Macroprudential Bank Capital Regulation in a Competitive Financial System*

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Abstract

We propose a tractable general equilibrium framework to analyze the effectiveness of bank capital regulations when banks face competition from outside investors. Our analysis shows that increased competition can not only render previously optimal bank capital regulations ineffective but also imply that, over some ranges, increases in capital requirements cause more banks in the economy to engage in value-destroying risk-shifting. To avoid this perverse outcome, the regulator has to set capital requirements high enough, so that risk-shifting activities become less profitable from a banker’s private perspective than socially valuable banking activities. Our model generates a set of novel implications that highlight the dependencies between optimal bank capital regulation and the comparative advantages of various institutions in the financial system.

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1 Introduction

The recent financial crisis has put bank capital regulation at the forefront of political and academic debates. One of the main concerns motivating bank regulation is the notion that banks may have incentives to take excessive risks when the government implicitly guarantees banks’ survival in bad states of the world. While bailing out banks might be welfare enhancing ex post, it naturally distorts the ex ante risk-reward trade-off and can cause excessive risk taking. Given these concerns many academics and politicians call for substantial increases in equity capital requirements (see, e.g., Admati, DeMarzo, Hellwig, and Pfleiderer, 2011). However, opponents of such changes highlight the potential negative ramifications for credit extension and growth, arguing that restrictive bank regulation might shift more activities outside of the regulated banking system (see, e.g., Adrian and Ashcraft, 2012).

As regulatory changes fundamentally alter the environment in which financial institutions operate, it may be dangerous to evaluate new capital regulations based entirely on partial equilibrium models that abstract from competition between banks and other financial institutions and the endogenous nature of prices and quantities. Relatedly, the recent push toward so-called macroprudential approaches to financial regulation reflects the view that regulation should not merely evaluate risks at the individual bank-level but rather address system-wide phenomena (see, e.g., Hanson, Kashyap, and Stein, 2011). In this paper we aim to take a step in this direction and develop a model of bank capital regulation that highlights the general equilibrium effects of regulatory changes when banks and other market participants are heterogeneous in both their capabilities and in the way they are regulated. We show that such heterogeneity in combination with competition can play a critical role in shaping the system-wide effects of changes in bank capital requirements and can induce non-monotonic relationships between regulatory capital requirements and banks’ risk taking.

The central building block for our analysis is a parsimonious general equilibrium model where capital requirements endogenously affect both the banking sector’s total funding capacity and banks’ return distributions from various investment strategies. The economy features heterogeneous investors, banks and outside investors, and a large number of firms in fixed supply that seek to obtain debt financing for investment projects of varying risk and quality: good projects that are safe and bad projects that can facilitate risk-shifting. Two assumptions are crucial. First, we assume that banks have an advantage over other market participants in collecting debt payments from firms. Such an assumption is common in
a variety of papers, see e.g., Diamond and Rajan (2001, 2005) and Bernardo and Welch (2013), and may result from relationship-specific bank-firm ties. Second, in our framework the banks’ debt holders are subject to a bailout guarantee, which allows bankers to raise debt financing at rates that reflect government guarantees rather than banks’ underlying asset risk. The insurance provision in turn destroys the role of debt as a disciplining device ex ante and gives banks an incentive to take on excessive risks.

In this setting, if the government does not impose regulation, the banking sector funds all projects in economy, both good and bad. Safe banks that invest in good assets coexist with highly levered, risky banks that invest in bad assets. This segmentation of the banking system into safe and risky institutions arises endogenously, as it maximizes the private sector’s use of implicit government guarantees. The bonds of bad issuers exhibit rational overpricing as implicitly insured banks become the marginal investors of these assets and bid prices up to the point where the cross-section of equilibrium yields is completely uninformative about underlying default risk.

To improve upon these outcomes, a regulator designs capital regulation that aims to prevent the funding of bad projects by banks while maintaining investment in socially valuable good projects. We make the natural assumption that the regulator cannot directly observe the quality of banks’ assets. Setting capital requirements has two important effects that the regulator must trade off. First, for a given level of bank equity, higher capital requirements imply that fewer firm projects can be funded. We refer to this as the funding capacity effect. Reducing the banking sector’s overall funding capacity generally affects the quality of the marginal project that banks fund, and, by generating funding scarcity, it creates a general equilibrium effect on the profits that banks can make given their investment opportunities. The welfare-implications of a decrease in funding capacity of the banking sector (credit supply) are naturally ambiguous. If decreased funding capacity reduces the financing of negative NPV projects then the corresponding reduction in credit supply is welfare-enhancing. A reduction of funding capacity is harmful, however, when banks, the most efficient financiers of projects, are constrained in their ability to fund all good projects. In that case, these projects can, at best, be funded by other, less efficient

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1. In Allen, Carletti, and Marquez (2011) market discipline comes from the asset side, not the liability side, inducing banks to hold positive levels of capital to be able to commit to monitor borrowers.
2. See e.g., Kareken and Wallace (1978) for the risk-taking incentives introduced by deposit insurance. In contrast, the seminal paper by Diamond and Dybvig (1983) highlights the key benefit of deposit insurance, namely preventing runs.
3. This result is consistent with observed heterogeneity in capital structure choices across banks (see, e.g., Gropp and Heider 2010). In Farhi and Tirole (2012) strategic complementarities can lead banks to correlate their risk exposures which highlights a different channel than the one that arises in our setting.
investors.

The second effect of higher capital requirements is a standard partial-equilibrium effect to which many proponents of stricter regulation refer: greater *skin in the game* reduces, ceteris paribus, the private incentives for risk-shifting. In general equilibrium, however, we show that, over a wide range, increases in capital requirements may induce some banks to switch from socially beneficial activities to risk-shifting behavior. Since we do not exogenously specify the return distributions of banks’ investment opportunities, we can show that competition from outside investors has an *asymmetric effect* on the returns banks generate from lending to good and bad borrowers. As outside investors, in contrast to banks, cannot rely on a government bailout, they are not willing to offer funding to bad borrowers that have risky, negative NPV projects. However, outside investors do compete with banks for good borrowers and thus depress the yields banks can charge these firms. Competition from non-banks thus affects the banks’ ranking of safe and risk-shifting investment strategies. This equilibrium effect is reminiscent of a popular argument that competition from the shadow banking system causes banks to “reach for yield” and for riskier investments in order to stay profitable (see, e.g., Becker and Ivashina [2013]).

The main result of the paper is driven by the interplay of the skin-in-the-game effect and the funding capacity effect. When increases in capital requirements constrain the banking sectors’ total funding capacity to the point where banks cannot fund all assets in the economy, banks may find it optimal to reduce investments only in good projects (in particular when competition from outside investors is fierce). In fact, after an increase in bank capital requirements a substantial fraction of banks may endogenously switch from safe investments to risky ones. As a result, small capital requirements may cause welfare losses even compared to an economy without any capital requirements. To prevent this adverse effect, capital requirements have to be high enough to ensure that risk-shifting strategies are less profitable for banks than socially valuable intermediation, the profitability of which is constrained by competition from non-bank investors. However, at this high level of capital requirements, banks may not have sufficient funding capacity to fund all good assets in the economy, so that second-best outside investors pick up the remaining funding of good projects, causing an inefficiency that cannot be removed by adjusting capital requirements. Interestingly, the funding terms for good issuers may also be non-monotonic, that is, enhanced capital requirements may *decrease* interest rates on good loans as good issuers face less competition from bad issuers. This result goes against the conventional wisdom that borrowers are hurt if capital regulation is more stringent. In our model, this is only strictly true for bad borrowers.
From a policy perspective, our results imply that substantial capital requirements may be warranted to be effective in achieving their task of a reduction in risk-shifting, in particular when fierce competition depresses banks profits from socially valuable intermediation. Surprisingly, our paper finds that small increases might not just be ineffective, but may actually lower welfare. Given that competition in standard banking activities has gone up over last few decades, the model predicts that non-monotonic relationships between capital requirements and risk taking are more likely to arise nowadays than in the past when banks were able to generate large profits from standard banking activities and thus had lower incentives to engage in risk-shifting. Further, this may mean that relative to the 1960s and 1970s significantly larger equity ratio requirements are needed to ensure that banks private ranking of investment opportunities is aligned with the social ranking and risk-shifting is prevented. More broadly, our results imply that capital requirements should not just be based on risk, but also on measures of competition, as both dimensions are relevant for the banks’ private ranking of investment opportunities.

For ease of exposition, we initially treat the amount of equity in the banking sector as fixed, which corresponds to an economy where banks face high costs of raising outside equity. However, we also show that, more generally, for any non-zero issuance cost, the endogenous adjustment of bank equity has ambiguous welfare consequences. Intuitively, banks have an incentive to raise equity only if the prevailing return on equity at the initial capital level outweighs the cost of raising additional equity. As long as banks’ average returns to risk-shifting are positive, an increase in bank equity can even be harmful, as it can allow for an expansion of banks’ risk-shifting activities. In this case, even more substantial capital requirements are required to curtail banks’ risk-taking.

The model thus shows that equity levels and equity ratios play distinct roles. Equity levels just determine the scale of banking operations, but do not affect the (privately optimal) ranking of investment. Increases in the levels of bank equity per se may be ineffective, since absent increases in equity ratio requirements banks can just lever up against additional equity, so that only the size of their total balance sheet changes while their leverage and riskiness is unaffected. Empirically, commercial banks tend to actively manage their balance sheets to maintain leverage ratios: after a positive shock to their assets in booms and the corresponding increase in their equity, commercial banks tend to raise additional debt to maintain target leverage ratios (see Figure 3 in Adrian and Shin).

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4 There are various frictions that can cause cost associated with raising equity. For example, Gennaioli, Shleifer, and Vishny (2013), attribute these cost to risk aversion on the part of households (while bankers are risk-neutral). Baker and Wurgler (2013) find empirical evidence for high cost of raising equity; as reflected in the low risk anomaly of banks’ stock returns.
As a result, in booms, banks may just use the increases in equity to expand their balance sheets and the scale of their risk-taking activities, which is consistent with the notion of aggregate expansions in risk-taking during booms when banks’ equity values go up.

**Related literature.** To the best of our knowledge, this paper is the first to provide a general equilibrium framework that allows analyzing the effects of bank capital regulations when banks and other market participants are heterogeneously skilled and regulated and compete in financial markets. The importance of institutional heterogeneity in the financial sector has been recently stressed by Hanson, Shleifer, Vishny, and Stein (2014). In their framework, institutions endogenously differ in terms of assets and liabilities, but they abstract from risk-taking incentives induced by bail-out guarantees, the very focus of our paper.

Although several recent papers on bank capital regulation also feature general equilibrium settings, these papers abstract from competition by non-bank investors and typically assume that banks either have monopolistic access to certain borrowers or compete only amongst each other (see, e.g., Begenau, 2013, Nguyen, 2014). As a result, these papers do not find the non-monotonic effect of capital requirements on banks’ risk taking that we highlight in this paper.

To highlight the mechanism of our results, our model isolates the debt’s role on risk-taking incentives, but does not explicitly account for the (social) benefits of bank debt. A large literature following Diamond and Dybvig (1983) and also Gorton and Pennacchi (1990) argues that banks can provide liquidity services via deposits, i.e., maturity transformation. In this context, DeAngelo and Stulz (2013) argue that stringent capital requirements may impede the banking sector’s ability to produce valuable liquid claims, causing potentially important social costs that need be considered in the overall trade-off calculation. While we do not include such liquidity considerations in our framework, the highlighted non-monotonic effect of capital requirements would survive provided that risk-taking is a relevant concern.

Further, a large set of papers in the micro-theory literature analyzes bank capital regulation but abstracts from the general equilibrium effects that are present in our model (see, e.g., Pennacchi, 2006, Mehran, Acharya, and Thakor, 2013). Harris and Raviv (2014) also shows that, under some circumstances, increasing capital requirements reduces welfare. The result is driven by the interaction between the regulator’s desire to prevent excessive
risk taking by banks, while, at the same time, ensuring that they disclose early on when they are in trouble. Plantin (2014) analyzes optimal bank capital regulation in the presence of shadow banking activities, but considers a partial equilibrium model that abstracts from competition. In his setting banks are not competing with the shadow banking system but rather can engage in shadow banking activities themselves (see also Ordoñez (2014), Moreira and Savov (2013)). Consequently, relaxing capital requirements may be beneficial, as it reduces banks’ incentives to circumvent regulation by engaging in shadow-banking activities. Gornall and Strebulaev (2013) develop a quantitative trade-off model of bank capital structure in which only highly levered banks can pass on tax benefits of debt to firms.

