Bank regulation with private-party risk assessments

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Abstract

In this paper we propose a general equilibrium framework to analyze the effectiveness of bank capital regulations when the banking sector faces competition from unregulated investors. In our model an implicit bail-out guarantee for banks may generate excessive incentives to invest in high risk, negative NPV projects. When competition from unregulated investors is low, the banking sector has a natural incentive to first fund positive NPV projects and to only engage in risk-shifting when the banking sector’s funding capacity exceeds the supply of positive NPV projects. This “natural pecking order” of bank investment allows regulation in the form of simple equity-capital ratio requirements to be effective. However, when banks face sufficiently strong competition from unregulated investors, they weakly prefer to focus on the funding of high-risk issuers, since government-insured banks have a natural comparative advantage in that market. This “reverse pecking order” of bank investment renders simple capital regulation to be ineffective and may even cause equity injections to be locally counterproductive. However, we show that contingency of capital regulation on credit ratings can restore the natural pecking order of bank investment and thereby increase the efficiency of capital requirements.

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1 Introduction

The recent financial crisis has put the regulation of financial institutions to the forefront of the public debate and academic interest (see Admati, DeMarzo, Hellwig, and Pfleiderer (2011)). At least two lessons justify this prominence. First, the bulk share of losses caused by banks are ultimately borne by the tax payer, and not the private investors in banks, which is often referred to as “privatizing gains and socializing losses.” Secondly, the importance of a well-functioning banking system and prevention of future crisis is of first-order importance given the enormous spill-over effects on the entire economy.

One of the most central tasks of regulators is therefore to deter risk-shifting in the banking sector given the implicit bailout guarantee for too-big-to-fail institutions. Regulators across the world aim to tame this incentive via equity capital regulations that are a function of the types of investments that banks undertake. In particular, since the introduction of the Basel I framework, credit ratings have played an important role in bank regulation as “objective” measures of credit risk. This role has been confirmed in the Basel III (2011) guidelines, which still rely on credit ratings as measures of creditworthiness. In contrast, in the United States, the Dodd Frank Act mandates that regulators abandon the regulatory reliance on credit ratings and find new approaches to regulate banks based on alternative measures, such as market-based risk measures. Interestingly, this disagreement between the Dodd Frank Act and Basel III has created a divergence in reform proposals in the US and other parts of the world and has generated a heated debate over the cost and benefits of regulatory reliance on credit ratings and other risk measures.

Given that regulatory changes fundamentally alter the environment in which financial markets operate, it is crucial to develop economic frameworks that facilitate a rigorous analysis of alternative policy proposals. A key feature that such frameworks have to capture is that fact that banks and other market participants are not only heterogeneously informed but also differentially regulated. Given the competitive nature of financial markets, such homogeneity may have non-trivial implications for equilibrium prices and allocations. However, incorporating such heterogeneity into a general equilibrium analysis is typically a challenging task.

1 Section 939A of the Dodd-Frank Wall Street Reform and Consumer Protection Act (2010), entitled Review Of Reliance On Ratings, states that each federal agency “…shall modify any such regulations identified by the review conducted under subsection (a) to remove any reference to or requirement of reliance on credit ratings and to substitute in such regulations such standard of credit-worthiness as each respective agency shall determine as appropriate for such regulations.”
In this paper we aim to make a step in this direction by developing a parsimonious general equilibrium model with heterogeneous investors, regulated banks, and unregulated household investors. The features of our economy can be described as follows. A continuum of firms in fixed supply may obtain debt financing for investment projects of varying quality from outside investors, banks or household investors. For ease of exposition, projects are either low-risk and NPV positive (good) or high risk and NPV negative (bad). Banks have a comparative advantage over household investors in screening firm types and collecting payments from firms (similar to Diamond and Rajan (2001)). This valuable role of the banking sector makes it optimal for the government to “bail-out” failed banks ex post, i.e., guarantee deposits with tax-payers’ money. On the other hand, the guarantee destroys the role of debt as a disciplining device for bankers and sows the seeds for asset substitution.

In an unregulated economy, the banking sector invests in both good and bad projects: “safe” banks with good assets and fully levered “risky” banks with bad assets coexist. Prices of good and bad assets are set such that the expected return on equity for both bank types are equalized. Bad issuers’ projects are funded (despite their negative NPV) as sufficient bank leverage in combination with a government put makes investment in these projects privately optimal. This rational overpricing of bad asset is one of the main ingredients of our analysis. It will also imply that market prices (yields) of bad assets are completely uninformative about the nature of risk in an unregulated economy.

The regulator tries to design capital regulation ex ante to avoid risk-shifting by banks without prohibiting socially valuable investment by banks. We make the natural assumption that the regulator cannot directly observe the quality of assets. However, regulation can be made contingent on risk assessments provided by a private, profit-maximizing credit rating agency (CRA). These ratings are used not only by the regulator, but also by household investors to guide their investment decisions. The CRA is paid by the issuer, so that their incentives are naturally aligned.

Our main findings with regards to the effectiveness of capital requirements can be summarized as follows. When competition for assets by household investors is low, that is, when banks’ comparative advantage in loan collection is sufficiently large, the banking sector’s investment policy follows a “natural pecking order” in the following sense: consider an increase in capital requirements starting from the unregulated economy. As soon as capital requirements are sufficiently high so that the banking sector is constrained in

\footnote{If a regulator could observe the riskiness of assets directly in a timely and unambiguous way, then he could simply prescribe banks which assets to hold and which assets not to hold.}
funding all assets in the economy, the banking sector first starts to reduce the funding of bad issuers. Since bad projects are the marginal projects of the banking sector, they pin down the expected return on bank equity. At some point, when capital requirements are sufficiently stringent, none of the bad projects are funded, but the banking sector still has sufficient funding capacity to finance all of the good projects in the economy. This is the optimal level of capital requirements which can achieve the first-best outcome.

However, when competition for assets by household investors is sufficiently high, the natural pecking order of bank investment reverses. Now, as soon as capital requirements are sufficiently strong so that the banking sector is constrained in funding all assets in the economy, the banking sector will first start to reduce funding in good projects. The intuition for this reversal is simple: in the absence of competition from unregulated investors, investment with the highest social NPV can also offer the highest private NPV for banks. This ordering no longer obtains, when unregulated investors compete for good assets. Competition depresses possible yields in the market for good assets, but not in the market for bad assets, where government-insured banks are insulated from outside competition, since other investors cannot rely on a government bail-out. This effect is related to a popular argument that competition from “the shadow banking system” causes banks to “reach for yield” and for riskier investments in order to stay profitable. Under this reverse pecking order regime, a local increase in bank equity (via an injection) can even be harmful, as it may allow the expansion of bank investment in bad assets (given a certain leverage-ratio constraint). To prevent the reverse pecking order, capital requirements have to be so strong that asset substitution is no longer privately optimal. However, at this level, banks may not be able to fund all good assets in the economy, so that the second-best investors, i.e., household investors, pick up the remaining good projects, which causes an inefficiency. In this regime, the credit rating agency facilitates competition from outside investors by providing them with information on the quality of assets. However, due to the presence of a reverse pecking order, simple capital regulation may not achieve the first-best outcome.

Next, the question arises whether the regulator can improve upon this outcome by relying on credit ratings. Without rating contingent regulation, the CRA is only concerned with households’ ability to fund issuers, since banks will rely on their own information. Thus, the CRA can only extract surplus from the good issuers that are funded by household investors. In contrast, with rating contingent regulation, the CRA also extracts value from issuers funded by banks, since banks in this case can only invest in rated assets. Since, the CRA’s profit is tied to the surplus generated by the issuer, it always generates weakly
higher profits by inducing funding of good projects. The rating agency’s disclosure policy (combined with rating-contingent regulation) thus restores the “natural” pecking order. Only if capital regulation is sufficiently lax, so that the banking sector has more funds available than are needed to fund all good issuers, the CRA will add bad issuers into the pool of highly rated securities. This implies that rating inflation with rational investors is an artifact of insufficient capital regulation. An increase in capital requirements might not only reduce the risk-shifting incentive of banks, but also cause the credit rating agency to report truthfully. Under optimum rating contingent regulation, the first best outcome can be implemented.

Our findings thus indicate that the Dodd-Frank Act’s mandate to remove any references to ratings in regulation might have harmful consequences. Moreover, our model highlights another potential point of concern in the implementation of this mandate. If ratings are replaced by market prices (yields) in regulation, the regulator has to take into account that market prices of investments reflect the bail-out guarantee if banks are marginal in pricing assets, i.e., highly risky, bad assets are rationally overpriced. That is, prices imply negative expected average returns on risky assets, a feature that is absent in intermediary asset pricing models such as [2012], where risky assets’ expected returns always carry a weakly positive risk-premium that compensates risk averse intermediaries for their exposure to aggregate risk.

2 Setup

The general structure of our economy is as follows. Firms may obtain debt financing for investment projects of varying quality from outside investors, banks or household investors. Banks have a comparative advantage over household investors in screening and collecting payments from firms. This special role of the banking sector makes it optimal for the government to “bail-out” failed banks ex post, i.e., guarantee deposits with taxpayers’ money. The regulator tries to design regulation ex ante to avoid excessive risk-taking by banks without prohibiting socially valuable investment. Regulation can be made contingent on risk assessments of funded investments (firms) provided by a profit-maximizing rating agency. These ratings are used not only by the regulator, but also by household investors to guide their investment decisions.

To illustrate the forces at play in this economy, we consider a two-period, discrete-

\[3\] This can be understood as a generalization of the results in [Opp, Opp, and Harris (2013)].
state economy in which the aggregate state of the world is either “high” \((s = H)\) or “low” \((s = L)\). The ex-ante probability of the high state is denoted by \(p_H\). For ease of exposition, all agents in the economy (which we describe below) are risk-neutral and discount their respective payoffs at a discount rate of 0.

