Optimal Insurance with Counterparty Default Risk

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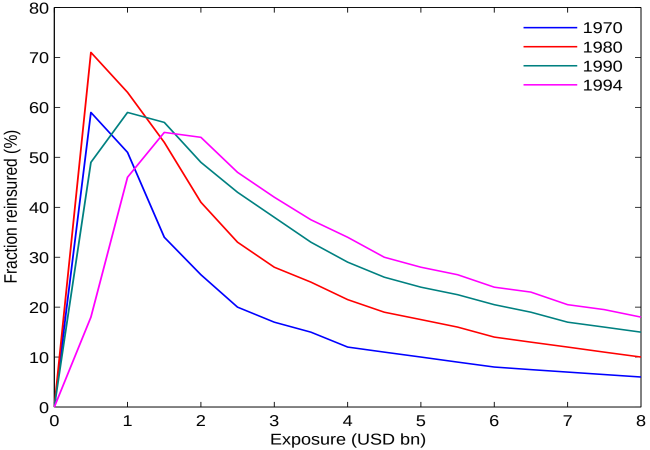
Insurance Markets and Catastrophe Risk Conference
NBER
May 11, 2012

OUTLINE

OUTLINE

REINSURANCE PURCHASES



Source: Guy Carpenter & Co., see Froot (2001)

CDS SPREADS

TRX P&C Re Index

Company	Net Premiums (USD)	1Y CDS Spread	Weight	Index contrib
Munich Re	28,384m	26.26	28.92%	7.59
Swiss Re	23,770m	42.20	24.22%	10.22
Berkshire Hathaway	10,650m	107.36	10.85%	11.65
Hannover Re	10,640m	37.22	10.84%	4.03
Lloyd's of London	8,593m	145.86	8.76%	12.78
SCOR	7,826m	39.21	7.97%	3.13
Everest Re	3,505m	45.22	3.57%	1.61
XL Capital	2,402m	55.81	2.45%	1.37
Renaissance Re	1,354m	95.02	1.38%	1.31
Ace	1,019m	38.12	1.04%	0.40
			Index value	**54.09**

Source: Thomson Reuters; January 1, 2010.

RELATED LITERATURE

Limited risk sharing of large (catastrophic) risks

- Froot/O'Connell (1997), Froot (2001): market frictions, exogenous contract

Insurance cycles

- Gron (1994), Winter (1994), Cummins/Danzon (1997): insolvency risk, no contract design

Optimal insurance / risk sharing (Arrow, Borch, Raviv, etc.)

- Tapiero/Kahane/Jaques (1986): mutual insurance
- Schlesinger/Schulenburg (1987), Doherty/Schlesinger (1990): three-state model, exogenous default
- Cummins/Mahul (2003): heterogeneous beliefs, exogenous default
- Dana/Scarsini (2007): background risk

OUTLINE

SETUP

One-period, no discounting

Insurance buyer

- utility u $(u' > 0, u'' < 0)$, random wealth $W \geq 0$
- insurable risk, X, valued in $[0, \overline{x}]$
- premium $P \geq 0$, indemnity $I(x)$ on $\{X = x\}$ $(0 \leq I(x) \leq x)$

$$W - X - P + I(X)$$

Insurer

- risk-neutral, random assets $A \geq 0$
- receives premium P, pays $I(X)$

$$A + P - I(X)$$

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$$(W - X - P + I(X))\, 1_{\{A+P-I(X))\geq 0\}}$$
$$+$$
$$(W - X - P + (A + P)\,\gamma)\, 1_{\{A+P-I(X)<0\}}, \quad (0 \leq \gamma \leq 1)$$

