Optimal Domestic Sovereign Default*

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Abstract

Infrequent but dramatic episodes in which governments default outright on their domestic creditors are an important and largely unexplained historical fact. This paper proposes an incomplete-markets, heterogeneous-agents model in which a utilitarian government chooses to default when the distributional benefits of default outweigh the costs in terms of hindered access to the vehicle for private self-insurance and government tax smoothing. We solve a Markov equilibrium in which the endogenous distribution of non-state-contingent public debt holdings across private agents interacts with the government’s default choice and the equilibrium risk premium. A quantitative analysis calibrated to data from Spain shows that debt, the concentration of its distribution across agents, and spreads rise sharply before default occurs. The debt ratio reaches 55% of GDP, less than in the data, but the low default frequency (1.13%), the ratio of domestic to total debt (75%) and the dynamics of spreads are in line with the data.

Keywords: Public debt, sovereign default, debt crisis, European crisis

JEL Classifications: E6, E44, F34, H63

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1 Introduction

A key finding of the seminal cross-country historical review of public debt data going back to 1750 by Reinhart and Rogoff [28] is that there were 68 episodes of *de jure* sovereign default on domestic debt (i.e. outright defaults by means other than *de facto* defaults via inflation).\(^1\) Furthermore, they document that, while these domestic defaults are less frequent than the more familiar external defaults, they are at least as important in terms of magnitude and the associated macro-economic instability, and many of them triggered external defaults. *De jure* domestic defaults took place via mechanisms ranging from forcible conversions to lower coupon rates to unilateral reduction of principal and suspension of payments. For example, Argentina defaulted three times on its domestic debt between 1980 and 2001. Two of these defaults coincided with external defaults (1982 and 2001) and a large scale domestic default in 1989 did not involve external debt. Other examples of recent domestic defaults include countries in Africa, Europe and the Middle East.

Understanding the dynamics of domestic public debt and default is critical because domestic debt accounts for a large fraction of total public debt in most countries (almost two-thirds on average), and, as Reinhart and Rogoff also noted, a heavy domestic debt burden can even help explain external defaults that occur when the external debt ratio seems low. In 2011, the global market of local-currency government bonds was valued at about U.S.$30 trillion, roughly 50 percent of the world’s GDP and 6 times larger than the market for investment-grade sovereign debt denominated in foreign currencies. Moreover, gross general government debt as a share of GDP reached about 100 percent for advanced economies, compared with 40 percent for developing economies.\(^2\)

The ongoing debt crisis in the Eurozone highlights further the pressing need to study *de jure* domestic sovereign default, because three features particular to the Eurozone make the potential sovereign default by some of its members more akin to a domestic default than an external default: First, a large fraction of Euro-zone governments’ bonds are held within Europe, so default by some European countries can be viewed as a (partial) domestic

\(^1\)Reinhart and Rogoff also noted in motivating their work that data on domestic public debt are hard to obtain, even tough domestic holdings of public debt are large. Domestic debt in our model corresponds to debt held by domestic residents, which is in line with the definition that some data sources use (OECD, for example, reports marketable central government debt held by country residents). But the breakdown of public debt in terms of the residence of holders is not always available or reliable. Alternatively, Reinhart and Rogoff defined domestic public debt based on whether the debt is issued under home-country legal jurisdiction or abroad, typically under New York or London law. These two definitions of domestic debt are correlated, but not perfectly, and in some episodes can look very different (as with the famous case of Mexico’s Tesobonos in 1994, which were issued under Mexican jurisdiction but with significant holdings outside of Mexico).

\(^2\)Global bond market and debt ratios are from *The Economist*, Feb. 11, 2012, based on data from Bank of America Merril Lynch and IMF.
default from the point of view of the Eurozone as a whole. Second, members of the Euro area share a common currency, which prevents them from unilaterally reducing the real value of debt through inflation (i.e. implementing country-specific de facto defaults). Third, our model is in line with key concerns in the handling of the Eurozone debt crisis related to the distributional implications of a default by one country on all of the Eurozone, and to the costs of the potentially long-lasting damage to the market of government bonds. On the other hand, the model is not like the Eurozone in that it assumes one single fiscal authority that collects all government revenues, whereas in the Eurozone this only applies to seigniorage collected by the European Central Bank, while tax revenues are controlled by each member nation. There are agreements that coordinate tax policies, and the recent accord to tighten fiscal ties brings Europe closer to being in line with our assumptions, but as of today the Eurozone still falls short from being characterized by a single fiscal authority.

Table 1: Euro Area Fiscal and Debt Situation in 2011

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<tr>
<td>France</td>
<td>98.62</td>
<td>31.87</td>
<td>53.30</td>
<td>50.60</td>
<td>-2.70</td>
<td>0.71</td>
</tr>
<tr>
<td>Germany</td>
<td>86.88</td>
<td>51.0</td>
<td>42.80</td>
<td>44.50</td>
<td>1.80</td>
<td>0.00</td>
</tr>
<tr>
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<td>165.10</td>
<td>30.4</td>
<td>44.80</td>
<td>42.40</td>
<td>-2.40</td>
<td>13.14</td>
</tr>
<tr>
<td>Ireland</td>
<td>112.57</td>
<td>14.42</td>
<td>44.80</td>
<td>34.90</td>
<td>-10.00</td>
<td>6.99</td>
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<tr>
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<td>127.74</td>
<td>55.4</td>
<td>45.10</td>
<td>46.20</td>
<td>1.20</td>
<td>2.81</td>
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<tr>
<td>Portugal</td>
<td>111.94</td>
<td>36.7</td>
<td>45.40</td>
<td>45.00</td>
<td>-0.40</td>
<td>7.63</td>
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<tr>
<td>Spain</td>
<td>74.13</td>
<td>58.5</td>
<td>42.70</td>
<td>35.70</td>
<td>-7.00</td>
<td>2.83</td>
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Note: Author’s calculations based on OECD Statistics, Eurostat and ECSB. “Gov. Debt” refers to Total General Government Gross Financial Liabilities; “Held by Res.” corresponds to Gross government debt held by domestic non-financial corporations, financial institutions, other government sectors, households and non-profit institutions; “Gov. Non-Int. Out.” is government non-interest outlays calculated as General Government Total Outlays minus interest payments; “Gov. Rev.” corresponds to general government revenues. “Prim. Balance” corresponds to the primary balance calculated as total revenues minus government outlays net of interest payments. “Sov Spreads” correspond to the difference between interest rates of the given country and Germany (for bonds of similar maturity). For a given country i, they are computed as $\frac{1+r_i}{1+r_{Ger}} - 1$.

3In the model we study, all the debt is repudiated when the government defaults, but it could be extended to consider an environment with partial defaults.

4This also applies to other default events in economies with currency boards (e.g. Argentina prior to the 2001 default) or economies that use a hard currency as medium of exchange (e.g. Ecuador or Panama). In contrast, the United States, which is not in much better fiscal shape than the Eurozone, has a flexible exchange rate and its currency is used globally, so the scope for a de facto default through inflation and devaluation is much larger.
Table 1 highlights some of the main facts of the Eurozone public debt and fiscal flows. This table shows that in 2011 the debt ratios of countries at the epicenter of the Eurozone debt crisis (Greece, Ireland, Italy, Portugal and Spain) ranged from 74 to 165 percent of GDP, and their spreads v. Germany were large, ranging from 2,800 to 13,100 basis points. Debt ratios in the large core countries, France and Germany, were also high at 98 and 87 percent respectively. Figure 1 shows that both debt ratios and spreads grew rapidly in most countries starting around the 2008 financial crisis (except in Italy, where the debt ratio was already high but spreads widened also after 2008). The fractions of each country’s debt held by residents of the same country ranged from 30 percent in Greece to 58 percent in Spain. At the level of the Eurozone as a whole, Lojsch, Rodríguez-Vives and Slavík [20] report that 48 percent of total government debt issued by Euro area countries was held by Euro area residents as of 2010, and 99.1 percent of this debt was denominated in Euros.\footnote{The 48 percent is only for Euro area members. The fraction is likely to be much higher if we include public debt holdings of European countries that are not Euro area members (particularly Denmark, Sweden, Switzerland, Norway, and the United Kingdom).}

Figure 1: Eurozone Debt Ratios and Spreads
Unfortunately, as Reinhart and Rogoff [28] highlighted, the record of *de jure* domestic sovereign defaults is a “forgotten history” that has remained largely unexplored. In this paper, we aim to make some progress towards explaining this phenomenon. In particular, we propose a model with heterogeneous agents and incomplete asset markets in which domestic defaults emerge as an optimal decision by a utilitarian government that trades off the benefits of default from the perspective of income and wealth re-distribution, with the costs in terms of hindered access to the asset that serves as the vehicle for private self-insurance and government tax smoothing. Intuitively, default occurs when the net welfare loss (i.e. the difference between the value of repayment and the value of default) that default imposes on those that are the government’s creditors is lower than the net gain provided to those that benefit from default. In contrast with external default models, however, the government’s social welfare function depends on the utility of all domestic agents, including domestic government bond holders.

The problem is complex, because under incomplete markets the dynamics of the wealth distribution (i.e. the distribution of public bond holdings across private agents), the probability of default, default risk premia, and the social welfare function are all endogenous and interact with each other. Moreover, even with purely idiosyncratic individual income shocks, sovereign default represents a form of aggregate risk, which implies that the household cross-section distribution of holdings of government bonds and income varies over time, making the distribution itself a state variable.

In the model, as in standard models of heterogeneous agents, government debt enhances the liquidity of households by providing a means for self-insurance and consumption smoothing. For simplicity, we assume that public debt is the only asset available to households and that households face idiosyncratic income shocks subject to a borrowing limit, as is also typical in standard heterogeneous agents models. The government faces an exogenous stochastic stream of expenditures, which is a second form of aggregate uncertainty, levies lump-sum taxes that are uniform across agents, and follows a fiscal reaction function to set the amount of one-period, non-state-contingent debt that is issued to finance its primary balance while retaining the option to default. We assume lump-sum taxes to highlight the role of the government’s distributional incentives for repayment without blurring the picture with the effects of tax distortions, and we use a fiscal reaction function to simplify in a tractable way the law of motion of the supply of public debt under repayment, while keeping it in line with standard tax smoothing principles and strong empirical evidence supporting the existence of fiscal reaction functions.

The government decides whether to default or not by comparing the social welfare attainable under repayment with that under autarky. To determine these payoffs, the government
uses a utilitarian social welfare function that aggregates the net welfare of repayment versus default across individuals displaying a preference for redistribution (i.e. relatively heavier weight on poorer agents). Taxes are also determined by the default decision, because under autarky they need to adjust to match the realization of stochastic government expenditures, and under repayment they adjust to match any difference between these expenditures and the financing allowed by the evolution of government debt, as pinned down by the fiscal reaction function, and the equilibrium price of debt.

The net welfare gain of debt repayment v. default is not equally spread among the population, because of endogenous inequality in the wealth distribution due to the incomplete asset market structure. Domestic default has important distributional consequences because defaults are not selective in the model (i.e. the government cannot default on a particular borrower or group of borrowers), and holdings of public debt are unevenly distributed. If the government defaults, the fact that debt service expenses are skipped implies a short-run cut in lump-sum taxes, which represents a relatively larger benefit, in utility terms, for relatively poorer agents. Default also wipes out public debt holding, and thus reduces the wealth of those with positive holdings of debt, which include both domestic and foreign creditors, and this hurts more relatively richer agents. These distributional effects represent the benefits of default as perceived from the standpoint of a sovereign with a utilitarian welfare function that weighs the poor more heavily. The sovereign, however, trades these benefits against two important costs of default: Since once in autarky the government cannot issue new debt, default prevents the government from accessing debt markets to smooth taxation, and it prevents private agents from trading in the only asset that serves as the vehicle for self-insurance. These costs are likely to be significant, as the high estimates of the social value of government debt produced by Aiyagari and McGrattan [3] in a model of heterogeneous agents without default risk suggest.