While we don’t explicitly model signals about risks of banks’ assets, one can view our analysis as conditional on observable measures of risks. As the recent literature points out, however, the use of private risk assessments in capital regulation is controversial (see, e.g., Opp, Opp, and Harris 2013; Becker and Opp 2013). In particular, ratings are problematic, as they do not (intend to) measure systematic risk exposures (see Iannotta and Pennacchi 2012).

Our paper is organized as follows. We discuss the structure of the economy and our modeling assumptions in Section 2. In Section 3 we present the general equilibrium implications of capital regulation which are the main results of our paper. Section 4 discusses two extensions of our basic model, most notably the ability of banks to raise additional equity. Section 5 concludes.

2 Model setup

The general structure of our economy is as follows. Firms may obtain debt financing for investment projects of varying quality from banks or non-bank investors. We refer to the latter as outside investors. Banks differ from outside investors in two important ways. First, banks are intrinsically better at collecting payments from firms (as in Diamond and Rajan 2001). Secondly, banks are subject to a bail-out guarantee, that is, bank debt, including deposits, is guaranteed by taxpayers’ money (which in our setting can be motivated by banks’ special collection skill in the first place). The regulator designs bank

5 In an empirical study, Kisin and Manela (2013) estimate the shadow cost of existing capital regulation by exploiting a regulatory loophole that allowed banks to bypass capital requirements. They find that the compliance costs of pre-crisis regulation are small.
capital regulation ex ante to avoid excessive risk-taking while maintaining socially valuable intermediation by the banking sector.

It is important to note that although we distinguish in our model for expositional simplicity between “banks” and “outside investors” the two characteristic features that we associate with banks ((1) skill and (2) bailouts) are in reality not necessarily only fulfilled by banks nor do all banks in reality necessarily satisfy these characteristics. In that sense, the banks in our model could be interpreted more broadly as “systemically important financial institutions” (SIFIs). For example, if an insurance company provides socially valuable intermediation services and would get bailed out by the government in case it cannot satisfy its liabilities (such as AIG), then this institution would also be subsumed by the label “bank” in our analysis.

To illustrate the forces at play in this economy, we consider a two-period, discrete-state economy in which the aggregate state of the world is either “high” \( s = H \) or “low” \( s = L \). The ex-ante probability of the high state is denoted by \( p_H \). For ease of exposition, all agents in the economy (which we describe below) are risk-neutral and discount their respective payoffs at a discount rate of 0.

### 2.1 Firms

There is a continuum of firms of measure one. Each firm is owned by a cashless entrepreneur who seeks debt financing from banks or outside investors. The entrepreneur has access to a risky project that requires an initial investment of 1 and may either succeed or fail. If the project succeeds, the firm’s net cash flow \( Z \) at the end of the period is \( R > 1 \). In case of failure, the cash flow satisfies \( Z = 0 \).

Firms differ with regard to their probabilities of default. In particular, there are two types of different quality \( q \in \{g, b\} \) with respective average default probabilities \( d_g \) and \( d_b \). Thus, the NPV of a type \( q \) project is given by

\[
V_q = R (1 - d_q) - 1. \tag{1}
\]

We assume that type \( g \) firms are “good,” i.e., \( V_g > 0 \) whereas type-\( b \) are “bad”, i.e., have negative NPV projects. The fraction of good types in the population \( \pi_g \) is common

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6 In other words, the average default probabilities \( d_g \) and \( d_b \) are unconditional with respect to information about the aggregate state \( s \).
knowledge to all parties at date 0. In addition, each firm knows its own type.

To make our setup interesting, we assume that the projects are differentially exposed to the aggregate risk indicated by the state \( s \in \{ H, L \} \). State-contingent default probabilities are denoted by \( d_s^q \) so that \( d_q = p_H d_q^H + (1 - p_H) d_q^L \).

**Assumption 1** Good projects \((g)\) always succeed in both aggregate states of the world. Bad projects \((b)\) always succeed in the high aggregate state \((H)\), but always default in the low aggregate state \((L)\).

\[
d_H^g = d_H^b = d_L^g = 0, \quad d_L^b = 1.
\]

We make this extreme assumption to capture two important features. First, good and bad securities look identical in the good state of the world, i.e., \( d_H^g = d_H^b = 0 \) (for simplicity we assume they do not default at all). Secondly, only bad securities are exposed to catastrophe risk, i.e., they don’t pay out in the bad state. Thus, “bad” securities may be interpreted as *economic catastrophe bonds* in the sense of Coval, Jurek, and Stafford (2009). This simple binary-type framework covers the essential tradeoffs of much more general firm-type distributions.

### 2.2 Investors

#### 2.2.1 Bankers

The economy consists of a measure one of competitive ex-ante identical bankers (also referred to as banks) each with initial wealth, that is, *inside* equity, of \( \bar E_0 \). To economize

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7 Our setup implicitly equalizes safe projects with good projects and risky projects with bad projects, which is a typical assumption for models that analyze risk-shifting. If good projects were also risky, then banks’ investments in these assets would still be beneficial from a welfare-perspective (as the projects are positive NPV and banks are more efficient than outside investors). The key tension in our model arises when high-risk projects are negative NPV from a social perspective (as considered in our setup), so that there is a potential misalignment between the interests of banks and the public. Further, without loss of generality, we can ignore safe, bad projects, as these projects would never be funded by banks or any other investor.

8 While we assume a continuum of banks, our qualitative results only require a finite number of banks behaving competitively in the asset market. This interpretation is more realistic, since we also assume that banks are too-big-too-fail. From a technical perspective, however, a finite number of banks would introduce cumbersome indivisibilities in the optimal asset allocation among banks.
on notation, we omit bank-specific subscripts, even when we refer to a specific bank. For ease of exposition, it useful to start with the assumption that banks cannot raise outside equity from outside investors (as in Gennaioli, Shleifer, and Vishny (2013)), but are free to pay out their equity in the form of cash dividends $Div_0$. In Section 4.1 we relax this assumption and analyze the model in the presence of equity issuances. In addition to their own equity, banks may raise government-insured deposits from outside investors at time 0, denoted as $D_0$. The funding side implies that a bank possesses $M_0$ dollars for investment purposes, with

$$M_0 = E_0 + D_0,$$  \hspace{1cm} (4)

where $E_0 = \bar{E}_0 - Div_0$, the amount of inside equity left after paying out dividends. In Section 4.1 we will allow banks to raise additional equity at cost $C_E$, which may be interpreted as a negative dividend. The bank may either fund firm projects (see Section 2.1) or invest in a storage technology, which we will label “cash.” Total investment in cash is given by $C_0$.

For ease of exposition, we assume that there is no information asymmetry between banks and issuers, i.e.,

**Assumption 2** The bank observes the project type of any firm in which it invests and can collect payments from firms at no cost.

Let $A_0 = M_0 - C_0$ denote the total amount invested by a bank in firms’ securities, i.e., non-cash assets, and define the bank’s (net) equity ratio by

$$e \equiv \frac{E_0}{A_0}.$$  \hspace{1cm} (5)

The equity ratio relates the investment of the banker (after the dividend payout) to the total book value of non-cash assets, $A_0$. Starting from an initial endowment $\bar{E}_0$, the bank can effectively choose any equity ratio $e \in [0, 1]$ through the appropriate choice of dividends, $Div_0$, and cash investments, $C_0$. By choosing to hold enough cash as a reserve, i.e., $C_0 = D_0$, the bank has no net leverage ($e = 1$). By paying out all equity as dividends, the bank becomes fully levered ($e = 0$). This simple connection between asset allocation and funding choices highlights the important relationship between reserves (cash) and capital (equity) requirements.

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9 We omit the distinction between FDIC insured deposits and any short-term debt that is implicitly guaranteed by the government. $D_0$ refers to the sum of both funding sources.
Let $x_j \geq 0$ denote the fraction of non-cash assets ($A_0$) that the bank invests in security $j$, and let $r^s_j$ denote the state-contingent rate of return on security $j$. Then the rate of return on the portfolio of non-cash assets is given by $r^s_A = \sum x_j r^s_j$. The rate of return on bank equity, $r^s_E$, in each aggregate state $s$ is given by

$$r^s_E = \max \left\{ \frac{(1 + r^s_A) A_0 + C_0 - D_0}{E_0} - 1, -1 \right\} = \max \left\{ \frac{r^s_A}{e}, -1 \right\}.$$  

This formula reflects the limited liability of equity holders and that their investment is $E_0$, which may be interpreted as the book value of equity. Each bank chooses its equity investment, $E_0$, its equity ratio, $e$, and its portfolio $\{x_j\}$ to maximize the market value of its equity, which is given by

$$E_0^M = \max_{e, E_0, \{x_j\}} E_0 \cdot E \left( 1 + \max \left\{ \frac{\sum x_j r^s_j}{e}, -1 \right\} \right).$$  

2.2.2 Outside investors

Outside investors are deep-pocketed and always behave competitively if they have an investment opportunity with non-negative NPV. Unlike banks, outside investors observe firms’ types. However, compared to banks, they are at a disadvantage as they incur an additional cost $c \geq 0$ per unit of investment which reflects cost associated with collecting payments, as in Diamond and Rajan (2001). While in practice such cost presumably varies across assets (loans, mortgages, securities) our model only considers one cost parameter $c$ for simplicity. Due to risk-neutrality and the lack of a government bailout guarantee, competitive outside investors require an expected return of 0 on any asset. Given a collection cost $c$, outside investors thus value a bond of quality $q$ with face value $N_q$ and unconditional default probability $d_q$, as

$$PV_O(q) = N_q (1 - d_q) - c,$$

where the subscript $O$ indicates outside investors. Note that, if $q$ is a bad firm, $PV_O(q) < V_b + 1 - c < 1$, since $V_b < 0$, so that outside investors will not fund bad projects. We interpret $c$ as a measure of competition from outside investors (see, e.g., Petersen and Rajan (1995) or Rajan and Zingales (2001)). If $c > V_g$, the NPV of the good project,

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10 When the bank does not default, the overall portfolio return on all bank assets in each state satisfies the standard decomposition: $r^s_A = er^s_E + (1 - e) r^s_D = er^s_E$

11 Of course, there also exist specialized outside investors that are more skilled than banks in certain asset classes (e.g., private equity firms, venture capitalists, etc.). However, for the purpose of our analysis, we can ignore projects in which banks are inferior and thus do not invest under any circumstances.
outside investors cannot exert any competitive pressure. However, if \( c < V_g = R - 1 \), competitive outside investors are willing to provide financing to good firms at face value \( 1 + c \), which effectively implies that good types have a (type-dependent) outside option of \( V_g - c \).

### 2.3 Regulator

The regulator aims to maximize (expected) welfare \( W \) defined as the NPV of funded securities in the economy net of collection cost \( c \) for projects funded by outside investors, i.e.,

\[
W = \mu_g V_g + \mu_b V_b + \mu_{g,O} (V_g - c) \tag{9}
\]

where \( \mu_q \) represents the mass of funded projects of type \( q \) funded by banks and \( \mu_{g,O} \) the mass of good projects funded by outside investors. As a benchmark, it is useful to define the first-best outcome \( W^* \) given by

\[
W^* = \pi_g V_g, \tag{10}
\]

which represents the welfare that can be achieved if banks hold all good assets in the economy (and no bad assets are financed).

The need to regulate results from the fact that the government in our model finds it optimal to bail out banks that cannot repay their depositors (debtor). This protection by the taxpayer allows banks to engage in asset substitution, i.e., funding of bad issuers, without negatively affecting depositors, thus increasing banks’ incentive to do so. Banks in our model are more efficient in collecting loans than other investors, which in turn motivates the government to ensure their survival. Thus, exactly those financial institutions that can fulfill socially valuable functions may be the ones that behave most opportunistically, as they can count on government support. Apart from this model-motivated rationale for bailouts, it seems widely accepted that government bailouts of large, insolvent financial institutions are optimal to avoid triggering a cascade of defaults by those institutions’ counterparties, their counterparties, etc., that could result in a system-wide financial crisis and recession. Modeling such a process and the “ex-post” optimality of the bailout decision explicitly is possible, but would clutter the model considerably without adding

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12 Even absent a bailout guarantee, an asset substitution problem may arise after the bank has issued debt. However, without a bailout guarantee, incentives for risk shifting would be reduced since debt holders would require higher yields in advance (or not even invest).
any additional insights regarding the “ex-ante” choice of bank capital requirements. We also note that all of our results are robust to the assumption that bailouts occur with certainty: probabilistic bailouts would simply be reflected in the repayment terms demanded by rational bank debt holders.

