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Who Should Pay for Credit Ratings and How?
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ABSTRACT

This paper analyzes a model where investors use a credit rating to decide whether to finance a firm. The rating quality depends on the unobservable effort exerted by a credit rating agency (CRA). We analyze optimal compensation schemes for the CRA that differ depending on whether a social planner, the firm, or investors order the rating. We find that rating errors are larger when the firm orders it than when investors do. However, investors ask for ratings inefficiently often. Which arrangement leads to a higher social surplus depends on the agents' prior beliefs about the project quality. We also show that competition among CRAs causes them to reduce their fees, put in less effort, and thus leads to less accurate ratings. Rating quality also tends to be lower for new securities. Finally, we find that optimal contracts that provide incentives for both initial ratings and their subsequent revisions can lead the CRA to be slow to acknowledge mistakes.

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1 Introduction

Virtually every government inquiry into the 2008 and 2009 financial crisis has assigned some blame to credit rating agencies. For example, the Financial Crisis Inquiry Commission (2011, p. xxv) concludes that “this crisis could not have happened without the rating agencies”. Likewise, the United States Senate Permanent Subcommittee on Investigations (2011, p. 6) states that “inaccurate AAA credit ratings introduced risk into the U.S. financial system and constituted a key cause of the financial crisis”. In announcing its lawsuit against S&P, the U.S. government claimed that “S&P played an important role in helping to bring our economy to the brink of collapse”. But the details of the indictments differ slightly across the analyses. For instance, the Senate report points to inadequate staffing as a critical factor, the Financial Crisis Inquiry Commission highlights the business model that had firms seeking to issue securities pay for ratings as a major contributor, while the Department of Justice lawsuit identifies the desire for increased revenue and market share as a critical factor. In this paper we explore the role that these and other factors might play in creating inaccurate ratings.

We study a one-period environment where a firm is seeking funding for a project from investors. The project’s quality is unknown, and a credit rating agency can be hired to evaluate the project. That is, the rating agency creates value by generating information that can lead to more efficient financing decisions. The CRA must exert costly effort to acquire a signal about the quality of the project, and the higher the effort, the more informative the signal about the project’s quality is. The key friction is that the CRA’s effort is unobservable, so a compensation scheme must be designed to provide incentives to the CRA to exert it. We consider three settings, where we vary who orders a rating — a planner, the firm, or potential investors.

This simple framework makes it possible to directly address the claims made in the government reports. In particular, we can ask: how do you compensate the CRA to avoid

1The United States Senate Permanent Subcommittee on Investigations (2011) reported that “factors responsible for the inaccurate ratings include rating models that failed to include relevant mortgage performance data, unclear and subjective criteria used to produce ratings, a failure to apply updated rating models to existing rated transactions, and a failure to provide adequate staffing to perform rating and surveillance services, despite record revenues”. Financial Crisis Inquiry Commission (2011) concluded that “the business model under which firms issuing securities paid for their ratings seriously undermined the quality and integrity of those ratings; the rating agencies placed market share and profit considerations above the quality and integrity of their ratings”. The United States Department of Justice Complaint (2013) states that because of “the desire to increase market share and profits, S&P issued inflated ratings on hundreds of billions of dollars’ worth of CDOs”. 
shirking? Does the issuer-pays model generate more shirking than when the investors pay for ratings? In addition, in natural extensions of the basic model we can see whether a battle for market share would be expected to reduce ratings quality, or whether different types of securities create different incentives to shirk.

Our model explains five facts about the ratings business, documented in the next section, in a unified fashion. The first fact is that rating mistakes are in part due to insufficient effort by rating agencies. The second is that outcomes and accuracy of ratings do differ depending on which party pays for a rating. Third, increases in competition between rating agencies are accompanied by a reduction in the accuracy of ratings. Fourth, ratings mistakes are more common for newer securities with shorter histories that can be studied than for more established types of securities. Finally, revisions to ratings are slow.

We begin our analysis by characterizing the optimal compensation arrangement for the CRA. The need to provide incentives for effort requires setting the fees that are contingent on outcomes — the issued rating and the project’s performance —, which can be interpreted as rewarding the CRA for establishing a reputation for accuracy. Moreover, as is often the case in this kind of models, the problem of effort under-provision argues for giving the surplus from the investment project to the rating agency, so that the higher the CRA’s profits, the higher the effort it exerts.

We proceed by comparing the CRA’s effort and the total surplus in this model depending on who orders a rating. We find that under the issuer-pays model, the rating is acquired less often and is less informative (i.e., the CRA exerts less effort) than in the investor-pays model (or in the second best, where the planner asks for a rating). However, the total surplus in the issuer-pays model may be higher or lower than in the investor-pays model, depending on the agents’ prior beliefs about the quality of the project. The ambiguity about the total surplus arises because even though investors induce the CRA to exert more effort, they will ask for a rating even when the social planner would not. So the extra accuracy achieved by having investors pay is potentially dissipated by an excessive reliance on ratings.

We also extend the basic setup in four ways. The first extension explores the implications of allowing rating agencies to compete for business. An immediate implication of competition is a tendency to reduce fees in order to win business. But with lower fees comes lower effort on project evaluation. Hence, this framework predicts that competition tends to lead to less accurate ratings.

Second, we analyze the case when the CRA can misreport its information. We show
that although the optimal compensation scheme is different than without the possibility of misreporting, our other results extend to this case.

The third extension considers the accuracy of ratings on different types of securities. We suppose that some types of investment projects are inherently more difficult for the CRA to evaluate — presumably because they have a short track record that makes comparisons difficult. We demonstrate that in this case it is inevitable that the ratings will deteriorate.

Finally, we allow for a second period in the model and posit that investment is needed in each of the two periods, so that there is a role for ratings in both periods. The need to elicit effort in both periods poses a problem. The most powerful way to provide incentives for the accuracy of the initial rating requires paying the CRA only when it announces identical ratings in both periods and the project’s performance matches these ratings. Paying the CRA if it makes a ‘mistake’ in the initial rating (when a high rating is followed by the project’s failure) would be detrimental for the incentives in the first period’s effort. However, not paying to the CRA after a ‘mistake’ will result in zero effort in the second period, when the rating needs to be revised. Balancing this trade-off involves the fees in the second period after a ‘mistake’ being too low ex-post, which leads to the CRA being slow to acknowledge mistakes.

While we find that our simple model is very powerful in that it explains the five aforementioned facts using relatively few assumptions, our approach does come with several limitations. For instance, due to complexity, we do not study the problem when multiple ratings can be acquired in equilibrium. Thus we cannot address debates related to rating shopping — a common criticism of the issuer-pays model. Also, we assume that the firm has the same knowledge about the project’s quality ex ante as everyone else. Without this assumption the analysis becomes much more complicated, since in addition to the moral hazard problem on the side of the CRA there is an adverse selection problem on the side of the firm. We do offer some cursory thoughts on this problem in our conclusions.

The remainder of the paper is organized as follows. The next section documents the empirical regularities that motivate our analysis, and compares our model to others in the literature. Section 3 introduces the baseline model. Section 4 presents our main results about the CRA compensation as well as comparison between the issuer-pays and investor-pays models. Section 5 covers the four extensions just described. Section 6 concludes, and proofs are given in the Appendix.

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2See the literature review below for discussion of papers that do generate rating shopping. Notice, however, that even without rating shopping we were able to identify problems with the issuer-pays model.
2 Motivating Facts and Literature Review

Given the intense interest in the causes of the financial crisis and the role that official accounts of the crisis ascribe to the ratings agencies, it is not surprising that there has been an explosion of research on credit rating agencies. White (2010) offers a concise description of the rating industry and recounts its role in the crisis. To understand our contribution, we find it helpful to separate the recent literature into three sub-areas.

The first consists of the empirical studies that seek to document mistakes or perverse rating outcomes. There are so many of these papers that we cannot cover them all, but it is helpful to note that there are five facts that our analysis takes as given. So we will point to specific contributions that document these particular facts.

First, the question of who pays for a rating does seem to matter. The rating industry is currently dominated by Moody’s, S&P, and Fitch Ratings which are each compensated by issuers. So comparisons of their recent performance does not speak to this issue. But Cornaggia and Cornaggia (2012) provide some evidence on this question by comparing Moody’s ratings to those of Rapid Ratings, a small rating agency which is funded by subscription fees from investors. They find that Moody’s ratings are slower to reflect bad news than those of Rapid Ratings.

Jiang, Stanford, and Xie (2012) provide complementary evidence by analyzing data from the 1970s when Moody’s and S&P were using different compensation models. In particular, from 1971 until June 1974 S&P was charging investors for ratings, while Moody’s was charging issuers. During this period the Moody’s ratings systematically exceeded those of S&P. S&P adopted the issuer-pays model in June 1974, and from that point forward over the next three years their ratings essentially matched Moody’s.

Second, as documented by Mason and Rosner (2007), most of the rating mistakes occurred for structured products that were primarily related to asset-backed securities. As Pagano and Volpin (2010) note, the volume of these new securities increased tenfold between 2001 and 2010. As Mason and Rosner emphasize, the mistakes that happened for these new products were not found for corporate bonds where CRAs had much more experience. In addition, Morgan (2002) argues that banks (and insurance companies) are inherently more opaque than other firms, and this opaqueness explains his finding that Moody’s and S&P differ more in their ratings for these intermediaries than for non-banks.

Third, some of the mistakes in the structured products seem to be due to insufficient monitoring and effort on the part of the analysts. For example, Owusu-Ansah (2012)
shows that downgrades by Moody’s tracked movements in aggregate Case-Shiller home price indices much more than any private information that CRAs had about specific deals.

Interestingly, the Dodd-Frank Act in the U.S. also presumes that shirking was a problem during the crisis and takes several steps to try to correct it. First, section 936 of the Act requires the Securities and Exchanges Commission to take steps to guarantee that any person employed by a nationally recognized statistical rating organization (1) meets standards of training, experience, and competence necessary to produce accurate ratings for the categories of issuers whose securities the person rates; and (2) is tested for knowledge of the credit rating process. The law also requires the agencies to identify and then notify the public and other users of ratings which five assumptions would have the largest impact on their ratings in the event that they were incorrect.

Fourth, revisions to ratings are typically slow to occur. This issue attracted considerable attention early in the last decade when the rating agencies were slow to identify problems at Worldcom and Enron ahead of their bankruptcies. But, Covitz and Harrison (2003) show that 75% of the price adjustment of a typical corporate bond in the wake of a downgrade occurs prior to the announcement of the downgrade. So these delays are pervasive.

Finally, it appears that competition among rating agencies reduces the accuracy of ratings. Very direct evidence on this comes from Becker and Milbourn (2011) who study how the rise in market share by Fitch influenced ratings by Moody’s and S&P (who had historically dominated the industry). Prior to its merger with IBCA in 1997, Fitch had a very low market share in terms of ratings. Thanks to that merger, and several subsequent acquisitions over the next five years, Fitch substantially raised its market share, so that by 2007 it was rating around 1/4 of all the bonds in a typically industry. Becker and Milbourn exploit the cross-industry differences in Fitch’s penetration to study competitive effects. They find an unusual pattern. Any given individual bond is more likely to be rated by Fitch when the ratings from the other two big firms are relatively low. Yet, in the sectors where Fitch issues more ratings, the overall ratings for the sector tend to be higher.

This pattern is not easily explained by the usual kind of catering that the rating agencies have been accused of. If Fitch were merely inflating its ratings to gain business with the poorly performing firms, the Fitch intensive sectors would be ones with more ratings for these under-performing firms and hence lower overall ratings. This general increase in ratings suggests instead a broader deterioration in the quality of the ratings, which would be expected if Fitch’s competitors saw their rents declining; consistent with this view, the forecasting power of the ratings for defaults also decline.
Our paper is also related to the many theoretical papers on rating agencies that have been proposed to explain these and other facts. However, we believe our paper is the only one that simultaneously accounts for the five facts described above.

Two papers that are perhaps closest to ours are Opp, Opp, and Harris (2012) and Bolton, Freixas, and Shapiro (2012). Opp, Opp, and Harris explain rating inflation by building a model where ratings not only provide information to investors, but are also used for regulatory purposes. As in our model, expectations are rational and a CRA’s effort affects rating precision. But unlike us, they assume that the CRA can commit to exert effort (or, equivalently, that effort is observable), and they do not study optimal contracts. They find that introducing rating-contingent regulation leads the rating agency to rate more firms highly, although it may increase or decrease rating informativeness.