There are also distributional effects at work if the government repays. Conditional on public debt repayment, taxes represent a larger fraction of total disposable income for those with low debt holdings. Hence, these agents are better off if the government finances debt repayments by borrowing more. Moreover, the tax smoothing that the government can undertake by borrowing more when it repays also facilitates private consumption smoothing, and the access to a supply of assets for self-insurance is valued by agents seeking to build precautionary savings. The cost of increasing the debt is that expected future taxes are higher and a default more probable. We find that in the early stages of public debt accumulation, the government can smooth taxation effectively and borrowing costs remain low. In contrast, as debt grows large, a “debt overhang effect” takes over: the higher default probability results in higher risk premia that impede the government from smoothing taxation, because higher
tax revenues are needed to keep up with debt service.

We study the quantitative predictions of the model using a set of parameter values calibrated to match relevant statistics for Spain, one of the large Eurozone members currently facing problems in sovereign debt markets. In the equilibrium produced by this baseline calibration, a large fraction of households are credit constrained, and thus repaying the debt is highly beneficial for the reasons described above. Still, higher public debt increases the need for tax revenues in the future and over time wealth inequality increases. Default thus arises along the equilibrium path when the debt is high enough and the distribution of public bond holdings is sufficiently concentrated. Government debt is about 34 percent of GDP on average, but once the debt reaches about 55 percent of GDP, and conditional on the debt being concentrated in a small fraction of agents, the government finds it optimal to default. This occurs with a frequency of about 1.13 percent. These results contrast with typical findings of the quantitative literature on external sovereign default, where exogenous default costs in the form of exclusion of stochastic length and convex, increasing income costs are generally needed to produce non-negligible debt ratios at realistic default frequencies. The model is also in line with the data in producing rising spreads between the interest rate on public debt and the risk-free rate as debt builds up leading to a default episode, and it matches the observed ratio of public debt held by domestic agents to total debt.

The analysis undertaken in this paper aims to contribute to three important strands of the Macroeconomics literature: (1) Studies of the wealth distribution and private savings decisions based on heterogeneous agents models with incomplete asset markets; (2) work in the International Macro field on quantitative models of sovereign default; and (3) macro-public finance studies on fiscal solvency and public debt dynamics.

The model we propose introduces sovereign default into a variant of the workhorse heterogeneous agents model of the wealth distribution under incomplete markets. To date, however, the literature on this subject has not examined jointly domestic public debt subject to default risk and the evolution of the wealth distribution, particularly the distribution of government bond holdings. Our model is similar to those studied by Aiyagari and McGrattan [3] and Heathcote [17] in that government debt plays a key role as the vehicle private agents use for self insurance (against idiosyncratic shocks in Aiyagari and McGrattan, and both idiosyncratic and aggregate in Heathcote) and the distribution of debt holdings across agents is endogenous. However, they abstract from sovereign default and emphasize the stationary equilibrium, while in our analysis the transitional dynamics of the wealth distribution are a key determinant of the government’s default. A related literature initiated by Aiyagari, Marcet, Sargent and Seppala [2] studies optimal taxation and public debt dynamics with aggregate uncertainty and incomplete markets, but in a representative-agent
environment. Pouzo [27] incorporates default and renegotiation into a similar framework. Corbae, D’Erasmo and Kuruscu [10] examine optimal taxation in a setting with heterogeneous agents but without default, where there is an endogenous feedback loop connecting the wealth distribution and the set of optimal policies.\(^6\)

Relative to the external sovereign default literature, our work adds to the growing quantitative studies that up to now have been largely based on the classic model of Eaton and Gersovitz [13], in which a benevolent government maximizing the utility of a representative agent has the option to default on external lenders (see, for example Aguiar and Gopinath [1], Arellano [4]).\(^7\) Some studies in this area have examined the role of tax and expenditure policies and settings with foreign and domestic lenders, but always maintaining the representative agent assumption (e.g. Cuadra, Sanchez and Sapriza [11]), and recently Dias, Richmond and Wright [12] have examined the benefits of debt relief from the perspective of a global social planner with utilitarian preferences. Another important strand of the external default literature focuses on the consequences of default on domestic agents, the role of secondary markets and discriminatory v. nondiscriminatory default (see for example Guembel and Sussman [16], Broner, Martin and Ventura [9] and Gennaioli, Martin and Rossi [14]). As in these studies, default in our setup is non-discriminatory, because the government sells bonds to both foreign and domestic agents, but when it defaults it cannot discriminate between them. Our analysis differs in that the default decision is driven by the dynamics of the distribution of debt among domestic agents, and that these dynamics are also influenced by default expectations, because they determine risk premia, which affect savings decisions.

Finally, this paper is also related to classic studies of public debt dynamics and fiscal solvency which generally assume that debt is always honored (e.g. Barro [6], Bohn [8]). In particular, the law of motion driving the supply of government debt in our model follows from Bohn’s seminal work on fiscal reaction functions. This ensures that the government’s intertemporal government budget constraint holds in general, because the reaction function satisfies Bohn’s sufficiency condition for fiscal solvency, so that default in our model is always a strategic choice, and not a necessity imposed by inability to pay. In the quantitative analysis, the reaction function is parametrized using empirical estimates taken from the large literature that has provided evidence showing that the data of several industrial and emerging economies fits well Bohn’s fiscal reaction function (e.g. Mendoza and Ostry [23]).

The rest of this paper is organized as follows: Section 2 describes the model and defines the recursive Markov equilibrium we solve for. Section 3 provides two simple examples,

\(^6\) The algorithm we use to solve the model extends the one they proposed, which in turn is an extension of the algorithm proposed by Krusell and Smith [19]. Bachmann and Bai [5] use a similar algorithm in a model with aggregate uncertainty, heterogeneous agents and endogenously determined government policies. 

\(^7\) See Panizza, Sturzenegger and Zettelmeyer [25] for a recent review of this literature.
one that illustrates the mechanics of the distributional default incentives in the absence of
uncertainty, and one that quantifies the social value of public debt as a vehicle for self-
insurance in a setup without default risk. Section 4 discusses the calibration to data from
Spain and examines the models quantitative implications. Section 5 provides conclusions,
and we end with an Appendix that provides details on the solution method.

2 A Model of Domestic Sovereign Default

We study a small open economy populated by a continuum of agents with aggregate unit mea-
sure and a utilitarian government. There is also a pool of risk-neutral international investors
modeled in the same way as in the Eaton-Gersovitz class of external default models, that face
an opportunity cost of funds equal to the world-determined real interest rate. These investors
are introduced so as to simplify the pricing condition for government debt. Domestic agents
are ex-ante identical but heterogeneous ex-post because of two forms of non-insurable shocks:
income fluctuations that are purely idiosyncratic, and aggregate shocks in the form of fluc-
tuations in government expenditures and the possibility of sovereign default. Asset markets
are incomplete because the government issues one-period, non-state-contingent bonds and
this is the only vehicle of savings. The government also levies lump-sum taxes and chooses
whether to repay public debt or not. Both domestic households and international investors
participate in the market of government debt, and the government cannot discriminate one
from the other when it chooses to default.

2.1 Households

The preferences of individuals are defined with the standard CRRA utility function of the
heterogeneous agents literature:

\[ U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c_t) = c_t^{1-\sigma} / (1 - \sigma) \]

where \( \beta \in (0,1) \) is the discount factor, \( c_t \) is individual consumption and \( \sigma \) is the coefficient
of relative risk aversion.

Every period, each agent receives a stochastic income realization drawn from a bounded, non-negative set: \( y_t \in \mathcal{Y} \). Aggregate income is non-stochastic, and idiosyncratic income evolves as first-order Markov process with realization set given by \( \{\bar{y}, \ldots, \bar{y}\} \) and a tran-
sition probability matrix defined as \( \pi(y_{t+1}, y_t) \) with stationary distribution \( \pi^*(y) \). In the
quantitative exercise, this Markov processes is calibrated to represent an AR(1) process in
logs with coefficients set to empirical estimates of the individual earnings process:

\[
\log(y_{t+1}) = (1 - \rho_y) \log(\mu_y) + \rho_y \log(y_t) + u_t
\]

where \(|\rho_y| < 1\) and \(u_t\) is i.i.d. over time and distributed normally with mean zero and standard deviation \(\sigma_u\). The discrete Markov representation of this process is obtained using the well-known quadrature method developed by Tauchen [29].

Households can hold positions in one-period, discount government bonds defined by \(b_{t+1} \in B \equiv [0, \infty)\). Since agents are not allowed to take short positions in these bonds, and there is no other asset in the model, effectively they are not allowed to borrow. The price of these bonds is labeled \(q\). The distribution of households over debt and income (i.e. the wealth distribution) at a point in time is defined as \(\Gamma_t(b, y)\).

The government can choose to default on its outstanding debt depending on the solution of its optimization problem, which we describe later. If the government chooses not to default, each individual agent’s budget constraint is:

\[
c_t + q_t b_{t+1} = y_t + b_t - \tau_t.
\]

(1)

At the beginning of period \(t\), the agent collects income from the payout on its individual holdings of government debt \(b_t\) and from its idiosyncratic income realization, and pays lump-sum taxes \(\tau_t\). This net-of-tax resources are used to pay for consumption and the agent’s purchases of new government bonds \(b_{t+1}\). If the government defaults, the agents’ budget constraint becomes simply:

\[
c_t = y_t - \tau_t.
\]

(2)

Thus, default wipes out the agents’ holdings of government bonds and the public debt market freezes, so the agent cannot purchase new bonds.\(^8\)

2.2 Government

Every period, the government issues bonds from positions defined over a non-negative set \(B_{t+1} \in B \equiv [0, \infty)\), collects revenues from lump sum taxes and pays for exogenous stochastic expenditures \(g_t\). We abstract from modeling government welfare and entitlement payments, and simply roll net out these transfers from lump sum taxes (i.e. these taxes represent the

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\(^8\)While we leave the option of partial default for future research, it is important to note that our environment is capable of accommodating partial default (or reductions in the real value of the debt via inflation). In that case, bond positions would be reduced proportionally by the fraction defaulted by the government and the marginal benefit of one more unit of savings will take this into account the partial repayment of the government. Moreover, the evolution of the wealth distribution will be consistent with the partial default outcome.
gap between all tax revenue and transfer payments). We do not restrict the sign of \( \tau_t \), so when \( \tau_t < 0 \) we will refer to it as a lump-sum transfers.

Government expenditures evolve according to a finite, discrete Markov process with realizations defined over the set \( \mathcal{G} = \{g, \ldots, g\} \) and associated transition probability matrix \( F(g_{t+1}, g_t) \).

Government expenditure shocks are persistent but independent of households’ income shocks. As with the income shocks, in the quantitative analysis we set the Markov process of \( g \) to represent an AR(1) log process estimated with actual data:

\[
\log(g_{t+1}) = (1 - \rho_g) \log(\mu_g) + \rho_g \log(g_t) + \epsilon_t
\]

where \( |\rho_g| < 1 \) and \( \epsilon_t \) is i.i.d. over time and distributed normally with mean zero and standard deviation \( \sigma_\epsilon \).

As in [3], we simplify the heterogeneous agents setting with public debt, which is more complex than in their setup because of the presence of aggregate risk, by abstracting from modeling explicitly the government’s decision to issue debt. In contrast with Aiyagari and McGrattan, however, we do not assume an exogenous constant level of \( B \), but instead adopt a law of motion to represent the government’s debt issuance decision in reduced form. In particular, we assume a debt law of motion derived from Bohn’s ([7], [8]) fiscal reaction function (FRF) framework, which is characterized by a conditional positive, linear response of the primary fiscal balance to increases in public debt, controlling also linearly for the cyclical components of government purchases and output. Bohn showed that the FRF is sufficient to guarantee that the government’s intertemporal government budget constraint holds, which ensures that if default is an equilibrium outcome in our model, it is because of strategic reasons and not because of inability to pay. Moreover, Bohn demonstrated that the FRF can be derived as the optimal policy in a simple government optimization setup consistent with Barro’s [6] classic tax smoothing theory, and indeed tax smoothing will be found to be a feature of our model’s equilibrium dynamics.

There is also ample empirical evidence in support of the FRF in historical U.S. data going back to 1790 (Bohn [7], [8]) and in cross-country panel and time-series studies (Mendoza and Ostry [23], Mauro et al. [22], Ghosh et al. [15]).

