Measuring the Financial Soundness of U.S. Firms, 1926–2012*

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Abstract

In the large literature modeling and measuring the effects of financial frictions on business cycles, a key aggregate state variable is the distribution of individual firms' financial soundness. This distribution determines how financial frictions amplify and propogate business cycle shocks through their effect on the decisions of financially unsound firms. We propose a simple, transparent, and robust method for retracing quantitatively the history of the distribution of firms' financial soundness during U.S. business cycles over most of the last century for a broad cross section of firms. We highlight three main findings for this key aggregate state variable. First, the three worst recessions between 1926 and 2012 coincided with sharp deteriorations in the financial soundness of all firms, but other recessions did not. Second, fluctuations in total asset volatility, rather than fluctuations in leverage, at the firm level appear to drive most of the variation in the distribution of firms' financial soundness. These fluctuations in firms' total asset volatility are only partly due to a change in the volatility in the component of returns common across firms — the majority are in fact due to fluctuations in level of volatility that is idiosyncratic to each firm. Finally, the distribution of financial soundness for large financial firms largely resembles that for large nonfinancial firms.

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1 Introduction

A large literature in macroeconomics argues that financial frictions impair the decisions of financially unsound firms and play a key role in amplifying and propagating business cycle shocks. Papers in this literature include Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Cooley and Quadrini (2001), Cooley, Marimon, and Quadrini (2004), Jermann and Quadrini (2012), Covas and Den Haan (2011), and Kahn and Thomas (Forthcoming) and many others.[1] One key feature of models in this literature is that the distribution of financial soundness across firms in the economy at any point in time is an aggregate state variable that has important consequences for the response of the macroeconomy to a variety of aggregate shocks. In these models, heterogeneous firms choose output, employment, and investment, and, due to distorted incentives or expected bankruptcy costs, financial frictions adversely affect the decisions of financially unsound firms. As a result, negative macroeconomic shocks are greatly amplified and propagated when they simultaneously deteriorate the distribution of financial soundness across firms.

Motivated by the literature cited above, in this paper, we propose a contribution to the measurement of this key aggregate state variable, the *aggregate distribution of firms' financial soundness*. We start by developing and empirically validating a procedure for measuring the financial soundness of each publicly traded firm in the U.S. economy that we call *Distance to Insolvency*, or *DI*. This measure is theoretically grounded in a simple structural model of firms' based on the work of Leland (1994). In this model, DI measures a firm's leverage adjusted for the volatility of its underlying assets. Put differently, for each firm in the economy, DI is a measure of the adequacy of that firm's equity cushion relative to its business risk. We show theoretically that one can approximate a firm's DI using data on the inverse of the volatility of individual firms' equity returns and we show empirically that the signal of a firms' financial soundness derived from the volatility of its equity returns is highly correlated with alternative signals of firms' financial soundness studied in the literature including those derived from credit ratings, bond spreads, spreads on credit default swap rates, and bond default rates for those firms and time periods for which these alternative signals are available.

We then use data on equity volatility to approximate DI on a firm-by-firm basis monthly for all publicly traded firms in the CRSP daily database, and, in this way, we create a monthly time series for the cross-section distribution of DI across all publicly

[1] Ben Bernanke delivered a speech summarizing this literature in 2007 available at http://www.federalreserve.gov/newsevents/speech/bernanke20070615a.htm

traded firms in the U.S. economy over the 87 years from 1926-2012. This exercise allows us to make two contributions to measurement. First, we are able to construct a theoretically grounded measure of the financial soundness of a broader cross section of firms over a longer historical time period than has been done previously in the literature with alternative market signals of firms' financial soundness. In particular, we are able to directly compare the evolution of the distribution of financial soundness across firms during the Great Depression with that during the postwar period including the most recent financial crisis. Second, we can use our structural model to present a novel decomposition of the proximate drivers of movements in this cross section distribution of firms' financial soundness into movements due to changes in firms' leverage and to changes in two components of firms' business risk: a component of risk that is common to all firms and a component of risk that is idiosyncratic to each firm.

We emphasize three main empirical findings regarding this evolution of the cross-section distribution of firms' financial soundness.

Our first main finding is that over the 1926-2012 time period, there are a number of episodes in which the entire distribution of measured DI across publicly traded firms in the United States deteriorated sharply. We term these episodes *insolvency crises*. In particular, we define insolvency crises as months in which the median measured DI in that month's cross section of measured DI across firms drops to a level normally associated with extreme financial distress, and the 95th percentile of measured DI drops to a level normally associated with junk credit ratings or worse. We find that the largest recessions in our sample, namely 1932–1933, 1937, and 2008, are closely associated with insolvency crises. However, we do not find significant insolvency crises in other recessions outside of these three. This includes even the deep recessions of the late 1970s and early 1980s. These findings are not sensitive to the cutoffs used to define insolvency crises — the insolvency crises of 1932–33, 1937, and 2008 are quite distinctive events in the data. These findings are consistent with the hypothesis that a sharp deterioration in the financial soundness of most, if not all firms played a major role in three of the largest recessions in U.S. history. At the same time, these findings cast doubt on the importance of changes in the distribution of financial soundness for a broad cross section of firms for U.S. postwar recessions outside of the most recent one.[2]

To obtain our second main empirical finding we decompose the proximate drivers of changes in the distribution of measured DI across firms into movements due to changes in firms' leverage and changes to two components of firms' business risk: a component of risk

[2]See Giesecke, Longstaff, Schaefer, and Strebulaev (2011) for a related quantitative finding based on bond default rates.

that is common to all firms and a component of risk that is idiosyncratic to each firm. A significant literature points to the buildup of leverage across firms as a key precursor to the start of a financial crisis (see, for example, Kindleberger and Aliber, 2005, and Reinhart and Rogoff, 2009). Typically, in the literature cited in our first paragraph, the proximate cause of the deterioration of firms' financial soundness is a further increase in firms' leverage precipitated by a drop in the value of firms' assets or collateral. The impact of changes in firms' business risk or asset volatility on firms' financial soundness, on the other hand, has been examined more closely only recently, for example, by Christiano, Motto, and Rostagno (2010), Gilchrist, Sim, and Zakrajsek (2010), Rampini and Viswanathan (2010), Arellano, Bai, and Kehoe (2011), and others.[3] For the time period 1972–2012, we can use accounting data from COMPUSTAT in order to measure separately the leverage and asset volatility components of DI. This allows us to examine the role of changes in firms' leverage versus changes in firms' asset volatilities in accounting for the changes in the distribution of firms' measured DI over this time period, in particular during the insolvency crisis of 2008.

Contrary to many theories of financial crises cited above, we find that the deterioration of firms' measured DI during the insolvency crisis of 2008 was mainly due to an increase in asset volatility for all firms. The contribution of the increase in leverage, induced either by "excessive borrowing" in advance of the crisis or a fall in asset values during this insolvency crisis, was relatively small. In fact, over the entire period for which we have the COMPUSTAT accounting data needed to compute firms' leverage, we find that changes over time in the distribution of measured DI across firms are mainly a result of changes in the volatility of firms' underlying assets rather than of changes in firms' leverage.[4] Moreover, one of the striking features of the insolvency crises that we observe in the data is that all firms appear to be impacted, including firms that have little or no financial leverage.[5] In the most recent financial crisis in particular, we find that the deterioration in DI that occurred for firms with no long term financial debt is very similar to that which occurred for firms with long-term financial debt. Our second finding complements recent work by Bloom (2009) and others, which demonstrates large contractions in aggregate

[3]A notable earlier exception is Williamson (1987) who introduces business cycle variation in project risk into the costly state verification framework of Townsend (1979).

[4]Related findings appear in the contemporaneous work by Choi and Richardson (2013) on the decomposition of levered equity returns with stochastic volatility. They study the movement of equity volatility on a firm-by-firm basis rather than the movement of the overall distribution of equity volatility across firms. Their findings corroborate the greater relative contribution to changes in equity volatility of changes in asset volatility relative to changes in leverage, except, in their firm level data, for the most levered firms.

[5]In our structural model of firms' distance to insolvency, fixed operating costs, i.e. operating leverage, are mathematically equivalent in the theory to financial leverage.

activity from changes in asset volatility directly.[6]

The rapidly growing literature in macroeconomics that examines the role of changes in business risk as measured by changes in asset volatility in driving macroeconomic fluctuations can be divided into two parts. One part of this literature emphasizes factors such as time variation in the market price of risk[7] or in aggregate disaster risk[8] that impact the volatility of a common component of the innovations to asset values for all firms. A second part of this literature emphasizes variation in the distribution of financial soundness across firms due to time variation in the variance of the idiosyncratic or firm-specific component of innovations to asset valuations for all firms.[9] Following Campbell et al. (2001), Gilchrist, Sim, and Zakrajsek (2010), and Kelly et al. (2012), we decompose the volatility of innovations to firms' asset values into a portion of that volatility due to a common component of these innovations and a portion of the volatility due to an idiosyncratic component of these innovations. With this procedure, we are able to further decompose movements in the distribution of firms' DI into components due to changes in leverage, changes in aggregate risk (the volatility of a common component of innovations to the value of firms' assets), and changes in idiosyncratic risk (the volatility of the idiosyncratic component of innovations to the value of firms' assets for all firms). Consistent with the results in Kelly et al. (2012), we find that fluctuations in the volatility of the idiosyncratic component of innovations to firms' asset values for all firms account for most of the movement in the cross section distribution of financial soundness across firms, particularly in the most recent financial crisis. Movements in the common component of innovations to the value of firms' assets are less important as a driver of movements in the cross section distribution of firms' financial soundness over time.

For our third main empirical finding, we examine the differences in the movements of the distribution of financial soundness for large financial firms versus large non-financial firms. The macroeconomic literature cited above highlights the role of financial frictions facing *all firms* in shaping business cycles. Another large literature in macroeconomics makes the case that frictions facing *financial intermediaries* play perhaps an even larger role in shaping the evolution of the macroeconomy. According to this literature, recessions can be caused by a deterioration in the financial soundness of financial intermediaries

[6]Recent related empirical work by Jurado, Ludvigson, and Ng (2013) provides a measure of uncertainty by measuring the volatility of the unforecastable component of macroeconomic time series.

[7]See, for example, the models of intermediary asset pricing in Brunnermeier and Sannikov (2012), He and Krishnamurthy (2013), and Muir (2013), or the model of equilibrium credit spreads and consumption volatility in Gomes and Schmid (2010).

[8]See, for example, Rietz (1988), Barro (2006), Gabaix (2012), and Gourio (Forthcoming).

[9]See, for example, Christiano et al. (2010), Christiano et al. (Forthcoming), and Arellano et al. (2012).

alone, due to their central role in reallocating resources in the economy.[10] One of the main virtues of our proposed method for measuring the financial soundness of firms is that it can easily be applied to financial as well as nonfinancial firms even though the type and reporting of leverage in financial statements vary considerably across the two types of firms. We find that the financial soundness of large publicly traded financial firms closely resembles that of large nonfinancial firms for the entire period of 1962 to July 2007.[11] In particular, in advance of the most recent financial crisis, we find that the market perceived both large financial and non-financial firms to be at a historically high level of financial soundness. Empirically, then, we see little or no evidence that the many changes in financial regulation that occurred over this time period played a significant role in leading large financial institutions to take on significantly more leverage than their non-financial peers in the run-up to the most recent crisis once leverage is adjusted by the market's perceptions of each type of firms' business risk.

Taken together, our findings also suggest that measures of leverage traditionally used in macroeconomic analysis behave very differently over time from measures of financial soundness, such as ours, that adjust leverage for business risk. On the basis of these findings, we argue that in order to understand insolvency crises, one must account for changes in firms' asset volatility over and above changes in firms' leverage. Specifically, one must account for changes in the volatility of the idiosyncratic component of firms' asset volatility that simultaneously impact all firms in the economy, whether they have financial leverage or not.

We present our measurement procedure in the remainder of this paper as follows. We first offer a theoretical analysis of DI in the context of the structural credit risk model of Leland (1994). Our main result is that a firm's DI is bounded above by the inverse of its instantaneous equity volatility. Moreover, this bound is tight if the firm's creditors are alert and aggressive in quickly forcing the firm into bankruptcy when it becomes insolvent. Because this result is based only on elementary properties of the value of the firm's equity as a function of the value of the firm's assets in the model, we conjecture that it should generalize to a wider class of structural credit risk models. Based on these findings, in our empirical work we approximate a firm's DI with the inverse of its equity volatility.

Next, we validate our equity-based measure of DI by showing empirically that it is

[10] An important early paper is Bernanke (1983). Gertler and Kiyotaki (2010) surveys recent theoretical contributions, and the empirical experience with financial crises is described in Reinhart and Rogoff (2009). Additional recent work focusing on the distinct role of the intermediary sector in business cycle contractions in real activity includes Cúrdia and Woodford (2009), Brunnermeier and Sannikov (2012), Gilchrist and Zakrajšek (2012), Gertler and Karadi (2011), and Rampini and Viswanathan (2012).

[11] The number of publicly traded financial firms is very small before 1962 and thus we are not able to present meaningful results for the 1926-1961 time period.

highly correlated with leading alternative empirical measures of firms' financial soundness, whenever data are available for these alternative measures. The alternative measures that we consider include credit ratings, option-adjusted bond spreads, credit default swap rates, aggregate bond default rates, and the measure of firms' *Distance to Default* based on the structural credit risk model of Merton (1974), which is commonly used in forecasting individual firms' bond default and bankruptcy rates (see, for example, Duffie, 2011; Sun, Munves, and Hamilton, 2012). These correlations are strong and relatively stable both in the cross section at a point in time and across time, including even the turbulent financial crisis of 2008. Our findings here that equity volatility and credit spreads are closely associated corroborates finance industry pricing and hedging practices as summarized in the CreditGrades model described in Finkelstein et al. (2002), as well as the findings in Campbell and Taksler (2003), Schaefer and Strebulaev (2008), and Gilchrist, Sim, and Zakrajsek (2010). We see these results as validating our use of firms' inverse equity volatility to measure their DI for those firms and time periods for which these alternative measures of financial soundness are not available.

We use our measurement procedure to compute the cross section distribution of DI for each month from 1926 through 2012 for all publicly traded U.S. firms in the daily U.S. Stocks database maintained by the Center for Research in Security Prices (CRSP) at the University of Chicago. We find that over the entire time period, the distribution of measured DI across firms is approximately lognormal, so it can be summarized well in a low-dimensional manner by two of its moments.[12] Moreover, we find that fluctuations in the median (or mean) of this lognormal distribution account for most of the movements in the entire distribution — the fluctuations in the cross-sectional standard deviation of these lognormal distributions are relatively small. Our median statistic for the DI distribution offers a one dimensional measure of insolvency crises which can be used to complement indicator measures of financial crises such as that in Schularik and Taylor (2012).[13]

There is a large literature in macroeconomics that uses movements in indices of credit spreads in bond markets as a proxy for financial conditions impacting firms' financial soundness.[14]. We see a very close correspondence between our equity-based measure of

[12]See Kelly et al. (2012), which studies the factor structure of individual firm volatilities, and who were the first to our knowledge to establish the approximate lognormality of the distribution of firm level volatilities. In contemporaneous work, Kelly et al. (2013b) study the evolution of the cross section distribution of volatility and relate this distribution to the firm size distribution in a spatial network model.

