Credibility and the Maturity of Government Debt

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Motivation

- The maturity of government debt varies across countries and time
- Policy makers actively manage maturity of debt
- Lack of positive theory of the maturity of government debt
What we do

- Document regularities of maturity of government debt and link
  - Maturity is shorter when inflation is high
  - Debt is lower when inflation is high

- Develop model of maturity of debt based on government credibility
  - Credible governments have low inflation, long maturity, and high debt
  - Non-credible government have high inflation, short maturity, and low debt
Literature

- Ramsey Problems: Optimal policy government problems with incomplete markets and commitment
  
  Buera and Nicolini (real debt and rich maturity); Lustig, Yeltekin and Sleet (nominal debt and limited maturity)

- Markov Problems with Debt
  
  Martin; and Diaz-Gimenez, Giovanetti, Marimon and Teles (only short nominal debt)
Empirical Regularities

- Data for 14 OECD countries for 1960-2010
- Series on public government debt, debt maturity, and inflation
- Maturity and inflation varies across countries and years
- All countries have periods of high inflation and low maturity
Maturity and Inflation: United States

The graph shows the relationship between maturity and inflation in the United States from 1960 to 2010. The x-axis represents the years, and the y-axes represent maturity and inflation levels.

The maturity line (blue) fluctuates significantly, with peaks around 1970, 1980, and 1990, and a decrease towards 2010. The inflation line (red) also shows variability, with notable spikes around 1970 and 1980, followed by a decline towards the end of the period.

The graph suggests that there is a complex interplay between maturity and inflation over time.
Maturity and Inflation: Other countries

**US**

- **Maturity**
- **Inflation**

**Germany**

- **Maturity**
- **Inflation**

**Italy**

- **Maturity**
- **Inflation**

**Spain**

- **Maturity**
- **Inflation**
Maturity and Inflation: Pooled data

Inflation and maturity are negatively correlated
Debt and Inflation: United States

Debt/GDP and Inflation over time in the US.
Debt and Inflation: Pooled data

The graph shows a scatter plot with the x-axis representing Inflation Groups and the y-axis representing Maturity. The data points are scattered above a downward-sloping line, indicating a negative correlation between Inflation Groups and Maturity. The points are labeled as Debt/GDP and Fitted values.
Empirical Regularities

- High inflation associated with lower maturity
- High inflation associated with lower debt
Model Economies: Two Extremes

- Credible Ramsey government can commit to future policies
- Non-credible Markov government cannot commit to future policies
Model Environment

- Consumers have preferences over cash and credit goods and labor
- Government finances government purchases with labor taxes, money creation, and nominal debt
- Nominal bonds are uncontingent with short and long duration
Consumers

- Preferences over cash $c_{1t}$ and credit $c_{2t}$ goods and labor $\ell_t$

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, \ell_t) \]

- Cash-in-advance constraint with inherited money $M_{t-1}$

\[ P_t c_{1t} \leq M_{t-1} \]

- Holds short $B_{St}$ and long $B_{Lt}$ government debt, so budget

\[ P_t c_{1t} + P_t c_{2t} + M_t + Q_{St} B_{St} + Q_{Lt} L_t = (1 - \tau_t) P_t \ell_t + B_{St-1} + B_{Lt-1} + M_{t-1} \]

\[ B_{Lt} = \delta B_{Lt-1} + L_t \]

- Long bonds are perpetuities with coupons that decay at rate $\delta$

- Firms are competitive with $y_t = \ell_t$
Government

- Spending follows exogenous process $g_t$

- Finances with labor taxes $\tau_t$, money $M_t$, short $B_{St}$, long debt $B_{Lt}$

\[
P_t \tau_t \ell_t + M_t + Q_{St}B_{St} + Q_{Lt}B_{Lt} = P_t g_t + B_{St-1} + (1 + \delta Q_{Lt})B_{Lt-1} + M_{t-1}
\]

