Contracting on Credit Ratings: Adding Value to Public Information

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Abstract

We provide a rationale for the use for credit ratings even when ratings contain no new information about a risky security (such as in the case of sovereign debt). We develop a contracting model in which a principal contracts with an agent who invests in a risky bond. Some information about the state is known to both principal and agent, but unverifiable to a third party and therefore non-contractible. A credit rating on the bond provides a verifiable signal about state. We show that the principal uses the credit rating to reward the agent for holding a riskless bond in a bad state. A more precise rating improves both the payoff of the principal and total surplus, but the agent loses rents from renegotiation. As a result, the agent’s payoff may decrease. In an economy with a continuum of principal-agent pairs, the use of the rating in a contract in turn determines the equilibrium price of the risky asset. We establish that widespread use of credit ratings may increase asset volatility and discuss the determinants of optimal rating precision.

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1 Introduction

There are many contexts in which market participants use and react to the issuance of reports based on already known, publicly-available information, a noteworthy example are credit ratings on sovereign bonds. While many models of credit ratings on companies assume the rating agency possesses information not already reflected in market prices, such a claim is difficult to make for government debt. A credit rating merely provides a summary of information already available, and yet markets react to it.\footnote{For example, Brooks, et al., find that downgrades of sovereign debt adversely affect both the level of the domestic stock market and the dollar value of the country’s currency.}

In this paper, we pursue a novel explanation for the existence of this (seemingly) redundant information aggregation and reporting: Agents use ratings to improve incomplete contracts. Consider an investor who delegates the management of her portfolio and wants to provide incentives to a manager who is prone to moral hazard. An incentive contract based on portfolio outcomes, may not be precise enough to ensure that the manager always acts in her best interests. However, an additional signal (such as a credit rating) can provide her with a useful tool to improve on the contract.

Our framework is (loosely) based on Aghion and Bolton (1992). Briefly, this is a stylized incomplete contracting model between a principal and an agent in which states are observable, but not verifiable. There are two states, good and bad, and two feasible actions: hold a risky bond or hold a riskless asset instead. The agent’s preferred action depends on the realization of a stochastic private benefit. There is a potential inefficiency in that the principal and agent may end up preferring different state-contingent actions. We interpret a credit rating as a verifiable signal about the state. Contracts may be written on this verifiable signal, potentially improving efficiency in the contracting relationship.

In our model, the principal chooses the wages to offer the agent based on the return delivered by the agent and the credit rating of the risky bond. We begin by assuming that the agent’s action do not affect the return on the bond (i.e., the agent is atomistic in the context of the market for the bond). We show that in general, the principal rewards the agent for holding the riskless bond when the state is bad and the bond has a low rating. If the state is bad and the bond has a high rating, the principal is more tolerant. The key intuition is that the principal must induce the agent to hold the risky bond when the state is good. Boosting the reward for holding the riskless bond in the bad state requires correspondingly increasing the reward for holding the risky bond in the good state, providing a trade-off for the principal. The optimal contract, of course, depends on the precision of the credit rating.
When the rating is sufficiently precise, the reward for holding the riskless bond when the state is bad and the credit rating on the risky bond is low is positive and increasing in signal precision.

The use of the credit rating strictly improves both the principal’s payoff and the total surplus. Both those values also increase in the precision of the credit rating. However, the agent’s payoff may or may not increase. If the contract induces an inefficient action, there are gains to renegotiation. We assume that the agent, the point person making the actual investment decision, has all the bargaining power in any renegotiation. Improving ex ante inefficiency reduces the need for renegotiation. The overall effect on the agent’s payoff may be negative when the possibility of renegotiation is high. Therefore, the credit rating also serves as a device to transfer surplus from the agent to the principal.

We then turn to the market-wide equilibrium implications of credit ratings. Specifically, we consider a continuum of principal-agent pairs in a delegated portfolio management sector. The equilibrium price of the risky bond depends in part on the aggregate mass of agents in this sector. Taking the price of the risky asset as given, each principal benefits from using the credit rating. However, when all principals do so, that affects the price of the bond. In particular, even when fundamentals are fixed, the price of the risky bond now depends on its credit rating. Therefore, asset prices are more volatile than justified by fundamentals when credit ratings are widely used in contracts.

The aggregate effects of contracting on credit ratings on the price of the asset therefore imply that, even absent any direct cost to producing more precise credit ratings, it may be optimal for a rating to have some noise in it. We briefly consider three welfare criteria a planner may have in mind: maximizing total surplus, maximizing investor welfare, or maximizing surplus with a penalty for asset price volatility.

Our focus on the use of non-informative credit ratings to mitigate contracting frictions is novel. Other work on non-informative ratings includes Boot, Milbourn, and Schmeits (2006), who present a framework in which a firm’s funding costs depend on the market’s beliefs about the type of project being chosen. The credit rating agency, by providing a rating, allows infinitesimal investors to coordinate on particular beliefs when multiple equilibria are possible. Further the credit watch procedure provides a mechanism to monitor the firm if it can improve the payoff of its project. Manso (2014), also considers how a credit rating might have real effects, in a model with multiple equilibria self-fulfilling beliefs. In his framework changes in a firm’s credit rating affects its ability to raise capital, which then reinforce the original rating.

Much of the work in the literature considers credit ratings that communicate new infor-
information about the firm to the market. For example, Opp, Opp and Harris (2013) illustrate how the use of ratings by regulators might have pernicious effects, and Fulghieri, Strobl and Xia (2013) consider rating manipulation by the credit rating agency itself. Mathis, McAndrews and Rochet (2009) demonstrate that when the flow income from new transactions is high, a rating agency’s concern for future reputation no longer acts as a disciplining device.

Rating shopping has been examined by Bolton, Freixas and Shapiro (2012) in a world with some boundedly rational consumers who trust the assigned rating. Competition between credit rating firms induces inefficiency, and ratings are more likely to be inflated in booms. Skreta and Veldkamp (2009) examine a similar friction and show that issuers have an incentive to produce complex assets when some consumers are naïve. Subsequent work by Sangiorgi and Spatt (2013) considers rating shopping when all consumers are rational, with the key friction being opacity about how many ratings an issuer has actually obtained. In equilibrium, too many ratings are obtained. While we have a single rating in our model, we consider the effect on different users of the rating, such as firms and portfolio managers.

Donaldson and Piacentio (2013) also consider the effect of credit ratings in contracts, and suggest that investment mandates based on ratings lead to inefficiency. Researchers have also considered the optimal degree of coarseness (see Goel and Thakor (2013) and Kartasheva and Yilmaz (2013)).

We build on the large literature on optimal contracts in a delegated portfolio problem. Bhattacharyya and Pfleiderer (1985) consider such a problem with asymmetric information. In a normal exponential framework, they find that quadratic contracts lead to true information revelation. Dybvig and Spatt (1986) demonstrates that if the manager and the investor have similar preferences then risk-sharing can be efficient. By contrast, we show that risk aversion on the part of the manager can lead to optimal contract to include risk limits. Similar to Grinblatt and Titman (1989), we impose limited liability on the agent. In this case, the agent has an incentive to take on excessive risk as he is not penalized in the bad states. They propose a contract that if a portfolio is constructed so that the loss to agents is larger than the gain, excessive risk taking can be curbed. Stoughton (1993) considers moral hazard in which the manager expends effort to get better information about the risk asset. He characterizes the effect of two types of incentive contracts: linear and quadratic in realized payoffs. He finds that as the principal approaches risk neutrality, the quadratic contract induces the first best.

In other work, Admati and Pfleiderer (1997), Lynch and Musto (1997), and Das and Sundaram (1998) consider the use of benchmark evaluation measures. Innes (1990) provides optimal contracts in a limited liability setting when there is moral hazard on the part of the
principal. Finally, Palomino and Prat (2003) consider a delegated portfolio problem in which the manager can affect the risk of the portfolio. They find that the optimal contract is a fixed fee plus a bonus if the payout is above a certain threshold. Interestingly, the optimal contract may induce either insufficient or excessive risk.

There is little work on the equilibrium effects of delegated portfolio management. A notable exception is a recent paper by Guerrieri and Kondor (2012) who model fund managers with career concerns where some managers have private information on bond risk. Managers get fired based on past performance, which means that when default risk is high, return on bonds is high to compensate managers for the risk of being fired. The reputational premium also amplifies price volatility.

We introduce our model in Section 2. In Section 3, we demonstrate the optimal contract for a single principal-agent pair, holding the price of the risky bond as fixed for each state and credit rating. We then step back to exhibit the equilibrium effects of the contract Section 4. We conclude with a discussion of the implications of our findings in Section 6.

2 Model

There is a continuum of principals and a continuum of agents, each of mass 1. Principals and agents are matched in pairs, so that each principal transacts exclusively with one agent, and vice versa. Each principal is a bond investor and each agent a portfolio manager. The principal-agent pairs constitute the delegated portfolio management sector of the economy.

The basic contracting problem between each principal and agent is as follows. There are two states, \( g \) and \( b \). The probability of state \( g \) is \( \phi \). State \( g \) (the “solvent” state) is a state in which a risky bond is likely to pay off, whereas state \( b \) (the “default” state) is one in which it is less likely to do so. There are two assets available for investment, a risky bond and a riskless one.\(^2\) Each portfolio manager can hold up to one unit of one of the two securities, and has two available actions. Action \( a_1 \) corresponds to holding the risky bond and action \( a_2 \) to holding the riskless one. If the investor could directly control the agent’s action, she would prefer to hold the risky bond in state \( g \) (i.e., action \( a_1 \) in state \( g \)) and the riskless bond in state \( b \) (action \( a_2 \) in state \( b \)).