[13]Schularik and Taylor (2012) dates financial crises based on both qualitative and quantitative data. Jordà et al. (2011), Jordà et al. (2012), and others use such measures to examine the role of financial crises in aggregate fluctuations.

[14]See, for example, Bernanke, Gertler, and Gilchrist (1999) and, more recently, Gilchrist and Zakrajšek (2012)

the distribution of financial soundness across firms and these indices of bond spreads in the period 1926 to 1941 and again from 2000 through the present. In particular, the three episodes that we identified as insolvency crises associated with large recessions are also periods with very elevated bond spreads.[15] We find, however, that the deterioration of the distribution of financial soundness across firms that occurred in our three insolvency crises impacted a broader cross section of firms than simply those that have financial debt.

The remainder of this paper is organized as follows. In section 2, we describe the theory underlying our measurement procedure. In section 3 we compare the empirical performance of our measure of firms' distance to insolvency to alternative measures of firms' financial soundness. We then turn to an analysis of the characteristics of the distribution of distance to insolvency across firms as our aggregate state variable of interest. In section 4 we present our empirical results regarding our three questions on the relationship between insolvency crises and business cycles. We conclude in section 5 with a discussion of the implications of these findings for business cycle research.

2 The Theory Underlying Our Measurement

Our empirical work has its theoretical foundations in the structural models of firms' credit risk pioneered by Merton (1974) and Leland (1994). In those models, a key state variable summarizing a firm's financial soundness is Distance to Insolvency, or DI: a measure of the firm's leverage adjusted by the volatility of innovations to the market valuation of its underlying assets. DI is a key state variable both in a statistical sense — it summarizes the probability that the firm will become insolvent in the future[16] — and in an economic sense because it summarizes the distortions to equity holders' incentives that potentially arise when the firm becomes financially distressed.[17]

[15] In related work, Gilchrist, Sim, and Zakrajsek (2010) find a close correspondence between firm-specific equity volatility and firm-specific bond spreads over the full 1963 to 2009 time period.

[16] A large literature uses a related measure of firms' leverage adjusted for asset volatility computed from the Merton model to forecast firms' bond default and bankruptcy rates in a reduced-form manner. Duffie (2011) clearly describes one way in which this procedure can be implemented. Moody's Analytics (a subsidiary of the credit rating agency) has sold the results from a related model under the brand name *Expected Default Frequency*, or EDF, for over a decade. The specification of their model and its empirical implementation are described in Sun, Munves, and Hamilton (2012).

[17] To generate real costs of financial distress, these models rely on some violation of the Modigliani and Miller theorem (Modigliani and Miller, 1958). Myers (1977) is an early contribution characterizing the cost of debt financing arising from suboptimal investment. Townsend (1979) studies optimal financing under asymmetric information and shows that debt financing minimizes monitoring costs. Diamond and He (forthcoming) shows that investment distortions due to debt overhang vary with debt maturity, and they derive the optimal debt maturity structure. Villamil (2008) presents a survey of some of the important theoretical work that derives violations of Modigliani and Miller from the underlying constraints on contracting and information. Recent work by Hackbarth et al. (2006), Almeida and Philippon (2007),

In this section we use a straightforward extension of Leland's (1994) structural model of credit risk in order to derive two approximation results that dramatically simplify measurement relative to what has been done in the academic literature and in commercial applications. We show that one can approximate a firm's DI simply with the inverse of its instantaneous equity volatility. Specifically, we show that in the Leland's model of credit risk, at any point in time, inverse equity volatility *is an upper bound on the firm's DI*. Second, if the firm's creditors are aggressive in forcing the equity holders to file for bankruptcy as soon as the firm is insolvent, then *this upper bound is tight*.[18] We argue that because these findings rely on just a few elementary properties of the value of equity, they are likely to hold in a broad class of models.

2.1 Distance to Insolvency: Definition

To define terms, we make use of the following notation. On the left-hand side of its balance sheet, the firm has assets that yield at time $t \geq 0$ a stochastic cash flow denoted by y_t. Let V_{At} be the market value of the assets' future cash flows, measured using state-contingent prices. On the right-hand side of its balance sheet, the firm has liabilities, that we model as a deterministic sequence of cash flows $\{c_t, t \geq 0\}$ that the equity holders of the firm are contractually obligated to pay if they should wish to continue as owners of the firm. Let V_{Bt} be the market value of the liabilities' future cash flows, valued as if they were default free. Of course, since the firm may default on its liabilities, V_{Bt} is larger than the market value of the firm's debt. We say that a firm is *solvent* if its underlying assets are worth more than the promised value of its liabilities, $V_{At} \geq V_{Bt}$, and *insolvent* otherwise. Let the *asset volatility*, σ_{At}, be the (instantaneous) annualized percentage standard deviation of innovations to V_{At}, representing the business risk that the firm faces. Let the *leverage* be the percentage gap between the value of the firm's underlying assets and the firm's liabilities, $\frac{V_{At}-V_{Bt}}{V_{At}}$. A firm's *Distance to Insolvency*, or DI, is defined as the ratio of our measure of leverage to our measure of asset volatility, both dated at a point in time t:

$$DI_t \equiv \left(\frac{V_{At} - V_{Bt}}{V_{At}}\right)\frac{1}{\sigma_{At}}. \tag{1}$$

Chen et al. (2009), Chen (2010), Bhamra, Kuehn, and Strebulaev (2010), and Gomes and Schmid (2010) emphasizes the time-varying nature of the costs of financial distress.

[18] Black and Cox (1976) pioneered the study of structural models of credit risk in which creditors add bond provisions to force equity to exercise their right to limited liability when the firm becomes insolvent. Longstaff and Schwartz (1995) build on the Black and Cox model to incorporate both default and interest rate risk.

This ratio corresponds to the drop in asset value that would render the firm insolvent, measured in units of the firm's asset standard deviation.

We illustrate these concepts graphically in Figure 1. The solid blue line in the figure denotes the evolution of the value of the firm's assets, V_{At}, over time. The solid blue line ends at the current time t. The solid red line denotes the value of the firm's promised liabilities V_{Bt}. The black arrow denotes the distance between V_{At} and V_{Bt} at time t. The dashed blue lines denote standard error bands around the evolution of V_{At+s} going forward at different time horizons $s > 0$. The likelihood that the firm becomes insolvent in the near term depends on both the distance between V_{At} and V_{Bt}, measured here in percentage terms by the firm's leverage, and the volatility in percentage terms of innovations to the value of the firm's assets. We combine these two factors into DI, which serves as simple one-dimensional index of the firm's financial soundness.

2.2 Distance to Insolvency: Measurement

Calculating a firm's DI is challenging in practice because it requires one to measure separately the market value and volatility of a firm's underlying assets, V_{At} and σ_{At}, and the value of its liabilities, V_{Bt}. The former are not directly observable, and the latter is subject to deficiencies and inconsistencies in accounting measures of firms' liabilities across countries, time, and industries.

One approach to this measurement problem, pioneered by Merton (1974) and Leland (1994), is to use a specific structural model of the cash flows from the firm's assets and accounting data on the firm's liabilities, together with assumptions about the interest rates and risk prices used to discount those cash flows. Equipped with such a model, one can derive formulas for the value of the firm's equity at t, denoted by V_{Et}, and the standard deviation of the innovations to the logarithm of V_{Et}, denoted by σ_{Et}, as functions of the asset value and volatility and the firm's liabilities, V_{At}, σ_A, and V_{Bt}. Given market data on the firm's equity value and equity volatility, and accounting data on the firm's liabilities, one can then invert these formulas to uncover the unobserved asset value V_{At} and asset volatility σ_{At}. Duffie (2011) clearly describes one way in which this procedure can be implemented using the Merton model.

Our theoretical results below show that one can approximate DI in a simple way using only equity volatility data.

The benchmark Leland Model. Let interest rates and the market price of risk be constant. On the left-hand side of the firm's balance sheet, the cash flows derived from the firm's underlying assets (lines of business) follow a geometric Brownian motion with constant volatility. In this case, the market value of the firm's asset, V_{At}, also follows a geometric Brownian motion with constant volatility σ_A. In particular, fluctuations in V_{At} are driven entirely by fluctuations in the firm's projected cash flows. On the right-hand side of its balance sheet, the firm has liabilities given by a perpetual constant flow of payments $c > 0$. Hence, the present value of these payments is constant and equal to $V_B = c/r$, where $r > 0$ denotes the interest rate.

Equity holders have limited liability, in that they can choose to stop making the contractual liability payments, in which case they default and assets are transferred to creditors. Creditors are protected by covenants, allowing them to force equity holders into default if the value V_{At} of the assets falls below some exogenously given threshold, which we assume is lower than V_B. Using standard arguments, one can show that, when the value of assets falls below some endogenous threshold $V_A^\star \leq V_B$, either equity holders exercise their right to default or creditors exercise their protective covenants. The value of equity can be written as $V_{Et} = w(V_{At})$, for some continuous function $w(V_A)$ with three key properties presented in the next lemma and illustrated in Figure 2.

Lemma 1. *In the Leland (1994) structural model, the value of equity, $w(V_A)$, is greater than* $\max\{0, V_A - V_B\}$*, nondecreasing, convex, and satisfies $w'(V_A) \leq 1$ as well as $w(V_A^\star) = 0$.*

The lower bound, $\max\{0, V_A - V_B\}$, follows from the limited liability assumption: the value of equity has to be greater than zero, and it also has to be greater than $V_A - V_B$, its value under unlimited liability. Moreover, in line with the original insights from Merton (1974), the value of equity inherits the standard convexity properties of call options.[19] Note in particular that $w'(V_A) \leq 1$, which follows from the fact that the option value of limited liability falls as the value of the firm's assets rises. Finally, the value of equity must be zero at the default point, $V_A^\star$.

Armed with these basic properties for the value of equity, we develop our two approximation results that relate distance to insolvency and leverage adjusted for asset volatility. We show each result in turn.

Proposition 1. *In a Leland (1994) structural model, Distance to Insolvency bounded above by the inverse of equity volatility:*

$$DI_t = \left(\frac{V_{At} - V_B}{V_{At}}\right)\frac{1}{\sigma_A} \leq \frac{1}{\sigma_E}.$$

[19]The Merton model differs from the Leland model only in the assumption that the cash flows on liabilities are simply a single cash flow required at a specific date T in the future. This lemma also applies to the value of equity in the Merton model at dates $t < T$ with the change that $V_A^\star = 0$.

Proof. To prove this result, note first that, by Ito's formula, the volatility of equity solves:

$$\sigma_{Et} = \frac{w'(V_{At})}{w(V_{At})}\sigma_A V_{At} \Longrightarrow \frac{1}{\sigma_{Et}} = \frac{w(V_{At})}{w'(V_{At})}\frac{1}{\sigma_A V_{At}}.$$

By Lemma 1 we have that $w(V_{At}) \geq V_{At} - V_{Bt}$, and $w'(V_{At}) \leq 1$, and the results follow. $\square$

Next, consider the question of whether this upper bound on a firm's DI is tight. To do this, recall that $V_A^\star$ is the threshold asset value at which equity exercises its option to default: it gives up control of the firm's assets in exchange for abandoning the firm's liabilities. We use $V_{At}^\star$ to define the concept of *Distance to Default*, or DD, in our benchmark Leland model as

$$DD_t = \left(\frac{V_{At} - V_A^\star}{V_{At}}\right)\frac{1}{\sigma_A}. \tag{2}$$

Note that default is distinct from insolvency in our theory and that quite generally a firm's DD exceeds its DI. This is because equity may not walk away immediately from an insolvent firm, but will not choose default if the firm is solvent. With this definition we have our second proposition.

Proposition 2. *In a Leland (1994) structural model, the inverse of a firm's equity volatility lies between its Distance to Insolvency and its Distance to Default:*

$$DI_t \leq \frac{1}{\sigma_{Et}} \leq DD_t.$$

Proof. This proposition follows from the convexity of the value of the firm's equity as a function of the value of the firm's assets at each time t and because $w(V_A^\star) = 0$. $\square$

We illustrate the proof of these two propositions in Figure 2. At time t, the value of the firm's equity as a function of the value of its assets is a convex function with slope less than or equal to one that lies above the horizontal axis (exceeds zero) and the line $V_{At} - V_B$ giving the value of the firm's equity under unlimited liability. The value of the firm's equity hits the horizontal axis at the default point $V_A^\star$. Define X_t to be the point at which the tangent line to the value of equity V_{Et} at the current asset value V_{At} hits the x-axis. All these lines and points are drawn in this figure.

By the convexity of $w(V_A)$, we have $V_{At}^\star \leq X_t \leq V_{Bt}$. Simple algebra then delivers that

$$\frac{1}{\sigma_{Et}} = \left(\frac{V_{At} - X_t}{V_{At}}\right)\frac{1}{\sigma_A},$$

which proves the result.

With these two results, we have that the inverse of a firm's equity volatility, $1/\sigma_E$, is an accurate measure of Distance to Insolvency if the Distance to Insolvency and the Distance to Default are close to one another. That is, the bound is tight if creditors quickly force insolvent firms into default. Therefore, as an empirical matter, the economics of creditors' incentives to force a firm that is insolvent into bankruptcy as soon as possible to avoid further costs of financial distress suggests that firms with alert and aggressive creditors should satisfy this condition.Regardless of the tightness of the bound, our procedure for measuring DI should be conservative in identifying firms' financial distress because inverse equity volatility is an upper bound on DI.

Although we have established our approximations in the context of a simple model, our results rely on just a few elementary properties of the value of equity, which are likely to hold in a broad class of models used in applied work. First, the proof requires that the value of equity be a convex function of the value of assets with slope less than one, a property that is typical of structural credit risk models. Second, the proof requires that the value of equity is the only state variable following a diffusion. Thus, our results hold if there are other state variables, for the interest rate, market price of risk, or liability payments, as long as these are "slower moving" in the sense of being continuous-time Markov chains.[20]

Based on these theoretical results, in the empirical work that follows, we approximate DI by the inverse of equity volatility, $1/\sigma_E$. We term this approximation "measured DI". We estimate $1/\sigma_E$ by the inverse of realized volatility, which we compute from the CRSP database on daily equity returns for each firm and each month from 1926 to 2012.[21,22] One may argue that, since the concept of financial soundness is fundamentally forward looking, DI should be measured using implied instead of realized volatility. An important drawback of using implied volatility, however, is that it is available only for selected stocks, and only for recent dates. Moreover, in Appendix C, we compare the distribution

[20] See for example Hackbarth, Miao, and Morellec (2006) and Chen (2010) for versions of Leland's model in which the firm's cash flow process follows, under the risk-neutral measure, a modulated geometric Brownian motion. See Chen et al. (2009) for a structural model with time-variation in the market price of risk and in the variance of the idiosyncratic component of business risk.

