another factor that would endogenously push down the value of properties that end up delinquent or in foreclosure, namely the reduced incentive for the owner to maintain the property.

25 For details, see the report by Pinto (2011) who compiled these data from MBA sources.
at the national level, and based on the lack of correlation with returns on T-Bills, Stocks, and Bonds, argue that this volatility is almost entirely associated with idiosyncratic risk. This value is also consistent with the estimates reported by Case and Shiller (1989). Based on this evidence, we target an annual volatility of $\sigma_x = 0.15$. While our calibration cannot of course pin down all of the parameters of the distribution, we choose the parameters values $a = 1.329$, $b = 2.232$, $\bar{x} = 0.743$ jointly to match $E(x) = 1$, $\sigma_x = 0.15$ and an equilibrium default rate of $d = 0.5\%$.

In our model, the decision to default is triggered only by negative equity. To calibrate the share of defaults that end up in costly foreclosures, $\phi$, we use the results in the literature that track mortgages that enter foreclosure proceedings in detail. Foreclosure proceedings are better understood as a two-step procedure. The first step involves the borrower technically choosing to default by missing some of the scheduled payments. Foreclosure is the second step and results when the borrower either cannot take actions to prevent the foreclosure, or intentionally allows them to occur. Ambrose and Capone (1998) studied default resolutions using a large sample of FHA-insured mortgages between 1988 and 1994. They estimated that only 32% of all defaults are resolved with the lender involving in a real-estate-owned (REO) sale. The rest of the defaults are resolved by the borrower either by reinstating their mortgages out of default, or by selling them, most likely at a price lower than the original purchase price, and remitting the proceeds to the lender. In the latter case lenders, in most situations, waive their rights to a deficiency. In the light of this evidence, we use a value of $\phi = 0.30$ as the share of defaults that result in costly foreclosures. In any case, our results are essentially insensitive to the choice of $\phi$, since from the risk-neutral lenders’ point of view, $\phi \gamma$ represents the effective cost of foreclosures and $\gamma$ is used to target the aggregate default rate.

In the next section, we discuss the steady-state predictions of our model.

6.2 The Model Fit and Steady State Results

The first column of Table 4 and Figure 4 provide an overview of the economy at the steady state, which we take as a representation of the economy at “normal times”, before the home price index experienced a dramatic increase.

The steady state features a unit house price of $P = 4.83$ and a risk-free rate of 4%, the latter being a target in our calibration exercise. The average LTV for borrowers at the steady state is about 31%. Our model is too stylized to distinguish between LTV at origination and LTV for an outstanding mortgage. Since all mortgages are effectively re-financed every period, we think that this value should be interpreted as the average LTV for all outstanding mortgages, not LTV at origination. At the steady state, 47% of all home-

Quoting Case and Shiller (1989), "Individual housing prices are like many individual corporate stock prices in the large standard deviation of annual percentage change, close to 15 percent a year for individual housing prices."
owners in the economy have outstanding mortgages. The probability of a mortgage going into an eventual (costly) foreclosure is about 0.15%, about a third of 0.5% rate of switching to the default state. Figure 4 provides a snapshot of the distribution of LTV, housing, consumption and net financial assets (defined as $a_{t+1} - z_{t+1}Ph_{t+1}$).

We compare these results with a counterfactual economy in which LTV limits are exogenously relaxed all the way to 99% ($\zeta = 0.99$), but where there is no government bailout. One way to think about this equilibrium is to assume that the threshold rule for government bailout is either absent or involves a very high default rate. In an economy where financial institutions do not expect to be bailed out, mortgages are offered at rates that accurately reflect default risk and foreclosure costs. Households thus internalize those costs and do not choose high LTV loans even if they are allowed to do so, which in turn keeps the aggregate foreclosure rate low.