The source of agency conflict is that the portfolio manager receives a private monetary benefit \( m \) if he chooses the risky bond in state \( b \). The private benefit corresponds to either synergies with his other funds (“soft money”) or side transfers that he obtains from a firm

\(^2\)While nothing in the model depends on it, it is useful to think of the risky bond as being issued by a sovereign government.
if he places its bonds in an investor’s portfolio. The private benefit is realized only after the contract is signed, and at the contracting stage both parties view it as random.

There are four dates in the model, \( t = 1 \) through 4. Figure 1 shows the sequence of events in the model.

\[
\begin{array}{cccc}
  t = 1 & t = 2 & t = 3 & t = 4 \\
  \text{Principal and agent sign contract} & (i) \text{State } g \text{ or } b \text{ revealed} & (i) \text{Possible renegotiation} & \text{Output realized;} \\
  & (ii) \text{Size of private benefit } m \text{ realized} & (ii) \text{Agent takes action } a_1 \text{ or } a_2 & \text{Payoffs earned} \\
  & (iii) \text{Contractible signal } h \text{ or } \ell \text{ obtained} & (iii) \text{Market price of risky asset determined} & \\
\end{array}
\]

**Figure 1: Sequence of Events**

At time 1, the principal offers the agent a contract which offers a wage at time \( t = 4 \). At time 2, the state \( s \) is revealed to both parties. However, the state is not verifiable, so not directly contractible. Instead, a credit rating on the risky bond is available. The credit rating is verifiable, so serves as a contractible signal \( \sigma \). We do not model the source of the credit rating, but allow the rating to be exogenously noisy. Specifically, the signal \( \sigma \) takes on one of two values, \( h \) or \( \ell \). The signal is potentially informative, with \( \text{Prob}(\sigma = h \mid s = g) = \text{Prob}(\sigma = \ell \mid s = b) = \psi \geq \frac{1}{2} \). Two special cases of interest are: (i) \( \psi = \frac{1}{2} \), a completely uninformative signal, which is equivalent to the principal and agent being only able to contract on the final output, and (ii) \( \psi = 1 \), a perfectly informative signal, which is equivalent to principal and agent being able to contract directly on the state.

Also at time 2, the size of the agent’s private benefit from holding the risky bond in state \( b \) is realized. We denote this private benefit as \( m \). At time 1, \( m \) has a distribution \( F(\cdot) \) that is known to both parties. The distribution \( F \) is continuous and atomless, and has a density \( f \) and support \( [0, M] \). The realization of \( m \) is independent of the state or credit rating, and the realizations across different agents are also independent. As is customary, the private benefit \( m \) is not verifiable, so cannot be contracted on.

Importantly, the contract is written before the state and credit rating are realized. We have in mind a situation in which contracts are written on a periodic basis (say once a year), whereas the state (which reflects market and global events) can change frequently in between. The credit rating need not be known as soon as the state is revealed, but it must be known
before the agent takes an action. The private benefit reflects the effect of market events on other assets held by the agent or other payments the agent receives from his relationships, so is known only when the state itself is revealed.

At time 3, the agent must take an action $a_1$ or $a_2$. Sometimes, the contract will induce the incorrect action (holding the risky bond in state $b$), and yet $m$ will be low enough that there are gains to renegotiation. We assume that the opportunity to renegotiate is stochastic, and occurs with probability $\rho$ (so with probability $1 - \rho$, there is no renegotiation). We expect $\rho$ to be high, for example, if the agent is a private wealth manager, and negotiates separate contracts with each of his clients. Conversely, if the agent is a bond fund manager with dispersed investors all signing the same contract, $\rho$ will be low.

When renegotiation is feasible, the agent has all the bargaining power. The agent makes a take-it-or-leave-it offer to the principal that specifies both the action the agent will take and a new wage contract for the agent. If the principal accepts, the old contract is torn up and the new one holds. If the principal rejects, the old contract remains in force. Any gains to trade at the renegotiation stage are therefore captured by the agent.\(^3\)

After any renegotiation, the agent takes an action. The action is not directly observed by the principal, and although the state is known to both principal and agent, it is not verifiable by a third party. Therefore, the contract cannot depend on either state or action, but can depend on the output obtained at time 4 and on the credit rating revealed at time 2.

The last event at time 3 is the determination of the price of the risky bond. We defer a discussion of how the price is determined to Section 4. In the contracting problem, each principal and agent is atomistic and takes the price under different scenarios as given.

The risky bond has a gross payout $R^g$ in state $g$, and $R^b$ in state $b$. For the contracting problem, we focus on the net return, which the investor is more concerned with. We allow the price of the bond to vary based on the credit rating, so that, when state is $s$ and the credit rating is $\sigma$, the net payoff (i.e., the gross payout minus the price) denoted as $r^\sigma_s$. Investing in the risk-free bond generates the same net financial payoff $r^f \in (0, r^g)$ in both states.

Both parties are risk-neutral, and there is no discounting. The payoff to the principal is the output generated by the agent less the total compensation (i.e., the wage and any additional compensation agreed to in the renegotiation) paid to the agent. The payoff to the agent is the sum of the wage, any additional renegotiated compensation at time 3, and any

\(^3\)Suppose, instead, we gave all bargaining power at the renegotiation stage to the principal. This would be equivalent to allowing the principal to write a contract after the state were known, going against the spirit of the idea that contracts are revised only at periodic intervals, whereas the state may change rapidly in between contract revisions.
private benefits he may garner. The agent has limited liability that requires the wage in any state to be non-negative. The agent’s reservation utility is zero, so any contract that satisfies limited liability is also individually rational. We assume that if the principal invested on her own, she does not have access to the risky bond, so that her reservation payoff is \( r^f \) (what she can earn by investing in the riskless bond). In what follows, we assume that \( R^g \) is high enough (given the other parameters) to ensure that the principal’s individual rationality constraint is satisfied. When the outcome is realized at time 4, the agent is paid the wage specified by the contract signed at time 1 plus any extra surplus he is promised in the renegotiation, and the principal keeps the remainder of the output.

Because each principal and agent treat \( r^s \) as given for each \( \sigma \) and \( s \), in this section and the next section, we directly make the following assumptions. In Section 4, where we endogenize the price of the risky asset, we translate these assumptions to assumptions on the primitives of the model.

**Assumption 1**

(i) \( r^g_\sigma > r^f > r^b_\sigma \) for each \( \sigma = h, \ell \).

(ii) \( M \geq r^f - \min\{r^b_h, r^b_\ell\} \).

(iii) \( \frac{f'(x)}{f(x)} < \frac{2}{r^f - \min\{r^b_h, r^b_\ell\}} \) for all \( x \in [0, M] \).

The first part of the assumption is standard, and implies that action \( a_1 \) (buying the risky bond) maximizes output in state \( g \), whereas action \( a_2 \) (buying the riskless bond) maximizes output in state \( b \). Part (ii) ensures that for some realizations of \( m \), the agency conflict between principal and agent can be large. Finally, part (iii) is a technical condition that ensures a unique solution to the contracting problem. The condition implies that the prior distribution over \( m \) does not increase too rapidly at any point.

### 2.1 Optimal Contract Without a Credit Rating

To establish a benchmark for the optimal contract, we first consider an economy without a credit rating; i.e., without a contractible signal. In this setting, a contract is defined by \( W = \{w^g, w^f, w^b\} \); that is, a wage for every net payoff that the agent may obtain.

Absent renegotiation, the optimal behavior of the agent is as follows. In state \( g \), the agent takes action \( a_1 \) if \( w^g \geq w^f \) and action \( a_2 \) otherwise. As is customary, when the agent is indifferent, we break the tie in favor of the action preferred by the principal. In state \( b \), the agent takes action \( a_1 \) if \( w^f < w^b + m \), and action \( a_2 \) if \( w^f \geq w^b + m \). The mass of agents taking action \( a_2 \) in this state is therefore \( F(w^f - w^b) \).
Of course, if \( m \in (w^f - w^b, r^f - r^b) \), there are gains to renegotiation in state \( b \). If renegotiation is feasible (i.e., with probability \( \rho \)), the agent will switch to the efficient action \( a_2 \), rather than persist with action \( a_1 \). However, recall that the agent has all the bargaining power at this stage, and therefore captures any improvement in surplus at this stage. The principal is held down to her pre-renegotiation payoff. Therefore, in computing the principal’s payoff and the optimal contract, we can ignore the possibility of renegotiation. When we determine the agent’s payoff, we will come back to renegotiation.

Ignoring renegotiation, the ex ante payoff of the principal may be written as

\[
\Pi_n = \phi[(r^g - w^g)1_{\{w^g \geq w^f\}} + (r^f - w^f)1_{\{w^g < w^f\}}] + (1 - \phi)[(r^f - w^f)F(w^f - w^b) + (r^b - w^b)(1 - F(w^f - w^b))].
\] (1)

Here, \( 1_x \) is in an indicator variable that takes the value one if the event \( x \) occurs, and zero otherwise. The subscript \( n \) indicates the economy with no credit rating. The principal chooses \( \{w^g, w^f, w^b\} \) to maximize the payoff in equation (1).

We first show that it is always optimal to set \( w^g = w^f \), to induce the agent to purchase the risky bond in state \( g \). Further, an optimal contract sets \( w^b \) to zero.

**Lemma 1** In an optimal contract,

1. \( w^g = w^f \), and the agent takes action \( a_1 \) in state \( g \).
2. \( w^b = 0 \).