[21] The CRSP daily data set on equity returns includes NYSE daily data beginning December 1925, Amex (formerly AMEX) daily data beginning July 1962, NASDAQ daily data beginning December 1972, and ARCA daily data beginning March 2006. We estimate σ_E by the square root of the average squared daily returns in the month. We annualize this standard deviation by multiplying by $\sqrt{252}$ where 252 is the average number of trading days in a year.

[22] One could also compute realized volatility using a range of alternative methods including a rolling window of returns, or the latent-variable approach of stochastic volatility models. We have chosen our measure primarily to ensure that it does not use overlapping daily data and for the convenience of correspondence with the monthly calendar.

of realized versus implied volatilities, for the available data, and we show that the two track each other closely. We conclude that the benefits of using realized volatility largely outweigh the costs.

3 Distance to Insolvency and Alternative Measures of Financial Soundness

In this section we compare our measure of DI, based on equity volatility, to leading alternative measures of firms' financial soundness for those time periods in which we have data for these alternative measures.

First, we construct a mapping between the level of measured DI and Standard and Poor's credit ratings in the cross section. We use this mapping to interpret the level of measured DI in terms of these credit ratings.

We next validate our calibration of measured DI using credit ratings by comparing it to option adjusted bond spreads and credit default swap rates. Our structural model, all of these market signals are signals of firms' underlying DI and thus should be highly correlated. Specifically, we compare the median measured DI to the median option-adjusted bond spreads, and to the median credit default swap rate, every month within portfolios of firms sorted by credit ratings.[23] This comparison is informative because these market-based measures are more responsive to market conditions than are slow-moving credit ratings.

We find that there is a strong linear relationship between the logarithm of median measured DI for these portfolios and the logarithm of median option-adjusted bond spreads and the logarithm of median credit default swap rates. We find that this relationship is roughly stable both in the cross section and in the time series. This approximate stability over time of the relationship between firms' measured DI and bond spreads and CDS spreads is important because we argue that one can construct a measure of insolvency crises based on the unconditional level of an economy-wide measure of DI.

Next we show a strong monotonic relationship between DI and Black and Scholes' Distance to Default (DD) for our portfolios of firms by credit ratings. Given the large empirical literature that uses Black and Scholes' Distance to Default as an indicator

[23]By organizing firms into these portfolios, we are able to reduce the impact of sampling error in the estimation of firms' equity volatility on the empirical relationship between these alternative measures. We see this use of portfolios rather than individual firms as particularly important for our empirical purposes since we are interested in measuring the distribution of financial soundness across a broad cross section of firms rather than in measuring the financial soundness of any particular firm.

of firms' bond default and bankruptcy risk, we interpret this finding as indicating that measured DI should also be a strong indicator of firms' bond default and bankruptcy risk.

3.1 Measured DI and Credit Ratings

To interpret the level of measured DI, we first study its cross-sectional relationship with credit ratings. Specifically, we compare the inverse of firms' equity volatility to their credit ratings as reported quarterly in COMPUSTAT. We pool all firm-month observations from 1985 to the present for which we simultaneously have a credit rating from COMPUSTAT and daily stock return data from CRSP. Each month, we place firms into credit ratings bins and then compute the median measured DI for all firm-month observations by ratings bin.

In Figure 3, we plot the median of the cross-sectional distribution of firms' measured DI conditional on Standard & Poors (S&P) credit rating. The figure reveals a clear monotonic relationship between the two: highly rated firms have a higher median measured DI.[24] We emphasize four cutoffs. For highly rated firms (A and above), the median measured DI is 4. For firms at the margin between investment grade and speculative grade (BBB- vs. BB+), the median DI is 3. For firms that are vulnerable (in the B range), the median measured DI is 2, whereas for firms that have filed for bankruptcy and/or have defaulted (C or D),the median measured DI is 1.

In further support of our calibration, in Figure 4, we plot the frequency in the pooled firm-month data of firms having an investment grade rating (BBB- and above) conditional on values of measured DI.[25] The frequency of firms having an investment grade rating increases sharply with measured DI for rated firms: it is less than 15% if measured DI is below 1, and more than 80% if measured DI is above 4. For measured DI's between 1 and 2, this probability is 30%, for measured DI's between 2 and 3, it is just under 50% and finally for measured DI's between 3 and 4, it is 65%. Thus, a measured DI below 1 strongly indicates that a firm has a speculative grade rating, and a measured DI above 4 strongly indicates a firm has an investment grade rating.

Finally, we also consider the frequency in the pooled firm-month data of firms being what S&P calls "highly vulnerable" (a rating of CC and below), conditional on values of DI. For firms with measured DI less than 1, this frequency is about 10% and for firms with measured DI's between 1 and 2, it is about 1.4%. To interpret these conditional

[24] In fact, our data indicates that monotonicity holds for all percentiles. This means that, in the cross-section, a higher credit rating corresponds to a higher measured DI, in the sense of first-order stochastic dominance.

[25] It is important to note that the unconditional distribution of firms' credit ratings is biased towards higher ratings since firms select into being rated.

probabilities, note that the unconditional probability of a rating CC and below is very small, about 0.75%. Taking this into account, a firm with measured DI below 1 is thirteen times more likely to be highly vulnerable than a randomly chosen firm. A firm with a measured DI between 1 and 2 is twice more likely to be highly vulnerable.

Given these findings, we propose the following benchmark calibration to interpret the level of measured DI:

- measured DI above 4: good and safe.
- measured DI of 3: borderline between investment and speculative grade.
- measured DI of 2: vulnerable.
- measured DI below 1: highly vulnerable.

3.2 Measured DI and Bond Credit Spreads

We now consider the relationship between measured DI and credit spreads in bond yield data. Bank of America-Merrill Lynch (BAML) calculates daily data on option-adjusted bond spreads[26] for a large universe of corporate bonds whose yields underlie BAML's corporate bond indices. BAML then groups firms into portfolios by rating class, for the seven ratings classes AAA to CCC and below and reports an index of the option-adjusted spread on bonds of firms in each portfolio. These daily data on option adjusted bond spreads by ratings class are available from 1997 to 2012.[27] We compute monthly averages of daily option-adjusted spreads on these indices and, in Figure 5, we plot the logarithm of these option adjusted bond spreads against the logarithm of median measured DI for firms in the same ratings class bin in the same month. We plot separately the pre–August 2007 data (with blue triangles) and post–August 2007 data (with red circles). Note two sources of variation are shown in this figure: variation in measured DI and bond spreads across credit ratings classes at a point in time and variation over time in measured DI and bond spreads by ratings class.

Clearly, credit spreads are decreasing in measured DI, and the relationship is linear in logs. This relationship is relatively tight: the R^2's from a regression of log option-adjusted spread on log measured DI for data pre– and post–August 2007 are 0.74 and 0.79, respectively. For another way to see that measured DI is strongly indicative of credit spreads, consider that having a low measured DI and a low credit spread is very

[26]The option adjustment here is intended to correct bond spreads for features of corporate bonds, such as callability, that do not correspond to default risk and yet might impact observed bond spreads.

[27]These data are available in the data repository FRED at the Federal Reserve Bank of St. Louis.

rare. In particular, no portfolio has a measured DI below 1 and an option-adjusted spread below 400bp. Likewise, it is very rare to have a high measured DI (above 4) and a high credit spread (above 400bp). We conclude from the data on measured DI and option-adjusted spread, that measured DI captures a significant amount of the information in credit spreads, and this helps to validate DI as a measure of financial soundness.

Note as well that the linear relationship in logs between measured DI and bond spreads is relatively stable in the data pre–August 2007 and post–August 2007. We interpret this finding as indicating that during the financial crisis of 2008, both measured DI and bond spreads as indicators of financial soundness deteriorated over time in the same relative proportion as they do typically at a point in time across the spectrum of firms of different credit qualities.

3.3 Measured DI and Credit Default Swap Rates

In the past decade, a broad market in credit default swaps has emerged. Credit default swaps have a payoff, contingent upon default, that is equal to the value of the defaulted bond relative to its face value. Thus, CDS rates offer a natural market-based measure of corporate default risk which we can compare to measured DI.

We use data from Markit on single-name five-year CDS rates from 2001 through 2011. We construct monthly averages of daily swap rates by firm. We then merge this CDS swap rate data with our monthly DI data from CRSP by CUSIP using Markit's Reference Entity Dataset, then hand and machine-check the results of our merge. Finally, we bin firms by ratings class into seven ratings classes from AAA to CCC and below to reduce noise, and we compute the median CDS rate and median measured DI monthly by rating class.

Figure 6 plots measured DI versus CDS rates by ratings class for 2001–2011. We use a log scale because the relationship between measured DI and CDS rates is, like the relationship between measured DI and option-adjusted bond spreads, close to log linear. The plot shows a clear negative relationship between CDS rates and measured DI. It also adds further credibility to our calibration, since relatively few observations with a measured DI less than 2, and very few observations with a measured DI less than 1, correspond to a CDS rate below 400bp. Conversely, very few observations have a measured DI greater than 4 and a CDS rate above 200bp. We separate the data pre–August 2007 and post–August 2007 to support the claim that our calibration in levels that are constant over time. The R^2 from a regression of log CDS rate on log measured DI is 0.76 for the data pooled by credit rating pre–2007 and 0.67 post–2007.[28] Although the

[28]The same regression using firm-level data for the whole sample yields similar coefficients and an R^2 of over 30%.

slope and intercept coefficients differ across these two samples, our calibration appears robust, since in both time periods a measured DI below 1 corresponds to a CDS rate of 400bp, and a DI above 4 corresponds to a CDS rate below 200bp.

3.4 Measured DI and Bankruptcy

We now consider the relationship between DI and bankruptcy measures. We first compare our measure of DI to *Black and Scholes' Distance to Default* (DD). A large empirical literature in corporate finance examines the performance of DD as an indicator of the likelihood that a firm will declare bankruptcy and/or default on a bond. Duffie et al. (2009) and Duffie et al. (2007) document the economic importance of distance to default in determining default intensities.[29] Duffie (2011) is an important recent survey of such work. Moody's Analytics produces and sells estimates of the likelihood that publicly traded firms will default on their bonds using a similar methodology (Sun, Munves, and Hamilton, 2012). Campbell et al. (2008) provide further evidence on the role of equity volatility in forecasting firms' financial distress in a reduced form framework.

To calculate DD, we use data on a firm's equity value and volatility, together with accounting data on the firm's liabilities, and we follow a procedure outlined in Duffie (2011) based on Black and Scholes' option pricing formula. See Appendix B for details.[30] In Figure 7 we show a scatter plot of our computed DD against measured DI, monthly from December 1985 to December 2012, for the seven rating classes AAA to CCC and below. While the scale obviously differs, since our DI is measured in levels and DD is measured in logs, the figure shows a clear monotonic relationship between the two. Note, however, that since this relationship is non-linear, one would have to adjust the specification of the bankruptcy or default prediction regressions run in the literature cited above to perform a comparison of the accuracy of our measure of DI in default prediction.

To confirm that measured DI has implications for real outcomes, we also briefly document the relationship between measured DI and bankruptcy.[31]For our purposes, we establish two facts. First, in the cross section, we show that DI decreases monotonically

[29] Duffie et al. (2007) report that a 10% reduction in distance to default causes an estimated 11.3% increase in default intensity, and that distance to default is the most economically important determinant of the term structure of default probabilities.

[30] Note that this Distance to Default, which is the one commonly used in the literature, is measured in "log" units. In equation (2) we defined a related measure in levels to make it directly comparable to DI.

[31] See Bharath and Shumway (2008) for a systematic empirical comparison of structural and nonstructural models of default prediction. There is a large debate in the default prediction literature on the relative benefits of structural vs. non-structural approaches. The recent evidence in Schaefer and Strebulaev (2008) suggests that structural models capture credit risk well, and that any remaining mispricing is likely due to non-credit factors such as bond market illiquidity.

as firms become closer and closer to bankruptcy. This corroborates our calibration. Then, in the time series, we show that the fraction of firms with low measured DI is strongly correlated with the aggregate default rate. Thus, even though measured DI is based on market prices, and is thus driven by both fundamental risk and potentially time-varying risk premia, measured DI and actual default events are related both in the cross section and in the time series.

We first examine the evolution of measured DI as a firm progresses toward bankruptcy. To do so, we merge the data on bankruptcy filings by publicly traded firms collected by Chava and Jarrow (2004) with that in the UCLA-LoPucki bankruptcy database. In Figure 8, we show the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the distribution of the distance to insolvency for those firms that end up filing for bankruptcy in the 36 months prior to filing for bankruptcy or being delisted. As one can see, these percentiles decline monotonically as bankruptcy approaches. A year prior to bankruptcy, 90% of these firms have a measured DI that is below the cutoff of 3 for investment grade, and 50% are near the cutoff of 1 for being highly vulnerable. At bankruptcy, all firms have a measured DI below 2, associated with being vulnerable, and nearly all firms have a measured DI below 1, associated with being highly vulnerable.

Next, we consider the relationship between the distribution of measured DI and aggregate default rates. We use Exhibit 30 in Moody's (2012), which documents annual issuer-weighted corporate default rates for all rated corporations. For comparability, we construct an annual series of measured DI by computing firm-level volatilities over an annual window. In Figure 9, we plot the fraction of firms with measured DI less than 1, against Moody's aggregate default rate series. The figure reveals that the two series are highly correlated, with a correlation of 0.82. Even if we use the fraction of firms with annual measured DI less than 2, the correlation of this fraction with Moody's annual default rates is 0.72. Thus, we conclude that the fraction of firms with low measured DI is highly correlated with realized annual default rates.

4 Financial Soundness, 1926–2012

We now use our measure of DI to retrace the history of U.S. firms' financial soundness, from 1926 to the present. Our interest is to characterize the evolution of the cross-sectional distribution of measured DI across firms at a monthly frequency over this time period. We first show that this distribution is approximately lognormal each month from 1926 to 2012 and hence can be characterized by two if its moments. We next show that most of the movements in this distribution are accounted for by changes in the cross-sectional mean (and median) of log measured DI rather than by changes in its cross-sectional dispersion. We then define episodes that we term *insolvency crises* in terms of movements in the distribution of distance to insolvency across firms and examine our three empirical questions regarding the relationship between insolvency crises and business cycles over this long historical time period.

4.1 A Lognormal Approximation

Figure 10 displays the cross-sectional distribution of measured DI across firms over the 1926–2012 period, by plotting the time series of the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentile cutoffs for firms' measured DI. To analyze this distribution in a simple way, we first argue that the log of measured DI is approximately normally distributed. Figure 10 suggests this, since it shows that the distribution of measured DI appears to fan out at high levels: the higher percentiles cutoffs are further apart than the lower ones. It is intuitive that taking logs would make the percentile cutoffs more evenly distributed.

