Optimal Taxation of Entrepreneurial Capital with Private Information

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Abstract

This paper studies optimal taxation of entrepreneurial capital with private information and multiple assets. Entrepreneurial activity is subject to a dynamic moral hazard problem and entrepreneurs face idiosyncratic capital risk. We first characterize the optimal allocation subject to the incentive compatibility constraints resulting from private information. The optimal tax system implements such an allocation as a competitive equilibrium for a given market structure. We consider several market structures that differ in the assets or contracts traded, and obtain three novel results. First, the intertemporal wedge on entrepreneurial capital can be negative, as more capital relaxes the entrepreneur's incentive compatibility constraints. Second, *differential asset taxation* is optimal. Marginal taxes on financial assets depend on the correlation of their returns with idiosyncratic capital risk, which determines their hedging value. Entrepreneurial capital always receives a subsidy relative to other assets in bad states. Third, if entrepreneurs are allowed to sell equity, the optimal tax system embeds a prescription for *double taxation* of capital income- at the firm level and at the investor level.

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1 Introduction

A basic tenet in corporate finance is that incentive problems due to informational frictions play a central role in entrepreneurial activity (Tirole, 2006). Empirical evidence on financing and ownership patterns provides strong support for this view (Fazzari, Hubbard and Petersen, 1988). Yet, all recent papers on optimal *asset* taxes in the presence of informational frictions limit their analysis to economies with incentive problems in the supply of *labor*.[1] This is somewhat surprising, given that entrepreneurs hold over 40% of household wealth in the US economy (Cagetti and De Nardi, 2008) and face substantial idiosyncratic risk (Moscowitz and Vissing-Jorgensen, 2002).[2] This paper takes a first step in analyzing optimal taxation in an economy with entrepreneurial activity.[3]

Our main assumption is that entrepreneurial activity is subject to a dynamic moral hazard problem. Specifically, expected returns to capital positively depend on entrepreneurial effort, which is private information. Entrepreneurial earnings and investment are observable. The dependence of returns on effort implies that capital is agent specific and generates idiosyncratic *capital risk*. This structure of the moral hazard problem encompasses a variety of more specific frictions, such as private benefit taking or choice of projects with lower probability of success that deliver benefits in terms of perks or prestige to the entrepreneur studied in corporate finance, as well as environments with non-contractible earnings.[4] The approach used to derive the optimal tax system builds on the seminal work of Mirrlees (1971), and extends it to a dynamic setting. First, we characterize the constrained-efficient allocation, which solves a planning problem subject to the incentive compatibility constraints resulting from the private information. We then construct a tax system that implements such an allocation as a competitive equilibrium for a given market structure. A market structure specifies the feasible trades between agents and the distribution of ownership rights, which are treated as exogenous. A tax system implements the constrained-efficient allocation if such an allocation constitutes a competitive equilibrium under this tax system for the assumed market structure. The only a priori restriction on taxes is that they must depend on observables. The resulting tax system optimizes the trade-off between insurance and incentives. The analysis considers fiscal implementation in two different market structures, which both allow for multiple assets.

A crucial property of the environment is that idiosyncratic returns to capital, which depend on entrepreneurial effort, differ from aggregate capital returns, which are exogenous to the individual entrepreneur. The optimal allocation displays a positive wedge between the aggregate return to capital and the entrepreneurs' intertemporal marginal rate of substitution.[5] However, this *ag-*

[1] Kocherlakota (2010) provides an excellent presentation of this literature, known as New Dynamic Public Finance.

[2] Entrepreneurs are typically identified with households who hold equity in a private business and play an active role in the management of this business. Cagetti and De Nardi (2008) document, based on the Survey of Consumer Finances (SCF), that entrepreneurs account for 11.5% of the population and they hold 41.6% of total household wealth. Using the PSID, Quadrini (1999) documents that entrepreneurial assets account for 46% of household wealth. Moscowitz and Vissing-Jorgensen (2002) identify entrepreneurial capital with private equity, and they document that its value is similar in magnitude to public equity from SCF data.

[3] Entrepreneurial models with moral hazard and endogenous investment have been neglected by the dynamic contracting literature in macroeconomics. An exception is Kahn and Ravikumar (1999). They focus on an implementation with financial intermediaries and rely on numerical simulations. They do not provide an analytical characterization of the wedges associated with the constrained-efficient allocation.

[4] We prove this formally in Appendix C.

[5] Golosov, Kocherlakota and Tsyvinski (2003) show that this wedge is positive for a large class of private information economies with idiosyncratic *labor* risk.

gregate intertemporal wedge is not related to the entrepreneurs' incentives to exert effort, since the individual intertemporal rate of transformation differs from the aggregate. The *individual* intertemporal wedge can be *positive or negative.* The intuition for this result is simple. More capital increases an entrepreneur's consumption in the bad states, which provides insurance and undermines incentives. On the other hand, expected capital returns are increasing in entrepreneurial effort. This effect relaxes the incentive compatibility constraint and is shown to dominate when the spread in capital returns is sufficiently large or when the variability of consumption across states is small at the constrained-efficient allocation.

The properties of optimal capital income taxes depend on the effect of asset holdings on incentives in each particular market structure. In any market structure, a tax on entrepreneurial capital is necessary to implement the constrained-efficient allocation. The optimal marginal tax on *entrepreneurial* capital is increasing in earnings, when the individual intertemporal wedge is negative, decreasing when it is positive. The incentive effects of agent specific capital holdings provide the rationale for this result. When the intertemporal wedge is negative (positive), more capital relaxes (tightens) the incentive compatibility constraint, and the optimal tax system encourages (discourages) entrepreneurs to hold more capital by reducing (increasing) the after tax volatility of capital returns.

