Copula-Based Models of Systemic Risk in U.S. Agriculture: Implications for Crop Insurance and Reinsurance Contracts

Barry K. Goodwin

North Carolina State University

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Introduction

Salient facts about the US federal crop insurance program

- \$114 billion in liability in 2011
- Total premium in 2011 was \$12 billion
- Premium subsidy \$7.42 billion
- Implies 62% subsidy
- Subsidy paid as a percentage of premium such that rising prices (which we have seen in recent years) imply much larger costs to taxpayers
- Touted as a "public–private" partnership
- Latest CBO score \$91 billion over 10 years
- Governed by complex (and favorable to companies) reinsurance agreement
- Recent calls for Congress to raise guarantee to 90-95% ("shallow losses") of expected revenue

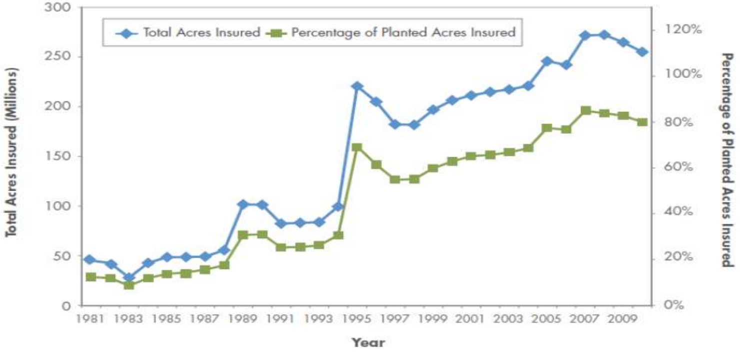
US Crop Insurance Statistics Today

Federal Crop Insurance Corp
Summary of Business Report for 2009 thru 2012
As of 05-07-2012

(Net Acre and Dollars in Thousands)

Combined Business:	2009 Crop Year To Date	2010 1 Year Ago To Date	2010 Crop Year To Date	2011 1 Year Ago To Date	2011 Crop Year Prev Week	2011 Crop Year To Date	2012 Crop Year Prev Week	2012 Crop Year To Date
Policies with Premium	1,171,943	1,140,390	1,140,647	217,383	1,151,149	1,151,251	225,353	227,261
Units with Premium	2,729,603	2,572,407	2,573,110	576,441	3,317,087	3,317,698	643,259	647,031
Net Acres Insured	264,774	256,155	256,242	82,329	265,358	265,405	84,869	84,774
Liability	79,575,395	78,068,379	78,100,849	15,164,828	114,078,351	114,104,391	17,799,962	17,968,961
Total Premium	8,950,769	7,590,029	7,593,600	1,578,411	11,953,074	11,954,936	1,979,808	1,993,996
Subsidy	5,426,897	4,708,328	4,710,764	1,004,978	7,451,273	7,452,384	1,255,046	1,263,781
Indemnity	5,228,182	4,188,111	4,246,916	74,619	10,734,592	10,750,475	59,447	82,837
Loss Ratio	0.58	0.55	0.56	0.05	0.90	0.90	0.03	0.04

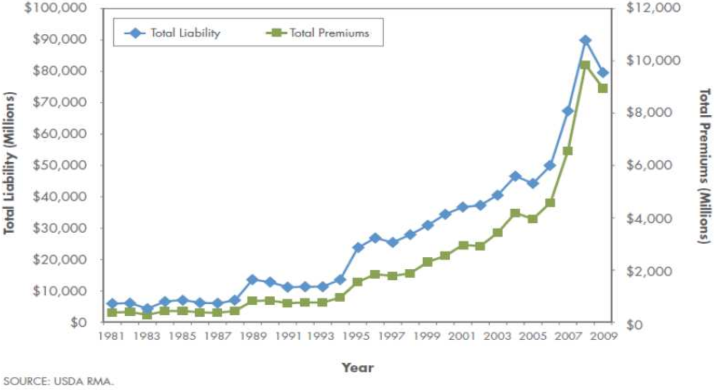
US Crop Insurance Statistics: Participation



SOURCE: Data on insured acres were obtained from the US Department of Agriculture (USDA) Risk Management Agency (RMA); data on annual acres planted to crops were obtained from the USDA National Agricultural Statistical Service.