More formally, consider the following empirical diagnostic for a lognormal distribution, in the spirit of the Kolmogorov Smirnov specification test. If the cross-sectional distribution of DI were truly lognormal, with the estimated cross-sectional mean and standard deviation in each month, then the transformed variable

$$N\left(\frac{\log\left(DI_t\right) - \text{mean}_t}{\text{standard deviation}_t}\right) \tag{3}$$

should be uniform, where $N(\cdot)$ is the cumulative distribution of a standard normal distribution. Figure 11 plots the 5th, 10th, 25th, 50th, 75th, 90th, and 95th empirical percentile cutoffs of this transformed variable over time, for each month 1926–2012. If measured DI were truly lognormal in the cross section, then the transformed variable should be uniform and its percentiles should be exactly equal to 0.5, 0.25, 0.5, 0.75, 0.90, and 0.95 in each month. One sees in the figure that this is approximately true: the empirical percentile cutoffs do not deviate much from these values. We thus conclude that measured DI is approximately lognormal in the cross section each month. This is convenient, since we can then approximately characterize the entire distribution each month with its mean or median and standard deviation.

Figure 12 plots the time series of the cross-sectional mean and standard deviation of log measured DI. One sees that there are some fluctuations in the standard deviation over time, notably as stocks from new exchanges are added to CRSP in July 1962, December 1972, and March 2006. However, these fluctuations are much smaller than those of the mean. This suggests that time variations of the cross-sectional mean of log measured DI account for most of the time variations of the entire cross-sectional distribution. To illustrate this, consider Figure 13, which shows the true 95th percentile of log measured DI (in red) versus an approximate 95th percentile, calculated assuming that the log of measured DI is normally distributed with a *constant* standard deviation, equal to the 1926–2012 average. The 95th percentile of a lognormal distribution is equal to $mean_t + N^{-1}(0.95) \times standard\ deviation_t$, so in principle it could be quite sensitive to fluctuations in standard deviation. One sees, however, that, empirically, most variations of the 95th percentile are accounted for by variations of the mean.

4.2 Insolvency Crises and Recessions

From now on we study time variation in one moment of the measured DI distribution: the median. We use our calibration to focus on one particular cutoff for median measured DI and define a *deep* insolvency crisis as one in which the median measured DI falls below 1. In deep crises, half of publicly traded firms have a measured DI associated with a highly vulnerable credit rating.

Although we point to particular dates as insolvency crises based on this calibrated cutoff, the entire time series of the median measured DI can also be used as a continuous measure of the financial soundness of all U.S. firms. This is an advantage over discrete, or "indicator" measures. The median is also a useful summary statistic for the entire distribution. Indeed, recall that log measured DI is approximately lognormal, so the log of median is approximately equal to the mean of log. Moreover, we established that the cross-sectional standard deviation fluctuates much less than the mean over time. Thus, in a deep insolvency crisis, the large negative shift in median measured DI is associated with approximately parallel negative shifts, in a log scale, of all other percentiles of measured DI.

To assess the size of these negative shifts, consider the following calculation. In a deep crisis, log measured DI is approximately normally distributed, with a cross-sectional mean equal to zero, the log of the median, and a standard deviation that is roughly equal to 0.61 (1.8 in levels), its historical average. Hence, during a deep crisis about $N\left(\frac{\log 3}{0.61}\right) \simeq 96\%$ of publicly traded firms have a measured DI below 3 associated with speculative grade rating, and about $N\left(\frac{\log 2}{0.61}\right) \simeq 87\%$ of publicly traded firms have a measured DI below 2 associated with a vulnerable credit rating. Hence, deep financial crises are also broad.

Figure 14 plots the median measured DI over time against a log scale and shows the *deep insolvency crises* that occurred in October of 1929 and the Great Depression, the fall of 1937, and the fall of 2008. During these times, 50% of firms became highly vulnerable, with a measured DI below 1. Thus, in relation to business cycles over this time period, the worst recessions (the Great Depression and the Great Recession) coincide with deep insolvency crises. One can also see in Figure 13 that, in the 1932–1933 and 2008 insolvency crises, 95% of firms had a measured DI below 2, well below the cutoff of 3 for investment grade. Thus, these crises are even broader than suggested by the lognormal approximation with constant cross-sectional standard deviation.

This finding that the recessions of 1932–1933, 1937, and 2008 are distinctive in being associated with insolvency crises is not particularly sensitive to the thresholds used to define insolvency crises. In the other recessions marked in Figure 14, median measured DI fell a bit below 2 in the recessions of the early and mid 1970's and in the 2001 recession.

In this paper so far, we measure the cross section distribution of the financial soundness of firms using market signals gathered from the public equity markets. There is also a large literature that looks to indices of corporate bond spreads to measure the financial condition of firms. Conceptually, within the context of a structural credit risk model such as the Leland model, a firm's equity volatility and bond spreads should both be signals of that firm's unobserved underlying distance to insolvency, and hence, we should see that indices of bond spreads rises markedly in the same insolvency crises that we identify with our equity-based measure of the cross section distribution of firms' distance to insolvency.

We find this to be the case. We have already shown the close correspondence of measured DI and bond spreads for the period 1997-2012 using indices of option-adjusted bond spreads con structured by the Bank of America-Merrill Lynch. We now consider the correlation between median measured DI and two other indices of bond spreads that cover a longer time period. One index of bond spreads that we consider is the index of option-adjusted spreads constructed by Gilchrist and Zakrajšek (2012) (GZ spread) covering the period 1973-2010.[32] The other index that we consider is the spread between the index of yields on Baa corporate bonds constructed by Moody's and the yield on long-term government debt covering the period 1926-2012.[33]

Given the linear relationship between the log of corporate bond spreads and the log of inverse equity volatility observed in Figure 5, we compare the level of these spreads to the level of equity volatility for the median firm in the cross section each month over the 1926 through 2012 time period.

[32] The data for this GZ spread is available on the AEA website.

[33] We collect the data from the FRED website at the St. Louis Federal Reserve Bank. We take the difference between the Moody's Seasoned Baa Corporate Bank yield monthly 1926-2012 and the 10 year constant maturity Treasury rate monthly 1962-2012 or the yield on Long-Term U.S. Government Securities monthly 1926-1961.

We show this comparison between our equity-based measure of the distribution of financial soundness across firms and these indices of corporate bond spreads in Figure 15. The Moody's Baa bond spread and the GZ spread are both plotted using the scale on the left vertical axis (in percentage points). The annualized equity volatility of the median firm each month is plotted using the scale on the right vertical axis (again in percentage points).

In this figure, we see a close correspondence between our equity based measure of the distribution of financial soundness across firms and these indices of bond spreads in the period 1926 to 1941 and again from 2000 through at least 2010. In particular, the three episodes that we identified as insolvency crises associated with large recessions (episodes in which the equity volatility of the median firm exceeds 100%) are also periods with very elevated bond spreads. We see these data as confirming our main finding that there are three large insolvency crises over the period 1926-2012 associated with three large recessions.

It is noteworthy that there are several episodes in which equity volatility spikes upward markedly but bond spreads do not. These episodes include October 1929, September 1946, October 1987, and October 1998. These episodes are also distinguished by the fact that these spikes in equity volatility do not correspond to recessions. One potential interpretation of these episodes worth pursuing in future research is that our equity-based measure of firms' financial soundness based on realized volatility can spike up due to a few large movements in stock prices over a few days in times in which market participants do not perceive that this elevated volatility will persist and hence they do not price it into bond spreads. In principle, in the context of a Leland-type structural model, longer-term bond spreads should reflect market participants' perceptions of firms' asset volatility over a longer-term horizon. Under this interpretation, our equity-based measure when computed using realized volatility over a one-month horizon may be subject to important measurement errors if movements in realized volatility are very transient.[34]

The correspondence between equity volatility and bond spreads is much less tight in the period 1941 to 2000. Both bond spreads and median equity volatility are low during this time period (in a long historical perspective). Part of the difficulty here may be a problem of composition — the universe of firms issuing bonds included in these bond spread indices during this time period may be quite different than the universe of firms with publicly traded equity. As evidence in favor of the hypothesis that composition

[34]There is a considerable literature that looks to model the dynamics of equity volatility econometrically. We have chosen not to so here because we do not wish to impose a parametric structure on the dynamics of equity volatility at this point in our analysis.

problems are impacting the bond data, consider the substantial differences between the Moody's Baa spread and the GZ spread over the period 1973 to 2001 that then largely disappear after 2001 when the bond market became broader.

We see two reasons that our use of equity data to measure the distribution of financial soundness across firms adds to measurement over and above what can be done with bond spreads. First, by using equity data, we can measure changes in the financial soundness of firms that do not have long term debt (bonds or bank debt) during insolvency crises (which are also periods of elevated bond spreads). Data on such firms, which we discuss below, helps shed light on the question of whether it is frictions in credit markets themselves that are leading to these signals of financial distress among firms or issues centered on the financial soundness of the firms themselves. Second, using our structural model, we can decompose the observed movements in equity volatility (and correlated bond spreads) into components due to changes in leverage, changes in a common or aggregate component of business risk, and changes in the variance of firm-specific or idiosyncratic business risk. We examine these questions in the next section.

4.3 Leverage versus Asset Volatility

Given the definition of leverage adjusted for asset volatility in our simple structural model and the relationship of this concept to DI, an insolvency crisis can occur for two reasons: one due to an increase in leverage (a drop in the equity cushion, $\frac{V_{At}-V_{Bt}}{V_{At}}$) and the other due to an increase in asset volatility (an increase in business risk, σ_{At}). In this section, we decompose DI into its leverage and asset volatility components to study the contribution of each to the level of DI over time. We provide evidence that the contribution of changes in asset volatility to movements in the distribution of DI across firms is substantially more important than the contribution of leverage. We also document that the changes in equity volatility that we observe are mainly due to changes in the volatility of idiosyncratic innovations to firms' equity values rather than changes in common or aggregate innovations to equity values.

To decompose movements in the distribution of DI across firms into components due to changes in leverage and changes in asset volatility, we take advantage of our previous finding that most of the movements in the cross section distribution of DI across firms can be summarized by movements in the mean of this distribution. If we use the mean of the distribution of the log of DI across firms as a summary statistic for the position of the entire cross section distribution, then we can decompose these movements in the mean of the log of DI into movements in the mean of the log of leverage and movements in the mean of the log of asset volatility.[35]

We begin with a simple benchmark, assuming unlimited liability, and then compare asset volatility under this assumption to asset volatility using Black and Scholes' model to compute the value of the option to default.[36] Under unlimited liability, $V_{At} = V_{Et} + V_{Bt}$, and thus we can decompose log DI into leverage and asset volatility simply using

$$\log\left(\frac{1}{\sigma_{Et}}\right) = \log\left(\frac{V_{At} - V_{Bt}}{V_{At}}\right) + \log\left(\frac{1}{\sigma_{At}}\right). \tag{4}$$

Specifically, in this decomposition, the mean of log DI, which we have argued is a good proxy for the position of the entire cross-sectional distribution of DI, is equal to the mean of log leverage and the mean of the log of firms' inverse asset volatility.

We use quarterly COMPUSTAT data on total liabilities as an estimate of V_{Bt}, and we use daily equity values at the end of the quarter to compute V_{Et}. Note that although our estimate of V_{Bt} based on book values might be slow moving, our estimate of leverage moves on a daily basis due to fluctuations in V_{Et}.Figure 16 plots the mean log distance to insolvency and asset volatility terms in equation (4), for the 1972–2012 time period. Clearly, most of the changes in the level of DI are due to changes in the level of asset volatility, especially in more recent data.

To show more clearly the relative contribution of leverage and asset volatility, Figure 17 calculates a counterfactual time series by shifting the median $\log(1/\sigma_E)$ up by a constant, so that it has the same historical mean as the median $\log(1/\sigma_A)$, thus obtaining a "constant leverage" measure of $\log(1/\sigma_E)$. The figure strongly suggests that most of the variation in DI is accounted for by variation in asset volatility. Of particular interest is the role of leverage versus asset volatility in the insolvency crisis of 2008. Figure 17 shows that this crisis was almost entirely due to an increase in asset volatility. This is in contrast to common narratives in the financial press and academic literature, which emphasize the role of an increase in leverage due to a fall in asset values in driving the deterioration in financial soundness in 2008.

One may wonder whether these decomposition results are biased by our assumption of unlimited liability. To address this concern, Figure 18 plots the mean of the log of option-adjusted asset volatility computed using Black and Scholes' model and the mean of the log of asset volatility computed using the unlimited liability model. As is clear, the option adjustment does not substantially alter the result.

As a second piece of evidence of the major contributing role of asset volatility over

[35] This result greatly simplifies our analysis relative to the analysis in Choi and Richardson (2013) which focuses on modeling the dynamics of equity volatility for individual firms.

[36] See Appendix B for details.

leverage in determining firms' financial soundness in 2008, we use the decomposition under unlimited liability to compare the percentiles of the cross-sectional distribution of DI in October 2008 with the cross-sectional distribution of DI in October 2008 that would have occurred if leverage for each firm had remained at its level from October 2007 and only asset volatility had risen to its level in October 2008.

These percentiles are shown in Figure 19. The first column of colored bars shows the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the cross-sectional distribution of DI in October 2007. The second column of colored bars shows the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the cross-sectional distribution of DI in October 2008. The third column of colored bars shows the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles of the cross-sectional distribution of DI computed firm-by-firm using that firm's leverage in October 2007 and its asset volatility in October 2008. As is clear in the figure, the percentiles of this counterfactual cross-sectional distribution shown in the third column are quite similar to those found for the actual distribution in October 2008 (shown in the second column) and quite different from those found for the cross-sectional distribution in October 2007 (shown in the first column). Thus, this cross-sectional decomposition provides further evidence that the collapse in the distribution of DI in the fall of 2008 is, in an accounting sense, primarily due to an increase in asset volatility rather than an increase in leverage.

As a final piece of evidence on the relative importance of changes in firms' leverage versus changes in their asset volatility in accounting for movements in firms' DI we consider the evolution over the 1972-2012 time period of median measured DI for firms that have no long-term debt versus firms that do have long-term debt. We use data from Compustat to divide firms into these two categories.[37] Many of those firms that have no long term financial debt are in the pharmaceutical or computer industries.

In Figure 20, we show the median of DI (on a log scale) for those firms with no long term debt (in blue) and those firms with long term debt (in red). As is apparent in the figure, the collapse of median measured DI in the recent financial crisis was quite similar for these two groups of firms. Clearly, the level of firms' financial debt was not critical in determining the extent to which their measured DI declined in this time period. We also see that firms with no debt had substantially lower DI in the period surrounding the boom and bust of the tech industry in the late 1990's and early 2000's as would be expected given the concentration of firms with no long term debt in the computer industry.