- The value of the debt burden depends on $Q_{Lt}$

- Can only issue debt

$B_{Lt} \geq 0, B_{St} \geq 0$
Constraints on government problem

- Market clearing
  \[ c_{1t} + c_{2t} + g_t = \ell_t \]

- Money-bond arbitrage condition
  \[ U_{1t} \geq U_{2t} \]

- Other first order conditions from competitive equilibrium
  \[- \frac{U_{lt}}{U_{2t}} = (1 - \tau_t)\]

- Labor taxes distort MRS ≠ MRT
- Positive nominal interest \((Q_{St} < 1)\) distort cash/credit choice
Costs and Benefits of Inflation

- **Costs of ex-post inflation:** consumer must use inherited cash for current cash purchases
  - Keeps Markov inflation rate finite

- **Costs of ex-ante inflation:** Distorts the consumption of cash vs. credit goods
  - Stops Ramsey from using inflation to make nominal debt mimic state contingent

- **Benefits of ex-post inflation:** Inflate away real value of outstanding nominal debt

- **Benefits of ex-ante inflation:** Manipulate bond prices + Seigniorage
Ramsey

- Can commit to not inflating away or diluting debt

- Main result: Optimal to use only long debt and have low inflation
  - Long debt allows better tax smoothing as it provides insurance
  - Low inflation implies small distortions to cash/credit

- As in Lustig, Sleet, and Yeltekin JME
Ramsey: Long debt provides insurance

Lifetime implementability constraint

\[ E_t \frac{1}{U_{2t}} \sum_{r=t} \beta^{r-t} [U_{1r} c_{1r} + U_{2r} c_{2r} + U_{\ell r} \ell_r] = \]

\[ \frac{U_{1t} M_{t-1}}{U_{2t} P_t} + \frac{B_{St-1}}{P_t} + [1 + \delta Q_{Lt}] \frac{B_{Lt-1}}{P_t} \]

PV revenues - PV expenditures = value money + value short + value long

- When \( g \) increases the PV expenditures go up making LHS falls
- Tax smoothing implies keep PV revenues as constant as possible
- RHS needs to fall but \( M_{t-1}, B_{St-1}, \) and \( B_{Lt-1} \) are uncontingent
Ramsey: Long debt provides insurance

- RHS of implementability constraint

\[
\frac{U_{1t}}{U_{2t}} \frac{M_{t-1}}{P_t} + \frac{B_{St-1}}{P_t} + \left[1 + \delta Q_{Lt}\right] \frac{B_{Lt-1}}{P_t}
\]

- How make RHS fall when \( g \) increases?

  - Increase inflation when \( g \) increases

    Inflate away debt by increasing \( P_t \) \( \rightarrow \) this is costly so don’t do much

  - Hold portfolio that has mostly long

    When \( g_t \) increases \( Q_{Lt} \) falls because \( c_{1t} \) and \( c_{2t} \) fall

- Fluctuations in value of long debt provides insurance

\[
Q_{Lt} = E \left[ \beta \frac{U_{2t+1}P_t}{U_{2t}P_{t+1}} (1 + \delta Q_{Lt+1}) \right]
\]

\[ABKR ()\]
Main problem: Government cannot commit to not inflate and not dilute long debt

Main result: Optimal to use only short debt and have high inflation

Why prefer short debt?
  - Ability to dilute long debt exacerbates commitment problem relative to short

Why high inflation?
  - Ex-post benefits of inflating away debt balanced with ex-post costs
Markov

- States \( z = \{ b_S, b_L, g \} = \left\{ \frac{B_{St-1}}{M_{t-1}}, \frac{B_{Lt-1}}{M_{t-1}}, g_t \right\} \)

- Current government controls:
  - Current policy: \( x = \{ \tau, b'_S, b'_L, \gamma \} \), where \( \gamma = \frac{M_t}{M_{t-1}} \)
  - Indirectly current allocations and prices through CE: \((c_1, c_2, \ell, Q_S, Q_L, p)\)

- Takes as given future government policy rules \( x(b'_S, b'_L, g') \)