Cornaggia and Cornaggia (2012) find evidence directly supporting the prediction of the Opp, Opp, and Harris (2012) model. Specifically, it seems that Moody’s willingness to grant inflated ratings (relative to a subscription-based rating firm) is concentrated on the kinds of marginal investment grade bonds that regulated entities would be prevented from buying if tougher ratings were given by Moody’s. We agree that regulations can influence ratings, but we see our results complementing the analysis in Opp, Opp, and Harris (2012) and providing additional insights on issues they do not explore.

Bolton, Freixas, and Shapiro (2012) study a model where a CRA receives a signal about a firm’s quality, and can misreport it (although investors learn about a lie ex post). Some investors are naive, which creates incentives for the CRA — which is paid by the issuer — to inflate ratings. The authors show that the CRA is more likely to inflate (misreport) ratings in booms, when there are more naive investors, and/or when the risks of failure which could damage CRA reputation are lower. In their model, both the rating precision and reputation costs are exogenous. In contrast, in our model the rating precision is chosen by the CRA; also, our optimal contract with performance-contingent fees can be interpreted as the outcome of a system in which reputation is endogenous. Similar to us, the authors predict that competition among CRAs may reduce market efficiency, but for a very different reason than we do: the issuer has more opportunities to shop for ratings and to take advantage of naive investors by only purchasing the best ratings. In contrast, we assume rational expectations, and predict that larger rating errors occur because of more

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3While not applied to rating agencies, there are a number of theoretical papers on delegated information acquisition, see, for example, Chade and Kovrijnykh (2012), Inderst and Ottaviani (2009, 2011) and Gromb and Martiní (2007). Our paper is also broadly related to the literature on media biases — see, e.g., Mullainathan and Shleifer (2005) and references therein.
shirking by CRAs.

Our result that competition reduces surplus is also reminiscent of the result in Strausz (2005) that certification constitutes a natural monopoly. In Strausz this result obtains because *honest* certification is easier to sustain when certification is concentrated at one party. In contrast, in our model the ability to charge a higher price increases rating *accuracy* even when the CRA cannot lie.

Skreta and Veldkamp (2009) analyze a model where the naiveté of investors gives issuers incentives to shop for ratings by approaching several rating agencies and publishing only favorable ratings. They show that a systematic bias in disclosed ratings is more likely to occur for more complex securities — a finding that resembles our result that rating errors are larger for new securities. Similar to our findings, in their model, competition also worsens the problem. They also show that switching to the investor-pays model alleviates the bias, but as in our set up the free-rider problem can then potentially eliminate the ratings market completely.

Sangiorgi and Spatt (2012) have a model that generates rating shopping in a model with rational investors. In equilibrium, investors cannot distinguish between issuers who only asked for one rating, which turned out to be high, and issuers who asked for two ratings and only disclosed the second high rating but not the first low one. They show that too many ratings are produced, and while there is ratings bias, there is no bias in asset pricing as investors understand the structure of equilibrium. While we conjecture that a similar result might hold in our model, the analysis of the case where multiple ratings are acquired in equilibrium is hard since, unlike in Sangiorgi and Spatt, the rating technology is endogenous in our setup.

Similar to us, Faure-Grimaud, Peyrache, and Quesada (2009) study optimal contracts between a rating agency and a firm, but their focus is on providing incentives to the firm to reveal its information, while we focus on providing incentives to the CRA to exert effort. Goel and Thakor (2011) have a model where the CRA’s effort is unobservable, but they do not analyze optimal contracts; instead, they are interested in the impact of legal liability for ‘misrating’ on the CRA’s behavior.

As we later discuss, the structure of our optimal contracts can be endogenously embodying reputational effects. Other papers that model reputational concerns of rating agencies include, for example, Bar-Isaac and Shapiro (2010), Fulghieri, Strobl, and Xia (2011), and Mathis, McAndrews, and Rochet (2009).

Finally, our analysis is also relevant for the many policy-oriented papers that discuss
potential reforms of the credit rating agencies. Medvedev and Fennell (2011) provide an excellent summary of these issues. Their survey is also representative of most of the papers on this topic in that it identifies the intuitive conflicts of interest that arise from the issuer-pays model, and compares them to the alternatives problems that arise under other schemes (such as the investor-pays, or having a government agency issue ratings). But all of these analyses are partly limited by the lack of microeconomic foundations underlying the payment models being contrasted. By deriving the optimal compensation schemes, we believe we help clarify these kinds of discussions.

3 The Model

We consider a one-period model with one firm, a number \( n \geq 2 \) of investors, and one credit rating agency. All agents are risk neutral and maximize expected profits.

The firm (the issuer of a security) is endowed with a project that requires one unit of investment (in terms of the consumption good) and generates the end-of-period return, which equals \( y \) units of the consumption good in the event of success and 0 in the event of failure. The likelihood of success depends on the quality of the project, \( q \).

The quality of the project can be good or bad, \( q \in \{g, b\} \), and is unobservable to everyone.\(^4\) A project of quality \( q \) succeeds with probability \( p_q \), where \( 0 < p_b < p_g < 1 \). We assume that \(-1 + p_by < 0 < -1 + p_gy\), so that it is profitable to finance a high-quality project but not a low-quality one. The prior belief that the project is of high quality is denoted by \( \gamma \), where \( 0 \leq \gamma \leq 1 \).

The CRA can acquire information about the quality of the project. It observes a signal \( \theta \in \{h, \ell\} \) that is correlated with the project’s quality. How informative the signal is about the project’s quality depends on the level of effort \( e \geq 0 \) that the CRA exerts. Specifically,

\[
Pr\{\theta = h|q = g, e\} = Pr\{\theta = \ell|q = b, e\} = \frac{1}{2} + e, \tag{1}
\]

where \( e \) is restricted to be between 0 and 1/2. Note that if effort is zero, the conditional distribution of the signal is the same regardless of the project’s quality, and therefore the signal is uninformative. Conditional on the project being of a certain quality, the probability of observing a signal consistent with that quality is increasing in the agent’s

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\(^4\)We discuss what happens if the issuer has private information about its type in the conclusions.
effort. So higher effort makes the signal more informative in Blackwell’s sense.\footnote{See Blackwell and Girshick (1954), chapter 12.}

Exerting effort is costly for the CRA, where $\psi(e)$ denotes the cost of effort $e$, in units of the consumption good. The function $\psi$ satisfies $\psi(0) = 0$, $\psi'(e) > 0$, $\psi''(e) > 0$, $\psi'''(e) > 0$ for all $e > 0$, and $\lim_{e \to 1/2} \psi(e) = +\infty$ (which is a sufficient but not necessary condition to guarantee that the project’s quality is never learned with certainty). The assumptions on the second and third derivatives of $\psi$ guarantee that the CRA’s and planner’s problems, respectively, are strictly concave in effort. We also assume that $\psi'(0) = 0$ and $\psi''(0) = 0$, which guarantee an interior solution for effort in the CRA’s and planner’s problems, respectively.

To keep the analysis simple, we will assume that the CRA cannot lie about a signal realization so the rating it announces will be the same as the signal. We describe what happens if we dispose of this assumption in Section 5.2. While allowing for misreporting changes the form of the optimal compensation to the CRA, it does not affect any other key results, as we illustrate in the Appendix. We also assume that the CRA is protected by limited liability, so that all payments that it receives must be non-negative.

The firm has no internal funds, and therefore needs investors to finance the project.\footnote{We make this assumption for expositional convenience. Our results would not change if the firm had initial wealth which is strictly smaller than one — the amount of funds needed to finance the project.} Investors are deep-pocketed so that there is never a shortage of funds.\footnote{It is not necessary for our results to assume that each investor has enough funds to finance the project alone. As long as each investor has more funds than what the firm borrows from him in equilibrium, our results still apply.} They behave competitively and will make zero profits in equilibrium.

We will consider three scenarios depending on who decides whether a rating is ordered — the social planner, the issuer, or each of the investors. Let $X$ refer to the identity of the player ordering a rating. The timing of events, illustrated in Figure 1, is as follows.

At the beginning of each period, the CRA posts a rating fee schedule — the fees to be paid at the end of the period, conditional on the history. Each investor announces financing terms (interest rates) conditional on a rating or the absence of one. When $X$ is the firm, it might not be able to pay for a rating if the fee structure requires payments when no output is generated, as it has no internal funds. Thus we also assume that in this case each investor offers rating financing terms that specify the return paid by the issuer when it has output in exchange for the investor paying the fee on the issuer’s behalf. Then $X$ decides whether to ask for a rating, and chooses whether to reveal to the public that a rating has
The CRA sets history-contingent fees. 

- Investors announce rating-contingent financing terms. 
- If $X$ is the firm, investors also announce interest rates for financing the rating fees.

- $X$ decides whether to order a rating. 

- If the rating is ordered, the CRA exerts effort, reveals the rating to $X$, who decides whether to announce it to other agents.

- The firm decides whether to borrow from investors in order to finance the project.

- If the project is financed, success/failure is observed. 
- The firm repays investors, the CRA collects the fees.

Figure 1: Timing.

been ordered. If a rating is ordered, the CRA exerts effort and announces the rating to $X$, who then decides whether it should be published (and hence made known to other agents).

The firm decides whether to borrow from investors in order to finance the project given the interest rates.\(^8\) If the project is financed, its success or failure is observed. The firm repays investors, and the CRA collects its fees.\(^9\)

We are interested in analyzing Pareto efficient perfect Bayesian equilibria in this environment. We will compare effort and total surplus depending on who orders a rating. The rationale for considering total surplus comes from thinking about a hypothetical consumer who owns both the firm and CRA, in which case it would be natural for the social planner to maximize the consumer's utility. In our static environment, we will not always be able to Pareto rank equilibria depending on who orders the rating. However, it can be shown that constraints that lead to a lower total surplus in the static model, lead to Pareto dominance in a repeated infinite horizon version of the model.

4 Analysis and Results

Before deriving any results, it will be convenient to introduce some notation. First, let $\pi_1$ denote the ex-ante probability of success (before observing a rating), so $\pi_1 = \mu g + \mu b(1 - \gamma)$. Next, let $\pi_h(e)$ denote the probability of observing a high rating given effort $e$, that is, $\pi_h(e) = (1/2 + e)\gamma + (1/2 - e)(1 - \gamma)$. The probability of observing a low rating given effort $e$ is then $\pi_l(e) = 1 - \pi_h(e)$. Also, let $\pi_{h1}(e)$ and $\pi_{h0}$ denote the probabilities of

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\(^8\)We assume that if the firm is indifferent between investors’ financing terms, it obtains an equal amount of funds from each investor. If each investor can fund the project alone, this is also equivalent to the firm randomizing with equal probabilities over which investor to borrow from.

\(^9\)We assume that $X$ can commit to paying the fees due to the CRA, and that the firm can commit to paying investors.
observing a high rating followed by the project’s success/failure given effort $e$: $\pi_{h1}(e) = p_y(1/2 + e)\gamma + p_b(1/2 - e)(1 - \gamma)$ and $\pi_{h0}(e) = (1 - p_y)(1/2 + e)\gamma + (1 - p_b)(1/2 - e)(1 - \gamma)$.

The probability of observing a high rating bears directly on the earlier discussion of the possibility that rating agencies issued inflated ratings for securities that eventually failed. In our model, when the CRA puts insufficient effort, its ratings will be unreliable. Thus, for bad projects, the under-provision of effort will lead to a more likely (incorrect) assignment of high ratings. The assumed connection between the CRA’s effort and the signal distribution given by (1) implies that the probability of giving a high rating to a bad-quality project is the same as the probability of giving a low rating to a good-quality project. Thus unconditionally the high rating is produced more often if less effort is put in whenever $\gamma < 1/2$. (Formally, $\pi_h'(e) < 0$ for $\gamma < 1/2$.) The cutoff value of $1/2$ arises because of the symmetric structure of (1). If instead we had adopted a more general signal structure such as $\Pr\{\theta = h|q = g, e\} = \alpha + \beta_h e$ and $\Pr\{\theta = h|q = b, e\} = \alpha - \beta e$, then the cutoff value for $\gamma$ that governs when erroneous ratings will be too high would differ. In particular, the lower the ratio $\beta_h/\beta_e$ (i.e., the more important is the CRA’s effort in detecting bad projects relative to recognizing good ones), the higher will be the cutoff.\(^{10}\)

Another important basic observation about the structure of our problem is that because producing a rating is costly, it cannot be optimal to pay to produce one if the information is not used. This implies that if the CRA exerts positive effort, then the project must be financed after the high rating and not financed after the low rating.

