Exchange Rate Flexibility under the Zero Lower Bound: the Need for Forward Guidance *

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Abstract

Macroeconomic theory says that when a country is vulnerable to idiosyncratic macro shocks, it should have its own currency and a flexible exchange rate. But recently in many countries, interest rates have been pushed down close to the lower bound, limiting the ability of policy-makers to accommodate shocks, even in countries with flexible exchange rates. This paper argues that if the zero bound constraint is binding and policy lacks an effective 'forward guidance' mechanism, a flexible exchange rate system may be inferior to a single currency area, even in face of country-specific macro shocks. With monetary policy constrained by the zero bound, under flexible exchange rates, the exchange rate exacerbates the impact of shocks. Remarkably, this may hold true even if only a subset of countries are constrained by the zero bound, and other countries freely adjust their interest rates under an optimal targeting rule. In a zero lower bound environment, in order for a regime of multiple currencies to dominate a single currency, it is necessary to have effective forward guidance in monetary policy.

Keywords:Zero Lower Bound, Monetary Policy, Optimal Currency Area, Forward Guidance JEL: E2, E5, E6

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1 Introduction

The theory of the optimal currency area (e.g. Mundell 1961, Kenen, 1969) says that a country vulnerable to idiosyncratic macro shocks should have its own independent monetary policy and a flexible exchange rate. If a country has a flexible exchange rate, then in face of a negative demand shock, the domestic interest rate can be reduced, allowing an exchange rate depreciation, which ensures faster adjustment in relative prices and quantities. Within a single currency area, this adjustment mechanism is absent. Much of the criticism of the Eurozone is built on that logic. When one country in the Eurozone goes into recession, it cannot offset this through a fall in its exchange rate. In the recent European crisis, the lack of independent monetary policy has been identified as one of the biggest hindrances to a faster adjustment in the current account and economic activity of Southern European countries.

But an important feature of the recent crisis, in both Europe and elsewhere, is that the normal functioning of monetary policy has been severely circumscribed by the zero bound constraint. In the Eurozone, and many other countries, interest rates have been at historically low levels, and have been unable to respond adequately to the scale of the downturns in the real economy. Arguably, the Eurozone and many other regions have been stuck in a liquidity trap.

The main aim of this paper is to show that in a liquidity trap, the standard reasoning in favour of multiple currencies and flexible exchange rates may be incorrect. When monetary policy is constrained by the zero bound on interest rates, and policy-makers lacks effective forward guidance, then paradoxically, it may be better to have a single currency than a regime of multiple currencies and a flexible exchange rate. Remarkably, this conclusion may still hold even if only a subset of countries in the region are constrained by the zero bound, and the other countries are free to follow optimal monetary policy rules. Equivalently, our analysis says that if regions in a single currency area experience large negative demand shocks which leads to policy rates being constrained by the zero bound, then they may in fact be better inside the single currency area than if they had kept their own independent currency and a floating exchange rate.

To give an intuition into these results, take a simple New Keynesian open economy model and assume there are country-specific demand shocks. Then under 'normal' times, when nominal interest rates are positive and monetary policy follows an inflation targeting rule, then a negative demand shock is followed by an exchange rate depreciation, which limits the impact of the shock.

Now, however, take a negative demand shock in the case where monetary policy is constrained by the zero bound. Then the exchange rate does not help to offset the impact of the shock. Rather, the exchange rate moves in the 'wrong direction', *exacerbating* the effects of the shock. Since at the zero bound, conventional monetary policy becomes temporarily powerless, and if forward guidance is ineffective, monetary authorities have no tools to independently affect the exchange rate so as to prevent this undesirable response. By contrast, a single currency area eliminates the possibility of perverse exchange rate adjustment, and achieves a superior sharing of macro risk among regions.

In a sense, the elimination of independent currencies acts as a commitment technology, removing the possibility of *perverse* adjustment of exchange rates following country specific shocks, whether the zero bound constraint on nominal interest rates is binding or not.

We present the argument in three stages. First we use a stylized 'canonical' two country New Keynesian model where countries may be subject to demand shocks arising from temporary changes in the rate of time preference (savings shocks). In the first case, monetary policy is governed by a simple Taylor rule, which applies so long as nominal interest rates are positive. In a multiple currency, flexible exchange rate version of the model, when the Taylor rule is operative, a country-specific savings shock elicits a compensating nominal and real exchange rate depreciation for the affected country. If, in the same circumstance, the region were governed by a single currency area, a real depreciation would require a relative domestic price deflation, which would be more costly and prolonged.

Now however assume that interest rates are constrained by the zero bound. In this case, the country experiencing the large savings shock will experience relative price deflation, pushing up its relative real interest rate[1], and generating a nominal and real exchange rate *appreciation* . This appreciation exacerbates the effect of the original shock. By contrast, the *relative* real interest rate and real exchange rate adjustment process under a single currency area is the same, whether or not the zero bound constraint applies. As a result, in a zero bound environment, adjustment to country-specific shocks is more efficient in a single currency area than under multiple currencies with flexible exchange rates. With flexible exchange rates, the endogenous movement in the exchange rate acts as a destabilizing mechanism at the zero lower bound.

We then extend this analysis to the case where monetary policy is chosen optimally in a cooperative framework, and some countries may not be constrained by the zero bound. Remarkably, we find that the same argument applies. That is, it may

[1]This response of real interest rates is very similar to those identified in the closed economy literature on the zero bound constraint (see in particular, Christiano et al 2011, and Eggertson, 2011).

be better to have a single currency area than a system of multiple currencies with flexible exchange rates, even when only one of the two countries is in a liquidity trap, and the other country follows an optimal monetary policy to maximize a weighted sum of each country's welfare. The logic here is in fact the same as in the previous case. While an optimal monetary policy can alleviate the impact of perverse movements in the exchange rate, it may still be better not to have had any exchange rate adjustment at all, when the affected country is at the zero bound.

Finally, we extend the model to allow for 'forward guidance' in monetary policy. Here, both countries have full commitment to determine the path of interest rates both during the life of the shock and after the expiry of the shock. In this case, we find that the traditional logic is restored. Optimal forward guidance can ensure that the country affected by the shock promises highly accommodative monetary policy in the future, after the shock ends, and if this promise is credible, it achieves an immediate contemporaneous movement of exchange rates in the right direction. By doing so, it can improve the adjustment process, compared with than in a single currency area. An optimal policy, with effect forward guidance, multiple currencies, and flexible exchange rates, is in general better than an equivalent policy under a single currency area.

Hence, a key message of the paper is that forward guidance is a particularly critical element in monetary policy making in open economies with flexible exchange rates, when the zero bound constraint is likely to be binding. By contrast, without absent effective forward guidance, a single currency area acts as an inbuilt commitment mechanism guaranteeing that a country pushed into a liquidity trap will experience future inflation, reducing the impact of the shock on current inflation. By contrast, with multiple currencies, flexible exchange rates, and no commitment, there is no such ability to guarantee future inflation for the affected country.

The paper is related to the recent literature on monetary and fiscal policy in a liquidity trap. In particular, with the experience of Japan in mind Krugman (1998), Eggertson and Woodford (2003, 2005), Jung, Teranashi, and Watanabe. (2005), Svensson (2003), Auerbach and Obstfeld (2004) and many other writers explore how monetary and fiscal policy could be usefully employed even when the authorities have no further room to reduce short term nominal interest rates. Recently, a number of authors have revived this literature in light of the very similar problems recently encountered by the economies of Western Europe and North America. Papers by Christiano, Eichenbaum and Rebelo (2009), Devereux (2010), Eggertson (2009), Cogan et al. (2008) have explored the possibility for using government spending expansions, tax cuts, and monetary policy when the economy is in a liquidity trap. Bodenstein, Erceg, and Guerrieri (2009) is an example of a fully specified two country DSGE model to examine the international transmission of standard business cycle shocks when one country is in a liquidity trap. In addition, Werning (2012) explores optimal monetary and fiscal policy in a continuous time model in face of zero lower bound constraints. Correa et al. (2012) explore a set of alternative fiscal instruments that can be used as a substitute for monetary policy in a zero lower bound situation.

The counterintuitive implications of the zero lower bound outlined in this paper parallel in part the surprising results that in a closed economy, some typically expansionary policies may be contractionary. An example is given of the contractionary effects of tax cuts in Eggertson (2010)

Some recent papers consider international dimensions of optimal policy in a liquidity trap. Jeanne (2009) examines whether either monetary policy or fiscal policy can implement an efficient equilibrium in a 'global liquidity trap' in a model of one-period ahead pricing similar to that of Krugman (1998). Fujiwara et al (2009) use numerical results to describe optimal monetary policy responses to asymmetric natural interest rate shocks. Nakajima (2008) and Fujiwara et al (2011) examine optimal policy responses to technology shocks in a model without home bias. Our model incorporates home bias in a way that implies that demand shocks require relative price changes. Cook and Devereux (2011) and Fujiwara and Ueda (2012) examine the fiscal policy multiplier in an open economy subject to the zero lower bound constraint. Farhi and Werning (2013) provide a general comparison of fiscal multipliers in a currency union both at and away from the zero lower bound. Finally, a more directly related paper is Cook and Devereux (2013), which looks at optimal monetary and fiscal policy in a flexible exchange rate version of a model similar to the one in the present paper.

The paper is structured as follows. The next section sets out the two country basic model. Section 3 shows some properties of the model solution. Then section 4 shows the main result of the paper in a simple setting with arbitrary monetary rules that may be constrained by the zero bound. Section 5 extends the argument to a situation where monetary policy is chosen optimally to maximize a weighted sum of each country's welfare, but again constrained by the zero lower bound, and without commitment. Section 6 extends the analysis to allow for monetary policy commitment. Some conclusions then follow.

2 A two country model

Take a standard two country New Keynesian model, denoting the countries as 'home' and 'foreign'. Utility of a representative infinitely lived home household evaluated from date 0 is:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t (U(C_t, \xi_t) - V(N_t)) \tag{1}$$

where U, and V represent the utility of the composite home consumption bundle C_t, and disutility of labor supply N_t. The variable ξ_t represents a shock to preferences or a 'demand' shock . We assume that $U_{12} > 0$. A positive ξ_t shock implies that agents become temporarily more anxious to consume today rather than the future. A negative ξ_t shock implies agents wish to defer consumption to the future, and so will increase their desired savings.

Composite consumption is defined as

$$C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}, \quad v \geq 1$$

where $\Phi = \left(\frac{v}{2}\right)^{\frac{v}{2}} \left(1 - \left(\frac{v}{2}\right)\right)^{\frac{v}{2}}$, C_H is the consumption of the home country composite good, and C_F is consumption of the foreign composite good. If $v > 1$ then there is a preference bias for domestic goods (home bias). Consumption aggregates C_H and C_F are composites, defined over a range of home and foreign differentiated goods, with elasticity of substitution θ between goods. Price indices for home and foreign consumption are:

$$P_H = \left[\int_0^1 P_H(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}, \qquad P_F = \left[\int_0^1 P_F(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}},$$

while the aggregate (CPI) price index for the home country is $P = P_{Ht}^{v/2} P_{Ft}^{1-v/2}$ and with identical 'home bias' for the foreign country, the foreign CPI is $P^* = P_F^{*v/2} P_H^{*1-v/2}$

Demand for each differentiated good ($j = H, F$) is

$$\frac{C_j(i)}{C_j} = \left(\frac{P_j(i)}{P_j}\right)^{-\theta}$$

The law of one price holds for each good so $P_j(i) = SP_j^*(i)$ where S_t is the nominal exchange rate (home price of foreign currency).

The household's implicit labor supply at nominal wage W_t is:

$$U_C(C_t, \xi_t) W_t = P_t V'(N_t). \tag{2}$$

Optimal risk sharing implies

$$U_C(C_t, \xi_t) = U_C(C_t^*, \xi_t^*) \frac{S_t P_t^*}{P_t} = U_C(C_t^*, \xi_t^*) T_t^{(v-1)}, \tag{3}$$

where $T = \frac{SP_F^*}{P_H}$ is the home country terms of trade.

Nominal bonds pay interest, R. Then the home consumption Euler equation is:

$$\frac{U_C(C_t, \xi_t)}{P_t} = \beta R_t E_t \frac{U_C(C_{t+1}, \xi_{t+1})}{P_{t+1}}. \tag{4}$$

Foreign household preferences and choices can be defined exactly symmetrically.