Insurer

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$$(A + P - I(X))^+$$

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$$\widetilde{W} := W - X - P + I(X)$$

$$\widetilde{W}(\gamma) := W - X - P + (A + P)\,\gamma$$

Insurer

- risk-neutral, random assets $A \geq 0$
- receives premium P, pays $I(X)$

$$\widetilde{A} := A + P - I(X)$$

OPTIMAL INSURANCE CONTRACTS

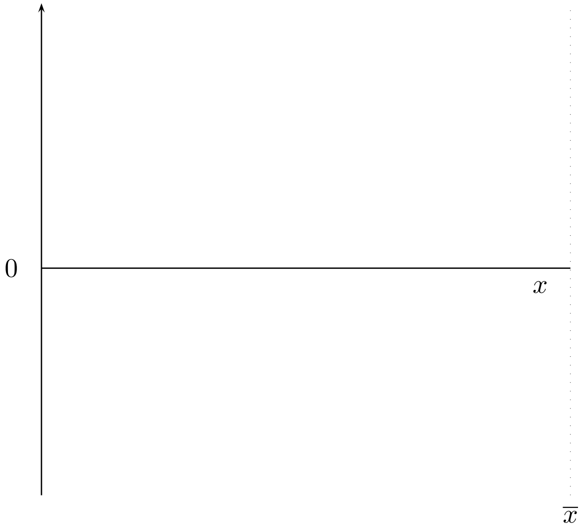
(P^*, I^*) solution of the following problem for different values of $\underline{v}$

$$\begin{cases} \sup_{(P,I)\in\mathbb{R}_+\times\mathcal{A}} E\left(u\left(1_{\{\widetilde{A}\geq 0\}}\widetilde{W} + 1_{\{\widetilde{A}<0\}}\widetilde{W}(\gamma)\right)\right) \\ E\left(\max\{\widetilde{A}, 0\}\right) \geq \underline{v} \end{cases}$$

Admissible contracts

- indemnity measurable function satisfying $0 \leq I(x) \leq x$ for all $x \in [0, \overline{x}]$
- focus on contracts at least as good as $(P = 0, I = 0)$

TRADITIONAL CASE



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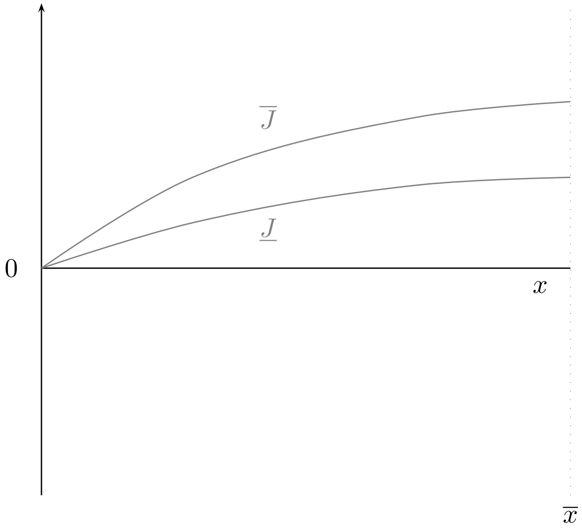

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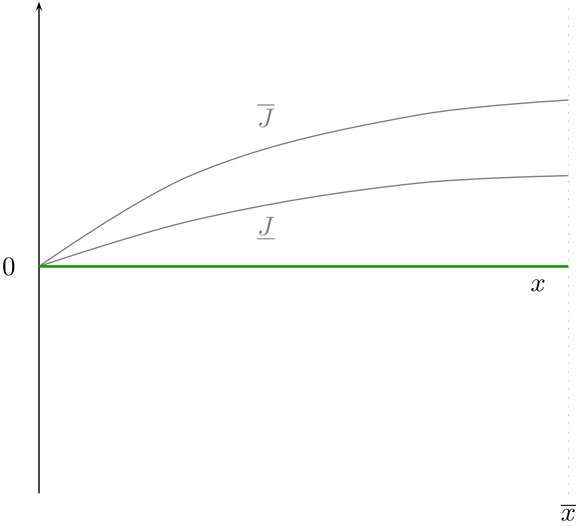