### 2.1 Firms

There is a continuum of firms of measure one that form the set \(\Omega\). Each firm is owned by a cash-less entrepreneur who seeks debt financing from outside investors (banks or households). The entrepreneur has access to a risky project that requires an initial investment of 1 and may either succeed or fail. If the project succeeds, the firm’s net cash flow at the end of the period is \(R > 1\). In case of failure, the cash flow is 0.

Firms differ with regard to their probability of default. Default probabilities conditioned on firm type are denoted by \(d_n\), where \(n \in \{g, b\}\) denotes the type of a project, and where \(g\) represents “good,” i.e., positive NPV, and \(b\) stands for “bad”, i.e., negative NPV projects. The NPV of a type \(n\) project is given by

\[
V_n = R (1 - d_n) - 1. \tag{1}
\]

The fraction of good types in the population \(\pi_g\) is common knowledge to all parties at date 0. In addition, each firm knows its own type. The average project with default probability \(\bar{d} = \pi_g d_g + (1 - \pi_g) d_b\) is assumed to have negative NPV, i.e., \(R (1 - \bar{d}) < 1\).

State-contingent default probabilities of type \(n\) in state \(s\) are denoted by \(d^s_n\) so that

\[
d_n = p_H d^H_n + (1 - p_H) d^L_n.
\]

**Assumption 1** Good projects \((g)\) always succeed in both aggregate states of the world. Bad projects \((b)\) always succeed in the high aggregate state \((H)\), but always default in the low aggregate state \((L)\).

\[
\begin{align*}
d^H_g &= d^H_b = d^L_g = 0, \quad (2) \\
d^L_b &> 0. \quad (3)
\end{align*}
\]

We make this extreme assumption to capture two important features. First, good
and bad securities look identical in the good state of the world, i.e., \( d^H_q = d^H_b = 0 \) (for simplicity, we assume they don’t default at all). Secondly, only bad securities are exposed to catastrophe risk, i.e., they don’t pay out in the bad state. Thus, “bad” securities may be interpreted as economic catastrophe bonds in the sense of Coval, Jurek, and Stafford (2009).

2.2 Investors

2.2.1 Bankers

The economy consists of a measure one of competitive ex-ante identical bankers (also referred to as banks) each with initial wealth, i.e., inside equity, of \( E_0 \). To economize on notation, we omit bank-specific subscripts, even when we refer to a specific bank. For ease of exposition, we start with the assumption that banks cannot raise outside equity from households, but are free to pay out their equity in the form of cash dividends \( Div_0 \). However, we discuss in section 4.3 under which conditions the results from this analysis apply when banks can issue outside equity.

In addition to their own equity, banks may raise “government-insured” deposits from households at time 0, denoted as \( D_0 \). The funding side implies that a bank possesses \( M_0 \) dollars for investment purposes

\[
M_0 = E_0 + D_0, \tag{4}
\]

where \( E_0 = \bar{E}_0 - Div_0 \), the amount of inside equity left after paying out dividends. The bank may either fund firm projects (see Section 2.1) or invest in a storage technology, which we will label “cash.” Total investment in cash is given by \( C_0 \). However, since the bank also possesses an advantage in terms of screening and monitoring projects relative to households (see Diamond and Rajan (2001) for a similar assumption), it can be more efficient if banks finance firm projects rather than investing in cash. Specifically, we make the following assumption.

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5 In our paper, we implicitly equalize safe projects with good projects and risky projects with bad projects. This need not be the case in reality. If all good projects were also riskier, private interests of the banking sector and the public would be aligned. However, all that essentially matters in our setup is, that there exist potential projects that are highly risky and not worth funding.

6 This is akin to the total net worth of the banking sector as in Viswanathan and Rampini (2013).

7 While this assumption is not uncommon in the corporate finance literature (see Hart and Moore (1998)), we will revisit this important point in an extension of the model.

8 It is important to note that banks may create social value through liquidity services even if they do not generate any value on the asset side, i.e., invest all assets in cash, \( C_0 = M_0 \).
Assumption 2. The bank observes the project type of any firm in which it invests and can collect firm cash flows, both at no cost.

Let \( A_0 = M_0 - C_0 \) denote the total amount invested by a bank in firms’ securities, i.e., non-cash assets, and define the bank’s equity ratio by

\[
e = \frac{E_0}{A_0}.
\]

The equity ratio relates the investment of the banker (after the dividend payout) to the total value of non-cash assets, \( A_0 \). Starting from an initial endowment \( E_0 \), the bank can effectively choose any equity ratio \( e \in [0, 1] \) through the appropriate choice of dividends, \( Div_0 \), and cash investments, \( C_0 \). By choosing to hold enough cash as a reserve, i.e., \( C_0 = D_0 \), the bank has no net leverage (\( e = 1 \)). By paying out all equity as dividends, the bank becomes fully levered (\( e = 0 \)). This simple connection between asset allocation and funding choices highlights the important relationship between reserves (cash) and capital (equity) requirements.

Let \( x_j \geq 0 \) denote the fraction of non-cash assets \( (A_0) \) that the banks invests in security \( j \), and let \( r_j^s \) denote the state-contingent rate of return on security \( j \). Then the rate of return on the portfolio of non-cash assets satisfies \( r_A^s = \sum x_j r_j^s \). The rate of return on bank equity, \( r_E^s \), in each aggregate state \( s \) is given by

\[
r_E^s = \max \left\{ \left( 1 + \frac{r_A^s}{e} \right) \frac{A_0 + C_0 - D_0}{E_0} - 1, -1 \right\} = \max \left\{ \frac{r_A^s}{e}, -1 \right\}.
\]

This formula reflects the limited liability of equity holders and that their investment is \( E_0 \), which may be interpreted as the book value of equity.\(^9\) Given their investment \( E_0 \), each bank maximizes the market value of equity, which is given by

\[
E_0^M = \max_{e, E_0, \{x_j\}} E_0 \cdot \mathbb{E} \left( 1 + \max \left\{ \frac{\sum x_j r_j^s}{e}, -1 \right\} \right).
\]

2.2.2 Households

Household investors are deep-pocketed and always behave competitively if they have an investment opportunity with non-negative NPV. However, they are at a disadvantage

\(^9\) When the bank does not default, the overall portfolio return on all bank assets in each state satisfies the standard decomposition: \( r_A^s = er_E^s + (1 - e) r_D^s = er_E^s \)
relative to banks in two ways. First, we assume that they incur an additional cost $c$ per unit of investment they provide to issuers. The value of $c$ determines by how much the banking system is affected by competition from arms-length investors (see Petersen and Rajan (1995) or Rajan and Zingales (2001)). Secondly, households are uninformed, i.e., they cannot observe the types of firm projects. Thus, in the absence of other signals (such as ratings), firm funding by households is impossible, since the average NPV of firms’ projects is negative. Throughout the analysis we maintain the following assumption.

**Assumption 3** Households believe that the rating agency never deliberately rates a good issuer (type $g$) badly (rating $B$).

In other words, households believe that the rating agency would never deliberately downward-bias ratings. This assumption is useful as it eliminates unreasonable equilibria. However, since households are assumed to be rational, we will naturally ensure that households’ beliefs will be consistent with the equilibria we discuss in the analysis below.

Note that if banks did not have a comparative advantage relative to households, the government could always optimally mandate that banks do not invest in non-cash assets. As a result, firm funding would be fully efficient outside of the banking sector. The tension in our model arises since banks have a comparative advantage in investing on the one hand, but incentives for risk-shifting on the other hand.

### 2.3 Credit rating agency

As in Opp, Opp, and Harris (2013), we model a monopolistic credit rating agency (or CRA) as a strategic information provider but introduce some simplifications that increase the tractability of the analysis. We assume that the credit rating agency observes the firm types perfectly and at zero cost, so that it just has to decide on what information it discloses.\(^\text{10}\) Given the binary signal ($g$ for good projects and $b$ for bad projects), we can restrict the credit rating agency to report either $G$ or $B$. We denote by $\Omega_G \subseteq \Omega$ the set of $G$-rated securities. Moreover, motivated by the reduced-form implication of the repeated game considered in Opp, Opp, and Harris (2013), we assume that the rating agency can commit to disclosure policies.

\(^{10}\) Introducing costly information acquisition as in Opp, Opp, and Harris (2013) would not affect our qualitative results.
The ratings may be used by rational household investors to guide their investment decisions and potentially by the regulator to design regulation. Since the credit rating agency relies on an issuer-pays model, each issuer $i$ that obtains a rating incurs a fee $f_i$ that is paid as a fraction of the issuer’s equity value. The assumption that the CRA is paid in this way is merely a modeling device that simplifies the analysis, as the cashless issuer otherwise would need to raise additional funds to pay the fee to the CRA upfront. Specifically, we model the fee as a fraction $\gamma$ of the private surplus generated through financing the rated project:

$$f_i = \gamma \mathbb{E}(\max\{Z^s_i - N_i, 0\})$$

and $Z^s_i \in \{0, R\}$ represents the cash flow of firm $i$ in state $s$ and $N_i$ denotes the face value for a bond issued by firm $i$. For unfunded securities, we simply set $N_i = R$ which implies $f_i = 0$. As a result, the profits of the CRA are given by:

$$\Pi_R = \int_{\Omega_C} f_idi.$$  

The CRA aims to maximize its payoff through the appropriate choice of its disclosure policy.