If the regulator could directly observe the quality of the asset, regulation would be trivial, as the regulator could simply mandate banks to invest only in good assets. To study the non-trivial case, we make the following assumption:

**Assumption 3** The regulator can observe all the exogenous parameters in the economy but, regarding bank assets, it can distinguish only between cash and firm projects. In particular, the regulator cannot distinguish between firm projects of different types \((g, b)\).

Motivated by existing regulation, we posit that the regulator sets regulatory rules ex ante in the form of minimum capital requirements, i.e.,

\[
e \geq e_{\text{min}}.
\]  

Our analysis will reveal that properly designed capital requirements are able to induce the first best outcome in the economy under certain parameter constellations. In those circumstances, our focus on simple minimum capital requirements is not even restrictive from a mechanism design perspective. Moreover, given the prevalence of minimum capital requirements in practice, our normative analysis produces positive predictions “on the side” by illustrating the non-trivial comparative statics of economic outcomes with respect to minimum capital requirements.

### 3 Analysis

Before describing the formal equilibrium definition, we summarize the timing of the moves by the players in the just-described economy.

1. The regulator sets minimum capital requirements \(e_{\text{min}}\).

2. Each bank decides on its dividend strategy, i.e., it sets \(E_0 = \bar{E}_0 - \text{Div}_0\).

13 In Section 4.2 we allow for parameter uncertainty about the fraction of good types.
3. Firms attempt to raise financing from banks and outside investors.

4. Firms that obtain financing invest in their projects.

5. Nature determines the aggregate state of the world \( s \in \{H, L\} \).

6. Project payoffs for all financed firms are realized.

Since the relevant information set of each player is a singleton, our game formally represents an extensive form game with perfect information (under exogenous uncertainty). Our notion of equilibrium is subgame perfection.

**Definition 1** *Equilibrium*

a) Given its information, the regulator chooses minimum equity capital requirements \( e \geq e_{\text{min}} \) to maximize expected welfare \( W \).

b) Each firm \( i \) maximizes its expected value of profits \( \mathbb{E}(\max\{Z_i^s - N_i, 0\}) \) by selling debt with the lowest face value \( N_i \) that results in raising 1 unit of capital.

c) Each bank maximizes the market value of equity, \( E_0^M \), by choosing its equity investment \( E_0 \leq \bar{E}_0 \), its equity buffer \( e \geq e_{\text{min}} \), and its portfolio strategy \( x_j \geq 0 \).

d) Outside investors invest in risky firm projects if and only if they expect to break-even.

Our equilibrium analysis focuses on the relevant aggregate implications of regulation for investment and funding terms. It is important to note that the total amount of deposits is indeterminate. Suppose \( D_0 > 1 \). Then banks could always borrow more, say raise debt to \( D_0 + \Delta D \) and invest the additional amount \( \Delta D \) in cash. This would keep \( e \) unchanged. For the purpose of our analysis, the outcomes are “essentially” equivalent.

The following analysis proceeds in four steps. In Section 3.1, we derive basic properties of the equilibrium, in particular how the valuation of risky assets is affected by the implicit bail-out guarantee for banks. These insights are essential for the remaining equilibrium analysis. In Section 3.2, we study the unregulated economy (in which \( e_{\text{min}} = 0 \)) as a benchmark. Since this economy features inefficiencies as a result of the bail-out guarantee, we then study the effectiveness of capital regulation to reduce or eliminate asset substitution and the role of competition from outside investors (see Section 3.3).
3.1 Equilibrium properties

In this section, we will derive fundamental characterizations of equilibrium bank behavior given an exogenous capital requirement constraint $e_{\text{min}}$.

**Lemma 1** Banks never have a strict preference to choose $\text{Div}_0 > 0$.

**Proof:** Inspection of the bank’s objective function $E_0 \cdot (1 + \frac{1}{e} \mathbb{E} \max \{\sum x_j r_j^*, -e\})$ s.t. $e \geq e_{\text{min}}$ reveals that profits scale with the initial investment $E_0$. Since the bank is infinitesimal, changes in $E_0$ do not affect $r_j^*$. If the maximum possible return on risky assets satisfied $\mathbb{E} \max \{\sum x_j r_j^*, -e\} < 0$, the bank could simply invest all its assets in cash (rather than paying out the dividend) to ensure it keeps its equity, which formally implies that $e \to \infty$. If $\mathbb{E} \max \{\sum x_j r_j^*, -e\} \geq 0$, it clearly has no strict incentive to reduce $E_0$ by paying out dividends. ■

Intuitively, if returns on risky assets are not sufficiently attractive, the bank could always invest in cash rather than paying out dividends. Thus, without loss of generality, we can assume that no bank ever pays out dividends, i.e., we set

$$E_0 = \bar{E}_0.$$  \hfill (12)

This allows us to write the decision problem of the banks solely in terms of the leverage choice $e$ and the portfolio choice $\{x_j\}$. Since banks are thus identical in terms of scale (and all other characteristics), the following lemma is immediate.

**Lemma 2** All banks share the same equilibrium expected return on equity, i.e., $\mathbb{E} [r_E^*] = \bar{r}_E \geq 0$ (even if different bank strategies $(e, \{x_j\})$ coexist).

**Proof:** Omitted. ■

An essential ingredient for the following analysis is how banks value assets. Banks’ marginal valuations depend on their overall portfolio strategy, in particular, whether the bank defaults in the low state or not. A bank that chooses an equity ratio and investment portfolio such that $e < -r_A^L = -\sum x_j r_j^L$ will default in the low state; otherwise the bank survives (see equation 6).
Lemma 3 Consider a bank that has an equity ratio $e$ and holds risky assets generating state-dependent returns $r^A_s$. Then the bank values a security with state contingent cash flows $CF^s$ as

$$V_B = \begin{cases} 
\frac{CF^H}{1+r^H_A} & \text{if } e < -r^L_A, \\
\frac{\mathbb{E}[CF^A_s]}{1+\mathbb{E}[r^A_s]} & \text{if } e \geq -r^L_A.
\end{cases}$$

(13)

Proof: See Appendix. ■

This lemma illustrates an important ingredient for the remaining analysis of the paper. A “safe” bank ($e \geq -r^L_A$) simply values the asset according to its unconditional payoff, similar to an outside investor. In contrast, a “risk-shifting” bank, whose equity buffer is small ($e < -r^L_A$), cares only about payoffs in the high state of the world. Any additional payoff generated in the low state of the world simply reduces the required subsidy by taxpayers (to pay bond holders), but does not affect equity value.

Based on the above valuation techniques, the following lemma reveals that we can restrict our attention to “pure portfolios.”

Lemma 4 Pure Portfolios:

a) There is no equilibrium in which safe banks fund bad issuers.

b) There is no equilibrium in which risk-shifting banks fund good issuers.

Proof: See Appendix. ■

The intuition for this lemma is simple. Safe banks care about the payoffs of securities in both states of the world. Since the NPV of a bad security is negative, they will never be willing to fund the investment. In contrast, risk-shifting banks care only about the high state of the world. Therefore, the payoff of the good security in the low state is of no value to a risk-shifting bank.

Proposition 1 Risk-shifting banks always operate at maximum leverage, i.e., $e = e_{\min}$.

Safe banks operate at maximum leverage when the expected return to equity is positive ($\bar{r}_E > 0$) and are indifferent among leverage ratios that satisfy the regulatory constraint ($e \geq e_{\min}$) when the expected return to equity is zero.

If an issuer of quality $q$ is funded in equilibrium by regulated banks, then its face value $N_q$
satisfies

\[ N_g (e_{\min}, \bar{r}_E) = 1 + e_{\min} \bar{r}_E, \quad (14) \]
\[ N_b (e_{\min}, \bar{r}_E) = 1 + e_{\min} \frac{\bar{r}_E + 1 - p_H}{p_H}. \quad (15) \]

**Proof:** If a good issuer \((d_g = 0)\) is funded by banks, then it must be financed by a safe bank (see Lemma 4). Setting the required funding, i.e., one unit, equal to the valuation by non-defaulting banks (see equation 13) yields \(N_g = 1 + \mathbb{E} [r_A^s]\). Since for a non-defaulting bank, \(e \bar{r}_E = \mathbb{E} [r_A^s]\) (see equation 6), it immediately follows that \(N_g = 1 + e \bar{r}_E\). Suppose that \(\bar{r}_E > 0\) in equilibrium. Then banks will try to invest as much as possible given their equity endowment \(\bar{E}_0\), i.e., set \(e = e_{\min}\), so that \(N_g = 1 + e_{\min} \bar{r}_E\). If \(\bar{r}_E = 0\), then \(N_g = 1\) regardless of the actual choice of \(e \geq e_{\min}\).

If a bad issuer \((d_b^H = 0)\) is funded by banks, then it must be financed by a risk-shifting bank (see Lemma 4). Setting the required funding, i.e., one unit, equal to the valuation by defaulting banks (see equation 13) yields \(N_b = 1 + r_A^H\). Since for a defaulting bank, \(e \bar{r}_E = \mathbb{E} [r_A^s]\) (see equation 6), we obtain that \(N_b (e, \bar{r}_E) = 1 + e \frac{\bar{r}_E + 1 - p_H}{p_H}\). If \(\bar{r}_E > 0\), then again we obtain \(e = e_{\min}\). Now, suppose \(\bar{r}_E = 0\) and all other risk-shifting banks choose \(e > e_{\min}\) so that \(N_b (e_{\min}, \bar{r}_E) = 1 + e \frac{1 - p_H}{p_H}\), then lowering \(e\) to \(e_{\min}\) would allow a single bank to lower the face value, attract as much bad issuers as its funding capacity, and make profits. This contradicts the fact that \(\bar{r}_E = 0\). Hence, it must be that in equilibrium \(e = e_{\min}\). ■

Intuitively, risk-shifting banks directly benefit from leverage and always opt for the maximum leverage given the regulatory constraints, i.e., \(e = e_{\min}\). Maximum leverage in the banking system also provides the lowest possible funding terms for issuers (holding the equilibrium rate of return \(\bar{r}_E\) fixed). Thus, market prices (yields) of good and bad issuers directly reflect regulatory constraints of banks in the form of \(e_{\min}\). Due to the unit size of investment, the face value \(N_q\) can be interpreted as the (gross) promised yield to maturity on an issue of type \(q\). These funding terms are expressed as a function of the endogenous equilibrium rate of return on bank equity.

The equilibrium rate of return of the banking sector is determined by the banking sector’s funding capacity \(A_{\max} (e_{\min})\), the distribution of firm types \(\pi_q\), and the private
ranking of investments. First, the funding capacity of the banking sector is given by

\[ A_{\text{max}}(e_{\text{min}}) = \frac{\bar{E}_0}{e_{\text{min}}}. \]  

(16)
i.e., the inside equity levered up to the maximum. We call this first and immediate effect of regulation the “funding capacity effect.” Second, capital regulation \( e_{\text{min}} \) differentially affects the private returns that bankers can extract from their investments in good and bad assets. Thus, capital regulation does not only determine the “funding capacity,” but may also impact the “private ranking” of assets. Let \( \bar{r}_E(e_{\text{min}}) \) denote the return on equity of a bank that is fully invested in type-\( q \) firms assuming that the bank can extract all the surplus:

\[ \bar{r}_E^q(e_{\text{min}}) = \min \left\{ \frac{R-1}{e_{\text{min}}} \right\}, \]
\[ \bar{r}_E^b(e_{\text{min}}) = p_H \frac{R-1}{e_{\text{min}}} + (1 - p_H)(-1) \]

(17)
(18)
Since banks do not necessarily extract all the surplus in equilibrium, it is important to note that \( \bar{r}_E^q(e_{\text{min}}) \) does not generally represent the equilibrium rate of return \( \bar{r}_E(e_{\text{min}}) \), which by Lemma 2 is equalized across all banks. The return functions \( \bar{r}_E^q \) and \( \bar{r}_E^b \) feature two properties, which are crucial for the further analysis. First, competition differentially affects the maximum returns possible from good investments and bad investments: The competition parameter \( c \) caps the returns that banks may extract from good types who can bypass the banking system and obtain financing from outside investors, but it does not affect \( \bar{r}_E^b(e_{\text{min}}) \) as bad firms can only be funded by banks with a bailout guarantee. Second, capital requirements \( e_{\text{min}} \) differentially affect the returns \( \bar{r}_E^q(e_{\text{min}}) \) and \( \bar{r}_E^b(e_{\text{min}}) \).