To study the implications of moral hazard for optimal asset taxes, we examine two different market structures in which entrepreneurs can trade multiple securities, in addition to hold entrepreneurial capital.[6] In the first, entrepreneurs can trade an arbitrary set of risky financial securities, which are in zero net supply and can be contingent on the realization of a variety of exogenous shocks. In addition, the entrepreneurs can trade a risk-free bond. We show that the optimal tax system equates the after tax returns on all assets in each (idiosyncratic) state. The optimal marginal tax on risk-free bonds is decreasing in entrepreneurial earnings, while the optimal marginal taxes on risky securities depend on the correlation of their returns with idiosyncratic risk. Moreover, entrepreneurial capital is subsidized relative to other assets in the bad states. These predictions give rise to a novel theory of *optimal differential asset taxation*, in which the optimal marginal tax on any asset depends on the hedging value of that asset for the entrepreneur. In the second market structure we consider, entrepreneurs can sell shares of their capital and buy shares of other entrepreneurs' capital. Viewing each entrepreneur as a firm, this arrangement introduces an equity market with a positive net supply of securities. We show that the optimal tax system then embeds a prescription for optimal *double taxation of capital*- at the firm level, through the marginal tax on entrepreneurial earnings, and at the investor level, through a marginal tax on stocks returns. Specifically, it is necessary that the tax on earnings be "passed on" to stock investors via a corresponding tax on dividend distributions to avoid equilibria in which entrepreneurs sell all their capital to outside investors. In such equilibria, an entrepreneur exerts no effort and thus it is impossible to implement the constrained-efficient allocation. Since, in addition, marginal taxation of dividends received by outside investors is necessary to preserve their incentives, earnings from entrepreneurial capital are subject to double taxation.

The differential tax treatment of financial securities and the double taxation of capital income in the United States and other countries have received substantial attention in the empirical public finance literature, since they constitute a puzzle from the standpoint of optimal taxation models that abstract from incentive problems.[7] The optimal tax system in our implementations is designed to ensure that entrepreneurs have the correct exposure to their idiosyncratic capital

[6] Most analyses of optimal asset taxes with private information only consider market structures in which only one type of asset is available, either aggregate capital or a risk-free bond.

[7] See Gordon and Slemrod (1988), Gordon (2003), Poterba (2002) and Auerbach (2002)

risk to preserve incentive compatibility. Holdings of additional assets affect this exposure in a measure that depends on their correlation with entrepreneurial capital returns, and thus should be taxed accordingly. The ability to sell equity introduces an additional channel through which entrepreneurs can modify their exposure to idiosyncratic risk. A tax on dividend distributions is then required to optimally adjust the impact of a reduction in the entrepreneurs' ownership stake on their exposure to idiosyncratic risk. This explains the need for double taxation of capital.

Another important property of these implementations is that the optimal marginal taxes do not depend on the *level* of asset holdings. Consequently, total entrepreneurial asset holdings need not be observed by the government or other private agents to administer the optimal tax system. For example, if assets are traded via financial intermediaries, the asset taxes could be levied according to the marginal schedule prescribed by the government (depending only on observables) on any unit transaction and collected at the source. This property has important implications for the role of tax policy in implementing optimal allocations. Even under the same informational constraints as private agents, the government can influence the portfolio choices of entrepreneurs through the tax system.

This paper is related to the recent literature on dynamic optimal taxation with private information. Albanesi and Sleet (2006) and Kocherlakota (2005b), focus on economies with idiosyncratic risk in labor income and do not allow agents to trade more than one asset. They show that the optimal marginal tax on capital income is decreasing in income in economies with labor risk, and this property holds independently of the nature of the asset. Golosov and Tsyvinski (2007) analyze fiscal implementations in a Mirrleesian economy with hidden bond trades and show that a linear tax on capital is necessary, since competitive insurance contracts fail to internalize the effect of the equilibrium bond price on incentives. Farhi and Werning (2010) study optimal estate taxation in a dynastic economy with private information. They find that the intertemporal wedge is negative if agents discount the future at a higher rate than the planner and that this implies the optimal estate tax is progressive. Grochulski and Piskorski (2005) study optimal wealth taxes in economies with risky human capital, where human capital and idiosyncratic skills are private information. The paper is also related to the corporate finance literature on agency and optimal investment (Tirole 2006, Fishman and DeMarzo, 2006). Limited commitment is another friction typically associated with entrepreneurial activity (Quadrini, 1999, and Cagetti and De Nardi, 2006). Cagetti and De Nardi (2004) explore the effects of tax reforms in a quantitative model of entrepreneurship where limited commitment gives rise to endogenous borrowing constraints. Vereshchagina and Hopenhayn (2009) study a model of occupational choice with borrowing constraints to explain why self-financed entrepreneurs may find it optimal to invest in risky projects offering no risk premium. Finally, Angeletos (2007) studies competitive equilibrium allocations in a model with exogenously incomplete markets and idiosyncratic capital risk.

The plan of the paper is as follows. Section 2 present the economy and studies constrained-efficient allocations and the incentive effects of capital. Section 3 investigates optimal taxes. Section 3 concludes. All proofs can be found in the Appendix.

2 Model

The economy is comprised of a continuum of unit measure of ex ante identical entrepreneurs who live for two periods. Their lifetime utility is:

$$U = u(c_0) + \beta u(c_1) - v(e),$$

where, c_t denotes consumption in period $t = 0, 1$ and e denotes effort exerted at time 0, with $e \in \{0, 1\}$. We assume $\beta \in (0, 1)$, $u' > 0$, $u'' < 0$, $v' > 0$, $v'' > 0$, and $\lim_{c \to 0} u'(c) = \infty$.