The steady-state results with high LTV limits under the assumption of no bailouts are summarized in the second column of Table 4. It is striking that relaxing the borrowing limit $\zeta$ has a negligible effect on prices: The house price does not change and the risk-free interest rate goes up by 3 basis points. In the new steady state, average LTV moves significantly, up to 42%, and percent borrowing increases modestly up to 50%. Due to higher LTV limits and the resulting greater default risk, foreclosure rates go up to about 2%. Although this is effectively a substantial increase, arguably, its level is not nearly so high as bring on government intervention. Measured in terms of lifetime consumption equivalent, steady-state welfare goes down by about 1.1%, compared to the welfare level in the benchmark. It is clear, however, that this is not entirely driven by a reduction in consumption. Although the foreclosure probability, and consequently the deadweight losses go up, per-capita consumption declines by less than 0.15%. On the other hand, consumption gini goes up from 0.132 to 0.147 and housing gini goes up from 0.157 to 0.164. Rising consumption and housing inequality is also evident when we compare Figures 4 and 5. We conclude that the only significant negative effect relaxed borrowing limits have on the economy is an increase in consumption and housing inequality. Interestingly, the impact of the higher incidence of foreclosures is negligible.

6.3 Transitional Dynamics, Bailout, and Crisis

How does an exogenous relaxation of LTV limits lead to an increase in house prices followed by a crash when the government bailout policy is in place? This is a question about how the economy responds to the policy over time, therefore, we need to go beyond a steady-state analysis. It is also implausible to have a permanent bailout policy with relaxed LTV limits. Presumably the large costs will prompt a reversion to strict credit limits. The appendix section contains a re-definition of variables and the notion of recursive
competitive equilibrium for the dynamic version of our model.

Our thought experiment is to start with the steady-state under the benchmark calibration with \( \zeta = 0.80 \) and exogenously increase it to \( \zeta = 0.99 \). With financial institutions serving a positive measure of agents, any single institution would have an incentive to deviate from the low-default accurate mortgage pricing described in the previous section, and capture a large share of the market by undercutting its competitors with mortgages that have lower interest rates and high LTVs. Because of this, equilibrium will be characterized by 99 percent LTV mortgages with interest rates at the risk-free rate.

We should note that despite having a heterogeneous agent model, we have not incorporated realistic frictions or heterogeneity that would result in a slower or less extreme outcome in terms of quantities. As soon as the bailout policy is perceived to be in place, essentially everyone in the economy jumps immediately to a 99 percent LTV mortgage at the risk-free rate, with the result that the aggregate incidence of negative equity ex post is extremely high. Thus we do not get realistic quantities such as average LTVs, or the fraction of loans that end up “under water.” We would argue, however, that even though much smaller fractions of buyers and lenders took advantage of the situation, those who did were the price-setters. Reported house prices are determined by those who choose to transact, and transactions would have been dominated by those who chose to avail themselves of the opportunity.

Under relaxed LTV limits, does there exist a competitive equilibrium in which bailout happens at some future date \( T^* > 0 \)? It turns out, if we abstract from explicitly modeling how expectations in the economy are formed, there are \textit{countably many} such equilibria. Indeed, under the assumption that the LTV limit \( \zeta \) is loose enough and the subsidy \( \eta \) is high enough, we can illustrate quantitatively that there is such an equilibrium for any given \( T > 0 \).

To understand this result, suppose that \( \eta = 1 \), and all agents in the economy expect that the foreclosure rate will exceed the threshold level so that a bailout will occur in some future period \( T^* \). Due to the bailout policy in place and given the expectations, all lenders will offer mortgages at the risk-free rate in period \( T^* - 1 \). Since borrowing rate reflects neither the risk nor the foreclosure costs, regardless of the choice of LTV ratio, the risk-averse borrowers will choose the mortgage contract offered with the highest LTV in period \( T^* - 1 \). Consequently, if \( \zeta \) is high enough, the foreclosure rate in period \( T^* \) will be high enough to trigger a bailout, rationalizing the expectations formed in the first place.