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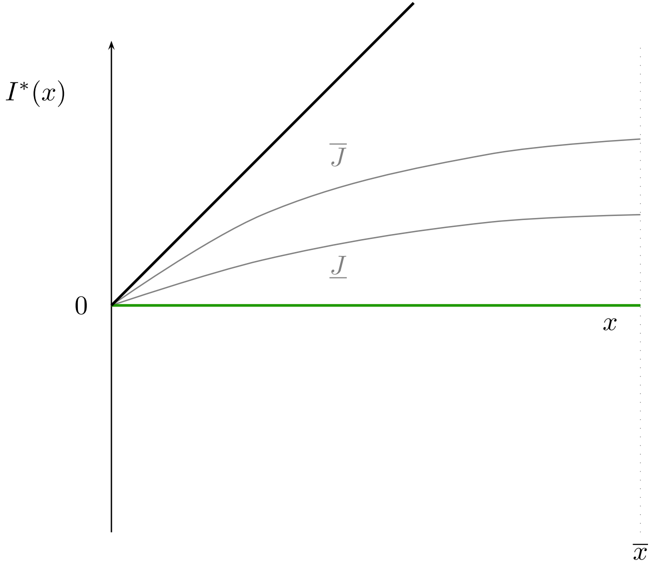

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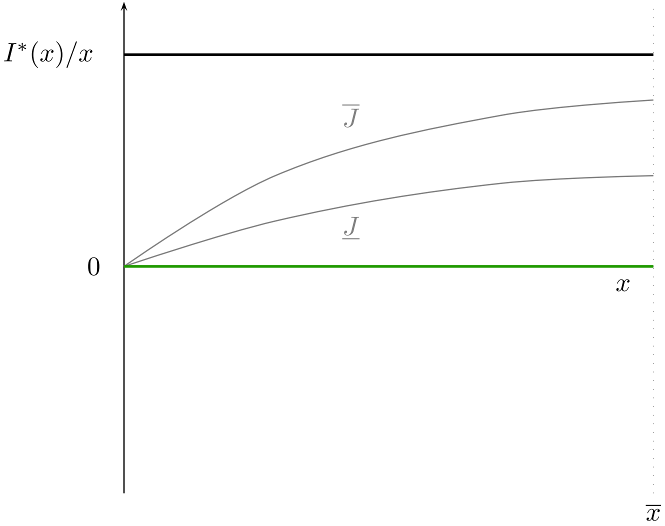
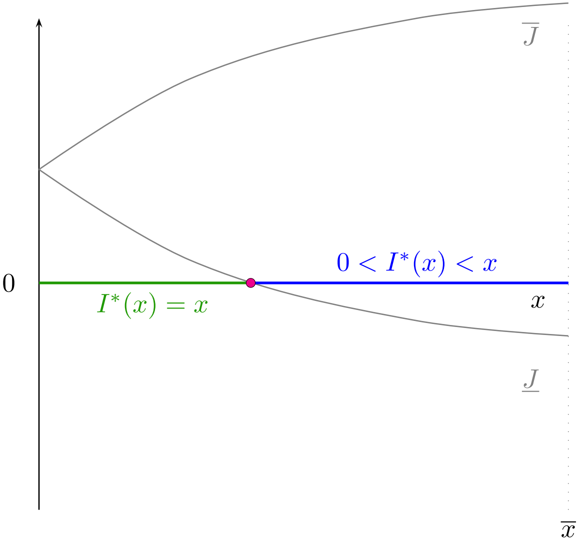


TRADITIONAL CASE (REGULATORY CONSTRAINT ON P, Raviv)



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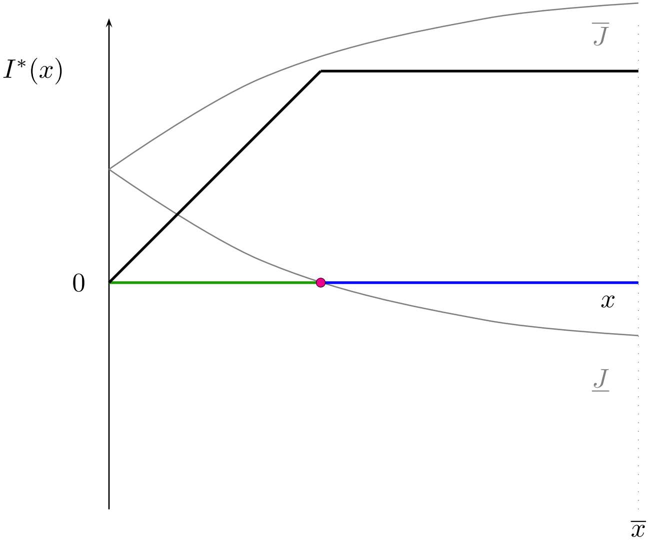