Consider state \( g \). It cannot be optimal to pay a wage \( w^g > w^f \), because reducing the wage to \( w^g = w^f \) strictly improves the principal’s payoff without affecting the agent’s action in either state. Similarly, if \( w^g < w^f \), by raising \( w^g \) to have it equal \( w^f \), the principal induces the agent to switch from action \( a_2 \) to action \( a_1 \) in this state. The principal’s payoff in state \( g \) thereby improves by \( r^g - r^f > 0 \).

Next, consider state \( b \). The output is higher when the agent takes action \( a_2 \) in this state, rather than action \( a_1 \). Recall that the output \( r^b \) (and hence the wage \( w^b \)) is obtained only if action \( a_1 \) is taken. Intuitively, it can never be optimal to pay the agent to take an output-reducing action, so it must be that \( w^b = 0 \).

Lemma 1 implies that the principal has a payoff \( r^g - w^f \) in state \( g \). Consider her payoff in state \( b \). With probability \( F(w^f - w^b) \), the agent invests in the riskless asset, and the principal earns \( r^f - w^f \). With probability \( 1 - F(w^f - w^b) \), the agent invests in the risky asset. The contract stipulates that the principal will earn \( r^b - w^b \) in this case. When renegotiation is
feasible, the agent will renegotiate and will switch to the riskless asset. However, as the agent has all the bargaining power at this stage, there is no increase in the principal’s payoff.

Substituting $w^g = w^f$ and $w^b = 0$ into the payoff function in equation (1) and taking the derivative with respect to $w^f$, we obtain the following first-order condition:

$$(r^f - w^f - r^b)f(w^f) - F(w^f) = \frac{\phi}{1 - \phi}. \quad (2)$$

Lemma 1 pins down the values of $w^g$ and $w^b$ in an optimal contract, so all that remains is to determine $w^f$. Define a threshold value $\bar{\phi}$ as follows:

$$\bar{\phi} = \frac{1}{1 + \frac{1}{(r^f - r^b)f(0)}}.$$ 

The threshold $\bar{\phi}$ depends on $r^b$. Since $r^b$ is taken as given by each principal-agent pair, we suppress this dependence in the notation.

We show that the optimal value of $w^f$ depends on $\phi$, the probability of the solvent state.

**Lemma 2** In the absence of a credit rating on the risky bond, in the optimal contract:

1. If $\phi < \bar{\phi}$, then $w^f$ satisfies the first-order condition in equation (2) above. In this case, $w^f < r^f - r^b$. Further, $w^f$ is strictly increasing in $r^f$ and strictly decreasing in $\phi$ and $r^b$.

2. If $\phi \geq \bar{\phi}$, it is optimal to set $w^f = 0$.

The intuition behind this proposition is as follows. Increasing $w^f$ above zero has two effects. On the positive side, it increases the probability the agent will take the “correct” action $a_2$ in state $b$. However, it also increases the agent’s wage in state $g$ — recall that inducing action $a_1$ in state $g$ requires setting $w^g \geq w^f$. Whether this is worthwhile for the principal naturally depends on the probability of state $g$. Suppose, for example, $\phi$ is approximately (but strictly less than) 1. The principal is willing to let all agents take the incorrect action $a_2$ in state $b$, since this state occurs so rarely, rather than pay the agent a wage above zero in state $g$. Conversely, if $\phi$ is low (specifically, below $\bar{\phi}$), it is worthwhile to induce more agents to take the correct action in state $b$. As $\phi$ increases, $w^f$ naturally decreases.
3 Optimal Contract Using Credit Rating

Now, consider the economy when a credit rating (i.e., a contractible signal) on the risky bond is available. The compensation of the agent can now depend both on the realized output and on the credit rating. A contract is therefore characterized by $W = (w^g_h, w^g_\ell, w^f_h, w^f_\ell, w^b_h, w^b_\ell)$, where $w^\sigma_\sigma$ denotes the compensation to the agent when the credit rating is $\sigma$ and the output is $r^\sigma$.

Consider first state $g$. With probability $\psi$ the signal is $h$, and with probability $1 - \psi$ the signal is $\ell$. When the signal is $\sigma$, the agent takes action $a_1$ if $w^g_\sigma \geq w^f_\sigma$, and action $a_2$ otherwise. If the principal induces the action $a_1$, her payoff is $r^g_\sigma - w^g_\sigma$, and if she induces the action $a_2$, her payoff is $r^f_\sigma - w^f_\sigma$. Again ignoring renegotiation, her expected payoff in state $g$ is therefore

$$
\pi^g = \psi((r^g_h - w^g_h)1_{\{w^g_h \geq w^f_h\}} + (r^f - w^f_\sigma)1_{\{w^g_h < w^f_h\}}) + (1 - \psi)((r^g_\ell - w^g_\ell)1_{\{w^g_\ell \geq w^f_\ell\}} + (r^f_\ell - w^f_\ell)1_{\{w^g_\ell < w^f_\ell\}}) \quad (3)
$$

Next, consider state $b$. The signal is $h$ with probability $1 - \psi$ and $\ell$ with probability $\psi$. Given signal $\sigma$, the agent takes the action $a_2$ if $w^f_\sigma \geq w^b_\sigma + m$ and action $a_1$ if $w^f_\sigma < w^b_\sigma + m$. Therefore, the principal’s payoff in state $b$ is

$$
\pi^b = (1 - \psi)((r^f - w^f_\sigma)F(w^f_\sigma - w^b_\sigma) + (r^f_\ell - w^f_\ell)(1 - F(w^f_\ell - w^b_\ell))) + \psi((r^f - w^f_\ell)F(w^f_\ell - w^b_\ell) + (r^f_\ell - w^f_\ell)(1 - F(w^f_\ell - w^b_\ell))). \quad (4)
$$

The principal’s overall payoff is

$$
\Pi_c = \phi \pi^g + (1 - \phi) \pi^b, \quad (5)
$$

where the subscript $c$ indicates the economy with a credit rating. The principal chooses the various wage levels $\{w^g_h, w^g_\ell, w^f_h, w^f_\ell, w^b_h, w^b_\ell\}$ to maximize $\Pi_c$.

As an analog to Lemma 1, we show that it must be that $w^g_\sigma = w^f_\sigma$ and that $w^b_\sigma = 0$ for each credit rating $\sigma$. The intuition is exactly similar as in the case with no credit rating.

**Lemma 3** The optimal contract sets $w^g_\sigma = w^f_\sigma$ and $w^b_\sigma = 0$ for each credit rating $\sigma = h, \ell$. 

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We can therefore write the principal’s payoff as

\[\Pi_c = \phi(r^g - \psi w^f_h - (1 - \psi) w^f_f) + (1 - \phi)(1 - \psi)\left[(r^f - w^f_f)F(w^f_f) + r^b_h(1 - F(w^f_f))\right]
+ (1 - \phi)\psi[(r^b - w^f_f)F(w^f_f) + r^b_\ell(1 - F(w^f_\ell))]\].

That is, Lemma 3 reduces the problem to two choice variables, \(w^f_h\) and \(w^f_\ell\).

The first-order conditions for an interior optimum are \(\frac{\partial \Pi_c}{\partial w^f_\sigma} = 0\) for each \(\sigma = h, \ell\). Simplifying, we obtain

\[(r^f - w^f_h - r^b_h)f(w^f_h) - F(w^f_h) = \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} \quad (7)\]
\[(r^f - w^f_\ell - r^b_\ell)f(w^f_\ell) - F(w^f_\ell) = \frac{\phi}{1 - \phi} \frac{1 - \psi}{\psi} \quad (8)\]

When \(\psi = \frac{1}{2}\) (i.e., the credit rating is completely uninformative about the state), it is immediate to see that the first-order conditions reduce to equation (2), the corresponding first-order condition in the economy without a credit rating.

Define a threshold precision for the credit rating \(\psi\) as follows:

\[\bar{\psi}(r^b) = \frac{1}{1 + \frac{1 - \phi}{\phi} (r^f - r^b)g(0)}, \quad (9)\]

where \(r^b\) is a generic payoff to the risky asset in state \(b\).

Next, for each \(\sigma = h, \ell\), define \(\psi^\sigma = \bar{\psi}(r^b_\sigma)\) to be the value of this threshold given credit rating \(\sigma\). As \(\phi\) (the probability of the solvent state) increases, \(\psi^h\) decreases and \(\psi^\ell\) increases.

In the limit as \(\phi\) approaches 1, \(\psi^h\) approaches zero and \(\psi^\ell\) tends to one.

Broadly, the optimal contract when a credit rating is available rewards the agent for avoiding the risky bond in state \(b\) when its credit rating is low. If the signal embodied in the credit rating is sufficiently informative about the state (i.e., \(\psi\) is sufficiently high), the agent receives a positive wage \(w^f_\ell\) for buying the riskless asset when the risky bond has a low credit rating, and a zero wage \(w^f_h\) for the same action when the risky bond has a high credit rating.

Intuitively, this corresponds to inducing the agent to tilt toward the risky bond when it has a high credit rating and steering clear of the risky bond when it has a low credit rating. Of course, as in the case of the economy with no credit rating, rewarding the agent for holding the riskless bond in turn translates to a higher compensation for the agent when he holds the risky bond in state \(g\) (i.e., an increase in \(w^g_\sigma\)).

The exact ranges of signal precision under which different compensation levels are paid
are outlined in Proposition 1.

**Proposition 1** In the optimal contract in the economy with a credit rating:

(i) If \( \psi < 1 - \psi^h \), then \( w^f_h \) satisfies the first-order condition (7) and decreases in \( \psi \). Conversely, if \( \psi \geq 1 - \psi^h \), then \( w^f_h = 0 \).