We begin by considering as a useful benchmark, the first-best case, where the CRA’s effort is observable, and the social planner decides whether to order a rating.\(^{11}\) There are three cases to consider: (i) do not acquire a rating and do not finance the project, (ii) do not acquire a rating and finance the project, and (iii) acquire a rating and finance the project only if the rating is high. Combining the three options, the total surplus in the first-best case is $S^{FB} = \max\{0, -1 + \pi_1 y, \max_e - \psi(e) - \pi_h(e) + \pi_{h1}(e)y\}$. Denote the first-best effort by $e^{FB}$. Notice that $e^{FB} = 0$ in cases (i) and (ii), and $e^{FB} = e^* > 0$ in case (iii), where $e^* \equiv \arg\max_e - \psi(e) - \pi_h(e) + \pi_{h1}(e)y$. The following lemma shows which of the three cases occurs depending on the prior $\gamma$.

**Lemma 1** There exist thresholds $\underline{\gamma}$ and $\bar{\gamma}$ satisfying $0 < \underline{\gamma} < \bar{\gamma} < 1$, such that

1. $e^{FB} = 0$ for $\gamma \in [0, \underline{\gamma}]$, and the project is never financed;

\(^{10}\)Formally, $\pi_h(e) = (\alpha + \beta_h e)\gamma + (\alpha - \beta e)(1 - \gamma)$, which is decreasing in $e$ if and only if $\gamma < \beta_h/(\beta_e + \beta_h)$.

\(^{11}\)In fact, it is easy to check that when effort is observable, the total surplus is the same regardless of who orders a rating.
(ii) $e^{FB} = 0$ for $\gamma \in [\bar{\gamma}, 1]$, and the project is always financed;
(iii) $e^{FB} > 0$ for $\gamma \in (\bar{\gamma}, \bar{\gamma})$, and the project is only financed after the high rating.

The intuition behind this result is quite simple. If the prior belief about the project quality is close to either zero or one, then it does not pay off to acquire additional information about the quality of the project.

We now turn to the analysis of the more interesting case when the CRA’s effort is unobservable, and payments are subject to limited liability. The CRA will now choose its effort privately, given the fees it expects to receive at the end of the period.

We will first characterize the (constrained) Pareto frontier in this setup. Then, depending on which player orders the rating, we will consider an equilibrium where the total surplus is maximized. Each of the equilibria will lie at a different point on this Pareto frontier, and in some cases inside it.

In order to construct the Pareto frontier, we need to analyze the optimal contract (fee structure) that provides the CRA with incentives to exert effort. The best way to provide incentives is to pay fees contingent on possible outcomes. If the CRA is asked for a rating, there are three possibilities: the rating is high and the project succeeds, the rating is high and the project fails, and the rating is low (in which case the project is not financed). Let $f_{h1}$, $f_{h0}$, and $f_{l}$ denote the fees that the CRA receives in each scenario.

We assume that the CRA posts a fee schedule. We will first consider an alternative setting the social planner chooses the fee structure, which allows us to write a standard optimal contracting problem. Then we will explain why this formulation is equivalent a setup where the CRA chooses the fees.

On the Pareto frontier, the value to one party is maximized subject to delivering at least certain values to other parties. Investors behave competitively and thus always earn zero profits. Therefore, we can maximize the value to the firm subject to delivering at least a certain value $v$ to the CRA.

As before, there are three options available — do not acquire a rating and do not finance the project, do not acquire a rating and finance the project, and acquire a rating and finance the project only if the rating is high. Let $u(v)$ denote the value to the firm under the third alternative, given that the value to the CRA is at least $v$. Since investors earn zero profits, the firm extracts all the surplus generated in production, net of the expected fees paid to
the CRA. Then the Pareto frontier can be written as max \{0 - v, -1 + \pi_1 y - v, u(v)\}, where

\[
\begin{align*}
  u(v) &= \max_{e,f_{h1},f_{h0},f_\ell} -\pi_h(e) + \pi_{h1}(e)y - \pi_{h1}(e)f_{h1} - \pi_{h0}(e)f_{h0} - \pi_{\ell}(e)f_{\ell}, \\
  \text{s.t.} & \quad -\psi(e) + \pi_{h1}(e)f_{h1} + \pi_{h0}(e)f_{h0} + \pi_{\ell}(e)f_{\ell} \geq v, \\
  \psi'(e) &= \pi'_{h1}(e)f_{h1} + \pi'_{h0}(e)f_{h0} + \pi'_{\ell}(e)f_{\ell}, \\
  e &\geq 0, \ f_{h1} \geq 0, \ f_{h0} \geq 0, \ f_{\ell} \geq 0.
\end{align*}
\] (2)

Constraint (3) ensures that the CRA’s profits are at least \(v\). Constraint (4) is the CRA’s incentive constraint, which reflects the fact that CRA chooses its effort privately. Accordingly, this constraint is obtained by maximizing the left-hand side of (3) with respect to \(e\). Finally, the constraints in (5) reflect limited liability and the nonnegativity of effort.

Our first main result demonstrates how the optimal compensation scheme must be structured in order to provide incentives to the CRA to exert effort.

**Proposition 1 (Optimal Compensation Scheme)** Suppose the project is financed only after the high rating. Define the cutoff value \(\hat{\gamma} = 1/(1 + p_b/p_g)\).

(i) If \(\gamma \geq \hat{\gamma}\), then it is optimal to set \(f_{h1} > 0\) and \(f_\ell = f_{h0} = 0\).

(ii) If \(\gamma \leq \hat{\gamma}\), then it is optimal to set \(f_\ell > 0\) and \(f_{h1} = f_{h0} = 0\).

The proposition states that there is a threshold level for the prior belief, above which the CRA should be rewarded only if it announces the high rating and it is followed by success, and below which the CRA should be rewarded only if it announces the low rating. Notice that, quite intuitively, the CRA is never paid for announcing the high rating if it is followed by the project’s failure. The proof relies on the standard maximum likelihood ratio argument: the CRA should be rewarded for the event whose occurrence is the most consistent with its exerting effort, which in turn depends on the agents’ prior.

The feature of the model that the fees are contingent on the rating and the project’s performance warrants a discussion. One might argue that in reality CRAs are mostly compensated upfront. In the static model, an up-front fee will never provide the CRA with incentives to exert effort — the CRA will take the money and shirk. In a repeated setting, it is possible to create incentives using an up-front payment as long as its size depends on the past outcomes. To be more precise, the optimal compensation structure written in a recursive form will require the CRA’s ‘promised values’ (future present discounted profits) to depend on histories. Using an argument similar to the one in Proposition 1, these values will optimally rise after \(h1\) and \(\ell\) and fall after \(h0\). Thus even if the fees are restricted to be
paid upfront in each period, the CRA will be motivated to exert effort by the prospect of higher future profits — via higher future fees — that follow from developing a ‘reputation’ by correctly predicting the firm’s performance. The fee structure in our static model can then be viewed as a shortcut for such a reputation mechanism.

The next proposition derives several properties of the Pareto frontier which will be important for our subsequent analysis.

**Proposition 2 (Pareto Frontier)** Suppose the project is financed only after the high rating.

(i) There exists $v^*$ such that for all $v \geq v^*$ $e(v) = e^*$, but $u(v) < 0$.

(ii) There exists $v_0 > 0$ such that (3) is slack for $v < v_0$ and binds for $v \geq v_0$. Moreover, $e(v_0) > 0$.

(iii) Effort and total surplus are increasing in $v$, strictly increasing for $v \in (v_0, v^*)$.

Part (i) of the proposition says that there exists a threshold promised value, $v^*$, above which the first-best effort is implemented. However, the resulting profit to the firm is strictly negative, violating individual rationality, and so this arrangement cannot be sustained in equilibrium. It will be handy to denote the highest value that can be delivered to the CRA without leaving the firm with negative profits by $\bar{v} \equiv \max\{v|u(v) = 0\} < v^*$.

There is an interesting economic reason why implementing the first-best effort requires the firm’s profits to be negative. Suppose for concreteness $\gamma \geq \hat{\gamma}$ (the other case is similar), so that the CRA gets paid after history $h_1$. Then the intuition is as follows. When effort is observable, the problem can be recast as saying that the firm chooses to acquire information itself rather than delegating this task to the CRA. But when the firm is making the effort choice, it accounts for two potential effects of increasing effort. One benefit is the increased **probability** that a surplus is generated. The other is that investors will lower the interest rate to reflect a more accurate rating, leading to an increase in the size of the surplus. When the CRA is doing the investigation and its effort is unobservable, the CRA internalizes the fact that more effort generates a higher probability of the fee being paid. But it cannot get a higher fee based on higher effort. So the only way to induce the CRA to exert the first-best level effort is to set an extraordinarily generous fee that leaves the firm with negative profits.\(^\text{12}\)

\(^{12}\)Formally, the firm’s problem in the first-best case is $\max_{e} -\psi(e) + \pi_{h_1}(e)(y - R(e))$, where the interest rate $R(e)$ solves the investors’ break even condition $-\pi_h(e) + \pi_{h_1}(e)R(e) = 0$. This implies that $1/R(e)$ equals the conditional probability of success given the high rating, $\pi_{1|h}(e) = \pi_{h_1}(e)/\pi_h(e)$, which is strictly increasing in effort. The CRA’s problem is $\max_{e} -\psi(e) + \pi_{h_1}(e)f_{h_1}$, where $f_{h_1}$ does not depend on $e$. Thus,
Part \((ii)\) of Proposition 2 identifies the lowest value that can be delivered to the CRA on the Pareto frontier. This value, denoted by \(v_0\), is strictly positive. So the rating agency will still be making profits and will exert positive effort. It immediately follows from \((ii)\) that for \(v \leq v_0\) \(u(v)\) does not depend on \(v\) and hence is constant; while if \(v > v_0\), constraint \((3)\) binds, which means that \(u(v)\) must be strictly decreasing in \(v\).

Finally, part \((iii)\) shows that the higher the CRA’s profits, the higher the total surplus, and the higher the effort. This is an important result, and will be crucial for our further analysis. Intuitively it follows because unobservability of effort leads to its under-provision. To implement the highest possible effort, one needs to set the fees as high as possible, extracting all surplus from the firm and giving it to the CRA. However, as part \((i)\) implies, implementing the first-best level of effort would result in negative profits to the firm. Combining \((i)\) and \((iii)\) tells us that the level of effort that can be implemented is strictly smaller than the first-best one.

Notice also that the firm’s profits are maximized at \(v_0\). (This follows immediately from part \((ii)\) of Proposition 2.) Thus the firm prefers a less informative rating than is socially optimal (as effort at \(v_0\) is lower than that at \(\bar{v}\) or \(v^*\)), but the firm still prefers to have a informative rating (because effort is positive at \(v_0\)).

The function \(u(v)\) is graphed in Figure 2. Recall that \(u(v)\) only describes the part of in order to induce \(e^{FB}\), \(f_{h1}\) must exceed \(y - R(e)\), leaving the firm with negative profits: \(\pi_{h1}(e)(y - R(e) - f_{h1}) < 0\).
the Pareto frontier which corresponds to the situation when the project is financed after
the high rating and not financed after the low rating. The whole Pareto frontier is given by
\[
\max \{-v, -1 + \pi_1 y - v, u(v)\},
\]
and the corresponding total surplus is \[
\max \{0, -1 + \pi_1 y, v + u(v)\}.
\]

To summarize, the fact that the CRA chooses its effort privately (and is protected by
limited liability) has the following implications. First, the optimal compensation must
involve outcome-contingent fees, which can be interpreted as rewards for establishing a
good reputation. Second, the CRA exerts less effort, and hence there are more rating
errors compared to the case when the CRA’s effort is observable. These results are general
— they do not depend on who orders a rating, and they will also hold in the extensions of
the basic model that we will consider in Section 5.

