A Model of Secular Stagnation

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Secular Stagnation Hypothesis

I wonder if a set of older ideas ... under the phrase secular stagnation are not profoundly important in understanding Japan’s experience, and may not be without relevance to America’s experience

- Lawrence Summers

Original hypothesis:
- Alvin Hansen (1938): Suggests a permanent demand recession
- Reduction in population growth and investment opportunities
- Concerns of insufficient demand ended with WWII and subsequent baby boom

Secular stagnation resurrected:
- Lawrence Summers (2013)
- Highly persistent decline in the natural rate of interest
- Chronically binding zero lower bound

Goal here:
- Formalize these ideas in a simple model
- Propose a OLG model in the spirit of Samuelson (1958)
Preview of Results

Negative natural rate of interest can be triggered by

- Deleveraging shock
- Slowdown in population growth
- Increase in income inequality
- Fall in relative price of investment

Deflation steady state

- Permanently binding zero lower bound
- Permanent deflation
- Permanent shortfall in output from potential

Paradoxes and policy responses

- Paradox of thrift, toil and flexibility
- Raising the inflation target good but better be high enough
- Fiscal expansions (debt or spending)
ECONOMIC ENVIRONMENT

Endowment economy

- Time: $t = 0, 1, 2, \ldots$

- Goods: consumption good ($c$)

- Agents: 3-generations: $i \in \{y, m, o\}$

- Assets: riskless bonds ($B^i$)

- Technology: exogenous borrowing constraint $D$
Objective function:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o} U = \mathbb{E}_t \left\{ \log (C_t^y) + \beta \log (C_{t+1}^m) + \beta^2 \log (C_{t+2}^o) \right\}$$

Budget constraints:

$$C_t^y = B_t^y$$
$$C_{t+1}^m = Y_{t+1}^m - (1 + r_t)B_t^y + B_{t+1}^m$$
$$C_{t+2}^o = Y_{t+2}^o - (1 + r_{t+1})B_{t+1}^m$$
$$(1 + r_t)B_t^i \leq D_t$$
Consumption and Saving

Credit-constrained youngest generation:

\[ C_t^y = B_t^y = \frac{D_t}{1 + r_t} \]

Saving by the middle generation:

\[ \frac{1}{C_t^m} = \beta E_t \frac{1 + r_t}{C_{t+1}^o} \]

Spending by the old:

\[ C_t^o = Y_t^o - (1 + r_{t-1})B_{t-1}^m \]
Determination of the Real Interest Rate

Asset market equilibrium:

\[ N_t B_t^y = -N_{t-1} B_t^m \]
\[ (1 + g_t) B_t^y = -B_t^m \]

Demand and supply of loans:

\[ L_t^d = \frac{1 + g_t}{1 + r_t} D_t \]
\[ L_t^s = \frac{\beta}{1 + \beta} (Y_t^m - D_{t-1}) - \frac{1}{1 + \beta} \frac{Y_t^{o}}{1 + r_t} \]
Determination of the Real Interest Rate

Expression for the real interest rate:

$$1 + r_t = \frac{1 + \beta (1 + g_t)D_t}{\beta Y_t - D_{t-1}} + \frac{1}{\beta Y_t - D_{t-1}}$$

Determinants of the real interest rate:

- Tighter collateral constraint reduces the real interest rate
- Lower rate of population growth reduces the real interest rate
- Higher income in the middle-generation reduces real interest rate
- Higher income in the old-generation increases real interest rate
Effect of a Deleveraging Shock

Impact effect:
- Collateral constraint tightens from $D_h$ to $D_l$
- Reduction in the loan demand and fall in real rate
- Akin to Eggertsson and Krugman (2012)

Delayed effect:
- Next period, shift out in loan supply
- Further reduction in real interest rate
- Novel effect from Eggertsson and Krugman (2012)
- Potentially powerful propagation mechanism
Effect of a Deleveraging Shock

- Loans
- Gross Real Interest Rate
- Loan Supply
- Loan Demand

Diagram showing the effect of a deleveraging shock on loans and interest rates.
Does inequality affect the real interest rate?

- Our result due to intergenerational income inequality that triggers borrowing and lending
- What about inequality across a given cohort?

Generalization of endowment process:

- High-type households with high income in middle period
- Low-type households with low income in middle period
- Both types receive same income in last period
Credit-constrained middle income:

- Fraction $\eta_s$ of middle income households are credit constrained.
- True for low enough income in middle generation and high enough income in retirement.
- Fraction $1 - \eta_s$ lend to both young and constrained middle-generation households.

Expression for the real interest rate:

\[
1 + r_t = \frac{1 + \beta}{\beta} \frac{(1 + g_t + \eta_s) D_t}{(1 - \eta_s) (Y_{t}^{m,h} - D_{t-1})} + \frac{1}{\beta} \frac{Y_{t+1}^{o}}{(Y_{t}^{m,h} - D_{t-1})}
\]
Price Level Determination

Euler equation for nominal bonds:

\[
\frac{1}{C_t^m} = \beta E_t \frac{1}{C_{t+1}^o} (1 + i_t) \frac{P_t}{P_{t+1}}
\]

\[i_t \geq 0\]

Lower bound on steady state inflation:

\[\bar{\Pi} \geq \frac{1}{1 + r}\]

- If steady state real rate is negative, steady state inflation must be positive
- No equilibrium with stable inflation
- But what happens when prices are NOT flexible and central bank does not tolerate inflation?
- Then the central bank’s refusal to tolerate high enough inflation will show up as a permanent recession.
Aggregate Supply