[37] Our data here are quarterly. The number of firms in CRSP/Compustat with no long term debt starts at roughly 150 in 1972, rises to nearly 1400 in the late 1990's and falls to between 800 and 900 over the past decade. In contrast, the number of firms with long-term debt is roughly 1700 in 1972, rises to nearly 5700 in the late 1990's, and falls to roughly 2700 at the end of the sample.

We now turn to an examination of the relative magnitude of changes in the volatility of the common, or aggregate component, of firms' equity returns versus changes in the median of the volatility of the idiosyncratic component in firms' equity returns in accounting for movements in the median of the distribution of firms' DI as measured by the (inverse of) the total volatility of their equity returns.[38]

To do this decomposition, first, every month, we regress the daily returns on each stock in CRSP on the daily returns for the equally weighted portfolio and compute the volatility of the idiosyncratic component of that firm's equity returns as the volatility over the month of the daily residuals from this regression. This regression has the form

$$r_{itk} = \alpha_{it} + \beta_{it} r_{EWtk} + \epsilon_{itk}$$

where the subscript i denotes a firm and k denotes the day within the month t, the subscript EW denotes the equally weighted portfolio in CRSP, and the coefficients α_{it} and β_{it} are estimated separately for every firm i and every month t. We let σ_{iTt} denote our measure of the volatility in month t of the daily total returns r_{itk} for firm i in month t and let σ_{iIt} denote our measure of the volatility in month t of the daily residuals ϵ_{itk} for firm i in month t.

To illustrate the importance of movements in the distribution of the volatility of the idiosyncratic component of firm's equity returns σ_{iIt} in accounting for movements in median DI measure as the median of the inverse of the total volatility of firms' equity returns $1/\sigma_{iTt}$, in Figure 21, we plot median DI computed using the total volatility of firms' equity returns (in blue) and the median of a counterfactual measure of firms' DI computed using only the volatility of the idiosyncratic component of firms' equity returns $1/\sigma_{iIt}$. This counterfactual measure of median DI is computed as if the volatility of the common or aggregate component of firms' equity returns was always set equal to zero for all firms. As can be seen clearly in the figure, this counterfactual measure of median DI using $1/\sigma_{iIt}$ is quite close to our actual measure of median DI over the entire 1926-2012 time period. We interpret this finding as implying that movements in the volatility of the idiosyncratic component of firms' equity returns accounts for the bulk of the movements in the distribution of DI across firms over this entire time period.

[38]The decomposition of volatility into an aggregate, industry, and firm-specific component is the focus of Campbell, Lettau, Malkiel, and Xu (2001). Their Figures 2-4 graph this decompostion and show the importance of firm-specific volatility in overall firm volatility. They also show a substantial correlation between the three volatility series. Kelly et al. (2012) is a more recent examination of the evolution of the cross section distribution of firms total equity volatility as well as the evolution of the cross section distribution of the volatilities of the idiosyncratic component of firms' equity returns. They examine several proxies for the aggregate component of firms' equity returns including Fama-French factors and a principal components decomposition of returns. We present a simplified version of their procedure.

4.4 Financial versus Nonfinancial Firms

A large literature in macroeconomics and finance argues that, when financial intermediaries are financially unsound, they amplify and propagate negative shocks to the real economy. In fact, a commonly held view is that the weak financial soundness of financial intermediaries was the root cause of the large recessions of 1932–1933, 1937, and 2008. A growing literature also argues that changes in regulation and/or the introduction of new financial products changed the risk-taking behavior of financial institutions. Given the prominent role that large financial institutions played in the recent crisis, we focus on the largest financial firms and compare them to their large non-financial peers.

To shed light on the relative financial soundness of large financial versus large nonfinancial firms over time, we compare the distribution of measured DI over time for large financial and large nonfinancial firms.[39] An advantage of our measure is that we do not require accounting or market value information for liabilities, which are hard to measure properly for financial firms. Like any market-based measure, however, our measure of DI based on equity volatilities is influenced by the presence (implicit or explicit) of government subsidies.[40] We also acknowledge that the use of market-based signals for regulation is subject to the usual caveats regarding adverse feedback loops between agents' actions and market prices.[41]

We begin by classifying financial firms as those firms in CRSP with an Standard Industrial Classification (SIC) code in the range of 6000–6999, and comparing the median measured DI for financial firms and non-financial firms. We measure the DI of these financial firms in exactly the same way as we do for all firms.

Figure 22 plots the median measured DI for the 50 largest financial and nonfinancial firms by market capitalization from 1962 to 2012, the period for which there are enough large firms of each type. The main message from this graph is that the median financial soundness of large financial and large nonfinancial firms was quite similar over this time period. From the evidence in Figure 22, it seems hard to argue that changes in bank regulation or financial innovations over the 1962-2007 time period led large financial firms to add leverage relative to their business risk in a manner different from their large nonfinancial peers.

One challenge in interpreting this graph is that many firms with SIC codes from 6000

[39] See Giammarino, Schwartz, and Zechner (1989) for an early contribution using a structural model of default to consider market-implied valuations of bank assets and the value of deposit insurance.

[40] Kelly, Lustig, and Van Nieuwerburgh (2013a) and Lustig and Gandhi (forthcoming) present evidence that goverment subsidies are evident in bank stock returns and option prices.

[41] See Bond, Goldstein, and Prescott (2010), which provides an equilibrium analysis of the use of market signals in regulation.

to 6999 are not banks, or at least the type of financial firms frequently discussed in the context of the most recent financial crisis (firms in these SIC codes include health insurers, property firms such as Public Storage, and, most recently, ETFs.) We address this challenge next by studying a set of firms that we call *government-backed large financial institutions*, or GBLFIs. This set of institutions comprises the 18 bank holding companies that currently participate in the Federal Reserve's annual stress tests and eight large financial institutions that failed during the crisis (AIG, Bear Stearns, Fannie Mae, Freddie Mac, Lehman, Merrill Lynch, Wachovia, and Washington Mutual). The full list of GBLFIs together with the dates for which data on their equity returns are available, is provided in Table 1.

Figure 23 plots the median measured DI for the GBLFIs and the 50 largest nonfinancial firms by market capitalization from 1962 to 2012. Again, there does not seem to be evidence in market prices of increased risk taking by the GBLFIs relative to non-financial firms over the period July 1962–July 2007. Thus, again it seems hard to argue that changes in bank regulation or financial innovations over the 1962-2007 time period led these large financial firms to add leverage relative to their business risk in a manner different from their large nonfinancial peers. However, it does appear that the distance to insolvency for the GBLFIs deteriorated relative to their nonfinancial peers starting in August of 2007 and fell to an extremely low level in the depth of the crisis from October 2008 to March 2009. Thus it appears that these firms suffered a sharper deterioration in the financial soundness in the crisis than did their large non-financial peers. Moreover, these firms have been slower to recover their DI since that time. Unfortunately, we cannot conduct a similar analysis for the Great Depression because the largest banks at that time were not traded on exchanges and thus are not in the CRSP database.

One goal of financial regulation is to identify relatively weak financial institutions in the cross section either before a crisis begins or during the crisis. We are skeptical that regulators can achieve this goal because we find that most of the movements of measured DI, even for the GBLFIs are systemic in nature — measured DI for all of these institutions moves closely together. Figure 24 plots the 90th, 50th, and 10th percentiles of the distribution of measured DI for the GBLFIs. The figure presents clear evidence that the cross-sectional variation in measured DI for these GBLFIs in any given month is quite small relative to the movement in the distribution of DI over time: during this time period, the risk that any one GBLFI is unsound compared with the others is small relative to the risk that the whole group of GBLFIs becomes unsound together. This pattern is particularly apparent in the fall of 2011: these figures indicate that the whole group of GBLFIs was nearly as unsound at that time as the group was in early 2008 or mid-2009.

5 Conclusion

This paper is intended as a contribution to measurement: we propose a simple and transparent method for measuring the financial soundness of firms that can be broadly applied to all publicly traded firms in the economy.

We identify three recessions in which a macroeconomic downturn coincides with or follows shortly after a substantial insolvency crisis: 1932–1933, 1937, and 2008. We find that the other recessions in this time period are not associated with significant deteriorations or insolvency crises. Of course, since our findings uncover only a correlation (or lack thereof) between insolvency crises and recession, they do not establish causation. We do, however, see our findings as consistent with the hypothesis that financial frictions may have played a significant role in the recessions of 1932–1933, 1937, and 2008, and that financial frictions (as envisioned by current theories) did not play a significant role in other recessions during this time period. We hope that our research will provoke more detailed studies of the differences between these three recessions and other recessions to see if a stronger empirical and theoretical basis for causal links between financial frictions and the evolution of the macroeconomy can be developed.

A decomposition of our distance to insolvency measure into its leverage and asset volatility components attributes almost all of the 2008 insolvency crisis to an increase in asset volatility, or business risk. Moreover, we find that the majority of the movement in the distribution of volatilities across firms is due to movement in the distribution of the volatility of the idiosyncratic component of fluctuations in firms' value. Distortions to managerial and equity holder decisions occur when the likelihood of insolvency is high for either reason. Thus, considering only the effects of leverage on agency costs may leave out quantitatively important variation due to time-varying asset volatility. We see this question as being of first-order importance for future research in order to understand the sources of these large changes in asset volatility.

We also find little or no evidence that the evolution of financial soundness across large financial firms differs meaningfully from that of large non-financial firms in the period 1962-2007. This finding casts doubt on the hypothesis that the market perceived that changes in financial regulation and financial innovation played an important role in shaping large financial firms' leverage and risk taking in advance of the most recent crisis relative to the choices made by their large non-financial peers.

Finally, we find it distressing that government-backed large financial institutions continued to appear weak in terms of their financial soundness long after the summer of 2007, in spite of the heightened regulatory scrutiny they received following the 2008 financial crisis. Why these firms continued to look financially weak relative to their peers for so long is an open question that calls for further research.

A Leland (1994) structural model

Under the true "physical" measure, the value of the firm's assets, V_A, follows a geometric Brownian motion with drift μ_A and volatility σ_A. The firm pays a dividend δV_A per period. Under the risk-neutral measure, the value of the firm's assets follows

$$dV_{At} = (r - \delta)V_{At}\, dt + \sigma_A V_{At} dB_t^Q.$$

The intuition for the risk-neutral drift of $r - \delta$ is simply that, under the risk-neutral measure, the expected return from buying the assets at V_{At}, selling at V_{At+dt}, and receiving the dividend flow $\delta V_{At}\, dt$ should be equal to rdt. Assume that the equity holders have to pay c (per unit of time) to the debt holders until either (i) equity holders choose to default or (ii) creditors exercise their right to force equity holders to default, when the value of the assets reach a protective covenant threshold V_A^P. Let τ_P be the first time the asset value falls below the protective covenant threshold, V_A^P. The problem of the equity holder is to choose a stopping time τ in order to solve

$$w(V_A) = \sup_\tau \mathbb{E}^Q \left[\int_0^{\tau \wedge \tau_P} (\delta V_{At} - c)\, e^{-rt}\, dt \right].$$

Consider equity holders starting with two different initial levels of assets, $V_{A0} < V'_{A0}$. Clearly, the equity holders starting with V'_{A0} can always mimic the policy of equity holders and creditors starting at V_{A0} and would earn a higher payoff, implying that $w(V_A)$ is non-decreasing. This also shows that an optimal policy is of the threshold form: there is a $V_A^\star$ such that when $V_A \leq V_A^\star$, equity holders default, or are forced into default by creditors, and continue operating the firm otherwise. Thus, the Bellman equation for the value of equity is

$$V_A \leq V_A^* : w(V_A) = 0$$
$$V_A \geq V_A^* : rw(V_A) = -c + \delta V_A + w'(V_A)(r - \delta)V_A + w''(V_A)\frac{\sigma_A^2}{2}V_A^2.$$

A particular solution to the second-order ordinary differential equation (ODE) is $V_A - V_B$, where $V_B = c/r$. The general solution of the corresponding homogeneous ODE is of the form $K_1 V_A^\phi + K_2 V_A^{-\theta}$, where K_1 and K_2 are constant, and ϕ and θ are the positive roots of

$$\phi^2\frac{\sigma_A^2}{2}+\phi\left(r-\delta-\frac{\sigma_A^2}{2}\right)-r=0 \tag{5}$$

$$\theta^2\frac{\sigma_A^2}{2}-\theta\left(r-\delta-\frac{\sigma_A^2}{2}\right)-r=0. \tag{6}$$

When $V_A \to \infty$, the value of equity has to asymptote to $V_A - V_B$, implying that $K_1 = 0$. The constant K_2 is found by value matching $w(V_A^*) = 0$, which delivers

$$K_2 = f(V_A^\star) \text{ where } f(x) \equiv -\left(x - V_B\right) x^\theta.$$

The optimal threshold maximizes $f(x)$ subject to $x \geq V_A^P$. Differentiating $f(x)$ with respect to x reveals that it is hump shaped and reaches a unique maximum at $\frac{\theta}{1+\theta}V_B$. Therefore, the optimal threshold is

$$V_A^\star = \max\left\{V_A^P, \frac{\theta}{1+\theta}V_B\right\} \text{ and } w(V_A) = V_A - V_B - (V_A^\star - V_B)\left(\frac{V_A}{V_A^\star}\right)^{-\theta}.$$

Convexity follows because $V_A^\star \leq V_B$ by our assumption that $V_A^P \leq V_B$. Simple calculation shows that $w'(V_A^*) \geq 0$ and that $w'(\infty) = 1$, implying that $w(V_A)$ is nondecreasing and has a slope less than one.

B Calculating V_A and σ_A using Black and Scholes

Following the empirical literature, we can use the Black and Scholes's model to calculate asset values, V_A, and volatilities, σ_A, using data on equity values and accounting data on liabilities. We use the values of V_A and σ_A can then be used to calculate an estimate of DI as defined in equation (1), or to calculate Black and Scholes (1973)'s DD, as defined in Section 3.4.

The algorithm. calculate Black and Scholes (1973)'s DD using an iterative algorithm that closely follows Vassalou and Xing (2004) and Duffie (2011). For each publicly traded firm in our sample, we assume that the value of the assets, V_A, is a geometric Brownian motion with volatility σ_A. We view equity as a call option with an underlying equal to the value of the assets, V_A, a maturity equal to one year, and a strike price equal to V_B. Under these assumptions, the Black and Scholes' formula implies that the value of equity is given by:

$$V_E = N(d_1)V_A - N(d_2)V_B e^{-r}, \tag{7}$$

where $N(\cdot)$ is the cumulative distribution function of a standard normal random variable, and

$$d_1 \equiv \frac{\log(V_A) - \log(V_B) + r + \frac{\sigma_A^2}{2}}{\sigma_A}, \text{ and } d_2 = d_1 - \sigma_A. \tag{8}$$

The iterative algorithm. We initialize our iterative algorithm for calculating V_A and σ_A by setting $V_A^{(0)} = V_E + V_B$, where V_E is the market capitalization of the firm, calculated using the CRSP data on price and number of shares outstanding. Following the literature, we take V_B to be the the sum of short term liabilities, and half of long-term liabilities, as given in COMPUSTAT. Consistent with Gilchrist and Zakrajsek (2012), for each firm we restrict attention to those times during which quarterly data are available, and we linearly interpolate between points to obtain daily data.