Source: Smith (2012)

US Crop Insurance Statistics: Liability and Premium



Source: Smith (2012)

Revenue Insurance

- 75% of liability is in the form of revenue coverage
- Based on product of (correlated) yields and prices
- Revenue coverage introduced in 1990s and quickly became most prominent plan
- Duplicates (in part at least) operation of private market (options) with a subsidized plan
- Basic rating model
  - RMA yield rates based on 40–year average of loss-costs
  - Calibrates RMA rates to a truncated normal distribution for yields
  - Uses futures and option prices to derive log-normal distribution for prices
  - Combines various assumptions or estimates of correlation
  - Uses Iman–Conover procedure with normal score function to generate correlated yield/price draws (more on this below)

The Prominence of Crop Revenue Insurance

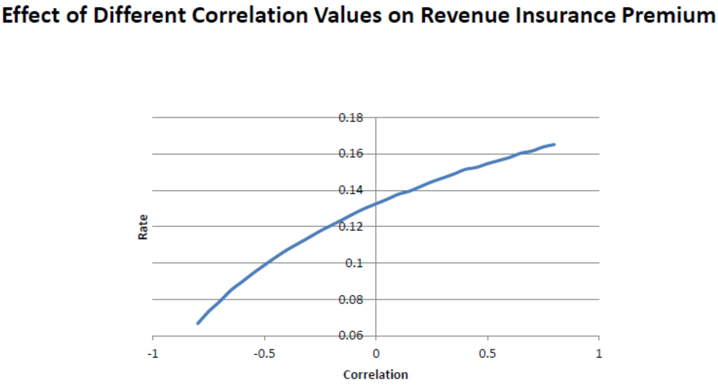
Federal Crop Insurance Corporation
Crop Year Statistics for 2011
As of: May 7, 2012
Nationwide Summary - By Insurance Plan

Ins Plan	Pol Sold	Pol Earn Prem	Pol Indem	Units Earn Prem	Units Indem	Net Acres	Liabilities	Total Premium	Subsidy	Cost Share	State Sbsdy	Prem Dscnt	Indemnity	Loss Ratio
AGR	419	399	7	399	7	0	356,825,237	13,261,937	7,109,434	0	0	0	564,838	.04
AGRLT	531	494	28	494	28	0	122,097,282	5,502,267	3,006,546	0	0	0	1,566,630	.28
APH	272,542	92,653	24,264	207,087	43,926	13,738,907	9,605,136,345	733,857,729	473,594,324	0	0	0	716,242,531	.98
AQU	161	91	3	199	3	0	24,339,167	798,734	491,845	0	0	0	146,819	.18
ARH	2,255	2,126	684	3,136	856	72,274	373,224,147	32,163,673	18,506,537	0	0	0	33,986,445	1.06
DOL	26,410	11,824	1,231	19,797	1,779	698,082	3,197,073,296	101,988,984	73,176,267	0	0	0	58,209,416	.57
GRIP	2,337	1,860	560	2,984	952	431,127	487,467,847	38,802,637	17,258,267	0	0	0	11,433,397	.29
GRIPH	16,160	12,438	4,755	24,538	9,423	3,444,220	3,938,902,352	390,347,674	172,842,072	0	0	0	119,449,565	.31
GRP	12,226	9,697	1,041	13,040	2,259	1,429,568	887,006,983	26,455,491	14,053,151	0	0	0	14,217,424	.54
PRV	1,563	1,242	202	1,524	278	161,447	140,825,516	11,420,095	7,130,287	0	0	0	4,011,936	.35
RAINF	13,885	11,722	9,485	70,086	42,831	30,884,217	486,404,714	106,154,957	57,886,284	0	0	0	177,665,043	1.67
RP	1,155,191	750,792	277,496	2,365,461	698,763	167,169,067	78,246,955,655	9,277,850,645	5,799,666,363	0	0	0	8,256,851,408	.89
RPHPE	34,051	27,054	7,379	83,447	17,912	6,015,283	3,393,812,842	232,003,053	140,636,300	0	0	0	179,636,021	.77
TDO	3,806	3,429	73	4,676	81	0	1,206,614,339	20,098,462	15,408,239	0	0	0	2,799,554	.14
VEGAT	2,022	1,738	179	3,980	355	3,532,860	36,542,822	5,329,137	2,923,396	0	0	0	4,540,060	.85
YDO	10,542	4,333	1,641	7,958	2,441	509,136	456,852,421	32,937,581	19,558,242	0	0	0	58,715,109	1.78
YP	512,821	219,359	64,542	508,892	127,081	37,318,991	11,144,310,294	925,963,094	629,136,648	0	0	0	1,110,437,877	1.20
	2,066,922	1,151,251	393,570	3,317,698	948,975	265,405,179	114,104,391,259	11,954,936,150	7,452,384,202	0	0	0	10,750,474,073	.90