Entrepreneurs are endowed with K_0 units of the consumption good at time 0 and can operate a production technology at time 1. Denoting with K_1 the amount invested at time 0, that is physical capital devoted to the entrepreneurial activity, then output at time 1 is given by:

$$Y_1 = F(K_1, x) = K_1 (1 + x),$$

where x is the random net return on capital. Capital returns are stochastic, with probability distribution:

$$x = \begin{cases} \overline{x} \text{ with probability } \pi(e), \\ \underline{x} \text{ with probability } 1 - \pi(e), \end{cases} \tag{1}$$

with $\bar{x} > \underline{x}$ and $\pi(1) > \pi(0)$. The first assumption implies that $E_1(x) > E_0(x)$, where E_e denotes the expectation operator for probability distribution $\pi(e)$. Hence, the expected capital returns are increasing in effort[8]. Effort can be thought as being exerted at time 0 or at the beginning of time 1, before capital returns are realized.

The entrepreneurial activity is subject to a dynamic moral hazard problem. Effort is *private information.* Output, the production function F, and the distribution of capital returns conditional on effort, and the realized value of x are *public information.* Since the dependence of output on physical capital is deterministic and x is observed, the level of K_1 is also public information.

The resulting formulation for the incentive problem is new to the dynamic optimal taxation literature. It is straightforward adaptation of Rogerson's (1985) dynamic moral hazard model to a setting with *idiosyncratic capital risk*, more suitable for analyzing the incentive problems faced by entrepreneurs. In Rogerson's model, effort is private information, output is observable and depends stochastically on effort and the probability distribution of output given effort is known. Here, output corresponds to Y_1 and depends deterministically on entrepreneurial capital K_1 and stochastically on effort, the entrepreneur's hidden action. This model is also a generalization of the framework is presented in Holmstrom and Tirole (1997).

This formulation of the moral hazard problem, though simple, encompasses a variety of more specific agency problems with contractable capital studied in the corporate finance literature. For example, moral hazard can result from the entrepreneur's ability to choose projects with lower probability of success that deliver private benefits, such as perks or prestige, to the entrepreneur, when the project choice is not observed or contactable (Tirole, 2006). Interpreting $\bar{x}$ as success and $\underline{x}$ as failure, $K_1(1 + \underline{x})$ represents the salvage value of the entrepreneurial activity. High effort would then correspond to the choice of a project with higher probability of success and low effort as a choice of project with lower probability of success and private benefit $v(1) - v(0)$. The private benefit is the utility gain corresponding to perks for the entrepreneur or to lower utility cost of operating the inferior project. More generally, our formulation of the moral hazard problem also encompasses environments with unobservable capital returns, in which the the entrepreneur can divert part of the output for her own private benefit.[9] Appendix C presents a version of the model in which entrepreneurial capital returns, x, are not observed based on Fishman and DeMarzo (2006)[10], and shows that the main properties of the optimal allocation derived in Section 2.1, are unchanged.

[8]Appendix B analyzes a more general version of the production technology, allowing for decreasing returns to capital and $x \in \{x_1, x_2, ...x_N\}$ with $N > 2$.

[9]Quadrini (2004), Clementi and Hopenhayn (2006) and Gertler (1992) consider variants of this incentive problem.

[10]Tirole (2006) shows that this framework is also isomorphic to a class of costly state verification models.

2.1 Constrained-Optimal Allocation

The constrained-optimal allocation is the solution to a particular contracting problem. A planner/principal maximizes an entrepreneur's expected lifetime utility by choice of a state contingent consumption and effort allocation. The planner's problem is:

$$\{e^*, K_1^*, c_0^*, c_1^*(\underline{x}), c_1^*(\bar{x})\} = \arg \max_{e\in\{0,1\}, K_1\in[0,K_0],\ c_0, c_1(x)\geq 0} u(c_0) + \beta E_e u(c_1(x)) - v(e) \quad \text{(Problem 1)}$$

subject to

$$c_0 + K_1 \leq K_0, \tag{2}$$

$$E_e c_1(x) \leq K_1 E_e(1+x), \tag{3}$$

$$\beta E_1 u(c_1(x)) - \beta E_0 u(c_1(x)) \geq v(1) - v(0), \tag{4}$$

where E_e denotes the expectation operator with respect to the probability distribution $\pi(e)$. Constraints (2)-(3) can be interpreted as a participation or resource feasibility constraint for the principal[11], while (43) is the incentive compatibility constraint, arising from the unobservability of effort. The value of the optimized objective for Problem 1 is denoted with $U^*(K_0)$.

Proposition 1 *An allocation* $\{e^*, K_1^*, c_0^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ *that solves Problem* 1 *with* $e^* = 1$ *satisfies:*

$$\frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))} = \frac{\left[1 + \mu\frac{(\pi(1)-\pi(0))}{\pi(1)}\right]}{\left[1 - \mu\frac{(\pi(1)-\pi(0))}{(1-\pi(1))}\right]} > 1, \tag{5}$$

$$u'(c_0^*) E_1\left[\frac{1}{u'(c_1^*(x))}\right] = \beta E_1(1+x), \tag{6}$$

where $\mu > 0$ *is the multiplier on the incentive compatibility constraint (43).*

Equation (5) implies that $c_1^*(\bar{x}) > c_1^*(\underline{x})$ – there is *partial insurance*. Equation (6) determines the intertemporal profile of constrained-efficient consumption. By Jensen's inequality, (6) immediately implies:

$$u'(c_0^*) < \beta E_1(1+x) E_1[u'(c_1^*(x))]. \tag{7}$$

Equation (7) suggests the presence of a wedge between the entrepreneurs' intertemporal rate of substitution and $E_1(1+x)$, which corresponds to the aggregate intertemporal rate of transformation. However, entrepreneurs face an idiosyncratic process for capital returns, corresponding to $(1+x)$. Hence, the individual intertemporal rate of transformation is given by the stochastic variable $1+x$. It is then useful to distinguish the intertemporal wedge with respect to the individual return to capital, from the wedge with respect to the aggregate return to capital. We introduce these notions in a formal definition.