(ii) If \( \psi > \psi^\ell \), then \( w^f_\ell \) satisfies the first-order condition (8) and increases in \( \psi \). Conversely, if \( \psi < \psi^\ell \), then \( w^f_\ell = 0 \).

(iii) If \( \psi > \frac{1}{2} \) and \( r^h_h = r^h_\ell \), then either (i) \( w^f_h < w^f_\ell \) or (ii) \( w^f_h = w^f_\ell = 0 \).

The intuition behind the structure of the optimal contract is similar to that in the no-credit-rating case. First, consider the case that the risky bond obtains the low credit rating \( \ell \). Increasing \( w^f_\ell \) has two effects: (i) In state \( b \), it induces the agent to hold the riskless bond more often (i.e., for a larger set of private benefit realizations), which increases the principal’s payoff (ii) In state \( g \), the incentive compatibility constraint to induce the agent to hold the risky bond is \( w^g_\ell \geq w^f_\ell \). Therefore, increasing \( w^f_\ell \) implies that the principal has to pay the agent a higher amount \( w^g_\ell \) to induce him to hold the risky bond in state \( g \), which reduces the principal’s payoff. When \( \psi \) is high (say close to 1), the second effect is unimportant because the risky bond is highly unlikely to obtain a low credit rating in state \( g \). Therefore, \( w^f_\ell \) is reasonably high. Conversely, when \( \psi \) is low, the first effect is less important (the risky bond may get a high credit rating even in state \( b \)) and the second effect more important (the risky bond may get a low credit rating even in state \( g \)), so that \( w^f_\ell \) is set to zero.

The intuition for setting \( w^f_h > 0 \) is similar. On the one hand, in state \( b \) it induces the agent to hold the riskless bond for a higher range of private benefit realizations. On the other, it requires the principal to increase \( w^g_h \) correspondingly, which lowers her payoff in state \( g \). When \( \psi \) is high, the latter effect dominates, because the risky bond is very likely to obtain the high credit rating in state \( g \). Conversely, when \( \psi \) is low, the first effect dominates, so the principal sets \( w^f_h \) to a positive number.

In part (iii) of Proposition 1, to establish a benchmark for the optimal wages \( w^f_\sigma \), we consider the case in which \( r^h_h = r^h_\ell \); that is, the net return on the risky bond in state \( b \) does not depend on its rating.\(^4\) Suppose in addition that \( \psi = \frac{1}{2} \); i.e., the credit rating is uninformative about state. An inspection of the first-order conditions for interior \( w^f_\sigma \) immediately shows

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\(^4\)Here, we assume \( r^h_h = r^h_\ell \) only to build some insight into the contracting problem. As we show in Section 4, we expect that in equilibrium it will be the case that \( r^h_h < r^h_\ell \).
that the optimal wage $w^f_\sigma$ is the same for both credit ratings and equal to the optimal wage $w^f$ in the no-credit-rating case.

When the rating is informative ($\psi > \frac{1}{2}$), the effects are quite different. If $\psi$ is sufficiently high that $w^f_\ell > 0$, then $w^f_h$ is necessarily smaller; i.e., in state $b$, the agent is paid a greater amount to avoid the risky bond when its credit rating is low. Figure 2 illustrates this case. Recall that $w^g_\sigma = w^f_\sigma$, so another implication is that, in state $g$, the agent is paid a greater amount to hold the risky bond when its rating is low.

![Graph showing $w^f_h$ and $w^f_\ell$ vs $\psi$](image)

This figure illustrates one case of Proposition 1 (iii). If $r^h_b = r^\ell_b$ and in addition if $\psi_h > \frac{1}{2}$, both $w^f_h$ and $w^f_\ell$ are positive and equal at $\psi = \frac{1}{2}$. The wage $w^f_\ell$ increases as $\psi$ increases to 1, but $w^f_h$ decreases and hits zero at $\psi_h$.

Figure 2: Illustration of how $w^f_h, w^f_\ell$ react to changes in $\psi$

If, on the other hand, the rating $\psi$ is not precise enough to make it worthwhile to offer a positive level of $w^f_\ell$, it must be that $w^f_h$ also equals zero. When $r^h_b = r^\ell_b$, it follows that $\psi_h + \psi_\ell = 1$, so if $\psi_\ell > \frac{1}{2}$, it must be that $\psi_h < \frac{1}{2}$. In that case, $w^f_h = 0$ for all $\psi$ between $\frac{1}{2}$ and 1.

### 3.1 Comparison to Surplus-Maximizing Outcome

Holding fixed $r^b_\sigma$ for each $\sigma$, consider the surplus-maximizing outcome for a single principal-agent pair. In state $g$, regardless of the credit rating $\sigma$, it is clearly surplus-maximizing for the agent to take action $a_2$ (hold the risky bond). In state $b$, the total surplus is $r^f$ if the riskless bond is held and $r^b_\sigma + m$ if the risky bond is held. Therefore, the surplus-maximizing
outcome is for the agent to take action $a_1$ if $m > r^f - r_b$, and $a_2$ otherwise. That is, if the private benefit of holding the risky bond is sufficiently high, the agent should hold the risky bond; otherwise, the agent should buy the riskless bond.

Consider a limiting case in which $\psi = 1$. That is, the signal fully reveals the state: A high signal implies the state is $g$ and a low one that the state is $b$. Effectively, the principal can now write a state-contingent contract. Even in this limiting case, the presence of a stochastic private benefit leads to the realized surplus remaining below the maximal level. To realize the maximal surplus through actions directly induced by the contract, the principal must set $w_h^f = r^f - r_b^b$. However, this implies that whenever the agent’s private benefit is smaller than $r^f - r_b$, the agent receives some of the surplus. The principal therefore finds it optimal to set $w_h^f < r^f - r_b$, which implies that if the private benefit is in the range $(w_h^f, r^f - r_b)$, the contract fails to induce the surplus-maximizing action.

**Proposition 2** Suppose $\psi = 1$, so that the signal is fully revealing and $\rho < 1$, so that renegotiation sometimes fails. When the credit rating is low (i.e., $\sigma = \ell$), the wage $w_h^f$ remains below $(r^f - r_b^b)$, so the surplus remains below the maximal attainable amount.

It is important in the proposition that renegotiation sometimes fails. If $\rho = 1$, the agent can always renegotiate the contract before taking an action. In this case, as long as there is an improvement in surplus from renegotiation, the agent will take the right action after renegotiation, even though the original contract at time 1 will sometimes induce the inefficient action in state $b$.

### 3.2 Principal and Agent Payoffs

Given Lemma 3, an optimal contract can be characterized just by the wages $(w_h^f, w_h^l)$, which apply when the agent delivers the riskless return. The optimal values of these wages are shown in Proposition 1. Given an optimal contract $(w_h^f, w_h^l)$, the principal’s payoff is

$$\Pi_c = \phi(r^g - \psi w_h^f - (1 - \psi)w_h^l) + (1 - \phi)(1 - \psi)((r^f - w_h^f)F(w_h^f) + r_h^b(1 - F(w_h^f)))$$
$$\hspace{1cm} + (1 - \phi)\psi((r^h - w_h^f)F(w_h^f) + r_h^b(1 - F(w_h^f))).$$

(10)

Consider the agent’s payoff. In state $g$, the agent takes action $a_1$ and obtains $w_h^g$ when the credit rating is $\sigma$. Using Lemma 3, the agent’s expected payoff in state $g$ is $\psi w_h^g + (1 - \psi)w_l^g$.

In state $b$, the time 1 contract induces the agent to take action $a_1$ if $m > w_h^f$ (recall that $w_h^b = 0$) and action $a_2$ if $m < w_h^f$. However, if renegotiation is feasible and $m \in (w_h^f, r^f - r_b)$,
there is are gains to trade at the renegotiation stage. In this circumstance, the agent will make a take-it-or-leave-it offer to the principal specifying the following new contract:

(i) The agent will switch from action $a_1$ to action $a_2$ (i.e., the agent will hold the riskless bond instead of the risky bond)

(ii) The principal will retain a payoff of $r^b_\sigma$, the payoff she would have obtained in the absence of renegotiation.

(iii) The agent will obtain a total payoff $r^f - r^b_\sigma$, capturing all the extra surplus from renegotiation.

Since the principal is indifferent between the old contract and the new contract, she will accept the new contract.