Figures 6, 7 and 8 demonstrate such a transition. For these results, it is assumed that \( T^* = 10 \), taking into account that house prices had been going over the long-term trend for about a decade right before the crisis. The overall trend in house prices exhibit a striking resemblance to the U.S. data, at least qualitatively. Due to favorable mortgage interest rates, households bid up house prices dramatically in period \( t = 9 \), by 18%, relative to the benchmark level. Naturally this spills over to all periods prior to bailout. In anticipation of
a high house price in period $t = 9$, all else equal, expected high capital gains increases demand for housing for all periods $t \in \{0, \ldots, 9\}$. In period $T^* = 10$, there is a huge drop in house price (about 18%) to a level slightly lower than benchmark steady state. It takes another 10 years for the house prices to get back to the benchmark level.

Figure 8 shows that the percent of mortgages that end up in foreclosures starts to climb up gradually in period 0, but remain less than 4% until the bailout period $T^*$. In the bailout period, foreclosure rate makes a big jump, up to 25% followed by a drop down to a level close to the benchmark level. The percent borrowing declines and average LTV (conditional on borrowing) goes up slightly as the borrowing limits are relaxed in period 0. Both of these variables jump up significantly in the period before the bailout. As mentioned previously, the starkness of these results reflect the lack of realistic frictions in the model, but (we conjecture) likely do not affect the conclusions about the price impact, on the assumption that prices are driven by those who transact in the market.

The risk-free rate and per-capita consumption starts declining in period 0 as soon as the LTV limits are relaxed. Due to the anticipated bailout and the large taxes to finance it, households, on average, save for the bad times to smooth consumption. Clearly, lower interest rate is a market response to this change. Right before the crisis, risk-free rate jumps up by 5 basis points due to excessive borrowing, thanks to favorable mortgage terms. Per-capita consumption drops by about 0.5% during the crisis. Both the risk-free rate and consumption revert back to the benchmark levels only after 20 years, as illustrated in figures 6 and 7. From a date-0 perspective, the overall decline in welfare, measured in terms of lifetime consumption units, is about 0.4%.

7 Conclusion

It is widely believed that a relaxation of lending standards, through a rapid expansion of the subprime market and availability of high-LTV loans, was the dominant force that paved the way to the crisis. Many observers also cite “Too Big to Fail” (i.e. the government’s unwillingness to allow large financial institutions to fail or incur enormous losses) as a key factor in those institutions’ increasing leverage—both their own and those of their clients. The main contribution of this paper is to directly link these phenomena both to each

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27Observe that there is a jump in period 0 before the ramp-up of house prices. This is an artifact of a low number of build-up periods. Increasing $T^*$ to 20 or 30 leads to a smooth increase in house prices.

28We have also experimented with an alternative scenario where the all agents anticipate that the LTV limits are relaxed in period $T^* - 1 = 9$ instead of period 0. Coupled with the assumption that large intermediaries engage in Bertrand competition, the equilibrium outcome is an anticipated crisis in period $T^*$. Just like in the static model, an equilibrium with low foreclosure rate and no bailout cannot be supported since in period $T^* - 1$, every intermediary would then have an incentive to capture the entire market by offering mortgages with rates arbitrarily close to the risk-free rate. The quantitative results in this scenario, in particular house price effects, are virtually indistinguishable from the baseline results presented here, except that during the first $T^*$ periods, average LTV rates remain slightly lower in comparison.
other and to a quantitatively large endogenous boom and bust in house prices and price/rent ratios. That is, credit limits in our model are not arbitrary frictions, but a welfare-improving response to the inability of government to allow large financial institutions to fail, and their relaxation has major adverse consequences. This contrasts with many models in the literature in which credit limits are imposed or removed arbitrarily and are actually welfare-reducing when in place.

At a normative level, our counterfactual exercise suggests that if the government could have pre-committed to allow lenders to fail, relaxed lending standards would likely have had little impact. Default risk would have been internalized, so that the higher LTV limits would have been non-binding. That is, the increase in housing demand would not have occurred in a mortgage market where the borrowing rates accurately reflect the default risk and the costs of foreclosure. By the same token, the government’s inability to commit to letting these institutions fail by itself would have been inconsequential had stricter LTV limits (and, more generally, tighter underwriting standards) been adhered to.