- By altering debt, \( b'_S \) and \( b'_L \) indirectly influences future
  - Government policies \( x' \)
  - Future allocations and prices \((c'_1, c'_2, \ell', Q'_S, Q'_L, p')\)
Markov: Price of long debt

- Markov government every period manipulates $Q_L$ with its policies

$$Q_L = E \left[ \frac{U'_2(b'_S, b'_L, g')}{U_2(1 + \pi'(x; b'_S, b'_L, g'))} \left[ 1 + \delta Q'_L(b'_S, b'_L, g') \right] \right]$$

where inflation

$$1 + \pi'(x; b'_S, b'_L, g') = \gamma \frac{c'_1(b'_S, b'_L, g')}{c_1}$$

- To reduce $Q_L$ current government
  - Chooses high money growth rate
  - Passes to future government high debt which reduces $Q'_L(b'_S, b'_L, g')$

- Want high $Q_L$ when issuing long: want low $Q_L$ when owe long

$$g + b_S + (1 + \delta Q_L)b_L = \tau \ell + \gamma Q_S b'_S + \gamma Q_L b'_L + (\gamma - 1)$$
Markov Problem

Using all the CE conditions we rewrite the government budget

Government chooses $c_1, c_2, l, b'_S, b'_L$

$$V_M (b_S, b_L, g) = \max \left[ U (c_1, c_2, l) + \beta EV_M (b'_S, b'_L, g') \right]$$

$$c_1 + c_2 + g = l$$

$$U_1 \geq U_2$$

$$U_2 c_1 + U_2 c_2 + U_l l + \beta E \left[ U'_1 c'_1 \right] + \beta E \left[ (U'_2 c'_1)(b'_S + [1 + \delta Q'_L] b'_L) \right]$$

$$= U_2 c_1 \left[ 1 + b_S + \left( 1 + \delta \frac{E U'_2 c'_1 [1 + \delta Q'_L]}{E U'_1 c'_1} \right) b_L \right]$$

Government takes as given future functions determining allocations
Quantitative Results

- Shocks: $g = \{g_H, g_L\}$ and serially correlated

\[
U(c_1, c_2, \ell) = \frac{\left(\left[\left(1 - \gamma\right) c_1^\eta + \gamma c_2^\eta\right]^{(1-\psi)}/\eta \left(1 - \ell\right)^\psi\right)^{1-\sigma} - 1}{1 - \sigma}
\]

- Parameters: $\beta = 0.9774$, $\sigma = 4.25$, $\eta = -4.25$, $\gamma = 0.9763$, $\psi = 0.3$, $\delta = 0.8$
Quantitative Results

<table>
<thead>
<tr>
<th></th>
<th>Inflation</th>
<th>Duration</th>
<th>Long Share</th>
<th>Debt/Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ramsey</td>
<td>0.01%</td>
<td>4.7</td>
<td>0.99</td>
<td>47%</td>
</tr>
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<td></td>
<td>(1.06)</td>
<td>(0.13)</td>
<td>(0.03)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>Markov</td>
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<td>1.00</td>
<td>0.0</td>
<td>36%</td>
</tr>
<tr>
<td></td>
<td>(1.99)</td>
<td>(0.01)</td>
<td>(0.0)</td>
<td>(0.02)</td>
</tr>
</tbody>
</table>

Ramsey: Low inflation and long duration

Markov: High inflation and short duration
Markov: Transition Paths

With low debt have negative inflation (Friedman rule)
Accumulate debt and eventually switch to high inflation
Markov: Transition Paths

![Labor tax (%)](chart1)

![Money growth rate](chart2)

![Short interest rate](chart3)

![Long interest rate](chart4)
Markov: Transition Paths

Labor tax revenue

Seigniorage income

Net revenues from debt
Markov Counterfactual: Why prefer short?

Red: At period 1 government can only issue long debt
Blue: Standard Markov

G-Shock

Inflation rate (%)

Real short debt

Real value of long debt

ABKR ()
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Markov Counterfactual: Why prefer short?