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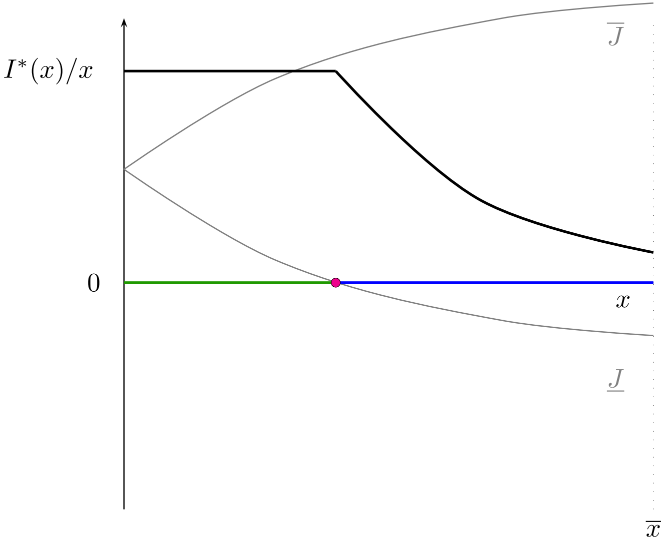

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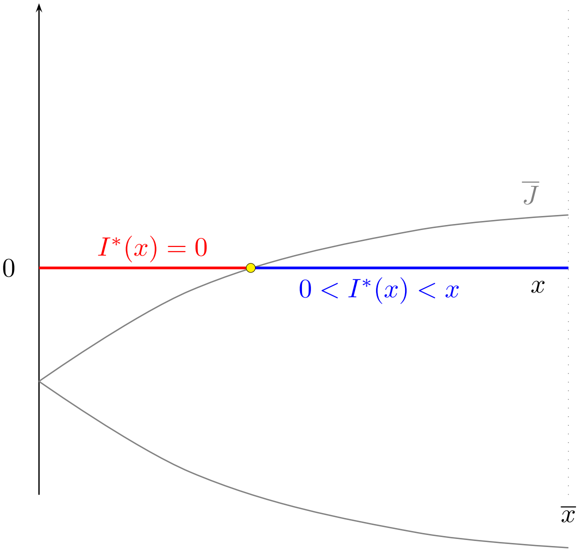

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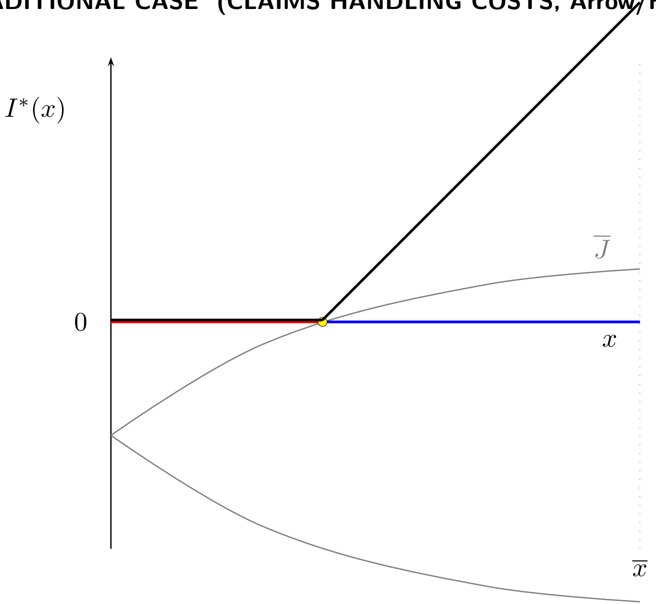