Taking the model more literally, an alternative to the direct supervision of risk would be a market share limitation on financial institutions, including GSE’s such Fannie Mae and Freddie Mac. The high-default “bad equilibrium” involves firms becoming large by undermining credit standards, thereby generating a sort of “race to the bottom” in credit quality. Size limitations could lead those institutions to plausibly expect a response to aggregate adverse outcomes more like that seen during the Savings & Loan crisis, when hundreds of small institutions were allowed to fail, thereby reducing the risk-taking that would lead to such an outcome.

In short, if the government takes the view that housing market should be regulated through a policy that protects too-big-to-fail lenders in the event of a crisis, it is essential that this policy be coupled either with strict controls on leverage and other sources of credit risk, or with a mechanism that induces proper pricing of risk, at least for those large firms. With such mechanisms in place, the equilibrium that results in a crisis and subsequent bailout is eliminated. Alternatively, a government that is unable or unwilling to restrict high-risk activities by large institutions, should either find a way to bind itself not to bail out failing institutions, or, alternatively, limit the size of institutions in terms of market share.

Our main technical contribution is to illustrate that a fairly standard model with homogeneous and rational beliefs can generate asset price movements that appear to deviate substantially from fundamentals over a number of periods. These deviations are not “bubbles” in the standard sense of that term. House prices are distorted by an implicit subsidy that we presume to be unsustainable, but is responsible for a boom-bust cycle. These findings provide an alternative (not necessarily mutually exclusive) to the view that beliefs were irrational or otherwise non-standard and heterogeneous.

We also find that a boom in house prices can be detrimental to welfare. Since housing is used as collateral
for borrowing, an increase in its price would effectively act to loosen credit constraints in the economy. However, we show that when these price increases are driven by the market and policy failures depicted in our model, the distortionary effects can be very large quantitatively. This prediction contrasts with some of the literature that suggests, either directly or indirectly, that there could be “welfare-improving booms” in the real-estate market.

While our analysis captures many characteristics of the mortgage and housing markets which we think were relevant for the recent mortgage crisis, due to technical reasons and expositional simplicity, we have abstracted from some potentially important features. First, the relatively short period of large increase in house price index over its long-term trend led us to refrain from modeling the production side of the housing market explicitly. Second, for reasons of tractability we do not model realistic frictions, including costs of housing adjustment or mortgage refinancing, that would result in more realistic dynamics. Extensions along these lines would contribute to our understanding of housing and mortgage markets as well as their interaction with policy.
References


Table 1: Parameter Values for the Two-Period Model

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>( \bar{z} )</th>
<th>( \bar{x} )</th>
<th>( \gamma )</th>
<th>( \sigma_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>7.42</td>
<td>106.07</td>
<td>0.6</td>
<td>1.4</td>
<td>0.00695</td>
<td>0.063</td>
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<tr>
<td>II</td>
<td>1.60</td>
<td>21.43</td>
<td>0.778</td>
<td>2.5</td>
<td>0.09829</td>
<td>0.135</td>
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</table>