2.1 Firms

Each firm i employs labor to produce a differentiated good, so that its output is

$$Y_t(i) = N_t(i),$$

Profits are $\Pi_t(i) = P_{Ht}(i)Y_t(i) - W_t H_t(i)\frac{\theta-1}{\theta}$ indicating a subsidy financed by lump-sum taxation to eliminate steady state first order inefficiencies. Each firm re-sets its price according to Calvo pricing with probability $1-\kappa$. Firms that adjust set a new price given by $\widetilde{P}_{Ht}(i)$:

$$\widetilde{P}_{Ht}(i) = \frac{E_t \sum_{j=0} m_{t+j}\kappa^j \frac{W_{t+j}}{A_{t+j}} Y_{t+j}(i)}{E_t \sum_{j=0} m_{t+j}\kappa^j Y_{t+j}(i)}. \tag{5}$$

where the stochastic discount factor is $m_{t+j} = \frac{P_t}{U_C(C_t,\xi_t)} \frac{U_C(C_{t+j},\xi_{t+j})}{P_{t+j}}$. In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht} = [(1-\kappa)\widetilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{6}$$

The behaviour of foreign firms and the foreign good price index may be described analogously.

2.2 Market Clearing

Equilibrium in the market for good i is

$$Y_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} \left[\frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^*\right],$$

Aggregate market clearing in the home good is:

$$Y_{Ht} = \frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^*. \tag{7}$$

Here $Y_{Ht} = V_t^{-1} \int_0^1 Y_{Ht}(i)di$ is aggregate home country output, where we have defined $V_t = \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} di$. It follows that home country employment (employment for the representative home household) is given by $N_t = \int_0^1 N(i)di = Y_{Ht}V_t$.

An equilibrium in the world economy with positive nominal interest rates may be described by the equations (2), (4), (5), (6), and (7) for the home country, and the analogous equations for the foreign economy. Together with (3), and for given values of V_t and V_t^*, given monetary rules (to be discussed below), these equations determine an equilibrium sequence for the variables $C_t, C_t^*, W_t, W_t^*, S_t, P_{Ht}$, P_{Ft}^*, $\widetilde{P}_{Ht}$, $\widetilde{P}_{Ft}^*$, R_t, R_t^*, and N_t, N_t^*.

3 The Effects of Savings Shocks

Define $\sigma \equiv -\frac{U_{CC}\overline{C}}{U_C}$ as the inverse of the elasticity of intertemporal substitution in consumption, $\phi \equiv -\frac{V''\overline{H}}{V'}$ as the elasticity of the marginal disutility of hours worked, and we assume that $\sigma > 1$. In addition, $\varepsilon_t = \frac{U_{C\xi}}{U_C} \ln(\xi_t)$ is the measure of a positive demand shock in the home country, with an equivalent definition for the foreign country.

For this section and the next section, we make a simplifying assumption about the nature of preference shocks. We assume that the shock is unanticipated, and reverts back to zero with probability $1 - \mu$ in each period. This assumption implies that under independent monetary policy and flexible exchange rates, there are no predetermined state variables in the model. Hence, all endogenous variables in the world economy will inherit the same persistence as the shock itself, in expectation. Thus, for any endogenous variable x_t, we may write $E_t(x_{t+1}) = \mu x_t$. After the shock expires, all variables will then revert to their zero initial equilibrium. This property does not carry over to the single currency area, since in that case, the lagged terms of trade becomes an independent state variable (as shown below).

3.1 The World and Relative Economy

We derive a log-linear approximation of the model as in Clarida et al. (2002) and Engel (2010). Let $\widehat{x}_t$ be the percentage deviation of a given variable x_t from its non-stochastic steady state level. In the analysis below, each variable will be described in this way, except for the nominal interest rate and inflation rates, which are defined in levels, and ε_t, which is already defined in terms of deviation from the steady state value of zero. We define the term $D \equiv \sigma v(2-v) + (1-v)^2 > 1$. In addition , define $\zeta \equiv \frac{(v-1)}{D}$. The parameter, $0 \leq \zeta \leq 1$, measures the intensity of home bias. In the absence of home bias, $\zeta = 0$; under full home bias $\zeta = 1$.

We express the approximations in terms of world *averages* and world *relatives* . Thus, the world average (relative) value for variable x is given by $x^W = \frac{1}{2}(x + x^*)$ $(x^R = \frac{1}{2}(x - x^*))$.

From (3) and (7), as well as the equivalent condition for the foreign country, the partial solutions for the terms of trade and relative consumption are:

$$\widehat{\tau}_t = 2\frac{\sigma}{D}\widehat{y}_t^R - 2\zeta\varepsilon_t^R \tag{8}$$

$$\widehat{c}_t^R = 2\zeta\widehat{y}_t^R + \frac{2v(2-v)}{D}\varepsilon_t^R \tag{9}$$

Given relative income, and assuming $v > 1$, a negative ε_t shock, reduces relative demand for the home good, and generates a terms of trade deterioration. A rise in relative income also generates a terms of trade deterioration, and leads to a rise in relative home consumption.

Using these conditions in combination with a linear approximation of (2), (4), (5), and (6), we can derive the following forward looking inflation equations and open economy IS relationships, expressed in terms of world averages and world relatives. It is convenient to write this system in 'gap' terms, where we define the variable $\widetilde{x} = x - \bar{x}$ as the gap between the log of a variable and the log of its flexible price analogue $(\bar{x})$. The only exceptions are for inflation, which is written in logs, since its flexible price value is zero, and the nominal interest rate, which is expressed in levels[2]. The world average equations are:

$$\pi_t^W = k((\phi + \sigma)\widetilde{y}_t^W) + \beta E_t \pi_{t+1}^W \tag{10}$$

$$\sigma E_t(\widetilde{y}_{t+1}^W - \widetilde{y}_t^W) = r_t^W - E_t \pi_{t+1}^W - \bar{r}_t^W \tag{11}$$

The coefficient k depends on the degree of price rigidity. The term $\bar{r}_t^W$ is the world average interest rate that would apply in a flexible price equilibrium, which we term the 'natural' interest rate. In the Appendix we show that $\bar{r}_t^W$ is expressed as:

$$\bar{r}_t^W = \rho + \frac{(1-\mu)\phi}{\sigma + \phi}\varepsilon_t^W. \tag{12}$$

[2]Note that $\pi^W = 0.5(\pi + \pi^*)$, where π and π^* are the home and foreign PPI inflation rates.

A negative shock to ε_t^W leads to a fall in the world natural interest rate when $\mu < 1$.

Now let $\sigma_D \equiv \frac{\sigma}{D}$,where $\sigma \geq \sigma_D \geq 1$. Then the world 'relative' equations are written as:

$$\pi_t^R = k((\phi + \sigma_D)\widetilde{y}_t^R + \beta E_t \pi_{t+1}^R \tag{13}$$

$$\sigma_D E_t(\widetilde{y}_{t+1}^R - \widetilde{y}_t^R) = r_t^R - E_t \pi_{t+1}^R - \bar{r}_t^W \tag{14}$$

where $\bar{r}_t^R$ represents the natural world relative interest rate, defined in the Appendix as:

$$\bar{r}_t^R = \frac{(1-\mu)\zeta\phi}{\sigma_D + \phi}\varepsilon_t^W. \tag{15}$$

A negative ε_t^R shock reduces the world relative natural interest rate when $\mu < 1$ and when there is home bias in the consumption bundle.

Equations (10)-(11) and (13)-(14) describe the response of the world economy to the savings shock through the world average inflation rate and output gap movements, and the world relative inflation rate and output gap movements. Note that the degree of home bias does not affect aggregate average outcomes, so that there is a dichotomy in the solution of aggregate and relative models. The effect of home bias on relative outcomes is summarized by the two parameters ζ and σ_D [3].

The solutions to (10)-(11) and (13)-(14) will depend on the rules for monetary policy, captured by the world average and relative nominal interest rates, given by r_t^W and r_t^R. We now turn to this question.

4 The Simplest Case: Comparing Interest Rate Rules with a Zero Bound Constraint

4.1 Separate Currencies with Independent Monetary Policies

We begin with a simple case to show the essence of the argument. Assume that outside of the zero lower bound, monetary policy is characterized by an interest rate

[3]The first parameter measures the size of home bias, and represents the direct impact of relative demand shocks on relative inflation and output. When home bias is absent, $(v-1) = \zeta = 0$, and relative demand shocks have no impact on relative demand allocations. The second parameter, σ_D, the intertemporal elasticity of relative demand, governs how intensely relative demand responds to adjustments in the relative interest rates. In the presence of home bias, $\sigma_D < \sigma$, so relative demand responds more to interest rate changes than average demand, since relative interest rate movements result in real exchange rate adjustments and expenditure switching across countries.

rule[4] . Under separate currencies, each country sets its own interest rate. We assume a simple Taylor rule described (for the home economy) as:

$$r_t = \rho + \gamma\pi_t \tag{16}$$

Here, monetary policy targets the rate of PPI inflation. With separate currencies, using (16) and the analogous foreign condition we have $r_t^W = \rho + \gamma\pi_t^W$, and $r_t^R = \gamma\pi_t^R$. Combining these two expressions with (10)-(14), we can derive the solutions for the four variables π^W, y^W, π^R, and y^R. Then using (8) and (9) we can obtain solutions for the terms of trade and relative consumption. The nominal exchange rate, s_t, may be then obtained from the condition

$$s_t - s_{t-1} = \pi_t^R + \tau_t - \tau_{t-1}. \tag{17}$$

In this example, the solution for world averages is the same under multiple currencies or a single currency area. We therefore focus only on the characteristics of world relatives. Also, with multiple currencies, under the assumed stochastic characteristics of the ε_t shock, the model is entirely stationary; there are no predetermined state variables, and all endogenous variables take on the persistence characteristics of the ε_t shock. From (13), we can then describe a relationship between relative PPI inflation and the relative output gap as:

$$\pi_t^R = \frac{k(\phi + \sigma_D)}{(1 - \beta\mu)} \widetilde{y_t}^R \tag{18}$$

A rise in the relative home output gap leads to a rise in relative home country inflation.

Likewise, from (14), we obtain a relationship between relative inflation and the output gap, conditional on the relative natural interest rate $\bar{r}_t^R$. We have to be careful here however, since (14) depends on the policy rule, and we want to take account of the possibility that the policy rule may be constrained by the zero lower bound. Using the definitions of world averages and relatives, this implies that we impose the conditions:

$$r_t = r_t^W + r_t^R = \text{Max}(0, \rho + \gamma\pi_t) \tag{19}$$

$$r_t^* = r_t^W - r_t^R = \text{Max}(0, \rho + \gamma\pi_t^*) \tag{20}$$

Assume first that neither condition is binding. Then we can substitute $r_t^R = \gamma\pi_t^R$ into (14), substituting also for $\bar{r_t}^R$ and take expectations, obtaining:

$$\pi_t^R = -\frac{(1-\mu)}{(\gamma - \mu)} \left(\sigma_D \widetilde{y_t}^R - \frac{\zeta\phi}{\sigma_D + \phi} \varepsilon_t^R \right) \tag{21}$$

[4] In section 5 and 6 below, we allow for monetary policy to be set optimally.

Figure 1 illustrates the determination of relative inflation and output gaps when monetary policy is not constrained by the zero bound. Equation (18) is upward sloping in π^R and $\widetilde{y}^R$ space, and (21) is downward sloping, since a rise in relative output, which is temporary, leads to a fall in the natural real interest rate, and under the monetary rule (16) relative inflation must fall so as to accommodate this.

Now we see the impact of a relative savings shock, given by a negative ε_t^R. For $\upsilon > 1$, this shifts (21) to the left, leading to a fall in both inflation and the output gap.

Given independent monetary policies and the monetary rule (16), the fall in relative inflation is associated with exchange rate and terms of trade adjustment. From the Euler equations for bond pricing (4), expressed in relative terms, combined with the risk sharing condition (3), we obtain the following relationship between relative inflation and the terms of trade:

$$\gamma\pi_t^R = E_t(\pi_{t+1}^R + \tau_{t+1} - \tau_t) \tag{22}$$

The right hand side of this equation is simply the expected change in the nominal exchange rate, so that (22) is just the uncovered interest rate parity condition. Imposing the stationarity condition gives us the solution for the terms of trade:

$$\tau_t = -\frac{\gamma - \mu}{1 - \mu}\pi_t^R \tag{23}$$

Since $\gamma > \mu$, the fall in inflation implies a terms of trade deterioration. In response to the savings shock, home relative inflation falls, so that when $\gamma > \mu$, the home relative interest rate falls, which facilitates a terms of trade deterioriation. We also get a nominal exchange rate depreciation, since from (17) and (23) we have

$$s_t - s_{t-1} = -\frac{\gamma - 1}{1 - \mu}\pi_t^R$$

The full solution for the terms of trade can be derived as:

$$\widehat{\tau} = \frac{-k\phi(\gamma - \mu)}{\sigma_D(1 - \beta\mu)(1 - \mu) + (\gamma - \mu)k(\sigma_D + \phi)}2\zeta\varepsilon_t^R \tag{24}$$

Now contrast this response to that constrained by the zero interest bound. In this case, nominal interest rates are equal, and zero, in both the home and foreign country. Then (14) must reflect this additional constraint. Again, imposing stationarity, we derive the relationship:

$$\pi_t = -\frac{1 - \mu}{\mu}\left(\sigma_D\widetilde{y}_t^R - \frac{\zeta\phi}{\sigma_D + \phi}\varepsilon_t^R\right) \tag{25}$$

This is a upward sloping locus in π^R-$\widetilde{y}^R$ space. A rise in current relative inflation raises anticipated future relative inflation, when $\mu > 0$. This reduces the home relative real interest rate, and raises the relative home output gap [5]. We further make the assumption that $\frac{1-\mu}{\mu}\sigma_D > k(\phi + \sigma_D)1 - \beta\mu$, so that the slope of (25) exceeds that of (18).