Intuitively, the more of their own capital that banks have to coinvest the relatively less attractive risk-shifting is, i.e., formally

\[ \frac{d\bar{r}_E^b(e_{\text{min}})}{de_{\text{min}}} < 0. \]

In fact, when capital requirements are sufficiently stringent, i.e., \( e_{\text{min}} > \hat{e} \), the private returns to banks from risk-shifting are negative so that holding cash is preferred. The threshold \( \hat{e} < 1 \) satisfies:

\[ \hat{e} = \frac{p_H}{1 - p_H} (R - 1)^{14} \]

(19)
The definition of \( \bar{r}_E^q(e_{\text{min}}) \) is a first step towards characterizing \( \bar{r}_E \) since the return \( \bar{r}_E^q(e_{\text{min}}) \) will define \( \bar{r}_E \) if an asset \( q \) is the marginal funded asset of the banking sector:

\[ \text{14 The fact that the bad project has negative NPV implies that } p_H (R - 1) - (1 - p_H) < 0, \text{ so that } \hat{e} < 1. \]
Definition 2 An asset of quality $q$ is the banking sector’s marginal funded asset if $\bar{r}_E^q > 0$ and $0 < A_{\text{max}} - \sum_{j: \bar{r}_E^j > \bar{r}_E^q} \pi_j \leq \pi_q$.

The marginal asset of the banking sector, or henceforth simply the marginal asset, has two intuitive properties. First, the asset must offer potential positive returns since banks can always use the storage technology. Second, the marginal asset is the most attractive asset whose investment is constrained by the available funding capacity $A_{\text{max}}$. Formally, $A_{\text{max}}$ is sufficient to finance all assets that are ranked above asset quality $q$, satisfying $\bar{r}_E^j > \bar{r}_E^q$, but the remaining funds, $A_{\text{max}} - \sum_{j: \bar{r}_E^j > \bar{r}_E^q} \pi_j$, are insufficient to finance all projects of type $q$. An increase in $A_{\text{max}}$ thus implies a one-to-one increase in the funding of the marginal asset $\mu_q$, i.e., $m_q \equiv \frac{d\mu_q}{dA_{\text{max}}} = 1$.

By definition of the marginal asset, if type $q$ assets are marginal, a fraction of type $q$ projects remains unfunded. In this case, competition of firms for funding by banks implies that type-$q$ firms’ participation constraint binds in equilibrium. This in turn determines the expected return on equity of banks investing in asset type $q$ and hence the expected return of the entire banking sector (by Lemma 2). In contrast, if the banking sector’s funding capacity exceeds the supply of investment opportunities with a non-negative return, competition of banks for firm projects drives down profits of the banking sector to zero. Thus,

Lemma 5 The equilibrium expected return of all banks satisfy:

$$\bar{r}_E (e_{\text{min}}) = \begin{cases} 0 & \text{if } A_{\text{max}} (e_{\text{min}}) > \sum_{j: \bar{r}_E^j > \bar{r}_E^q} \pi_j \\ \bar{r}_E^q (e_{\text{min}}) & \text{if } m_q = 1. \end{cases}$$

Technically, the rent distribution between firms and banks is not uniquely pinned down in the knife-edge case when the sectors’ funding capacity is just enough to finance the marginal asset. In this case, we assume that banks appropriate all rents. For example, if $A_{\text{max}} = 1$ and $\bar{r}_E^q > \bar{r}_E^b > 0$, we set $\bar{r}_E (e_{\text{min}}) = \bar{r}_E^b$. We will now consider the implications of Lemma 5 in a sequence of economies.
3.2 Unregulated Economy

We first characterize an unregulated economy to study the need for regulation. Formally, this corresponds to the equilibrium (see Definition 1) in which the regulator is constrained to choose $e_{\text{min}} = 0$.

**Proposition 2** In the unregulated economy:
1) all bad projects are financed by fully levered, risk-shifting banks,
2) all good projects are financed by safe banks,
3) all firm types can obtain financing at a face value of $N = 1$,
4) outside investors do not invest in firm projects,
5) social welfare is given by $W = \pi_g V_g + \pi_b V_b$.

**Proof:** See main text.

The intuition for these results is simple. Issuers are in short supply relative to bank funds as banks face no leverage restrictions, i.e., $A_{\text{max}} > \pi_g + \pi_b = 1$. As a result, the expected return on bank equity satisfies $\bar{r}_E = 0$ (by Lemma 2). The banking sector is segmented: Safe banks finance good issuers at $N_g = 1$ (by Proposition 1 setting $\bar{r}_E = 0$) while risk-shifting banks fund bad assets at $N_b = 1$ as well. This segmentation of the banking system maximizes the (private) benefits from asset substitution (see also equation 4). However, due to competition among banks, issuers of both project types can capture the entire (private) surplus. In an unregulated economy, the yields are completely uninformative about the default risk of an asset: For both issuers, the yield is zero. Bad assets are rationally inflated due to limited liability and the presence of the bailout guarantee for banks. At these prices, outside investors cannot compete with banks and play no role in this economy.

3.3 Regulated Economy

The just-described unregulated economy features “cheap financing for all firms” regardless of their quality and hence welfare losses relative to first best. Risk-shifting banks are willing to finance every project in the economy because they only care about outcomes in the good aggregate state. Thus, investment is too high relative to first best. Any type of

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15 Here, the mass of (infinitely-levered) risk-shifting banks goes to zero in such a way that exactly a mass $\pi_b$ of type-$b$ firms can be financed.
(successful) capital regulation $e \geq e_{\text{min}}$ must therefore limit excessive investment (without prohibiting positive NPV investments). To understand the role of outside investors in our analysis, it is useful to split the analysis into two steps. First, we assume that outside investors’ collection cost is prohibitively high, i.e., $c > V_g = R - 1$, so that they would never be able to break-even on their investments (Section 3.3.1) and only banks can invest. The subsequent Section 3.3.2 analyzes the case in which competition by outside investors affects the equilibrium outcome.

### 3.3.1 Banking Economy

When banks do not face competition from outside investors ($c > R - 1$), banks rank good projects above bad projects for all levels of capital requirements as $\bar{r}_E(g_{\text{min}}) = \frac{R-1}{e_{\text{min}}} > \bar{r}_E(b_{\text{min}})$. However, bad projects are still more attractive than investing in cash as long as $e_{\text{min}} < \hat{e}$. Using this case distinction of whether bad projects are worth funding from a private perspective or not, we can now characterize the comparative statics of capital requirements.

**Proposition 3** Comparative statics of capital requirements $e_{\text{min}}$.

The “Natural Pecking Order” Regime: If $e_{\text{min}} \leq \hat{e}$, the banking system will fund as many good issuers as possible, i.e., $\mu_g = \min \{A_{\text{max}}, \pi_g\}$ and invest the remaining funds in bad issuers, i.e., $\mu_b = \min \{A_{\text{max}}, 1\} - \mu_g$. Equilibrium values of $A_{\text{max}}, \bar{r}_E, \mu_g, \mu_b, N_g,$ and $N_b$ as functions of $e_{\text{min}}$ are shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>$e_{\text{min}} &lt; \bar{E}_0$</th>
<th>$e_{\text{min}} \in \left[\bar{E}_0, \frac{E_0}{\pi_g}\right]$</th>
<th>$e_{\text{min}} &gt; \frac{E_0}{\pi_g}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{max}}$</td>
<td>$A_{\text{max}} &gt; 1$</td>
<td>$A_{\text{max}} \in [\pi_g, 1]$</td>
<td>$A_{\text{max}} &lt; \pi_g$</td>
</tr>
<tr>
<td>$\mu_g$</td>
<td>$\pi_g$</td>
<td>$\pi_g$</td>
<td>$A_{\text{max}}$</td>
</tr>
<tr>
<td>$\mu_b$</td>
<td>$\pi_b$</td>
<td>$A_{\text{max}} - \pi_g$</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{r}_E$</td>
<td>0</td>
<td>$\bar{r}<em>E(e</em>{\text{min}})$</td>
<td>$\bar{r}<em>E(g</em>{\text{min}})$</td>
</tr>
<tr>
<td>$N_g$</td>
<td>1</td>
<td>$\left(1 - p_H\right) \left(1 - e_{\text{min}}\right) + p_H R$</td>
<td>$R$</td>
</tr>
<tr>
<td>$N_b$</td>
<td>$1 + e_{\text{min}} \frac{1-p_H}{p_H}$</td>
<td>$R$</td>
<td>N/A</td>
</tr>
</tbody>
</table>

The “Safe Banks” Regime: If $e_{\text{min}} > \hat{e}$, banks will fund as many good projects as possible, i.e., $\mu_g = \min \{A_{\text{max}}, \pi_g\}$ and never invest in bad issuers. Equilibrium values of $A_{\text{max}}, \bar{r}_E,$
\[ \mu_g, \mu_b, \text{ and } N_g \text{ as functions of } e_{\min} \text{ are shown in the following table.} \]

<table>
<thead>
<tr>
<th>( e_{\min} &lt; \frac{\bar{E}_0}{\pi_g} )</th>
<th>( e_{\min} &gt; \frac{\bar{E}_0}{\pi_g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{\max} )</td>
<td>( A_{\max} &gt; \pi_g )</td>
</tr>
<tr>
<td>( \mu_g )</td>
<td>( \pi_g )</td>
</tr>
<tr>
<td>( \mu_b )</td>
<td>0</td>
</tr>
<tr>
<td>( \bar{r}_E )</td>
<td>0</td>
</tr>
<tr>
<td>( N_g )</td>
<td>1</td>
</tr>
</tbody>
</table>

Welfare is given by

\[ W = \mu_g V_g + \mu_b V_b. \]

**Proof:** See Appendix. ■

It is instructive to illustrate the insights of this Proposition with an example. The example plotted in Figure 1 features parameters that imply \( \bar{E}_0 < \hat{e} < \frac{\bar{E}_0}{\pi_g} \). In the upper left panel, we plot the returns \( \bar{r}_E^g(e_{\min}) \) and \( \bar{r}_E^b(e_{\min}) \) and the equilibrium rate of return for \( \bar{r}_E \). In the lower left panel, we plot the funding decisions of the banking sector given its funding capacity \( A_{\max} \). Funding decisions for a given \( e_{\min} \) are indicated by the vertical length (see arrows). The resulting welfare is plotted in the upper right panel. The funding terms for good and bad issuers implied by the equilibrium rate of return are plotted in the lower right panel. The “natural pecking-order regime” applies for \( e_{\min} \leq \hat{e} \); the safe banks regime applies for \( e_{\min} > \hat{e} \).