Table 2: Equilibrium Outcomes for the Two-Period Model

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>( z )</td>
<td>( \Delta P% )</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>0.3</td>
</tr>
<tr>
<td>0.85</td>
<td>0.81</td>
<td>0.5</td>
</tr>
<tr>
<td>0.9</td>
<td>0.95</td>
<td>1.3</td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
<td>9.1</td>
</tr>
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Table 3: Parameter Values-Benchmark Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Target Moments/Source</th>
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</thead>
<tbody>
<tr>
<td>Exogenously Calibrated Parameters</td>
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<td></td>
</tr>
<tr>
<td>Coefficient of Risk Aversion (( \sigma ))</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>Capital Share in Production (( \alpha ))</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>Depreciation Rate (( \delta ))</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Loan-to-Value Limit (( \zeta ))</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>Foreclosed/Default Ratio (( \phi ))</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Jointly Calibrated Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount Rate (( \beta ))</td>
<td>0.96</td>
<td>( r = 0.04 )</td>
</tr>
<tr>
<td>Default Cost (( \gamma ))</td>
<td>0.04</td>
<td>25% foreclosure discount</td>
</tr>
<tr>
<td>Share of Housing in Utility (( \theta ))</td>
<td>0.16</td>
<td>Expenditure Share=14.10%</td>
</tr>
<tr>
<td>Shock Distribution Parameter (( \omega ))</td>
<td>1.33</td>
<td>( d = 0.5% )</td>
</tr>
<tr>
<td>Shock Distribution Parameter (( \epsilon ))</td>
<td>2.23</td>
<td>( \mathbb{E}(x) = 1 )</td>
</tr>
<tr>
<td>Shock Distribution Parameter (( \xi ))</td>
<td>0.74</td>
<td>( \sigma_x = 0.15 )</td>
</tr>
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Table 4: Steady-state Results under Benchmark Calibration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Benchmark (ζ = 0.80)</th>
<th>High LTV Limit and No Guarantees (ζ = 0.99)</th>
</tr>
</thead>
<tbody>
<tr>
<td>House Price (P)</td>
<td>4.83</td>
<td>4.83</td>
</tr>
<tr>
<td>Risk-free Rate (r)</td>
<td>4.00%</td>
<td>4.03%</td>
</tr>
<tr>
<td>Average LTV (z)</td>
<td>0.31</td>
<td>0.42</td>
</tr>
<tr>
<td>Percent Borrowing</td>
<td>47.21%</td>
<td>50.24%</td>
</tr>
<tr>
<td>Percent Foreclosed</td>
<td>0.15%</td>
<td>2.03%</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.17</td>
<td>1.16</td>
</tr>
<tr>
<td>Consumption Gini</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>House Gini</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Welfare</td>
<td>1.07</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Figure 1: Real Home Price and Building Cost Indices since 1890.

Note.– This data set is compiled by Robert Shiller and is available online at http://www.irrationalexuberance.com/index.htm.
Figure 2: Real Home Price/Rent Index.

Note.– This is the ratio of BLS Owners’ Equivalent Rent of Residences and Case & Shiller Home Price Index.

Figure 3: Example Distributions of x.
Figure 4: Steady-state Distributions under Benchmark Calibration

Figure 5: Steady-state Distributions under High LTV limits and No Bailout
Figure 6: Percent change in house price (relative to benchmark) and risk-free rate over the transition

Figure 7: Percent change in consumption (relative to benchmark) and average LTV ratio over the transition
Figure 8: Percent borrowing and percent foreclosed over the transition
A Proof of Proposition 1

If \( \eta = 1 \), the profit for a firm of size \( \sigma \), as a function of \( \rho \) and \( z \) is simply \( \sigma \rho z P \). In other words, since the mortgages are guaranteed, any \( \rho > 0 \) results in a profit. So we only need to check whether for some \( \rho > 0 \) and \( \zeta^* \) sufficiently close to one, borrowers will prefer this high-risk mortgage to one fairly priced with \( \eta = 0 \).

First, we show that under these circumstances agents' utility is increasing in \( z \). We can eliminate \( P \) from the problem by working with \( v \equiv b/h \) rather than with \( z \). Then the consumer’s expected utility is

\[
E\{u\} = G(z) u(y + h\rho (1 - \rho) - T) + \int_{v}^{\infty} u(y + h (x - v\rho) - T) dG(x).
\]

It is straightforward to see that at \( \rho = 0 \) \( E\{u\} \) is strictly increasing in \( v \). Consequently for some \( \rho > 0 \) it is still increasing in \( v \). And since \( E\{u\} \) is obviously decreasing in \( \rho \), a mortgage with smaller \( v \) and higher \( \rho \) will clearly be rejected in favor of one with higher \( v \) and sufficiently small \( \rho \). Finally, from Proposition 3 we know that if \( \eta = 1 \), as \( \zeta^* \) approaches one, \( P \) approaches \( \bar{x} \) and the default probability \( G(\zeta^* P) \) approaches one, so any threshold \( f^* < 1 \) that triggers bailouts will be exceeded.