Now look at the result of a relative savings shock where the zero bound constraint binds. Figure 2 shows that this again leads to a fall in relative home inflation and the relative home output gap. In contrast to the case where (16) applies, at the zero bound, the fall in relative inflation tends to magnify the fall in relative output. The fall in relative inflation causes a rise in home relative real interest rates, which causes a secondary fall in home relative demand. So long as the above stability condition holds, this process converges at a lower relative inflation and relative output gap.

What does this imply for the exchange rate and the terms of trade? Note that, although the zero bound is binding in both countries, the arbitrage conditions (3) and (4) still apply. This means that (22) still holds, but with the left hand side equal to zero. Under that stationarity condition, then we have

$$\tau = \frac{\mu}{1-\mu}\pi_t^R \tag{26}$$

Then, the terms of trade must *appreciate*. The full solution is written as:

$$\widehat{\tau} = \frac{k\phi\mu}{\sigma_D(1-\beta\mu)(1-\mu) + (\gamma-\mu)k(\sigma_D+\phi)}2\zeta\varepsilon_t^R$$

The difference with the previous case is that the fall in relative home inflation leads to a rise in the home relative real interest rate, which leads to an appreciation in the terms of trade. From (17) and (26), there is also a nominal exchange rate *appreciation*. The initial impact of the shock on the nominal exchange rate satisfies: [6]

$$s_t - s_{t-1} = \frac{1}{1-\mu}\pi_t^R$$

This is the essence of the argument over the merits of flexible exchange rates at the zero bound. Because movement in relative inflation lead to perverse movements

[5] Note that it as in Eggertson and Woodford, (2003), it is critical for this argument that the liquidity trap will expire in expectation. This ensures that the current inflation rate is pinned down by the expectation that prices and inflation will be uniquely determined along a stable manifold in the future.

[6] Because nominal interest rates are equal, the effect of the shock is to cause a one-time appreciation in the nominal exchange rate, and the subsequent anticipated movement in the exchange rate is zero, so that uncovered interest rate parity is still satisfied.

in relative real interest rates at the zero bound, relative prices move in the 'wrong direction'. The appreciation in the terms of trade exacerbates rather than ameliorates the impact of the initial relative savings shock.

4.2 Single Currency Area

Now assume that both countries are part of a single currency area. To investigate this, we can use the results of Benigno (2004). In a single currency area monetary policy for the whole region is governed by the condition $r_t^W = \rho + \gamma \pi_t^W$, and by definition, both countries face the same nominal interest rate, so that $r_t^R \equiv 0$. At first glance, it looks as if the solution should be the same as the multiple currency case under the zero lower bound. But this is incorrect, since as shown in Benigno (2004), without an endogenous nominal exchange rate, there is another initial condition imposed on the dynamics of the terms of trade in a single currency area. This is simply given by (17), but setting the left hand side to zero so that

$$\tau_t = \tau_{t-1} - \pi_t^R \tag{27}$$

Since there is only one nominal interest rate, the relative interest rate equations for nominal bond rates do not impose any additional constraints on the model. But we can combine (8) (rewritten in 'gap' terms) with (27) to obtain a separate relationship between inflation and the output gap implied by the single currency area. This is:

$$\pi_t^R = -2\left(\sigma_D \widetilde{y}_t^R - \frac{\zeta\phi}{\sigma_D + \phi}\varepsilon_t^R\right) + 2\left(\sigma_D \widetilde{y}_{t-1}^R - \frac{\zeta\phi}{\sigma_D + \phi}\varepsilon_{t-1}^R\right) \tag{28}$$

This replaces (21) as an equilibrium condition under the single currency area. Note that it doesn't contain any parameters relevant to the monetary rule, and also, it is a dynamic equation; movements in relative inflation and output gaps won't satisfy the same stationarity characteristics as those under the multiple currency regime. The first point is obvious - there is only an aggregate monetary policy in the single currency area, and relative inflation is independent of the area-wide policy rule (under the symmetry assumptions we've made so far). The second follows since in a single currency area, the terms of trade can change only due to movements in domestic prices indices that occur gradually. This becomes an important distinction between the single currency area and the multiple currency regime at the zero lower bound - the dynamic properties of a single currency area ensure that the impact of the shock does not dissipate immediately after the shock expires. As we will see below, this gives a 'proxy' commitment aspect to the single currency area that doesn't exist naturally under the multiple currency regime.

We may combine (18) and (28) as the equations determining the dynamics of relative inflation and the relative output gap following a savings shock. But before that, it's worth noting that while (28) gives a negative relationship between relative inflation and the relative output gap much like (21), the slope of the relationship is larger in the case of (28), since $2 > \frac{1-\mu}{\gamma-\mu}$. This implies that in 'normal' times, when the zero bound is not binding, a fall in relative inflation in a multiple currency area has a greater stabilizing effect on the output gap than in a single currency area. Under flexible exchange rates, a fall in relative inflation precipitates a fall in the home relative interest rate, and an exchange rate depreciation for the home economy. This can't happen in a single currency area.

Combining (18) and (28), we may solve for the dynamics of inflation. The solution is given by:

$$\pi_t^R = \lambda \pi_{t-1}^R - \chi 2(\varepsilon^R - \varepsilon_{t-1}^R) \tag{29}$$

where $0 < \lambda < 1$, and $\chi = -\frac{k}{2}\zeta\frac{\phi}{\Delta_{D1}} < 0$, with $\Delta_{D1} \equiv \sigma_D(1 - \beta\lambda + \beta(1-\mu)) + \frac{k}{2}(\phi + \sigma_D)$. The immediate effect of a savings shock is to reduce relative home country inflation. With a single currency area, this causes a terms of trade depreciation. The solution for the terms of trade is given as:

$$\widehat{\tau}_t = \lambda \widehat{\tau}_{t-1} + \chi 2\varepsilon^R \tag{30}$$

4.3 Comparison

Let"s now compare the properties of the flexible exchange rate system with the single currency area. First focus on the case where the Taylor rule applies and the zero bound constraint is not binding. To compare the responses across the two regimes, we perform a simple numerical simulation. We make the following calibration assumptions; $\beta = 0.99$, $k = 0.05$, $\sigma = 2$, $\phi = 1$, and $v = 1.5$. In addition, we assume that there is a home country savings shock which persists with probability $\mu = 0.7$. Finally, assume a monetary rule $\gamma = 3$. The Figure illustrates the responses of π^R, y^R, and τ, following the shock $\varepsilon = -0.5$[7]

With flexible exchange rates, relative inflation and relative output fall following the savings shock. The terms of trade depreciates sharply. By reducing the relative

[7]We follow this calibration everywhere below except when stated otherwise. Figure 3 illustrates the impact effect, and the *expected* response of each variable following the shock. Since the shock continues with probability μ, then under the flexible exchange rate the ex-post response is constant, so long as the shock continues.

price of the home good, this acts to limit the fall in relative inflation and relative output. Under the single currency area, the terms of trade also depreciates, but this is muted and gradual. In order to achieve this depreciation, relative domestic inflation must fall much more sharply than under flexible exchange rates. At the same time, relative output falls by more under the single currency area.

Figure 3 illustrates the traditional conclusions of the merits of multiple currencies and a flexible exchange rate in comparison with a single currency area. A country-specific demand shock requires an adjustment in relative prices. It is better to facilitate this adjustment with changes in the nominal exchange rate. A single currency area, by definition, can't achieve any nominal exchange rate adjustment. While the single currency has no consequences for overall world aggregates, it leads to an excessive volatility in relative output, and insufficient flexibility in relative prices.

Now look at the comparison when the zero bound constraint is binding. Figure 4 shows the effect of a negative demand shock, but now assuming that both countries are constrained by the zero lower bound. The single currency area responses are the same as Figure 3, since responses of relative inflation and output gaps in the single currency area don't depend on the monetary rule. But under flexible exchange rates, the terms of trade appreciates sharply, and there is a large fall in relative inflation, much larger for the flexible exchange rate than under the single currency area. This then leads to a dramatic reversal in the comparison between flexible exchange rates and a single currency when the zero bound constraint is binding. Relative output falls by substantially more with flexible exchange rates than the single currency area.

We conclude that when the zero lower bound is binding, flexible exchange rates do not act so as to stabilize the response to country specific shocks, and in fact impart greater relative macro instability than would exist under a single currency area. With flexible exchange rates, the exchange rate response compounds the original shock. In a single currency area, relative inflation rates move slowly, but move in a stabilizing fashion.

As we show below, a critical feature of this comparison is that monetary policy contains no commitment to future actions. The interest rate rule (16) implies that when the shock expires, both countries will return to a zero inflation steady state. Then monetary policy cannot be relatively more expansionary in the home country, either at the time of the shock, or after the expiry of the shock. Thus, there is no policy response that can ensure the exchange rate moves in a stabilizing direction. By contrast, paradoxically, the single currency area has an inbuilt commitment, because relative prices can only change through domestic inflation, and even after the shock expires, the relative price of the home good will continue to be lower, as the terms of trade adjusts back to the steady state. This point is made clearer in Section 6 below.

5 Extension to Optimal Monetary Policy

In the previous section we employed an arbitrary interest rate rule, and assumed that the zero lower bound constraint was binding in both countries or not at all. We now extend the argument to allow for the possibility that one country may not be constrained by the zero bound, and instead sets monetary policy optimally, assuming that central banks cooperate to maximize welfare. Remarkably, we find that even in this case, the single currency area may outperform the multiple currency case, when the zero bound is binding. That is, a single currency area, in which the whole region is at the zero bound, may dominate a multiple currency flexible exchange rate system, even when only one country is at the zero bound in the latter case. For now, we continue to assume that there can be no policy commitment; we characterize an optimal policy under discretion.

Optimal policy with flexible exchange rates will clearly differ from that for the single currency area, since in the former case there is potentially an additional policy instrument. But the notable result that we will show here is that the optimal setting even for the *world average interest rate* may differ between the multiple currency case and the single currency area.

Again, assume that the savings shock comes exclusively from the home country, so that $\varepsilon_t < 0$ and $\varepsilon_t^* = 0$, which obviously implies that $\varepsilon_t^W < 0$ and $\varepsilon_t^R < 0$. In addition, assume that the shock from the home country is large enough so that the world average natural interest rate is negative; i.e. $\bar{r}_t^W = \rho + \frac{(1-\mu)\phi}{\sigma+\phi}\varepsilon_t^W < 0$. From (12), then, the home natural interest rate, $\bar{r}_t = \bar{r}_t^W + \bar{r}_t^R$, must be negative, but the foreign natural interest rate $\bar{r}_t^* = \bar{r}_t^W - \bar{r}_t^R$, will depend on the degree of home bias v. For $v = 1$, the natural interest rates are identical, while for $v = 2$, the foreign natural interest rate is equal to ρ, and unaffected by the home country shock. Figure 5 shows that for a negative home country shock $\varepsilon_t < 0$, there is a critical value of v, denoted $\bar{v}$, such that for $v < \bar{v}$, $(v \geq \bar{v})$, the foreign natural interest rate is negative (positive).

First, we discuss the optimal policy with multiple currencies. Asecond order approximation to global welfare, in each period, for the model set out in section 2 may be written as[8]:

$$V_t = -(\widehat{y}_t^R)^2 \cdot \frac{\sigma_D + \phi}{2} - (\widehat{y}_t^W)^2 \cdot \frac{\sigma + \phi}{2} - \frac{\theta}{4k}(\pi_t^W + \pi_t^R)^2 - \frac{\theta}{4k}(\pi_t^W - \pi_t^R)^2 \quad (31)$$

Thus, the social welfare function faced by the policy maker depends upon average and relative world output gaps and inflation rates.