Clearly, our assumption of limited liability plays an important role in these results.
Without it, it would be possible to punish the CRA in some states and achieve the first
best for all \( v \). In particular, selling the project to the CRA and making it an investor would
provide it with incentives to exert the first-best level of effort.\(^{13}\)

Finally, notice that since it is optimal to give all profits to the CRA as long as the
project is financed only after the high rating, the fees set by the CRA will coincide with
those set by the social planner. Hence, in this case stating the problem as if the planner
chooses fees is equivalent to considering the problem where the CRA chooses them.

Next, we consider how the equilibria will vary depending on who (\( X \)) orders the rating.
For each \( X \), the equilibrium will correspond to a different point \((v, u(v))\). Moreover, whether
\( X \) even chooses to order a rating can differ across agents.

### 4.1 The Social Planner Orders a Rating

Let us start with the case where the planner gets to decide whether a rating is ordered. Re-
call that we are considering equilibria where the total surplus is maximized. It immediately
follows from Proposition 2 that if the project is financed only after the high rating, then
the planner will choose the point \((\bar{v}, u(\bar{v}))\) on the frontier. This corresponds to maximum
feasible CRA profits and effort, and zero profits for the firm. The implemented effort, which
we denote by \( e^{SB} \) (where \( SB \) stands for the second best), is strictly smaller than \( e^{FB} \). We
summarize these results in the following proposition.

\(^{13}\)However, forcing rating agencies to co-invest does not appear to be a practical policy implication, as
it would require them to have implausibly large levels of wealth, given that they rate trillions of dollars' worth of securities each year.
Figure 3: The total surplus (left) and effort (right) as functions of the prior belief $\gamma$. Parameter values: $y = 1.8$, $p_g = .8$, $p_b = .2$, $\psi(e) = 3e^5$.

**Proposition 3 (X = Planner)** If the social planner is the one who decides whether a rating should be ordered, then

(i) The maximum total surplus in equilibrium is $S^{SB} = \max\{0, -1 + \pi_1 y, \bar{v} + u(\bar{v})\}$;

(ii) $e^{SB} \leq e^{FB}$, $S^{SB} \leq S^{FB}$, with strict inequalities if $e^{FB} > 0$.

Figure 3 uses a numerical example to compare the total surplus and effort in the first- and second-best cases as functions of $\gamma$, depicted with solid black and dashed gray lines, respectively. The thin dotted line in the left panel is $-1 + \pi_1 y$, the total surplus if the project is financed without a rating. The total surplus if the project is not financed without a rating is zero. Therefore, the total surplus in the first-best case, $S^{FB}$, is the upper envelope of three lines, $0$, $-1 + \pi_1 y$, and $\bar{v} + u(\bar{v})$. Similarly, the total surplus in the second-best case, $S^{SB}$, is the upper envelope of $0$, $-1 + \pi_1 y$, and $\bar{v} + u(\bar{v})$.

From Figure 3 it is apparent that the planner decides not acquire a rating for some values of $\gamma$ when one would be acquired if effort were observable. The reduced propensity to get the rating occurs because the total surplus from acquiring the rating is lower. Thus, graphically, the interval on which the upper envelope of the three lines equals $\bar{v} + u(\bar{v})$ is smaller than that in the first-best case.
4.2 The Issuer Orders a Rating

Next, we consider the case where the firm decides whether to order a rating. Recall that the CRA sets the fee schedule, and hence it will post the highest fee that the firm is willing to pay. The firm’s willingness to pay equals its profit if it chooses not to order a rating. Without a rating, investors finance the firm’s project if and only if $-1 + \pi_1 y > 0$. Since investors break even, the firm’s profit in this case is $u \equiv \max\{0, -1 + \pi_1 y\}$. Thus, if a rating is acquired in equilibrium, the firm receives $u$, and the corresponding value to the CRA is $v^{iss} \equiv \max\{v | u(v) = u\} \leq \bar{v}$, with strict inequality if $-1 + \pi_1 y > 0$ since $u(v)$ is strictly decreasing in $v$ for $v > v_0$. Denote the total surplus and effort in the issuer-pays case by $S^{iss}$ and $e^{iss}$, respectively. Recall from Proposition 2 that the total surplus and effort are increasing in $v$. This leads us to the following result:

Proposition 4 ($X = \text{Issuer}$) Suppose the firm decides whether to order a rating. Then

(i) The maximum total surplus in equilibrium is $S^{iss} = \max\{0, -1 + \pi_1 y, v^{iss} + u(v^{iss})\}$;
(ii) $e^{iss} \leq e^{SB}, S^{iss} \leq S^{SB}$, with strict inequalities if $e^{SB} > 0$ and $-1 + \pi_1 y > 0$.

As usual, the firm will decide not to ask for a rating if the prior belief $\gamma$ is sufficiently close to zero or one. Moreover, since the implemented effort with the firm choosing whether to request a rating is lower relative to when the planner picks, rating acquisition will occur on a smaller set of priors in the former case than in the latter.

The total surplus and implemented effort in the case when the issuer orders a rating are depicted with solid gray lines on Figure 3. As described in Proposition 4, when $-1 + \pi_1 y > 0$, the total surplus and effort are lower than when the planner orders a rating. Notice that $e^{iss}$ decreases with $\gamma$ when $-1 + \pi_1 y > 0$ because the firm’s outside option is $-1 + \pi_1 y$, and $\pi_1$ increases with $\gamma$.

To summarize, the issuer-pays model leads to lower rating precision and total surplus than the planner would attain, because the option of receiving financing without a rating reduces the firm’s willingness to pay for a rating. That is, our model predicts that the issuer-pays model is indeed associated with more rating errors than is socially optimal. As we will see in the next section, in some cases the rating errors are also larger than when the investors order ratings.

14This argument relies on the assumption that the firm can credibly announce that it did not get rated. Without this assumption the issuer’s payoff is still strictly positive when $-1 + \pi_1 y > 0$, although it is lower than $-1 + \pi_1 y$ — see Claim 1 in the Appendix.
4.3 Investors Order a Rating

Consider finally the case when each investor decides whether to order a rating. We will show that this case results in a lower total surplus relative to the planner’s case because investors are competing over the interest rates that they offer to the firm conditional on the rating. As we will see, the comparison of the total surplus and effort relative to the issuer-pays case will depend on the prior $\gamma$.

The assumption that investors who do not pay for a rating can be excluded from learning it is critical. If this is not the case and the spread of information cannot be precluded, investors will want to free-ride on others paying for a rating. As a result, no rating will be acquired in equilibrium, and investors will make their financing decisions solely based on the prior. Until the mid 1970s, the investor-pays model was widely used. However, the rise of photocopying made protecting the sort of information described above became increasingly impractical, which arguably resulted in the switch to the issuer-pays model.

The following proposition identifies the total surplus in the investor-pays case.

**Proposition 5 (X = Investors)** Suppose each investor decides whether to order a rating. Then $S^{\text{inv}} = \max\{0, v^{\text{inv}} + u(v^{\text{inv}})\}$, where $v^{\text{inv}} = \bar{v} (= v^{\text{iss}})$ if $-1 + \pi_1 y \leq 0$, and $v^{\text{inv}} \in (v^{\text{iss}}, \bar{v})$, otherwise.

Notice the differences between the expressions for $S^{SB}$, $S^{iss}$, and $S^{\text{inv}}$ in Propositions 3, 4, and 5, respectively. When investors pay, the term $-1 + \pi_1 y$ does not appear in the expression for the surplus. For $\gamma$ sufficiently close to one, it is socially optimal not to ask for a rating and always finance the project, so that $S^{SB}$ (and $S^{iss}$) equal $-1 + \pi_1 y$. Investors, however, will choose to ask for a rating even when it is inefficient.\(^{15}\)

The intuition is as follows. If the project is financed without a rating, then all surplus from the production, $-1 + \pi_1 y$, goes to the firm, while the CRA earns nothing. The CRA can try to sell a rating; it would not succeed if the planner controls whether it should be ordered, unless the generated surplus is at least $-1 + \pi_1 y$ (or unless the firm’s profit is at least that amount, in case the firm orders a rating). However, when investors order a rating, they are not concerned with the total or the firm’s surplus. They make zero profits, and they can always pass along the costs of getting a rating to the firm, while the CRA generates profits.

But why do investors necessarily choose to order a rating if they earn zero profits either way? To show that this must be the case, suppose to the contrary that no one asks for

\(^{15}\)Thus in this case equilibrium payoffs actually lie inside the (constrained) Pareto frontier.
a rating regardless of what the fees are. Then we prove that if fees are low enough, one investor could generate profits by ordering a rating, hiding it from other investors, only investing if it is high, but charging the same or a slightly lower rate of return as other investors. Knowing this, the CRA can set fees low enough to entice someone to ask for a rating and hence break any equilibrium where no one is ordering a rating.

The other difference in the expressions for surpluses is that the promised value to the CRA in the investor-pays case is $v^{\text{inv}}$, which lies in between $v^{\text{iss}}$ and $\bar{v}$, strictly so whenever it is optimal to finance the project ex-ante, i.e., when $-1 + \pi_1 y > 0$. Therefore by Proposition 2, the implemented effort (and hence the rating precision) if investors ask for a rating is lower than if the planner asks for a rating, but higher than if the issuer does. The reason for $v^{\text{inv}} < \bar{v}$ is that the option to finance without a rating caps interest rates, and therefore caps fees that investors are willing to pay to the CRA. (This interest rate cap is $\hat{R} = 1/\pi_1$, which solves $-1 + \pi_1 \hat{R} = 0$.) And $v^{\text{inv}} > v^{\text{iss}}$ because the firm pays the same rate of return to investors as if there was no rating ($\hat{R}$, defined above), but receives financing less often — only when the rating is high (without a rating, it would be financed with probability one). Hence the firm’s profits are lower when the investor-pays than when the issuer does, $u(v^{\text{inv}}) < u(v^{\text{iss}})$, which in turn implies that $v^{\text{inv}} > v^{\text{iss}}$.

The total surplus and effort in the case when investors order a rating are plotted with dashed-dotted black lines on Figure 3. As one can see, the comparison between the total surplus in the issuer-pays and investor-pays cases depend on the prior belief about the project’s quality. When the project is not profitable to finance ex-ante, i.e., when $-1 + \pi_1 y \leq 0$, the total surplus and effort in both models are equal, and coincide with what the planner achieves. However, when $-1 + \pi_1 y > 0$, the issuer-pays model leads to a lower total surplus than the investor-pays model for intermediate values of $\gamma$, but performs better if $\gamma$ is sufficiently high. Also note that $e^{\text{inv}}$ decreases with $\gamma$ when $-1 + \pi_1 y > 0$, because $\hat{R} = 1/\pi_1$ decreases with $\gamma$, which means the fees that investors pay to the CRA are falling as $\gamma$ rises.

Corollary 1 in the Appendix formally states the comparison of the total surplus and effort in the different models. To summarize, the investor-pays model yields higher rating accuracy than the issuer-pays model, but lower than under the planner. The reason is that investors do not care about the firm’s outside option, but the option to finance without a rating caps interest rates, and hence fees. On the other hand, investors ask for a rating too often, even when it is socially inefficient to do so.
5 Extensions

We now consider four variants of the baseline model. Our first extension explores the effect of allowing more than one rating agency. Next, we consider the implications of allowing the CRA to misreport its information. Third, we look at differences in ratings for securities which differ in their ease of monitoring. The last modification introduces a second period in the model so that the propensity to downgrade a security can be studied.

5.1 Multiple CRAs

If multiple ratings are acquired in equilibrium, the problem becomes quite complicated. In particular, contracts will depend on CRAs’ relative performance (i.e., a CRA’s compensation would in part depend on other CRAs’ ratings). In fact, it may advantageous to order an extra rating only to fine-tune the contract structure, while planning to ignore that rating for the purpose of the financing decision. Furthermore, because different CRAs rely on models and data that have common features, it would seem doubtful that the signals from the various CRAs would be conditionally independent. This adds further modelling complications, but also implies the benefits of having more information will be smaller if the signals are more correlated. Finally, if ratings are acquired sequentially and are only published at the end, in the issuer-pays model the firm’s decision whether to acquire the second rating will depend on its first rating. Since this rating is the firm’s private information, it introduces an adverse selection problem. For all these reasons, the analysis of this problem is sufficiently complicated that we leave it for future research.

Instead, as a first step, we restrict our attention to the case when, even though there are multiple rating agencies, only one rating is acquired in equilibrium. (Of course, this may or may not happen in equilibrium, so we simply operate under the assumption that it does.)