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9We also need to truncate the process at an upper bound \( \bar{y} \leq y - \zeta \) where \( \zeta \) is a small number. This is so as to prevent situations in which households that draw the lowest income level and have zero bond holdings consume below \( \zeta \), which drives them close to the Inada condition of the CRRA function where marginal utility explodes. Since \( \zeta \) is a small number, this restriction has negligible effects on the properties of the approximated government expenditures process. In fact, in the current calibration this is not a binding constraint (see Section 4 for details).

10This approach to adopt a fiscal reaction function to characterize the law of motion of debt is also akin to the approach followed in several monetary DSGE models in which the Taylor rule is introduced to characterize the evolution of the central bank’s policy instrument, instead of explicitly modeling the bank’s underlying optimization problem.
In our model, since aggregate income is non-stochastic, entitlement payments are netted out of taxes, and government expenditures are stochastic, the FRF takes this form:

\[(\tau_t - g_t) = \alpha_0 + \alpha_1 (g_t - \bar{g}) + \rho B_t,\]  

(3)

where \(\bar{g}\) denotes the long-run average of government expenditures. As noted above \(\rho > 0\) is sufficient for fiscal solvency, \(\alpha_1\) is generally negative because of the use of government expenditures to lean against business cycles, and \(\alpha_0 < 0\) because it is necessary to support a positive long-run average of public debt.\(^{11}\)

Using the FRF to substitute for the primary balance in the government budget constraint \((\tau_t - g_t = B_t - q_t B_{t+1})\), and abstracting for simplicity from the effect of time-series variations in borrowing costs, we obtain the following law of motion for debt:\(^{12}\)

\[B_{t+1} = (1 - \rho^B)\mu_B + \rho^B B_t + \alpha^g (g_t - \bar{g})\]  

(4)

where \(\mu_B\) is the long-run average of government debt, \(\rho^B \propto (1 - \rho)\) and \(\alpha^g \propto -\alpha_1\).

The government is not committed to repay the debt. At the beginning of each period, it chooses to default or not on the outstanding debt \(B_t\), and the choice is denoted by the binary variable \(d\) (with \(d = 1\) being default). When deciding to default or not, the government behaves as a benevolent planner who maximizes a standard utilitarian social welfare function. This social welfare function aggregates the utility of each individual agent, identified by its wealth and income \((b, y)\), assigning to each agent a weight according to this joint cumulative distribution function:

\[\omega(b, y) = \sum_{y_i \leq y} \pi^*(y_i) \left(1 - e^{-\frac{b}{\bar{w}}}\right),\]  

(5)

In the income dimension, the distribution is given by the long-run distribution of income \(\pi^*(y)\), and in the bonds dimension the distribution is given by an exponential function with parameter \(\bar{\omega}\), which summarizes the government’s concern for wealth inequality. A lower \(\bar{\omega}\) implies that the government puts more weight on the utility of the relatively poor, and thus cares more for wealth inequality.

After the default decision is made, the level of taxes and debt are determined. If the government does not default at the beginning of \(t\), the law of motion implied by the FRF determines the supply of new public debt to be issued \(B_{t+1}\) and taxes are set as needed to

\(^{11}\)Stationarity also requires \(\rho^B\) to be both positive and larger than the interest rate, while fiscal solvency only requires the former (see Bohn [8]).

\(^{12}\)The exact law of motion would feature time varying coefficients that change with bond prices: \(\rho_t^B = (1 - \rho)/q_t\), \(\alpha_t^g = -\alpha_1/q_t\), and \(\mu_t^B = -\alpha_0/[q_t - (1 - \rho)]\), but the one with time-invariant coefficients is more tractable and converges to the same long-run average.
satisfy the government budget constraint:

\[ \tau_t^{d=0} = B_t + g_t - q_t B_{t+1}. \] (6)

If the government defaults, the market for government bonds closes and re-opens the following period. At default, all public debt contracts are broken and the government cannot discriminate a particular household or set of households when setting taxation, debt and default policies. Taxes are again set to satisfy the government budget constraint, which in this case becomes:

\[ \tau_t^{d=1} = g_t. \] (7)

2.3 International Investors

International investors are modeled as in the standard Eaton-Gersovitz model of external sovereign default. They are risk-neutral agents with access to unlimited funds and willing to invest in the domestic debt market an amount \( \hat{B}_{t+1} \), which defines also the economy’s net foreign asset position, as long as the expected return is equal to their opportunity cost of funds (i.e. the international rate of return \( \bar{r} \)). We assume that the size of the domestic debt market is small relative to the available funds to investors, so in equilibrium, bond prices will be such that expected profits \( \Omega_t \) for the marginal international investor are zero. Expected profits at date \( t \) are given by:

\[ \Omega_t = -q_t \hat{B}_{t+1} + \frac{(1 - p_t)}{(1 + \bar{r})} \hat{B}_{t+1} \]

where \( p_t \) is the probability of default at \( t + 1 \) perceived as of date \( t \). The term \( -q_t \hat{B}_{t+1} \) represents the cash flow lent to the government at date \( t \) and the term \( \frac{(1 - p_t)}{(1 + \bar{r})} \hat{B}_{t+1} \) is the expected present value of the payout on government debt at \( t + 1 \), which is expected to occur with probability \( (1 - p_t) \).

2.4 Timing of transactions

The timing of decisions and market participation in the model is as follows:

1. At the start of date \( t \), the distribution of wealth and income across households is given by \( \Gamma_t(b, y) \).
2. The realization of \( g_t \) is observed.
3. The government chooses to default or not
• If the government chooses not to default \( d_t = 0 \), the government repays its outstanding debt \( B_t \), the market of government bonds opens, new debt is issued according to the law of motion for \( B_{t+1} \), and taxes are determined according to equation (6). Also, households decide how much to save \( b_{t+1} \) taking as given the price \( q_t \).

• If the government defaults \( d_t = 1 \), the debt market closes and taxes are determined according to equation (7).

4. Agents pay taxes and consume, and date \( t \) ends.

2.5 Recursive Markov Equilibrium given Government Policies

We start by characterizing a Recursive Markov Equilibrium (RME) given a set of government policy rules, which include the default decision rule (which will be endogenized in the following Section), and the fiscal policy rules driving the supply of public debt (eq. (4)) and taxes (eqs. (6) and (7)). As is standard practice, the recursive notation denotes any variable \( x_t \) or \( x_{t+1} \) as \( x \) and \( x' \) respectively.

The aggregate state variables of the economy at any date \( t \) are \( \Gamma, B, \) and \( g \). The default decision rule that agents take as given in recursive form is defined as the recursive binary function \( d(\Gamma, B, g) \). Agents also take as given a conjectured law of motion for the wealth-income distribution \( \Gamma' = H^{d' \in \{0, 1\}}(\Gamma, B, g, g') \), where \( d' \) denotes the default decision the following period. The expected default probability (i.e. the probability of default at \( t + 1 \) evaluated as of \( t \)) can then be represented in recursive form as the function \( p(\Gamma, B, g) \) defined by:

\[
p(\Gamma, B, g) = \sum_{g'} d(\Gamma', B', g') F(g', g).
\]

For a current triple of aggregate states \( (\Gamma, B, g) \), the default probability is formed by pinning down the relevant values of \( \Gamma' \) and \( B' \) using \( H^{d' \in \{0, 1\}}(\Gamma, B, g, g') \) and the debt law of motion (4) respectively, and then taking the expected value of \( d(\Gamma', B', g') \) over \( g' \) using the transition probabilities \( F(g', g) \). Since \( d \) is a binary function equal to 1 if the government chooses to default and 0 otherwise, this expected value equals the cumulative probability of \( F(g', g) \) across the realizations of \( g' \) for which \( d(\Gamma', B', g') = 1 \).

2.5.1 Households’ Problem

The state variables relevant for the optimization problem of an individual agent include the agent’s own bond holdings and income \( (b, y) \) and the aggregate states \( \Gamma(b, y), B, \) and \( g \). In addition to \( d(\cdot) \) and \( \Gamma' = H^{d' \in \{0, 1\}}(\cdot) \), agents take as given the fiscal policy rules for taxes and
public debt in recursive form, a recursive bond pricing function \( q(\Gamma, B, g) \), and the Markov processes of \( y \) and \( g \). All these objects allow agents to project the evolution of aggregate states and bond prices, so that the continuation value for an individual household if the government has chosen to repay \( (d(\Gamma, B, g) = 0) \) can be characterized by the solution to the following problem:

\[
\begin{align*}
V^{d=0}(b, y, \Gamma, B, g) &= \max_{c \geq 0, b' \geq 0} \left\{ u(c) + \beta E_{(y', g')} [V(b', y', \Gamma', g')] \right\} \\
\text{s.t.} & \left\{ \begin{array}{l}
\tau^{d=0} = B + g - q(\Gamma, B, g)B' \\
\Gamma' = H^{d \in \{0, 1\}}(\Gamma, B, g, g').
\end{array} \right.
\end{align*}
\] (8)

where \( V(b', y', \Gamma', g') \) without superscript is the next period’s continuation value for the household before the default decision of the government has been made that period. The continuation value if the government chooses to default is simply:

\[
\begin{align*}
V^{d=1}(y, g) &= u(y - \tau^{d=1}) + \beta E_{(y', g')} [V^{d=0}(0, y', 0, g')] \\
\text{s.t.} & \left\{ \tau^{d=1} = g. \right.
\end{align*}
\] (9)

Finally, the continuation value at date \( t \) evaluated before the default decision has been made is given by:

\[
V(b, y, \Gamma, B, g) = (1 - d(\Gamma, B, g))V^{d=0}(b, y, \Gamma, B, g) + d(\Gamma, B, g)V^{d=1}(y, g). \] (10)

The solution to the above dynamic programming problem yields the individual decision rule \( b' = h(b, y, \Gamma, B, g) \) as well as value functions \( V(b, y, \Gamma, B, g), V^{d=0}(b, y, \Gamma, B, g) \) and \( V^{d=1}(y, g) \). By combining the household bond decision rule, the Markov transition matrices of \( y \) and \( g \), and the government default decision, we obtain the transition function \( \Gamma' = H^{d \in \{0, 1\}}(\Gamma, B, g, g') \). If \( d(\Gamma', B', g') = 0 \), for \( B_0 \subset B, Y_0 \subset Y, \Gamma' \) is:

\[
\Gamma'(B_0, Y_0) = \int_{y' \in Y_0, b' \in B_0} \left\{ \int_{Y, B} I_{\{b' = h(b, y, \Gamma, B, g) \in B_0\}} \pi(y', y) d\Gamma(b, y) \right\} db' dy'. \] (11)

Note that a state dated \( t + 1, g' \), is an argument of \( H^{d \in \{0, 1\}} \) because the realization of \( \Gamma' \)
arrives after $d'$, which depends on $g'$. If $d(\Gamma', B', g') = 1$, for $Y_0 \subset Y$, $\Gamma'$ is given by:

\[
\Gamma'([0], Y_0) = \int_{y' \in Y_0} \left\{ \int_{Y, B} \pi(y', y) d\Gamma(b, y) \right\} db' dy',
\]

and zero otherwise. This is because at default all households’ bond positions are set to zero, and hence $\Gamma'$ is determined only by the evolution of the income process (i.e. if the government defaults, $\Gamma'(b, y) = \pi^*(y)$ for $b = 0$ and zero for any other value of $b$).

A central feature of the individual agent’s problem is that the possibility of default affects the agent’s optimal plans. This effect is easy to identify by analyzing the first-order condition for $b'$ of the problem that defines $V^{d=0}(b, y, \Gamma, B, g)$ (assuming the value functions are differentiable):

\[
-u'(c') q_{(\Gamma, B, g)} + \beta E \left[ y'_{(y, g)} \right] [V_1(b', y', \Gamma', g')] \leq 0, \quad = 0 \text{ if } b' > 0
\]

where $V_1(\cdot)$ denotes the derivative of the corresponding value function with respect to its first argument. Using the envelope condition, this condition can be expressed as:

\[
u'(c') \leq \beta E_{(y', g')} [1 - d(\Gamma', B', g')] \frac{u'(c')}{q(\Gamma, B, g)}
\]

which holds with equality if $b' > 0$. The term in the right-hand-side of this expression shows that, in assessing the marginal benefit of holding an extra unit of government bonds, agents take into account the default decision at $t + 1$. In states in which there is default, $d(\Gamma', B', g') = 1$ and agents assign zero marginal benefit to holding bonds. If there is no default, the marginal benefit of saving in bonds is $u'(c') \frac{q_{(\Gamma, B, g)}}{q(\Gamma, B, g)}$, which includes the default risk premium embedded in bond returns $(1/q(\Gamma, B, g))$ even when the government does repay.