### 2.4 Regulator / Government

Formally, the government chooses regulatory restrictions the objective of which is to maximize welfare $W$ defined as the NPV of funded securities in the economy net of collection cost $c$ for projects funded directly by households. It is useful to define the first-best outcome $W^*$ given by

$$W^* = \pi_y V_g.$$  

11 Note that $B$-rated securities will have negative NPV and, hence, cannot be funded by households. Banks, however, may fund these securities, because they can default on their creditors in case the project fails, i.e., asset substitution may make these securities a viable option for banks.

12 Such a setup is considered in Opp, Opp, and Harris (2013) and a previous version of this paper. Although modeling the fee as an upfront cash payment makes the analysis less tractable it leads to similar results, since the CRA in both cases has incentives to put the issuer in the position to pay the largest possible fee (which implies that the CRA cares about the issuer’s pre-fee value).

13 This can be interpreted as the outcome of the bargaining game considered in Hart and Moore (1998). With probability $\gamma$, the CRA makes a take-it-or-leave-it-offer to the issuer and with probability $1 - \gamma$, the issuer makes a take-it-or-leave-it-offer to the CRA.
which represents the welfare that can be achieved if banks hold all good assets in the economy (and finance none of the bad assets).

The need to regulate results from the assumption that the government finds it ex post optimal to bail-out banks that cannot repay their depositors (debtors). Within our model, a bailout is ex-post optimal given the banks’ superior ability to collect cash flows. This protection by the taxpayer allows banks to engage in asset substitution, i.e., funding of bad issuers, without negatively affecting depositors. This eliminates the disciplining effect of debt in our setup.

If the regulator could directly observe the quality of the asset, regulation would be trivial, as the regulator could simply mandate banks to invest only in good assets. To study the realistic and non-trivial case, we make the following assumption:

**Assumption 4** Firm projects of different types \((g, b)\) are observationally equivalent to the regulator. The regulator can only distinguish between cash and firm projects.

Since the regulator can also observe (and use) ratings, the regulator’s information set \(F = (F^{-\rho}, \rho)\) is identical to the one of the household investor where the tuple \(F^{-\rho} = (\delta_n, R, \pi_g, \pi_H, \gamma, \bar{E}_0)\) lists the exogenous parameters in the economy and \(\rho\) represents ratings. Conditional on this information set \(F\), we posit that the regulator sets regulatory rules in the form of minimum capital requirements, i.e.,

\[
e \geq e_{\text{min}}(F).
\]

To understand the role of ratings in regulation, we separate our analysis into two parts, i.e., simple regulation based on \(F^{-\rho}\) (see Sections 4.1 and 4.2.1) and rating-contingent regulation based on \(F\) (see Section 4.2.2). While the choice of this regulatory tool seems ad hoc, our analysis will reveal that the proper design of rating contingent capital requirements is able to induce the first best outcome in the economy. Put differently, our focus on minimum capital requirements is not restrictive from a theoretical perspective. Moreover, given the prevalence of minimum capital requirements in practice, our normative analysis

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14 As a result, bank debt can (at least partially) be interpreted as government debt (see Wolf, 2013).
15 Even absent a bailout guarantee, an asset substitution problem may arise after the bank has issued debt. However, without a bailout guarantee, incentives for risk shifting would be reduced since debt holders would require higher yields in advance (or not even invest).
16 In an extension of the paper, we consider parameter uncertainty about the fraction of good types in the economy.
17 We do not claim that minimum capital requirements are the only way to achieve first best outcomes.
also produces *positive* predictions “on the side” by illustrating the non-trivial comparative 
statics of economic outcomes with respect to minimum capital requirements.

Throughout the analysis, we assume the following logical order of events at the beginning 
of the first period. The regulator moves first and sets the regulatory regime. Then, 
the CRA, if it operates, announces a disclosure policy, and the issuers decide whether to 
purchase a rating. After ratings are disclosed, a centralized market for bank funding opens. 
Thereafter, a centralized market for direct funding by households opens. The sequential 
opening of markets for bank funding and direct funding by households not only yields a 
very tractable setup but also reflects the notion that banks have a first-mover advantage 
in access to issuers.

3 Analysis

Before describing the formal definition of equilibrium, we want to emphasize an important 
feature of the just described economy: Banks, issuers and rating agencies can all identify 
the respective issuer types, whereas the regulator and the household investor cannot. Our 
equilibrium concept is a Perfect Bayesian Equilibrium which is defined as follows:

**Definition 1**  *Perfect Bayesian Equilibrium*

a) Each firm maximizes its expected value of profits \( \mathbb{E}(\max\{Z_i^* - N_i, 0\}) \) net of issuing 
cost \( f \) by selling debt with the lowest face value \( N_i \) that results in raising 1.

b) Each bank maximizes the expected value of equity, \( E_0^M \), by choosing its investment 
\( E_0 \leq \bar{E}_0 \), its equity buffer \( e \geq e_{\min}(F) \), and its portfolio strategy \( x_j \geq 0 \).

c) Households provide deposits and invest in risky firm projects if and only if they expect 
to break-even with expectations conditional on their information set \( F \) that satisfy Bayes’ 
rule taking as given the disclosure policy of rating agencies and the behavior of banks.

d) Rating agencies report the set of G-rated firms \( \Omega_G \) to maximize profits \( \Pi_R \) given the 
behavior of households and regulatory capital requirements \( e_{\min}(F) \).

e) The regulator chooses minimum equity capital requirements \( e \geq e_{\min}(F) \) to maximize 
welfare \( W \).

Our equilibrium analysis focuses on the relevant aggregate implications of regulation 
for investment and funding terms. It is important to note that the total amount of deposits 
is indeterminate. Suppose \( D_0 > 1 \). Then banks could always borrow more, say raise debt
to $D_0 + \Delta D$ and invest the additional amount $\Delta D$ in cash. This would keep $e$ unchanged. For the purpose of our analysis, the outcomes are essentially equivalent.

The following analysis proceeds in three steps. In Section 3.2, we illustrate how the valuation of risky assets is affected by the implicit bail-out guarantee for banks. These insights are essential for the remaining equilibrium analysis. In Section 3.3, we study the unregulated economy as a benchmark (in which $e_{\text{min}} = 0$). Since this economy features inefficiencies as a result of the bail-out guarantee, we then study the effectiveness of capital regulation to eliminate / reduce asset substitution.

### 3.1 Preliminaries

In this section, we will develop a few preliminary results that will facilitate exposition in the analysis going forward.

**Lemma 1** Banks never have a strict preference to choose $\text{Div}_0 > 0$.

**Proof:** Inspection of the bank’s objective function $E_0 \cdot \left(1 + \frac{1}{e} \mathbb{E} \max \left\{ \sum x_j r_j^* - e \right\} \right)$ s.t. $e \geq e_{\text{min}}$ reveals that profits scale with the initial investment $E_0$. Since the bank is infinitesimal, changes in $E_0$ do not affect $r_j^*$. If the maximum possible return on risky assets satisfied $\mathbb{E} \max \left\{ \sum x_j r_j^* - e \right\} < 0$, the bank could simply invest all its assets in cash (rather than paying out the dividend) to ensure it keeps its equity, which formally implies that $e \to \infty$. If $\mathbb{E} \max \left\{ \sum x_j r_j^* - e \right\} \geq 0$, it clearly has no strict incentive to reduce $E_0$ by paying out dividends. ■

Intuitively, if returns on risky assets are not sufficiently attractive, the bank could always invest in cash rather than paying out dividends. Thus, without loss of generality, we can assume that no bank ever pays out dividends, i.e., we set

$$E_0 = \bar{E}_0.$$  \hspace{1cm} (12)

This allows us to write the decision problem of the banks solely in terms of the leverage choice $e$ and the portfolio choice $\{x_j\}$. Since banks are thus identical in terms of scale (and all other characteristics), the following lemma is immediate.

**Lemma 2** All banks share the same equilibrium expected return on equity, i.e., $\mathbb{E} [r_E^*] = \bar{r}_E$ (even if different bank strategies $(e, \{x_j\})$ coexist).
Proof: Omitted.

### 3.2 Valuation and Funding Terms

An essential ingredient for the following analysis is how banks and households value assets. Due to risk-neutrality and competition, household investors require an expected return of 0 on any asset. Since households incur a cost \( c \) when collecting the face value of debt, they will value a bond \( i \) with face value \( N_i \) and default probabilities given their information set, \( \mathbb{E}(d_i|F) \), as

\[
V_H(i) = N_i (1 - \mathbb{E}(d_i|F)) - c.
\]  

(13)

Banks’ marginal valuations depend on their overall portfolio strategy, in particular, whether the bank defaults in the low state or not. A bank that chooses an equity ratio and investment portfolio such that \( e < -r^L_A = -\sum x_j r^L_j \) will default in the low state; otherwise the bank survives (see equation 6).

**Lemma 3** Consider a bank that has an equity ratio \( e \) and holds risky assets generating state-dependent returns \( r^*_A \). Then the bank values an asset \( i \) with default probabilities \( d^L_i \) and \( d^H_i \) as

\[
V_B(i) = \begin{cases} 
\frac{N_i (1 - d^H_i)}{1 + r^H_A} & \text{for } e < -r^L_A, \\
\frac{N_i (1 - d^L_i)}{1 + \mathbb{E}[r^H_A]} & \text{for } e \geq -r^L_A.
\end{cases}
\]

(14)

**Proof:** See Appendix.

This lemma illustrates an important ingredient for the remaining analysis of the paper. A non-defaulting bank simply values the asset according to its unconditional default probability \( d_i \), similar to a household investor. In contrast, a defaulting bank, whose equity buffer is small \( (e < -r^L_A) \), cares only about payoffs in the high state of the world, i.e., \( N_i (1 - d^H_i) \): Any additional payoff generated in the low state of the world would simply reduce the required subsidy by taxpayers (to pay bond holders), but does not affect equity value.