Correlation/Dependence

- Correlation relationships play very important role in pricing revenue coverage
- Livestock margin plans have $1.1 billion in liability
- Recently introduced livestock margin plans involve multiple overlapping options contracts
- Margin plans:
  - Cover margin between input prices and output price (e.g., cattle prices, hog prices, milk prices, corn prices, soybean meal prices)
  - Structured as Asian option
  - Requires estimation of a large number of different correlation relationships
  - Little attention has been paid to how these dependent relationships should be modeled
- Remember that price insurance is readily available in the private markets (options markets) but without taxpayer subsidies

Revenue Rate and Correlation



Objectives of this Analysis

- In recognition of the crucial role played by correlation in establishing premium rates:
  - Evaluate how different assumptions about the nature of the joint probability distribution function may affect the pricing of insurance contracts
  - Demonstrate more flexible (data-driven) approaches to modeling joint distributions
  - Consider copula models that remain tractable for high–dimension problems

Copulas

- A copula is a function that joins the marginal distribution functions to form the multivariate distribution function
- Copulas make use of the fact that if $x \sim F$ then $F(x) \sim U(0,1)$
- To define a copula, consider m uniform (on the unit interval) random variables $u_1, u_2, ..., u_m$. The joint cdf of m uniform random variables is:

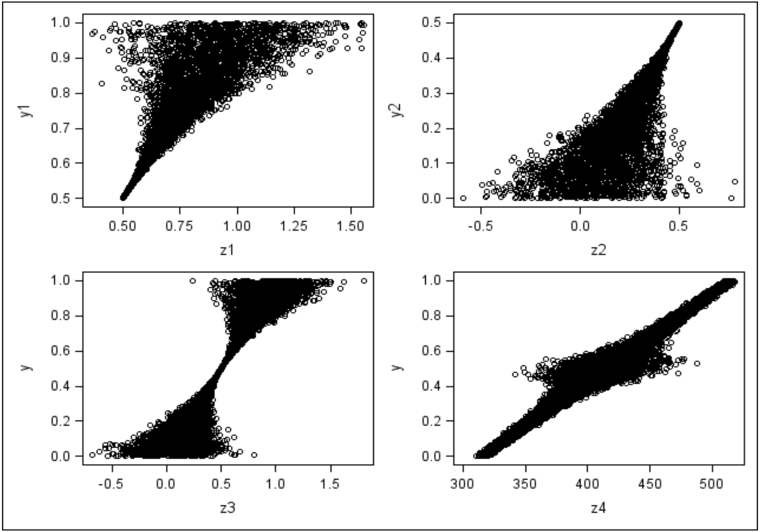
$$C(u_1, u_2, ..., u_m) = \text{Prob}(U_1 \leq u_1, U_2 \leq u_2, ...U_m \leq u_m)$$

  where C is a "copula," which is unique for continuous cdf's
- For an m-variate function F, the copula associated with F is a distribution function $C : [0,1]^m \rightarrow [0,1]$ that satisfies

$$F(y_1, ..., y_m) = C(F_1(y_1), ..., F_m(y_m); \theta),$$

  where θ is a set of parameters that measures dependence.