At step n of our algorithm, we have a candidate time series $V_A^{(n)}$ for the value of the assets on each day of our sample. Given $V_A^{(n)}$, we obtain a daily time series for asset realized volatility, $\sigma_A^{(n)}$, by computing the annualized square root of the average squared daily returns on the assets during the month. Given $V_A^{(n)}$ and $\sigma_A^{(n)}$, we use equation (8) to calculate a time series for $d_1^{(n)}$ and $d_2^{(n)}$. To deal with firms with small liabilities, we cap $\log(V_A^{(n)}/V_B)$ at 4 (the results turn out to be largely insensitive to this). In applying the formula, we take the interest rate to be the one-year Treasury constant-maturity (daily frequency) from the Federal Reserve's H.15 report. We then use equation (7) to obtain a new candidate time series for the value of the assets:

$$V_A^{(n+1)} = (1-\omega)V_A^{(n)} + \omega \frac{V_E + N\left(d_2^{(n)}\right) V_B e^{-r}}{N\left(d_1^{(n)}\right)},$$

where ω is a relaxation parameter, which we set equal to 0.2 to improve convergence. We terminate our algorithm when the norm of $(V_A^{(n+1)} - V_A^{(n)})/V_A^{(n)}$ is less than 10^{-5}. Convergence occurs for over 95% of the stocks in the sample.

An estimate of DI. Using the values of V_A and σ_A implied by Black and Scholes (1973), and the liability V_B measured as above, one obtains an estimate of DI using equation (1).

Black and Scholes' DD. Following the literature, the Black and Scholes' and Merton distance to default is defined as

$$DD \equiv \frac{\log(V_A) - \log(V_B) + \mu_A - \frac{\sigma_A^2}{2}}{\sigma_A},$$

where V_A and σ_A are calculated using the iterative algorithm described above, and μ_A is the mean return on the assets over the time period.

C Realized versus Implied Volatility

As a robustness check on our empirical implementation of our DI measure, we compare median DI computed using realized and option-implied volatilities from option metrics. We focus on the median log DI since, as argued in Section 4.1, its fluctuations account for most of the fluctuations in the overall DI distribution. Figure 25 plots the time series of the median log DI measured using implied and realized volatility from OptionMetrics for the available data from 1996 to 2013. We use their daily data for both series to ensure that the same firms are included in both samples. We use the implied volatility from OptionMetrics' standardized, at the money, options with 30 days to maturity, and pool both calls and puts. The figure shows that realized volatility closely tracks fluctuations of implied volatility.

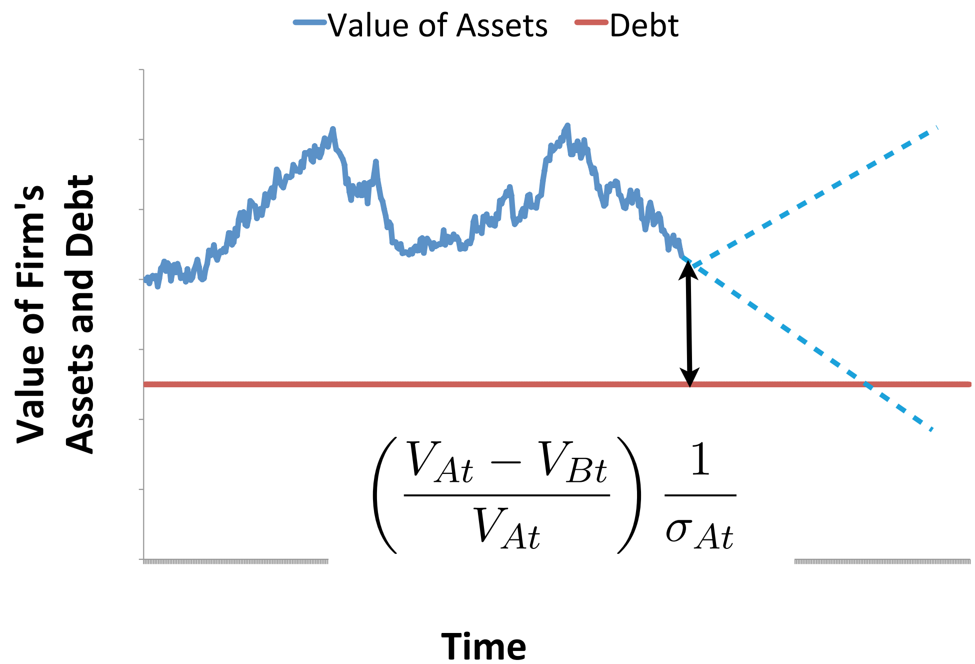

Figure 1: The value of equity as a function of the value of assets.

$$\left(\frac{V_A - V_B}{V_A}\right)\frac{1}{\sigma_A} \leq \frac{1}{\sigma_E} = \left(\frac{V_A - X}{V_A}\right)\frac{1}{\sigma_A} \leq \left(\frac{V_A - V_{A^*}}{V_A}\right)\frac{1}{\sigma_A}$$

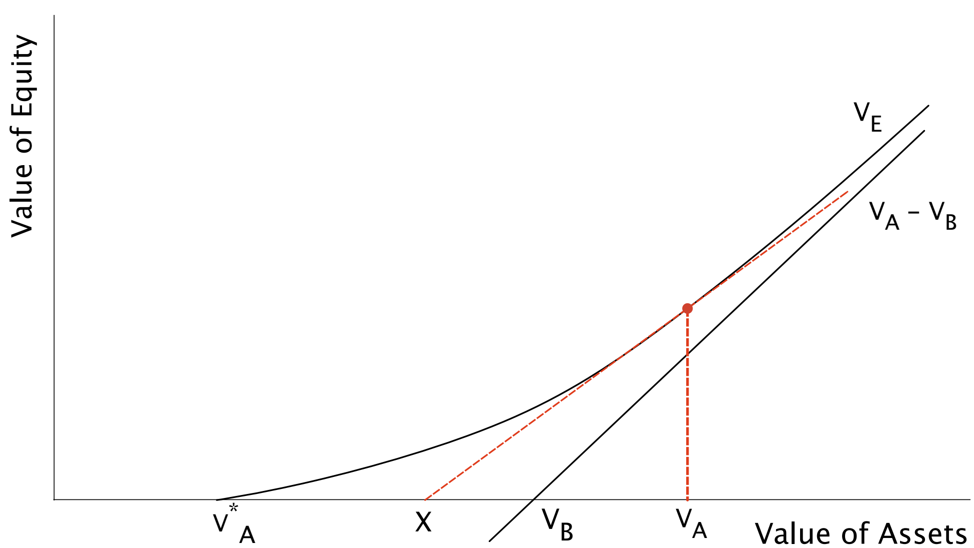

Figure 2: The value of equity as a function of the value of assets.

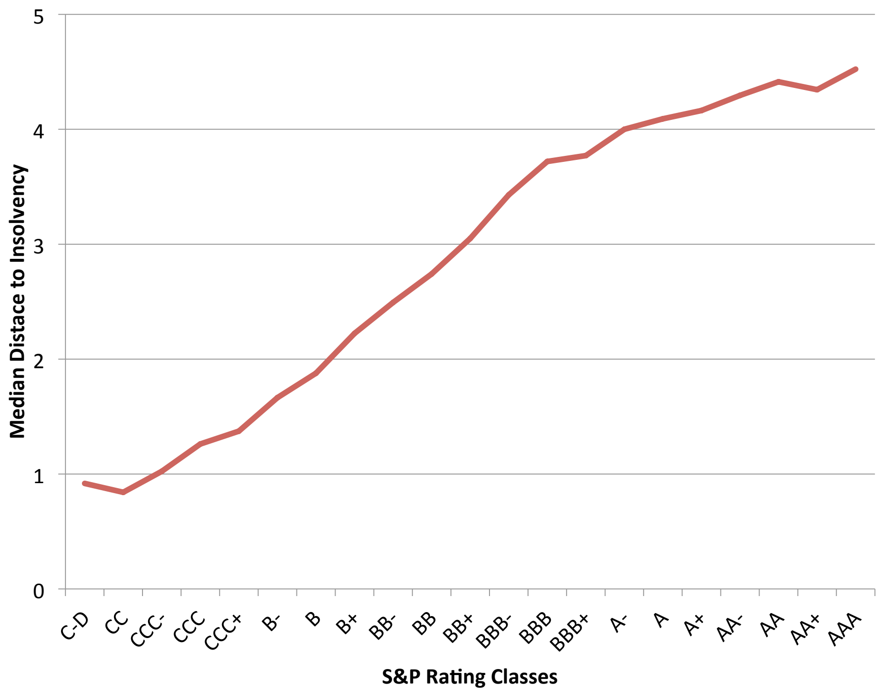

Figure 3: The empirical relationship between credit rating and measured DI.

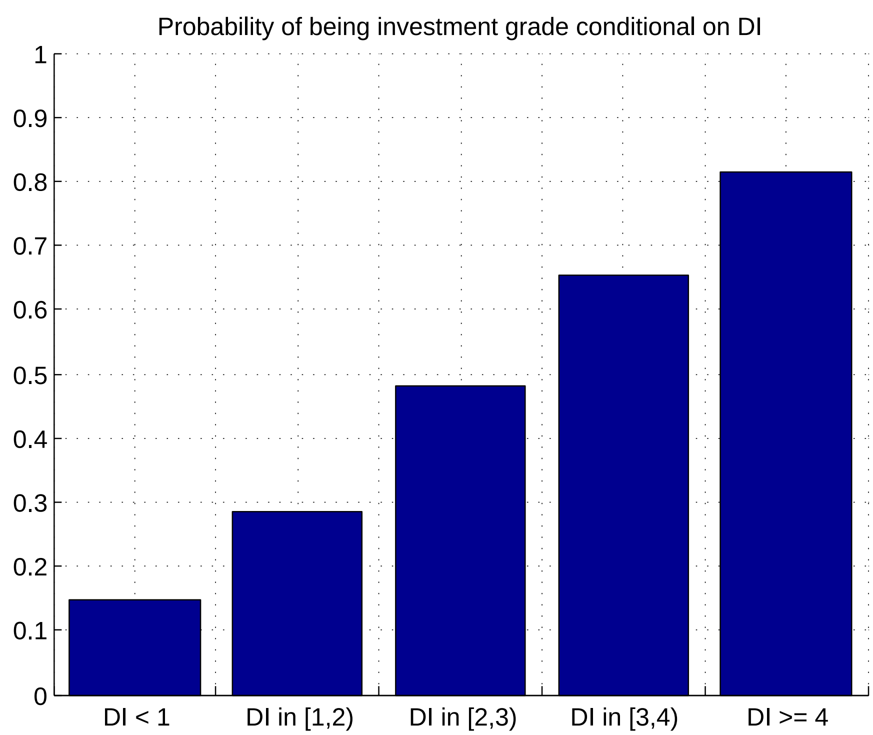

Figure 4: The empirical relationship between measured DI and credit rating.

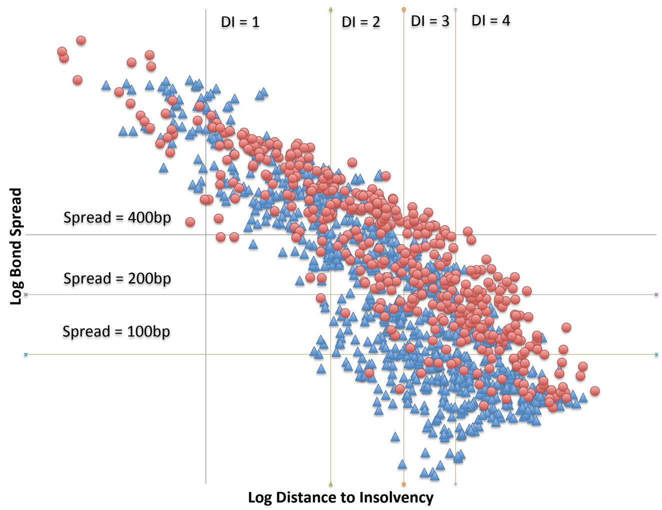

Figure 5: A scatter plot of monthly measured DI versus monthly averages of option-adjusted spreads for the Bank of America-Merrill Lynch corporate bond indices by ratings class for January 1997–December 2012, in log scale. Each point represents a single month and data for one of seven ratings from AAA to CCC and below. Pre–August 2007 data points are blue triangles, and post–August 2007 data points are red circles.

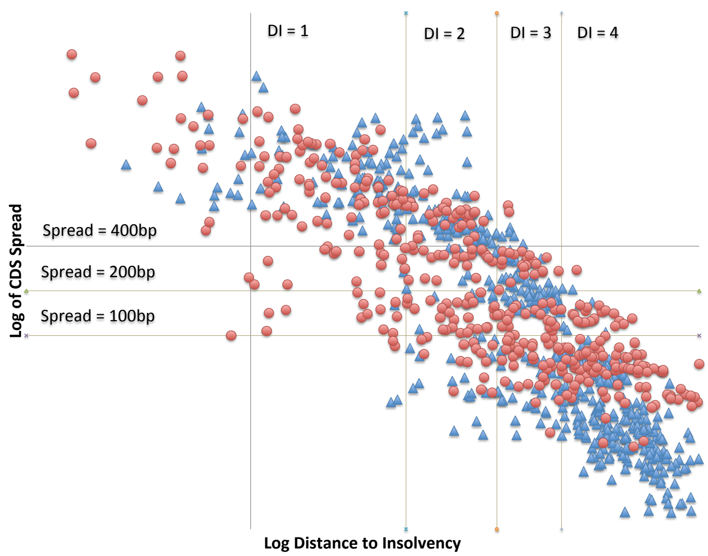

Figure 6: A scatter plot of monthly measured DI vs. monthly of averages of 5 year single name CDS rates for 2001–2011, in log scale. Data is pooled by credit rating. Each point represents a single month and one of seven ratings classes from AAA to CCC and below. Pre–August 2007 data points are blue triangles, and post–August 2007 data points are red circles.