We modify the timing of our original model as follows. The game starts by CRAs simultaneously posting fees. The issuer then chooses which CRA to ask for a rating. Under these assumptions the problem becomes very simple to analyze. CRAs compete in fees, which leads to maximizing the issuer’s profits. Recall from Proposition 2 that the firm’s profits are maximized at \( v_0 \). Hence, the total surplus in this case, denoted by \( S_{iss}^{\text{many}} \),

\[ S_{iss}^{\text{many}} \]

\[ \text{An example of a paper that considers relative performance incentives is Che and Yoo (2001).} \]

\[ \text{Of course, now there are more players in the game. If there are } N \text{ CRAs and the firm randomizes between whom to ask for a rating if it is indifferent, then each CRA receives } v_0/N \text{ in expectation. The} \]

\[ \]
equals \( \max\{0, -1 + \pi_1 y, v_0 + u(v_0)\} \). Let \( e_{\text{iss many}} \) denote the corresponding level of effort. Since \( v_0 < v_{\text{iss}} \), it immediately follows from part (iii) of Proposition 2 that \( e_{\text{iss many}} \leq e_{\text{iss}} \) and \( S_{\text{iss many}} \leq S_{\text{iss}} \), with strict inequalities if \( e_{\text{iss}} > 0 \).

We find this extension interesting because it suggests that a battle for market share and desire to win business will lead to lower fees, which means less accurate ratings and lower total surplus. However, the firm’s surplus is higher despite the lower overall surplus. Also note that despite the possibility of Bertrand competition, the CRAs still make positive profits, because \( v_0 > 0 \).

It is instructive to compare the outcomes of the issuer-pays model and the planner’s problem with multiple CRAs. The planner will always want to order the most precise rating possible. This will prevent the CRAs from attempting to undercut each others’ fees, because doing so will not gain them any business. Therefore, the optimal level of effort in this case will be the same as with one CRA. Hence the problem of increased rating errors associated with competition is specific to the issuer-pays model.\(^{18}\)

### 5.2 Misreporting a Rating

We next return to our original model with one CRA. So far we assumed that the CRA cannot misreport its signal; now we relax this assumption and suppose that the CRA can lie. In addition to moral hazard, this creates an adverse selection problem. Solving for the optimal contract requires imposing additional constraints to our optimal contracting problem (2)–(5).

It is easy to see that if the CRA intends to lie, the most profitable way to do so is a double deviation: exert no effort, and always report whatever rating yields the highest expected fee. This should not be surprising because if the CRA intends to misreport, exerting effort is wasteful. Thus, the additional constraint that needs to be imposed in order to deliver a truthful report is

\[
- \psi(e) + \pi_{h1} f_{h1} + \pi_{h0} f_{h0} + \pi_{f\ell} f_{\ell} \geq \max\{\pi_1 f_{h1} + \pi_0 f_{h0}, f_{\ell}\},
\]

frontier on Figure 2 shows the surplus division between the CRA whose rating is ordered and the firm after the outcome of the randomization is observed, with other CRAs (as well as investors) receiving zero profits.

\(^{18}\)We do not explore the effects of competition in the investor-pays model because it is impossible to do so without checking investors’ deviations that involve the acquisition of multiple ratings (i.e., analyzing out of equilibrium behavior where different investors acquire ratings from different CRAs).
which is equivalent to imposing the following two constraints:

\[-\psi(e) + \pi_{h1} f_{h1} + \pi_{h0} f_{h0} + \pi_{\ell} f_{\ell} \geq \pi_{1} f_{h1} + \pi_{0} f_{h0}, \quad (7)\]
\[-\psi(e) + \pi_{h1} f_{h1} + \pi_{h0} f_{h0} + \pi_{\ell} f_{\ell} \geq f_{\ell}. \quad (8)\]

The left-hand side of (6) shows the CRA’s payoff if it exerts effort and truthfully reports the acquired signal. The right-hand side is the value from exerting no effort and always reporting the rating that delivers the highest expected fee (or randomizing between the two, if the fees are the same).

The next proposition shows how the optimal compensation must be structured if the possibility of misreporting is present.

**Proposition 6 (Optimal Compensation under Misreporting)** Suppose the project is financed only after the high rating. Then for each \( \gamma \) it must be the case that \( f_{h1} > 0, f_{\ell} > 0, \) and \( f_{h0} = 0 \). Furthermore, (7) binds for \( \gamma \geq \hat{\gamma} \) and (8) binds for \( \gamma < \hat{\gamma} \), so long as the implemented effort is below the first-best level \( e^* \).

Recall from Proposition 1 that when the CRA cannot misreport its signal, only one of the two fees, \( f_{h1} \) or \( f_{\ell} \), is strictly positive. The situation is different with the possibility of misreporting: both \( f_{h1} \) and \( f_{\ell} \) must be strictly positive. The reason for paying in both cases is intuitive. In particular, without misreporting the CRA would only be paid for issuing a high rating followed by success if the prior about the project’s quality is high enough. But if the CRA can misreport its signal, it would always issue a high rating given this compensation scheme. To prevent the CRA from lying, it must be also paid for issuing a low rating.

Since constraint (6) binds, the total surplus generated if the rating is ordered when the CRA can lie is lower than in the case when it cannot lie. Also, the range of priors for which the rating will be ordered (by any agent) is smaller than when the CRA cannot lie. This is not surprising since essentially the option to lie gives the CRA leverage that allows it to extract fees in order to tell the truth. These fees were previously unnecessary and mean that the agents now become more cautious about using the CRA.

While the optimal compensation scheme is affected by the possibility of misreporting, our other results still apply — the proofs that require modification are provided in the Appendix.
5.3 New Securities

We will now use our results from Section 5.2 to analyze the case where the CRA must rate new securities. Suppose some types of investment projects are inherently more difficult for the CRA to evaluate — presumably because they have a short track record that makes comparisons difficult, and there is no adequate rating model that has been developed yet. One way to model this in our framework is to parametrize the cost of effort as $\psi(e) = A\varphi(e)$, with $A > 0$, and think of a new type of security as the one with a higher value of $A$.\(^{19}\)

A higher value of $A$ means that it is more costly for a CRA to obtain a rating of the same quality for a new security, or, alternatively, paying the same cost would produce a less accurate rating.

Suppose that $A$ increases to $A'$. We consider two scenarios. In the first scenario, the reduction in $A$ is unanticipated. In this case, fees remain unchanged. If the CRA cannot misreport a rating, the CRA’s incentive constraint immediately implies that it will exert less effort. Now consider a more interesting case when the CRA can misreport its rating. Claim 2 in the Appendix shows that in this case constraint (6) with $A'$ instead of $A$ becomes violated (recall from Proposition 6 that it was binding with $A$). Thus, when the CRA realizes that the cost of evaluating the security is higher than expected, its optimal response is to exert zero effort and always report either $h$ or $\ell$, depending on the prior. In particular, when the prior is above $\hat{\gamma}$, the CRA always reports the high rating.

Now consider the second scenario where the shift in $A$ is anticipated, and thus rating fees change appropriately. Claim 3 in the Appendix shows that it is optimal to implement lower effort with $A'$ than with $A$, which results in larger rating inaccuracies. (This result holds regardless of whether the CRA can or cannot misreport its rating.) Intuitively, since the marginal cost of information acquisition is higher, it is optimal to implement a lower level of effort.\(^{20}\)

Thus, our model predicts that under both scenarios the quality of ratings deteriorates for new securities.

\(^{19}\)We do assume that everything else, in particular, parameters $p_b$, $p_g$, $y$, and $\gamma$ remain the same.

\(^{20}\)The result that information acquisition is decreasing in the cost parameter is also obtained in Opp, Opp, and Harris (2012). However, in their case this result is obvious since the CRA can commit to any level of effort, and will choose less effort if its marginal cost is higher. This result is similar to our result in the case of an unanticipated change without the possibility of misreporting. With an anticipated change, the result is less straightforward since fees are optimally chosen, but nonetheless the new optimal fee structure results in lower effort.
5.4 Delays in Downgrading

Finally, we demonstrate that a straightforward extension of our model can explain delays in downgrading. Suppose that there are two periods. The project requires investment in both periods, and the project quality is the same in both periods. The CRA exerts effort in each period to rate the project. In the optimal contract, all payments to the CRA will be made at the end of the second period, conditional on the history. Denote these payments by $f_{i,j}$, where $i, j \in \{h1, h0, \ell\}$, whenever positive effort is exerted in both periods.

Suppose the CRA announced the high rating in period 1, which was followed by the project’s failure. We call this outcome a ‘mistake’, because the project’s performance did not match the rating. After such a mistake, the CRA might be expected to downgrade the security (i.e., announce the low rating in period 2). By the same argument as in Proposition 1, to provide incentives for the second period effort at this point the CRA should be paid either $f_{h0, h1} > 0$ or $f_{h0, \ell} > 0$. However, it is straightforward to show that the best way to provide incentives for effort in period 1 is to pay $f_{h1, h1}$ or $f_{\ell, \ell}$ rather than $f_{h0, h1}$ or $f_{h0, \ell}$. That is, in order to create powerful incentives for the initial rating, the contract should reward the CRA for issuing the same rating in both periods, and having the rating match the project performance in both periods. But if a mistake is made, it becomes optimal to create incentives for the second period to recognize the mistake and possibly change the rating.

This intuition suggests that there is a trade-off between providing incentives for effort in period 1 (the initial rating) and effort in period 2 after a mistake. The optimal contract is designed to balance this trade-off. The desire to support effort in period 1 makes fees, and thus also effort in period 2 after a mistake too low ex post. This means that if the agents were to renegotiate fees after a mistake, the fees would be set to a higher level. (Of course, ex ante it is optimal to commit not to renegotiate fees.) As a result of too low fees ex post, the probability of not downgrading conditional on the project quality being bad is too high ex post. Hence, the CRA will appear too slow to acknowledge mistakes. Remarkably, this inertia seems to be a very general property of an optimal compensation scheme. We want to stress that such delays in downgrading are not inefficient — quite the opposite, they arise as part of an optimal arrangement.
6 Conclusions

We develop a parsimonious optimal contracting model that addresses multiple issues regarding ratings performance. We show that when the CRA’s effort is unobservable, a rating is less precise, and is acquired less often (on a smaller set of priors) than in the first-best case. Giving all surplus to the CRA maximizes rating accuracy and total surplus.

Regarding the question of pros and cons of the issuer- and investor-pays model, we find that in the issuer-pays model the rating is less accurate than in the second-best case. The reason is that the option to finance without a rating puts a bound on the firm’s willingness to pay for one. The investor-pays model generates a more precise rating than the issuer-pays model, although still not as precise as what the planner could attain. However, investors tend to ask for a rating even when it is socially inefficient, in particular, when the prior about the project quality is sufficiently high. In addition, the investor-pays model suffers from a potential free-riding problem, which can collapse security rating all together.

We show that battle for market share by competing CRAs leads to less accurate ratings, which yields higher profits to the firm. We also find that rating errors tend to be larger for new securities. Finally, we demonstrate that optimal provision of incentives for initial rating and revision naturally generates delays in downgrading.

While we view the mileage that is possible with our very parsimonious framework as impressive, there are many ways in which the modelling can be extended. Perhaps most natural would be to allow the firm to have superior information about its investment opportunities relative to other agents. While a general analysis of moral hazard combined with adverse selection is typically quite complicated, there are a few things we can see in some interesting special cases.

First, suppose that the firm knows the quality of its project perfectly. Then if a separating equilibrium exists, the bad type must receive no financing, since investors know that the bad project has a negative net present value. If the firm has no initial wealth as in our original model, there is no way to separate the two types of firms in equilibrium. The reason is that the only (net) payment that the firm can possibly make occurs when project succeeds, and either both types will want to make such a payment, or neither one will. Thus only a pooling equilibrium exists, and the analysis is essentially the same as in our original model. By continuity, the same will be true if the initial wealth is positive but sufficiently small.

If the firm has sufficient internal funds (but not enough to fund the project), then even
in the absence of a rating agency investors can separate firms with different information about their projects. They could do so by requiring the issuer to make an upfront payment in addition to a payment in the event of success (or, equivalently, requiring the issuer to invest its own funds into the project).