**Natural pecking-order regime:** For \( e_{\min} < \bar{E}_0 \) banks invest in all assets in the economy, including the assets of bad issuers (\( \mu_g = \pi_g \) and \( \mu_b = \pi_b \)) - risk-shifting occurs. Since \( e_{\min} < \bar{E}_0 \), the associated funding capacity \( A_{\max} > \pi_g + \pi_b = 1 \) implies that banks can fund all firm projects in the economy. Since \( e_{\min} < \hat{e} \) and hence \( \bar{r}_E^b > 0 \) (see upper left panel) banks want to finance all projects. In this region, welfare is constant and equal to the unregulated economy. The equilibrium rate of return satisfies \( \bar{r}_E = 0 \) (by Lemma 5). Once \( e_{\min} \) starts to constrain total bank investment, i.e., \( e_{\min} > \bar{E}_0 \), the bad asset becomes the marginal asset (since \( 0 < \bar{r}_E^b < \bar{r}_E^g \)) and the equilibrium rate of return now satisfies \( \bar{r}_E(e_{\min}) = \bar{r}_E^b(e_{\min}) \). Graphically, the marginal asset can be identified by the color (in this case red) of the asset right beneath the funding capacity \( A_{\max} \). Investment in these bad assets determines the outside option of “safe banks.” Bad issuers offer as much as possible to attract investment, i.e., \( N_b = R \), which pins down the expected rate of return.
Figure 1. The figures illustrate the effect of varying equity ratio requirements, \( e_{\text{min}} \), on the ROE (upper left panel), the funding capacity \( A_{\text{max}} \) (lower left panel), welfare \( W \) (upper right panel) and the funding terms for good issuers \( N_g \) and bad issuers \( N_b \) (lower right panel). The parameters of the economy are chosen as follows: \( p_H = 0.5 \), \( R = 1.5 \), \( c = 0.7 \), \( \pi_g = 0.25 \), and \( \bar{E}_0 = 0.2 \). In this case, \( \bar{E}_0 = 0.2 < \hat{e} = 0.5 < \frac{\bar{E}_0}{\pi_g} = 0.8 \). The natural pecking order applies for \( e_{\text{min}} < 0.5 \). The “safe banks” regime applies for \( e_{\text{min}} > 0.5 \).

on equity \( \bar{r}_E = \bar{r}_E^b (e_{\text{min}}) \) for risk-shifting banks. Good issuers promise a face value \( N_g < R \) that implies the same expected return on equity for safe banks (by Lemma 2). Increases in \( e_{\text{min}} \) decrease \( A_{\text{max}} \) and hence reduce one-for-one investment in bad issuers. The reduction of negative NPV investments increases welfare (see upper right panel). Interestingly, an increase in capital requirements decreases the funding cost for good issuers in this region (see lower right panel) since the larger coinvestment \( e_{\text{min}} \) decreases the private returns that can be reaped by risk-shifting banks. Thus, higher capital requirements effectively decrease the competition that good issuers face from bad issuers.
Safe Banks regime: At $e_{\text{min}} = \hat{e}$, risk-shifting is no longer privately optimal since $\bar{r}_E < 0$ (see upper left panel) so that banks optimally invest only in good assets (lower left panel). The abandonment of risk-shifting leads to a discontinuous increase in welfare (upper right panel). Since the funding capacity exceeds the supply of projects with positive return, i.e., the supply of good projects, for $\hat{e} < e_{\text{min}} < \frac{\bar{E}_0}{\pi_g}$, banks compete for good assets and the resulting equilibrium rate of return is zero (upper left panel). As all good projects and no bad projects are funded in this region, first-best welfare $W^* = \pi_g V_g$ is achieved. Significant further increases in capital requirements, i.e., such that $e_{\text{min}} > \frac{\bar{E}_0}{\pi_g}$ or equivalently $A_{\text{max}} < \pi_g$, start constraining investment in good issuers (lower left panel) and hence decrease welfare (upper right panel). The good asset becomes the marginal asset, so that good issuers need to promise their entire returns, $N_g = R$, to attract funding (lower right panel).

The effects of capital requirements in the absence of competition from outsiders can be summarized as follows: Sufficiently lax capital requirements, $e_{\text{min}} < \min \{\hat{e}, \bar{E}_0\}$, produce the same inefficient outcome as in the unregulated economy $W = \pi_g V_g + \pi_b V_b$. Capital requirements have a positive effect on welfare until investment in good issuers is constrained, i.e., $e_{\text{min}} \geq \frac{\bar{E}_0}{\pi_g}$. Counter to conventional wisdom, the effect of bank capital requirements on funding costs for good issuers may be non-monotonic in general equilibrium. While capital requirements lead to higher funding cost for good types holding $\bar{r}_E$ fixed, the endogenously determined rate of return $\bar{r}_E (e_{\text{min}})$ may counter this effect.

**Proposition 4** First-best welfare $W^*$ can be achieved by setting $e_{\text{min}} = \frac{\bar{E}_0}{\pi_g}$. The minimum equity capital requirement that achieves first-best welfare is given by: $e^{*}_{\text{min}} = \min \{\hat{e}, \frac{\bar{E}_0}{\pi_g}\}$

**Proof:** If $\frac{\bar{E}_0}{\pi_g} \leq \hat{e}$, $e_{\text{min}} = \frac{\bar{E}_0}{\pi_g}$ implies Case (3) of the “Natural Pecking Order” Regime in Proposition 3 and $A_{\text{max}} = \pi_g$. If $\frac{\bar{E}_0}{\pi_g} > \hat{e}$, $e_{\text{min}} = \hat{e}$ implies the “Safe Banks” Regime and $A_{\text{max}} > \pi_g$. ■

This result follows directly from the comparative statics of capital requirements shown in Proposition 3. It reveals that in the absence of competition from outside investors, first best welfare can be achieved by simple capital regulation. The main ingredient for this result is the “natural pecking order.” Banks have an incentive to finance good assets first; bad issuers only represent the second best option. By constraining the size of the banking sector via capital regulation such that just a mass $A_{\text{max}} = \pi_g$ of projects can be financed, the first-best outcome can be implemented. Such capital regulation is akin to an aggregate lending constraint. A crucial ingredient for this simple macroprudential regulation is that
banks cannot raise additional equity and that the regulator knows the fraction of good projects. In Sections 4.1 and Section 4.2, we discuss robustness of the results with regards to these important extensions of the baseline model.

### 3.3.2 Banks vs. Outside Investors

We now assume that the cost of investing for outside investors is not prohibitively high, so that they may purchase good securities, i.e., \( c < R - 1 = V_g \). Competition from outside investors for good issuers implies that the face value for good issuers funded by banks is capped at \( \bar{N}_g = 1 + c \) which in turn caps the rate of return on good assets at \( c \), so that \( \bar{r}_E^g = \frac{\min(c, R-1)}{e_{\min}} = \frac{c}{e_{\min}} \). Note that this competition channel only applies to good investments, since outside investors never invest in bad issuers. If competition is sufficiently strong, i.e., \( c < p_H (R - 1) \), the private ranking of securities by the banking sector determined by \( \bar{r}_E^b \), depends on the leverage constraint \( e_{\min} \). In particular, for capital requirements below a threshold \( \tilde{e} \), the (maximum) private return on bad assets exceeds the (maximum) private return on good assets, i.e., \( \bar{r}_E^b > \bar{r}_E^g \) (see upper left panel of Figure 2). The threshold \( \tilde{e} \) is given by

\[
\tilde{e} = \hat{e} - \frac{c}{1 - p_H}^{16} \tag{22}
\]

Intuitively, laxer capital requirements (lower \( e_{\min} \)) and higher competition for good assets (lower \( c \)) make risk-shifting relatively more attractive. In the extreme case as outside investors become perfectly competitive, i.e., \( c = 0 \), banks can no longer make any profits with good issuers, so that the threshold \( \tilde{e} \) coincides with the zero profit condition for bad investments, i.e., \( \tilde{e} = \hat{e} \) (see equation 19). Only when \( e_{\min} > \tilde{e} \) is the social ranking of good and bad assets restored. We call the ranking \( \bar{r}_E^b > \bar{r}_E^g \) the “reverse pecking order.”

In our setup, banks have by assumption an absolute advantage over outside investors for both investments in good assets (due to collection cost \( c \)) as well as in bad assets (due to the bail-out guarantee). However, when \( e_{\min} < \tilde{e} \), banks have a comparative advantage in investing in bad assets. Then, the “natural pecking order” of investment (described in the previous section) is reversed as shown in the following proposition:

**Proposition 5** Comparative statics of \( e_{\min} \) in the presence of competition:

Regime “Reverse Pecking Order”: If \( e_{\min} < \tilde{e} \), banks fund as many bad issuers as possible, i.e., \( \mu_b = \min \{ A_{\max}, \pi_b \} \) and invest the remaining funds in good issuers, i.e.,
Figure 2. The figures illustrate the effect of varying equity ratio requirements, $e_{\text{min}}$, on the ROE (upper left panel), the funding decisions $\mu_g, \mu_b$ and capacity $A_{\text{max}}$ (lower left panel), welfare $W$ (upper right panel) and the funding terms for good issuers $N_g$ and bad issuers $N_b$ (lower right panel). The parameters of the economy are chosen as follows: $p_H = 0.5, R = 1.7, c = 0.05, \pi_g = 0.5, E_0$ and $e = 0.2$. If $E_0 < e_{\text{min}} < E_0/\pi_b$, the good asset is the marginal funded asset. If $E_0/\pi_b < e_{\text{min}} < \tilde{e}$, the bad asset is the marginal funded asset. The reverse pecking order applies for $e_{\text{min}} < 0.6$.

$\mu_g = \min \{A_{\text{max}}, 1\} - \mu_b$. Outside investors fund the remaining good issuers, i.e., $\mu_{g,O} = \min \{1 - A_{\text{max}}, \pi_g\}$. Values of $A_{\text{max}}, \tilde{r}_E, \mu_g, \mu_{g,O},$ and $\mu_b$ as functions of $e_{\text{min}}$ are given in
the following table.

<table>
<thead>
<tr>
<th>$e_{\text{min}} &lt; \bar{E}_0$</th>
<th>$e_{\text{min}} \in [\bar{E}_0, \frac{E_0}{\pi_b}]$</th>
<th>$e_{\text{min}} &gt; \frac{E_0}{\pi_b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{max}}$</td>
<td>$A_{\text{max}} &gt; 1$</td>
<td>$A_{\text{max}} &lt; \bar{E}_0$</td>
</tr>
<tr>
<td>$\mu_g$</td>
<td>$\pi_g$</td>
<td>$A_{\text{max}} - \pi_b$</td>
</tr>
<tr>
<td>$\mu_{g,o}$</td>
<td>$1 - A_{\text{max}}$</td>
<td>$\pi_g$</td>
</tr>
<tr>
<td>$\bar{r}_E$</td>
<td>$\bar{r}<em>E (e</em>{\text{min}})$</td>
<td>$\bar{r}<em>E (e</em>{\text{min}})$</td>
</tr>
<tr>
<td>$N_g$</td>
<td>$1 + e_{\text{min}} \frac{1-p_H}{p_H}$</td>
<td>$1 + e_{\text{min}} \frac{1-p_H}{p_H}$</td>
</tr>
<tr>
<td>$N_b$</td>
<td>$1 + c$</td>
<td>$1 + c$</td>
</tr>
</tbody>
</table>

If $e_{\text{min}} < \tilde{e}$, the “Natural Pecking Order” regime obtains.
If $e_{\text{min}} > \bar{e}$, the “Safe Banks” regime obtains.
In the last two regimes, $\bar{r}_E = \frac{c}{e_{\text{min}}}$ when $e_{\text{min}} > \frac{E_0}{\pi_g}$. All other outcomes in these regimes are as shown in 20 and 21, respectively.

Proof: We only prove the “Reverse Pecking Order” regime.

Case 1. If $e_{\text{min}} < \bar{E}_0$, $A_{\text{max}} > 1$, so banks are competing for all (good and bad) assets of firms. As a result, banks make zero profits ($\bar{r}_E = 0$) and all firms are financed.

Case 2. If $e_{\text{min}} > \bar{E}_0$, i.e., $A_{\text{max}} < 1$, the banking sector cannot finance all risky assets. Since the amount that may be extracted from bad issuers is greater than for good issuers, i.e., $\bar{r}_E > \bar{r}_E$, all bad issuers are financed by banks. Banks invest the remaining funds in good issuers, i.e., $A_{\text{max}} - \pi_b$. The good types that are unfunded by banks are financed by outside investors (with mass $1 - A_{\text{max}}$).

Case 3. Banks only invest in bad assets, since $\bar{r}_E > \bar{r}_E$. All good assets are funded by outside investors.

This Proposition reveals that intense competition from outside investors for good assets causes banks to switch their business models from primarily funding good assets (see Proposition 5) to primarily funding bad assets. The banks’ “outside option” is now given by the return it can extract from good assets $\frac{c}{e_{\text{min}}}$. Perversely, this enables bad issuers to secure funding terms satisfying $N_b < R$ when banks compete for their assets, i.e., $e_{\text{min}} < \frac{E_0}{\pi_b}$ (see Proposition 1). In contrast, when $e_{\text{min}} > \bar{e}$, the economy behaves similarly to the case without outside investors.

Figure 2 illustrates the results of Proposition 5 by plotting a parametrization that

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implies $\bar{E}_0 < \frac{\bar{E}_0}{\pi_b} < \tilde{e}$. The setup of the figure is identical to Figure 1, however, there are a few important differences since $\bar{r}^g_E$ is not always above $\bar{r}^b_E$. The “Reverse Pecking Order” regime applies for $e_{\min} < \tilde{e}$.