Under multiple currencies, the cooperative optimal monetary policy under discretion involves maximizing (31) subject to (10)-(11) and (13)-(14), taking as given the expected future values of all variables, as well as the non-negativity constraints on nominal interest rates. The Appendix derives the full solution. Here we give an intuitive description of the solution for home and foreign interest rates, following Cook and Devereux (2013). Define Ψ and Ψ^* respectively as the multiplier on the non-negativity constraint for the home and foreign interest rate. Thus, the optimal solution must have the property that $\Psi \geq 0$, $r_t \geq 0$, and $\Psi \cdot r_t = 0$, and similarly for the foreign multiplier and foreign interest rate. Then, in the Appendix, we show that the optimal cooperative policy under discretion and multiple currencies requires that the following two conditions be satisfied:

$$\Omega_D(r_t^R - \bar{r}_t^R) = \Psi_t - \Psi_t^* \quad (32)$$

$$\Omega(r_t^W - \bar{r}_t^W) = \Psi_t + \Psi_t^* \quad (33)$$

where Ω_D and Ω are composite terms which have the following property; $\Omega_D < \Omega$ for $1 < v \leq 2$, $\Omega_D = \Omega$ for $v = 2$ [9]. From (32) and (33) we can show that, under the assumptions $\varepsilon_t < 0$ $\varepsilon_t^* = 0$ and $\bar{r}_t^W < 0$, the solution has the following properties; a) the home country is always constrained by the zero bound, so that $r_t = 0$, b) there exists a critical value $\hat{v} < \bar{v}$, such that for $1 \leq v \leq \hat{v}$, $r_t^* = 0$, while for $\hat{v} < v < 1$, the foreign interest rate solution is:

$$r_t^* = \bar{r}_t^* + \frac{\Omega_D - \Omega}{\Omega_D + \Omega}\bar{r}_t \quad (34)$$

[8] See Cook and Devereux 2013.

[9] The exact expressions are

$$\Omega = \frac{\sigma + \phi}{\sigma} \frac{(1 - \beta\mu) + k\theta(\sigma + \phi)}{\sigma(1 - \beta\mu)(1 - \mu) - k\mu(\sigma + \mu)},$$

$$\Omega_D = \frac{\sigma_D + \phi}{\sigma_D} \frac{(1 - \beta\mu) + k\theta(\sigma_D + \phi)}{\sigma_D(1 - \beta\mu)(1 - \mu) - k\mu(\sigma_D + \mu)}$$

.

Thus, for v sufficiently greater than unity, the foreign country will choose to set its interest rate above zero, even if the world natural interest rate is negative. Moreover, because the second expression on the right hand side of (34) is positive, the foreign country may set a positive interest rate, even if its own natural interest rate is below zero.

The intuition behind this result is that in a cooperative optimal monetary policy outcome, under multiple currencies and the zero bound constraint applying to the home country, it may be optimal for the foreign country to raise policy rates in order to reduce the appreciation of the home exchange rate. This limits the deflation and fall in output in the home country. At the same time, for v sufficiently high, the home terms of trade appreciation may generate inflation and a positive foreign output gap, so an increase in the foreign policy rate may also be warranted on that account.

This solution may be expressed in a different way. Note that the average world interest rate is $r_t^W = \frac{r_t + r_t^*}{2}$, so we may characterize the behaviour of the average world interest rate under multiple currencies and flexible exchange rates as:

$$r_t^{W,mc} = \max(0, \bar{r}_t^W - \frac{\Omega}{\Omega_D + \Omega}\bar{r}_t) \tag{35}$$

Under the single currency area, the cooperative optimal monetary policy chooses only a single world interest rate subject to (10) and (11), and the non-negativity constraint. This policy differs from the multiple currency area case only in that it ignores the path of relative world output and relative world inflation, focusing only on the optimal path of world averages. It is immediate that the solution will be:

$$r_t^{W,sca} = \max(0, \bar{r}_t^W) \tag{36}$$

Comparing (35) and (36) we see that under optimal monetary policy, average policy interest rates will be higher in a multiple currency area than in a single currency area. This is because under the multiple currency area it may be optimal to raise foreign policy rates, even though the world natural rate is negative, in order to reduce the appreciation of the home country terms of trade.

Figure 6 illustrates the response of average and relative world output, inflation, and the terms of trade under optimal monetary policy, in the multiple currency case, and the single currency area. We choose the same parameter and shocks as before, assuming that the world natural interest rate falls below zero. The home bias parameter v is set so that $v > \hat{v}$, and the optimal response of the foreign country is to set a positive policy interest rate. Then, since average world interest rates are higher under multiple currencies than in the single currency area (zero), average world output and inflation falls by more in the response to the savings shock. Thus, due to the perverse response of the exchange rate under a multiple currency regime, at the zero lower bound, *overall world output* will fall by more with multiple currencies and flexible exchange rates than under a single currency area, *even when only the home country is constrained by the zero lower bound*. Figure 6 likewise shows that relative output and inflation also falls by more in the multiple currency case than in the single currency area. Finally, even under the optimal policy rule, the terms of trade still appreciates under the multiple currency case, while as before, the terms of trade in the single currency area depreciates [10].

These results indicate that, even when policy is set optimally and only one country in a multiple currency area is constrained by the zero lower bound, there is a perverse response of relative prices and output gaps under flexible exchange rates. How does this comparison translate into welfare terms? We can construct a welfare comparison by computing the loss associated with a savings shock using the welfare function (31). To do this, we take the expected loss following a shock which follows the persistence properties described above. In particular we assume again that the shock such that the world natural interest rate is negative, and the shock persists with probability μ in each period in the future. Welfare in each regime is constructed by computing the discounted expected value of losses starting from the period of the shock. In the multiple currency case, expected welfare is constant in each future period, since the shock is either the same, with probability μ, or zero, with probability $1 - \mu$, and if it is zero, all future gaps are closed. In the single currency area, welfare evolves over time, as the terms of trade gradually adjusts to the shock, and welfare doesn't go to zero after the shock expires, since the terms of trade continues to be away from its steady state level, and only gradually converges back to steady state.

Figure 7 illustrates the welfare comparison for different values of the persistence parameter μ, and for two different values of v. The Figure illustrates the value of

$$\frac{|\mathrm{V(mc)}|}{|\mathrm{V(sca)}|} - 1$$

where V(mc) (VL(sca)) is the welfare function under multiple currencies (single currency area). For $v = 1$, there is no difference between the two measures. For $v > 1$, the loss from the multiple currency area increases, relative to that under the single currency area, as μ rises. The welfare comparison between the multiple currency case and the single currency area involves a trade-off. On the one hand, under multiple currencies, the savings shock leads the home terms of trade to appreciate, and

[10]This is not necessarily the case. In section 6 below, we show that in an optimal discretionary equilibrium with relatively transitory shocks, the home terms of trade under multiple currencies may depreciate, but does so at a rate less than would take place in the single currency area.

relative output is destabilized during the period of the shock. But under a single currency area, the shock causes a persistent movement in the terms of trade, relative output, and inflation rates, even after the shock ends. For low values of μ, the welfare effect of the second factor dominates, and the multiple currency area, under the optimal choice of monetary policy, is preferable. But for high μ, the first factor becomes more important, and the multiple currency area is worse in welfare terms, *even if monetary policy is chosen optimally.* This is clear from Figure 6 which shows the expected values of the home and foreign interest rate[11], where the calibration for this Figure involves $v = 1.5$ and $\mu = 0.7$. Hence, for this case, the foreign country is actively adjusting its interest rate. Despite this, welfare is higher in the single currency area.

6 The Need for Forward Guidance

A key aspect of the comparison so far has been that monetary policy is myopic; we have allowed no possibility for policy-makers to commit to future actions. In general, the literature on the zero lower bound in closed economy settings has stressed the benefits of *forward guidance.* As shown by Krugman (1999), Eggertson and Woodford (1983), and Jung et al. (2005), with full commitment, an optimal monetary policy can significantly alleviate the consequences of the zero lower bound constraint. This is done by promising to follow, in the future, after the conditions leading to the zero bound have elapsed, a more expansionary policy than would otherwise be appropriate for the economy's conditions at that time. In Eggertson and Woodford (2003) and Jung et al. (2005), this involves keeping a zero interest rate policy for a period of time *after* the shock which drives the natural interest rate below zero has disappeared.

We now extend the analysis to allow for commitment in monetary policy. Again, we focus on cooperative optimal monetary policy, but assume that policymakers can choose a path of interest rates for each country (in the multiple currency case) or a world interest rate (in the single currency case) that will hold for current and future periods, subject to the zero bound constraint. To simplify the analysis, we focus on a special case of perfect foresight, where the preference shock is known to last a fixed number of periods. Specifically, assume that at time $t = 1$, there is a shock to home preferences $\varepsilon < 0$, which drives world natural interest rates below zero, and further, it is known at time 1 that the shock will last exactly T periods. Thus, the shock leads to a fall in the world natural interest rate below zero for T periods. Then, in both the single currency case and the case of multiple currencies, an optimal monetary

[11] Note that the *actual* value of the home policy rate is zero so long as the savings shock continues.

policy at time $t = 1$ is characterized by a path of interest rates for all $t \geq 1$.

We focus on a case where $v \geq \hat{v}$, which implies that in the equilibrium of the optimal discretionary monetary policy, the foreign country would set a positive interest rate. In this case, we find that it is also optimal for the foreign country to pursue a positive interest rate in the optimal policy problem with commitment. So again, we're focusing on a case where only the home country is currently constrained by the zero bound.

In the multiple currency case, an optimal policy with commitment involves a path of interest rates r_t^W and r_t^R for $t \geq 1$ to minimize the discounted sum of losses given by:

$$\begin{aligned} V_0 &= -\sum_{t=0}^{\infty} \beta^t \left[(\widehat{y}_t^R)^2 \cdot \frac{\sigma_D + \phi}{2} + (\widehat{y}_t^W)^2 \cdot \frac{\sigma + \phi}{2} \right] \\ &\quad - \sum_{t=0}^{\infty} \beta^t \left[\frac{\theta}{4k} (\pi_t^W + \pi_t^R)^2 + \frac{\theta}{4k} (\pi_t^W - \pi_t^R)^2 \right] \end{aligned} \tag{37}$$

subject to (10)-(11) and (13)-(14) for each period t, as well as the non-negativity constraints on national nominal interest rates. In the single currency area case, the optimal policy involves a choice of the path of r_t^W for $t \geq 1$ to minimize (37) subject to (10)-(11) and the non-negativity constraint on r_t^W.

The first order conditions for the optimal policy with commitment are quite familiar from previous literature. The conditions are described fully in the Appendix. Here we illustrate the results and comparisons in Figures 8-11. The Figure are based on the assumption that at time $t = 1$, there is an unanticipated savings shock in the home country equal to $\varepsilon = -0.5$, which is known to persist for three periods (so that $T = 3$). After that $\varepsilon = 0$. The Figures use the calibration used in Section 5, and assume that $v = 1.5$.

For comparison, Figures 8 and 9 illustrate the case of discretionary monetary policy, under multiple currencies and the single currency area. The qualitative features are the same as in Section 5. Under multiple currencies, there is no persistence beyond $t = 3$. In response to the savings shock, the home country policy rate is stuck at the zero bound for 3 periods only, while as shown in the previous section for relatively high values of v, the foreign policy rate is positive. Relative inflation is negative for three periods. In the single currency area, the world interest rate is at the zero bound for three periods, but does not converge immediately to the steady state natural rate. This is because the home terms of trade ends the third period above its steady state, and the home country must experience some relative inflation in order to converge back to steady state. This is achieved by having a world interest rate lower than the steady state, which, in conjunction with a home country terms of trade above its steady state, facilitates more inflation in the home country. This mechanism illustrates the in-built commitment dynamic of the single currency area - producing relative home country inflation after the expiry of the shock, even in a discretionary equilibrium.

Figures 10 and 11 now focus on the commitment equilibrium under the multiple currency case and the single currency area. Under commitment, the policymaker can choose the whole future path of interest rates so as to produce the desired outcome, announcing interest rates to hold even after the shock elapses. We see that under the multiple currency area, there is a dramatic difference from the discretionary outcome. In particular, the home country keeps its policy rate at zero for an additional two periods, even after the shock expires. Moreover, the foreign country's interest rate, while still rising immediately following the shock as in the case of discretion, converges to steady state only gradually. This conjunction of policy announcements sharply reduces the deflation experienced in the home country, and as a result, the home terms of trade experiences an immediate depreciation, as would occur under the outcome outside of the zero bound. The fall in world average output and world relative output is now much smaller than under discretion. More importantly, the outcomes under multiple currencies with full commitment are better than those under the single currency area with commitment, which is illustrated in Figure 11. This Figure shows that, with full commitment, the single currency area policy rate remains at zero for one period after the shock expires. The home terms of trade depreciates, but by less than that under multiple currencies with full commitment. The movement in the average world interest rate means that average world output falls by about the same amount as under multiple currency areas, but the inability to fully adjust the home terms of trade leads to a greater fall in relative world output. Thus, with full commitment, the macro outcomes under the multiple currency area, even constrained by the zero lower bound, seem to dominate those of the single currency area.