**2.5.2 International Investors**

International investors maximize profits taking bond prices and the default probability as given. The recursive formulation of expected profits when they buy government debt $\hat{B}'$ is:

\[
\Omega(\Gamma, B, g) = -q(\Gamma, B, g) \hat{B}' + \frac{(1 - p(\Gamma, B, g))}{(1 + \bar{r})} \hat{B}'
\]

The first-order-condition of this maximization problem yields the standard arbitrage condition typical of models of external default:

\[
q(\Gamma, B, g) = \frac{(1 - p(\Gamma, B, g))}{(1 + \bar{r})}.
\]
Hence, risk-neutral arbitrage against the opportunity cost of funds requires a wedge between the price at which foreign investors are willing to buy government debt \( q(\cdot) \) and the price of international bonds \( 1/(1 + \bar{r}) \) that compensates them for the risk of default implied by the default probability.

### 2.5.3 Recursive Markov Equilibrium with given government policies

Define aggregate consumption as:

\[
C = \int_{y \times B} c \ d\Gamma(b, y),
\]

where \( c \) corresponds to individual consumption of domestic households. Similarly, define aggregate (nonstochastic) income as:

\[
Y = \int_{y \times B} y \ d\Gamma(b, y),
\]

and the domestic demand for government bonds as:

\[
B^d = \int_{y \times B} b \ d\Gamma(b, y)).
\]

With these definitions at hand, we can define a recursive equilibrium given government policies as follows:

**Definition:** Given a default decision rule \( d(\Gamma, B, g) \), a law of motion of debt \( B'(B, g) \) implied by (4), and tax policies \( \tau^{d \in \{0, 1\}} \) defined by (6) and (7), a **Recursive Markov Equilibrium** (RME) is defined by a value function \( V(b, y, \Gamma, B, g) \) with associated household decision rule \( b' = h(b, y, \Gamma, B, g) \), a transition function for the wealth-income distribution \( H^{d \in \{0, 1\}}(\Gamma, B, g, g') \), a default probability function \( p(\Gamma, B, g) \), and a price function \( q(\Gamma, B, g) \) such that:

1. Given prices and policies, \( V(b, y, \Gamma, B, g) \) and \( h(b, y, \Gamma, B, g) \) solve the households’ problem.

2. The foreign investors’ arbitrage condition holds

\[
q(\Gamma, B, g) = \frac{(1 - p(\Gamma, B, g))}{(1 + \bar{r})}
\]

3. If \( d(\Gamma', B', g') = 0 \), the transition matrix of the wealth distribution \( H^{d \in \{0, 1\}}(\Gamma, B, g, g') \)
is given by:

$$\Gamma'(B_0, \mathcal{Y}_0) = \int_{y' \in \mathcal{Y}_0, b' \in B_0} \left\{ \int_{y \in \mathcal{Y}, b \in B} I_{\{y' = h(b, y; \Gamma, B, g) \in B_0\}} \pi(y', y) d\Gamma(b, y) \right\} db' dy'$$

for $B_0 \subseteq B, \mathcal{Y}_0 \subseteq \mathcal{Y}$.

4. The government budget constraints (6) and (7) hold.

5. The market of government bonds clears:  

$$\hat{B} = B^d - B.$$  

(21)

6. The aggregate resource constraint is satisfied. In the no default periods, this constraint is given by:

$$C + g = Y + \hat{B} - q(\Gamma, B, g)\hat{B},$$

(22)

and in the default states it is:

$$C + g = Y.$$  

(23)

### 2.6 Recursive Markov Equilibrium with Endogenous Policies

We now characterize a Recursive Markov Equilibrium in which the government formulates its default decision optimally. Since there is no commitment on the part of the government, it optimizes its default policy every period. By construction, this equilibrium is a subset of the RME in which $d(B, \Gamma, g)$ is the solution of a government maximization problem. The evolution of debt and taxes are consistent with the rule presented in equation (4) and the government budget constraints (6) and (7).

If $B > 0$ at the beginning of period $t$, the government chooses whether to default or not. The optimal default decision is the solution to the following problem:

$$\max_{d \in \{0,1\}} \{ W^{d=0}(B, \Gamma, g), W^{d=1}(g) \}$$

(24)

where

$$W^{d=0}(B, \Gamma, g) = \int_{\mathcal{Y} \times B} V^{d=0}(b, y, \Gamma, B, g) d\omega(b, y),$$

$\{ W^{d=0}(B, \Gamma, g), W^{d=1}(g) \}$

Note that as long $\hat{B} > 0$ the country is a net borrower (i.e. the amount of bonds issued by the government is lower than the domestic demand of government bonds) and when $\hat{B} < 0$ the country becomes a net saver. We will also denote the fraction of domestic debt (as a fraction of the supply of government bonds) by

$$\frac{\int_{\mathcal{Y} \times B} b d\omega(b, y))}{B}.$$  

To be consistent with national accounting, since our model does not incorporate physical capital we normalize mean income $Y$ such that represents GDP net of fixed capital investment.
and

\[ W^{d=1}(g) = \int_{y=B} V^{d=1}(y, g) d\omega(b, y). \]

As noted earlier, the government’s payoff functions \( W^{d=0}(B, \Gamma, g), W^{d=1}(g) \) follow from a utilitarian social welfare function in which each agent’s utility is weighed using \( \omega(b, y) \). Notice that by defaulting the government not only removes all the dispersion in wealth (since everyone ends up with zero bond holdings), but it also affects the level of taxes and disposable income, effectively redistributing resources from those with a positive level of government debt to those with no government debt.

**Definition:** A Recursive Markov Equilibrium with Endogenous Policies is an RME in which the government default decision \( d(B, \Gamma, g) \) solves problem (24).

## 3 Distributional Incentives and the Value of Self-Insurance: Simple Examples

This Section provides two simple examples that illustrate two central elements of the model: The distributional incentives of the government when considering the default decision, and the social value of public debt as a vehicle for self-insurance. The first example isolates the distributional incentives by reducing the model to a one-period model without uncertainty, abstracting from the tradeoffs due to self-insurance and tax smoothing, and from the interaction between the wealth-income distribution, risk premia and default. The second example quantifies the welfare effects that would result from closing the market of government bonds as the only vehicle for precautionary savings in the stationary equilibrium of a variant of the model without default. This experiment is analogous to the one conducted by Aiyagari and McGrattan ([3]), except the model we examine abstracts from production and capital accumulation.

### 3.1 Distributional default incentives

Consider a one-period snapshot of a perfect-foresight variant of the model described in the previous Section. Assume that the government has issued an amount of debt \( B \), and that the distribution of wealth is exogenous and reduced to two types of households. A fraction \( \gamma \) of households are “low-wealth” (\( L \)) individuals who hold \( b^L \), and a fraction \((1 - \gamma)\) are “high-wealth” (\( H \)) agents who hold \( b^H = \frac{B - \gamma b^L}{1 - \gamma} \), which is the amount needed to clear the market of public debt. Income is non-stochastic and constant across households at the amount \( y \), but we allow for the possibility that default may entail an exogenous cost that reduces income.
by a fraction $\phi \geq 0$.

The budget constraints of the government and households under repayment are $\tau^{d=0} = g + B$ and $c^i = y - \tau^{d=0} + b^i$ (for $i = L, H$) respectively, and under default are $\tau^{d=1} = g$ and $c^i = (1 - \phi)y - \tau^{d=1}$ (for $i = L, H$) respectively. The utility function can be as in Section 2, but all that is necessary for the results in this example is that it be increasing and strictly concave.

Since we are focusing solely on the distributional incentives to default, we abstract from modeling how private agents choose $b^L$ and $b^H$, and how the corresponding equilibrium allocations are determined. Hence, we take as given an exogenous “decentralized” distribution of debt holdings that would be consistent with those allocations. This distribution is characterized by a parameter $\epsilon$ that divides bond holdings as follows: $b^L = B - \epsilon$ and $b^H = B + \frac{\gamma}{1 - \gamma}\epsilon$. Since we are still assuming agents cannot borrow, we need $\epsilon \leq B$, and notice that if $H$ types are truly the high-wealth agents we would also want $b^H \geq b^L$, which requires $\epsilon \geq 0$.

Combining these bond allocations with the budget constraints, decentralized consumption allocations under repayment are $c^L(\epsilon) = y - g - \epsilon$ and $c^H(\gamma, \epsilon) = y - g + \frac{\gamma}{1 - \gamma}\epsilon$, and under default they are $c^L = c^H = y(1 - \phi) - g$. Notice that under repayment, $\epsilon$ determines also the dispersion of consumption across agents, which increases with $\epsilon$, and under default there is zero consumption dispersion.

How does the decentralized dispersion of consumption supported by the above bond holdings ”chosen” by private agents differs from the socially-optimal, or efficient, consumption dispersion that a utilitarian social planner would choose? To answer this question we study the optimization problem of the social planner. The planner’s welfare weight on $L$-type households is denoted by $\omega$, which we refer to as the planner’s “preference for redistribution.” The optimal default decision solves:

$$\max_{d \in \{0, 1\}} \left\{ W^{d=0}_1(\epsilon), W^{d=1}_1(\phi) \right\},$$

where welfare under repayment is:

$$W^{d=0}_1(\epsilon) = \omega u(y - g + \epsilon) + (1 - \omega)u \left( y - g + \frac{\gamma}{1 - \gamma}\epsilon \right)$$

and under default is:

$$W^{d=1}_1(\phi) = u(y(1 - \phi) - g).$$

We approach the solution of the optimal default decision by treating the planner’s prob-

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15 We do this because, as shown later, distributional incentives alone cannot support equilibria with debt unless $\gamma$ exceeds the weight of $L$ type agents in the social welfare function, and because the default cost can serve as a proxy for features missing from this example that make default costly in the full model (e.g. self-insurance and tax smoothing).
lem as choosing the socially optimal consumption dispersion $\epsilon^{SP}$ as the value of $\epsilon$ that maximizes $W^{d=0}(\epsilon)$. Since the only instrument available to the government to improve consumption dispersion relative to what the decentralized allocations support is the default decision, the planner will repay only if doing so allows it to either attain the optimal $\epsilon^{SP}$ or get closer to it than by defaulting.

The optimality condition for the choice of $\epsilon^{SP}$ reduces to:

$$\frac{u'(c^H)}{u'(c^L)} = \frac{u'\left(y - g + \frac{\gamma}{1-\gamma}\epsilon^{SP}\right)}{u'\left(y - g - \epsilon^{SP}\right)} = \left(\frac{\omega}{\gamma}\right) \left(\frac{1-\gamma}{1-\omega}\right).$$

(28)

For a standard increasing, concave utility function, this condition implies that the socially optimal ratio of $c^L$ to $c^H$ increases as $\omega/\gamma$ rises (i.e. as the ratio of the planner’s preference for redistribution to the existing mass of $L$ type households rises). If $\omega/\gamma = 1$ the planner desires zero consumption dispersion, for $\omega/\gamma > 1$ the planner likes consumption dispersion to favor $L$ types so that $c^L > c^H$, and the opposite holds for $\omega/\gamma < 1$. As we show below, $\omega/\gamma \geq 1$ always leads the planner to default, because for any decentralized consumption dispersion $\epsilon > 0$, the consumption allocations feature $c^H > c^L$, while the socially efficient consumption dispersion requires $c^H \leq c^L$, and hence there is no way to implement $\epsilon^{SP}$ (since the only instrument is the default choice). Default is therefore a second-best policy that brings the planner the closest it can get to the level of social welfare associated with $\epsilon^{SP}$.