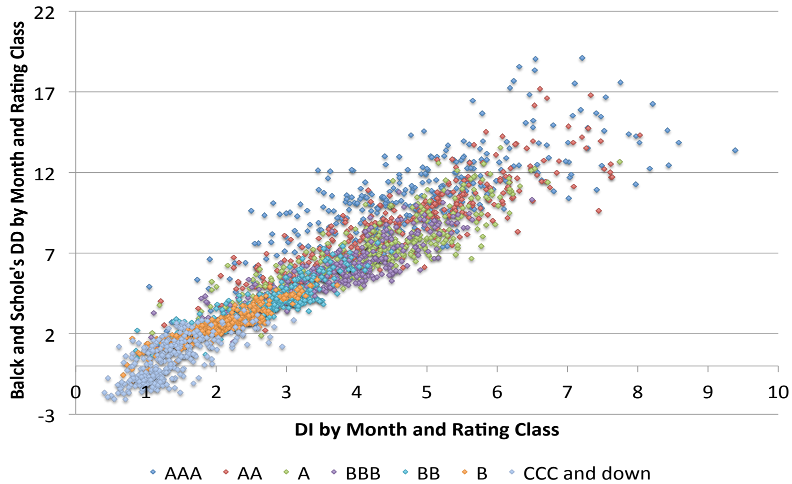

Figure 7: A scatter plot of monthly median measured DI versus monthly median Black and Scholes' Distance to Default by month and ratings class from December 1985 to December 2012. Each point represents a single month and data for one of seven ratings from AAA to CCC and below.

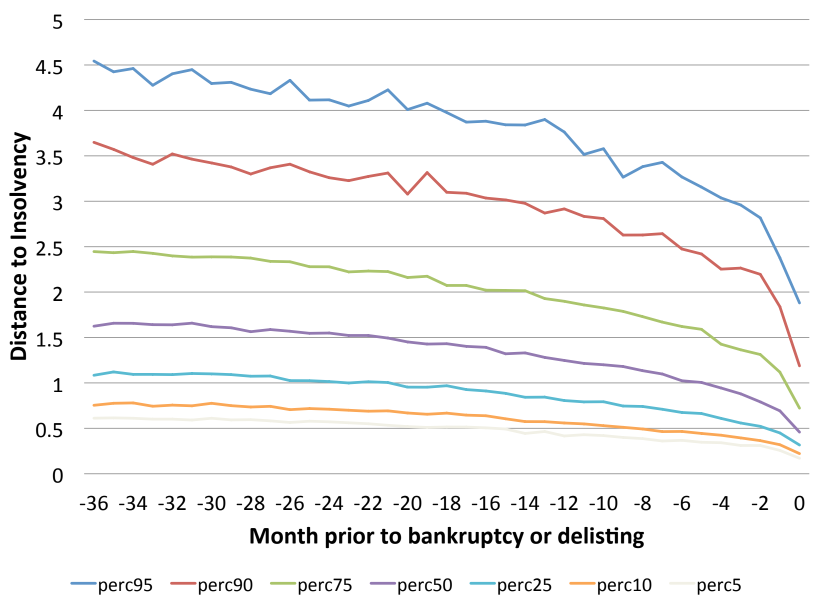

Figure 8: The distribution of measured DI for firms that declare bankruptcy in the 60 months prior to bankruptcy or delisting.

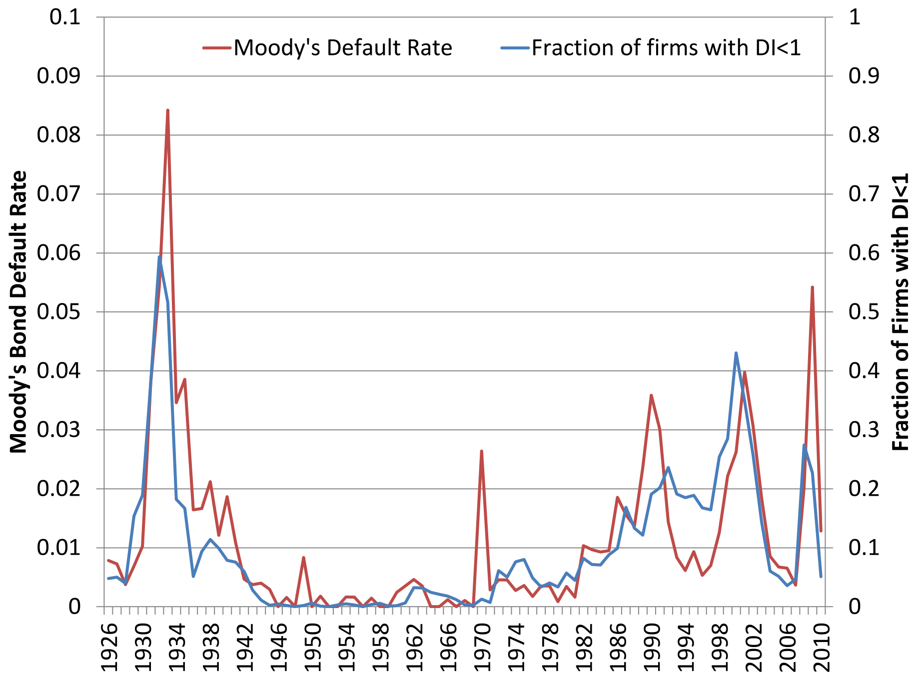

Figure 9: Annual measured DI versus annual issuer-weighted corporate default rates from Moody's Investor Service Annual Default Study 2012.

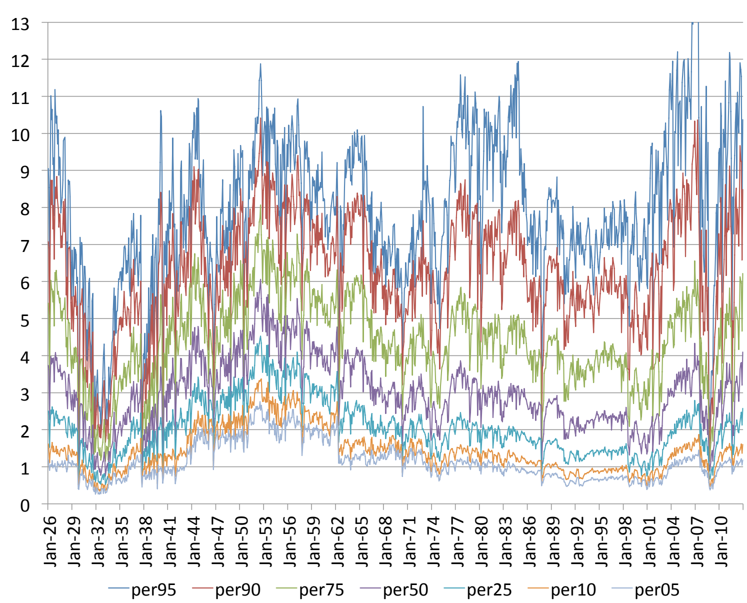

Figure 10: The distribution of measured DI, 1926–2012.

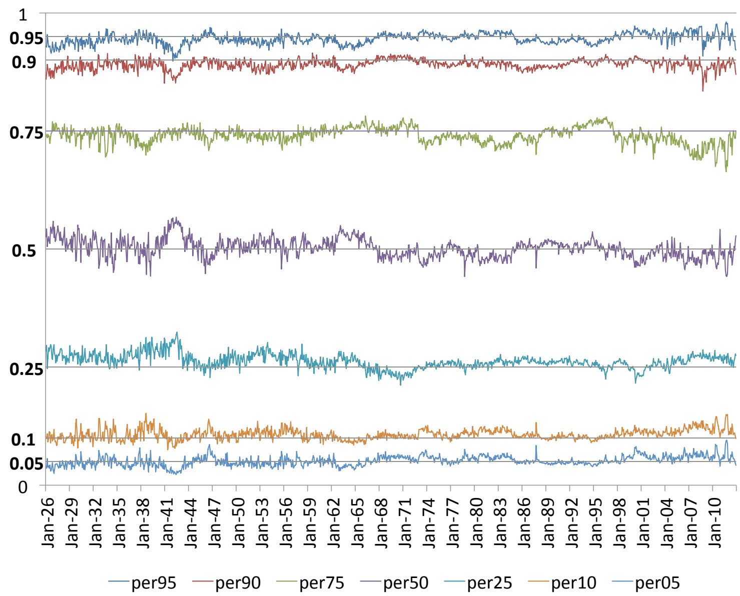

Figure 11: The y-axis plots the percentiles of a true lognormal distribution for $DI_t = 1/\sigma_{Et}$ with the estimated cross-sectional mean and standard deviation for each month, 1926–2012. The colored lines display the empirical percentile cutoffs on each date.

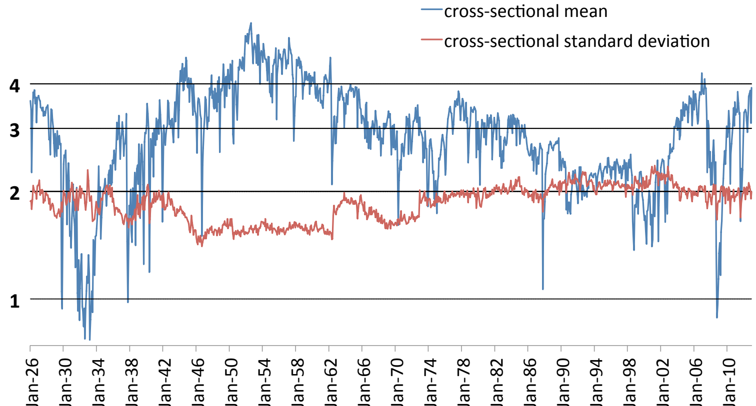

Figure 12: The mean and standard deviation of log measured DI, 1926–2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

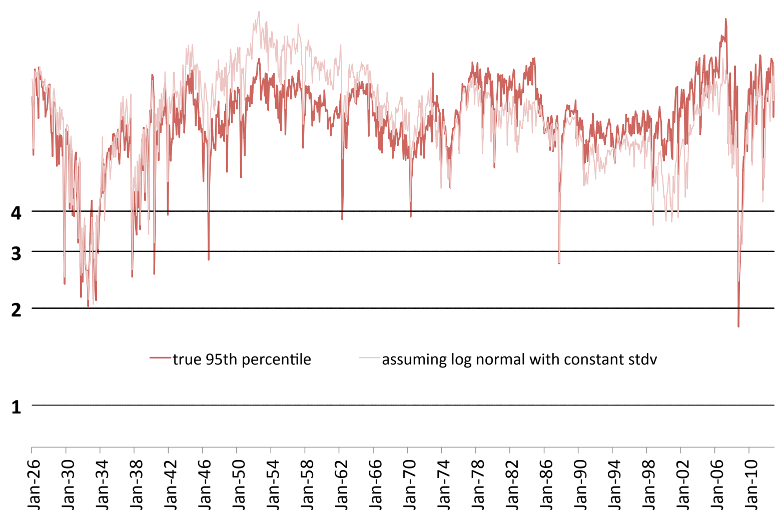

Figure 13: The 95th percentile of log measured DI 1926-2012 with time varying (red) versus constant (pink) standard deviation. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

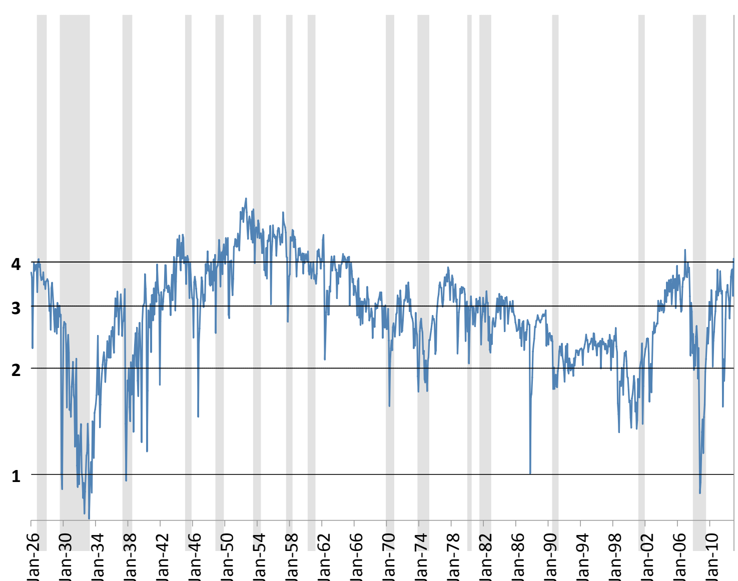

Figure 14: Deep and Broad Insolvency Crises: The log of the median measured DI, 1926-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale. Recessions are indicated by vertical gray bars. The median measured DI hits 1, associated with a highly vulnerable rating, in the Great Depression, 1937, 1987, and the Financial Crisis of 2008.

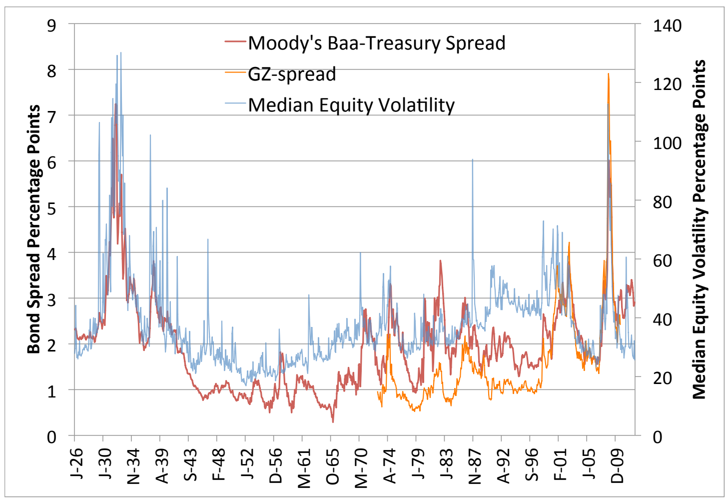

Figure 15: Bond Spread Indices and Median Equity Volatility: The Gilchrist-Zakrajsek bond spread index 1973-2010 (in orange) and the Moody's Baa - long term government debt spread index 1926-2012 (in red) in percentage points are plotted on the left axis. The level of equity volatility for the median firm in the cross section every month 1926-2012 is plotted in percentage points on the right axis.