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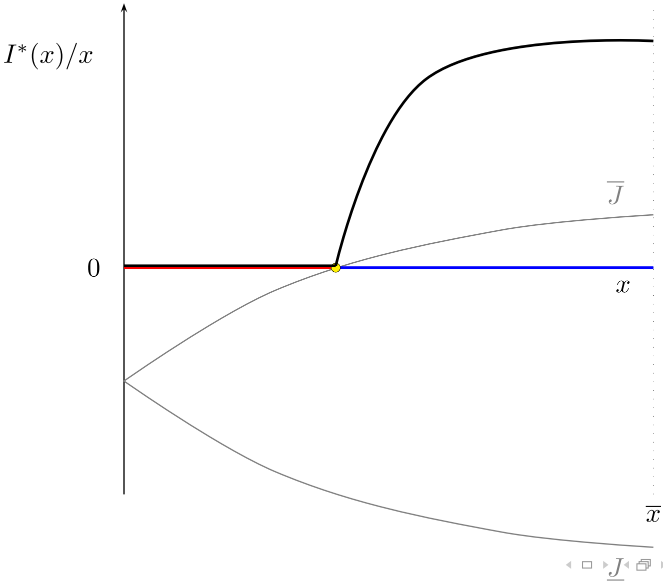
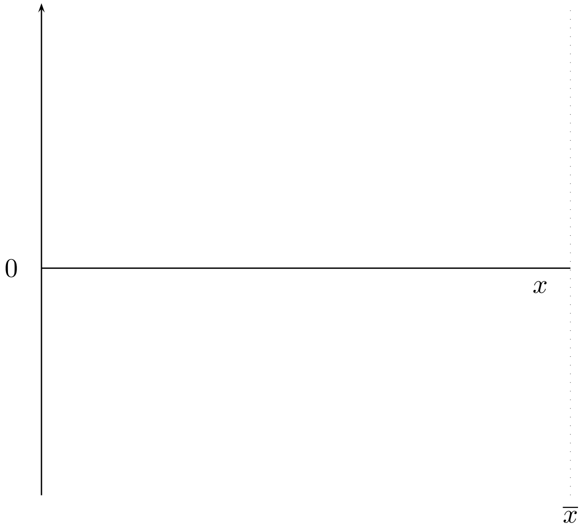


OUR SETTING



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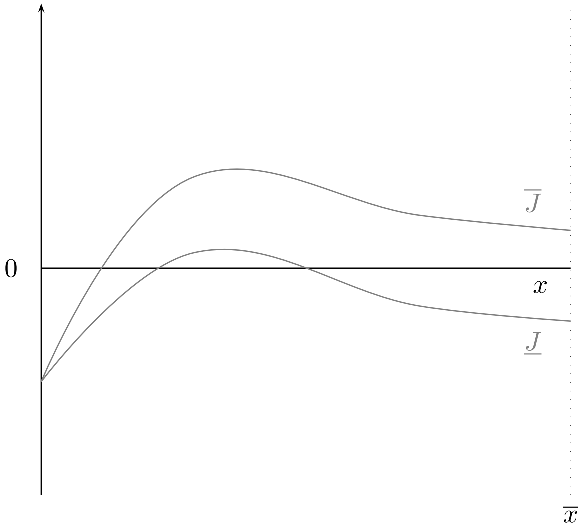

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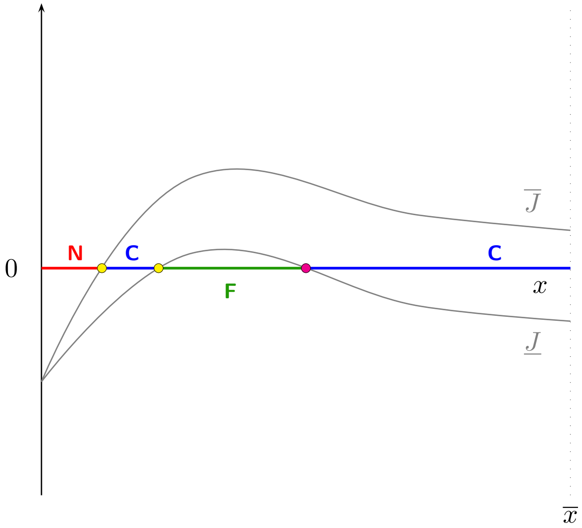