The choice of $\epsilon^{SP}$ and the default decision in the absence of default costs (i.e. $\phi = 0$) are illustrated in Figure 1. This Figure plots the functions $W^{d=0}(\epsilon)$ for $\omega \geq \gamma$. The value of social welfare at default and the values of $\epsilon^{SP}$ for $\omega \geq \gamma$ are also identified in the plot. Notice that the vertical intercept of $W^{d=0}(\epsilon)$ is always $W^{d=1}$ for any values of $\omega$ and $\gamma$, because when $\epsilon = 0$ there is zero consumption dispersion and that is also the outcome under default. In addition, the bell-shaped form of $W^{d=0}(\epsilon)$ follows from $u'(\cdot) > 0$, $u''(\cdot) < 0$.16

16Note in particular that $\frac{\partial W^{d=0}(\epsilon)}{\partial \epsilon} \geq 0 \iff \frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))} \geq \left(\frac{\omega}{\gamma}\right) \left(\frac{1-\gamma}{1-\omega}\right)$. Hence, social welfare is increasing (decreasing) at values of $\epsilon$ that support sufficiently low (high) consumption dispersion so that $\frac{u'(c^H(\epsilon))}{u'(c^L(\epsilon))}$ is above (below) $\left(\frac{\omega}{\gamma}\right) \left(\frac{1-\gamma}{1-\omega}\right)$.
Figure 2: Choice of $\epsilon^{SP}$ and default decision ($\phi = 0$)

Take first the case for $\omega > \gamma$. In this case, $\epsilon^{SP} < 0$, because condition (28) implies that the planner’s optimal choice features $c^L > c^H$. However, since default is the only instrument available to the government, these consumption allocations are not feasible, and by choosing default the government attains $W^{d=1}$, which is the highest feasible social welfare for $\epsilon \geq 0$. When $\omega = \gamma$, $\epsilon^{SP} = 0$, and default attains exactly the same level of welfare, so default is chosen and it also delivers the efficient level of consumption dispersion. In short, if the fraction of $L$ type agents does not exceed the planner’s preference for redistribution (i.e. $\omega \geq \gamma$), the government always defaults for any decentralized distribution of debt holdings determined by $\epsilon \in [0, B]$, and thus equilibria with debt cannot be supported.

Equilibria with debt can be supported when $\omega < \gamma$. In this case, the intersection of the downward-sloping segment of $W^{d=0}(\epsilon)$ with $W^{d=1}$ determines a threshold value $\hat{\epsilon}$ such that default is optimal only for $\epsilon < \hat{\epsilon}$. Default is still a second-best policy in this case, because with it the planner cannot attain $W^{d=0}(\epsilon^{SP})$, it just gets the closest it can get given that default is the only instrument it can use. As the Figure shows, for $\epsilon < \hat{\epsilon}$, the choice of repayment is preferable because $W^{d=0}(\epsilon) > W^{d=1}$. Thus, in this simple example, when default is costless, equilibria for which repayment is optimal requires two conditions: (a) that the government’s preference for redistribution be smaller than the fraction of $L$ type agents, and (b) that the debt holdings chosen by private agents do not produce consumption dispersion in excess of $\hat{\epsilon}$.

Scenarios that would feature $\omega < \gamma$ are not difficult to visualize. Consider, for example,
a social planner with weights that follow from the first-best, complete-markets equilibrium, in which the distribution of wealth matches the tail value of the agent’s endowments priced with Arrow securities. If both types of agents were ex-ante identical, then the social planner’s weights that would support the first-best competitive equilibrium would be $\omega = 0.5$, which implies a uniform distribution of wealth and equal consumption for both agent types. On the other hand, the decentralized distribution of wealth $\gamma$ that would result from having the same agents trade in some incomplete-markets environment (e.g. the stationary wealth distribution of a Bewley economy), would generally feature more $L$ than $H$ types ($\gamma > 0.5$). The planner of our example would then choose to support an optimal amount of consumption dispersion such that $c_L / c_H = \left[ \left( \frac{\omega}{\gamma} \right) \left( \frac{1-\gamma}{1-\omega} \right) \right]^{\frac{1}{\gamma}} < 1$.

Since this simple example focuses on the distributional default incentives abstracting from the other tradeoffs of repayment v. default present in the model of Section 2 (e.g. self insurance and tax smoothing), it is illustrative to examine how its implications would change if we use the exogenous output cost of default as a proxy for these extra tradeoffs.

The solutions of the model for $\phi > 0$ are shown in Figure 2. The key difference is that with the exogenous default cost it is possible to support repayment equilibria even when $\omega \geq \gamma$. Now there is a threshold value of consumption dispersion, $\hat{\epsilon}$, separating repayment from default decisions for all values of $\omega$ and $\gamma$. The government chooses to repay whenever the decentralized consumption dispersion $\epsilon$ exceeds the value of $\hat{\epsilon}$ for the corresponding values of $\omega$ and $\gamma$. It is also evident that the range of values of $\epsilon$ for which repayment is chosen widens as $\gamma$ rises relative to $\omega$. Thus, when default is costly, equilibria with repayment require only the condition that the debt holdings chosen by private agents, which are implicit in $\epsilon$, do not produce consumption dispersion in excess of the value of $\hat{\epsilon}$ associated with given values of $\omega$ and $\gamma$. Intuitively, the consumption of $H$ type agents must not exceed that of $L$ type agents by more than what $\hat{\epsilon}$ allows. If it does, the preference for redistribution of the planner takes over, and default is optimal.
3.2 Social value of the self-insurance asset

The results of the previous example illustrate the potentially key importance of default costs other than those related to default incentives in driving the domestic default decision. In this second example, we show that the portion of these costs associated with the adverse effects of default on the use of government debt as a self-insurance vehicle are likely to be large, and thus are an important element of our domestic default framework. To illustrate this point, we consider again a specialized variant of the model but now aimed at highlighting the social value of having access to government debt as the self-insurance asset. We take the model of Section 2 with the continuum of heterogeneous agents and the stochastic shocks, but abstract from default, so that we remove the distributional incentives, and quantify the social value of the self-insurance asset at the stationary equilibrium.

We assume also for simplicity that $g_t = \bar{g}$. In addition, we assume no access to international markets so that the exercise is comparable with similar exercises in the literature (e.g. Aiyagari and McGrattan ([3])). In recursive form, the government budget constraint at the stationary equilibrium becomes:

$$\tau = \bar{g} + B(1 - q(B))$$

where $B$ is the constant supply of government bonds and $\tau$ and $q(B)$ are the long-run
equilibrium value for taxes and the price of government bonds.

The households’ budget constraint and borrowing constraint, using the above government budget constraint to substitute for the lump sum taxes that agents pay, are:

\[ c = y + b - q(B)b' - [\bar{g} + B(1 - q(B))] \]
\[ b' \geq 0 \]

It is also illustrative to adopt the variable transformation \( \tilde{b} = (b - B) \), and use it to rewrite the above constraints:

\[ c = y + \tilde{b} - q(B)b' - \bar{g} \]
\[ \tilde{b}' \geq -B \]

This reformulation of the agents’ constraints makes it evident that the provision of public debt by the government helps relax the agents’ borrowing constraint, and thus lessen the burden of building precautionary savings.

To quantify the social value of public debt as the self-insurance vehicle, we define \( \alpha(b, y; B) \) as the compensating variation in consumption, constant across time and states of nature, that renders agents indifferent between living in the economy with public debt and living in one with financial autarky:

\[ \alpha(b, y; B) = \left[ \frac{V(b, y; B)}{V_{\text{aut}}(y)} \right]^{\frac{1}{\pi - 1}} - 1 \]

For a given supply of debt \( B \), we have a distribution of these welfare measures, which define the value of public debt for each individual agent with a particular individual wealth and income pair \((b, y)\). The social value of public debt is then computed by aggregating these individual welfare measures across agents using the utilitarian social welfare function as defined in Section 2:

\[ \bar{\alpha}(B) = \int \alpha(b, y; B) d\omega(b, y) \]

In Table 2, we report results for six experiments in which the ratio of debt to output ranges from 10 to 250 percent and the planner’s weights are set equal to the endogenous stationary wealth-income distribution of the competitive equilibrium (i.e. \( \omega(b, y) = \Gamma(b, y) \)). For each scenario, we report \( \bar{\alpha}(B) \) and the associated long-run averages of the price of bonds and lump-sum taxes, as well as the fraction of agents with \( \alpha(b, y; B) > 0 \) (i.e. the fraction of agents that would prefer living in the economy with public debt than in the one without).
Table 2: Welfare gains of government debt

<table>
<thead>
<tr>
<th>B/Y</th>
<th>q(B)</th>
<th>τ(B)</th>
<th>$\hat{\alpha}(B)$ (%)</th>
<th>hh's $\alpha &gt; 0$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1000</td>
<td>0.9998</td>
<td>0.1829</td>
<td>-0.6168</td>
<td>35</td>
</tr>
<tr>
<td>0.6333</td>
<td>0.9640</td>
<td>0.2057</td>
<td>-1.3729</td>
<td>50</td>
</tr>
<tr>
<td>0.9000</td>
<td>0.9591</td>
<td>0.2197</td>
<td>-1.4073</td>
<td>51</td>
</tr>
<tr>
<td>1.4333</td>
<td>0.9538</td>
<td>0.2491</td>
<td>-1.1837</td>
<td>52</td>
</tr>
<tr>
<td>1.9667</td>
<td>0.9510</td>
<td>0.2792</td>
<td>-0.7366</td>
<td>52</td>
</tr>
<tr>
<td>2.5000</td>
<td>0.9493</td>
<td>0.3096</td>
<td>0.0513</td>
<td>53</td>
</tr>
</tbody>
</table>

The result show that, for all but the highest debt ratio, average welfare would decline sharply if public debt is taken out (the costs range between 0.6 and 1.4 percent of permanent consumption). But we also find that there is an interior maximum for the welfare benefits of public debt, which is attained with a 90 percent debt ratio and a 1.4 percent gain. After that, the social value of public debt shrinks, and for a debt ratio of 250 percent agents in fact prefer financial autarky, with a welfare gain of about 0.05 percent. This occurs because at this high level of debt the real interest rate is high and taxes are also high. The former leads agents to build larger stocks of precautionary savings, which are costly to sustain in terms of foregone consumption, and the latter reduces disposable income. Note also that these calculations exclude the trade off vis-a-vis the government’s ability to smooth taxation, because we kept government expenditures constant. With this tradeoff at play, the benefits of having access to public debt would be even larger.

In summary, the results of this experiment show that, in the absence of default risk, the social value of public debt as a vehicle for self insurance is large, even when we take into account the equilibrium cost of servicing the debt. It is only at very high debt ratios that the debt service costs become large enough to overtake the benefits of the improved ability to self insure and the implicit relaxation of individual borrowing constraints that public debt provide. Thus, in the full model with the option to default the social planner will be trading off the distributional incentives to default against the non-trivial benefits of public debt as a key financial asset, and also against the benefits of smoothing taxes in the presence of fluctuating government expenditures. Moreover, in the model with default the government will be considering these tradeoffs off the stationary equilibrium, particularly in states in which debt grows large and concentrated enough to create strong default incentives, and also in the aftermath of defaults, when debt holdings are very low and the marginal benefit of precautionary savings is very high.
4 Quantitative Predictions of the Dynamic Model

In this Section, we study the quantitative predictions of the model based on a set of parameter values calibrated to data from Spain. We chose Spain because it is one of the key countries in the ongoing debt crisis of the Eurozone and we have information on the country’s individual earnings process that is an important element of the calibration, in addition to data on its public debt and fiscal flows. We also acknowledge, however, that all the data we have for Spain pertains to a period in which default on either domestic or external default has not been observed. According to Reinhart and Rogoff [28], Spain’s last default on its domestic sovereign debt was during the Spanish Civil War in 1936-1939, and consisted mainly of arrears in interest payments. This was also an episode of external default with suspension of debt payments, and the total debt ratio just before the default (in 1935) was 65.9 percent of GDP. Another limitation of our calibration using Spanish data only is that it falls short of our interpretation of the European crisis as a domestic default, which hinges partly on the notion that European-wide institutions are internalizing the tradeoffs of default across the Eurozone. Because of these limitations, we see the main goal of this exercise as to show whether the model can support an equilibrium with domestic default, and if so, to study its properties, rather than aiming to match the stylized facts of Spanish public debt accurately.