The substance of these results indicate that in exploring the benefits of multiple currencies and flexible exchange rates in an environment constrained by the zero bound, it is critical to have well functioning forward guidance as part of the policy toolkit. Table 1 makes this clear in terms of welfare evaluation.

The Table reports the discounted sum of losses under discretion and commitment, using an optimal policy in each case, for the multiple currency area and the single currency case, for the shock process and the calibration example described in the previous paragraphs. For the discretionary case, the single currency area still dominates, as implied by the previous section. But with full policy commitment, the

Table 1: Welfare Comparison

	SCA	Float
Discretion	-0.007	-0.017
Commitment	-0.0039	-0.002

Notes: Compares present value of welfare under optimal policy under discretionary and commitment policies under each regime

traditional result applies - when policy can effectively employ forward guidance, the welfare benefits of multiple currencies and flexible exchange rates re-emerge.

7 Conclusions

A growing recent literature has demonstrated that conventional responses to macroeconomic shocks can be substantially different when monetary policy is constrained by the zero bound on nominal interest rates (see e.g. Eggertson 2011, 2012, Cook and Devereux, 2013). This present paper extends that literature by showing that the conventional reasoning on the benefits of flexible exchange rates and the costs of a single currency area can be reversed in a situation of a liquidity trap. When monetary policy is ineffective, the conventional response of the exchange rate to aggregate demand shocks may be reversed, and the exchange rate exacerbates rather than ameliorates economic instability.

Our paper focuses on the role of the exchange rate as an equilibrating mechanism in a situation with country-specific shocks. Some recent literature argues that in a single currency area, there will exist tax-subsidy policies that can substitute for the absent exchange rate variation. Fahri et al. (2012) describe how a mix of tax and subsidies can achieve *Fiscal Devaluation* in a small economy, exactly replicating the effects of a nominal exchange rate devaluation. Therefore, if fiscal policy is sufficiently flexible, it can completely eliminate the loss of monetary autonomy implied by a fixed exchange rate regime. More generally, it has been established by Correa et al. (2012) that a combination of state-contingent taxes and subsidies can undo the effects of the zero bound and fully replicate the flexible price equilibrium in standard New Keynesian models. In the Appendix, we see how these results extend to our setting. We show that a combination of VAT adjustment and payroll tax changes can be used to ensure price stability and zero output gaps, achieving the fully optimal flexible price equilibrium in face of shocks which would drive the world interest rate in a single currency area, or the home interest rate in a multiple currency area, to the zero bound. But the key finding is that when monetary policy is constrained by the zero bound, fiscal adjustment will be required even in a situation of flexible exchange rates. We find that a fall in average and relative VAT taxes combined with a rise in average and relative payroll taxes achieves the first best outcome, leaving all gaps and inflation rates equal to zero. But the main result we show is that the tax-subsidy policy is the same for the multiple currency case and the single currency area. Hence, the key need for the optimal fiscal response is not the inability of exchange rates to adjust, but the zero lower bound constraint itself.

It is important to emphasize that the paper does not argue unconditionally for the benefits of a single currency area. For instance, we have entirely ignored a whole set of factors that have been identified as important in the Eurozone crisis, such as sovereign debt constraints, moral hazard elements of a single currency/single market, potential for bubbles in financial markets, and financial and banking fragility. In addition, we have shown that effective forward guidance in monetary policy can restore the traditional advantages of multiple currencies and flexible exchange rates. Hence, the key message of the paper is that, when monetary policy is constrained by the zero lower bound, the support for traditional policy conclusions is acutely dependent on the ability for policy-makers to make credible future commitments.