B Solving for \( P \) in the 2-Period Model

We have

\[
\int_{X} u(y + P(1 - h) + zPh + h(\max\{x - Pz, 0\} - \rho(zP)zP) - T) dG(x)
\]

\[
= G(zP) u(y + P(1 - h) + zPh - h\rho(zP)zP - T)
\]

\[
+ \int_{zP}^{\infty} u(y + P(1 - h) + hx - h\rho(zP)zP - T) dG(x).
\]

The FOC with respect to \( h \) is (imposing \( h = 1 \))

\[
h : 0 = G(zP) u'(y + zPh - h\rho(zP)zP - T)(-P(1 - z) - \rho(zP)zP)
\]

\[
+ \int_{zP}^{\infty} u'(y + h(x - \rho zP) - T)(-P + x - \rho(zP)zP) dG(x).
\]
The first condition can be written as

\[
G(zP) u'(y + zP h - h \rho(zP) zP - T) P z (1 - \rho(zP)) \\
+ \int_{zP}^{z} u'(y + h (x - \rho(zP) zP) - T) (x - \rho(zP) zP) dG(x) \\
= P \left( G(zP) u'(y + zP h - h \rho(zP) zP - T) + \int_{zP}^{z} u'(y + h (x - \rho(zP) zP) - T) dG(x) \right)
\]

or, more compactly,

\[
P = \frac{E\{u'(c) \frac{dc}{dh}\}}{E\{u'(c)\}}
\]

where

\[
\frac{dc}{dh} = \max\{x, zP\} - \rho(zP) zP
\]

### C Proof of Proposition 4

The proof relies on the following technical result.

**Lemma 1** Suppose that the parameters \((\eta, \gamma, \phi)\) are such that \(\frac{d(\rho(z))}{dz} \leq r + G(z)\) holds for each \(z \in [\bar{x}, \zeta]\).

Then, it is optimal to borrow up to the LTV limit for any \(I\), i.e. \(z'(I) = \zeta\) for each \(I \in \mathcal{I}\).

**Proof.** Let \((z^*, a^*, c^*, h^*)\) denote the optimal solution for some \(I \in \mathcal{I}\). We need to dismiss two possible cases: (i) \(z^* \in (\bar{x}, \zeta)\), and (ii) \(z^* = \bar{x}\) (without loss of generality, due to proposition 2).

Case (i): Suppose that \(z^* \in (\bar{x}, \zeta)\) holds. Given \(h^* > 0\) and that \(z^*\) is an interior solution, the first-order condition (12) can be simplified as

\[
u_1(c^*, h^*) = \beta \left( \int_{z^*}^{z} V'(I') dG(x) + \left. \frac{d(z' \rho(z'))}{dz'} \right|_{z'=z^*} E V'(I') \right)
\]

Imposing \(u_1(c^*, h^*) \geq \beta(1 + r) EV'(I')\) due to (14), and using \(\frac{d(\rho(z))}{dz} \leq r + G(z)\) for each \(z \in [\bar{x}, \zeta]\), we obtain the following inequality

\[
\beta \left( \int_{z^*}^{z} V'(I') dG(x) + (r + G(z^*) EV'(I') \right) \geq \beta(1 + r) EV'(I')
\]

Simplifying this expression further and using \(G(z^*) > 0\), we obtain

\[EV'(I'|x \geq z^*) \geq EV'(I').\]
Since $V(.)$ is strictly concave and $z^* < \zeta < x$, this is a contradiction.

Case (ii): It can be shown that $z^* = x$ solves the first-order conditions and the second-order condition reveals that $z^* = x$ is an inflection point, therefore we follow a different route, exploiting the strict concavity of the value function. More specifically, we show that a portfolio change, moving from $z^* = x$ to some $z \in [x, \zeta]$ (so that $\Delta z \equiv z - x$), and simultaneously increasing risk-free assets $a$ by the implied extra borrowing (i.e. $\Delta a = Ph^* \Delta z$), cannot make a household worse off. Combined with the result for Case (i), we will have shown that $z^* = x$ cannot be optimal.