4.1 Calibration

The model is calibrated to annual frequency. The parameter values that need to be set are those for the subjective discount factor $\beta$, the coefficient of relative risk aversion $\sigma$, the Markov processes of individual income ($\mu_y, \rho_y, \sigma_u$) and government expenditures ($\mu_g, \rho_g, \sigma_e$), the fiscal reaction function ($\mu_B, \rho_B, \alpha^g$), the opportunity cost of funds of foreign investors $\bar{r}$, and the preference for redistribution in the utilitarian welfare function $\omega$. The discrete Markov approximations of the processes of $y$ and $g$ estimated from data are created using Tauchen’s [29] method, using 11 equally spaced grid points for income and 25 points for the government expenditures process. Thus, the full set of parameters that need to be defined in order to solve the model are:

$$\Theta = \{\bar{r}, \beta, \sigma, \mu_y, \rho_y, \sigma_u, \mu_g, \rho_g, \sigma_e, \mu_B, \rho_B, \alpha^g, \omega\}.$$

We separate the parameters into two groups. The first group consists of all parameters except $\beta$ and $\omega$. The parameters in this group are calibrated to be consistent with standard parameters in the literature or independent targets from Spain or Industrial countries that do not require us to solve the model. The second group ($\beta$ and $\omega$) is calibrated to minimize the distance between two empirical moments taken from the data (the fraction of domestic debt...
and the average spread) and their counterparts in the model. Thus, to determine the values of these parameters we need to solve the model and compute the corresponding moments on the equilibrium path.

The calibration of the first group of parameters proceeds as follows: First, we set $\sigma = 1$ (i.e. log-preferences), which is a value in the range of those commonly used in DSGE models. Regarding the cost of funds of foreign investors, we set $\bar{r} = 0.020681$ to match the average yearly real return on German EMU convergence criterion government bond yields in the Eurostat database for the period 2002-2007 (these are secondary market returns, gross of tax, with around 10 years’ residual maturity and deflated with the German CPI deflated). We chose 2002-2007 because the starting point coincides with the introduction of the Euro and the last period corresponds to the last year before the financial crisis.

To calibrate the individual income process we proceed as follows. We set $\rho_y = 0.85$ (a standard value in the incomplete market literature, see for example Guvenen (2009)) and $\sigma_u = 0.2498$ to be consistent with Spain’s cross-sectional variance of log-wages (on average equal to 0.225 for the period 1994-2001) reported by Pijoan-Mas and Sanchez Marcos [26].

Average income is calibrated such that the aggregate resource constraint is consistent with the data when GDP is normalized to one. This implies that the value of households’ aggregate endowment must equal GDP net of fixed capital investment, since the latter is not explicitly modeled. The average for the period 1975-2011 results in a value for $\mu_y = 0.75$.

To calibrate the $g$ process we use data on government final consumption expenditures from National Accounts. We estimate $\rho_g$, $\sigma_e$ and $\mu_g$ by running an AR(1) on log-government expenditures to GDP for the period 1970-2011 using data from OECD Statistics. The first-order autocorrelation of the process in logs (controlling for a linear time trend) is estimated to be 0.92515 (this pins down $\rho_g$). The variance of the innovation of log government consumption over GDP is 0.028531 (this pins down $\sigma_e$) and the mean ratio for the period 1970-2011 equals 15.883 percent (this allows us to set $\mu_g$).

The parameters of the fiscal reaction function are set using the estimation results for industrial countries reported by Mendoza and Ostry [23], which are based on a panel with data for the period 1970-2005. They estimated $\rho = 0.022$ and $\alpha_1 = -0.21$. Using the growth-adjusted gross real interest rate for Spain for the period 2002-2007, which is equal to 0.9938, we obtain $\rho^b = (1 - \rho)0.9938 = 0.9719$ and $\alpha^g = -\alpha_1 0.9938 = 0.2087$. We set $\mu_B$ to

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17 Since the process is stationary, we use the fact that $\sigma^2_u = \text{Var}(\log(y))(1 - \rho^2_y)$ to pin down $\sigma^2_u$ once we have values for $\rho_y$ and $\text{Var}(\log(y))$.

18 Grids for $y$ and $g$ are equally spaced, centered at their min with their lowest and highest points set at 2.5 standard deviations from the mean in logs. This implies that $y = 0.2293$ and $g = 0.1916$. In this case the value of $c$ is not relevant since a household with the lowest endowment and not debt holdings can have positive consumption in a period where $\tau = g = \bar{g}$. The implied variances of the approximated $y$ and $g$ process are less than 1 percent away from their empirical counterpart.
match the average of the government gross financial liabilities as a share of GDP for Spain for the period 1980-2011. This ratio equals 54.63 percent.\footnote{The average for the period 2002-2011 is very similar at 56.53 percent.}

The calibrated law of motion of debt implies that debt issuance is increasing in current debt and the mean deviation of government expenditures. Absent a default, the level of government debt fluctuates around its unconditional mean $\mu^B$. Notice also that higher government expenditures induce the government to borrow more, and can push it towards debt levels where default is optimal. There is however, a debt threshold above which debt issuance falls for all values of current debt even when government expenditures are high. The threshold is at the point where the law of motion of government bonds crosses the 45° degree line. That is, since $\rho^B < 1$ and $g_t$ is mean reverting, $B_{t+1}$ crosses the 45° at $B = 0.7847$ even when $g_t = \bar{g}$ for all $t$.

The calibration targets for setting the values of $\beta$ and $\omega$ are the observed average fraction of domestic debt over total debt for 1981-2010, which equals 74.73 percent, and the average spread for Spanish government bonds relative to their German counterpart for the period 2002-2012, which equals 0.9133 percent.\footnote{The ratio of domestic debt to total debt corresponds to the fraction of marketable debt held by residents as a fraction of total marketable debt of central government debt. To compute the spread we use data from EMU convergence criterion government bond yields. The spread corresponds to the ratio between the yield on Spanish bond and German bonds of the same maturity.} To perform the calibration, we minimize the following loss function

$$J(\Theta) = [M^d - M^m(\Theta)]' [M^d - M^m(\Theta)] ,$$

where $M^m(\Theta)$ is a $2 \times 1$ vector of model moments and $M^d$ is the corresponding $2 \times 1$ vector of data moments. The model moments are averages obtained from 96 repetitions of 10,000 period simulations. The initial 20 percent of periods of each simulation are discarded to avoid a dependence on initial conditions. Since Spain has not defaulted on its debt during the period for which data is available, we use moments generated outside default events. Even though each moment generated by the model is a function of all parameters, we argue that the discount factor and the welfare weights are well identified using this set of moments since (everything else equal) the discount factor controls the demand for assets by domestic households and the value of $\omega$ determines the frequency of default by the government and thus is informative of debt prices and spreads.

Table 3 shows all the parameter values.
Table 3: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-Free Rate (%)</td>
<td>( \tilde{r} )</td>
<td>2.068 Real return German Bonds</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>( \sigma )</td>
<td>1.000 Standard Value</td>
</tr>
<tr>
<td>Autocorrel. Income</td>
<td>( \rho_y )</td>
<td>0.850 Spain Wage Data</td>
</tr>
<tr>
<td>Std Dev Error</td>
<td>( \sigma_u )</td>
<td>0.200 Spain Wage Data</td>
</tr>
<tr>
<td>Avg. Income</td>
<td>( \mu_y )</td>
<td>0.750 GDP net of Fixed Capital Investment</td>
</tr>
<tr>
<td>Autocorrel. G</td>
<td>( \rho_g )</td>
<td>0.925 Autocorrel. Government Consumption</td>
</tr>
<tr>
<td>Std Dev Error</td>
<td>( \sigma_e )</td>
<td>0.029 Std. Dev. Government Consumption</td>
</tr>
<tr>
<td>Avg. Gov. Consumption</td>
<td>( \mu_g )</td>
<td>0.159 Avg. G/Y Spain</td>
</tr>
<tr>
<td>Avg. Debt Rule</td>
<td>( \mu_B )</td>
<td>0.546 Avg. Debt over GDP Spain</td>
</tr>
<tr>
<td>Autocorrel. B</td>
<td>( \rho_B )</td>
<td>0.972 Autocorrel. Government Debt</td>
</tr>
<tr>
<td>Feedback G</td>
<td>( \alpha_a )</td>
<td>0.209 Feedback gov. expenditures on Debt</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>( \beta )</td>
<td>0.903 Avg. Ratio Domestic/Total Debt</td>
</tr>
<tr>
<td>Welfare Weights</td>
<td>( \omega )</td>
<td>0.003 Avg. Spread Spain (vs Germany)</td>
</tr>
</tbody>
</table>

Table 4 shows the targeted moments of the SMM exercise in the model and the data.

Table 4: Targeted Moments

<table>
<thead>
<tr>
<th>Moments (%)</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic/Total Debt</td>
<td>74.95</td>
<td>74.73</td>
</tr>
<tr>
<td>Avg. Spread Spain</td>
<td>1.27</td>
<td>0.91</td>
</tr>
</tbody>
</table>

With this calibration strategy, the model produces a fraction of domestic debt over total debt that is on target but over predicts the level of spreads.

The model is solved by extending the well-known Krusell-Smith methods for solving heterogeneous agents models with aggregate uncertainty. The model requires keeping track of the fact that the evolution of bond prices as well as government default choices depend on the wealth distribution of households, and in turn households need to forecast future prices in order to choose their level of savings and consumption. As is standard in models of heterogeneous agents with aggregate risk, the wealth distribution is an infinitely dimensional object, and hence it is not possible to include it as a state variable. To get around this issue, we extend the algorithm proposed by Krusell and Smith [18] and [19] to accommodate an environment with government default and conjecture that prices can be predicted by a finite set of moments. In particular, we assume that current prices and the default decision can be written as functions of a finite set of moments of the wealth distribution and the current realization of government expenditures. Appendix A1.1 describes the solution algorithm in...
detail, including a description of the aggregate law of motion of the wealth distribution and measures of fitness of the algorithm.

4.2 Policy, Payoff and Pricing Functions

We start by describing the quantitative properties of household and government behavior, and then move to a description of the evolution of the economy along the equilibrium path. As we show, the model under the baseline calibration supports an equilibrium path with episodes of domestic default, even in the absence of exogenous default costs in the form of credit market exclusion or income costs that are typically needed in models of external default.

Consider first the valuations that households with different bond holdings and income assign to the repayment and default options. At the household level, by standard arguments, the value under repayment is increasing in \( b \). Hence, we can define a repayment threshold as the value of household bond holdings \( \hat{b}(y, \Gamma, B, g) \) for given \( (y, \Gamma, B, g) \) such that \( b \geq \hat{b} \) implies that the household prefers repayment over default. That is,

\[
\hat{b}(y, \Gamma, B, g) = \{ b \in B : V^{d=0}(b, y, \Gamma, B, g) = V^{d=1}(y, g) \}.
\]

Households with \( b \geq \hat{b}(y, \Gamma, B, g) \) prefer repayment because \( V^{d=0}(b, y, B, g) \geq V^{d=1}(y, g) \). Interestingly, the threshold \( \hat{b}(y, \Gamma, B, g) \) is increasing in \( B \). At higher levels of government debt, the difference in taxes in the repayment v. default scenarios increases, because the higher debt induces a debt-overhang-like effect that makes servicing the debt more costly, so the threshold value of household wealth \( \hat{b} \) necessary to favor repayment increases. For example, at the calibrated equilibrium, when \( B = 0.33 \) (average debt), \( \hat{b}(y, \Gamma, B, g) = 0 \). On the other hand, when \( B = 0.55 \) (maximum value of debt observed), \( \hat{b}(y, \Gamma, B, g) = 0.21 \).

We can illustrate further the households preferences over default and repayment by computing a standard measure of compensating variation in consumption that equates expected utility in the two scenarios. In particular, we compute \( \alpha(b, y, \Gamma, B, g) \) as the fraction of consumption that would render an individual agent identified by wealth and income \( (b, y) \) indifferent between the repayment and the default options at the aggregate states given by \( (\Gamma, B, g) \):

\[
\alpha(b, y, \Gamma, B, g) = \exp \left( (V^{d=1}(y, g) - V^{d=0}(b, y, \Gamma, B, g))(1 - \beta) \right) - 1.
\]

A negative value of \( \alpha(b, y, \Gamma, B, g) \) implies that the household prefers repayment over default (i.e. it has to be compensated in order to attain the same utility under default). Next we want to examine how these valuations vary with the individual and aggregate states.
Figure 4 shows two intensity plots that illustrate how $\alpha(b, y, \Gamma, B, g)$ varies over $b$ and $y$, evaluating the welfare effects at the long run averages of $\Gamma$ and $g$. Panel (i) uses a low value of the supply of debt ($B_L$) and Panel (ii) uses a high value ($B_H$).