Note that renegotiation is feasible only with probability $\rho$. With probability $1 - \rho$, the old contract signed at time 1 stands, and the agent takes the action induced by that contract. Putting all this together, and recalling that $w^b_\sigma = 0$, the agent’s payoff in state $b$ given signal $\sigma$ and a realized private benefit $m$ may therefore be written as follows:

$$\tilde{u}(b, \sigma, m) = \begin{cases} 
w^f_\sigma & \text{if } m \leq w^f_\sigma \\
\rho(r^f - r^b_\sigma) + (1 - \rho)m & \text{if } m \in (w^f_\sigma, r^f - r^b_\sigma) \\
m & \text{if } m \geq r^f - r^b_\sigma.
\end{cases} \quad (11)$$

Taking an expectation over the private benefit $m$, the expected return in state $b$ when the signal is $\sigma$ is

$$u(b, \sigma) = w^f_\sigma F(w^f_\sigma) + \{\rho(r^f - r^b_\sigma) + (1 - \rho)E(m \mid m \in (w^b_\sigma, r^f - r^b_\sigma))\} \{F(r^f - r^b_\sigma) - F(w^f_\sigma)\} + E(m \mid m \geq r^f - r^b_\sigma) (1 - F(r^f - r^b_\sigma)). \quad (12)$$

The agent’s overall expected payoff is then

$$U = \phi(\psi w^f_h + (1 - \psi)w^f_\ell) + (1 - \phi)[(1 - \psi)u(b, h) + \psi u(b, \ell)]. \quad (13)$$

The total surplus generated in the transaction between the principal and agent is just the sum of the payoffs of the principal and agent. In state $g$, this sum is $r^g$, with the wage $w^g_\sigma$ being a pure transfer. In state $b$, if the signal is $\sigma$, the expected surplus generated is determined as follows. With probability $F(w^f_\sigma) + \rho(F(r^f - r^b_\sigma) - F(w^f_\sigma))$ the agent takes action $a_2$, and in the remaining cases takes action $a_1$. The expected surplus given state $b$ and
signal $\sigma$ is therefore

$$\tilde{T}(b, \sigma) = r^f((1 - \rho)F(w_f^f) + \rho F(r^f - r_b^h)) + r_b^h(1 - (1 - \rho)(1 - \rho)F(w_f^f) + \rho F(r^f - r_b^h)).$$

(14)

Observe that the wage level $w_f^f$ affects the total surplus through its effect on the probability that the agent will take any given action in state $b$. Although the cash compensation itself is indeed a pure transfer between principal and agent, its effect on the agent’s incentives implies that surplus is also affected.

The total surplus in the transaction between each principal-agent pair is now expressed as

$$T = \phi r^g + (1 - \phi)(1 - \psi)\tilde{T}(b, h) + \psi \tilde{T}(b, \ell).$$

(15)

We show that an increase in the precision of the signal $\sigma$ strictly improves the payoff of the principal and the total surplus in the transaction between principal and agent. However, it reduces the agent’s payoff when $\rho$ is high. Increasing the precision of the signal increases the likelihood of taking the correct action in each state. However, it also reduces the need for renegotiation. All else equal, a lower probability that renegotiation is necessary reduces the payoff of the agent. When renegotiation is highly likely to succeed, this effect dominates.

**Proposition 3** Suppose $r_b^h = r_b^\ell$ and $\psi > \psi_\ell$. Consider an increase in $\psi$, the precision of the signal. Then,

(i) The expected payoff of the principal, $\Pi_c$, strictly increases.

(ii) If $\rho < 1$ (i.e., if renegotiation can sometimes fail), the total surplus in each principal-agent transaction, $T$, strictly increases.

(iii) There exists a renegotiation probability $\bar{\rho}$ such that the expected payoff of the agent, $U$, decreases when $\rho < \bar{\rho}$ and increases when $\rho > \bar{\rho}$.

It is important to note that throughout this section, we treat the net return on the risky asset (particularly, $r_b^h$ and $r_b^\ell$) as fixed. As changing the precision of the signal affects the actions of the agent, in the market equilibrium it in turn affects the price of the risky asset, and hence the net return.
4 Market Equilibrium

The total demand for the risky bond from the delegated portfolio management (DPM) sector in any given scenario (i.e., state-signal pair) is given by the mass of agents who purchase the risky asset in that scenario. The market for the risky asset includes investors outside the DPM sector, who provide a net supply curve $S(\cdot)$ for the asset. If $q$ is the quantity demanded by the DPM sector, the price for the asset is determined by market-clearing. That is, the price $p$ satisfies $S(p) = q$, or $p = S^{-1}(q)$. Let $P = S^{-1}$ denote the inverse supply curve, so that we can write the price as $p = P(q)$.

**Assumption 2** (i) The supply curve from the non-DPM sector $S(\cdot)$ is strictly increasing and is invariant across state and credit rating.

(ii) $S^{-1}(0) > 0$.

(iii) $R^g - S^{-1}(1) > r^f$, and $R^b < S^{-1}(0)$.

(iv) $M \geq r^f - R^b + S^{-1}(1)$, and $F'(x) < \frac{2}{r^f - R^b + S^{-1}(1)}$ for all $x$.

The assumption that the supply curve from the non-DPM sector is invariant across state and credit rating allows us to focus on the effect of the contracting problem we consider on the price of the asset. To interpret parts (ii) through (iv), recall that $S^{-1}(q)$ is the market-clearing price of the risky asset when a mass $q$ of agents in the DPM sector hold the risky asset. Therefore, $S^{-1}(0)$ is the price when no agents in the DPM sector hold the asset, and $S^{-1}(1)$ when they all hold the asset. Part (ii) of the assumption ensures that the market-clearing price is positive regardless of the demand from the DPM sector (since the supply curve is upward-sloping, an increase in the mass of DPM agent holding the risky bond will increase the price above $S^{-1}(0)$). Part (iii) ensures that the net return on the risky bond, $r^g$, exceeds $r^f$. Further, $R^b < S^{-1}(0)$ implies that $r^b$ is negative (and so less than $r^f$) regardless of the mass of agents buying the risky bond. Finally, part (iv) ensures that part (iii) of Assumption 1 is satisfied.

The equilibrium price of the risky asset is now determined by the number of agents who buy the risky bond. In state $b$, the action of each agent depends on the realization of his private benefit $m$. As the private benefits are independent and identically distributed across agents, the mass of agents taking a particular action is determined by the distribution $F(\cdot)$ over relevant ranges. As Figure 3 shows, there are three relevant regions to consider.

Suppose the state is $b$ and the signal is $\sigma$. When $m \leq w^f_{\sigma}$, the agent will buy the riskless bond. When $m \in (w^f_{\sigma}, r^f - r^b_{\sigma})$, the time 1 contract induces the agent to buy the risky bond.
This figure shows the agent’s action after renegotiation in state \( b \), when the credit rating on the risky bond is \( \sigma \). The agent’s action depends on the realized level of the private benefit \( m \).

**Figure 3: Agent’s action in state \( b \)**

However, there are gains to trade from renegotiation, so whenever renegotiation is successful, the agent will switch to the riskless bond. Thus, only a proportion \( 1 - \rho \) of agents in this range will buy the risky bond. Finally, when \( m \geq r_f - r_b \sigma \), the agent will buy the risky bond.

Let \( p^s_\sigma \) denote the price of the risky bond in state \( s \) when the credit rating is \( \sigma \). The equilibrium prices for the risky asset under different scenarios are as follows.

**Proposition 4**

(i) In state \( g \), the price of the risky bond is invariant to the credit rating, and is given by \( p^g_h = p^g_\ell = P(1) \).

(ii) In state \( b \), if \( \psi > \psi_\ell \) the price of the risky bond depends on the credit rating, and solves the implicit equation

\[
p^b_\sigma = P(1 - (1 - \rho)F(w^f_\sigma) - \rho(F(r_f - r_b \sigma))). \tag{16}
\]

In state \( b \), we have \( r_b \sigma = R^b - p^b_\sigma \) and \( w^b_\sigma \) is in turn a function of \( r_b \sigma \). Therefore, equation (16) is an implicit equation.

Three observations are worth noting. First, \( p^b_\sigma < P(1) \) for each \( \sigma \), so the price of the risky bond is (as expected) lower in state \( b \) than in state \( g \). This statement is true even in the economy without a credit rating (or equivalently, when the credit rating is uninformative, with \( \psi = \frac{1}{2} \)). Can the contract be improved by allowing the wage to be contingent on the price of the risky bond, which is known before the action is taken? The answer is a clear “no,” because we are already contracting on the net return of the risky bond. Further, if the
agent earns a return other than \( r_f \), it immediately reveals that he invested in the risky bond.

Second, whenever in equilibrium \( w_f^h \neq w_f^l \), it will be the case that the price of the risky bond in state \( b \) will depend on the credit rating. Observe that, fixing a state, the payoff to the risky bond is the same across both credit ratings. Therefore, contracting on the credit rating introduces a volatility into the price of the risky bond even when the fundamentals are fixed. This additional volatility is potentially a social cost to the widespread use of ratings.

Third, in general we expect (and will show in an example in the next subsection) that \( w_f^b < w_f^h \) in equilibrium. This will in turn imply that \( p_h^b > p_f^b \). Therefore, in state \( b \), the risky bond has a higher price when it has a higher credit rating. Further, the credit spread across rating categories (which will be monotone in \( p_h^b - p_f^b \)) is higher in state \( b \) than in state \( g \) (in the model, the spread is zero in state \( g \) as the risky bond has the same price regardless of rating).

### 4.1 Example

We now present an example to illustrate our results. Let \( F \) be the uniform distribution over \([0, M]\). Then, \( F(x) = \frac{x}{M}, f(x) = \frac{1}{M} \), and \( f'(x) = 0 \) for all \( x \in [0, M] \).

Next, consider a linear net supply curve for the risky bond from the non-DPM sector, given by \( S = -a + kp \), where \( a, k > 0 \) and \( p \) is the price of the risky bond. The market price function is then given by \( P(q) = \frac{a+q}{k} \), where \( q \) is the mass of agents in the DPM sector who buy the asset. Consider a value \( M \geq r_f - R^b + \frac{1+a}{k} \). Then, part (iv) of Assumption 2 is satisfied.

Consider the first-order conditions for an interior solution to \( w_f^j \), equations (7) and (8). Substitute in for \( f(\cdot) \) and \( F(\cdot) \), and solve for \( w_f^h \) and \( w_f^l \). Further imposing limited liability to ensure that the wages are non-negative, we have

\[
w_f^h = \max \left\{ \frac{1}{2} \left( r_f - r_h^b - \frac{\phi}{1-\phi} \frac{\psi}{1-\psi} M \right), 0 \right\} \tag{17}
\]

\[
w_f^l = \max \left\{ \frac{1}{2} \left( r_f - r_l^b - \frac{\phi}{1-\phi} \frac{1-\psi}{\psi} M \right), 0 \right\} \tag{18}
\]

Observe that as \( \psi \to 1 \) (i.e., the signal becomes fully revealing), we have \( w_f^h = 0 \) and \( w_f^l \to \frac{1}{2} (r_f - r_l^b) \).