A more interesting but also a more complicated problem is when the firm has some private information about the project quality, but does not know it perfectly. In this case, even in the absence of internal funds it might be possible to use the CRA to separate different types of firms by inducing them to choose different fees and thus produce ratings with different degrees of precision. In particular, suppose that there are two types of firms, one being more optimistic about its project than the other, and there are no internal funds. Then one can show that in a separating equilibrium where both types get rated, the firm that has a lower prior about its quality must receive a more precise rating.

Notice that different rating precision means that the same signal for different types will lead to different posterior beliefs about the project’s quality. The different posteriors can be interpreted as reflecting different ratings. That is, with two signals there can be effectively four different ratings in equilibrium, associated with four different posteriors.

We leave a more complete treatment of this problem and the associated issues for future work.
A Appendix: Omitted Proofs

Proof of Lemma 1. The total surplus in the first-best case is \( S^{FB} = \max\{0, -1 + \pi_1 y, \max_e -\psi(e) - \pi_h(e) + \pi_{h1}(e)y\} \), where the third term can be rewritten as \( \max_e -\psi(e) + (1/2 + e)(-1 + p_g y)\gamma + (1/2 - e)(-1 + p_b y)(1 - \gamma) \). At \( \gamma = 0 \), the first term in the expression for \( S^{FB} \) exceeds the other two terms: \( 0 > -1 + \pi_1 y = -1 + p_b y \) and \( 0 > \max_e -\psi(e) + (1/2 - e)(-1 + p_b y) \). At \( \gamma = 1 \), the second term exceeds the other two terms: \( -1 + \pi_1 y = -1 + p_g y > 0 \) and \( -1 + p_g y > \max_e -\psi(e) + (1/2 + e)(-1 + p_b y) \). Hence at \( \gamma = 0 (\gamma = 1) \) it is optimal not to acquire a rating and never (always) finance the project.

Define \( \gamma^* \) such that \(-1 + \pi_1 y = (-1 + p_b y)\gamma + (-1 + p_b y)(1 - \gamma) = 0 \) at \( \gamma = \gamma^* \). We claim that at \( \gamma = \gamma^* \), the third term in the expression for \( S^{FB} \) exceeds the other two terms, and hence it is optimal to acquire a rating and only finance the project after the high rating. To see this, consider the first-order condition of the maximization problem in the third term,

\[
\psi'(e) = (-1 + p_g y)\gamma - (1 - p_b y)(1 - \gamma). \tag{9}
\]

The right-hand side of this equation is strictly positive at \( \gamma = \gamma^* \). Hence (9) has a unique solution \( e > 0 \) at \( \gamma^* \). Moreover, it is always possible to obtain zero surplus by choosing \( e = 0 \). Since the problem is strictly concave in effort, \( -\psi(e) - \pi_h(e) + \pi_{h1}(e)y \) must be strictly positive at the optimal \( e \).

Next, we show that the term \( \max_e -\psi(e) - \pi_h(e) + \pi_{h1}(e)y \) is strictly increasing and convex in \( \gamma \). It will then follow that it must single-cross 0 at \( \gamma \in (0, \gamma^*) \) and \(-1 + \pi_1 y \) at \( \gamma \in (\gamma^*, 1) \), proving the interval structure stated in the lemma. Indeed, by the Envelope theorem, \( \partial[\psi'(e) - \pi_h(e) + \pi_{h1}(e)y] / \partial \gamma = (1/2 + e)(-1 + p_g y) - (1/2 - e)(-1 + p_b y) > 0 \). Differentiating again yields \( \partial^2[\psi'(e) - \pi_h(e) + \pi_{h1}(e)y] / \partial \gamma^2 = (-1 + p_g y - 1 + p_b y)\partial e / \partial \gamma = (-1 + p_g y - 1 + p_b y)^2 / \psi''(e) \geq 0 \), where the last equality follows from differentiating (9) with respect to \( \gamma \), which completes the proof. \( \square \)

Proof of Proposition 1. Let \( \lambda \) and \( \mu \) denote the Lagrange multipliers on constraints (3) and (4), respectively. The first-order condition of problem (2)–(5) with respect to \( f_i, i \in \{h1, h0, \ell\} \) is

\[
(-1 + \lambda)\pi_i(e) + \mu \pi'_i(e) \leq 0, \quad f_i \geq 0,
\]

with complementary slackness. Dividing by \( \pi_i(e) \), one can see that the first-order condition that will hold with equality (resulting in the strictly positive corresponding fee) is the one that corresponds to the highest likelihood ratio, \( \pi'_i(e)/\pi_i(e) \). Straightforward algebra shows
that

\[
\frac{\pi'_h}{\pi_h} \geq \frac{\pi'_e}{\pi_e} \quad \Leftrightarrow \quad \gamma \geq \frac{1}{1 + \sqrt{p_y/p_b}},
\]

\[
\frac{\pi'_h}{\pi_h} > \frac{\pi'_h}{\pi_h} \quad \text{for all } \gamma,
\]

which completes the proof. \( \square \)

**Proof of Proposition 2.** In this proof, we only consider the case when \( \gamma \geq \hat{\gamma} \), as the other case is analogous.

(i) Define \( f^*_h = \psi'(e^*)/\pi'_h(e^*) \) — the fee that implements \( e^* \) — and let \( v^* = -\psi(e^*) + \pi_h(e^*)f^*_h \). Thus by construction \( e^* \) can be implemented at \( v = v^* \). For \( v > v^* \), it can be implemented by paying the fee \( f^*_h \) plus an upfront fee equal to \( v - v^* \).

We now show that \( u(v^*) < 0 \). Since \( u(v) = u(v^*) - (v - v^*) \) for \( v \geq v^* \), it will follow that \( u(v) < 0 \) for \( v \geq v^* \). Using (9), for the first-best effort to be implemented it must be the case that \( \pi'_h f^*_h = (-1 + p_y)\gamma - (-1 + p_b)(1 - \gamma) \). Substituting this into the firm’s payoff, obtain

\[
u(v^*) = -\pi_h + \pi_{h1}y - \pi_{h1}f^*_h = (1/2 + e)(-1 + p_y)\gamma + (1/2 - e)(-1 + p_b)(1 - \gamma)
-[(1/2 + e)p_y\gamma + (1/2 - e)p_b(1 - \gamma)]\frac{(-1 + p_y)\gamma - (-1 + p_b)(1 - \gamma)}{p_y\gamma - p_b(1 - \gamma)}
= \gamma(1 - \gamma)[p_b - p_y]y < 0,
\]

where the last equality follows from straightforward algebra.

(ii) Consider maximizing the firm’s payoff while omitting constraint (3). Using the incentive constraint \( \psi'(e) = \pi'_h(e) f_h \), the firm’s payoff can be written as \( -\pi_h(e) + \pi_h(e)y - \pi_h(e) f_h = (-1 + p_y)(1/2 + e)\gamma + (-1 + p_b)(1/2 - e)(1 - \gamma) - \pi_h(e)\psi'(e)/\pi'_h(e) \). The first-order condition with respect to effort is 0 = \([(-1 + p_y)\gamma - (-1 + p_b)(1 - \gamma)] - \psi'(e) - \psi''(e)\pi_h(e)/p_b(1 - \gamma)) \). The term in the square brackets is strictly positive, while the last two terms are zero at \( e = 0 \) by our assumptions \( \psi'(0) = \psi''(0) = 0 \). Thus \( e = 0 \) cannot maximize the firm’s profits, and hence the optimal level of effort when (3) does not bind is strictly positive.

To see that (3) indeed does not bind for \( v \) low enough, consider the CRA’s payoff \( \Pi(e) = -\psi(e) + \pi_h(e)\psi'(e)/\pi'_h(e) \), where \( \Pi(0) = 0 \). Differentiating yields \( \Pi'(e) = \psi''(e)\pi_h(e)/(p_b(1 - \gamma)) \geq 0 \), with strict inequality for \( e > 0 \). Therefore, at \( v = 0 \) (3) cannot bind, and by continuity it will not bind for \( v \) below some threshold value, which we
denote by $v_0$. Thus for $v \in [0, v_0]$ the optimal contract is the same, and the optimal level of effort is strictly positive.

(ii) For $v \leq v_0$, constraint (3) does not bind, and hence the total surplus and effort are constant. For $v \geq v^*$, $e = e^*$ and the total surplus equals $S^{FB}$. Suppose that $v \in (v_0, v^*)$. Then (3) holds with equality: $-\psi(e) + \pi_{h1}(e)\psi'(e)/\pi_{h1}'(e) = v$, where we used the incentive constraint to substitute for $f_{h1}$. Differentiating the left-hand side with respect to $e$ and using the fact that $\pi_{h1}'(e) = p_g\gamma - p_b(1 - \gamma)$ is independent of $e$, yields $\pi_{h1}(e)\psi''(e)/\pi_{h1}'(e)$, which is strictly positive for $e > 0$ as $\psi''(e) > 0$ and $\pi_{h1}' > 0$ for $\gamma \geq \hat{\gamma}$. Thus the optimal choice of $e$ must be strictly increasing in $v$. Since the total surplus $-\psi(e) - \pi_b(e) + \pi_{h1}(e)y$ is strictly increasing in $e$ for $e < e^*$, it follows that the total surplus is also strictly increasing in $v$. \hfill \Box

Proof of Proposition 4. Part (i) is shown in the main text. Part (ii) immediately follows from part (iii) of Proposition 2 and the fact that $v^{iss} \leq \bar{v}$, with strict inequality if $-1 + \pi_1y > 0$. Given our assumption that the firm can credibly announce that it did not get rated, it immediately follows that if $-1 + \pi_1y > 0$, then $\underline{u} = -1 + \pi_1y$, and thus $v^{iss} < \bar{v}$, as described in the main text. Claim 1 in this Appendix shows how the results change if the firm could not credibly reveal to investors that it did not order a rating. \hfill \Box

Proof of Proposition 5. Suppose first that $-1 + \pi_1y > 0$. We want to show that financing the project without a rating cannot happen in equilibrium. In particular, we will demonstrate that it cannot happen that no investor orders a rating when fees are sufficiently low, and thus the CRA can sell a rating to investors by posting fees low enough.

Suppose to the contrary that investors do not order a rating regardless of the fees. In such an equilibrium, the CRA and investors earn zero profits, while the firm captures all the surplus, $-1 + \pi_1y$. Investors always finance the project, and charge the (gross) rate of return $\bar{R} = 1/\pi_1$ that solves $-1 + \pi_1\bar{R} = 0$.

Consider the case $\gamma \geq \hat{\gamma}$ (the other case is analogous). Suppose the CRA were to offer a history-contingent fee $f_{h1}$ plus a flat fee $f$. The level of effort that these fees induce solves $\psi'(e)/\pi_{h1}'(e) = f_{h1}$. Consider a deviation by one investor who orders a rating, only invests if the rating is high, and asks the issuer for the same, or a slightly lower, rate of return as everyone else. Then after paying the flat fee, the deviating investor can generate profits equal to $\Pi(e) = -\pi_h(e) + \pi_{h1}(e)[\bar{R} - \psi'(e)/\pi_{h1}'(e)] = -\pi_h(e) + \pi_{h1}(e)[1/\pi_1 - \psi'(e)/\pi_{h1}'(e)]$, where $\pi_{h1}'(e) = p_g\gamma - p_b(1 - \gamma)$. Notice that $\Pi(0) = 0$. Moreover, $\Pi' = -\pi_h' + \pi_{h1}'/\pi_1 - [\psi' + \psi''\pi_{h1}/\pi_{h1}']$. Since $\psi'(0) = \psi''(0) = 0$, the second term is zero at $e = 0$, while straightforward
algebra shows that the first term is strictly positive. Thus \( \Pi'(0) > 0 \), as the marginal cost of implementing an arbitrarily small strictly positive level of effort is zero, while the marginal benefit is positive. Thus the deviating investor can generate strictly positive profits by requesting a rating, and will agree to any strictly positive flat fee \( f \) that results in expected cost strictly lower than these profits. The CRA earns \( f - \psi(e) + \pi_{h1}(e)\psi'(e)/\pi'_{h1}(e) \), where the last two terms go to zero as \( e \) goes to zero. Thus the CRA can sell a rating to investors by setting fees low enough.

We have shown that all investors not asking for a rating and always financing the project cannot be part of equilibrium. Next, we will show that if all investors order a rating, then \( v^{iss} < v^{inv} < \bar{v} \). After that, we will prove that it indeed must be the case that in equilibrium all investors order a rating, i.e., it cannot happen that some investors order it and others do not.