[11] The planner takes the initial distribution of capital as given. Given that the investment technology is linear in capital, the efficient distribution of capital is degenerate, with one entrepreneur operating the entire economy wide capital stock. Since this result is not robust to the introduction of any degree of decreasing returns, and this in turn would not alter the structure of the incentive problem, we simply assume that the planner cannot transfer initial capital across agents. In Appendix B, we generalize the model to allow for decreasing returns to entrepreneurial capital. In this setting, the optimal value of capital is strictly positive for all entrepreneurs.

Definition 1 *The* individual *intertemporal wedge, IW_K, is:*

$$IW_K = \beta E_1 u'\left(c_1^*\left(x\right)\right)\left(1+x\right) - u'\left(c_0^*\right). \tag{8}$$

The aggregate *intertemporal wedge, IW, is:*

$$IW = \beta E_1\left(1+x\right) E_1 u'\left(c_1^*\left(x\right)\right) - u'\left(c_0^*\right). \tag{9}$$

The *individual* intertemporal wedge is the difference between the individual marginal benefit of increasing capital by one unit and the individual marginal cost, given by the marginal utility of current consumption. By (8) and the definition of covariance, it immediately follows that:

$$\mathrm{IW}_K = \mathrm{IW} + \beta Cov_1\left(u'\left(c_1^*\left(x\right)\right), x\right). \tag{10}$$

By (7), the aggregate intertemporal wedge is positive. Equation (5) and strict concavity of utility imply: $Cov_1\left(u'\left(c_1^*\left(x\right)\right), x\right) < 0$. Then, it follows from equation (10) that $\mathrm{IW}_K <$IW and that the sign of IW_K can be *positive or negative.*

This result stands in contrast with the standard view of intertemporal wedges in private information economies. Rogerson (1985) shows that in repeated moral hazard models, the wedge between the individual marginal benefit of increasing wealth and the individual marginal cost is always positive. Golosov, Kocherlakota and Tsyvinski (2003) prove that this result holds very generally in economies with idiosyncratic labor income risk. The rationale for this result is the adverse effect of saving on incentives. Additional wealth reduces the dependence of future consumption on future income and, therefore, effort or labor supply. A positive intertemporal wedge signals that in addition to the private marginal cost of increasing savings, there is an additional efficiency cost stemming from the resulting adverse effect on incentives.

In this model, the sign of the individual intertemporal wedge is also related to the incentive effects of increasing individual holdings of entrepreneurial capital. The key difference is that an entrepreneur's marginal benefit from increasing capital depends on effort. To examine the role of this feature of the environment, it is useful to derive an expression for the individual intertemporal wedge from the first order necessary conditions for Problem 1:

$$\mathrm{IW}_K = \mu\left(\pi\left(1\right) - \pi\left(0\right)\right)\beta\left[u'\left(c_1^*\left(\underline{x}\right)\right)\left(1+\underline{x}\right) - u'\left(c_1^*\left(\bar{x}\right)\right)\left(1+\bar{x}\right)\right]. \tag{11}$$

This expression can be rewritten as:

$$\mathrm{IW}_K = \beta\mu\left[E_0\left(1+x\right)u'\left(c_1^*\left(x\right)\right) - E_1\left(1+x\right)u'\left(c_1^*\left(x\right)\right)\right]. \tag{12}$$

This expression clarifies that IW_K is positive when the marginal benefit of increasing entrepreneurial capital, $E_e\left(1+x\right)u'\left(c_1^*\left(x\right)\right)$, declines with effort. This implies that an additional unit of capital tightens the incentive compatibility constraint. Therefore, the marginal cost of increasing capital is greater than the entrepreneur's private cost, given by the utility value of forgone time 0 consumption, $u'\left(c_0^*\right)$, which corresponds to the positive wedge. This is the standard result in the literature. By contrast, IW_K is *negative* when the marginal benefit of increasing entrepreneurial capital rises with effort. This signals the presence of an additional *benefit* from increasing capital, stemming from the fact that more capital relaxes an entrepreneur's incentive compatibility constraint.

To explore the determinants of the impact of capital on entrepreneurial incentives, it is useful to rewrite (11) as follows:

$$\mathrm{IW}_K = \mu\left(\pi\left(1\right) - \pi\left(0\right)\right)\beta\left\{\left[u'\left(c_1^*\left(\underline{x}\right)\right) - u'\left(c_1^*\left(\bar{x}\right)\right)\right]\left(1+\underline{x}\right) - \left(\bar{x} - \underline{x}\right)u'\left(c_1^*\left(\underline{x}\right)\right)\right\}. \tag{13}$$

Equation (13) decomposes the individual intertemporal wedge into a *wealth effect*, which corresponds to the first term inside the curly brackets, and an opposing *substitution effect.* The wealth effect captures the adverse effect of additional capital on incentives, arising from the fact that more capital increases consumption in the bad state. This provides insurance and generates a negative relation between capital and effort. The substitution effect is linked to the positive dependence of expected returns on entrepreneurial effort. This tends to increase effort at higher levels of capital. The size of the wealth effect is positively related to the spread in consumption across states that drives the entrepreneurs' demand for insurance. The strength of the substitution effect depends on the spread in capital returns, which determines by how much the expected return from capital increases under high effort.

The *aggregate* intertemporal wedge is proportional to the difference between the entrepreneurs' intertemporal marginal rate of substitution and the aggregate intertemporal rate of transformation, which corresponds to $E_1(1+x)$. This wedge is always positive by (9). This wedge captures the incentive effects of increasing holdings of a risk-free asset with return equal to the expected return to entrepreneurial capital, $E_1(1+x)$. As in Rogerson (1985) and Golosov, Kocherlakota and Tsyvinski (2003), higher risk-free wealth always has an adverse effect on incentives, because it reduces the dependence of consumption on the realization of idiosyncratic capital returns, and therefore on effort, thus tightening the incentive compatibility constraint.