Now, consider the price of the risky asset in state \( b \). The mass of agents in the DPM sector buying the risky asset is \( q_f^b = 1 - (1-\rho)F(w_f^l) - \rho(F(r_f - r_f^b)) \). There are therefore two cases to consider:
1. Suppose $w_f = 0$. Then, $q_b = 1 - \rho \left( \frac{r_f - r_b}{M} \right) = 1 - \frac{\rho}{M} (r_f - R_b + p^b)$. Further, $p^b = q_b^b a + q_b^b$, so that $k p^b = a + q_b^b$. Solving the last equation for $p^b$, we obtain

$$p^b = \frac{a + 1 + \frac{\rho}{M} (R_b - r_f)}{k + \frac{\rho}{M}} \text{ for each } \sigma = h, \ell.$$  \hspace{1cm} (19)

2. Suppose $w_f > 0$. We illustrate the calculations for $w_f = \frac{1}{2} \left( r_f - r_h - \frac{\phi}{1 - \phi} - \frac{\psi}{1 - \psi} M \right)$. We have $q^b_h = 1 - \frac{\rho}{M} (r_f - R^b + p^b_h) - \frac{1 - \rho}{2M} \left( r_f - R^b + p^b_h - \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} M \right)$. Using $k p^b_h = a + q^b_h$ and solving for $p^b_h$, we have

$$p^b_h = \frac{a + 1 + \frac{\rho}{2M} (R^b - r_f) + \frac{1 - \rho}{2M} \frac{\phi}{1 - \phi} \frac{1 - \psi}{1 - \psi}}{k + \frac{1 - \rho}{2M}}.$$  \hspace{1cm} (20)

A similar calculation yields that when $w_f^b > 0$, the price of the risky bond is

$$p^b_\ell = \frac{a + 1 + \frac{\rho}{2M} (R^b - r_f) + \frac{1 - \rho}{2M} \frac{\phi}{1 - \phi} \frac{1 - \psi}{1 - \psi}}{k + \frac{1 - \rho}{2M}}.$$  \hspace{1cm} (21)

It is immediate that when both $w^f_h$ and $w^f_\ell$ are strictly positive, $p^b_\ell < p^b_h$ so that $r^b_\ell > r^b_h$.

We illustrate the effects of a changing precision of the credit rating in the context of a numeric example. We set $\phi = 0.8$, $\rho = 0.4$, $a = 2$, $k = \frac{1}{40}$, and $M = 60$. We vary $\psi$ from $\frac{1}{2}$ to 1 and solve for the equilibrium price for each $\psi$ and the corresponding optimal contract. Figure 4 shows the equilibrium prices and wages as $\psi$ changes.

![Figure 4: Prices and wages in state b as $\psi$ changes](image-url)
As seen in the left panel of the figure, for these parameters changing $\psi$ has no effect on the price in state $b$ when the credit rating is high. The optimal contract consistently sets $w^f_h = 0$ (so $\psi^h$ is less than $\frac{1}{2}$), and changing $\psi$ does not affect the proportion of agents who buy the risky asset. However, when the credit rating is low, the optimal contract sets a positive $w^f_\ell$ for $\psi > 0.84$. Beyond this point, as $\psi$ increases, the proportion of agents buying the risky asset at any given price decreases, so the equilibrium price also decreases.

![Figure 5: Volatility of prices in state b as \( \psi \) changes](image)

Figure 5 shows the resulting effect on the volatility of prices in state $b$. That is, we compute the standard deviation of the distribution that places mass $1 - \psi$ on $p^h_b$ and $\psi$ on $p^\ell_b$. Of course, when $\psi < 0.84$, the volatility is zero, as $p^h_b = p^\ell_b$. Similarly, it is zero at $\psi = 1$, because the distribution collapses to a single point. For $\psi$ between 0.84 and 1, volatility first increases and then decreases. The maximal volatility is a little over 1\% of the price of the risky bond in state $b$.

## 5 Optimal Precision of Credit Rating

We suggest that the benefit of a credit rating is that it improves contracting between an investor and a portfolio manager. In the usual framework in which a credit rating provides information about a risky corporate bond, the natural notion of efficiency relates to changes
in real output. The goal of a planner is to facilitate investment in positive NPV projects. In
our framework, in which the rating provides no new information, and the bond is potentially
a government bond, a standard notion of efficiency is total surplus in the transaction between
the principal and the agent. However, policy-makers have also focused on other measures.
We therefore consider three objectives for a planner: first, the payoff of the investors; second,
total surplus that comprises the aggregate principal and aggregate agent payoffs; and third,
trading off total surplus with changes in the volatility of the price of the risky bond.

Throughout, we take the excess supply curve from the non-DPM sector as given. To the
extent that the supply curve comes from investors in the market more broadly, a holistic
notion of welfare would include the welfare of non-DPM investors. We implicitly assume that
the non-DPM sector realizes zero surplus from trading with the DPM sector (i.e., they are
indifferent between trading and not at any given price), so changes in price do not affect the
welfare of these investors.

Only considering the aggregate welfare of the investors is consistent with the mandate of
the Securities and Exchange Commission, and so we label it the SEC criterion. Historically,
the commission has attempted to promulgate rules that benefit the investing public. Recall
from equation 10 that the payoff to the principal is simply:

$$\Pi_c = \phi (r^g - \psi w_h^f - (1 - \psi) w_f^f) + (1 - \phi)(1 - \psi) ((r_f - w_f^f)F(w_f^f) + r_h^b(1 - F(w_h^f))]$$

$$+ (1 - \phi)\psi [(r_h^b - w_f^f)F(w_f^f) + r_h^b(1 - F(w_h^f))].$$

The SEC, of course, will also take into account the effect of changes in signal precision on
the equilibrium prices.

When \(r_h^b\) and \(r_f^b\) are taken as given, as in the contracting problem for a single principal-
agent pair, the Envelope Theorem implies that \(\frac{d\Pi_c}{d\psi} = \frac{\partial \Pi_c}{\partial \psi}\). However, when a planner considers
aggregate welfare, it must take into account the effects on the overall economy as a result of a
change in the precision of the signal. In particular, changes in \(\psi\) imply changes in the optimal
contract, which in turn affects the price of the risky asset in different scenarios. Therefore,
at the level of the economy,

$$\frac{d\Pi_c}{d\psi} = \frac{\partial \Pi_c}{\partial \psi} + \frac{\partial \Pi_c}{\partial p_h^b} \frac{\partial p_h^b}{\partial \psi} + \frac{\partial \Pi_c}{\partial p_f^b} \frac{\partial p_f^b}{\partial \psi}. \quad (22)$$

A similar argument applies when total surplus is the measure of welfare. As changes in
price affect the contract and the action of the agent, they affect the surplus realized in the
transaction between the principal and agent.
Finally, it is useful to determine how changes in ratings’ precision affects price volatility. All agents in our model are risk-neutral (except perhaps the unmodeled generators of the excess supply curve), so volatility per se should have no effect on them. However, more generally, from an ex ante perspective, volatility unrelated to the fundamentals of an asset can be costly if prices are used to guide investment decisions or if it exacerbates systemic risk. Therefore, minimizing variability that is purely due to noise rather than fundamentals may provide benefits.

6 Discussion: Interpreting the Verifiable Signal as a Credit Rating

We interpret the verifiable signal in our model as a credit rating, and demonstrate its usefulness in contracting. There are many verifiable signals that may potentially be used in contracts. In this section, we briefly discuss why credit ratings may have become a more popular signal to contract on than some of the other possibilities.

If the principal and agent cannot directly contract on a state, then they may wish to use a contractible signal that is correlated with some states or subset of states. To determine the best contractible signal to use, both the principal and agent must weigh different factors. First, it is not often clear what states will be relevant so a signal has to be forward looking: For example, a firm that undertakes a new line of business after a contract has been signed may have payoffs that depend on states that were not obvious when the principal and agent agreed on the contract. Second, the signal cannot be to “volatile.” This is because contracts have to be enforceable, if signals change at too high a frequency relative to the actions of the agent it is difficult to determine if he (or she) behaved appropriately given the contract.

Various public signals might be of use in contracts: first, prices, price differences, or price indices; second, professional reports such as auditor reports or analysts’ assessments; and third, government indices or variables such as non-farm payroll and finally credit ratings. For the purposes of this discussion, we’ll consider a state or a set of states as ones pertaining to the payoff to an investor in a particular firm.

The relationship between price levels and payoff-relevant states is tenuous as the abject failure of asset pricing to establish prices levels attests. Price changes or relative price changes are potentially useful. The primary drawback from both of these is volatility.

Price volatility means that enforcing contracts written on prices is difficult: prices that fall and rise could give different actions to prices that rise and fall. Further, how do you
know what the agent knew when he took the action? One can imagine ways to control for volatility in contracts. In a CAPM world, price changes are driven by market movements in addition to idiosyncratic risk. Unless the principal strips out the systematic volatility (i.e., by also looking at the price change on an index) a contract that only looks at price changes will conflate the two sources of volatility. Relative prices i.e., the (perhaps beta adjusted) difference between an index and a firm’s returns could be used to condition on a firms’ performance. However, such measures will impose a significant computational burden on the contracting parties. It requires an estimate of betas, and potentially a conditional CAPM. Further, the maintained assumption has to be that the underlying firms’ returns are drawn from a stationary distribution. That is, any asset pricing model (which is imperative to distinguish between systematic and idiosyncratic risk) is inherently backward looking.