Output and labor demand:

\[ Y_t = L_t^\alpha \]
\[ \frac{W_t}{P_t} = \alpha L_t^{\alpha-1} \]

Labor supply:

- Middle-generation households supply a constant level of labor \( \bar{L} \)
- Implies a constant market clearing real wage \( \bar{W} = \alpha \bar{L}^{\alpha-1} \)
- Implies a constant full-employment level of output: \( Y_{fe} = \bar{L}^\alpha \)
Downward Nominal Wage Rigidity

Partial wage adjustment:

\[ W_t = \max \left\{ \tilde{W}_t, P_t \alpha \bar{L}^{\alpha - 1} \right\} \]

where \( \tilde{W}_t = \gamma W_{t-1} + (1 - \gamma) P_t \alpha \bar{L}^{\alpha - 1} \)

Wage rigidity and unemployment:

- If real wages exceed market clearing level, employment is rationed
- Unemployment \( U_t = \bar{L} - L_t \)
- Similar assumption in Schmitt-Grohe and Uribe (2013)
Derivation of Aggregate Supply

For positive steady state inflation:

\[ w_t = \bar{W} = \alpha \bar{L}^{(\alpha - 1)} \]
\[ Y_t = Y_{fe} \]

For steady state deflation:

\[ w_t = \gamma \frac{w_{t-1}}{\Pi_t} + (1 - \gamma) \bar{W} \]
\[ w_t = \alpha Y_t^{\alpha - 1} \]

- Upward sloping relationship between inflation and output
- Vertical line at full-employment
FULL EMPLOYMENT STEADY STATE

![Graph showing full employment steady state](image)
**Derivation of Aggregate Demand**

Monetary policy rule:

\[ 1 + i_t = \max \left( 1, (1 + i^*) \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \right) \]

Above binding ZLB:

\[ 1 + i^* \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} = \frac{1 + \beta (1 + g_t)D_t}{\beta} \frac{Y_t - D_{t-1}}{Y_t - D_{t-1}} \]

Binding ZLB:

\[ \frac{1}{\Pi_{t+1}} = \frac{1 + \beta (1 + g_t)D_t}{\beta} \frac{Y_t - D_{t-1}}{Y_t - D_{t-1}} \]
**Effect of a Collateral Shock**

- Gross Inflation Rate
- Aggregate Supply
- Deflation Steady State

- $AD_1$
- $AD_2$

Output

Gross Inflation Rate

0.80 0.85 0.90 0.95 1.00 1.05 1.10 1.15 1.20

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Properties of the Deflation Steady State

Long slump:
- Binding zero lower bound so long as natural rate is negative
- Deflation raises real wages above market-clearing level
- Output persistently below full-employment level

Existence and stability:
- Secular stagnation steady state exists so long as $\gamma > 0$
- Secular stagnation state state is determinate
- Contrast to deflation steady state emphasized in Benhabib, Schmitt-Grohe and Uribe (2001)
- Can do comparative statics!
**Paradox of Toil**

- **AD₂**
- **AS₁**
- **AS₂**

Graph showing the relationship between gross inflation rate and output with lines indicating deflation steady state and high productivity steady state.
Monetary Policy Responses

Forward guidance:
- Extended commitment to keep nominal rates low?
- Ineffective if households/firms expect rates to remain low indefinitely

Raising the inflation target:
- For sufficiently high inflation target, full employment steady state
- Law of the excluded middle or the timidity trap (Krugman (2014))
- Multiple steady states (two determinate, one indeterminate)
Raising the Inflation Target
Fiscal Policy Responses

Fiscal policy and the real interest rate:

\[ L^d_t = \frac{1 + g_t}{1 + r_t} D_t + B^g_t \]

\[ L^s_t = \frac{\beta}{1 + \beta} (Y^m_t - D_{t-1} - T^m_t) - \frac{1}{1 + \beta} \frac{Y^o_{t+1} - T^o_{t+1}}{1 + r_t} \]

- Higher government debt increases the interest rate by increasing demand for bonds
- Taxes on middle aged reducing loan supply: increase \( r_t \)
- Expected taxes on old increase loan supply: decrease \( r_t \)
- In AD-AS framework, gov. spending financed either by taxes or debt is expansionary
INCREASING GOVERNMENT SPENDING WITH TAX ON MIDDLE AGED

- Aggregate Supply
- Full Employment Steady State

Gross Inflation Rate vs. Output

- AD$_2$
- AD$_3$

Deflation Steady State
Incorporating Capital

Rental rate and real interest rate:

\[ r_t^k = p_t^k - p_{t+1}^k \frac{1 - \delta}{1 + r_t} \geq 0 \]

\[ r_{ss} \geq -\delta \]

- Assume that return on capital is realized in the same period as investment
- Negative real rate now constrained by fact that rental rate must be positive

Relative price of capital goods:

- Decline in relative price of capital goods lowers the real interest rate
- Global decline in price of capital goods (Karabarbounis and Neiman, 2014)
- Consistent with argument by Summers (2014)
Conclusions

Policy implications:

- Higher inflation target needed
- Limits to forward guidance
- Role for fiscal policy
- Avoid policies that tighten collateral constraint D? (i.e. capital requirements, etc.)

Key takeaway:

- NOT that we will stay in a slump forever
- Instead, the slump can be of arbitrary duration which has strong policy implications.
- Stakes are even higher for good aggregate demand management.