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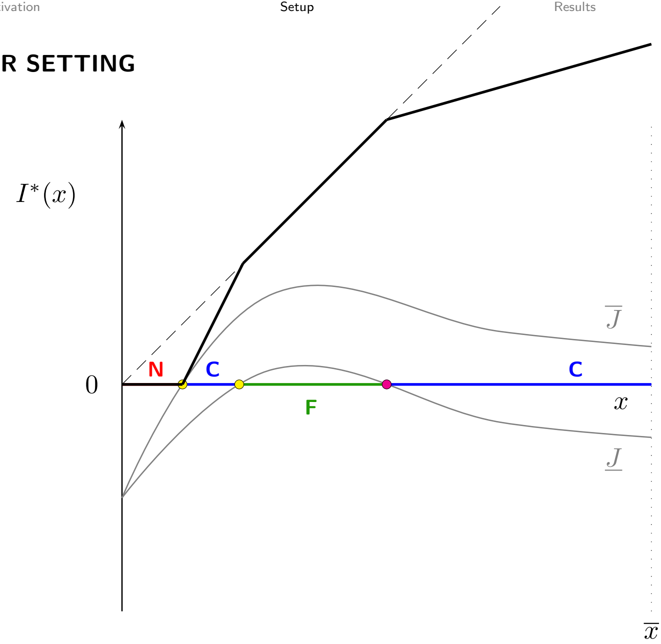

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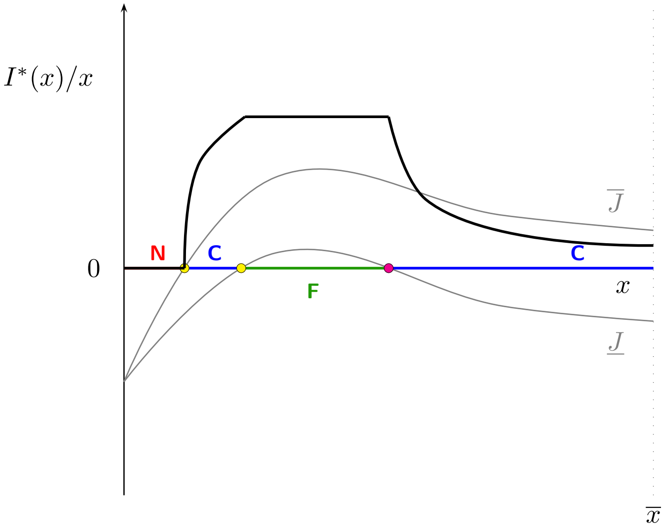

OUTLINE

GENERAL INSIGHTS

Optimal contracts

- Optimal (P^*, I^*)

$$P^* = E\left(I^*(X)\right) + (\underline{v} - E(A)) - E\left(\left(I^*(x) - (A + P^*)\right)^+\right),$$

- If no insurance optimal, then $E(u'(W - x)|X = x)$ is constant in x.
- With bankruptcy costs $(0 \leq \gamma < 1)$, any optimal contract must provide no insurance on a set of positive measure.

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Implications

- ⋆ deductibles without administrative costs (Raviv, 1979) or background risk (Gollier, 1996; Dana/Scarsini, 2007)
- ⋆ upper limits without regulatory constraints (Raviv, 1979; Jouini/al., 2008) or policyholder's limited liability (Huberman/al., 1983)

THE ROLE OF DEPENDENCE

Negative dependence

If W stochastically decreasing in X, then

- Any optimal contract entails a positive deductible $(0 \leq \gamma < 1)$ or a generalized deductible followed by coinsurance and full insurance $(\gamma = 1)$.