Next period resources for some realization of $x$ and given LTV ratio $z$ equals

$$I'(x, z) = \bar{w} + a(1 + r) + Ph^* (\max\{0, x - z\} - \rho(z)z)$$

Define the change in next period resources for some realization of $x$, given the proposed portfolio reallocation:

$$\Delta I(x) \equiv I'(x, z) - I'(x, \bar{x}) = Ph^* [\Delta z(1 + r) - (\rho(z)z - r\bar{x}) + \max\{0, x - z\} - \max\{0, x - x\}]$$

Straightforward (but cumbersome) algebra yields

$$E \Delta I = Ph^* \left[ \eta \left( G(z)z - \int_{\bar{x}}^{z} xdG(x) \right) - (1 - \eta)\gamma \phi \int_{\bar{x}}^{z} xdG(x) \right]$$

(18)

Since $\frac{d(\rho(z)z)}{dz} \leq r + G(z)$ holds for all $z \in [x, \zeta]$, equation (15) implies $\eta G(x) \geq (1 - \eta)\gamma \phi x g(x)$ for each $x \in [\bar{x}, z]$. Therefore, by integrating this inequality over $x \in [\bar{x}, z]$, we obtain

$$\eta \left( G(z)z - \int_{\bar{x}}^{z} xdG(x) \right) \geq (1 - \eta)\gamma \phi \int_{\bar{x}}^{z} xdG(x).$$

This inequality implies $E \Delta I \geq 0$ from (18).

Due to concavity of $V(.)$, we have

$$V(I'(x, z)) - V(I'(x, \bar{x})) \geq \Delta I(x)V'(I'(x, z)) \text{ for each } x \in [x, \bar{x}]$$

$$EV(I'(x, z)) - EV(I'(x, \bar{x})) \geq E[\Delta I(x)V'(I'(x, z))]$$.
Observe that
\[
\mathbb{E} [\Delta I(x)V'(I'(x, z))] = \text{cov}(\Delta I, V'(.) + \mathbb{E} \Delta I \mathbb{E} V'(I'(x, z))
\]

Since \( \Delta I(x) \) is weakly decreasing in \( x \), \( I'(x, z) \) is weakly increasing in \( x \), and the value function is concave, the covariance term is non-negative. We have also established that \( \mathbb{E} \Delta I \geq 0 \). Therefore, we have

\[
\mathbb{E} V'(I'(x, z)) - \mathbb{E} V'(I'(x, x)) \geq 0.
\]

The portfolio change considered leaves \( c^* \) and \( h^* \) intact, therefore we conclude that any such change in the portfolio makes the agent at least as well off. In our case of strict concavity of the value function, the agent strictly prefers the proposed change. ■

**Proposition 4** It is always optimal to choose \( z' = \zeta \) when (i) there is no cost of default, i.e. \( \gamma = 0 \), and when (ii) there is full subsidy, i.e. \( \eta = 1 \).

**Proof.** These results follow immediately from Lemma 1, since \( \frac{d(\rho(z)z)}{dz} = r + G(z) \) for each \( z \in [\underline{z}, \zeta] \) in case (i) and \( \frac{d(\rho(z)z)}{dz} = r \) for each \( z \in [\underline{z}, \zeta] \) in case (ii). ■

### D Equations for the Transition Analysis

#### D.1 Household’s Problem

Let \( \pi = \{P, \rho, r, \zeta\} \) summarize all time varying macroeconomic factors that influence the decision of a household: The sequence of house prices \( P = \{P_0, P_1, \ldots\} \), mortgage interest rate functions \( \rho = \{\rho_1, \rho_2, \ldots\} \), risk-free rates \( r = \{r_1, r_2, \ldots\} \), and the LTV limits \( \zeta = \{\zeta_1, \zeta_2, \ldots\} \). The following Bellman equation represents the recursive formulation of a household’s problem:

\[
V_t(I_t; \pi) = \max_{c_t, h_{t+1}, a_{t+1}, z_{t+1}} u(c_t, h_{t+1}) + \beta \mathbb{E} V_{t+1}(I_{t+1}(.), \pi) \tag{19}
\]