**Reverse pecking order regime:** For low $e_{\min}$, banks again invest in all assets in the economy, including the assets of bad issuers. Increasing $e_{\min}$ beyond $\bar{E}_0$ ($A_{\max} < 1$), however, leads to a situation where the reverse pecking order of bank investment becomes apparent. Since banks have a comparative advantage in investing in bad assets, i.e., $\bar{r}^b_E > \bar{r}^g_E$ for $e < \tilde{e}$, banks first reduce their investment in good issuers as regulation starts to constrain total bank investment. Since banks now face competition from outside investors for investment in good securities, banks rather maintain maximum investment in bad assets (that is, $\mu_b$ stays at $\pi_b$) and fund only $A_{\max} - \pi_b$ good assets. The good asset is the marginal asset of the banking sector (graphically indicated by the color grey just below the funding capacity). The remaining good assets are funded inefficiently (at cost $c$) by the outside investors, i.e., $\mu_{g,O} = 1 - A_{\max}$. As a result, increases in capital requirements initially lead to a welfare loss compared to the unregulated economy (see upper right panel). At the same time, the scarcity of total bank funds allows banks to generate positive expected returns on equity, $\bar{r}_E = \bar{r}^g_E = \frac{c}{e_{\min}}$ (see upper left panel). At $e_{\min} = \frac{\bar{E}_0}{\pi_b}$, the banking sector only finances bad assets and outside investors finance all good assets. This is the worst possible outcome in terms of welfare. Further increases in capital requirements constrain investments in bad assets, i.e., the bad asset becomes the marginal asset so that $\bar{r}_E = \bar{r}^b_E$. Welfare is increasing in capital requirements, since a fraction of bad assets now remains unfunded (although bad firms pledge the maximum face value $N_b = R$).

**Natural pecking order and Safe Banks regime:** Increasing capital requirements beyond $\tilde{e}$ will undo the reverse pecking order. The banking sector switches back to maximum possible investment in good assets, creating a discontinuous increase in welfare. However, as $\frac{\bar{E}_0}{\tilde{e}} < \pi_g$, not all good assets can be financed by banks. Thus, when $e_{\min} > \tilde{e}$, a fraction $\pi_g - A_{\max}$ is funded by outside investors. As $e_{\min}$ increases over this range, banks fund fewer and fewer good issuers while outside investors fund more and more. Although all good issuers are funded, welfare decreases over this range, since outside investors are not as efficient as banks. In this example, first-best welfare can never be achieved with simple capital requirements.

The possibility of a reverse pecking order has important implications for the design of optimum capital regulation.
Proposition 6 If $\bar{e} \leq \frac{F_0}{\pi_g}$, the first best outcome can be achieved by setting $e_{\min} = \frac{F_0}{\pi_g}$, i.e., $A_{\max} = \pi_g$. If $\bar{e} > \frac{F_0}{\pi_g}$, the first best outcome cannot be achieved. The second best outcome, $W = W^* - \min \left\{ \left( \pi_g - \frac{F_0}{\bar{e}} \right) c, \pi_b | V_b | \right\}$, is achieved by setting $e_{\min} = \left\{ \begin{array}{ll} 0 & \text{for } \pi_b | V_b | < \left( \pi_g - \frac{F_0}{\bar{e}} \right) c, \\ \bar{e} & \text{otherwise}. \end{array} \right.$

Proof: If $\bar{e} \leq \frac{F_0}{\pi_g}$, setting $e_{\min} = \frac{F_0}{\pi_g}$ implies that we are in the “natural pecking order” regime (by Proposition 5) and so the “optimality” results of the previous section apply. The first best outcome can be achieved by constraining investment of the banking sector to $A_{\max} = \pi_g$ (setting $e_{\min} = \frac{F_0}{\pi_g}$). However, when $\bar{e} > \frac{F_0}{\pi_g}$, i.e., $c$ sufficiently small, constraining the size of the banking sector to $A_{\max} = \pi_g$ is not enough since banks will “first” invest in bad issuers as long as there is a higher return to be made from bad issuers (reverse pecking order as in the upper left panel of Figure 2). In this case, if capital regulation aims to prevent any risk-taking one needs to set $e_{\min} \geq \bar{e}$. Since outside investors are inefficient at holding good assets, setting $e_{\min} = \bar{e}$ is the best possible choice among these choices. This results in a welfare loss of $\left( \pi_g - \frac{F_0}{\bar{e}} \right) c$ relative to first best, caused by the collection cost of outside investors. However, if this welfare loss is larger than the welfare loss in an unregulated economy, i.e., $\pi_b | V_b | < \left( \pi_g - \frac{F_0}{\bar{e}} \right) c$, it is optimal not to impose any capital regulation.

Intuitively, when the natural pecking order obtains at a level of $e_{\min}$ that constrains the funding capacity to $A_{\max} = \pi_g$, first best welfare can be obtained because banks use their funds in the socially desired way. If this condition is not satisfied, i.e., the reverse pecking order holds at $e_{\min} = \frac{F_0}{\pi_g}$, then the regulator decides between two second best solutions. Either the regulator sets $e_{\min} = \bar{e}$, just enough to give banks an incentive to use all their funds for good assets while the remaining good assets $\left( \pi_g - \frac{F_0}{\bar{e}} \right)$ are financed inefficiently (at cost $c$) from outside investors, or the regulator does not regulate at all and tolerates the financing of bad projects. The rationale for the latter outcome is simple: If the welfare loss from bad projects, $\pi_b | V_b |$, is sufficiently low, it is better from a welfare perspective to tolerate risk-shifting by banks than to impose capital regulation that prevents risk-shifting at the cost of pushing too many good assets, $\pi_g - \frac{F_0}{\bar{e}}$, into the hands of second-best investors. If risk-shifting is however a significant concern, as motivated by our analysis, i.e., $\pi_b | V_b | > \pi_g c$, then the regulator is always forced to make capital requirements substantial to avoid the pitfalls of the reverse pecking order outcome.
Our results thus have concrete normative implications to the extent that the social value contributed by banks (reflected in the parameter $c$) varies across asset classes. Capital requirements should not just be a function of risk (to the extent observable), but also relate to banks’ comparative advantages in managing assets (see Proposition 6). Intuitively, capital requirements should be more stringent for asset classes in which banks have a small efficiency advantage. In asset classes where banks are less efficient than outside investors optimal regulation would mandate that banks cannot hold any of these assets at all (since banks would only have private incentives to invest in these assets due to the distortionary effects of bailouts). Such banning can be effectively achieved in our framework by requiring 100% capital for a certain activity.

Concretely, one may argue that the efficiency advantage of banks in collection is greater for “whole loans” compared to securitized loans or proprietary trading. As a result, capital requirements should be smaller for “whole loans” than for securitized loan portfolios (holding risk measures constant). This is at odds with the current structure of capital requirements in which the securitization process in itself reduces capital requirements (see empirical evidence by Acharya, Schnabl, and Suarez (2013)).

4 Extensions

4.1 The Effect of Equity Issuances

So far, we have made the assumption that raising new equity is prohibitively costly. We now relax this assumption by allowing for the possibility of raising equity at constant marginal cost $C_E$. Raising an amount of equity $\Delta E$ results in a new funding capacity of:

$$A_{\max}(e_{\min}, \Delta E) = A_{\max}^0 + \frac{\Delta E}{e_{\min}}$$  

where $A_{\max}^0 = \frac{\bar{E}_0}{e_{\min}}$. In this context it is useful to define the expected return on banks’ book equity absent equity issuances as a benchmark. We denote this expected return by

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17 Empirically measuring the extent to which banks have efficiency advantages in various asset classes is not a trivial exercise (similar to measuring risk exposures). In particular, banks’ profit margins may be distorted due to bailouts and other types of government subsidies and thus do not necessarily provide an accurate measure of efficiency advantages.

18 For ease of exposition, we impose a constant marginal cost $C_E$. The source of this cost (asymmetric information, issuance fees, or risk-aversion (see e.g., Gennaioli, Shleifer, and Vishny (2013)) is irrelevant for the bank’s decision.
$\bar{r}_E$ going forward. Note that equity issuances do not have any effect on the return functions $\bar{r}_E^g(e_{\min})$ and $\bar{r}_E^b(e_{\min})$ but rather change the banking sector’s aggregate funding capacity $A_{\max}(e_{\min}, \Delta_E)$ and thereby can alter the banking sector’s marginal type of investment (cash, type $g$, or type $b$ projects). The marginal investment’s type in turn pins down banks’ expected return on book equity in equilibrium. In the following Proposition we characterize the symmetric equilibrium that arises when banks can raise additional equity.

**Proposition 7** Symmetric Equilibrium with Equity Issuances

1. If $\bar{r}_E^0(e_{\min}) \leq C_E$, no bank raises equity, $\Delta_E = 0$.

2. If $\bar{r}_j(e_{\min}) \leq C_E < \bar{r}_E^0(e_{\min})$, where $j \in \{g, b\}$ represents the non-marginal asset, banks choose $\Delta_E$ such that $A_{\max}(e_{\min}, \Delta_E) = 1 - \pi_j$.

3. If $A_{\max}^0 < 1$ and $\min \{\bar{r}_g(e_{\min}), \bar{r}_b(e_{\min})\} > C_E$, banks choose $\Delta_E$ such that $A_{\max}(e_{\min}, \Delta_E) = 1$.

**Proof:** Case 1): Marginal equity issuance cost exceed the expected marginal return on additional equity. Case 2): If $\bar{r}_j(e_{\min}) \leq C_E < \bar{r}_E^0(e_{\min})$, the marginal asset has the highest private value (since $\bar{r}_E^g(e_{\min}) > \bar{r}_j(e_{\min})$) and the banking sector’s funding capacity $A_{\max}^0$ is not sufficient to fund all assets of that type. Since the second-best asset satisfies $\bar{r}_j(e_{\min}) \leq C_E$, banks only have an incentive to raise equity up to the point where the funding capacity satisfies $A_{\max} = 1 - \pi_j$, i.e., they set $\Delta_E = (1 - \pi_j) e_{\min} - \bar{E}_0$. Any further increase would cause a drop in the return to $\bar{r}_j(e_{\min})$. Case 3): Since all real assets generate private returns in excess of $C_E$, banks raise just enough equity so to finance all assets in the economy, i.e., $A_{\max} = 1$, i.e., they set $\Delta_E = e_{\min} - \bar{E}_0$.

Thus, a necessary and sufficient condition for equity raising is $\bar{r}_E^0(e_{\min}) > C_E$. Cases 2) and 3) then determine the extent of additional equity raising. In Case 2) banks only fund the asset with the highest private value. As long as $e_{\min} \geq \hat{e}$, i.e., either the “Natural pecking-order” or “Safe banks” regime obtains, banks would raise just enough equity to finance all good assets (so that no assets are inefficiently financed outside the banking sector). However, when the “reverse pecking order” obtains, banks will raise equity to finance more bad assets, as their initial endowment $\bar{E}_0$ constrained their ability to fund all of the bad assets. Case 3) can occur only if $\bar{r}_E^b > 0$, or equivalently $e_{\min} < \hat{e}$. Then, a sufficiently low cost of raising equity implies that banks raise enough costly equity such that all assets in the economy are financed. In this case, the endogenous choice of raising equity will lead to the same financing decisions as in an unregulated economy.
Figure 3. The figures illustrate the effect of varying equity ratio requirements, $e_{\text{min}}$, on the ROE (upper left panel), the funding decisions $\mu_g, \mu_b$ and capacity $A_{\text{max}}$ (lower left panel), welfare $W$ (upper right panel) and the funding terms for good issuers $N_g$ and bad issuers $N_b$ (lower right panel). The parameters of the economy are chosen as follows: $p_H = 0.5$, $R = 1.7$, $c = 0.05$, $\pi_g = 0.5$, $\bar{E}_0 = 0.2$ and $C_E = 0.18$. If $\bar{E}_0 < e_{\text{min}} < \bar{E}_0/\pi_b$, the good asset is the marginal funded asset. In this region, banks will raise additional equity as long as $\bar{r}_g(e_{\text{min}}) > C_E$ (see upper left panel). If $\bar{E}_0/\pi_b < e_{\text{min}} < \bar{e}$, the bad asset is the marginal funded asset. Banks will raise additional equity as long as $\bar{r}_b(e_{\text{min}}) > C_E$ (see upper left panel). The endogenous raising of equity will generate increases in welfare relative to an environment with fixed equity $W^0$ when financing of good assets is increased and will decrease welfare when the financing of bad assets is increased (see upper right panel).