Different Copulas Imply Different Relationships



State Dependence in Spatial Correlation (Goodwin 2001):

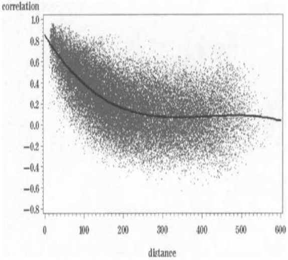


Figure 2. Pearson correlation coefficients vs. distance: normal yield years

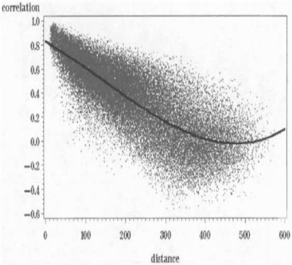


Figure 3. Pearson correlation coefficients vs. distance: extreme yield years

Systemic Risk and Agriculture:

- The most prominent aspect of risk in agriculture is weather
- Weather conditions are usually very systemic in major growing regions (an exception is hail insurance, which is sold in private markets)
- The diagrams presented above suggest that dependencies may be stronger during extreme events (e.g., drought and flooding)
- Many argue that systemic risk is so large that private insurance companies and reinsurers lack the capacity needed to provide coverage
- Many reasons to dispute this assertion, but it has worked well in Congress as a basis for arguing for high subsidies and an ever-expanding program
- Compare the $100 billion in total liability in crop insurance to the trillions held in credit default swaps

Higher Ordered Copulas ($k > 2$)

Aas, et al. (2009) noted that a multivariate density function for k random variables can be factored form as

$$f(x_1, x_2, \ldots, x_k) = f_k(x_k) \cdot f(x_{k-1}|x_k) \cdot \cdots \cdot f(x_1|x_2, \ldots, x_k).$$

This density is unique for a given ordering of variables and can also be expressed as

$$f(x_1, x_2, \ldots, x_k) = c_{1\ldots k}(F_1(x_1), \ldots, F_k(x_k)) \cdot \prod_{i=1}^{k} f_i(x_i)$$

In the case of two random variables:

$$f(x_1, x_2) = c_{12}(F_1(x_1), F_2(x_2)) \cdot f_1(x_1) f_2(x_2).$$

Thus, with rearranging, a bivariate conditional density can be written as

$$f(x_1|x_2) = c_{12}(F_1(x_1), F_2(x_2)) \cdot f_1(x_1).$$

Higher Ordered Copulas ($k > 2$)

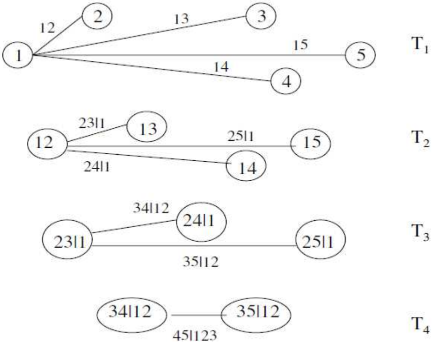
- Joe (1996) demonstrated that each of the terms can be decomposed into the product of a pair-wise copula and a conditional marginal density:

$$f(x|v) = c_{x,v_k|v_{-k}}(F(x|v_{-k}), F(v_k|v_{-k})) \cdot f(x|v_{-k}).$$

- However, this decomposition is dependent on how the conditional densities are arranged (i.e., the order)
- A set of k random variables can be arranged in $k!/2$ different ways
- Other alternatives to a more parsimonious problem are factor copulas (Patton)

Canonical Vines

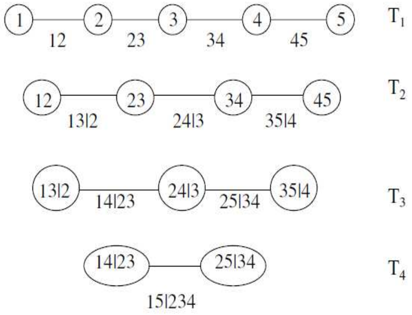
$$f_{12345} = f_1 \cdot f_2 \cdot f_3 \cdot f_4 \cdot f_5 \cdot c_{12} \cdot c_{13} \cdot c_{14} \cdot c_{15} \cdot c_{23|1} \cdot c_{24|1} \cdot c_{25|1} \cdot c_{34|12} \cdot c_{35|12} \cdot c_{45|123}$$