This observation will play a key role in the fiscal implementation of the optimal allocation. As we will show in section 3, the equilibrium after tax return on any risk free asset is equal to $E_1(1+x)$. The differential incentive effects of entrepreneurial capital and a risk less asset with the same expected return will lead to a prescription of optimal differential asset taxation.

It is not possible to generally sign the individual intertemporal wedge. However, we can derive an intuitive condition that guarantees $\mathrm{IW}_K < 0$. This condition simply amounts to the coefficient of relative risk aversion being weakly smaller than 1. No additional restrictions on preferences or the returns process are necessary.

Proposition 2 *Let $\sigma(c) \equiv -cu''(c)/u'(c)$ denote the coefficient of relative risk aversion for the utility function $u(c)$. Then, $IW_K < 0$ for $\sigma(c) \leq 1$.*

Portfolio theory offers an interpretation for this result. As shown in Gollier (2001), the amount of holdings of an asset increase in its expected rate of return when the substitution effect dominates, that is for $\sigma(c) < 1$. Since under high effort the rate of return on capital is higher than under low effort, a similar logic applies in this setting.[12] This interpretation however is incomplete, since an entrepreneur's effort and the returns to her capital are endogenous and linked via (1).

How relevant is this finding? The value of the coefficient of relative risk aversion is very disputed, due to difficulties in estimation. Typical values of $\sigma(c)$ used in macroeconomics with constant relative risk aversion preferences largely exceed 1. On the other hand, Chetty (2006) develops a new method for estimating this parameter using data on labor supply behavior to bound the coefficient of relative risk aversion. He argues that for preferences that are separable in consumption and labor effort, $\sigma(c) \leq 1$ is the only empirically relevant case. This finding suggests that low values of $\sigma(c)$, relatively to those used in macroeconomics, may be quite plausible.

Since Proposition 2 is merely sufficient, IW_K can be negative for $\bar{x} - \underline{x}$ large enough even if $\sigma(c) > 1$. For given spread in capital returns, it is more likely for IW_K to be negative when $\sigma(c)$

[12]Levhari and Srinivasan (1969) and Sandmo (1970) study precautionary holdings of risky assets and discuss similar effects.

Table 1: Numerical Example

σ	$[0.95, 8]$
$\{\underline{x}, \bar{x}\}$	$\{0.05, 0.38\}$
$E_1 x$	0.30
$E_0 x$	0.13
SD_1	0.12
γ	0.08
K_0	1
a, b	$0.25, 0.5$
β	$\frac{1}{1.3}$

large, since when risk aversion is high the optimal spread in consumption across states is be small. Hence, for $\sigma(c) > 1$, the sign of IW_K depends on the value of fundamental parameters that govern the variance of entrepreneurial earnings and an entrepreneur's risk aversion. We now turn to some numerical examples to explore the possibilities.

2.2 Numerical Example

Let $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ for $\sigma > 0$ and $v(e) = \gamma e$, $\gamma > 0$. Then, σ corresponds to the coefficient of relative risk aversion and γ is the cost of high effort. The probability a high capital returns depends linearly on effort, according to $\pi(e) = a + be$, with $a \geq 0$, $b > 0$ and $2a + b \leq 1$. The parameter b represents the impact of effort on capital returns. We consider values of a and b such that the standard deviation of x is equalized under high and low effort. This requires $a = 0.25$ and and $b = 0.5$, so that $\pi(1) = 0.75$ and $\pi(0) = 0.25$. We set $E_1 x = 0.3$ and fix $\{\underline{x}, \bar{x}\}$ so that the standard deviation of capital returns is equal to 14%. Finally, we set $\gamma = 0.08$ and let σ vary between 0.95 and 8. Initial capital is normalized to $K_0 = 1$. Parameter values are reported in Table 1[13].

Figure 1 displays the results. The left panel plots the individual intertemporal wedge (solid line) and the aggregate intertemporal wedge (dashed line). The right panel plots c_0^* (dashed line), $c_1^*(x)$ in each state (solid lines) and output $K_1^*(1+x)$ in each state (dotted lines). High effort is optimal for all parameter values reported.

The individual intertemporal wedge is non-monotonic in σ. It is negative and rising in σ for $\sigma \leq 1.6$, it then declines and starts rising again for σ approximately equal to 4, converging to 0 from below. IW_K is negative for high enough values of σ, since the spread across states in optimal consumption decreases with σ, for given spread in capital returns, which decreases the wealth effect as illustrated by equation (11). The individual intertemporal wedge is negative for all σ lower than 1.3, positive for values of σ between 1.3 and 2.2. A greater spread in capital returns would increase the range of values of σ for which IW_K is negative. The aggregate intertemporal wedge is always positive, but is also displays a non monotonic pattern in σ, initially rising and then declining in this variable. It tends to 0 for high enough values of σ, since the spread in consumption across

[13]If we identify entrepreneurial capital with private equity, then x corresponds to the net returns on private equity. Moskowitz and Vissing-Jorgensen (2002) estimate these returns using the Survey of Consumer Finances. They find that the average returns to private equity, including capital gains and earnings, are 12.3, 17.0 and 22.2 percent per year in the time periods 1990-1992, 1993-1995, 1996-1998. It is much harder to estimate the variance of idiosyncratic returns. Evidence from distributions of entrepreneurial earnings, conditional on survival, suggest that this variance is much higher than for public equity.