Another set of potentially contractible signals are generated by professional organizations such as auditors or analysts. Auditors provide information that is inherently backward looking. That is they verify cash flows and expenditures. The benefit of auditors is that the processes are very transparent but they do not provide guidance as to the future prospects of the firm. To some extent, an auditor is useful to determine if the correct action was taken in the past, but not to specify the correct future action. By contrast, analysts do provide an assessment of future prospects and could thus be useful in contracts that govern investments. However, analysts’ coverage tends to focus on equities and of those, larger issues. Thus, the investor who uses analysts’ recommendations has to restrict their investment universe.

Various government agencies provide information that can be used as a signal of the macro-state. They are somewhat less useful for use in investment contracts that concern firms or industries. First, the mapping between the macro-state and performance of any given firm is not clearly defined. Second, the government data aggregates a tremendous amount of information and the figures are randomly and frequently revised.

Credit ratings (at least the US model) have a few characteristics that make them extremely useful in contracts. First, they are stable in that they change relatively infrequently and are cycle neutral. Second, they are also forward looking, that is they are provide a business analysis of all the states that could be relevant to a particular investment: they can anticipate movements into a new industry, or out of an existing one and changes in the competitive environment. Third, most issues are rated by more than one agency. Fourth, the rating agencies do not sell investment services or actively participate in intermediation markets, therefore they do not present an obvious conflict of interest such as might arise with analysts. It is interesting to note that in comparison to firms, ratings on assets owned by SIVs are less useful than (say) auditor reports. This is because the underlying assets do not change.
Our model implies that credit ratings are used only because they improve contracting efficiency between principals and agents. In the absence of credit ratings, agents sometimes take an inefficient action. In addition, the threat to take an inefficient action results in loss of payoff to the principals. In short, if credit ratings did not exist, investors would have to invent them. Therefore, moves such as those in the European Union in 2012 to ban the use of credit ratings are short-sighted at best.

7 Conclusion

When contracts are incomplete, credit ratings have value even when they contain no new information about the issuer or the security being rated. They enable contracts to be written on a noisy signal about known but unverifiable states, improving efficiency when asset prices are given, and also increasing the payoff to the principal in the contract. However, when credit ratings are used in contracts economy-wide, there is a feedback effect leading to increased volatility of prices of risky assets.

The incomplete contracting approach suggests that credit ratings are not necessarily the most appropriate way for investors and managers to use in contracts governing investments in structured finance vehicles. Consider either a company or a state. Using a forward looking business model, a credit rating might provide a useful summary of states in which a government might change tax or monetary policies. Further, they might provide a useful summary of states in which a company obtains refinancing or sells assets to ensure its financial solvency. However, in the case of structured finance, vehicles comprise pools of assets, for which the servicer does not take analogous actions. It therefore suggests that for these types of assets, whose quality is sunk at the time of origination and no action can be taken to improve quality or viability, an auditor or a business entity that specializes in backward-looking analysis is most appropriate.
Appendix: Proofs

Proof of Lemma 1

(i) As observed in the text, in state $g$ the agent takes action $a_1$ if $w^g \geq w^f$ and action $a_2$ otherwise. In state $b$, the agent takes action $a_1$ if $w^f < w^b + m$, and action $a_2$ if $w^f \geq w^b + m$. It is immediate to see that it cannot be optimal to set $w^g > w^f$: Reducing $w^g$ to $w^f$ does not change the action in either state, and strictly reduces the amount paid to the agent in state 1.

Suppose $w^g < w^f$. Then, in state $g$, the agent takes action $a_2$, so that the principal’s payoff in this state is $r^g - w^f$. If the principal increases $w^g$ to set it equal to $w^f$, the agent switches to action $a_1$. The principal’s payoff in this state becomes $r^g - w^f$. Under Assumption 1 part (iii), $r^g > r^f$, so this strictly improves the principal’s payoff. Therefore, it must be that $w^g = w^f$, establishing parts (i) and (ii) of the Lemma.

In state 1, the total demand from the DPM sector for the risky asset is therefore 1 (all agents buy the risky asset). The price is therefore $P(1)$.

(ii) Increasing $w^b$ above zero has two effects: (i) the probability that the agent takes the inefficient action $a_1$ in state $b$ is $G(w^f - w^b)$; this probability increases in $w^b$ (ii) conditional on action $a_1$ being taken in state $b$, the principal’s payoff in that state is $r^f - w^f$, which decreases in $w^b$. Therefore, it must be optimal to set $w^b = 0$ (i.e., for the limited liability constraint to bind when the output is $r^b$).

Proof of Lemma 2

(i) Substituting $w^g = w^f$ and $w^b = 0$ into the principal’s payoff in equation (1), we obtain

$$
\Pi_n = \phi (r^g - w^f) + (1 - \phi)[(r^f - w^f)F(w^f) + r^b(1 - F(w^f))].
$$

The first-order condition is $\frac{\partial \Pi_n}{\partial w^f} = 0$, or

$$
-\phi + (1 - \phi)[-F(w^f) + (r^f - w^f - r^b)f(w^f)] = 0.
$$

Observe that it cannot be optimal to have $w^f > r^f - r^b$, as reducing $w^f$ to $r^f - r^b$ does not affect the agent’s action in any scenario but strictly improves the principal’s payoff. Suppose $w^f = r^f - r^b$. Then,

$$
\frac{\partial \Pi_n}{\partial w^f} = -\phi - (1 - \phi)F(r^f - r^b) < 0,
$$


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so it must be optimal to set \( w^f < r^f + r^b \).

Now, suppose \( \phi < \tilde{\phi} \). Suppose further that \( w^f = 0 \). Then,
\[
\frac{\partial \Pi_n}{\partial w^f} = -\phi + (1 - \phi)(r^f - r^b)f(0).
\]
The right-hand-side of the last equation is greater than zero if (and only if) \( \phi < \tilde{\phi} \), so in this case it is optimal to set \( w^f > 0 \). If \( w^f \in (0, r^f - r^b) \), the first-order condition in equation (2) must be satisfied.

Now, under Assumption 1 part (iii), the second derivative \( \frac{\partial^2 \Pi_n}{\partial w^f^2} \) is strictly negative for all \( w^f \), so the first derivative \( \frac{\partial \Pi_n}{\partial w^f} \) is strictly decreasing in \( w^f \). The comparative statics with respect to \( \phi, r^f, \) and \( r^b \) now follow.

(iii) Suppose \( \phi > \bar{\phi} \). Then, we have \( \frac{\partial \Pi_n}{\partial w^f} < 0 \) when \( w^f = 0 \), so it is optimal to set \( w^f = 0 \). At \( \phi = \bar{\phi} \), the first-order-condition is satisfied at \( w^f = 0 \), so again it is optimal to set \( w^f = 0 \).

**Proof of Lemma 3**

Suppose the credit rating on the risky bond is \( h \). Replacing \( w^g \) by \( w^g_h \), \( w^f \) by \( w^f_h \), and \( w^b \) by \( w^b_h \), the proof of Lemma 1 goes through exactly. A similar argument applies when the rating is \( \ell \).

**Proof of Proposition 1**

Given Lemma 3, the payoff function of the principal is simplified to the expression in equation (6), reproduced here for convenience.
\[
\Pi_c = \phi(r^g - \psi w^f_h - (1 - \psi)w^f_{b}) + (1 - \phi)(1 - \psi)[(r^f - w^f_h)F(w^f_h) + r^b_h(1 - F(w^f_h))] \\
+ (1 - \phi)\psi[(r^b - w^f_{\ell})F(w^f_{\ell}) + r^b_{\ell}(1 - F(w^f_{\ell}))].
\]

The first-order conditions are \( \frac{\partial \Pi_c}{\partial w^f_h} = 0 \) and \( \frac{\partial \Pi_c}{\partial w^f_{\ell}} = 0 \), or
\[
-\phi \psi + (1 - \phi)(1 - \psi)[(r^f - w^f_h - r^b_h)f(w^f_h) - F(w^f_h)] = 0 \quad (25)
\]
\[
-\phi(1 - \psi) + (1 - \phi)\psi[(r^f - w^f_{\ell} - r^b_{\ell})f(w^f_{\ell}) - F(w^f_{\ell})] = 0. \quad (26)
\]

(i) It is immediate that any \( w^f_h > r^f_h - r^b_h \) cannot be optimal; by simply setting \( w^f_h = 0 \), the principal strictly improves payoff in both states \( g \) and \( b \) when signal \( h \) is realized. Consider equation (25). If \( w^f_h = r^f_h - r^b_h \), the LHS is strictly negative, so it is optimal to reduce \( w^f_h \).

That is, it must be that at the optimum \( w^f_h < r^f_h - r^b_h \).
The second derivative is \( \frac{\partial^2 \Pi}{\partial w^2} = (1 - \phi)(1 - \psi)[(r_f - w_f - r_h^b)f'(w_h^f) - 2f(w_h^f)] \), which, given Assumption 1 part (iii), is strictly negative. Therefore, when a solution exists to the first-order condition (25), it provides the optimal level of \( w_h^f \).