Based on the just developed valuation techniques, the following lemma reveals that we can restrict our attention to “pure portfolios.”

**Lemma 4** Pure Portfolios:

a) There is no equilibrium in which non-defaulting banks fund bad issuers.
b) There is no equilibrium in which defaulting banks (with \( e > 0 \)) fund good issuers.

**Proof:** See Appendix. ■

The intuition for this lemma is simple. Non-defaulting banks care about the payoffs of securities in both states of the world. Since the NPV of a bad security is negative, they will never be willing to fund the investment. In contrast, defaulting banks care only about the high state of the world. Therefore, the payoff of the good security in the low state is of no value to a defaulting bank.

**Proposition 1** If an issuer of type \( i \) is funded in equilibrium by banks, then its face value satisfies:

\[
N_g(e, \bar{r}_E) = 1 + e \bar{r}_E, \quad \text{if } i = g, \\
N_b(e, \bar{r}_E) = 1 + e \frac{\bar{r}_E + 1 - p_H}{p_H}, \quad \text{if } i = b.
\]

**Proof:** If a good issuer (\( d_g = 0 \)) is funded by banks, then it must be financed by a non-defaulting bank (see Lemma 4). Setting the required funding, i.e., one unit, equal to the valuation by non-defaulting banks (see equation 14) yields \( N_g = 1 + \mathbb{E}[r_A^s] \). Since for a non-defaulting bank, \( e \bar{r}_E = \mathbb{E}[r_A^s] \) (see equation 6), the result immediately follows.

If a bad issuer (\( d_b^H = 0 \)) is funded by banks, then it must be financed by a defaulting bank (see Lemma 4). Setting the required funding, i.e., one unit, equal to the valuation by defaulting banks (see equation 14) yields \( N_b = 1 + r_A^H \). Since for a defaulting bank, \( \bar{r}_E = p_H \left[ \frac{r_A^H}{e} \right] + (1 - p_H)(-1) \) (see equation 6), the result follows. ■

Due to the unit size of investment, the face value can be interpreted as the (gross) yield to maturity. Note that these face values depend on the choice of leverage \( e \) and the equilibrium expected return on equity \( \bar{r}_E \) of the banking sector. As our subsequent analysis will reveal, the equilibrium rate of return of the banking sector is related to the relative scarcity of the banking sector and the firms.

### 3.3 Unregulated Economy

We first characterize an unregulated economy to study the need for regulation. Thus, the equilibrium is simply given by parts a) to d) of the equilibrium definition and setting \( e_{\min} = 0 \).
Proposition 2  In the unregulated economy:
1) all bad projects are financed by fully levered, defaulting banks,
2) all good projects are financed by non-defaulting banks
3) all firm types can obtain financing at a face value of $N = 1$,
4) households do not invest in firm projects and rating agencies do not operate
5) social welfare is given by $W = R (1 - \bar{d}) - 1 < 0$

Proof: See main text. ■

The intuition for these results is simple. Issuers are in short supply relative to bank funds as banks face no leverage restrictions. As a result, the expected return on equity satisfies $\bar{r}_E = 0$, which directly implies $N_g = 1$ (see Proposition 1). Defaulting banks go to the limit of $e = e_{\min} = 0$, implying that they fund bad assets at $N_b = 1$ as well. Issuers of both project types can capture all the (private) surplus. Since all projects are funded by banks and since banks do not need ratings, the CRA does not operate. In an unregulated economy, the yields are completely uninformative about the default risk of an asset: For both issuers, the yield is zero. Bad assets are rationally inflated due to limited liability and the presence of the bailout guarantee for banks. At these prices, rational household investors cannot compete with banks and play no role in this economy (even if they observed the type of the issuer).

4 Regulated Economy

The just described unregulated economy features welfare losses relative to first best. Bankers are willing to finance every project in the economy because they only care about outcomes in the good aggregate state. Thus, investment is too high relative to first best. Any type of (successful) capital regulation $e \geq e_{\min}$ must therefore limit excessive investment (without prohibiting positive NPV investments).

To understand the role of households and rating agencies in our analysis, it is useful to split the analysis into multiple steps. First, we assume that households’ collection cost is prohibitively high, i.e., $c > V_g = R - 1$, so that they would never be able to break-even on their investments (Section 4.1) and only banks can invest. The subsequent section (Section 4.2) analyzes the case in which competition by households affects the equilibrium outcome. Moreover, for each case, we first solve for optimum capital regulation that is not contingent on ratings (simple leverage constraints) to see the scope for making regulation
contingent on ratings.

4.1 Banking Economy

In the pure banking economy in which collection costs for households make it impossible for them to invest in firms, capital regulation $e \geq e_{\text{min}}$ limits the total amount of funds available for investment. The maximum amount of risky assets that can be financed is given by

$$A_{\text{max}} = \min \left\{ \frac{\bar{E}_0}{e_{\text{min}}}, 1 \right\}. \quad (17)$$

Investment in risky assets is capped by the supply of risky assets of 1, whenever capital requirements are sufficiently lax, i.e., $e_{\text{min}} < \bar{E}_0$. When $e_{\text{min}} > \bar{E}_0$, bank funds are short of the supply of all risky assets, so that some projects in the economy are unfunded. We denote by $\mu_i$ the mass of funded securities of type $i$. When regulation is sufficiently stringent, i.e., $e_{\text{min}} > \frac{\bar{E}_0}{\pi_g}$, the banking sector cannot even finance all good projects in the economy, i.e., $A_{\text{max}} < \pi_g$.

The just described cases will be important in the analysis going forward as they determine the rent distribution in the economy, which is reflected in the equilibrium rate of return on equity, $\bar{r}_E$. Given the expected return of equity $\bar{r}_E$ and $e_{\text{min}}$, the funding terms for issuers of each type are given by Proposition 1, i.e., $N_g(e_{\text{min}}, \bar{r}_E)$ and $N_b(e_{\text{min}}, \bar{r}_E)$. Note, that since the maximum pledgable face value is $R$, we can define the cutoff $\tilde{e}$ such that $N_b(\tilde{e}, 0) = R$:

$$\tilde{e} = \frac{p_H}{1 - p_H} (R - 1) < 1. \quad (18)$$

If the regulator mandates that $e_{\text{min}} \geq \tilde{e}$, bad projects are not attractive enough for banks to pursue asset substitution. We assume that, if indifferent, banks will not shift risk. Moreover, it is useful to define the maximum expected return on equity that can be achieved if banks engaging in asset substitution extract everything from bad issuers, i.e., $N_b(e_{\text{min}}, \bar{r}_E^b) = R$ for all $e_{\text{min}} \leq \tilde{e}$:

$$\bar{r}_E^b(e_{\text{min}}) = p_H \frac{R - 1}{e_{\text{min}}} + (1 - p_H)(-1) \quad (19)$$

Proposition 3 Comparative statics of capital requirements $e_{\text{min}}$.

The “Natural Pecking Order” Regime: If $e_{\text{min}} \leq \tilde{e}$, banks will fund as many good issuers

\footnote{The fact that the bad project has negative NPV implies that $p_H (R - 1) - (1 - p_H) < 0$, so that $\tilde{e} < 1$.}
as possible, i.e., \( \mu_g = \min (A_{\text{max}}, \pi_g) \) and invest the remaining funds in bad issuers, i.e., \( \mu_b = \max \{A_{\text{max}} - \pi_g, 0\} \). Equilibrium values of \( \bar{r}_E \), \( \mu_g \), and \( \mu_b \) as functions of \( e_{\text{min}} \) are shown in the following table.

<table>
<thead>
<tr>
<th>( \bar{r}_E )</th>
<th>( \mu_g )</th>
<th>( \mu_b )</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \pi_g )</td>
<td>( \pi_b )</td>
<td>( e_{\text{min}} &lt; \bar{E}_0, )</td>
</tr>
<tr>
<td>( \bar{r}_E )</td>
<td>( \pi_g )</td>
<td>( A_{\text{max}} - \pi_g )</td>
<td>( \bar{E}<em>0 &lt; e</em>{\text{min}} &lt; \frac{E_0}{\pi_g}, )</td>
</tr>
<tr>
<td>( \frac{R-1}{e_{\text{min}}} )</td>
<td>( A_{\text{max}} )</td>
<td>0</td>
<td>( e_{\text{min}} &gt; \frac{E_0}{\pi_g}, )</td>
</tr>
</tbody>
</table>

The “Safe Banks” Regime: If \( e_{\text{min}} > \bar{e} \), banks will fund as many good projects as possible, i.e., \( \mu_g = \min (A_{\text{max}}, \pi_g) \) and never invest in bad issuers. Equilibrium values of \( \bar{r}_E \), \( \mu_g \), and \( \mu_b \) as functions of \( e_{\text{min}} \) are shown in the following table.