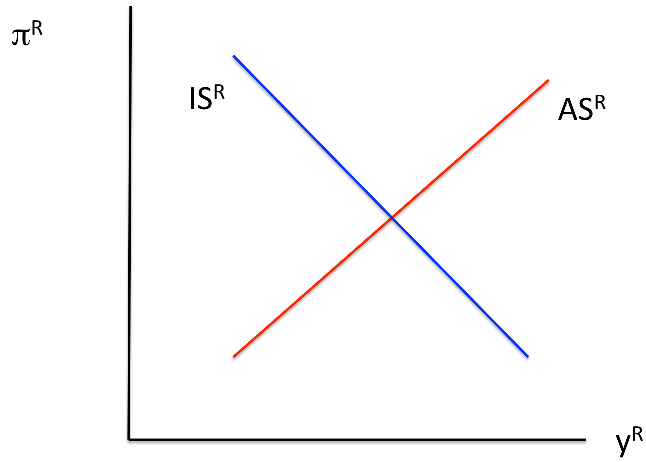


Figure 1: World Relatives under Normal Monetary Policy

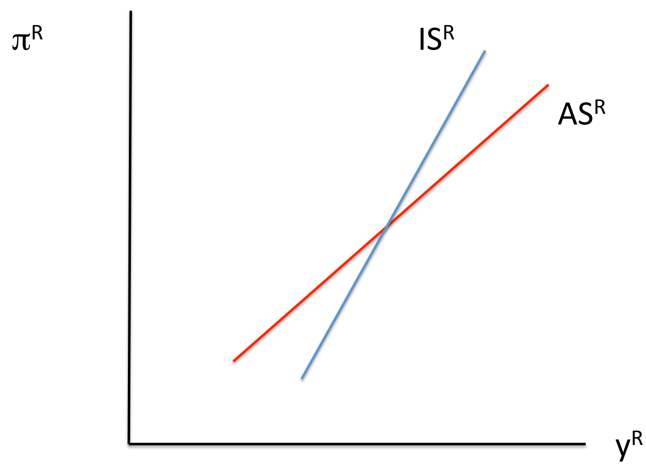


Figure 2: World Relatives at the zero-bound

Figure 3: A Home Shock, SCA vs Flex ER, Active Monetary Policy

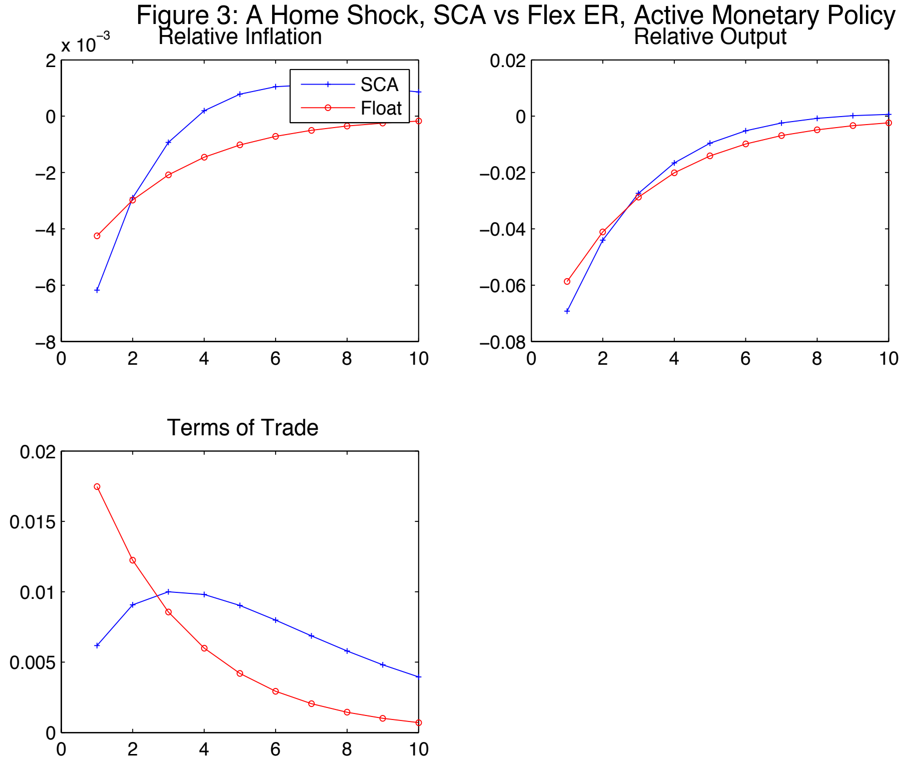


Figure 4: A Home Shock, SCA vs Flex ER, ZLB

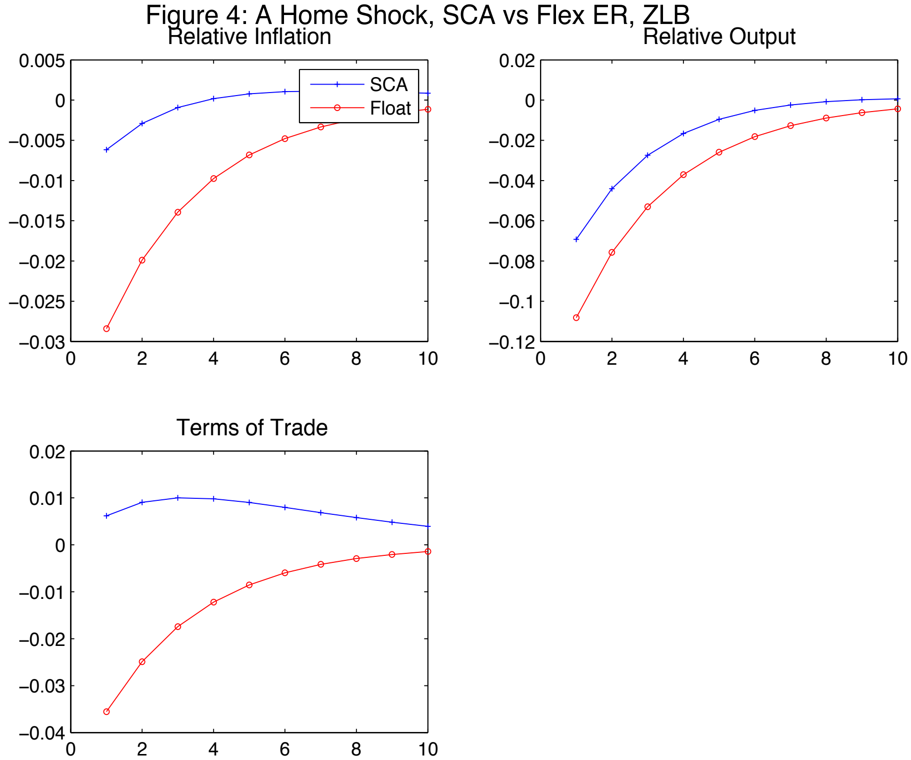


Figure 5: Natural Interest Rates

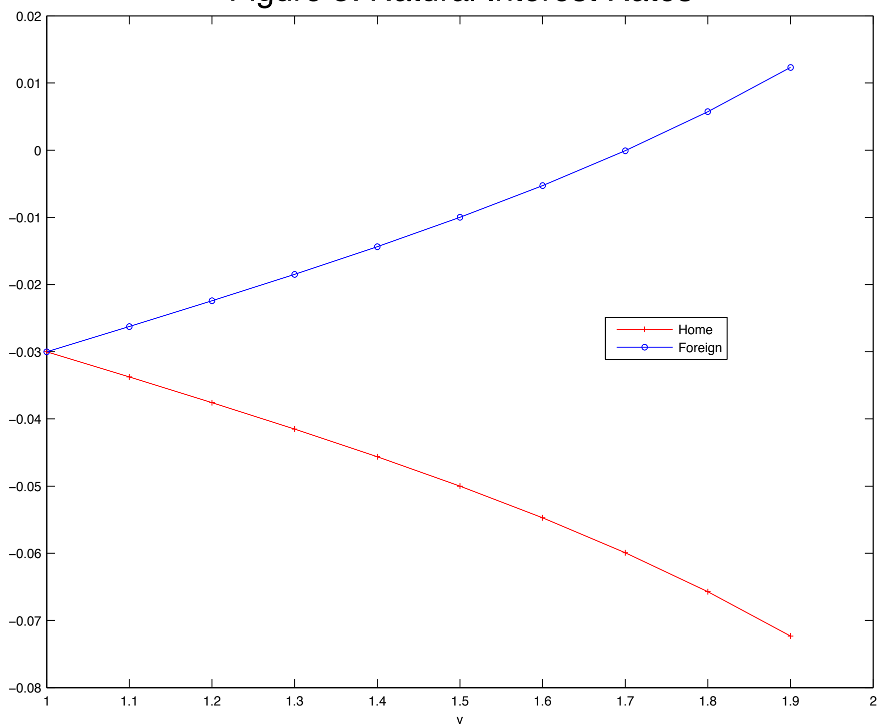


Figure 6: Optimal Policy

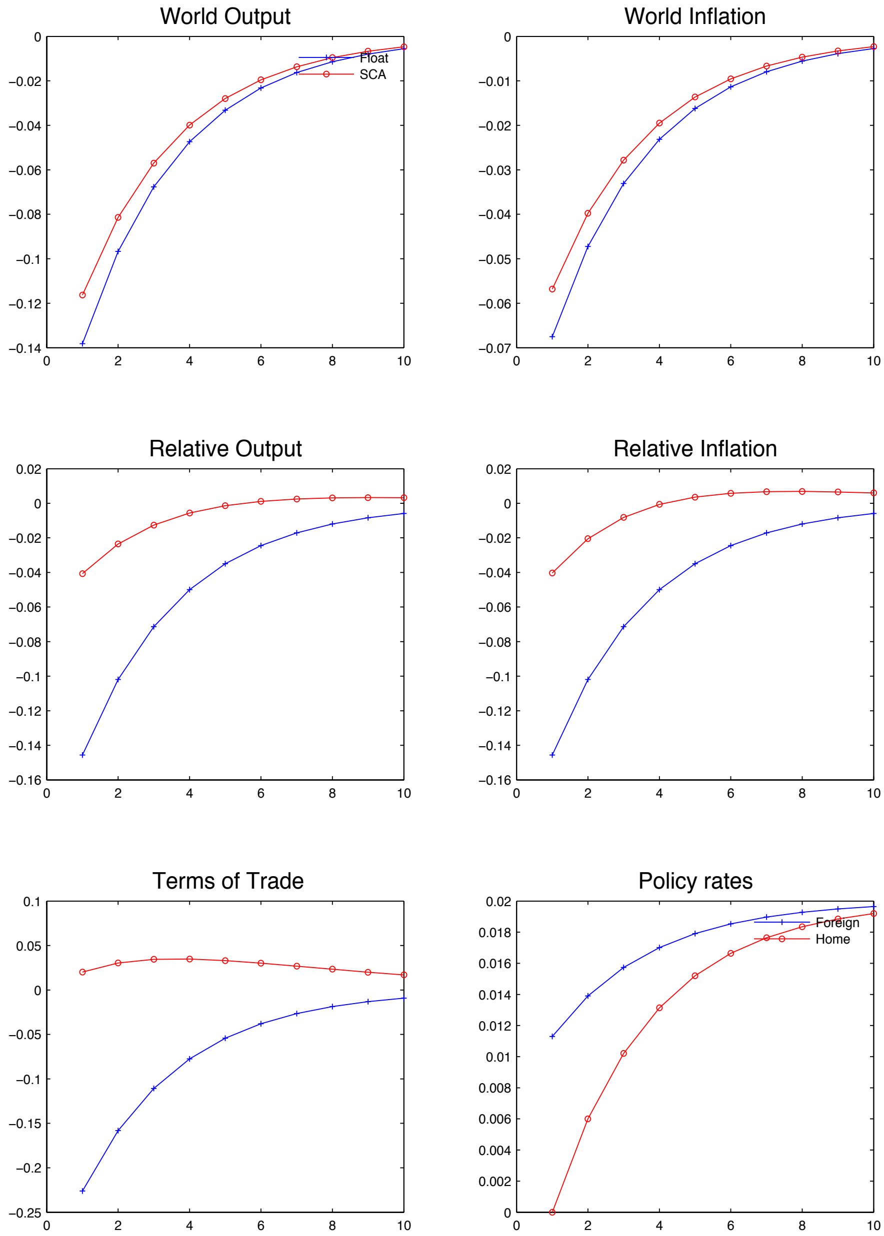


Figure 7: Welfare comparison with optimal Policy

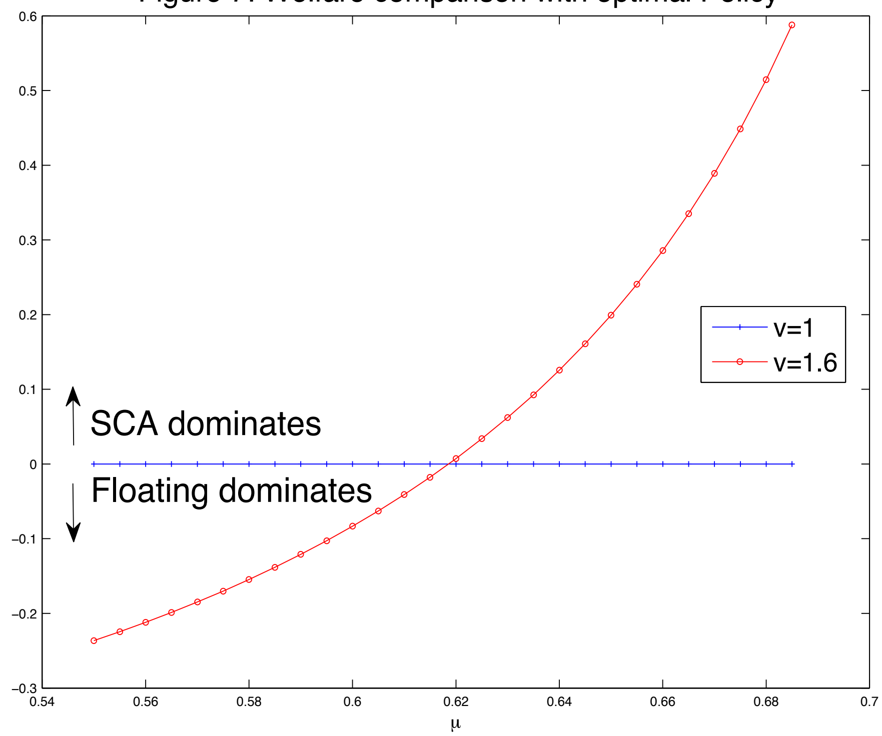


Figure 8: SCA Discretion

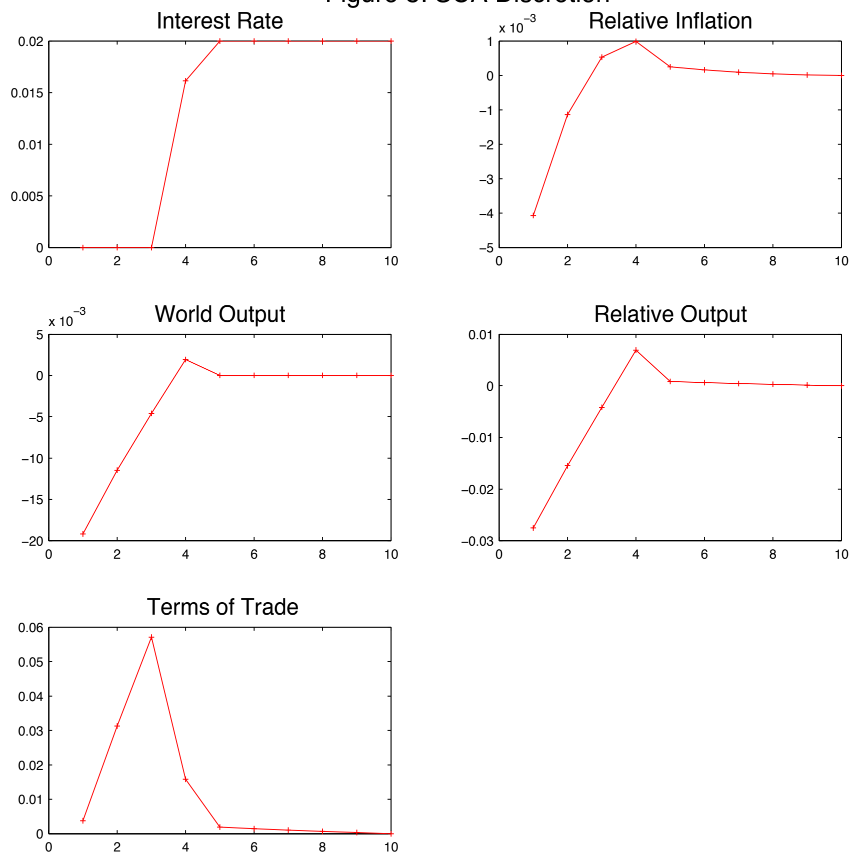


Figure 9: Floating, Discretion

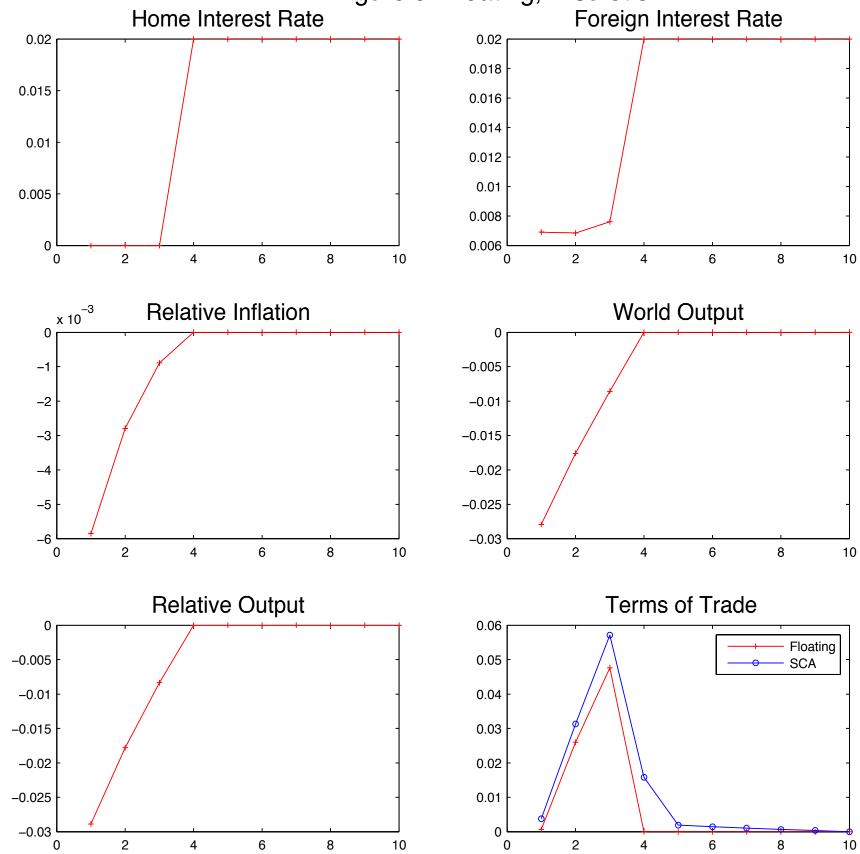


Figure 10: SCA Commitment

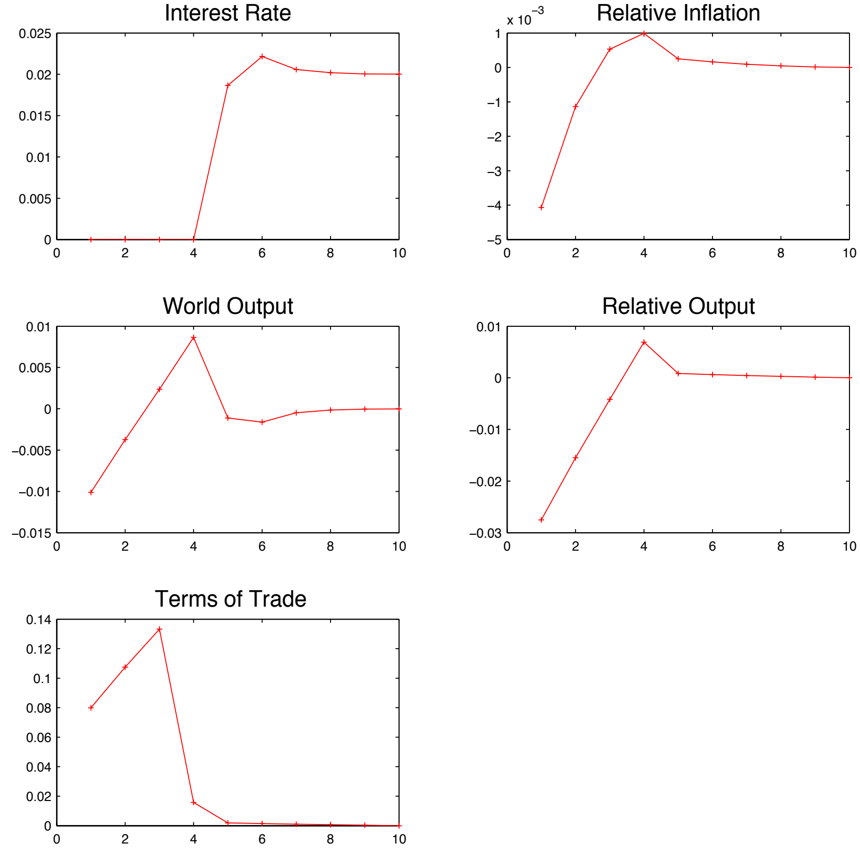
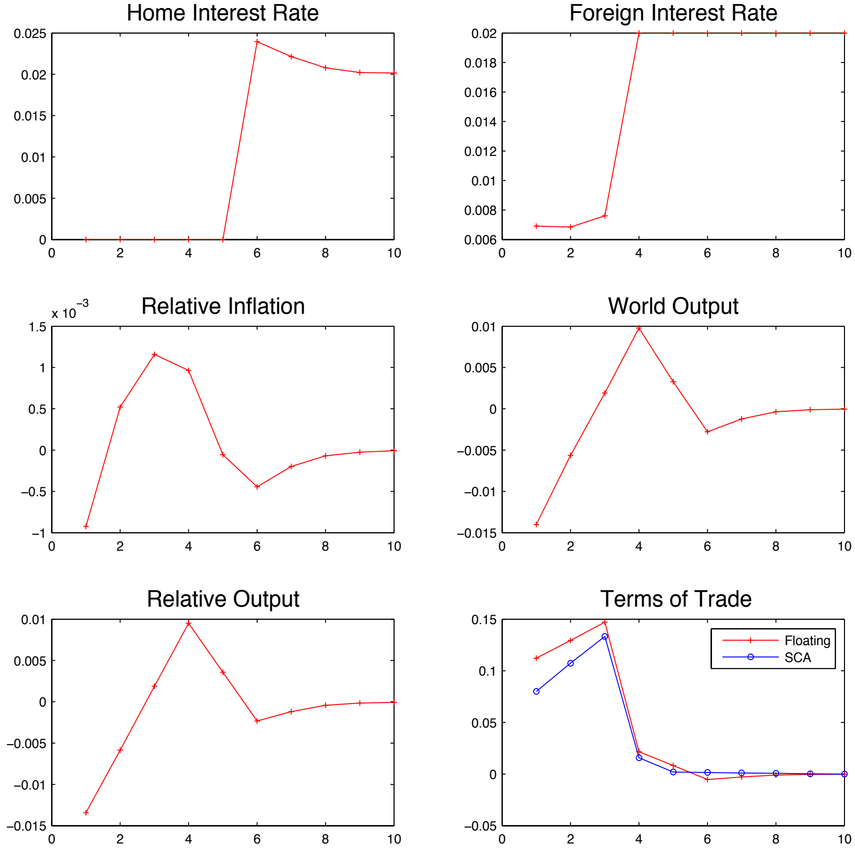


Figure 11: Floating, Commitment



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8 Appendix A

8.1 Equilibrium with Fully Flexible Prices

We first define the equilibrium of the model in a fully flexible price world equilibrium, where $\kappa = 0$ in each country. In that case, $P_{Ht}(i) = P_{Ht}$, $P_{Ft}(i) = P_{Ft}$, and $V_t = V_t^* = 1$. In addition (given the presence of optimal subsidies) we have $P_{Ht} = A^{-1}W_t$ and $P_{Ft}^* = A^{-1}W_t^*$. Letting a bar denote values in a flexible price world equilibrium, we may describe the equilibrium by the equations:

$$U_C(\bar{C}_t, \zeta_t)A = \bar{T}_t^{1-v/2}V'(\bar{N}_t), \qquad U_C^*(\bar{C}_t^*, \zeta_t^*)A^* = \bar{T}_t^{-(1-v/2)}V'(\bar{N}_t^*) \tag{38}$$

$$U_C(\bar{C}_t, \zeta_t)\bar{T}_t^{v-1} = U_C^*(\bar{C}_t^*, \zeta_t^*), \tag{39}$$

$$A\bar{N}_t = \frac{v}{2}\bar{T}_t^{1-v/2}\bar{C}_t + (1-\frac{v}{2})\bar{T}_t^{v/2}\bar{C}_t^*, \tag{40}$$

$$A^*\bar{N}_t^* = \frac{v}{2}\bar{T}_t^{-(1-v/2)}\bar{C}_t^* + (1-\frac{v}{2})\bar{T}_t^{-v/2}\bar{C}_t^* \tag{41}$$

This implicitly describes the efficient world equilibrium for consumption, output (or employment), and the terms of trade.

We analyze equations (38)-(41) by taking a linear approximation around the globally efficient steady state. For a given variable X, define $\bar{x}$ to be the log difference of the global efficient value from the non-stochastic steady state, except for ε_t (as defined in the text), and π_{Ht} and r_t, which refer respectively to the *level* of the inflation rate and nominal interest rate. Since the model is symmetric, we have $\overline{T} = 1$ in a steady state. Then we may express the linear approximation of (38)-(41) as:

$$\sigma\bar{c}_t - \varepsilon_t + \phi\bar{y}_t + (1 - \frac{v}{2})\bar{\tau}_t = 0 \tag{42}$$

$$\sigma\bar{c}_t^* - \varepsilon_t^* + \phi\bar{y}_t^* - (1 - \frac{v}{2})\bar{\tau}_t = 0 \tag{43}$$

$$\bar{y}_t = (\frac{v}{2}\bar{c}_t + (1 - \frac{v}{2})\bar{c}_t^*) + v(1 - \frac{v}{2})\bar{\tau}_t, \tag{44}$$

$$\bar{y}_t^* = (\frac{v}{2}\bar{c}_t^* + (1 - \frac{v}{2})\bar{c}_t) - v(1 - \frac{v}{2})\bar{\tau}_t, \tag{45}$$

$$\sigma\bar{c}_t - (\varepsilon_t - \varepsilon_t^*) - \sigma\bar{c}_t^* - (v-1)\bar{\tau}_t = 0, \tag{46}$$

We may solve the system (42)-(46) to obtain the first order solutions for consumption, output and the terms of trade in response to savings shocks. We can write home and foreign consumption responses to savings shocks as:

$$\bar{c}_t = \frac{1}{\phi + \sigma}\varepsilon_t^W + \left(\frac{1 + \phi v(2 - v)}{\sigma + \phi D}\right)\varepsilon_t^R$$

$$\bar{c}_t^* = \frac{1}{\phi + \sigma}\varepsilon_t^W - \left(\frac{1 + \phi v(2 - v)}{\sigma + \phi D}\right)\varepsilon_t^R$$

A savings shock reduces the efficient flexible price world consumption level, but the impact on individual country consumption depends on the source of the shock, and the degree of home bias in preferences.

The impact of savings shocks on flexible price output levels are likewise written as:

$$\bar{y}_t = \frac{1}{\phi + \sigma}\varepsilon_t^W + \left[\left(\frac{(v-1)}{\sigma + \phi D}\right)\right]\varepsilon_t^R$$

$$\bar{y}_t^* = \frac{1}{\phi + \sigma}\varepsilon_t^W - \left[\left(\frac{(v-1)}{\sigma + \phi D}\right)\right]\varepsilon_t^R$$

A world savings shock reduces equilibrium output in both countries. When there is home bias in preferences, so that $v > 1$, a relative home savings shock tends to reduce home output and raise foreign output.

Demand shocks also affect the flexible price efficient response of the terms of trade. We can show that:

$$\frac{\bar{\tau}_t}{2} = -\frac{\phi(v-1)}{\sigma + \phi D}\varepsilon_t^R$$

In response to a relative home country savings shock, relative home output falls, but the terms of trade deteriorates, in a fully flexible price equilibrium.