Source: Claudia Czado (Technische Universität Müunchen)

D Vines

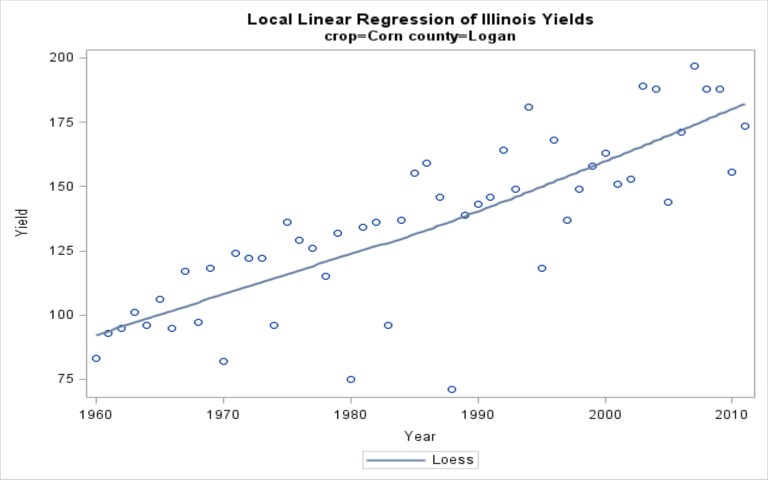
$$f_{12345} = f_1 \cdot f_2 \cdot f_3 \cdot f_4 \cdot f_5 \cdot c_{12} \cdot c_{23} \cdot c_{34} \cdot c_{45} \cdot c_{13|2} \cdot c_{24|3} \cdot c_{35|4} \cdot c_{14|23} \cdot c_{25|34} \cdot c_{15|234}$$



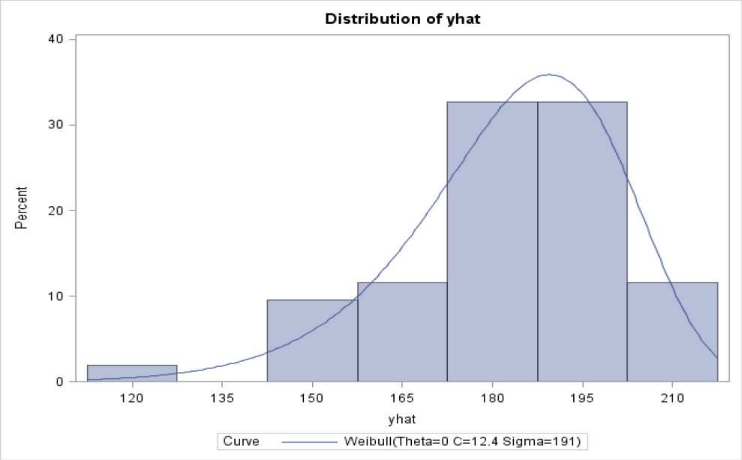
Source: Claudia Czado (Technische Universität Müunchen)

Normalizing Yields and Prices

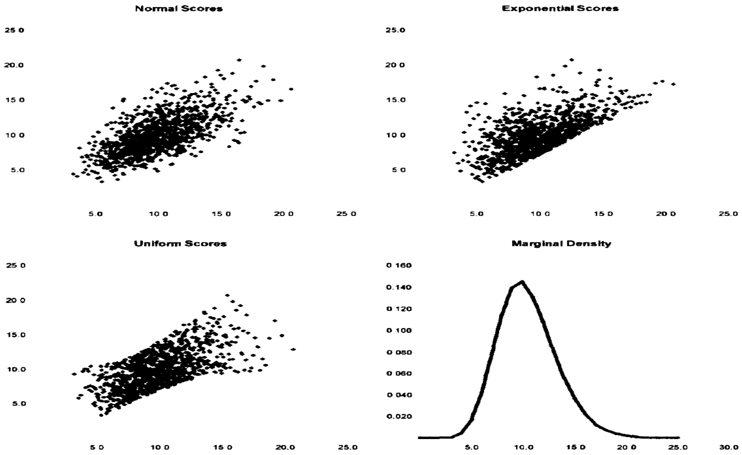
- Prices are easy—use futures and options
- Yields more problematic in light of substantial technological gains



Weibull Density for Yields



Iman-Conover with Different Scoring Functions



Source:
S. J. Mildenhall, demonstrates that IC with normal scores is equivalent to Gaussian copula.