First, we show that \( v^{inv} < \bar{v} \). Notice that in the investor-pays model the interest rates are used to finance the rating fees, and thus they must be as high as possible to maximize the value to the CRA and the total surplus. However, investors cannot charge a rate of return above \( \hat{R} = 1/\pi_1 \). Indeed, suppose they charge \( R_h > \hat{R} \). Then there is a profitable deviation by one investor, namely, do not order a rating and offer \( R' \in (\hat{R}, R_h) \). The firm prefers \( R' \) to \( R_h \), and this investor makes positive profits. Suppose that the restriction \( R_h \leq \hat{R} \) does not bind so that the second best is achieved. Since maximizing the total surplus means pushing the firm’s payoff to zero, \( \pi_{h1}(y - R_h) = 0 \), it follows that \( R_h = y > 1/\pi_1 = \hat{R} \), as \(-1 + \pi_1 y > 0 \). A contradiction. Thus \( v^{inv} < \bar{v} \).

To see that \( v^{inv} > v^{iss} \) when \(-1 + \pi_1 y > 0 \), notice that the firm pays the same rate of return \( \hat{R} \) as when it is financed without a rating, but it receives funding less often:
\[
\mu(v^{inv}) = \pi_{h1}(y - 1/\pi_1) < \pi_1(y - 1/\pi_1) = \mu(v^{iss}).
\]
Thus \( v^{inv} > v^{iss} \).

Now we will show that it must be the case that in equilibrium all investors ask for a rating.\(^{21}\) To the contrary, suppose that there is an equilibrium where \( k < n \) investors ask for a rating and \( n - k \) investors do not and always finance. Investor who do not ask for a rating (uninformed investors) must earn zero profit, and hence must charge \( \hat{R} = 1/\pi_1 \). Investor who ask for a rating and only invest if the rating is high (informed investors) also charge \( \hat{R} \). Recall our assumption that the firm borrows equal amounts (or one unit with equal probabilities) from investors between whose offers it is indifferent.\(^{22}\) Hence the firm borrows

---

\(^{21}\)In this case, the CRA charges \( \tilde{f}_i \) (where \( i = h1 \) if \( \gamma \geq \hat{\gamma} \) and \( i = \ell \) otherwise), each investor pays it, and the CRA exerts \( \epsilon \) that solves \( \psi'(\epsilon) = \pi_i'(\epsilon)\tilde{f}_i = \pi_i'(\epsilon)f_1 \).

\(^{22}\)Without this assumption, the proof applies with the modification that informed investors must charge \( 1/\pi_1 - \epsilon \), where \( \epsilon > 0 \) is arbitrarily small.
equally from all investors (informed and uninformed) when the rating is high, and borrows
equally from all uninformed investors when the rating is low. Denote \( \pi_\ell(e) \equiv \pi_1 - \pi_{h1}(e) \),
the probability that the low rating is followed by the project’s success. Then the expected
profit of an uninformed investor is \( \left[ -\pi_h + \frac{\pi_{h1}}{\pi_1} \right]/n + \left[ (1/2 + e) + \pi_{\ell1}/\pi_1 \right]/(n-k) < \left[ -\pi_h + \frac{\pi_{h1}}{\pi_1} \right]/n + \left[ -\pi_h + \frac{\pi_{\ell1}}{\pi_1} \right]/n = 0 \), as \( -\pi_h + \pi_{\ell1}/\pi_1 < 0 < -\pi_h + \frac{\pi_{h1}}{\pi_1} \). A contradiction.

Suppose now that \(-1 + \pi_1 y \leq 0\). We want to show that in this case \( S^{\text{inv}} = S^{SB} \).
Suppose first that if the planner is the one who orders a rating, then asking for the rating
and financing only after the high rating results in a negative total surplus. In this case, it
is optimal not to ask for a rating and never finance, so that \( S^{SB} = 0 \). If investors are the
ones who order a rating, then by definition \( S^{\text{inv}} \leq S^{SB} \). Ordering a rating cannot be part
of an equilibrium strategy, since it would result in a negative payoff to at least one player.
Hence in this case investors do not order a rating and never finance, so that \( S^{\text{inv}} = S^{SB} \).
Now suppose that \( S^{SB} = \bar{v} + u(\bar{v}) \). All surplus in the second-best case is captured by
the CRA, and the firm and investors earn zero. Clearly, this is also an equilibrium when
investors order a rating, and the one that maximizes the total surplus. Thus in this case
\( S^{\text{inv}} = S^{SB} \). \( \square \)

The following result summarizes comparison of effort and total surplus under the three
models, where \( X \) is the planner, the firm, or each of the investors.

**Corollary 1**

(i) If \(-1 + \pi_1 y \leq 0\) then \( S^{\text{inv}} = S^{iss} = S^{SB} \) and \( e^{\text{inv}} = e^{iss} = e^{SB} \).

(ii) Suppose \(-1 + \pi_1 y > 0\). Then
(a) \( e^{\text{inv}} < e^{SB} \) if \( e^{SB} > 0 \), and \( e^{\text{inv}} > e^{SB} \) if \( e^{SB} = 0 \);
(b) \( e^{\text{inv}} \to 0 \) and \( S^{\text{inv}} \to S^{SB}/2 \) as \( \gamma \to 1 \);
(c) \( S^{\text{inv}} < S^{SB} \);
(d) \( e^{iss} < e^{\text{inv}} \), with strict inequality if \( e^{\text{inv}} > 0 \);
(e) \( S^{iss} < S^{\text{inv}} \) if \( S^{\text{inv}} \geq -1 + \pi_1 y \), and \( S^{iss} > S^{\text{inv}} \) otherwise.

**Proof.** The only part remains to be proven is (ii)–(b). We first show that \( e^{\text{inv}} \to 0 \) as
\( \gamma \to 1 \). Recall that investors charge \( \hat{R} = 1/\pi_1 \). Their expected profits when \( \gamma \geq \hat{\gamma} \) equal
0 = \(-\pi_h + \pi_{h1}/\pi_1 - \pi_{h1} f_{h1} \). As \( \gamma \to 1 \), \(-\pi_h + \pi_{h1}/\pi_1 \to -(1/2 + e) + (1/2 + e) p_g / p_g = 0 \). Therefore \( f_{h1} \to 0 \) as \( \gamma \to 1 \), and so \( e^{\text{inv}} \to 0 \). Since the project is financed only after
the high rating, and the probability of this event goes to 1/2 as \( e \to 0 \), it follows that
\( S^{\text{inv}} \to (-1 + \pi_1 y)/2 = S^{SB}/2 \). At \( \gamma = 1 \), there is discontinuity in the total surplus and
$S^{inv} = -1 + \pi_1 y = S^{SB}$, because there is no reason to base the financing decision on a rating when investors are sure that the project is of good quality. \hfill \square

The argument behind the proof of Proposition 4 relied on the assumption that the firm can credibly announce that it did not get rated. The claim below demonstrates how the results change if we dispose of this assumption.

**Claim 1** Suppose that the firm cannot credibly reveal to investors that it did not order a rating. Then the maximum total surplus in the issuer-pays case is $\max\{0, -1 + \pi_1 y, \hat{v}^{iss} + u(\hat{v}^{iss})\}$, where $\hat{v}^{iss} = v^{inv}$.\textsuperscript{23}

**Proof.** If the issuer cannot credibly announce that it did not order a rating, investors’ contracts cannot distinguish between events when a rating has not been ordered, and when it has been ordered, but the firm chose not to reveal it. Furthermore, if the investors financed the project without a rating, then the firm with a low rating would choose not to announce it. Therefore the argument provided in the main text for showing that $u = -1 + \pi_1 y$ when $-1 + \pi_1 y > 0$ does not work.

Suppose that $-1 + \pi_1 y$, and investors finance only after the high rating, and do not finance if the rating is low or if there is no rating. Consider a potential deviation by one investor to offer financing regardless of the rating. The lowest interest rate that this investor can offer is $1/\pi_1$. Other investors ask for a lower interest rate if the rating is high (as the probability of success after the high rating exceeds the ex-ante probability of success), but offer no financing if the rating is low. The deviating investor will make negative profits if the issuer orders a rating, borrows from other investors if the rating is high, and only borrows from this investor if the rating is low, as the probability of success in the latter case is lower than the ex-ante one.

If the firm borrows from investors who only finance after the high rating, it earns a payoff of $\pi_{h1}(e)(y - R_h(e)) - \sum_i \pi_i(e)f_i$, where $R_h(e) = \pi_h(e)/\pi_{h1}(e)$ is the competitive gross interest if the rating is high, and $e$ is the level of effort implemented given the fees. If the firm borrows from the deviating investor after the high rating, it earns $\pi_{h1}(y - 1/\pi_1)$. The issuer will order a rating and choose the first option after the high rating — and thus the deviating investor will earn negative profits — if the first payoff exceeds the second one. (An implicit assumption is that the firm cannot commit not to borrow at a lower interest rate if one is available, and thus cannot commit to borrow from an uninformed investor.

\textsuperscript{23}The expressions for $v^{inv}$ and $S^{inv}$ remain the same.
investor at all states.) This restriction imposes an upper bound on the effort level that can be implemented in equilibrium. But in order to obtain an expression for \( \hat{\psi} \) (the CRA’s payoff corresponding to the highest effort), it is enough to note that the payoff to the firm in this equilibrium is \( u(\hat{\psi}) = \pi_{h1}(y - 1/\pi_1) = u(\nu_{inv}) \), see the proof of Proposition 5.

**Proof of Proposition 6.** The first-order conditions of problem (2)–(5) subject to additional constraints (7)–(8) with respect to \( f_i, i \in \{h1, h0, \ell\} \) — with \( \xi_h \) and \( \xi_\ell \) denoting the Lagrange multipliers on these constraints — can be written as

\[
-1 + \lambda + \xi_h + \xi_\ell - \xi_h \frac{\pi_1}{\pi_{h1}(e)} + \mu \frac{\pi'_{h1}(e)}{\pi_{h1}(e)} \leq 0, \quad f_{h1} \geq 0, \quad (10)
\]
\[
-1 + \lambda + \xi_h + \xi_\ell - \xi_h \frac{\pi_0}{\pi_{h0}(e)} + \mu \frac{\pi'_{h0}(e)}{\pi_{h0}(e)} \leq 0, \quad f_{h0} \geq 0, \quad (11)
\]
\[
-1 + \lambda + \xi_h + \xi_\ell - \xi_\ell \frac{1}{\pi_\ell(e)} + \mu \frac{\pi'_\ell(e)}{\pi_\ell(e)} \leq 0, \quad f_\ell \geq 0, \quad (12)
\]

all with complementary slackness. Straightforward algebra shows that \( \pi'_{h0}(e)/\pi_{h0}(e) < \pi'_{h1}(e)/\pi_{h1}(e) \) and \( \pi_0/\pi_{h0}(e) < \pi_1/\pi_{h1}(e) \) for all \( e \) and all \( \gamma \). Thus the left-hand side of (11) is always strictly smaller than the left-hand side of (10), and thus \( f_{h0} = 0 \).

To show that both \( f_{h1} \) and \( f_\ell \) must be strictly positive, suppose, for example, that \( f_\ell = 0 \). Then from (7), using \( f_{h0} = f_\ell = 0 \), we have \(-\psi(e) + \pi_{h1}(e)f_{h1} \geq \pi_1f_{h1}, \) or \(-\psi(e) - \pi_{\ell1}(e)f_{h1} \geq 0 \), where \( \pi_{\ell1}(e) \equiv \pi_1 - \pi_{h1}(e) \). But the left-hand side is strictly negative since \( e > 0 \) (which is the case when the project is only financed after the high rating). A contradiction. A similar argument supposing \( f_{h1} = 0 \) and using (8) also arrives to a contradiction.