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<thead>
<tr>
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</tr>
<tr>
<td>( \frac{R-1}{e_{\text{min}}} )</td>
<td>( A_{\text{max}} )</td>
<td>0</td>
<td>( e_{\text{min}} &gt; \frac{E_0}{\pi_g}, )</td>
</tr>
</tbody>
</table>

Proof: See Appendix. ■

When \( e_{\text{min}} \leq \min \left\{ \bar{e}, \frac{E_0}{\pi_g} \right\} \), asset substitution of banks occurs at least partially. Intuitively, sufficiently lax capital requirements might produce the same inefficient outcome as in the unregulated economy. When \( \bar{E}_0 < e_{\text{min}} < \min \left( \bar{e}, \frac{E_0}{\pi_g} \right) \), all good assets are financed, but only a fraction of bad assets. Investment in these bad assets determines the outside option of “safe banks.” Bad issuers offer as much as possible to attract investment, i.e., \( N_b = R \) which pins down the expected rate of return on equity \( \bar{r}_E = \bar{r}_E (e_{\text{min}}) \) for defaulting banks. Good issuers promise a face value \( N_g < R \) that implies the same expected return on equity for safe banks (by Lemma 2). Once \( e_{\text{min}} \geq \frac{E_0}{\pi_g} \), bank funds are in short supply to finance good assets, so that banks can extract the entire NPV, \( V_g = R - 1 \), from good issuers. Bad issuers cannot promise higher returns than good issuers and remain unfunded, i.e., \( \mu_b = 0 \). Thus, in the absence of competition for good issuers, banks’ portfolios exhibit a “natural” pecking order. First, banks try to fund as many good projects as possible and use excess funds \( A_{\text{max}} - \pi_g \) (if possible) to finance bad projects provided that asset substitution is privately attractive, i.e., \( e < \bar{e} \).

Figure 1 illustrates the results of Proposition 3 by plotting the expected return on
The figures illustrate the effect of varying equity ratio requirements, \( e \geq e_{\text{min}} \), on the expected return on bank equity (\( \bar{r}_E \)) and on funding volume (\( \mu_g \), \( \mu_b \), and \( \mu_g,H \)) in a pure banking economy. In panel (a) expected returns are plotted for two different scenarios, one with low initial equity capital (\( E_0^{\text{Low}} = 0.3 \)) and one with high initial equity capital (\( E_0^{\text{Low}} = 0.4 \)). In panel (b) funding volume is plotted for the case where equity capital is low (\( E_0 = 0.3 \)). Funding volume is broken down into funding of good and bad issuers by banks (\( \mu_g \) and \( \mu_b \) respectively). The remaining parameters of the economy are chosen as follows: \( p_H = 0.5 \), \( R = 1.45 \), \( \pi_g = 0.35 \).

**Figure 1.** Outcomes in a pure banking economy with capital regulation

bank equity and funding volume of good and bad issuers as a function of the minimum equity ratio \( e_{\text{min}} \). In panel (a) expected returns are plotted for two different scenarios, one with low initial equity capital (\( E_0^{\text{Low}} = 0.3 \)) and one with high initial equity capital (\( E_0^{\text{Low}} = 0.4 \)). In panel (b) funding volume is broken down into funding of good and bad issuers by banks (\( \mu_g \) and \( \mu_b \) respectively). For low \( e_{\text{min}} \) banks invest in all assets in the economy, including the assets of bad issuers (\( \mu_g = \pi_g \) and \( \mu_b = \pi_b \)) – risk-shifting occurs. However, since available bank funds exceed the supply of projects, competition between banks eliminates any excess returns (\( \bar{r}_E = 0 \)). Increasing \( e_{\text{min}} \) at some point starts to constrain total bank investment, and banks choose to first reduce their investment in bad issuers. The natural pecking order of bank investment applies. At this point, the scarcity of total bank funds allows banks to generate positive expected returns on equity (\( \bar{r}_E > 0 \)). However, once \( e_{\text{min}} \) exceeds \( \tilde{e} \), risky banks are not viable any more, and banks can only fund good issuers. Since bank capital is at that point sufficient to fund all good issuers, expected returns on bank equity drop back to zero. Further increases in \( e_{\text{min}} \) finally also constrain banks investment in good issuers. Again, bank capital becomes scarce and banks can generate positive expected returns on equity (\( \bar{r}_E > 0 \)). However, investment in good
issuers is inefficiently low and generates welfare losses ($\mu < \pi_g$.)

**Proposition 4** First-best welfare $W^*$ can be achieved by setting $e_{\text{min}} = \frac{\bar{E}_0}{\pi_g}$. The minimum capital requirement that achieves first-best welfare is given by: $e_{\text{min}}^* = \min \left( \bar{e}, \frac{\bar{E}_0}{\pi_g} \right)$

**Proof:** If $\bar{E}_0/\pi_g \leq \bar{e}$, $e_{\text{min}} = \bar{E}_0/\pi_g$ implies Case (3) of the “Natural Pecking Order” Regime in Proposition 3 and $A_{\text{max}} = \pi_g$. If $\bar{E}_0/\pi_g > \bar{e}$, $e_{\text{min}} = \bar{E}_0/\pi_g$ implies the “Safe Banks” Regime and $A_{\text{max}} = \pi_g$.

See Proposition 3. $lacksquare$

This result follows directly from the comparative statics of capital requirements shown in Proposition 3. It reveals that in the absence of competition from households, first best welfare can be achieved by simple capital regulation: Contingency on ratings is not necessary. The main ingredient for this result is the “natural pecking order:” Banks first have an incentive to finance good assets; bad issuers only represent the second best option. By constraining the size of the banking sector via capital regulation such that just a mass $\pi_g$ of projects can be financed, the first-best outcome can be implemented.

4.2 Banking and Households Economy

We now assume that the cost of investing for households is not prohibitively high, so that they may purchase good securities. However, since households cannot directly observe the quality of assets, they have to rely on risk assessments by the rating agency. We first consider, in the next subsection, the case of simple capital regulation in which regulation is not ratings-contingent.

4.2.1 Simple capital regulation

In this subsection, ratings only serve the purpose of providing information to households. As a result, the profit maximizing disclosure rule of a committed CRA facing rational investors is truthful reporting (see Opp, Opp, and Harris (2013)): The CRA will rate all good issuers and none of the bad issuers as $G$. The existence of competitive households provides an outside option for good issuers to obtain funding at $N_{g,H} = 1 + c$. As the rating agency collects a fee proportional to the value of the investment for the firm (after
financing), i.e., \( f = \gamma (R - N_{g,H}) \), the total financing cost (yield + issuing cost), \( c_T \) is given by:

\[
c_T = \gamma (R - 1) + (1 - \gamma) c
\]  

(22)

Intuitively, \( c_T \) is increasing in the collection cost \( c \) and the surplus that the CRA can extract from the issuer \( \gamma \).

Competition from households for good issuers results in an upper bound on the return on equity of \( \bar{r}_{E,g} = \frac{\bar{c}_T}{e_{\min}} \) that banks can achieve. Note that this competition channel only applies to good investments, since banks never face competition for investing in bad issuers. Recall that the upper bound on the expected return on equity for investments in bad issuers is given by \( \bar{r}_{E,b} = \rho_H (R - 1) - (1 - p_H) \) (see equation [19]). We can now solve for the threshold value of \( e_{\min} \) for which \( \bar{r}_{E,g} = \bar{r}_{E,b} \). We denote the solution \( \tilde{e}(c_T) \)

\[
\tilde{e}(c_T) = \frac{p_H (R - 1) - c_T}{1 - p_H}
\]  

(23)

where \( \tilde{e}(c_T) < \tilde{e}(0) = \tilde{e} < 1 \). When \( e_{\min} \) sufficiently low, i.e., \( e_{\min} < \tilde{e}(c_T) \), we have that \( \bar{r}_{E,b} > \bar{r}_{E,g} \).\(^{19}\) Intuitively, when leverage constraints are sufficiently lax (or equivalently competition from households is sufficiently high, i.e., \( c_T \) low), investment in bad issuers becomes more attractive. As a result, the “natural pecking order” of investment (described in the previous section) is reversed as shown in the following proposition:

**Proposition 5** Comparative statics of \( e_{\min} \) in the presence of competition by households:

Regime “Reverse Pecking Order”: If \( e_{\min} < \tilde{e}(c_T) \), banks fund as many bad issuers as possible, i.e., \( \mu_b = \min(A_{\max}, \pi_b) \) and invest the remaining funds in good issuers, i.e., \( \mu_g = \max(A_{\max} - \pi_b, 0) \). Households fund the remaining good issuers, i.e., \( \mu_{g,H} = \min(1 - A_{\max}, \pi_g) \).

\[
\begin{array}{ccccc}
\bar{r}_E & \mu_g & \mu_{g,H} & \mu_b & \text{Case} \\
0 & \pi_g & 0 & \pi_b & e_{\min} < \bar{E}_0, \\
\bar{r}_{E,g} = \frac{c_T}{e_{\min}} A_{\max} - \pi_b & 1 - A_{\max} & \pi_b & \bar{E}_0 < e_{\min} < \frac{\bar{c}_T}{\bar{e}_b}, \\
\bar{r}_{E,b}(e_{\min}) & 0 & \pi_g & A_{\max} & e_{\min} > \frac{\bar{c}_T}{\bar{e}_b}.
\end{array}
\]  

(24)

If \( \tilde{e}(c_T) < e_{\min} < \tilde{e} \), the natural pecking order regime obtains.

If \( e_{\min} > \tilde{e} \), regime “Safe Banks” obtains. In the last two regimes, \( \bar{r}_E = \frac{c_T}{e_{\min}} \) when \( e_{\min} > \)

\(^{19}\) Of course, this can only occur if \( \tilde{e}(c_T) \) is non-negative or equivalently \( c_T < p_H (R - 1) \).
$E_0 / \pi_g$. All other outcomes in these regimes are as in Proposition 5.

Proof:

Case 1. If $e_{\min} < \bar{E}_0$, banks are competing for all (good and bad) assets of firms. As a result, banks make zero profits ($\bar{r}_E = 0$) and all firms are financed.