Our Approach

- Use common methods (with Gaussian copula as benchmark) to estimate joint distributions for yields and prices
- Illustrative example considers a portfolio comprised of an equivalent number of acres of corn and soybeans for four prominent Illinois counties
- Consider a much broader approach to estimating the joint distribution—Vine Copulas
- Use model selection criteria to evaluate alternatives and to choose relevant copulas at each point in the vine
- Data–driven process to select from a range of 17 different copulas: Gaussian, Student t, Clayton, Gumbel, Frank, Joe, Clayton-Gumbel, Joe-Gumbel, Joe-Clayton, Joe-Frank, (With rotated versions of all)
- Estimate by standard ML procedures

Kendall's Tau for Observed Data and Clayton Copula Simulation

	C1	C2	C3	C4	S1	S2	S3	S4	CP	SP
C1		0.6878	0.7089	0.7602	0.3590	0.2941	0.4133	0.2640	-0.3273	-0.1991
C2	0.1720		0.7195	0.7707	0.2308	0.2624	0.3695	0.2262	-0.3710	-0.2549
C3	0.1585	0.1665		0.7164	0.2881	0.2534	0.4570	0.2564	-0.3439	-0.2157
C4	0.1574	0.1656	0.1546		0.2278	0.2624	0.3816	0.2383	-0.3469	-0.2308
S1	0.1544	0.1573	0.1516	0.1607		0.5701	0.5294	0.5762	-0.2021	-0.2006
S2	0.1587	0.1559	0.1595	0.1532	0.1625		0.6365	0.6652	-0.2609	-0.2926
S3	0.1659	0.1625	0.1635	0.1671	0.1666	0.1631		0.5641	-0.2685	-0.2278
S4	0.1603	0.1557	0.1586	0.1648	0.1482	0.1633	0.1713		-0.2730	-0.3469
CP	0.1634	0.1700	0.1579	0.1652	0.1586	0.1618	0.1683	0.1673		0.5520
SP	0.1653	0.1669	0.1637	0.1585	0.1587	0.1544	0.1603	0.1654	0.1575	

Results

Copula Specification	LLF	AIC	BIC
Gaussian	352.50	-615.01	-527.20
t-Copula	354.93	-617.86	-528.10
Clayton	56.48	-110.97	-109.01
Gumbel	33.95	-65.91	-63.96
Vine	365.20	-624.39	-520.97