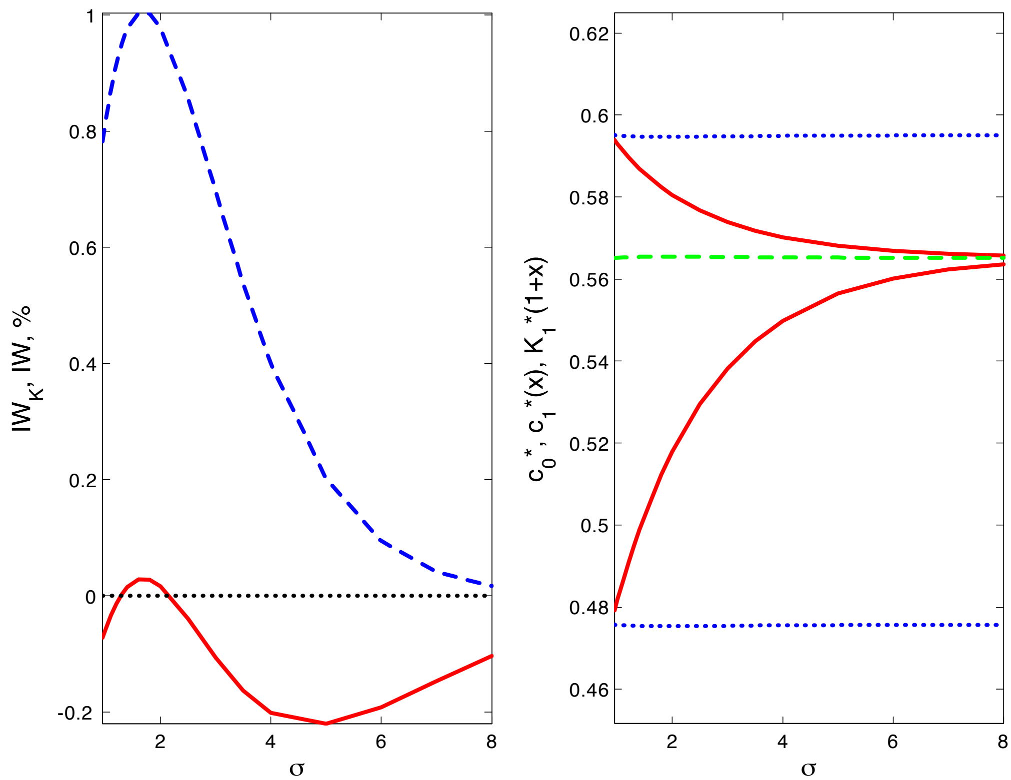

FIGURE 1: Constrained-efficient allocation

states is vanishingly small[14].

3 Optimal Differential Asset Taxation

We now consider the design of optimal tax systems. An optimal tax system implements the constrained-efficient allocation in a setting where agents can trade in decentralized markets and thus depends on the market structure. A market structure specifies the distribution of ownership rights, the feasible trades between agents and any additional informational assumptions beyond the primitive restrictions that comprise the physical environment. The tax system influences agents' choices by affecting their budget constraint. A tax system implements the constrained-efficient allocation if such an allocation arises as an equilibrium under this tax system for a given market structure. This requires that individuals find that allocation optimal given the tax system and prices, and that those prices satisfy market clearing. The only restriction imposed on candidate tax systems is that the taxes and transfers must be conditioned only on observables.

The benchmark market structure we consider allows entrepreneurs to choose capital and effort, as well as trade a set of financial securities in zero net supply. The set of financial securities is arbitrary and the implicit assumption is that they are costlessly issued. We first consider the case of a risk-free bond. This arrangement is a generalization of the market structure considered in

[14] The fact that IW_K starts rising for values of σ greater than 4 is due to the fact that for $\sigma \geq 4$, $c_1^*(\bar{x})$ is approximately constant, while $c_1^*(\underline{x})$ continues to rise with σ. By (11), this causes IW_K to rise. For higher values of σ than the ones reported, the optimal effort drops to 0. In that case, entrepreneurs are given full insurance and there are no intertemporal wedges.

Albanesi and Sleet (2006) and Kocherlakota (2005). We later allow for the possibility that the securities payoff is contingent on stochastic variables which can be correlated with idiosyncratic capital returns.

Decisions occur as follows. Entrepreneurs are endowed with initial capital K_0 and choose investment, K_1, and bond purchases, B_1, at the beginning of period 0. They then exert effort. At the beginning of period 1, x is realized and securities pay off. Finally, the government collects taxes and agents consume. The informational structure is as follows: K_1 and x are public information, while effort is private information. We assume that all bond purchases, B_1, are observable. The tax system is given by a time 1 transfer from the entrepreneurs to the government, conditional on observables, and represented by the function $T(B_1, K_1, x)$. We restrict attention to functions $T(\cdot)$ that are differentiable almost everywhere in their first argument and satisfy $E_1 T(B_1, K_1, x) = 0$. This restriction simply corresponds to the government budget constraint, since government consumption is zero. Let r denote to net return on the risk-free bond.

An entrepreneur's problem is:

$$\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}(B_0, K_0, T) = \arg \max_{K_1 \geq 0,\ B_1 \geq \bar{B},\ e \in \{0,1\}} U(e, K_1, B_1; T) - v(e), \qquad \text{(Problem 3)}$$

subject to

$$K_0 + B_0 - K_1 - B_1 \geq 0,$$

$$K_1(1+x) + (1+r)B_1 - T(B_1, K_1, x) \geq 0 \text{ for } x \in X,$$

where

$$U(e, K_1, B_1; T) = u(K_0 + B_0 - K_1 - B_1) + E_e u(K_1(1+x) + B_1(1+r) - T(K_1, B_1, x)).$$

The borrowing constraint, $B_1 \geq \bar{B}$, is imposed to ensure that an entrepreneur's problem is well defined. The debt limit, $\bar{B}$, corresponds to the "natural" limit that ensures that agents will be able to pay back all outstanding debt in the low state. The initial bond endowment, B_0, can be interpreted as a transfer from the government to the entrepreneurs.