Case 2. If $A_{\text{max}} < 1$, the banking sector cannot finance all risky assets. Since the amount that may be extracted from bad issuers is greater than for good issuers, i.e., $\bar{r}_{E,b} > \bar{r}_{E,g}$, all bad issuers are financed by banks. Banks invest the remaining funds in good issuers, i.e., $A_{\text{max}} - \pi_b$. The good types that are unfunded by banks are financed by households (with mass $1 - A_{\text{max}}$).

Case 3. Banks only invest in bad assets, since $\bar{r}_{E,b} > \bar{r}_{E,g}$. All good assets are funded by households.

This Proposition reveals that the intense competition from households for good assets causes banks to switch their business model from primarily funding good assets (see Proposition 5) to primarily funding bad assets. The banks’ “outside option” is now given by the return it can extract from good assets $\bar{r}_{E,\text{min}}$. This enables bad issuers to secure funding terms satisfying $N_b < R$ when banks compete for their assets, i.e., $e_{\min} < \bar{E}_0 / \pi_b$ (see Proposition 1). When $e_{\min} > \bar{e}(c_T)$, the economy behaves similarly to the case without households.

Figure 2 illustrates the results of Proposition 5 by plotting the expected return on bank equity and funding volume of good and bad issuers as a function of the minimum equity ratio $e_{\min}$. The figure shares similarities with figure 1 – however, there are a few important differences. For low $e_{\min}$ banks again invest in all assets in the economy, including the assets of bad issuers. However, increasing $e_{\min}$ at some point leads to a situation where the reverse pecking order of bank investment becomes apparent: banks first reduce their investment in good issuers as regulation starts to constrain total bank investment. Since banks now face competition from households for investment in good securities, banks rather maintain maximum investment in bad assets (that is, $\mu_b$ stays at $\pi_b = 0.65$.) At the same time, the scarcity of total bank funds allows banks to generate positive expected returns on equity ($\bar{r}_E > 0$). However, once $e_{\min}$ exceeds $\bar{e}(c_T)$, the normal pecking order obtains, and banks switch back to maximum possible investment in good assets, since stricter regulation starts to limit the benefits of doing risk-shifting. Once $e_{\min}$ exceeds $\bar{e}$ risky banks are not viable any more so that banks can only fund good issuers. Further increases in $e_{\min}$ finally also constrain banks investment in good issuers. At that point households invest in the remaining good issuers which generates welfare losses, since households are not as efficient.
The figures illustrate the effect of varying equity ratio requirements, \( e \geq e_{\text{min}} \), on the expected return on bank equity (\( \bar{r}_E \)), and on funding volume (\( \mu_g \), \( \mu_b \), and \( \mu_{g,H} \)) in an economy with competition between banks and household investors. In panel (a) expected returns are plotted for two different scenarios, one with low initial equity capital (\( \bar{E}_0^{\text{Low}} = 0.3 \)), and one with high initial equity capital (\( \bar{E}_0^{\text{High}} = 0.4 \)). In panel (b) funding volume is plotted for the case where equity capital is low (\( \bar{E}_0 = 0.3 \)). Funding volume is broken down into funding of good and bad issuers by banks (\( \mu_g \) and \( \mu_b \) respectively), and funding of good issuers by households (\( \mu_{g,H} \)). The remaining parameters of the economy are chosen as follows: \( p_H = 0.5, R = 1.45, \pi_g = 0.35, \gamma = 0.02, c = 0.02 \).

The possibility of a reverse pecking order has implications for the design of optimum capital regulation

**Proposition 6** If \( \tilde{e}(c_T) \leq \frac{\bar{E}_0}{\pi_g} \), the first best outcome can be achieved by setting \( e_{\text{min}} = \frac{\bar{E}_0}{\pi_g} \). If \( \frac{E}{\tilde{e}(c_T)} < \pi_g \), the first best outcome cannot be achieved. Then second best outcome is achieved by setting \( e_{\text{min}} = \tilde{e} \) and results in a welfare loss of \( (\pi_g - \frac{\bar{E}_0}{\tilde{e}}) c \)

**Proof:** See main text. □

If \( \tilde{e}(c_T) \leq \frac{\bar{E}_0}{\pi_g} \), setting \( e_{\text{min}} = \frac{\bar{E}_0}{\pi_g} \) implies that we are in the “natural pecking order” regime (by Proposition 5) and so the “optimality” results of the previous section apply. The first best outcome can be achieved by constraining investment of the banking sector to \( \pi_g \) (setting \( e_{\text{min}} = \frac{\bar{E}_0}{\pi_g} \)). However, when \( \tilde{e}(c_T) > \frac{\bar{E}_0}{\pi_g} \), i.e., \( c_T \) sufficiently small, constraining the size of the banking sector to \( A_{\text{max}} = \pi_g \) is not enough since banks will “first” invest in
bad issuers as long as there is a positive return to be made from bad issuers (reverse pecking order). In this case, leverage regulation needs to be so stringent that asset substitution is no longer privately attractive, i.e., \( e_{\min} = \tilde{e} \). Since \( \tilde{e} > \tilde{e}(c_T) > \frac{E_0}{\pi_g} \) in this case, banks cannot fund all good issuers in the economy. The remaining good issuers \( \pi_g - \frac{E_0}{\tilde{e}} \) are funded by households which incur a collection cost \( c \) per unit. The associated welfare loss implies that simple capital regulation could potentially be improved upon by rating contingent regulation.

4.2.2 Rating contingent capital regulation

We now allow the regulator to make regulation contingent on ratings. Note first, that if the rating agency always reported the truth, optimal regulation would be trivial. Define the rating specific equity capital ratio

\[
e(i) \equiv \frac{E_0}{A_i}, \quad i \in \{G, B\},
\]

where \( A_i \) is the dollar amount invested in \( i \)-rated securities. Banks are only allowed to invest in assets that the rating agency labels good, i.e., \( G \)-rated assets and are also subject to the equity capital ratio constraint \( e(G) \geq e_{\min}(G) > 0 \). We consider the relevant case where regulation has to take into account strategic choice of the disclosure policy by the credit rating agency.

**Lemma 5** In the presence of rating contingent regulation \( e(G) > e_{\min}(G) \), there is no equilibrium in which households fund good issuers, and a bank funds bad issuers.

**Proof:** See Appendix. ■

Lemma 5 shows that the phenomenon of a “reverse pecking order,” which may occur in a regime with pure capital regulation (see previous section), does not arise under rating contingent regulation.

**Proposition 7** Conditional on rating contingent regulation with \( e(G) > e_{\min}(G) > 0 \), the rating agency chooses to rate a mass \( \mu_g + \mu_{gH} \) of good issuers \( G \) and a mass \( \mu_b \) of bad issuers \( G \).
Regime a) If $\epsilon_{\text{min}}(G) < \bar{\epsilon}$, defaulting and non-defaulting banks are viable:

\[
\begin{array}{cccc}
\bar{\epsilon}_E & \mu_g & \mu_{g,H} & \mu_b \\
0 & \pi_g & 0 & \pi_b \\
0 & \pi_g & 0 & A_{\text{max}} - \pi_g \\
0 & A_{\text{max}} & 0 & 0 \\
\frac{c}{\epsilon_{\text{min}}(G)} & A_{\text{max}} & \pi_g - A_{\text{max}} & 0 \\
\end{array}
\]

Case

$\epsilon_{\text{min}}(G) < \bar{\epsilon}_0,$

$\bar{\epsilon}_0 < \epsilon_{\text{min}}(G) < \frac{\bar{E}_0}{\pi_g},$

$\epsilon_{\text{min}}(G) > \frac{\bar{E}_0}{\pi_g} \land c \geq \bar{\epsilon}_G,$

$\epsilon_{\text{min}}(G) > \frac{\bar{E}_0}{\pi_g} \land c < \bar{\epsilon}_G.$

where

$$\bar{\epsilon}_G \equiv (R - 1) \left(1 - \frac{A_{\text{max}}}{\pi_g}\right).$$

Regime b) If $\epsilon_{\text{min}}(G) \geq \bar{\epsilon}$, only non-defaulting banks are viable, so that $\mu_b = 0$. All other outcomes are the same as in the case $\epsilon_{\text{min}}(G) < \bar{\epsilon}$.

**Proof:** See Appendix □

The private profit-maximization interest of the CRA, which is tied to the surplus generated for the issuers, restores the “natural pecking order.” The CRA first has an incentive to rate good issuers well. However, if regulation is sufficiently lax, i.e., $\epsilon_{\text{min}}(G) < \min \left(\bar{\epsilon}, \frac{\bar{E}_0}{\pi_g}\right)$, so that banks can and want to engage in asset substitution, the CRA additionally rates bad issuers as $G$. Thus, suboptimal (lax) capital regulation will induce rating inflation as the CRA caters to the demands of regulated institutional investors. Proposition 7 thus highlights the complementarity of sound capital regulation and reliable signals as inputs to capital regulation.

Figure 3 illustrates the results of Proposition 7 by plotting the expected return on bank equity and funding volume of good and bad issuers as a function of the minimum equity ratio $\epsilon_{\text{min}}(G)$. A key difference relative to the outcomes under pure liability side regulation (see Proposition 5 and Figure 2) is the fact that the phenomenon of a ”reverse pecking order” of bank investment no longer obtains when regulators also use ratings to constrain the asset side of banks’ balance sheet. Banks rather follow the ”natural pecking order” and always maintain maximum possible investment in good assets. Interestingly, excess returns generated by banks are now generally zero, except for the case where $\epsilon_{\text{min}}$ is so large that it constrains bank investment in good assets. At that point, banks can

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20 This reveals that the results of Opp, Opp, and Harris (2013) extend to a setting with endogenous asset substitution.
The figures illustrate the effect of varying rating-contingent equity ratio requirements, \( e_G \geq e_{\text{min}} \), on the expected return on bank equity \((\bar{r}_E)\), and on the funding volume \((\mu_g, \mu_b, \text{and } \mu_{g,H})\). In panel (a) expected returns are plotted for two different scenarios, one with low initial equity capital \((\bar{E}_{0}^{\text{Low}} = 0.3)\), and one with high initial equity capital \((\bar{E}_{0}^{\text{Low}} = 0.4)\). In panel (b) funding volume is plotted for the case where equity capital is low \((\bar{E}_0 = 0.3)\). Funding volume is broken down into funding of good and bad issuers by banks \((\mu_g \text{ and } \mu_b \text{ respectively})\), and funding of good issuers by households \((\mu_{g,H})\). The remaining parameters of the economy are chosen as follows: \( p_H = 0.5, R = 1.45, \pi_g = 0.35, \gamma = 0.02, c = 0.02 \).