Loss Probabilities

Table 5: Simulated Probabilities of Loss Claim

Insurance Instrument	Clayton	Gumbel	Gaussian	t	Canonical Vine
..........................75% Revenue Guarantee					
Corn Revenue County 1	0.1117	0.1054	0.0257	0.0273	0.0553
Corn Revenue County 2	0.1078	0.0987	0.0195	0.0205	0.0467
Corn Revenue County 3	0.1101	0.1029	0.0217	0.0250	0.0470
Corn Revenue County 4	0.1153	0.1026	0.0198	0.0205	0.0503
Soybean Revenue County 1	0.0876	0.0849	0.0401	0.0452	0.0554
Soybean Revenue County 2	0.0831	0.0792	0.0297	0.0318	0.0408
Soybean Revenue County 3	0.0966	0.0927	0.0414	0.0436	0.0538
Soybean Revenue County 4	0.0869	0.0831	0.0306	0.0302	0.0380
Corn Revenue Total	0.1012	0.0756	0.0160	0.0185	0.0484
Soybean Revenue Total	0.0809	0.0688	0.0287	0.0319	0.0455
Total Revenue	0.0704	0.0337	0.0071	0.0128	0.0299
..........................95% Revenue Guarantee					
Corn Revenue County 1	0.3942	0.4305	0.3916	0.3947	0.4234
Corn Revenue County 2	0.3942	0.4212	0.3941	0.3954	0.4291
Corn Revenue County 3	0.3914	0.4236	0.3991	0.4040	0.4320
Corn Revenue County 4	0.4011	0.4261	0.3949	0.3921	0.4311
Soybean Revenue County 1	0.3590	0.3765	0.3519	0.3584	0.3953
Soybean Revenue County 2	0.3629	0.3803	0.3489	0.3527	0.4039
Soybean Revenue County 3	0.3657	0.3865	0.3612	0.3627	0.4135
Soybean Revenue County 4	0.3590	0.3801	0.3459	0.3519	0.3936
Corn Revenue Total	0.3960	0.4299	0.3945	0.3945	0.4292
Soybean Revenue Total	0.3612	0.3893	0.3529	0.3543	0.3986
Total Revenue	0.3392	0.3876	0.3551	0.3563	0.4038

Loss Probabilities

Table 6: Simulated Revenue Insurance Premium Rates

Insurance Instrument	Clayton	Gumbel	Gaussian	t	Canonical Vine
..........................75% Revenue Guarantee					
Corn Revenue County 1	0.0142	0.0113	0.0017	0.0020	0.0042
Corn Revenue County 2	0.0151	0.0111	0.0014	0.0017	0.0030
Corn Revenue County 3	0.0153	0.0118	0.0014	0.0020	0.0035
Corn Revenue County 4	0.0134	0.0099	0.0011	0.0013	0.0035
Soybean Revenue County 1	0.0125	0.0094	0.0025	0.0032	0.0041
Soybean Revenue County 2	0.0100	0.0072	0.0013	0.0016	0.0024
Soybean Revenue County 3	0.0113	0.0087	0.0022	0.0027	0.0037
Soybean Revenue County 4	0.0124	0.0084	0.0015	0.0015	0.0024
Corn Revenue Total	0.0102	0.0043	0.0009	0.0013	0.0032
Soybean Revenue Total	0.0088	0.0049	0.0012	0.0017	0.0028
Total Revenue	0.0070	0.0015	0.0003	0.0006	0.0017
..........................95% Revenue Guarantee					
Corn Revenue County 1	0.0628	0.0634	0.0365	0.0365	0.0466
Corn Revenue County 2	0.0622	0.0619	0.0346	0.0347	0.0446
Corn Revenue County 3	0.0625	0.0626	0.0361	0.0367	0.0458
Corn Revenue County 4	0.0619	0.0610	0.0345	0.0344	0.0457
Soybean Revenue County 1	0.0530	0.0522	0.0380	0.0401	0.0452
Soybean Revenue County 2	0.0509	0.0506	0.0353	0.0354	0.0418
Soybean Revenue County 3	0.0544	0.0542	0.0390	0.0399	0.0461
Soybean Revenue County 4	0.0529	0.0513	0.0351	0.0346	0.0405
Corn Revenue Total	0.0579	0.0559	0.0340	0.0343	0.0447
Soybean Revenue Total	0.0500	0.0478	0.0351	0.0358	0.0424
Total Revenue	0.0437	0.0392	0.0288	0.0297	0.0375

Conclusions

- An immense portfolio of taxpayer subsidized crop insurance is currently priced under strong assumptions regarding the correlation of yields and prices
- In some of the more complex cases—livestock margin insurance—a combination of many different overlapping contracts and correlation coefficients may be used
- The famous "function that felled Wall Street" (i.e., the Gaussian Copula) is applied in pricing these contracts
- Latest farm bill proposals may increase level of coverage to 95%
- We demonstrate that different copulas can yield very different loss probabilities and premium rates

Conclusions

- These differences are especially acute in the tails corresponding to large losses
- The validity and robustness of current assumptions should be evaluated in light of the many other alternative copulas that may be more appropriate
- As ever–more complicated instruments are introduced, correlation among dependent risks may become even more important