The features of these intensity plots follow from the discussion of the default threshold $\hat{b}(y, \Gamma, B, g)$. Consider first the variations along the dimension of government bond holdings. At low values of $b$, households prefer default to repayment ($\alpha(b, y, \Gamma, B, g) > 0$). These households would benefit from the reduction in taxes associated with default and would suffer minimum or null wealth losses. The opposite is true for agents with high values of $b$. We also observe, by comparing across panels (i) and (ii), that for a given pair $\{b, y\}$, $\alpha(b, y, \Gamma, B, g)$ is increasing in $B$. Since servicing the debt is more costly the larger is $B$, the compensation needed for households in order to achieve the same utility as under the default allocation increases (i.e. at higher $B$ more agents prefer default).

Comparing preferences for default v. repayment along the $y$ dimension is not as straight-

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21In this figure and the figures that follow, we evaluate the functions at the “average” wealth and income distribution. This distribution is the average over a simulated sample of 5,000 periods with no default. To be precise, this distribution is constructed as the average of $\Gamma_t(b, y)$ over a collection of $T = 5,000$ of simulated time periods (not necessarily contiguous) where the government chooses $d = 0$. The average distribution is $\sum_{t=1}^{T} \Gamma_t(b, y) / T$. 

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forward as in the $b$ dimension, because both the repayment value function and the default continuation value depend on $y$. The value function under default is increasing in income. The value of repayment is increasing in households’ “total resources,” defined as the sum of income $y$ and wealth $b$. The composition of “total resources” between $b$ and $y$ affects the level of utility under repayment, but in general, for any given $b$, we see that the value of repayment falls as $y$ rises (i.e. $\alpha(b, y, \Gamma, B, g)$ rises). This happens because as the share of $y$ in total resources increases, the differential in taxes across repayment and default becomes more relevant in determining preferences over government actions.

Figure 5 illustrates further the role of the interaction between $b$ and $y$ in driving $\alpha(b, y, \Gamma, B, g)$, but now by plotting the welfare measures as a function of $y$ and the supply of government debt $B$ for two values of individual bond holdings $b \in \{0, 1\}$ and $g = \mu_g$.

Figure 5: $\alpha(b, y, \Gamma, B, g)$ (for different $b$ at $g = \mu_g$)

Figure 5 shows that, for both values of $b$, $\alpha(b, y, \Gamma, B, g)$ is a concave function of $y$ when government debt is low ($B < 0.25$). This is because at low values of $B$, the government is able to engage in tax smoothing (i.e. reduce the level of taxes by repaying its existing debt and issuing more debt). The concavity emerges because this reduction in taxes is more important for those with low levels of income (for a given level of wealth). Households with low bond holdings and high income prefer repayment since they would like to save (the self-insurance
benefit of repayment). Their compensating equivalent is smaller in absolute terms than that for those with low income since their marginal utility of consumption is also smaller. We note that as the level of government debt increases, at low levels of household wealth \((b = 0)\), \(\alpha(b, y, \Gamma, B, g)\) becomes a convex function of income. As the level of \(B\) increases, issuing an extra unit of government debt becomes more costly (due to default risk), so taxes under repayment might be higher than under default. In those cases, households with no wealth and little or no income prefer default rather repayment, since they are not saving and their taxes go up. High-income households still prefer repayment, however, since that allows them to save. For higher levels of wealth \((b = 1)\), \(b\) is a higher fraction of “total resources.” For agents with low income, those assets are highly valued and hence repayment is strongly preferred, whereas as \(y\) rises households rely relatively less on bonds and more on income and they value repayment less.

To complete the analysis of the individual valuation of default v. repayment, Figure 6 displays how \(\alpha(b, y, \Gamma, B, g)\) varies in the \(\{y, g\}\) dimension evaluated at \(b = 0\) for different values of \(B\).

Figure 6: \(\alpha(b, y, \Gamma, B, g)\) (for different \(B\) at \(b = 0\))

Figure 6 shows that at high levels of debt, the value of defaulting increases sharply with \(g\) for low income agents. Since these agents have zero holdings of bonds, the higher taxes needed to both service the high debt and pay for higher \(g\) take a large share of their total
resources, and the pain grows increasingly large as \( g \) rises. These agents would be willing to pay more than 5 percent of consumption to remain in the default state. On the other hand, the pattern is the opposite for low \( B \), because in this case low income agents prefer that the government repays so that it can smooth the impact of higher \( g \) on their tax bill, and thus they prefer repayment and at an increasing rate as \( g \) rises. These opposing patterns of the valuation of default v. repayment as \( g \) rises reflect implicitly the value agents give to the government’s ability to smooth taxation, which dominates for \( B = B_L \), v. the debt overhang effect that dominates when \( B = B_H \).

The value that each agent assigns to the default and repayment options interact with the utilitarian welfare weights to determine the government’s default decision. Hence, we are also interested in studying the average (or “social”) value, in consumption compensating variation, that society would be willing to pay after a government repayment in order to achieve the allocation of default using the welfare weights as follows

\[
\bar{\alpha}(\Gamma, B, g) = \int_{B \times \mathbb{Y}} \alpha(b, y, \Gamma, B, g) d\omega(b, y).
\]

The properties of \( \bar{\alpha}(\Gamma, B, g) \) reflect the properties of the difference between the welfare under repayment and under default (i.e. \( W^{d=0}(\Gamma, B, g) - W^{d=1}(g) \)) and can be understood as the determinant of the government default decision. Figure 7 presents \( \bar{\alpha}(\Gamma, B, g) \) for three different values of \( g \).

Figure 7: \( \bar{\alpha}(\Gamma, B, g) \) (for different \( g \))

It is evident from Figure 7 that the social value of default v. repayment is increasing
in the level of government debt. As debt raises, the cost of servicing the debt (i.e. the 
taxes paid by households) increases and overcomes the benefit of providing the means of 
self-insurance to households and tax-smoothing of their tax burdens. This also explains why 
$\bar{\alpha}$ is in fact non-monotonic in $g$. At sufficiently high levels of debt, $\bar{\alpha}$ becomes increasing in $g$. In these cases, the price of debt (see Figure 10) is such that net resources the government 
obtains from borrowing ($qB' - B$) are negative, which impedes the government from lowering 
the level of taxes households pay. This is analogous to the debt overhang problem. When $B$ 
is high, interest rates are high and the government needs revenue to keep up with interest 
payments, so the continuation value is lower the more revenue is needed to pay also for more 
g, and this hampers the ability to use debt to smooth taxation, because instead taxes rise 
with debt. At these levels of $B$, some households have accumulated enough wealth that 
they assign a positive value to repayment because it maintains the market for assets open. 
However, the figure shows that the overhang effect dominates and $\bar{\alpha} > 0$, favoring default.

The average value of $\alpha$, while informative, hides the rich heterogeneity that we observe at 
the household level. Figure 8 presents the distribution of $\alpha$ that arises along the equilibrium 
path for different values of $B$ and $g$.

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**Figure 8: Distribution of $\alpha$ (for different $B$ and $g$)**

Consistent with Figure 7, Figure 8 shows that as the level of debt increases the distribu-

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22This distribution is derived from values of $\alpha(b, y, \Gamma, B, g)$ observed along the equilibrium path and the welfare weights $\omega(b, y)$. 
tion of $\alpha$ shifts to the right. It is notable that, when debt levels are relatively high $B = B_H$, approximately 50 percent of households would favor a default. In fact, a fraction of them would require about 8 percent of consumption compensating variation in order to be indifferent. However, a large portion of households still prefers repayment over default (these are households with relatively high level of income and wealth). Note also that when debt is low all households prefer repayment since 100 percent of the population is located at values equal or below 0.

Consider next the default policy of the government. Figure 9 presents the government default policy as a function of $B$ and $g$ (the light color area represents $d = 0$ and the dark area represents $d = 1$). This government policy function is derived from government optimization and is determined by the welfare values we just discussed.

Figure 9: Equilibrium Default Function $d(\Gamma, B, g)$

Note: Black area represents $d = 0$ and white area represents $d = 1$.

Figure 9 shows that the size of the government (as measured by the level of debt and government expenditures) influences the default decision. Government default is increasing in government expenditures and government debt. The connection between the distribution of repayment (weighted by the welfare weights) and the level of public debt that we presented before is implicit in this relationship.

Figure 10 shows the equilibrium price function for government bonds as a function of new debt issuance ($B'$). This price function features conventional properties of bond prices.
in models with sovereign default. In particular, the bond price decreases as the value of government debt $B$ increases (recall that $B'$ is increasing in $B$) and prices are lower the higher the level of government expenditures. Default risk induces a reduction in bond prices as $B$ increases. For example if $B$ is higher than or equal to 0.5 when $g = g_H$, we reach states where default happens in the following period with probability 1 and the bond price drops to zero.

Figure 10: Equilibrium Price Function $q(\Gamma, B, g)$

The government default decision and the corresponding bond prices imply a level of government taxes for different levels of debt under the two possible scenarios: repayment or default. Figure 11 displays the difference between the tax level under repayment and the tax level in default ($\tau^d=0(B^d=1(g))$).
Figure 11: Equilibrium Tax Differential $\tau_{d=0}^{d=1}(g)$

Figure 11 shows that the tax differential under repayment is negative when debt levels are below $B \leq 0.15$ for all levels of government expenditures, that is the level of taxes paid by households is smaller under repayment than under default. This again reflects the fact that at low levels of debt the government can engage in tax smoothing. A negative tax differential implies that the government is able to substitute current taxes for future debt service since total resources from debt issuance $qB'$ are larger than $B$ together with the fact that the government bond rule specifies $B' > B$ for low levels of $B$. Consistent with the consumption equivalent values presented in Figure 7, the difference is decreasing in the level of government expenditures. As debt increases, this differential becomes positive and increases rapidly in the default region, because the debt overhang effect takes over and impedes tax smoothing. Note that this jump is a result of the reduction in equilibrium bond prices given that there is a positive probability of a government default.

4.3 Default Dynamics

We now examine the paper’s key result: That default can arise in equilibrium as a result of the tradeoff between the distributional incentives and the incentives for tax smoothing and provision of a self insurance asset. To illustrate this point, we show the results of a representative simulation of the model between two default events, starting the simulation the period after the government has regained access to capital markets (which is by construction a single period, since there is no exclusion) and finish it right after the default event.

Panel (i) of Figure 12 displays the evolution of debt $B$ (government bond supply), $B^d$
(domestic demand) as well as the default decision of the government. Panel (ii) displays the evolution of government expenditures $g$, taxes $\tau$ and the primary balance $\tau - g$ (denoted “Prim. Bal.”).

Figure 12: Evolution of Debt, Taxes and Gov. Expenditures

This figure shows that the model is able to generate default along the equilibrium path. Government debt first goes through a phase in which it increases gradually, until it reaches a level of near 55 percent of GDP and default occurs. In the early stages (i.e. periods right after a default), the increase in government debt allows taxes to be lower than government purchases (i.e. tax smoothing). This is also reflected in the evolution of the primary balance up to period 20 (the government runs a deficit since $\tau < g$). As debt increases, however, further debt issuance brings in increases in borrowed resources ($qB'$) at a diminishing rate, shifting taxes to raise above expenditures to keep up with debt service (i.e. debt overhang). When the level of government debt is high enough and repaying the level of debt would imply a level of taxes much larger than the level of taxes in the case of no repayment, the government chooses to default (the government is running a primary surplus at this point of approximately 2.5% of GDP two periods before a default). During the default period, taxes equal the level of government expenditures $g$. Once the government re-enters the bond market, the cycle starts again with differences arising only depending on the path of government expenditure.

Notice an interesting similarity with the model of optimal taxation under incomplete
markets studied by Aiyagari et al. [2]. Volatile taxes are a feature of their model with exogenous bounds on government debt. In our economy, the upper bound on debt is derived endogenously by allowing the government to default.