(a) Expected Return on Bank Equity

(b) Funding Volume

Without rating contingent regulation, the CRA is only concerned with households ability to fund issuers, since banks will rely on their own information. Thus, the CRA can only extract surplus from the good issuers that are funded by households. Due to competition from households for the funding of good issuers, banks may therefore find it optimal to fund bad issuers first so that the reverse pecking order of bank investment may obtain (see Proposition 5 and Figure 2). On the other hand, with rating contingent regulation, the CRA also extracts value from issuers funded by banks, since banks now are regulated to only invest in rated assets. Given this, the CRA can also extract part of the private surplus generated through bank financing. However, the CRA can’t sustain equilibria in
which G-rated securities are inflated and households that lack a bailout insurance invest in these inflated securities. Thus, if the CRA decides to inflate ratings allowing banks to do risk-shifting, households will not pose a competitive threat to banks. Therefore, banks will again prefer to first invest in good issuers and will only invest in bad issuers if they have excess funds. This leads to the result that under optimal rating-contingent regulation, rating inflation can be prevented:

**Proposition 8** The optimal rating contingent regulation is to set

\[
e_{\text{min}}^*(G) = \min \left( \tilde{E}_0 \frac{\tilde{e}}{\pi_g}, \bar{e} \right) \tag{27}
\]

\[
e_{\text{min}}^*(B) = 1 \tag{28}
\]

The regulation achieves first-best investment.

**Proof:** See Proposition [7].

Thus, in contrast to simple capital regulation, a properly designed system of rating contingent capital regulation can achieve the first best outcome even when there is strong competition by household investors. We want to highlight that the first-best outcome could have also been implemented if the regulator imposed a “grading curve” on the CRA, i.e., to restrict the maximum allowable mass of G ratings to $\pi_g$ and restrict banks to only invest in G-rated assets.

### 4.3 Why not raise more equity?

One question our setup may raise is why $\bar{E}_0$ cannot be increased. We currently state that equity is given by “inside equity,” and that the firm cannot raise more equity externally. However, many academics argue that banks should just raise more equity and that this alone would solve the problems. Consistent with that argument, a pure capital ratio requirement of $e_{\text{min}} = 1$ in combination with raising $\bar{E}_0$ would achieve first-best in our setup: the banking sector cannot default since $E_0 = A_0$ and thus only invests in good assets. Further, total investment in good projects is never constrained since $E_0$ could be larger than $\pi_g$. Thus, rating contingent regulation would be superfluous. However, the following two points provide counterarguments.
Net-liquidity provision by banks would require $e_{\text{min}} < 1$. The very existence of banks is often motivated by the notion of liquidity provision in the spirit of Diamond and Dybvig (1983), or more recently Stein (2012). In the context of our model, if households received some additional utility from safe assets, liquidity provision by banks should be defined as the net-provision of safe assets which is given by $(1 - e) A_0$ (this is banks’ net debt). Raising debt and investing the proceeds in cash does not increase banks’ net-debt. Banks just take one unit of safe assets (cash) to generate another one. Thus, if the regulator sets $e_{\text{min}} = 1$, it essentially mandates that the banking sector does not generate any additional safe assets/liquidity, as the banking sector demands just as much liquidity (cash) as it generates. It follows that in the presence of demand for safe assets regulators should optimally consider setting $e_{\text{min}} < 1$ such that the banking sector actually may operate as a net liquidity provider.

Increasing equity $\bar{E}_0$ may be ineffective when $e_{\text{min}} < 1$. Given the first argument, consider a regulatory environment with equity-ratio constraints ($e < e_{\text{min}}$) and the case where $e_{\text{min}} < 1$ and $e_{\text{min}} < \bar{e} (c_T)$. Note that increasing $\bar{E}_0$ in this case can actually increase welfare losses. The more equity the banking sector has, the more aggregate investment in bad projects will occur (see results in section 4.2). Next, the question arises whether it is reasonable that the condition $\bar{e} (c_T) > e_{\text{min}}$ can be satisfied. Rearranging this inequality yields the restriction:

$$R - 1 > \frac{e_{\text{min}} (1 - p_H) + c_T}{p_H}.$$

Thus, as long as the NPV of good projects is sufficiently large, this restriction is satisfied. For example for $p_H = 0.8$, $c_T = 0.05$, and $e_{\text{min}} = 0.5$ (i.e., a 50% equity to asset ratio which corresponds to net liquidity provision of $0.5 \pi_g$ under optimal investment) the NPV for good issuers would have to be larger than 0.18. If we consider a parametrization with rare crises (say $p_L = 0.05$), we obtain a required NPV of 0.08.

If regulators allow banks to have positive net-debt and only regulate banks based on equity requirements (pure liability-side regulation), it is conceivable that banks still obtain incentives to focus on the funding of bad assets. Competition from non-banks for good assets can crowd out bank investment and can lead to excessive risk taking by banks even when equity cushions are large.


5 Conclusion

In this paper we proposed a general equilibrium framework to analyze the effectiveness of bank capital regulations in the presence of competition from unregulated investors. Our model highlights the importance of various general equilibrium effects that arise when regulated and unregulated market participants interact in financial markets. When competition from unregulated investors is low, the banking sector has a natural incentive to first fund positive NPV projects and to only engage in risk-shifting when the banking sector’s funding capacity exceeds the supply of good projects. This “natural pecking order” of bank investment allows simple capital regulation to be effective. However, when banks face sufficiently strong competition from unregulated investors, they weakly prefer to focus on markets for high-risk investments in which they are insulated from outside competition due to their monopoly on issuing government-insured debt. This “reverse pecking order” of bank investment renders simple capital regulation to be ineffective and may even cause equity injections to be locally counterproductive. However, we show that contingency of capital regulation on credit ratings may restore the natural pecking order of bank investment and can increase the efficiency of capital regulation.

A Proofs

A.1 Proof of Lemma 3

Proof: Let $r^i_s$ denote the return on asset $i$ in state $s$. To obtain marginal valuations we consider an $\varepsilon$-perturbation of the bank’s asset portfolio such that state-dependent returns on assets are given by

$$r^s_A(\varepsilon) = \varepsilon r^s_i + (1 - \varepsilon) r^s_A.$$  

(29)

Banks maximize their equity value and thus require that asset $i$ is priced such that a marginal perturbation keeps the expected return on equity unchanged. For a bank that defaults in the low state ($e < -r^L_A$), the expected return on equity given the perturbation is given by

$$\bar{r}_E(\varepsilon) = p_H \frac{\varepsilon r^H_i + (1 - \varepsilon) r^H_A}{e} - (1 - p_H).$$  

(30)

Imposing that $\frac{\partial \bar{r}_E(\varepsilon)}{\partial \varepsilon} |_{\varepsilon=0} = 0$ then implies that the bank is willing to purchase a marginal unit of asset $i$ as long as the return on asset $i$ in the high state satisfies $r^H_i = r^H_A$, or
1 + r_i^H = \frac{N_i(1-d_i)}{V_B(i)} = 1 + r_A^H. Solving the second equality for \( V_B(i) \) results in the upper branch of equation [14]

Similarly, for a bank that does not default in the low state \( e \geq -r_A^L \) the expected return on equity given the perturbation is given by

\[
\bar{r}_E(\varepsilon) = \varepsilon \frac{p_H r_i^H + (1-p_H) r_i^L}{e} + (1 - \varepsilon) \frac{p_H r_A^H + (1-p_H) r_A^L}{e}.
\] (31)

Imposing that \( \frac{d\mathbb{E}[r_E(\varepsilon)]}{de} \mid_{\varepsilon=0} = 0 \) then implies that the bank is willing to purchase a marginal unit of asset \( i \) as long as the average return on asset \( i \) has to satisfy \( \mathbb{E}[r_i^*] = \mathbb{E}[r_A^*] \). This implies the lower branch of equation [14].

### A.2 Proof of Lemma 4

**Proof:** With regard to the first statement, suppose there were an equilibrium in which some non-defaulting banks fund bad issuers. Then those non-defaulting banks must fund a mix of bad types and good types (or cash), since funding only bad types would imply that the bank defaults. Then by optimality, the expected return on bad and good assets (or on bad assets and cash) would have to be equal – otherwise it would be optimal to change the portfolio at the margin. Given that non-defaulting banks fund bad assets and some other assets (good assets or cash), which together generate expected returns \( \mathbb{E}[r_A^*] \), bad issuers’ bond prices must be equal to non-defaulting banks’ marginal valuation which is given by \( N_b (1-d_b) / (1 + \mathbb{E}[r_A^*]) \). However, equity holders require a weakly positive expected return, i.e., for a non-defaulting bank \( \mathbb{E}[r_E^*] = \mathbb{E}[r_A^*]/e \geq 0 \). Therefore for non-defaulting banks, \( V_B \leq N_b (1-d_b) \leq R (1-d_b) < 1 \) since \( N_b \leq R \) and the NPV of bad types is negative, so such a bank would never be willing to provide 1 unit of capital to bad issuers. This contradicts the supposition that some non-defaulting banks fund bad issuers.