Note that the LHS of equation (25) is strictly decreasing in \( w_h^f \). Therefore, for a non-negative solution to exist, it must be that the LHS is weakly positive when evaluated at \( w_h^f = 0 \). Evaluating it at \( w_h^f = 0 \), the LHS reduces to \( -\phi\psi + (1 - \phi)(1 - \psi)(r_f - r_h^b)f(0) \), which is weakly positive if and only if \( \psi \leq \psi_h \). If \( \psi > \psi_h \), the LHS is negative when evaluated at \( w_h^f = 0 \), so it is optimal to set \( w_h^f = 0. \)

To show that \( w_h^f \) is decreasing in \( \psi \) when \( \psi < \psi_h \), rewrite the first-order condition in equation (25) as shown in equation (7) in the text:

\[
(r_f - w_f - r_h^b)f(w_h^f) - F(w_h^f) = \frac{\psi}{1 - \phi} - \frac{\psi}{1 - \psi}.
\]

The LHS is strictly decreasing in \( w_h^f \). Now, observe that the RHS is strictly increasing in \( \psi \). Therefore, when \( \psi \) increases, \( w_h^f \) must fall for the equality to be maintained.

(ii) Follow the same steps as in part (i), considering \( w_f^b \) throughout. Evaluating the LHS of equation (26) at \( w_f^b = 0 \), we have \( -\phi(1 - \psi) + (1 - \phi)\psi(r_f - r_h^b)f(0) \), which is weakly positive if and only if \( \psi \geq \psi_f \). Therefore, if \( \psi \geq \psi_f \), the optimal value of \( w_f^b \) is given by the solution to equation (26), or equivalently to equation (8). Conversely, if \( \psi < \psi_f \), it is optimal to set \( w_f^b = 0. \)

The RHS of equation (8) is strictly decreasing in \( \psi \). A similar argument as at the end of part (i) establishes that \( w_f^b \) is strictly increasing in \( \psi \).

(iii) Suppose \( \psi > \frac{1}{2} \) and \( r_h^b = r_f^b = r_h^b \). Define \( \gamma(w) = (r_h - w - r_h^b)f(w) - F(w) \). Then, as observed in part (i), \( \gamma \) is strictly decreasing in \( w \). There are two cases to consider:

1. \( \gamma(0) \leq \frac{\phi}{1 - \phi} \frac{1 - \psi}{1 - \psi} \). Then, it follows that \( w_f^b = 0 \). However, observe that since \( \psi > \frac{1}{2} \), we also have \( \gamma(0) < \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} \), so that \( w_h^f = 0. \)

2. \( \gamma(0) > \frac{\phi}{1 - \phi} \frac{1 - \psi}{1 - \psi} \). Then, \( w_f^b > 0 \). There are two subcases to consider:

   (a) \( \gamma(0) \leq \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} \), in which case \( w_f^b = 0 \) so that \( w_f^b > w_h^f \).

   (b) \( \gamma(0) > \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} \). Then, \( w_h^f \) satisfies \( \gamma(w) = \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} \) and \( w_f^b \) satisfies \( \gamma(w) = \frac{\phi}{1 - \phi} \frac{1 - \psi}{1 - \psi} \).

As \( \gamma \) is strictly decreasing in \( w \), it follows that \( w_h^f < w_f^b \).

Proof of Proposition 2
Suppose $\psi = 1$, so that the signal fully reveals the state. Consider state $b$. The signal is then $\ell$ with probability 1 (so we can ignore the possibility of signal $h$). The first-order condition for an interior optimum for $w^f_\ell$, equation (8), reduces to

$$(r^f - w^f_\ell - r^b_\ell)f(w^f_\ell) = F(w^f_\ell).$$

(27)

It is immediate to see that if $w^f_\ell = r^f - r^b_\ell$, the LHS is zero whereas the RHS is positive, so that satisfying the first-order condition requires $w^f_\ell$ to be reduced below $r^f - r^b_\ell$.

If $m \in (w^f_\ell, r^f - r^b_\ell)$, in state $b$ the contract induces the agent to take action $a_1$ instead of the surplus-maximizing action $a_2$. With probability $\rho$, the agent will successfully renegotiate the contract and switch to the surplus-maximizing action $a_2$. However, with probability $1 - \rho$, the agent continues to take the inefficient action $a_1$, so that the realized surplus falls below the maximal level.

\textbf{Proof of Proposition 3}

(i) Consider the payoff of the principal, shown in equation (10). Let $\frac{d\Pi}{d\psi}$ denote the derivative with respect to $\psi$ when $w^f_h$ and $w^f_\ell$ are acknowledged as functions of $\psi$, and let $\frac{\partial \Pi}{\partial w^f_\ell}$ denote the same derivative holding $w^f_h$ and $w^f_\ell$ fixed. Then,

$$\frac{d\Pi}{d\psi} = \frac{\partial \Pi}{\partial \psi} + \frac{\partial \Pi}{\partial w^f_\ell} \frac{\partial w^f_\ell}{\partial \psi} + \frac{\partial \Pi}{\partial w^f_h} \frac{\partial w^f_h}{\partial \psi},$$

and

$$\frac{\partial \Pi}{\partial \psi} = \phi(w^f_\ell - w^f_h) + (1 - \phi)[(r^f - w^f_\ell)F(w^f_\ell) + r^b_\ell(1 - F(w^f_\ell))] - (r^f - w^f_h)F(w^f_h) - r^b_\ell(1 - F(w^f_h))].$$

(29)

Consider the derivative $\frac{\partial \Pi}{\partial w^f_\ell}$. There are two cases: (i) the first-order condition for an interior optimum, equation (7), holds with equality, in which case $\frac{\partial \Pi}{\partial w^f_\ell} = 0$, or (ii) $w^f_h = 0$, and further a small increase in $\psi$ has no effect on $w^f_\ell$, so again $\frac{\partial \Pi}{\partial w^f_\ell} = 0$. A similar argument holds for the derivative $\frac{\partial \Pi}{\partial w^f_h}$. Therefore, in argument similar to what is used to prove the Envelope Theorem, we have $\frac{d\Pi}{d\psi} = \frac{\partial \Pi}{\partial \psi}$.

Now, suppose $r^b_h = r^b_\ell = r^b$. We can write

$$\frac{\partial \Pi}{\partial \psi} = \phi(w^f_\ell - w^f_h) + (1 - \phi)[(r^f - r^b)(F(w^f_\ell) - F(w^f_h)) + w^f_h F(w^f_h) - w^f_\ell F(w^f_\ell)].$$

(30)

Define $\delta(w) = (r^f - r^b - w)F(w)$. Then, $\delta'(w) = (r^f - r^b - w)f(w) - F(w) > 0$ at $w = w^f_\ell$, whenever $w^f_\ell$ satisfies the first-order condition (8). Further, from Proposition 1, when $\psi > \psi_\ell$,
we have \( w^f_\ell > w^f_h \). Putting all this together, it follows that \( \frac{\partial \Pi_c}{\partial \psi} > 0 \).

(ii) Consider the effect of increasing \( \psi \) on the total surplus. In state \( g \), given the optimal contract, the agent takes the efficient action \( a_1 \), so in this state maximal surplus is realized regardless of the value of \( \psi \). In state \( b \), the time 1 contract induces an agent with a private benefit \( m \in (w^f_\ell, r^f - r^b_\ell) \) to take the inefficient action \( a_1 \). With probability \( \rho \), the agent renegotiates and switches to the efficient action \( a_2 \). Therefore, the probability that after renegotiation the agent takes the inefficient action in state \( b \) is \( (1 - \rho)F(r^f - r^b_\ell - w^b_\ell) \), which is strictly decreasing in \( w^b_\ell \). Now, when \( \psi \geq \psi_\ell \), an increase in \( \psi \) leads to an increase in \( w^b_\ell \) (see Proposition 1 part (i)), and therefore a reduction in the probability the agent takes an inefficient action in state \( b \). Therefore, it strictly increases the total surplus.

(iii) Consider two extreme scenarios. First, suppose \( \rho = 1 \); i.e., renegotiation is always feasible. Then, after renegotiation, the action is always first-best, so the agent’s payoff is maximized when the contract most often induces the incorrect action; i.e., when \( \psi = \frac{1}{2} \) and the signal is completely uninformative. Conversely, suppose \( \rho = 0 \), so that renegotiation is never feasible. Then, the agent’s payoff strictly increases in \( \psi \). By continuity, there exists a threshold \( \bar{\rho} \) such that the agent’s payoff increases in \( \psi \) if \( \rho < \bar{\rho} \) and decreases in \( \psi \) if \( \rho > \bar{\rho} \). ■

**Proof of Proposition 4**

(i) In state \( g \), all agents in the DPM sector buy the risky bond, regardless of rating. The total demand from the DPM sector is therefore 1, and the price of the bond is \( P(1) \).

(ii) Suppose the state is \( b \) and the signal is \( \sigma \). Agents who buy the risky bond consist of (a) agents with \( m \geq r^f - r^b_\sigma \) (b) agents with \( m \in (w^b_\sigma, r^f - r^b_\sigma) \) who fail to renegotiate to the efficient action \( a_2 \). The total mass of these agents is

\[
1 - F(r^f - r^b_\sigma) + (1 - \rho)(F(r^f - r^b_\sigma) - F(w^b_\sigma)) = 1 - (1 - \rho)F(w^b_\sigma) - \rho F(r^f - r^b_\sigma).
\]

The market-clearing price therefore satisfies \( p^b_\sigma = P(1 - (1 - \rho)F(w^b_\sigma) - \rho F(r^f - r^b_\sigma)) \). As \( r^b_\sigma \) is a function of \( p^b_\sigma \) and \( w^b_\ell \) in turn depends on \( r^b_\sigma \), the equation is an implicit equation; the solution yields the market-clearing price. ■
References