Figure 13 shows the evolution of debt and spreads around the default period.

Figure 13: Evolution of Government Debt and Spreads

Figure 13 shows that spreads stay at very low levels while government debt is below 40 percent of GDP. As the debt rises above 50 percent, bond interest rate increase sharply. The evolution of spreads is qualitatively consistent with the data observed for Spain before the start of the current crisis.

Changes in the supply of government bonds affect interest rates and the domestic demand for these bonds, which is one of the main determinants of the default decision. Figure 13 shows that when debt is low during tranquil times (i.e. when spreads are zero), domestic demand increases with the total supply of government bonds but at a decreasing rate. Then domestic demand increases much slower and the government places most of its new debt with foreign creditors. This is followed by a period of stable but high debt just below 50 percent of GDP, followed then by a rapid rise in debt supply, debt holdings of foreign and domestic agents, and spreads just before the default occurs. Domestic debt as a fraction of total debt is on average 74 percent but in the default period corresponds to 42.35 percent of the total debt. Right after the default, the domestic demand for government debt goes to zero since the debt market is closed.
Figure 8 presented how the distribution of the average consumption compensating variation changes as a function of $B$ and $g$ on the equilibrium path. We explore this further in Figure 14 by showing how $\bar{\alpha}(\Gamma, B, g)$ evolves over time together with the evolution of government debt and domestic asset demand.

Figure 14: $\bar{\alpha}(\Gamma, B, g)$, Debt and Domestic Demand

The value of $\bar{\alpha}(\Gamma, B, g)$ approaches zero (i.e. the point at which the government finds default optimal) as the level of government debt increases and the fraction of domestic debt decreases (note that domestic debt increases at a slower pace than $B$). The fraction of households in favor of default is one of the determinants of this result. Even tough $\omega(b, y)$ is exogenous, the planner uses these weights to weight the individual values over repayment and default that change over time as debt, taxes and interest rates change.

A key determinant of the value of $\alpha$ is the primary balance. We present in Figure 15 a scatter plot of $\bar{\alpha}(\Gamma, B, g)$ as a function of the primary balance $\tau - g$ during the no default periods (recall that GDP is normalized to 1 so the primary balance can be read as a fraction of GDP).
Figure 15: $\pi(\Gamma, B, g)$ and Primary Balance

Figure 15 shows that there is a clear positive relationship between the average value of $\alpha$ and the primary balance. The relationship is not linear, because when debt is low the government is able to smooth taxes but as debt increases it faces a debt overhang problem that generates a rapidly increasing primary deficit.

Next we examine the evolution of the model’s distribution of wealth. We focus on a set of statistics that summarize how the distribution changes as we move away from a default period into a new default. In particular, Figure 16 presents the standard deviation of $b$ (std($b$)), the fraction of households at $b = 0$ (Frac. $b = 0$) and the median value of $b$ (median($b$)).
Figure 16: Evolution of moments of the wealth distribution

Figure 16 shows that there is a considerable mass of agents at $b = 0$ along the equilibrium path and that this fraction is maximum right after a default and minimum before a new default. The risk of default together with their frequency keeps at least 25 percent of the households in any given period with $b = 0$. In period 94 (the default period) this fraction goes to 1 since the government wipes out all wealth in the economy. We also observe that the dispersion of bond holdings (one of the determinants of the dispersion of consumption) increases with the domestic demand of bonds and it is maximum in the period before default.

Since the distributional incentives example of Section 3 showed that the relative values of the planner’s weights and the actual wealth distribution are important for the default decision, it is interesting to analyze how the distribution of welfare weights $\omega(b, y)$ compares to the average distribution of income and wealth observed along the equilibrium path. Figure 17 presents the “average” distribution of households over wealth conditional on income $\Gamma$. This distribution is an average over a simulated sample of 5,000 periods (with no default) and shown in three panels each conditional on an income level.\textsuperscript{23}

\textsuperscript{23}To be precise, this distribution is constructed as the average of $\Gamma_t(b, y)$ over $T$ simulated time periods. The average distribution is given by $\frac{\sum_{t=1}^{T} \Gamma_t(b, y)}{T}$. We simulate the economy for 10,000 periods, drop the first 2,000 periods and collect $T = 5,000$ periods in which the government chooses $d = 0$. 

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This Figure shows that the model generates more households with low wealth (conditional on low income) that those reflected in the weight assigned by the planner when making the default decision. It also shows that the calibrated distribution of welfare weights is similar to the observed distribution of wealth conditional on average and high income. Recall that by construction both distributions coincide if we look at the distribution of income (conditional on wealth) since $\omega(b,y)$ is parametrized using $\pi^*(y)$ along the $y$ dimension. To quantify the distance between the two distributions we compute the average sum squared error and obtain:

$$\int_{B \times Y} [\Gamma(b,y) - \omega(b,y)]^2 dbdy = 0.0285$$

where $\bar{\Gamma}(b,y) = \sum_{t=1}^{T} \Gamma_t(b,y)/T$.

Finally, we present how the fraction of households that favors repayment evolves over time in Figure 18. Panel (i) presents this fraction conditional on income and Panel (ii) the overall fraction.
It is interesting, that along the equilibrium path, the fraction of households in favor of repayment (conditional on income) is increasing in income, that is the fraction of households with high income that prefers repayment over default is larger than that of low income as we approach a default. While the government runs a primary deficit, low income household benefit from repayment since they receive a net transfer from other households and foreign investors but that effect disappears as the government debt increases.

5 Conclusion

This paper provides a model of heterogeneous agents and incomplete asset markets in which infrequent domestic sovereign defaults occur as the optimal choice of a utilitarian social planner who values the welfare of all domestic agents, including its domestic creditors. The planner balances the tradeoff between distributional incentives to default and endogenous default costs associated with the role of public debt as a key financial asset that serves as the vehicle for self-insurance and tax-smoothing. In this model, an endogenous feedback mechanism links the evolution of the distribution of bond holdings across private agents, the government’s default probability, and the risk premium of government bonds. Default is optimal when the weighted net benefit of defaulting on the debt for relatively poor agents exceeds the corresponding weighed net loss imposed on relatively rich agents.
Defaults are not selective, and hence the choice of default versus repayment has important distributional implications, as well as effects on private self-insurance and government tax smoothing. In the short run, default implies a reduction in the level of lump-sum taxes common to all agents, as tax revenues to service a large debt stock at a high risk spread (i.e. a debt overhang) are avoided. On the other hand, default causes a reduction of wealth for agents with positive holdings of government debt and the loss of the vehicle that agents use to self-insure against idiosyncratic and aggregate shocks. If the government repays, it can use both new debt and taxes to manage aggregate shocks affecting government expenditures and service outstanding debt. Taxes represent a larger fraction of disposable income for those with low debt holdings, so (conditional on debt repayment) these agents are better off if the government finances its outlays by borrowing more. The cost of increasing the level of debt is that expected future taxes are higher and a default more probable.

The model’s quantitative predictions based on a calibration to data for Spain show that the model produces a high degree of concentration in the distribution of public debt holdings, with a large fraction of agents holding zero debt. The model supports debt ratios of up to 55% of GDP (34% on average) without relying on exogenous default costs in the form of exclusion or income costs. These debt ratios are lower than in the data, but the low frequency of domestic default (1.13%), the ratio of domestic to total debt (75%), and the dynamics of spreads in the periods leading to a default are all in line with the data.

In work in progress we are studying further the model’s quantitative implications and its ability to match the empirical regularities of debt dynamics. In particular, we are extending the analysis in three directions. First, to introduce other redistributive policies (e.g. proportional taxation) to analyze the interaction between them and default incentives. Second, to allow for partial default, which provides a natural framework to study a situation akin to *de facto* defaults via inflation, and also allows the model to capture the fact that, unlike external defaults, domestic defaults generally do not imply a substantial and generalized reduction in government debt obligations, but more often occur via mechanisms like forced reconversions or maturity extensions. Third, to examine the effects of exogenous default costs, which may increase the debt ratios supported by the model.
A1 Appendix

A1.1 Solution Method and Computational Algorithm

We extend the algorithm proposed by Krusell and Smith [19] to accommodate an environment with government default. We assume that current prices \( q \), and the default decision \( d \) can be written as a function of government debt \( B \), government expenditures \( g \) and a finite set of moments from the joint distribution of income and wealth \( M = \{m_1, \ldots, m_N\} \). We also let \( M' \) be a function of \( B, M \), and \( g \) and denote its transition matrix by \( M' = H^M(B, M, g) \). Note that \( H^M \) needs to be defined only for states when \( d' = 0 \) since after a government default every household debt position is set to zero.

In what follows we redefine the household and government problems.

A1.1.1 Household Problem

Given government policies, the problem of the household can be written as follows:

\[
V(b, y, B, M, g) = (1 - d(B, M, g))V^{d=0}(b, y, B, M, g) + d(B, M, g)V^{d=1}(y, g)
\]

where \( V^{d=0}(b, y, B, M, g) \) denote the continuation value if the government chooses not to default and \( V^{d=1}(y, g) \) is the continuation value if the government chooses to default. The continuation value in the case of no default is given by:

\[
V^{d=0}(b, y, B, M, g) = \max_{b' \geq 0} u(b + y - \tau - q(B, M, g)b') + \beta E_{y', g'}[V(b', y', B', M', g')|y, g]
\]

\[
\begin{align*}
B' &= (1 - \rho^B)\mu_B + \rho^B B + \alpha^g(g - \overline{g}) \\
\tau &= B + g - q(B, M, g)B' \\
M' &= H^M(B, M, g)
\end{align*}
\]

The value of default is defined as follows:

\[
V^{d=1}(y, g) = u(y - g) + \beta E_{y', g'}[V^{d=0}(0, y, 0, M_0, g)|y, g],
\]

where \( M_0 \) denotes the set of moments corresponding to a distribution of households where debt holdings for all households are \( b = 0 \).
A1.1.2 Government’s Default Decision

At any given distribution, the government evaluates

\[ W(B, M, g) = \max_{d \in \{0, 1\}} \left\{ \int V^{d=0}(b, y, B, M, g) d\omega(b, y), \int V^{d=1}(y, g) d\omega(b, y) \right\} \]

That is, after the government expending shock is realized, the government chooses to repay the debt and borrow again or default.

The optimal debt problem and the auxiliary problem to obtain market clearing are similarly to those previously defined. They are solved after the government has decided not to default.

A1.1.3 Computational Algorithm

1. Guess Aggregate Functions: \( q^0, H^{M,0}, d^0 \) (note that from \( d \) and the rule of \( B' \) we can obtain \( \tau \)).

2. Solve Household Problem.

3. Solve the government default problem. Obtain the new default decision rule \( d^1 \).

4. Simulate the economy for \( T \) periods:
   
   (a) Simulate a sequence of government expenditures \( \{g_t\}_{t=1}^T \).

   (b) Guess \( \Gamma_{t=1} \). From the initial distribution \( \Gamma_{t=1} \), compute the set of moments \( M_{t=1} \).

   (c) While \( t \leq T \):

      i. Using the government debt rule obtain \( B_{t+1} \), the corresponding price \( q_t \) and \( \tau_t \).

      ii. Using \( \Gamma_t \) and \( b' = h(b, y, B, M, g) \) obtain the new distribution \( \Gamma_{t+1} \) and compute the set of moments \( M_{t+1} \).

      iii. Solve the government default decision. If default, set \( \Gamma_{t+1} \) such that all agents hold zero debt. Return to 4(c)i.

5. Using the sequences of \( \{g_t, B_t, M_t\}_{t=\hat{t}}^T \) obtain the new aggregate function \( H^{M,1} \), where \( \hat{t} \) is a period \( \hat{t} > 1 \) such that the model is simulated long enough to make all results independent of the initial distribution (we use \( \hat{t} > 2,000 \)).

6. If \(|H^{M,1} - H^{M,0}| < \epsilon_M\) for \( \epsilon_M \) small continue, if not, set \( H^{M,0} = H^{M,1} \) and return to step 2.
7. Using the new government default decision rule $d^1$ (from step 3) obtain the new price function $q^1$. If $|q^1 - q^0| < \epsilon_q$ for $\epsilon_q$ small you have found an equilibrium. If not, return to step 2.
References