We now turn to the second statement. Conjecture an equilibrium in which a defaulting bank funds some good issuers. Then this bank must fund a mix of good types and bad types, since funding only good types would imply that the bank does not default. By optimality, the return on the bad and the good assets in the high state has to be equal to the bank’s overall non-cash asset return in the high state, \( r_A^H \); otherwise it would be optimal for the bank to change the portfolio at the margin by tilting it toward the asset with the higher return in the high state. Given this, by Lemma 3 a defaulting bank values a bond from a good issuer with face value \( N_g \) at \( N_g / (1 + r_A^H) \). Assume that the defaulting bank
makes an expected return on equity equal to \( \bar{r}_E \geq 0 \) with its strategy \((e, \{x_j\})\). This implies that the return on a good issuer in the high state must be \( r^H_A = e \frac{\bar{r}_E + (1-p_H)}{p_H} \). Now consider an alternative strategy for the bank: the bank chooses the same \( e \) as before, but only invests in the good assets that are priced at \( \tilde{P} \). Since, given the price \( \tilde{P} \), the expected return on a good asset is \( e \frac{\bar{r}_E + (1-p_H)}{p_H} \), the bank will make an expected return on equity equal to \( \frac{\mathbb{E}[r^*_A]}{e} = \frac{\bar{r}_E + (1-p_H)}{p_H} > \bar{r}_E \). Thus, the defaulting bank would want to deviate to this better strategy. This contradicts the initial conjecture that a defaulting bank funds some good issuers in equilibrium.

### A.3 Proof of Proposition 3

**Proof:** The “Natural Pecking Order” Regime: Since \( \epsilon_{\min} < \tilde{\epsilon} \), bad assets can be potentially funded by banks.

**Case 1.** If \( \epsilon_{\min} < \bar{E}_0 \) so that \( A_{\max} = 1 \), banks are competing for all (good and bad) assets of firms. As a result, banks make zero profits \((\bar{r}_E = 0)\) and all firms are financed.

**Case 2.** If \( \bar{E}_0 < \epsilon_{\min} < \frac{\bar{E}_0}{\pi_g} \) so that \( \pi_g < A_{\max} < 1 \), then good projects are in short supply whereas bad projects are in excess supply. This implies that bad projects have to deliver all their returns if they are funded in equilibrium, i.e., defaulting banks make an expected return on equity of \( \bar{r}_E^{b}(\epsilon_{\min}) \). Lemma 2 pins down the equilibrium rate of return:

\[
\bar{r}_E = \bar{r}_E^{b}(\epsilon_{\min})
\] (32)

**Case 3.** If \( \epsilon_{\min} > \frac{\bar{E}_0}{\pi_g} \) so that \( A_{\max} < \pi_g \), only \( \mu_g = A_{\max} \) good assets can be financed. Issuers compete for banks, i.e., banks extract all the rents, so that \( \bar{r}_E = \frac{(R-1)A_{\max}}{\bar{E}_0} = \frac{R-1}{\epsilon_{\min}} \).

The “Safe Banks” Regime: For \( \epsilon_{\min} > \tilde{\epsilon} \), only good projects can be financed.

**Case 1.** If \( \epsilon_{\min} < \frac{\bar{E}_0}{\pi_g} \) bank funds exceed total feasible (good) investment opportunities. Hence, issuers capture all rents, i.e., \( \bar{r}_E = 0 \), and all good projects are financed.

**Case 2.** See the “Natural Pecking Order” Regime, Case (3).

### A.4 Proof of Lemma 5

**Proof:** Conjecture that there is an equilibrium in which the regulator sets rating contingent regulation, households fund good issuers, and a bank funds bad issuers. First note
that issuers have to be highly rated to get financing at all: banks are forbidden to hold B-rated securities. Further, given their beliefs, households will also not fund B-rated issuers. Thus, we can condition the following analysis on the fact that all funded issuers must have received a G rating. Next, for the conjectured equilibrium to be existent it must be the case that the bad issuer doesn’t have a strict incentive to deviate, that is mimic good types and apply for direct household funding in the second funding round. Let \( N_{1,g}, N_{1,b} \) denote the type-dependent face values set in equilibrium in the first round, and let \( N_2 \) denote the face value set in the second round. Bad issuers have no strict incentive to deviate to the second round if and only if \( N_{b,1} \leq N_2 \). Thus, assume that \( N_{b,1} \leq N_2 \) is satisfied. Further, note that by Lemma 4 there are only equilibria in which defaulting banks fund bad issuers, and in these cases defaulting banks only invest in bad issuers. Assume that a defaulting bank of this kind makes an expected return on equity return \( \bar{r}_E \geq 0 \) (the weak inequality is a necessary condition for an equilibrium, since banks can always at least generate \( \bar{r}_E = 0 \)), then the equilibrium face value \( N_{b,1} = 1 + e^{\frac{r_E + (1 - p_H)}{p_H}} \). Now consider a deviation by that bank: it maintains the same \( e \) but only invests in good projects with face values \( N_{1,g} \). However, we know that \( N_{1,g} \geq N_2 \), since otherwise good types would have no incentives to get funding in the second round, through household investors. Thus, it must be the case that \( N_{1,g} \geq N_2 \geq N_{b,1} \). However, then the bank’s strategy of funding only good projects yields an expected return on equity equal to \( \frac{r_E + (1 - p_H)}{p_H} > \bar{r}_E \), which implies that the bank would want to deviate from its current strategy, thus destroying the equilibrium.

A.5 Proof of Proposition 7

Proof: Given Lemma 5 the rating agency can only choose disclosure policies that lead to the following equilibrium outcomes:

**OPTION 1: Banks invest in good and bad issuers; households don’t invest.**

Bad issuers can only be funded when \( e_{\min}(G) \leq \tilde{e} \), since bad issuers can only be funded by defaulting banks in equilibrium. The CRA rates a mass \( \pi_g \) of good issuers \( G \) and a mass

\[
\min \left\{ A_{\max} - \pi_g - \lim_{\varepsilon \searrow 0} \varepsilon, \pi_b \right\} \tag{33}
\]

of bad issuers \( G \). The rating agency strategically limits the supply of G-rated securities, so that total bank funds that can be invested in G-rated securities (that is, \( \frac{E_0}{\epsilon_{\min}(G)} \)) exceed the total supply of G-rated securities. Perfect competition among ex ante identical banks and excess supply of funds for G-rated issuers then drives down the borrowing cost for
each issuer type up to the point where banks break even on each type of issuer \((\bar{r}_E = 0)\). Since banks can distinguish good issuers from bad issuers there are separate face values set for each issuer type and these face values are set such that risky banks break even on bad issuers and safe banks break even on good issuers \((N_g = 1\) and \(N_b = 1 + e_{\min}(G) \frac{1-p_H}{p_H}\)). Thus, given \(e_{\min}(G) \leq \bar{c}\), the CRA’s profit is given by

\[
\Pi_R = \left\{ \gamma \left( \pi_g (R - 1) + \left( A_{\max} - \pi_g - \lim_{\varepsilon \to 0} \varepsilon \right) p_H \left( R - 1 - e_{\min} \frac{1-p_H}{p_H} \right) \right) \right\}. \tag{34}
\]

This option is only optimal when \(A_{\max} > \pi_g\), since it is otherwise dominated by option 2 below.

**OPTION 2: Banks invest only in good assets; households don’t invest.** The CRA only rates a mass \(\min \{ A_{\max} - \lim_{\varepsilon \to 0} \varepsilon, \pi_g \}\) of good issuers \(G\). Again this implies that bank capital is in excess supply relative to fundable securities such that banks make zero profits \((\bar{r}_E = 0)\). Thus, \(N_g = 1\) and the CRA’s profit is given by

\[
\Pi_R = \gamma \min \left\{ A_{\max} - \lim_{\varepsilon \to 0} \varepsilon, \pi_g \right\} (R - 1). \tag{35}
\]

**OPTION 3: Banks invest only in good asset and households invest in good assets.** This can only be optimal when \(A_{\max} < \pi_g\) since otherwise issuer’s NPV is always higher when funding is only provided by banks. Under this option the rating agency chooses to rate all good issuers \(G\) and to rate all bad issuers \(B\). Given the parameter restriction \(A_{\max} < \pi_g\), the banking sector as a whole is constrained by regulation and cannot fund all good issuers. Thus, banking sector funds for \(G\)-rated securities are scarce. Now households can become the marginal investors in \(G\)-rated issuers. In order to ensure that households can break even on good issuers, the face value on \(G\)-rated bonds has to be set such that households can recoup their debt collection cost \(c\). Thus, the face value will be set at \(N_g = 1 + c \leq R\). Given this face value for \(G\)-rated good assets, banks can generate a return on assets equal to \(c\), and a return on equity equal to \(\bar{r}_E = \frac{c}{e_{\min}(G)}\). In this case, the CRA’s profit is given by

\[
\Pi_R = \gamma \pi_g (R - 1 - c). \tag{36}
\]

Note that option 2 dominates option 3 only when \(A_{\max} < \pi_g\) and

\[
c > \bar{c}_G \equiv (R - 1) \left( 1 - \frac{A_{\max}}{\pi_g} \right). \tag{37}
\]
OPTION 4: Banks invest only in bad assets and retails don’t invest. This is always dominated by options 1, 2, or 3.

OPTION 5: No funding at all. This is always dominated by options 1, 2, or 3.
References