If monetary authorities could adjust nominal interest rates freely to respond to demand shocks, then the flexible price efficient global equilibrium could be sustained. We denote the interest rate that would sustain the flexible price efficient outcome as the 'natural interest rate'. Let ρ be the steady state value for the natural interest rate. Then a log linear approximation of (4) leads to the expressions for the flexible price equilibrium nominal interest rate in the home country as:

$$\bar{r}_t = \rho + \sigma E_t(\bar{c}_{t+1} - \bar{c}_t) - E_t(\varepsilon_{t+1} - \varepsilon_t) + E_t\pi_{Ht+1} + (1 - \frac{v}{2})E_t(\bar{\tau}_{t+1} - \bar{\tau}_t) \qquad (47)$$

Assume that an efficient monetary policy is to keep the domestic rate of inflation equal to zero. In addition, for now, assume that demand shocks follow an AR(1) process so that $\varepsilon_{t+1} = \mu\varepsilon_t + u_t$ and $\varepsilon^*_{t+1} = \mu\varepsilon^*_t + u^*_t$, where u_t and u^*_t are mean-zero and i.i.d., then the value of $\widetilde{r}_t$ when the right hand side is driven by demand shocks alone can be derived as:

$$\bar{r}_t = \rho + \left(\frac{\phi}{\phi + \sigma}\varepsilon_t^W + \left(\frac{\phi(v-1)}{\sigma + \phi D}\right)\varepsilon_t^R\right)(1-\mu) \qquad (48)$$

In similar manner, the foreign efficient nominal interest rate is:

$$\bar{r}_t^* = \rho + \left(\frac{\phi}{\phi + \sigma}\varepsilon_t^W - \left(\frac{\phi(v-1)}{\sigma + \phi D}\right)\varepsilon_t^R\right)(1-\mu) \qquad (49)$$

Natural interest rates respond to both aggregate and relative savings shocks. An aggregate savings shock raises global marginal utility and raises natural interest rates. A relative savings shock affects natural interest rates in separate ways in the two countries, but this depends upon the degree of home bias in preferences. With identical preferences across countries, the natural interest rate is independent of purely relative demand shocks, and is equalized across countries.

This discussion has direct bearing on the degree to which the zero bound constraint will bind across countries in response to time preference shocks (negative demand shocks) emanating from one country. In general, these shocks will have both aggregate and relative components. If there are full security markets and identical preferences, we see that natural interest rates are always equated across countries. But when $v \geq 1$ a home country shock has a smaller impact on the foreign natural interest rate, so the home country may be constrained by the zero lower bound, but the foreign country will not be so constrained.

8.2 Optimal Policy: Discretion

Cooperative policy under discretion (and multiple currencies) may be characterized by the solution to the following optimization problem:

$$\begin{aligned}
& \max_{\widehat{y}_t^R, \widetilde{y}_t^W, \pi_t^W, \pi_t^R, r_t^W, r_t^R} L_t \\
= \; & V_t + \lambda_{1t} \left[\pi_t^W - k(\phi + \sigma) \widetilde{n}_t^W - \beta E_t \pi_{t+1}^W \right] \\
& + \lambda_{2t} \left[\pi_t^R - k(\phi + \sigma_D) \widetilde{y}_t^R - \beta E_t \pi_{t+1}^R \right] \\
& + \psi_{1t} \left[\sigma E_t (\widehat{y}_{t+1}^W - \widehat{y}_t^W) - E_t \left(r_t^W - \widetilde{r}_t^W - \pi_{t+1}^W \right) \right] \\
& + \psi_{2t} \left[\sigma_D E_t (\widehat{y}_{t+1}^R - \widehat{y}_t^R) - E_t \left(r_t^R - \frac{\widetilde{r}_t^R}{2} - \pi_{t+1}^R \right) \right] \\
& + \gamma_{1t} \left(r_t^W + r_t^R \right) + \gamma_{2t} \left(r_t^W - r_t^R \right)
\end{aligned}$$

The policy optimum involves the choice of the output gaps, government spending gaps, inflation rates and the foreign interest rate to maximize this Lagrangian. The first two constraints are the inflation equations in average and relative terms. The second two constraints are the average and relative 'IS' equations. The final two constraints are the non-negativity constraint on the two policy interest rates.

The first order conditions of the maximization are:

$$-(\sigma_D + \phi) \widehat{y}_t^R = \lambda_{2t} k(\phi + \sigma_D) + \sigma_D \psi_2 \tag{50}$$

$$-(\sigma + \phi) \widehat{y}_t^W = \lambda_{1t} k(\phi + \sigma) + \sigma \psi_{1t} \tag{51}$$

$$k \lambda_{1t} = \theta \pi_t^W \tag{52}$$

$$k \lambda_{2t} = \theta \pi_t^R \tag{53}$$

$$\psi_{2t} = \gamma_{1t} + \gamma_{2t} \tag{54}$$

$$\psi_{2t} = \gamma_{1t} - \gamma_{2t} \tag{55}$$

$$\gamma_{1t} \geq 0, \quad (r_t^W + r_t^R) \geq 0, \quad \gamma_{1t}\ (r_t^W + r_t^R) = 0 \tag{56}$$

$$\gamma_{2t} \geq 0, \quad (r_t^W - r_t^R) \geq 0, \quad \gamma_{2t}\ (r_t^W - r_t^R) = 0 \tag{57}$$

These equations, in conjunction with (10)-(11) and (13)-(14), give the conditions determining average and relative output gaps; $\widehat{y}_t^R, \widehat{y}_t^W$, inflation rates; π_t^R, π_t^W, fiscal gaps; $\widehat{cg}_t^R, \widehat{cg}_t^W$, the Lagrange multipliers; $\lambda_{1t}, \lambda_{2t}, \psi_{1t}, \psi_{2t}$, and the value of either $r_t^W + r_t^R$, or γ_{1t} and $r_t^W - r_t^R$, or γ_{2t}, under optimal policy.