Since both \( f_{h1} > 0 \) and \( f_\ell > 0 \), constraints (10) and (12) must both hold with equality. Subtracting one from the other, obtain:

\[
\mu \left[ \frac{\pi'_{h1}(e)}{\pi_{h1}(e)} - \frac{\pi'_\ell(e)}{\pi_\ell(e)} \right] = \frac{\xi_h \pi_1}{\pi_{h1}(e)} - \frac{\xi_\ell}{\pi_\ell(e)}.
\]

Suppose that \( \gamma > \hat{\gamma} \), so that \( \pi'_{h1}(e)/\pi_{h1} - \pi'_\ell(e)/\pi_\ell > 0 \) (see the proof of Proposition 1), and (7) does not bind. Then \( \xi_h = 0 \) and the right-hand side of the above equation is non-positive. On the other hand, as long as the incentive constraint binds so that \( \mu > 0 \), the left-hand side of (7) is strictly positive, a contradiction. An analogous argument shows that \( \xi_\ell \) must be strictly positive when \( \gamma < \hat{\gamma} \). When \( \gamma = \hat{\gamma}, \pi'_{h1}(e)/\pi_{h1}(e) = \pi'_\ell(e)/\pi_\ell(e) \), so that incentives for effort can be provided equally well with \( f_{h1} \) and \( f_\ell \), and thus (6) can be
satisfied without any cost. Without loss of generality, we can assume that (7) is satisfied
with equality at $\gamma = \hat{\gamma}$.

Claim 2 Suppose $f_{h1}$ and $f_\ell$ are the optimal choices of fees in problem (2)−(6), where
$\psi(e) = A\varphi(e)$. If the CRA chooses effort facing such fees and $A' < A$, then (6) is violated,
and hence the optimal response of the CRA is to exert zero effort and always report $h$ if
$\gamma \geq \hat{\gamma}$ and $\ell$ otherwise.

Proof. The CRA’s profits if it chooses to exert effort are $\pi(A) \equiv \max_e -A\varphi(e) +
\pi_{h1}(e)f_{h1} + \pi_\ell(e)f_\ell$. By the Envelope theorem, $\pi'(A) = -\varphi(e) < 0$. Therefore the left-hand
side of (6) with $A'$ is strictly lower than that with $A$. Since the right-hand side of (6) does
not change, and the constraint was binding with $A$, it now becomes violated. Which report
the CRA makes then follows from Proposition 6.

Claim 3 Suppose that $\psi(e) = A\varphi(e)$. Then the optimal level of effort in problem (2)−(5)
strictly decreases with $A$.

Proof. We use strict monotone comparative statics results from (Edlin and Shannon,
1998) to show that $e$ is strictly decreasing in $a$. Define $a = 1/A$. Consider the case
$\gamma \geq \hat{\gamma}$ (the other case is analogous). Using Proposition 2 and substituting from (4),
problem (2)−(5) can be written as $\max_e -\pi_h(e) + \pi_{h1}(e)y - \pi_{h1}(e)\varphi'(e)/(a\pi'_{h1}(e))$ subject
to $-\varphi(e) + \pi_{h1}(e)\varphi'(e)/\pi'_{h1}(e) \geq va$, where $\pi'_{h1}(e) = p_y \gamma - p_h(1 - \gamma)$. Denote the objective
function by $F(e, a)$. Differentiating with respect to $e$, $F_e = -\pi'_{h1}(e) - \pi_{h1}(e)y - [\varphi'(e) +
\pi_{h1}(e)\varphi''(e)/\pi'_{h1}(e)]/a$. Since $\varphi'(e) > 0$ and $\varphi''(e) > 0$ for $e > 0$, and $\pi'_{h1} > 0$ for $\gamma \geq \hat{\gamma}$, it
follows that $F_{ea} > 0$. Next, differentiating the left-hand side of the constraint with respect
to $e$, obtain $\partial[-\varphi(e) + \pi_{h1}(e)\varphi'(e)/\pi'_{h1}(e)]/\partial e = \pi_{h1}(e)\varphi''(e)/\pi'_{h1}(e) > 0$ for $e > 0$. Thus,
the constraint can be written as $g(e) \geq va$, where $g$ is a strictly increasing function, or,
equivalently, $e \in \Gamma(a)$, where $\Gamma$ is nondecreasing in $a$ in the strong set order. Therefore the
optimal choice of effort is strictly increasing in $a$, or strictly decreasing in $A$.

A.1 Proofs in the Case of Misreporting

The proofs of Propositions 3 and 4 (as well as the proof of Claim 1) extend to the case
of misreporting without changes. The proofs of Propositions 2 and 5 and Claim 3 for this
case are provided below. When it is important to distinguish functions and variables with
and without misreporting, we mark those in the latter case with tilde.

First consider the payoff to the CRA, $-\psi(e) + \pi h_1(e) f_{h1} + \pi e(e) f_e$, which from Proposition
6 equals $\pi_{h1} f_{h1}$ if $\gamma \geq \hat{\gamma}$ and $f_e$ if $\gamma < \hat{\gamma}$. In the case of $\gamma \geq \hat{\gamma}$, (7) holding with equality
imply $\pi_{\ell} f_{\ell} = \psi + \pi_{\ell 1} f_{h1}$. Substituting this into the incentive constraint $\psi' = \pi_{h1} f_{h1} + \pi_{\ell} f_{\ell}$,
obtain $\psi' = \pi_{h1} f_{h1} + (\psi + \pi_{\ell 1} f_{h1}) \pi_{\ell}/\pi_{\ell}$ or $\psi' - \psi \pi_{\ell}/\pi_{\ell} = [\pi_{h1} + \pi_{\ell 1} \pi_{\ell}/\pi_{\ell}] f_{h1}$. We can then
express $f_{h1}$ and substitute it into the payoff to the CRA to express the latter as a function
of effort only. Similarly, for $\gamma < \hat{\gamma}$, (8) holding with equality implies $\pi_{h1} f_{h1} = \psi + \pi_{h} f_{e}$.
Substituting into the incentive constraint, obtain $\psi' - \psi \pi_{h1}/\pi_{h1} = f_{e} [\pi_{\ell} + \pi_{h} \pi_{h1}/\pi_{h1}]$. This
leads to the following expression for the payoff to the CRA as a function of effort only,
which we denote by $V(e)$:

$$V(e) \equiv \begin{cases} 
\pi_{1} \left[ \frac{\psi'(e) - \psi(e) \pi'_{h1}(e)}{\pi_{h1}(e) + \pi_{\ell 1}(e) \pi_{\ell}(e)/\pi_{\ell}(e)} \right], & \text{if } \gamma \geq \hat{\gamma}, \\
\psi'(e) - \psi(e) \pi'_{h1}(e)/\pi_{h1}(e) \pi_{\ell}(e) + \pi_{h}(e) \pi_{h1}(e)/\pi_{h1}(e), & \text{if } \gamma < \hat{\gamma}.
\end{cases} \quad (13)$$

Also denote $C(e) \equiv \psi(e) + V(e)$, the expected fees that implement effort $e$.

**Proof of Proposition 2 under Misreporting.** (i) Define $\tilde{v}^* = V(e^*)$. By construction,
$e^*$ can be implemented at $v = v^*$. For $v > v^*$, $e^*$ can be implemented by paying the
same history-contingent fees as at $v^*$ plus an upfront fee equal to $v - v^*$.

Next we show that $\tilde{u}(\tilde{v}^*) < 0$. Since $\tilde{u}(v) \leq u(v)$ for all $v$ and $\tilde{u}(v) = u(v) = S_{FB} - v$ for $v \geq \max\{v^*, \tilde{v}^*\}$, it follows that $\tilde{v}^* \geq v^*$. Thus $\tilde{u}(\tilde{v}^*) \leq \tilde{u}(v^*) \leq u(v^*) < 0$, where the
last inequality follows from part (ii) of Proposition 2.

(ii) Consider maximizing the firm’s payoff while omitting constraint (3). The firm’s
payoff can be written as $-\pi_{h}(e) + \pi_{h1}(e) y - \pi_{h1}(e) f_{h1} - \pi_{e}(e) f_{e} = (-1 + p_{y}) (1/2 + e) \gamma + (-1 + p_{y}) (1/2 - e) (1 - \gamma) - C(e)$. The first-order condition with respect to effort is $0 = [(1 - p_{y}) \gamma - (1 + p_{y}) (1 - \gamma)] - C'(e)$. The term in the square brackets is strictly positive,
while straightforward algebra shows that $C'(e)$ equals zero at $e = 0$ by our assumptions
$\psi(0) = \psi'(0) = \psi''(0) = 0$. Thus $e = 0$ cannot maximize the firm’s profits. The CRA’s
payoff at $e = 0$ is $V(0) = -\psi(0) + C(0) = 0$. Moreover, as we will show in the proof of part
(iii) below, $V(e)$ must be strictly increasing in $e$ for $e > 0$. Therefore, for $v$ below some
threshold value, denoted by $\tilde{v}_{0}$, (3) does not bind. Moreover, the optimal level of effort for
$v \leq \tilde{v}_{0}$ is strictly positive.

(iii) For $v \leq \tilde{v}_{0}$ effort is constant at $e(\tilde{v}_{0})$, and for $v \geq \tilde{v}_{0}$ it is constant at $e^*$. Suppose
that $v \in (\tilde{v}_{0}, \tilde{v}^*)$. To show that the implemented effort is strictly increasing in $v$ on this
interval, it is enough to show that $V(e)$ is strictly increasing in $e$. Since the total surplus $-\psi(e) - \pi_h(e) + \pi_{h1}(e)y$ is strictly increasing in $e$ for $e < e^*$, it will then follow that the total surplus is also strictly increasing in $v$.

We will only consider the case of $\gamma \geq \hat{\gamma}$, as the other case is analogous. The derivative of the numerator in the top expression in (13) with respect to $v$ is $\psi'' + (\pi'_e/\pi_e)^2 - \psi' \pi'_e/\pi_e$, which is strictly positive if $\pi'_e \leq 0$. As for the denominator, $\pi'_{h1} = \rho_h \gamma - p_g (1 - \gamma)$ and $\pi'_e = 1 - 2\gamma$ are independent of $e$. In addition, $\pi_{h1}(e)/\pi_e(e) = \pi_{1|e}(e)$, the probability of success conditional on the low rating, is strictly decreasing in $e$. Thus the denominator is strictly decreasing in $e$, while the numerator is strictly increasing in $e$ if $\pi'_e \leq 0$. Since for $v \in (\bar{v}_0, \bar{v}^*)$ the left-hand side of (13) is equal to $v$, the implemented effort is strictly increasing in $v$ if $\pi'_e \leq 0$.

Now suppose that $\pi'_e > 0$, and suppose that as $v$ increases, the optimal level of effort remains unchanged or falls. The latter is not possible, since it is feasible to increase all $f_i$, $i \in \{h1, h0, \ell\}$, by the same amount, thereby keeping $e$ unchanged; this dominates a lower effort since the total surplus is strictly increasing in effort for $e < e^*$. If effort does not change, then an increase in $v$ can be delivered by increasing all $f_i$ by the same amount. But as the proof of Proposition 6 shows, it is never optimal to increase $f_{h0}$ unless $e < e^*$. By keeping $f_{h0}$ unchanged and increasing only $f_{h1}$ and $f_\ell$, effort inevitably increases from (4) since $\pi'_{h1} > 0$ (for $\gamma \geq \hat{\gamma}$) and $\pi'_e > 0$ (by supposition). A contradiction. The above argument also implies that $V(e)$ must be strictly increasing in $e$ (even if $\pi'_e > 0$).

**Proof of Proposition 5 under Misreporting.** The proof is a straightforward modification of the proof of Proposition 5 without misreporting. The fact that the marginal cost of implementing an arbitrarily uninformative rating with the possibility of misreporting is zero was shown in the proof of part (ii) of Proposition 2 under misreporting: $C'(0) = 0$. □

**Proof of Claim 3 under Misreporting.** The proof is a straightforward extension of the proof of Claim 3 without misreporting. Let $a = 1/A$, and define $V_\varphi(e)$ as $V(e)$ given in (13) where $\psi$ is replaced by $\varphi$. Then the maximization problem (2)—(6) can be written as $\max_e -\pi_h(e) + \pi_{h1}(e)y - [V_\varphi(e) + \varphi(e)]/a$ subject to $V_\varphi(e) \geq va$. Denote the objective function by $\bar{F}(e, a)$. Differentiating with respect to $a$, $F_a = [V_\varphi(e) + \varphi(e)]/a^2$. Since $V_\varphi(e)$ and $\varphi(e)$ are both strictly increasing in $e$ (the former is shown in the proof of Proposition 2 under misreporting), it follows that $\bar{F}_{ea} > 0$. In addition, the constraint can be written as $e \in \Gamma(a)$, where $\Gamma$ is nondecreasing in $a$ in the strong set order. Therefore the optimal choice of effort is strictly increasing in $a$, or strictly decreasing in $A$. □
References