- In period 1 the government
  - Issues long debt (which implies that the state in period 2 for short is zero and state for long is large)
  - Wants a good price for the long debt so it reduces money growth in period 1 to get low inflation in period 2
  - to raise $Q_L$ for new issuances

- In period 2 (and on)
  - Switch portfolio towards short
  - Wants to dilute and inflate away the value of long debt and buy back long debt
  - Hence: prints money and induce high inflation in period 3
  - Borrow a low of short to induce future governments to inflate
  - all which reduce $Q_L$ for existing debt
Markov Counterfactual: Why prefer short?

- Labor tax (%)
- Money growth rate
- Short interest rate
- Long interest rate
Markov Counterfactual: Why prefer short?

Cash goods

Credit goods

Labor
Markov Counterfactual: Why prefer short?

Labor tax revenue

Seigniorage income

Net revenues from debt
Markov Counterfactual: Why prefer short?

- Extra revenue from debt and seignorage lowers need to tax labor
- In face of low tax, labor and consumption increases in short run

Summary:

- Having long term debt exacerbates commitment problems
- Strategic interactions among governments generates high inflation and uneven path for allocations
Future plans/Conclusion

- Data begs for model with time varying credibility

- Current approach: Ramsey with added sustainability constraint

\[
U(c_1(s'), c_2(s'), \ell(s')) + \beta V(s', \phi'(s'), Q'_L(s')) \geq V^{\text{markov}}(b_S, b_L, s')
\]

  - When constraint binds act like Markov
  - When constraint slack act like Ramsey
  - Difficult computationally

- Alternative: Time varying parameters

  - As these fluctuate sometimes behave like Ramsey or Markov
  - Candidates: costs of inflation, discount factor of government
Extra slides for Paco: Recursive Ramsey

To solve Ramsey we follow the approach of Lustig, et al

Change of variables:
Let $\phi_t$ be the expected present value of government surpluses before the shock at $t$ is realized

$$\Lambda_t + \beta \phi_{t+1} = E_t \sum_{t=r} \beta^{r-t} [U_{1r}c_{1r} + U_{2r}c_{2r} + U_{lr}l_r]$$

with

$$\phi_{t+1} = E_t \sum_{r=t+1} \beta^{r-t} [U_{1r}c_{1r} + U_{2r}c_{2r} + U_{lr}l_r] = E_t [\Lambda_{t+1} + \beta \phi_{t+2}]$$

Normalize nominal variables by $M_{t-1}$ and use CIA with equality

Lifetime implementability under Ramsey

$$\Lambda_t + \beta \phi_{t+1} = [U_{1t}c_{1t} + U_{2t}c_{1t} (b_{St-1} + [1 + \delta Q_{Lt}] b_{Lt-1})]$$
Under Ramsey government keeps promises of $\phi_t$ and $Q_{L,t}$

Ramsey state variables: $\{\phi, Q_L, s_{-1}\}$

Government chooses $c_1(s), c_2(s), l(s), \phi(s), Q_L(s), b_S, b_L$

$$V_R(\phi, Q_L, s_{-1}) = \max E \left[ U(c_1, c_2, l) + \beta V_R(\phi', Q'_L, s) \right]$$

$$c_1(s) + c_2(s) + g(s) = l(s)$$

$$U_2(s) - U_1(s) \leq 0$$

$$U_2(s) c_2(s) + U_L(s) l(s) + \beta \phi(s) = U_2(s) c_1(s) \left[ b_S + (1 + \delta Q_L(s)) b_L \right]$$

Promise keeping constraints and both $\phi$ and $Q_L$

$$\phi = E[U_1(s) c_1(s) + U_2(s) c_2(s) + U_L(s) l(s) + \beta \phi(s)|s_{-1})]$$

$$Q_L = \frac{E(U_2(s)c_1(s)[1 + \delta Q_L(s)]|s_{-1})}{E(U_1(s)c_1(s)|s_{-1})}$$