We now consider an example of the equilibrium implications of allowing the endogenous raising of equity by the banking sector. Figure 3 illustrates how welfare and funding volume of good and bad issuers change when equity issuance costs $C_E$ are reduced and the regulator exogenously sets capital requirements to a certain $e_{\text{min}}$. Otherwise, the example parameters are identical to Figure 2. The equity issuance cost $C_E$ is plotted in the upper left panel where it can be compared to the equilibrium rate of return. When $e_{\text{min}} < \bar{E}_0$, all assets in the economy are funded and $\bar{r}_E(e_{\text{min}}) = 0$. Thus, case 1) of Proposition 7 applies
and no additional equity is raised. For regulatory requirements $e_{\text{min}}$ just above $\bar{E}_0$, equity issuance cost $C_E$ is below the marginal expected return on book equity absent equity issuances (the black line indicating $\bar{r}^0_E = r^0_E$ is above $C_E$; see the upper left panel) and also below $\bar{r}^b_E$. Hence, case 3) applies and banks raise equity such that the total funding capacity $A_{\text{max}}(e_{\text{min}}, \Delta E)$ allows banks to finance a measure of $\pi_g + \pi_b = 1$ firms (see lower left panel). In this case additional raising of equity allows expansion of investment in good projects (see lower left panel). As a result, welfare is increased relative to the welfare in the absence of equity issuance denoted as $W^0$. However, there is also a region for capital requirements $e_{\text{min}}$ where lower equity issuance cost harms welfare. When $e_{\text{min}}$ is just above $\frac{E_o}{\pi_g}$, bad projects are the marginal type and banks generate higher returns on book equity than the equity issuance cost $C_E$ absent equity issuances ($\bar{r}^0_E = \bar{r}^b_E > C_E$). Moreover, $\bar{r}^0_E < C_E$ in this region. Case 2) of Proposition [7] then implies that banks raise additional equity to finance all bad projects in the economy and welfare is lower than when issuance cost is (prohibitively) high.

Optimal regulation in the presence of equity issuances. We now turn to the optimum regulatory response. For simplicity, we assume that the cost of raising equity is not a social cost (and can thus be ignored in the welfare analysis)[19]. The regulator has to account for the fact that in cases where optimal regulation worked through the funding capacity effect, his actions might be countered by the bank’s endogenous response to raise equity and lever up on the additional capital. This concern is of course only valid if we are in regime c) of Proposition [7]. However, the endogenous response of the banking sector might also increase welfare. This would happen when $\frac{E_o}{\hat{e}} < \pi_g$. In that case, a restrictive capital requirement $e_{\text{min}} = \hat{e}$ (which prevents financing of bad assets) would potentially no longer lead to insufficient financing of good projects.

Proposition 8 First-best welfare can be achieved if any of the following conditions are satisfied:

1) $\frac{E_o}{\hat{e}} \geq \pi_g$, then $e_{\text{min}} = \hat{e}$.

2) $\frac{E_o}{\pi_g} \geq \hat{e}$ and $\bar{r}^b_E \left( \frac{E_o}{\pi_g} \right) \leq C_E$, then $e_{\text{min}} = \frac{E_o}{\pi_g}$.

3) $\frac{E_o}{\hat{e}} < \pi_g$ and $C_E < \bar{r}^b_E (\hat{e})$, then $e_{\text{min}} = \hat{e}$.

4) $\exists \tilde{e} \geq \hat{e} : \bar{r}^b_E (\tilde{e}) > C_E > \bar{r}^b_E (\hat{e})$, and $\frac{E_o}{\tilde{e}} < \pi_g$, then $e_{\text{min}} = \tilde{e}$.

Proof: See main text. ■

[19] Of course, if raising equity was (partially) socially costly, this would not affect the private incentives of banks and their funding decisions for a given $e_{\text{min}}$ would be unaffected.
In the first two cases, the endogenous adjustment of equity does not play a role in equilibrium. In case 1) financing of bad assets is (privately) not attractive (since $e = \hat{e}$) and all good assets can already be financed (so that $\bar{r}_E = 0 < C_E$). In the second case, capital requirements are set such that the aggregate funding capacity is sufficient to finance all good projects without additional equity, i.e., $A_{\text{max}}^0 = \pi_g$. Since the associated capital requirements $\frac{E_0}{\pi_g}$ satisfy $\frac{E_0}{\pi_g} \geq \hat{e}$, banks prefer to invest in good assets (and finance all of them). Moreover, since $r_E^g \left( \frac{E_0}{\pi_g} \right) \leq \bar{r}_E^g \left( \frac{E_0}{\pi_g} \right) \leq C_E$, the maximum achievable return on bad assets is too low to make raising additional equity privately optimal.

In the final two cases, the regulator benefits from the fact that banks endogenously adjust equity in equilibrium. If the equity issue cost is sufficiently low, i.e., $C_E < r_E^g (\hat{e})$, the regulator’s optimal choice solely needs to ensure that bad projects are not worthwhile financing, i.e., $e_{\text{min}} = \hat{e}$. Banks will endogenously respond to such regulation by raising just enough equity to finance all the good projects in the economy (since the benefit $r_E^g (\hat{e})$ outweighs the cost). The final case applies when the cost of raising equity is intermediate. Then, the regulator sets $e_{\text{min}} = \bar{e} \geq \hat{e}$, so that good projects are weakly preferred, while ensuring that the private return on good projects $r_E^g (\bar{e})$ exceeds the cost of raising equity, whereas the return on bad projects is below the cost of raising equity.

Intuitively, endogenous adjustment of equity matters for the regulator’s decision only when the cost of raising equity is small. Then, the regulator has to respond by making capital requirements even higher so as to prevent banks from raising equity to finance additional bad projects.

### 4.2 Parameter Uncertainty

Up to now, we have made the simplifying assumption, that the regulator knows all exogenous parameters in the economy. In particular, knowledge about the fraction of good firms in the economy allowed the regulator to achieve the first best outcome when $\hat{e} \leq \frac{E_0}{\pi_g}$ by simply setting the capital requirement $e_{\text{min}}$ such that $A_{\text{max}} = \pi_g$, akin to an aggregate lending constraint (see Proposition 4). We will now relax this assumption by allowing for imperfect information about the distribution of firm types. Formally, this can be thought of as an additional stage in our game in which nature draws the realized fraction of good types $\tilde{\pi}_g$ from some distribution $f$ with support $[\pi_g, \bar{\pi}_g]$ after the regulator sets the regulatory regime. Of course, regulation cannot be allowed to be state-contingent.

To illustrate how our results would be altered with an uncertain fraction of good types,
we consider the case of a pure banking economy \( c > R - 1 \), so that the natural pecking order applies. Moreover, we assume that \( C_E \) is sufficiently high, so that raising equity does not occur in equilibrium. Finally, we make the expositional assumption that the distribution \( f \) is uniform over \([\bar{\pi}_g, \bar{\pi}_g]\).

Since \( \hat{e} \) depends only on project characteristics and not on the fraction of good and bad types, a conservative regulator can always prevent risk-taking by setting \( e_{\text{min}} = \hat{e} \). Let \( \hat{A}_{\text{max}} \) be the corresponding funding capacity. To make the problem interesting, assume that \( \bar{\pi}_g < \hat{A}_{\text{max}} < \bar{\pi}_g \).

For \( e_{\text{min}} = \hat{e} \), the expected welfare loss \( \Delta W \), relative to first best, is given by

\[
\Delta W (\hat{A}_{\text{max}}) = V_g \int_{\hat{A}_{\text{max}}}^{\bar{\pi}_g} (\bar{\pi}_g - \hat{A}_{\text{max}}) \, df (\bar{\pi}_g) \tag{24}
\]

\[
= \frac{V_g}{2} (\bar{\pi}_g - \hat{A}_{\text{max}})^2, \tag{25}
\]

where the second line follows from the assumption of a uniform distribution of \( \bar{\pi}_g \). In this case, welfare losses are due entirely to banks’ inability to fund all good projects when the realization of good types is sufficiently high, i.e., \( \bar{\pi}_g > \hat{A}_{\text{max}} \). In contrast, by setting \( e < \hat{e} \), the regulator makes it possible to reduce inefficiencies resulting from too little investment in good projects at the cost of allowing investment in bad projects (when the realization \( \bar{\pi}_g \) is sufficiently low). Given \( A_{\text{max}} \in \left( \hat{A}_{\text{max}}, \bar{\pi}_g \right) \), expected welfare losses can now be decomposed into these two sources:

\[
\Delta W (A_{\text{max}}) = V_g \int_{A_{\text{max}}}^{\bar{\pi}_g} (\bar{\pi}_g - A_{\text{max}}) \, df (\bar{\pi}_g) + |V_b| \int_{\hat{A}_{\text{max}}}^{A_{\text{max}}} (A_{\text{max}} - \bar{\pi}_g) \, df (\bar{\pi}_g) \tag{26}
\]

\[
= \frac{V_g}{2} (\bar{\pi}_g - A_{\text{max}})^2 + \frac{|V_b|}{2} (A_{\text{max}} - \bar{\pi}_g)^2. \tag{27}
\]

Taking first-order conditions with respect to \( A_{\text{max}} \) yields the trade-off solution \( A_{\text{max}}^* = \frac{\bar{\pi}_g V_g + |V_b|}{V_g + |V_b|} \in (\bar{\pi}_g, \bar{\pi}_g) \)\(^{21}\) and results in a welfare loss of

\[
\Delta W (A_{\text{max}}^*) = (\bar{\pi}_g - \bar{\pi}_g)^2 \frac{|V_b| V_g}{2 (V_g + |V_b|)}. \tag{28}
\]

Comparing \( \Delta W (\hat{A}_{\text{max}}) \) with \( \Delta W (A_{\text{max}}^*) \), we see that avoiding risk-taking altogether

\(^{20}\) If \( \hat{A}_{\text{max}} > \bar{\pi}_g \), then it would always be possible to achieve first best welfare by setting \( e_{\text{min}} = \hat{e} \). If \( \hat{A}_{\text{max}} < \bar{\pi}_g \), then setting \( e_{\text{min}} = \hat{e} \) is always dominated by setting \( e_{\text{min}} < \hat{e} \) such that \( A_{\text{max}} = \bar{\pi}_g \).

\(^{21}\) This assumes that the resulting value is greater than \( \hat{A}_{\text{max}} \).
by choosing $\hat{A}_{\text{max}}$ is strictly preferred to optimizing the tradeoff between funding more good assets but allowing some risk-taking by choosing $A^*_{\text{max}}$ if and only if
\[
\left(1 + \frac{\hat{A}_{\text{max}} - \bar{\pi}_g}{\bar{\pi}_g - \hat{A}_{\text{max}}}\right)^2 > 1 + \frac{V_g}{|V_b|}.
\]

Intuitively, a regulator prefers to avoid risk-taking altogether by choosing $\hat{A}_{\text{max}}$ if the minimum capital constraint that makes risk-taking unattractive, $\hat{e}$, is small (i.e., $\hat{A}_{\text{max}} = \frac{\hat{e}_0}{\hat{e}}$ is large) so that a large fraction of good projects is funded without risk-taking, or if the welfare loss from a bad project, $|V_b|$, is large relative to the gain from a good project, $V_g$. On the other hand, if $\hat{A}_{\text{max}}$ or $|V_b|$ is small, choosing the trade-off solution $A^*_{\text{max}}$ is optimal, even though this will result in financing bad projects when $\bar{\pi}_g < A^*_{\text{max}}$.

4.3 Robustness

In this section we discuss the robustness of our results to various alternative modeling assumptions.

**Competition and the pecking order.** Private rent extraction opportunities for banks are affected not only by competition but also by the underlying project payoff distribution. In our baseline model the good project always returns $Z^* = R$, whereas the bad projects only pays $R$ in the good aggregate state $H$, but zero otherwise. If one allows for more general payoff distributions, one can easily construct examples in which the “reverse pecking order” even obtains absent competition, for example, by raising the bad project’s payoff in the $H$-state above $R$. While competition from outside investors is not necessary to generate the reverse pecking order, we argue that competition from outside investors is a first-order economic force that influences banks’ profitability across different activities, and makes investments in risky, negative NPV activities relatively more attractive for banks.

The key ingredient for this asymmetric effect of competition is that outside investors are by definition not subject to a bailout guarantee. Hence outside investors’ ability to lever up and simultaneously invest in risky, negative NPV projects would be constrained by

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Recall that, given our terminology, any type of financial institution that is subject to a bailout guarantee would be considered a “bank”. 