Definition 2 *An equilibrium is an allocation $\{c_0, e, K_1, B_1, c_1(\underline{x}), c_1(\bar{x})\}$ and initial endowments B_0 and K_0 for the entrepreneurs, a tax system $T(K_1, B_1, x)$, with $T: [\bar{B}, \infty) \times [0, \infty) \times \{\underline{x}, \bar{x}\} \to \mathbb{R}$, government bonds B_1^G, and an interest rate, $r \geq 0$, such that: i) given T and r and the initial endowments, the allocation solves Problem 3; ii) the government budget constraint holds in each period; iii) the bond market clears, $B_1^G = B_1$.*

The restriction on the domain of the tax system is imposed to ensure that the tax is specified for all values of K_1 and B_1 feasible for the entrepreneurs. We now define our notion of implementation.

Definition 3 *A tax system $T: [\bar{B}, \infty) \times [0, \infty) \times \{\underline{x}, \bar{x}\} \to \mathbb{R}$ implements the constrained-efficient allocation, if the allocation $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$, the tax system T, jointly with an interest rate r, government bonds B_1^G, and initial endowments B_0 and K_0 constitute an equilibrium.*

There two components of the implementation. The requirement that, for given r, B_1^* and B_0, the allocation $\{K_1^*, e^*, c_1^*(x)\}$ is optimal for an entrepreneur under the tax system. The second requirement is that markets clear at r, B_1^* and B_0. Since the goods market clears by construction at the constrained optimal allocation, this restriction only applies to the bond market. We first discuss the entrepreneurs' problem for a given r, B_1^* and B_0 and then consider the equilibrium values of these variables.

Without loss of generality, we can restrict attention to tax systems of the form: $T(K_1, B_1, x) = \rho(x) + \tau_K(x) K_1 + \tau_B(x) B_1$. The natural borrowing limit then corresponds to $\bar{B} = -\frac{[K_1(1+\underline{x}-\tau_K(\underline{x}))-\rho(\underline{x})]}{1+r-\tau_B(\underline{x})}$. Let $B_1^* \geq \bar{B}$ be the level of bond holdings to be implemented. We begin our characterization with a negative result and identify a tax systems in the class $T(K_1, B_1, x)$ that does *not* implement the constrained-efficient allocation.

Let B_0 and $T(K_1^*, B_1^*, x)$ respectively satisfy:

$$c_0^* = B_0 + K_0 - K_1^* - B_1^*, \tag{14}$$

$$c_1^*(x) = K_1^*(1+x) + (1+r) B_1^* - T(K_1^*, B_1^*, x). \tag{15}$$

Then, K_1^* and B_1^* are affordable and, if they are chosen by an entrepreneur, incentive compatibility implies that high effort will also be chosen at time 1. Evaluating the entrepreneurs' Euler equation at $\{1, K_1^*, B_1^*\}$, we can write:

$$u'(c_0^*) = \beta E_1 [u'(c_1^*(x))(1+x-\tau_K(x))], \tag{16}$$
$$u'(c_0^*) = \beta E_1 [u'(c_1^*(x))(1+r-\tau_B(x))]. \tag{17}$$

The restrictions on $T(K_1^*, B_1^*, x)$ implied by (14)-(15) and (16)-(17) do not fully pin down the tax system and do not ensure that the constrained-efficient allocation is chosen by an entrepreneur. To see this, let $\tau_K(\bar{x}) = \tau_K(\underline{x}) = \bar{\tau}_K$ and $\tau_B(\bar{x}) = \tau_B(\underline{x}) = \bar{\tau}_B$, so that marginal asset taxes do not depend on x, with $\bar{\tau}_K$ and $\bar{\tau}_B$ that satisfy (16)-(17). Set $\bar{\rho}(x)$ so that (15) holds under $\bar{\tau}_K$, $\bar{\tau}_B$, and let $\bar{T}(K_1, B_1, x) = \bar{\rho}(x) + \bar{\tau}_K K_1 + \bar{\tau}_B B_1$. Such a tax system is fully specified and guarantees that the necessary conditions for the entrepreneur's problem are satisfied at the constrained-efficient allocation.

The Euler equation for entrepreneurial capital, (16), implies:

$$\bar{\tau}_K = \frac{\beta E_1 [u'(c_1^*(x))(1+x)] - u'(c_0^*)}{\beta E_1 u'(c_1^*(x))}.$$

It follows that $\bar{\tau}_K$ has the same sign as the individual intertemporal wedge, IW_K. By contrast, (17) implies that $\bar{\tau}_B$ is always positive, since the intertemporal wedge on the bond is positive. By construction, the following inequalities hold:

$$u'(c_0^*) \lesseqgtr \beta E_0 [u'(c_1^*(x))(1+x-\bar{\tau}_K)] \text{ if } \mathrm{IW}_K \gtreqless 0, \tag{18}$$
$$u'(c_0^*) < \beta(1+r-\bar{\tau}_B) E_0 u'(c_1^*(x)). \tag{19}$$

Since the incentive compatibility constraint is binding, entrepreneurs are indifferent on the margin between choosing high or low effort, these inequalities imply that under this tax system, the entrepreneurs' deviate from the constrained-efficient allocation and choose low effort, a level of bond holdings greater than B_1^* and a level of investment different from K_1^*. In particular, the will choose a level of investment lower/higher than K_1^* if the intertemporal wedge is negative/positive.

The result that non-state dependent marginal asset taxes induce agents to deviate for deviations from the constrained-efficient allocation is familiar from the analysis of economies with idiosyncratic labor risk, in Albanesi and Sleet (2006) and Kocherlakota (2005)[15]. A tax system with marginal

[15] Golosov and Tsyvinski (2006) derive a related result in a disability insurance model.

asset taxes that do not depend on the realization of individual risk at time 1 can ensure that given the right choice of assets entrepreneurs find it optimal to exert high effort, but it cannot prevent entrepreneurs from deviating *both* in their choice of assets and in the choice of effort from the constrained-efficient allocation. This is due to the fact that the marginal value of assets depends on the choice of effort, which generates a complementarity between the choice of portfolio and the choice of effort.