In the single currency area, cooperative policy is described in a similar manner, but involves only the choice of r_t^W subject to a non-negativity constraint.

8.3 Optimal Policy: Commitment

Cooperative policy under commitment with multiple currencies is characterized by:

$$\begin{aligned}
&\max_{\widehat{y}_t^R, \widetilde{y}_t^W, \pi_t^W, \pi_t^R, r_t^W, r_t^R} L_0 \\
= \quad & V_0 + \sum_{t=0}^{\infty} \lambda_{1t} \left[\pi_t^W - k(\phi + \sigma)\widetilde{n}_t^W - \beta E_t \pi_{t+1}^W\right] \\
& + \sum_{t=0}^{\infty} \lambda_{2t} \left[\pi_t^R - k(\phi + \sigma_D)\widetilde{y}_t^R - \beta E_t \pi_{t+1}^R\right] \\
& + \sum_{t=0}^{\infty} \psi_{1t} \left[\sigma E_t(\widehat{y}_{t+1}^W - \widehat{y}_t^W) - E_t \left(r_t^W - \widetilde{r}_t^W - \pi_{t+1}^W\right)\right] \\
& + \sum_{t=0}^{\infty} \psi_{2t} \left[\sigma_D E_t(\widehat{y}_{t+1}^R - \widehat{y}_t^R) - E_t \left(r_t^R - \frac{\widetilde{r}_t^R}{2} - \pi_{t+1}^R\right)\right] \\
& + \sum_{t=0}^{\infty} \gamma_{1t} \left(r_t^W + r_t^R\right) + \sum_{t=0}^{\infty} \gamma_{2t} \left(r_t^W - r_t^R\right)
\end{aligned}$$

where V_0 is defined in (37) of the text.

The policy optimum involves the choice of the output gaps, government spending gaps, inflation rates and the foreign interest rate to maximize this Lagrangian. The first two constraints are the inflation equations in average and relative terms. The second two constraints are the average and relative 'IS' equations. The final two constraints are the non-negativity constraint on the two policy interest rates.

The first order conditions of the maximization are, for time $t = 0$:

$$-(\sigma_D + \phi)\widehat{y}_0^R = \lambda_{2t} k(\phi + \sigma_D) + \sigma_D \psi_{2t} \tag{58}$$

$$-(\sigma + \phi)\widehat{y}_t^W = \lambda_{1t} k(\phi + \sigma) + \sigma \psi_{1t} \tag{59}$$

$$k\lambda_{1t} = \theta \pi_t^W \tag{60}$$

$$k\lambda_{2t} = \theta \pi_t^R \tag{61}$$

$$\psi_{2t} = \gamma_{1t} + \gamma_{2t} \tag{62}$$

$$\psi_{2t} = \gamma_{1t} - \gamma_{2t} \tag{63}$$

$$\gamma_{1t} \geq 0, \quad (r_t^W + r_t^R) \geq 0, \quad \gamma_{1t}\ (r_t^W + r_t^R) = 0 \tag{64}$$

$$\gamma_{2t} \geq 0, \quad (r_t^W - r_t^R) \geq 0, \quad \gamma_{2t}\ (r_t^W - r_t^R) = 0 \tag{65}$$

and for time $t > 0$, the conditions are:

$$-(\sigma_D + \phi)\widehat{y}_0^R = \lambda_{2t} k(\phi + \sigma_D) + \sigma_D \psi_{2t} - \sigma_D \psi_{2t-1} \tag{66}$$

$$-(\sigma + \phi)\widehat{y}_t^W = \lambda_{1t} k(\phi + \sigma) + \sigma \psi_{1t} - \sigma \psi_{1t-1} \tag{67}$$

$$k(\lambda_{1t} - \beta \lambda_{1t-1} - \psi_{1t-1}) = \theta \pi_t^W \tag{68}$$

$$k(\lambda_{2t} - \beta \lambda_{2t-1} - \psi_{2t-1}) = \theta \pi_t^R \tag{69}$$

$$\psi_{2t} = \gamma_{1t} + \gamma_{2t} \tag{70}$$

$$\psi_{2t} = \gamma_{1t} - \gamma_{2t} \tag{71}$$

$$\gamma_{1t} \geq 0, \quad (r_t^W + r_t^R) \geq 0, \quad \gamma_{1t}\ (r_t^W + r_t^R) = 0 \tag{72}$$

$$\gamma_{2t} \geq 0, \quad (r_t^W - r_t^R) \geq 0, \quad \gamma_{2t}\ (r_t^W - r_t^R) = 0 \tag{73}$$

These equations, in conjunction with (10)-(11) and (13)-(14) for each period t, give the conditions determining average and relative output gaps; $\widehat{y}_t^R, \widehat{y}_t^W$, inflation rates; π_t^R, π_t^W, fiscal gaps; $\widehat{cg}_t^R, \widehat{cg}_t^W$, the Lagrange multipliers; $\lambda_{1t}, \lambda_{2t}, \psi_{1t}, \psi_{2t}$, and the value of either $r_t^W + r_t^R$, or γ_{1t} and $r_t^W - r_t^R$, or γ_{2t}, under an optimal policy with commitment.

Again, under the single currency area, the optimal policy problem is simply a subset of the above problem, where r_t^W is chosen subject to a non-negativity constraint.

9 Appendix B. Adjustment with Fiscal Policy

Fahri et al. (2012) have described how a mix of tax and subsidies can achieve "Fiscal Devaluation" in a small economy, exactly replicating the effects of a nominal exchange rate devaluation. Therefore, if fiscal policy is sufficiently flexible, it can completely eliminate the loss of monetary autonomy implied by a fixed exchange rate regime. More generally, it has been established by Correa et al. (2012) that a combination of state-contingent taxes and subsidies can undo the effects of the zero bound, and fully replicate the flexible price equilibrium in standard New Keynesian models. A similar set of results applies to our model. We show below that a combination of VAT adjustment and payroll tax changes can be used to ensure price stability and zero output gaps, achieving the fully optimal flexible price equilibrium. But when monetary policy is constrained by the zero bound, fiscal adjustment will be required even in a situation of flexible exchange rates. So we need to identify the set of optimal fiscal instruments both in the single currency model as well as the model with flexible exchange rates. The main result we show is that the tax-subsidy mix is the same in both cases. We can express the extended model in terms of world averages and world relatives as follows:

$$\pi_t^W = k((\phi+\sigma)\widehat{y}_t^W + t_{VAT,t}^W + t_{WAGE,t}^W - \varepsilon_t^W) + \beta E_t \pi_{t+1}^W \tag{74}$$

$$\sigma E_t(\widehat{y}_{t+1}^W - \widehat{y}_t^W) = E_t(\varepsilon_{t+1}^W - t_{VAT,t+1}^W) - (\varepsilon_t^W - t_{VAT,t}^W) + E_t\left(r_t^W - E_t\pi_{t+1}^W - \rho\right) \tag{75}$$

$$\pi_t^R = k((\phi+\sigma_D)\widehat{y}_t^R - \zeta(\varepsilon_t^R - t_{VAT}^R) + t_{WAGE}^R) + \beta E_t \pi_{t+1}^R \tag{76}$$

$$\sigma_D E_t(\widehat{y}_{t+1}^R - \widehat{y}_t^R) = \zeta(E_t(\varepsilon_{t+1}^R - t_{VAT,t+1}^R) - (\varepsilon_t^R - t_{VAT,t}^R)) + E_t\left(r_t^R - \pi_{t+1}^R\right) \tag{77}$$

Here, $t_{VAT,t}^W$ ($t_{VAT,t}^R$) represents the world average (world relative) VAT tax at time t, assuming taxes are zero in steady state. Likewise $t_{WAGE,t}^W$ ($t_{WAGE,t}^R$) represent the world average (world relative) payroll tax. An increase in world average VAT tax raises consumer prices, shifting back labor supply and pushing up marginal costs for firms, thus reducing world output. A payroll tax also increases marginal costs and reduces output. At the same time, an expected rise in the VAT rate $E_t t_{VAT,t+1}^W - t_{VAT,t}^W$ will reduce the expected real interest rate, inclusive of taxes, and reduce the rate of growth of world consumption and output. The impact of world relative VAT and payroll taxes on world relative output can be explained in an analogous manner. We may rewrite these two equations systems in terms of inflation and output gaps

$$\pi_t^W = k((\phi+\sigma)\tilde{y}_t^W + t_{VAT,t}^W + t_{WAGE,t}^W) + \beta E_t \pi_{t+1}^W \tag{78}$$

$$\sigma E_t(\tilde{y}^W_{t+1} - \tilde{y}^W_t) = -E_t(t^W_{VAT,t+1}) - t^W_{VAT,t}) + E_t\left(r^W_t - E_t\pi^W_{t+1} - R^W_{N,t}\right) \quad (79)$$

$$\pi^R_t = k((\phi + \sigma_D)\tilde{y}^R_t + \zeta t^R_{VAT} - t^R_{WAGE}) + \beta E_t \pi^R_{t+1} \quad (80)$$

$$\sigma_D E_t(\tilde{y}^R_{t+1} - \tilde{y}^R_t) = -\zeta(E_t t^R_{VAT,t+1} - t^R_{VAT,t}) + E_t\left(r^R_t - \pi^R_{t+1} - R^R_{N,t}\right) \quad (81)$$

here $R^W_{N,t} = \rho + (1-\mu)\frac{\phi}{\sigma+\phi}\varepsilon^W_t$ is defined as the world average natural interest rate, and $R^R_{N,t} = (1-\mu)\zeta\frac{\phi}{\sigma_D+\phi}\varepsilon^R_t$ is the world relative natural real interest rate.

From these two equations, it is easy to see that the following combination of VAT and payroll tax changes can eliminate all gaps in a liquidity trap. The policy mix can be described by the following conditions:

$$t^W_{VAT} = \frac{\phi}{\sigma + \phi}\varepsilon^W_t \quad (82)$$

$$t^W_{WAGE} = -t^W_{VAT} \quad (83)$$

$$t^R_{VAT} = \frac{\phi}{\sigma_D + \phi}\varepsilon^R_t \quad (84)$$

$$t^R_{WAGE} = -\zeta t^R_{VAT} \quad (85)$$

In addition, these taxes are applied with the same persistence μ as the shock itself. This policy mix does the following. In terms of world averages, it combines a VAT tax cut with a payroll tax increase. The VAT tax cut takes on the same expected persistence as the negative demand shock, and so induces a fall in the expected real interest rate that cannot be achieved by a nominal interest rate cut. The VAT tax cut on its own however would be too expansionary, since with inflation and expected inflation maintained at zero, it would lead to a positive output gap. This must be offset by a payroll tax increase. In terms of world relatives, there is a relative VAT tax cut that is greater in the worst hit economy. That effectively achieves the relative price tilting that mimics the terms of trade depreciation that should take place in the fully efficient economy with fully flexible prices. Again, with $\upsilon > 1$ however, this would lead to a positive world relative output gap, and so must be countered by a rise in the relative payroll tax. The key feature of the solutions (82)-(85) is that they do not depend on the monetary rule. Thus, they are the same for the flexible exchange rate model and the single currency area. In both cases, they achieve the adjustment in world and relative output without any inflation adjustment, or nominal exchange rate adjustment. To see this, we note that the terms of trade (exclusive of VAT) in the presence of VAT changes can be written as

$$\hat{\tau}_t = 2\sigma_D \hat{y}^R_t - 2\zeta(\varepsilon^R_t - t^R_{VAT,t}) \quad (86)$$

It is easy to see from (80) and (86) that the adjustment of relative VAT rates obviates any movement in the terms of trade. Hence, if VAT rates are adjusted appropriately, neither domestic inflation or nominal exchange rate adjustment is necessary. But in contrast to the discussion on Fiscal Devaluations in the eurozone, we see that in a liquidity trap, the optimal fiscal policy mix is the same whether or not the region has a system of flexible exchange rates. Thus, the key constraint is the zero bound on interest rates, not the non-adjustability of the nominal exchange rate.