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rational market participants’ unwillingness to provide funds for such activities ex ante.\textsuperscript{23} We also note that if outside investors took on leverage to finance \textit{good} projects (for example private equity firms), they would behave identically to the outside investors in our model. Overall, a more competitive financial system that features investors that are not subject to government guarantees can thus reduce banks’ incentives to use scarce capital for socially beneficial intermediation.

\textbf{Bailouts and costs of financial distress.} In our baseline model we do not explicitly consider cost of financial distress in the social welfare function (see equation 9), although we motivate bailouts based on the negative repercussions of bank defaults. In the presence of financial distress costs (e.g. a reduction in bank value due to distress and/or negative externalities on other banks) such as in \textit{Gornall and Strebulaev} (2013), the regulator has a clear ex post incentive to bail out banks. As long as the financing of these bailouts is non-distortionary, it is ex-post efficient to always bail out banks. In this case, the \textit{direct} effect of distress cost is not reflected in welfare on the equilibrium path. However, distress cost still affect welfare \textit{indirectly} by causing bailouts that affect banks’ ex-ante investment decisions\textsuperscript{24}

\textbf{Regulatory tools.} In our main analysis, we only allow the regulator to condition capital requirements on one observable signal, namely whether an asset is cash or a real project. This assumption has several interpretations. First, one can interpret our analysis as conditional on observable signals, i.e., good and bad firms are observationally equivalent (at least to the regulator). Second, our analysis can inform the debate on the repercussions of the policy recommendation by \textit{Admati, DeMarzo, Hellwig, and Pfleiderer} (2011) who advocate elimination of risk-weights based on observable signals such as ratings\textsuperscript{25}

Which economic quantities or signals of economic activity \textit{within our model} would be of interest to the regulator? First, in our simple setting with two assets and two aggregate states, bond yields of firms are generally perfectly informative about project types (unless there are no capital requirements and all yields are nil). However, in a more complex

\textsuperscript{23} If market participants anticipate ex ante that outside investors will choose negative NPV projects after taking on leverage, they will refuse to provide such leverage to outside investors in the first place.

\textsuperscript{24} If the financing of bailouts is distortionary, e.g., via inefficient taxes, safety of the banking sector itself becomes desirable, implying an additional rationale for high capital requirements. This consideration would be particularly relevant in a setup with risky, positive NPV projects, i.e., when the financing of these projects may lead to ex post distress cost in downturns if banks are not sufficiently capitalized.

\textsuperscript{25} Some justification for eliminating risk-weights based on ratings is provided in \textit{Opp, Opp, and Harris} (2013), who show that the informativeness of ratings is affected by the regulatory use of those ratings.
economic environment with more assets and more states, bond yields will generally not be perfect signals of project types and social value, as prices are determined by multiple state-contingent payoffs. Moreover, if yields were used for regulatory purposes, we should expect rational banks to adjust their pricing behavior, in turn making yields less useful for regulation (an argument that goes back to the Lucas critique).

Second, the regulator could attempt to infer banks’ risk-taking behavior from unusually high profits in good times. Taxation of bank profits in good states of the world would therefore be another way to reduce incentives to engage in risk-taking. However, in a more complex setting, good bank performance could also be due to superior skill, creating another source of noise that would compromise the benefits of such a regulatory approach.26 Despite these caveats, we note however that it would be valuable to gauge the costs and benefits of more complex regulatory policies in models that are richer than ours.

5 Conclusion

In this paper we propose a general equilibrium framework to analyze the effectiveness of bank capital regulations when banks face competition from other investors, such as institutions in the shadow-banking system. Our model highlights the importance of general equilibrium effects that arise when regulated and unregulated market participants interact in financial markets, revealing that competition can induce a non-monotonic relationship between regulatory capital requirements and banks’ risk taking. We provide conditions under which there exist ranges of increases in capital requirements that cause more banks in the economy to engage in value-destroying risk-shifting. Equity issuances by banks, i.e., increases in equity levels, may have counterproductive effects from a welfare perspective, as banks may lever up on the additional capital to expand risk taking activities.

Moreover, our framework allows us to shed light on the impact of bank capital requirements on the borrowers’ loan terms, a question that is often raised in the public debate. While borrowers face strictly higher interest rates when capital requirements are increased in partial equilibrium, i.e., holding fixed the expected rate of return on equity of the banking sector, we show that higher capital requirements may have the desirable effect of better

26 Note also that a regulatory policy that inhibits competition from outside investors could also be beneficial in our stylized setting. However, the benefits of such a policy would again not be robust to slight variations of the economic environment. For example, if investors that are not subject to bailouts have comparative advantages in certain asset classes relative to banks, then broad limits on competition would likely have detrimental effects on welfare.
funding terms for good issuers in general equilibrium. Intuitively, higher capital requirements decrease the returns that banks can reap from risk-taking (bad investments), which depresses the equilibrium rate of return of the banking sector when the bad investment is funded at the margin.

Our results highlight the importance of evaluating regulatory reform proposals in economic frameworks that explicitly consider the endogenous nature of banks’ investment opportunities in the presence of competition from other players in the financial system. Concretely, our framework implies that optimal capital requirements should not just be a function of risk, but should also depend on the (social) efficiency advantage of the banking sector across different asset classes, such as whole loans vs. actively traded securities. In contrast to the Volcker rule which is only supposed to ban proprietary trading activities, this suggested regulatory approach (1) applies more broadly to all banking activities that can lead to risk exposures and (2) allows for continuous adjustments in capital requirements depending on banks’ efficiency advantages.

A Proofs

A.1 Proof of Lemma 3

Proof: Let \( r^s_i \) denote the return on asset \( i \) in state \( s \). To obtain marginal valuations we consider an \( \varepsilon \)-perturbation of the bank’s asset portfolio such that state-dependent returns on assets are given by

\[
r^s_A(\varepsilon) = \varepsilon r^H_i + (1 - \varepsilon) r^L_A.
\]  

(29)

Banks maximize their equity value and thus require that asset \( i \) is priced such that a marginal perturbation keeps the expected return on equity unchanged. For a bank that defaults in the low state (\( e < -r^L_A \)), the expected return on equity given the perturbation is given by

\[
\bar{r}_E(\varepsilon) = p_H \frac{\varepsilon r^H_i + (1 - \varepsilon) r^L_A}{e} - (1 - p_H).
\]  

(30)

Imposing that \( \frac{d\mathbb{E}[r_E(\varepsilon)]}{d\varepsilon} \big|_{\varepsilon=0} = 0 \) then implies that the bank is willing to purchase a marginal unit of asset \( i \) as long as the return on asset \( i \) in the high state satisfies \( r^H_i = r^L_A \), or \( 1 + r^H_i = \frac{C^F_H}{V_B(i)} = 1 + r^L_A \). Solving the second equality for \( V_B(i) \) results in the upper branch

\[ \text{See Neuhann and Saidi (2014) for an empirical analysis of the cost of restricting the scope of banking activities.} \]
of equation 13

Similarly, for a bank that does not default in the low state \( (e \geq -r_L^A) \) the expected return on equity given the perturbation is given by

\[
\bar{r}_E(\varepsilon) = \varepsilon \frac{p_H r_i^H}{e} + (1 - \varepsilon) \frac{(1 - p_H) r_i^L}{e} + (1 - \varepsilon) \frac{p_H r_A^H}{e} + (1 - p_H) r_A^L.
\] (31)

Imposing that \( \frac{d\mathbb{E}[\bar{r}_E(\varepsilon)]}{d\varepsilon}|_{\varepsilon=0} = 0 \) then implies that the bank is willing to purchase a marginal unit of asset \( i \) as long as the average return on asset \( i \) has to satisfy \( \mathbb{E}[r_i^s] = \mathbb{E}[r_A^s] \). This implies the lower branch of equation 13.

A.2 Proof of Lemma 4

Proof: With regard to the first statement, suppose there were an equilibrium in which some safe banks fund bad issuers. Then those safe banks must fund a mix of bad types and good types (or cash), since funding only bad types would imply that the bank defaults. Then by optimality, the expected return on bad and good assets (or on bad assets and cash) would have to be equal – otherwise it would be optimal to change the portfolio at the margin. Given that safe banks fund bad assets and some other assets (good assets or cash), which together generate expected returns \( \mathbb{E}[r_A^s] \), bad issuers’ bond prices must be equal to safe banks’ marginal valuation which is given by \( N_b (1 - d_b) / (1 + \mathbb{E}[r_A^s]) \). However, equity holders require a weakly positive expected return, i.e., for a safe bank \( \mathbb{E}[r_E^s] = \mathbb{E}[r_A^s] / e \geq 0 \). Therefore for safe banks, \( V_B \leq N_b (1 - d_b) \leq R (1 - d_b) < 1 \) since \( N_b \leq R \) and the NPV of bad types is negative, so such a bank would never be willing to provide 1 unit of capital to bad issuers. This contradicts the supposition that some safe banks fund bad issuers.

We now turn to the second statement. Conjecture an equilibrium in which a risk-shifting bank funds some good issuers. Then this bank must fund a mix of good types and bad types, since funding only good types would imply that the bank does not default. By optimality, the return on the bad and the good assets in the high state has to be equal to the bank’s overall non-cash asset return in the high state, \( r_A^H \); otherwise it would be optimal for the bank to change the portfolio at the margin by tilting it toward the asset with the higher return in the high state. Given this, by Lemma 3 a risk-shifting bank values a bond from a good issuer with face value \( N_g \) at \( N_g / (1 + r_A^H) \). Assume that the risk-shifting bank makes an expected return on equity equal to \( \bar{r}_E \geq 0 \) with its
strategy \((e, \{x_j\})\). This implies that the return on a good issuer in the high state must be \(r^H = e^{\bar{r}_{E} + (1 - p_H)}\), and the asset has an equilibrium price \(\tilde{P} = N_g / \left(1 + e^{\bar{r}_{E} + (1 - p_H)}\right)\). Now consider an alternative strategy for the bank: the bank chooses the same \(e\) as before, but only invests in the good assets that are priced at \(\tilde{P}\). Since, given the price \(\tilde{P}\), the expected return on a good asset is \(e^{\bar{r}_{E} + (1 - p_H)}\), the bank will make an expected return on equity equal to \(\frac{E[r^E]}{e} = \frac{\bar{r}_{E} + (1 - p_H)}{p_H} > \tilde{r}_{E}\). Thus, the risk-shifting bank would want to deviate to this better strategy. This contradicts the initial conjecture that a risk-shifting bank funds any good issuers in equilibrium.

\section*{A.3 Proof of Proposition 3}

\textbf{Proof:} The “Natural Pecking Order” Regime: Since \(e_{\min} < \hat{e}\), bad assets can be potentially funded by banks.

\textbf{Case 1.} If \(e_{\min} < \bar{E}_0\) so that \(A_{\max} = 1\), banks are competing for all (good and bad) assets of firms. As a result, banks make zero profits (\(\bar{r}_E = 0\)) and all firms are financed.

\textbf{Case 2.} If \(\bar{E}_0 < e_{\min} < \frac{E_0}{\pi_g}\) so that \(\pi_g < A_{\max} < 1\), good projects are in short supply whereas bad projects are in excess supply. This implies that bad projects have to deliver all their returns if they are funded in equilibrium, i.e., defaulting banks make an expected return on equity of \(\bar{r}_{E}^{b}(e_{\min})\). Lemma 2 pins down the equilibrium rate of return:

\[ \bar{r}_E = \bar{r}_{E}^{b}(e_{\min}) \]  

\textbf{Case 3.} If \(e_{\min} > \frac{E_0}{\pi_g}\) so that \(A_{\max} < \pi_g\), only \(\mu_g = A_{\max}\) good assets can be financed. Issuers compete for banks, i.e., banks extract all the rents, so that \(\bar{r}_E = \frac{(R - 1)A_{\max}}{E_0} = \frac{R - 1}{e_{\min}}\).

The “Safe Banks” Regime: For \(e_{\min} > \hat{e}\), only good projects can be financed.

\textbf{Case 1.} If \(e_{\min} < \frac{E_0}{\pi_g}\) bank funds exceed total feasible (good) investment opportunities. Hence, issuers capture all rents, i.e., \(\bar{r}_E = 0\), and all good projects are financed.

\textbf{Case 2.} See the “Natural Pecking Order” Regime, Case (3).

\section*{References}