subject to

\[
c_t + h_{t+1}[P_t - P_{t+1}z_{t+1}] + a_{t+1} \leq I_t \tag{20}
\]

\[
c_t, z_{t+1}, h_{t+1}, a_{t+1} \geq 0
\]

\[
P_{t+1}z_{t+1} \leq P_t \zeta_{t+1}
\]
where

\[ I_{t+1}(x, a_{t+1}, h_{t+1}, z_{t+1}) = \bar{w}_{t+1} + a_{t+1}(1 + r_{t+1}) + P_{t+1}h_{t+1}\left(\max\{0, x - z_{t+1}\} - \rho_{t+1}(z_{t+1})z_{t+1}\right) \]

D.2 Equilibrium

**Definition 2** A recursive competitive equilibrium with bailout policy \( \eta^*, \zeta^*, f^* \) consists of sequences of house prices \( P_t \), mortgage interest rates \( \rho_t \), risk-free rates \( r_t \), wages \( w_t \), tax rates \( \tau_t \), and LTV limits \( \zeta_t \); policy functions \( c_t, h_{t+1}, z_{t+1} : \mathcal{I} \rightarrow \mathbb{R}^+ \); distributions \( \mu_t \in \mathbb{P} \); and bailout period \( T^* \), such that

1. Given prices and taxes, policy functions solve the households’ problem (19).
2. Given factor prices, firms choose \( K_t \) and \( N_t \) to maximize profits, therefore, for each \( t \),
   \[
   F_K(K_t, N_t) = r_t \\
   F_N(K_t, N_t) = w_t
   \]
3. Intermediaries maximize profits: Given bailout period \( T^* \), mortgage interest rates satisfy \( \rho_t(z) = \rho(z; \eta = 0) \) for each \( t \neq T^* \) and \( \rho_t(z) = \rho(z; \eta = \eta^*) \) for \( t = T^* \).
4. Given bailout period \( T^* \), the LTV limits satisfy \( \zeta_t = \zeta \) for each \( t \leq T^* \) and \( \zeta_t = \zeta^* \) for each \( t > T^* \).
5. Given policy functions, prices clear all markets:
   
   (a) Labor market
   \[
   N_t = 1
   \]
   
   (b) House market
   \[
   \int h_{t+1}(I)d\mu_t = 1 \tag{21}
   \]
   
   (c) Capital market
   \[
   K_{t+1} = \int a_{t+1}(I)d\mu_t - P_{t+1} \int z_{t+1}(I)h_{t+1}(I)d\mu_t
   \]
   
   (d) Goods market
   \[
   C_t + K_{t+1} + DWL_t = Y_t + (1 - \delta)K_t
   \]
where dead-weight loss satisfies

$$DWL_{t+1} = \gamma \phi P_{t+1} \int h_{t+1} \left( \int_{\mathbb{Z}}^{z_{t+1}(I)} x dG \right) d\mu_t$$

6. Bailout period satisfies

$$T^* = \min\{t \geq 0 : \phi \int G(z_{t+1}(I))d\mu_t > f^*\} + 1.$$ 

7. Government runs a balanced budget: Tax rate satisfies $\tau_t = 0$ for each $t \neq T^*$ and

$$\tau_t w_1 N_t = \eta P_t \int h_I(I) \left[ \int_{\mathbb{Z}}^{z_{t}(I)} (z_I(x) - x) dG(x) + \gamma \phi \int_{\mathbb{Z}}^{z_{t}(I)} x dG(x) \right] d\mu_{t-1}$$

for period $t = T^*$.

8. The stationary distribution of households $\mu$ is generated by policy functions.

$$\mu_{t+1}(\mathcal{I}_0) = \int \left[ \int 1[I_{t+1}(x, a_{t+1}(I), h_{t+1}(I), z_{t+1}(I)) \in \mathcal{I}_0] dG(x) \right] d\mu_t \text{ for each } \mathcal{I}_0 \in \Sigma, \text{ each } t$$

where $1[.]$ is an indicator function, taking value 1 if the condition in brackets is true and 0 otherwise.