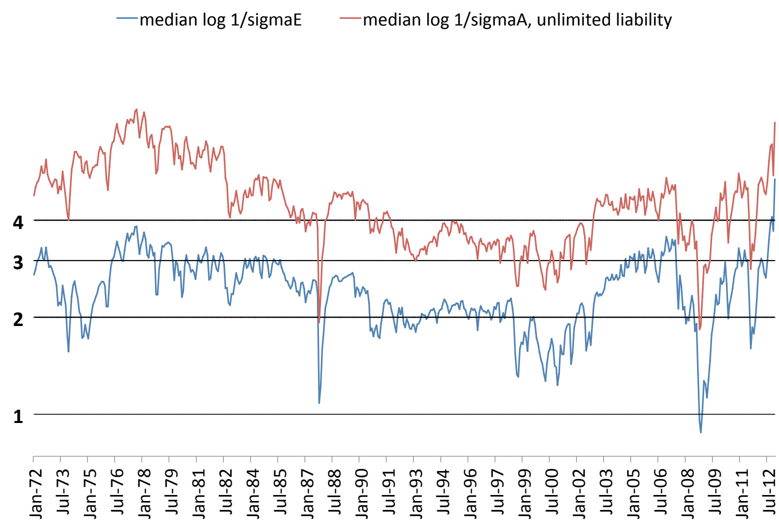

Figure 16: Leverage and asset volatility under the assumption of unlimited liability, 1971-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

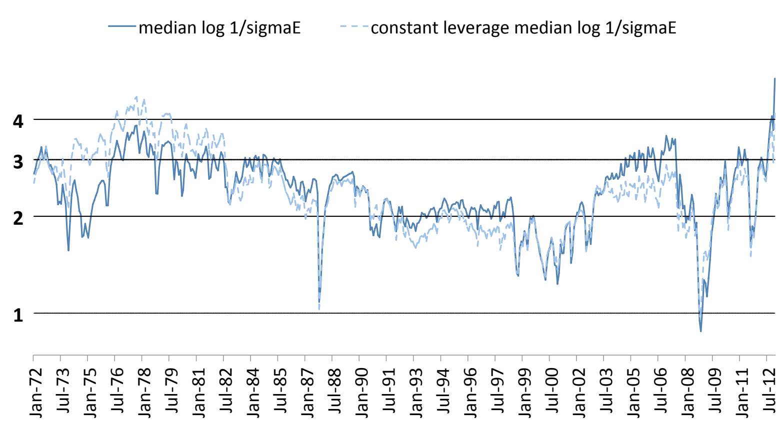

Figure 17: Measured DI versus constant leverage measured DI, under the assumption of unlimited liability, 1971-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

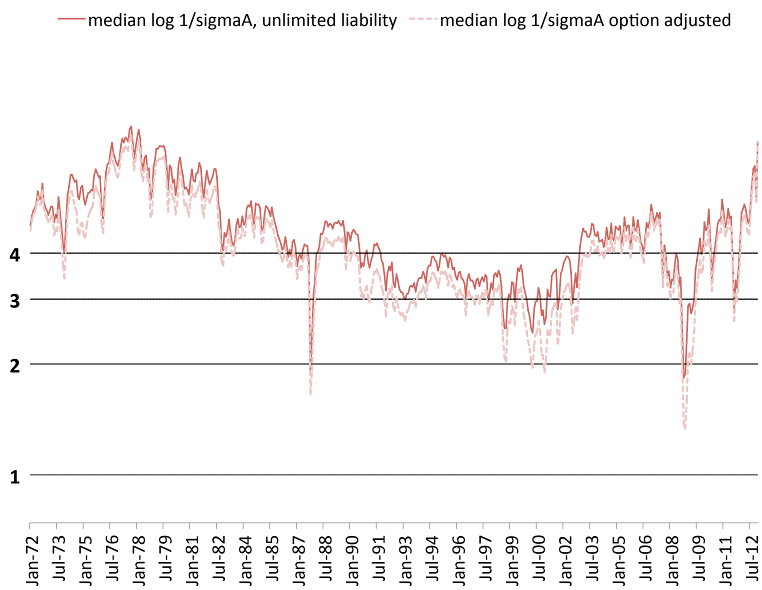

Figure 18: Asset volatility under the assumption of unlimited liability, and using Black and Scholes to compute the value of equity's default option, 1971-2012. In the calculation, we take V_B be equal to total liabilities, as in the unlimited liability calculations. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

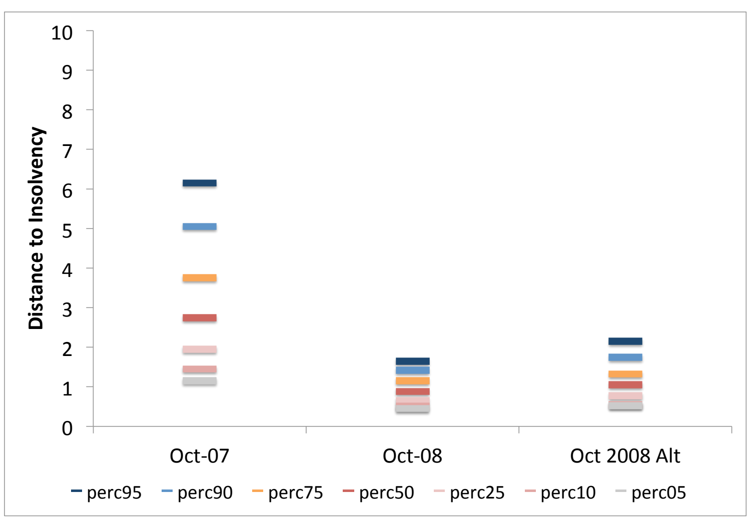

Figure 19: The percentiles of measured DI for all firms in October 2007 and October 2008 together with the counterfactual alternative percentiles of DI that would have arisen from October 2007 leverage and October 2008 asset volatility.

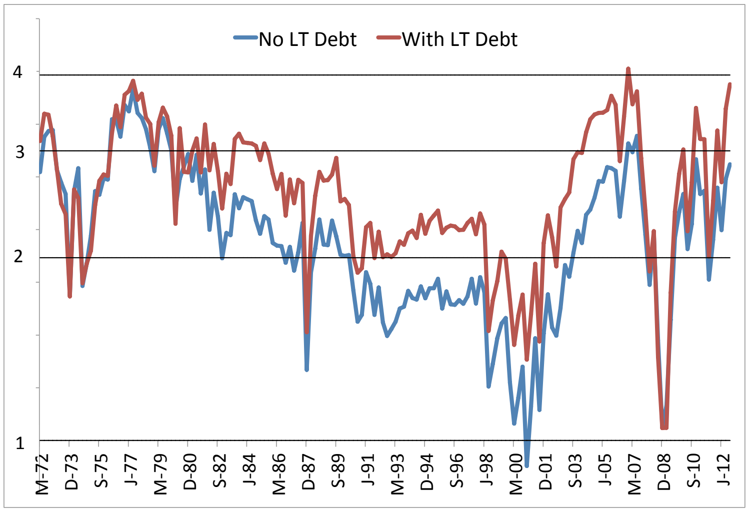

Figure 20: The median of log DI for those firms with no long term debt (in blue) and those firms long term debt (in red), 1972-2012.

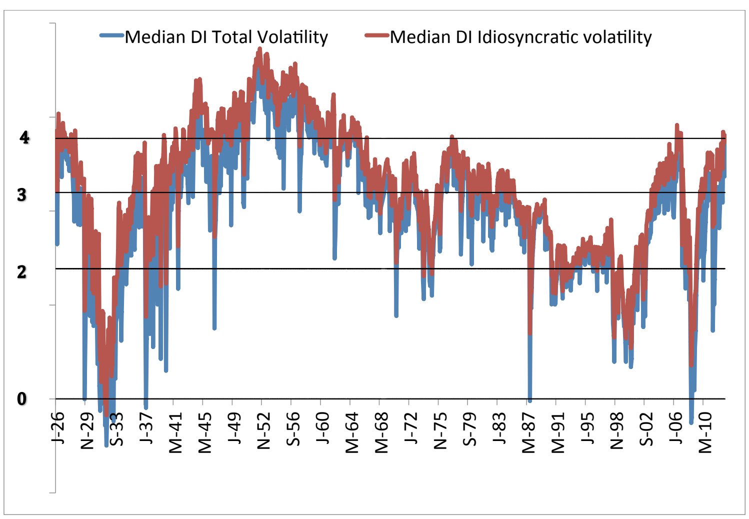

Figure 21: Median DI measured using firms' total equity volatility and Median DI using firms' idiosyncratic equity volatility. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

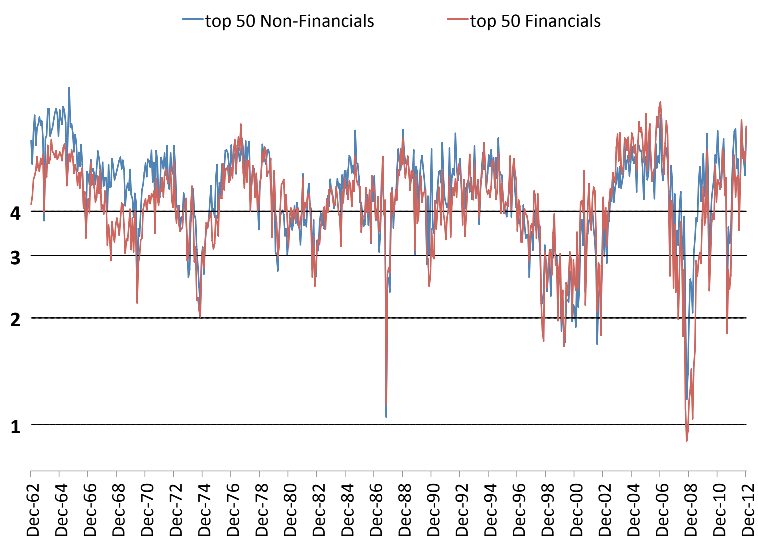

Figure 22: A comparison of the log median measured DI for the largest 50 financial and non-financial firms in terms of market capitalization, 1962-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

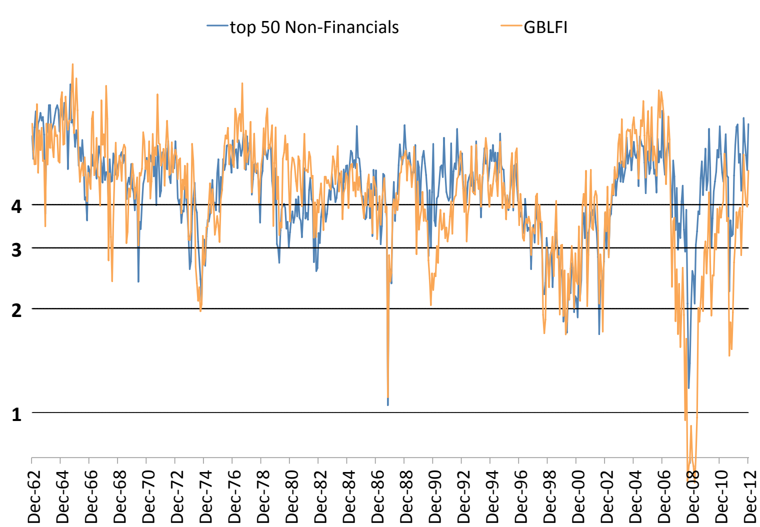

Figure 23: A comparison of the log median measured DI for the Government Backed Large Financial Institutions and the largest 50 non-financial firms in terms of market capitalization, 1962-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

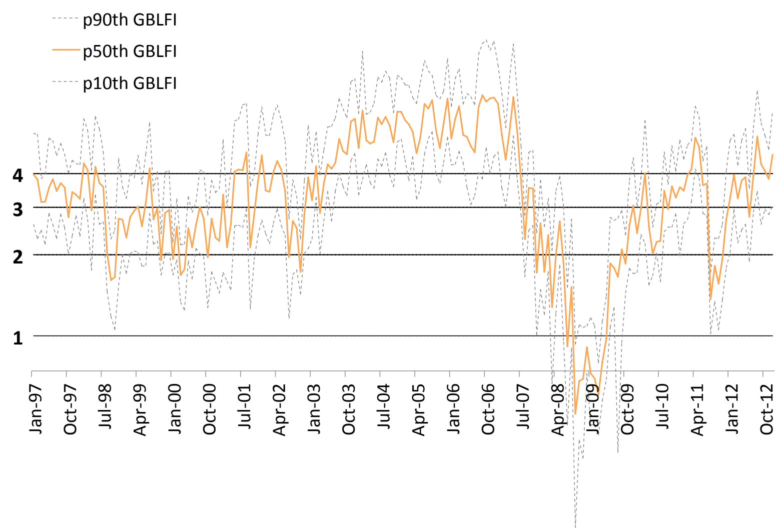

Figure 24: The 90th percentile, median, and 10th percentile of measured DI for the GBLFI's from 1997-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

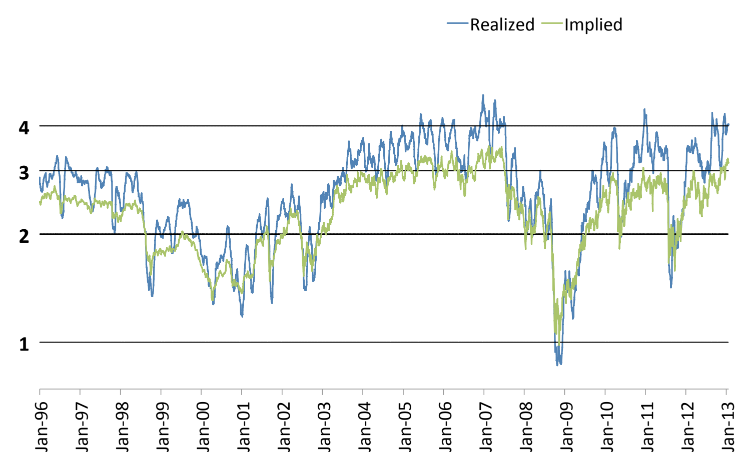

Figure 25: The mean of the log of inverse realized versus option implied volatilities, 1993-2012. The horizontal lines indicate the position of our benchmark cutoffs (DI=1,2,3,4) on the log scale.

Table 1: Government Backed Large Financial Institutions

Name		Sample
American Express	Amex	01/02/1969 to today
American Insurance Group (AIG)	AIG	12/14/1972 to today
Bank of America	BOA	01/02/1969 to today
Bank of New York	BONY	12/04/1969 to today
Branch Banking and Trust	BB&T	12/14/1972 to today
Bear Stearns	BST	10/29/1985 to 05/30/2008
Capital One	COF	11/16/1994 to today
City	C	12/31/1925 to today
Fifth Third Bancorp	FITB	04/23/1975 to today
Fannie Mae	FNMA	08/31/1970 to 07/07/2010
Freddy Mac	FRE	08/10/1989 to 07/07/2010
Goldman Sachs	GS	05/04/1999 to today
JP Morgan	JPM	03/05/1969 to today
Key Banks	KEY	02/23/1972 to today
Lehman Brothers	LEH	05/31/1994 to 09/17/2008
Merrill Lynch	MERRILL	07/27/1971 to 12/31/2008
MetLife	MET	04/05/2000 to today
Morgan Stanley	MS	03/21/1986 to today
PNC Financial Services	PNC	12/14/1972 to today
Regions Financial Corp	REG FIN	12/14/1972 to today
Suntrust Banks	SUNTRUST	07/01/1985 to today
State Street Boston Corporation	STATESTREET	12/14/1972 to today
US Bancorps	USB	12/14/1972 to today
Wachovia Corporation	WACH	12/14/1972 to 12/31/2008
Washington Mutual	WaMu	03/11/1983 to 09/26/2008

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