Positive dependence

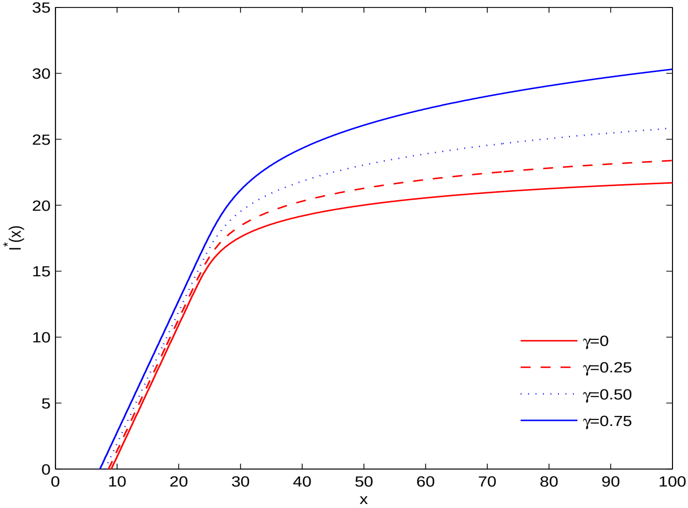
If W stochastically increasing in X,then

- Any optimal contract entails full insurance followed by coinsurance and no insurance (upper limit on coverage).

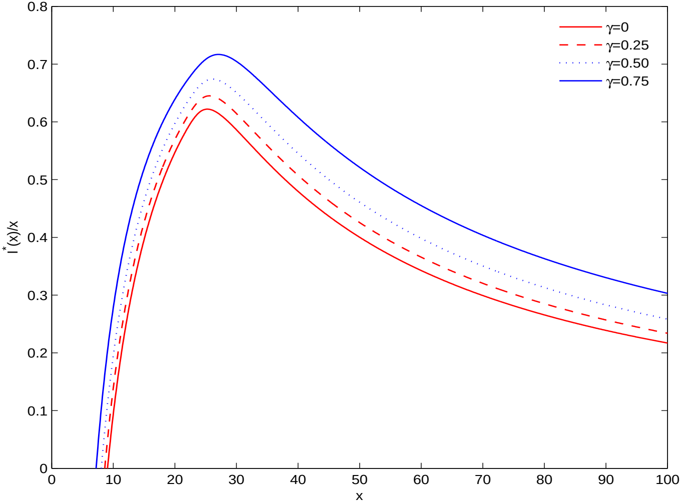
Independence, $W \perp X$

- With bankruptcy costs $(0 \leq \gamma < 1)$, any optimal contract entails a positive deductible.
- With no bankruptcy costs $(\gamma = 1)$, full insurance is optimal.

PARTIAL RECOVERY AND INDEMNITY SCHEDULE



PARTIAL RECOVERY AND INSURED FRACTION



COINSURANCE

Optimal coinsurance

When I^* differentiable and interior, it satisfies

$$I^{*\prime}(x) = \frac{\delta_0(x) - \delta_1(x) - h(x)(\delta_5(x,\gamma) - \delta_6(x,\gamma) + \delta_7(x,\gamma))}{\delta_0(x) - h(x)\left(\delta_2(x) + \delta_3(x,\gamma) + \delta_4(x,\gamma)\right)},$$

with $h(x)$ the hazard rate $f_A(a|x)/\mathbb{P}(A \geq a|X = x)$.

Implications

- ⋆ insured fraction may be tent-shaped
- ⋆ nonmonotonicity results of Schlesinger/vonSchulenburg (1987), Doherty/Schlesinger (1990) far from surprising
- ⋆ background risk and default risk *jointly* shape coinsurance rates

$$I^{*\prime}(x) = \frac{\delta_0(x) - h(x)\delta_5(x,\gamma)}{\delta_0(x) - h(x)\left(\delta_3(x,\gamma) + \delta_4(x,\gamma)\right)} \quad \text{when } A, W, X \text{ independent}$$

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CONCLUSION

Main findings

- contract design with endogenous counterparty risk
- limited liability, bankruptcy costs, role of dependence
- Pareto optimal insurance contracts
- existence, necessary and sufficient conditions

Positive and normative implications

- risk sharing patterns for high layers of exposures
- insolvency risk and insurance demand "puzzles"
- optimal design of (re)insurance programs

THANK YOU