In this economy, entrepreneurs can invest in more than one asset and thus the optimal deviation under $\bar{T}$ involves an extreme portfolio choice. We formally prove this result in the following lemma.

Lemma 1 *Under tax system $\bar{T}$, $\hat{e} = 0$. If $IW_K > 0$, $\hat{B}_1 = \underline{B}$ and $\hat{K}_1 > K_1^*$; if $IW_K < 0$, $\hat{K}_1 = 0$ and $\hat{B}_1 > B_1^*$.*

This lemma shows that rather than choose $\{1, K_1^*, B_1^*\}$, which is affordable and satisfies first order necessary conditions, entrepreneurs find it optimal to choose low effort and adjust their portfolio under the tax system $\bar{T}(K_1, B_1, x)$. Hence, it does not implement the constrained-efficient allocation. The lemma also illustrates that the consequences of adopting a tax system like $\bar{T}$ is which marginal asset taxes do not depend on the realization of individual risk at time 1 are particularly severe when the individual intertemporal wedge is negative. In this case, the entrepreneur's optimal deviation under $\bar{T}$ is to set investment in productive capital equal to 0.

We now construct a tax system that does implement the constrained-efficient allocation. The critical properties of this system are that marginal asset taxes depend on observable capital returns and that after tax returns are equalized across all assets, state by state.

Proposition 3 *A tax system $T^*(B_1, K_1, x) = \rho^*(x) + \tau_B^*(x) B_1 + \tau_K^*(x) K_1$, with $T^* : [\bar{B}, \infty) \times [0, \infty) \times \{\underline{x}, \bar{x}\} \to \mathbb{R}$, and an initial bond endowment B_0^* that satisfy:*

$$1 + r - \tau_B^*(x) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}, \tag{20}$$

$$1 + x - \tau_K^*(x) = \frac{u'(c_0^*)}{\beta u'(c_1^*(x))}, \tag{21}$$

$$c_1^*(x) = K_1^*(1 + x - \tau_K^*(x)) + B_1^*(1 + r - \tau_B^*(x)) - \rho^*(x), \tag{22}$$

and

$$c_0^* = B_0^* + K_0 - K_1^* - B_1^*, \tag{23}$$

ensure that the allocation $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ is optimal for entrepreneurs for some $B_1^* \geq \bar{B}$ and some $r \geq 0$.

The proof proceeds in three steps. It first shows that the only interior solution to the entrepreneur's Euler equations are B_1^* and K_1^* under T^*, and that local second order conditions are satisfied. It then shows that T^* admits no corner solutions to the choice of K_1 and B_1. Moreover, these results do not depend on the value of effort used to compute expectations over time 1 outcomes. Then, K_1^* and B_1^* are the unique solutions to an entrepreneur's portfolio problem irrespective of the value of effort that she might be contemplating. The last step establishes than $\rho^*(x)$ guarantees that, once K_1^* and B_1^*, have been chosen, high effort will be optimal.

The optimal tax system T^* has two main properties. It removes the complementarity between the choice of effort and the choice of capital and bond holdings, thus removing any incentive effects of the entrepreneurs' asset choice. This guarantees that the necessary and sufficient conditions for the *joint* global optimality of K_1^* and B_1^* are satisfied at all effort levels. Moreover, T^* equates after tax returns on all assets in each state. This renders entrepreneurs indifferent over the composition of their portfolio. The next corollary establishes that the tax system T^* implements the constrained-efficient allocation.

Corollary 1 *The tax system $T^*(K_1, B_1, x)$ and initial bond endowment B_0^* defined in Proposition 3, jointly with the allocation $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$, and government bonds B_1^G, with $B_0^* = B_1^* = B_1^G \geq \bar{B}$, a return r, constitute an equilibrium for the market economy with initial capital K_0.*

It is useful to note that if entrepreneurs are all ex ante identical, $B_1 = B_0 = 0$ in any competitive equilibrium if the government does not issue any bonds, so that $B_1^* = 0$. However, since the optimal tax system described in Proposition 3 can implement any level B_1 of bond holdings, it is possible to construct equilibria in which the government issues bonds B_1^G and $B_1^* = B_1^G$.[16]

The following proposition characterizes the properties of the optimal tax system.

Proposition 4 *The tax system $T^*(B_1, K_1, x)$ defined in Proposition 3 implies:*
i) $E_1\tau_K^*(x) = 0$;
ii) $E_1(x) = r - E_1\tau_B^*(x)$;
iii) $sign(\tau_K^*(\bar{x}) - \tau_K^*(\underline{x})) = sign(-IW_K)$;
iv) $\tau_B^*(\bar{x}) < \tau_B^*(\underline{x})$;
v) $\tau_B^*(\underline{x}) > \tau_K^*(\underline{x})$ *and* $\tau_B^*(\bar{x}) < \tau_K^*(\bar{x})$.

The average marginal capital tax is zero. Result ii) in proposition 4 implies that the expected after tax return on any risk-free asset is equal to the expected return on entrepreneurial capital. This implies that under T^*, the equilibrium values of r and $E_1\tau_B^*(x)$ are not separately pinned down. This indeterminacy does not affect the dependence of marginal bond taxes on x, which is governed by (21). Hence, without loss of generality we restrict attention to competitive equilibria with $r = E_1(x)$ and $E_1\tau_B^*(x) = 0$.

Result iii) states that the marginal capital tax is decreasing in capital returns, if the individual intertemporal wedge is positive, while it is increasing in capital returns if it is negative. The incentive effects of capital provide intuition for this result. Following the reasoning in section 2, when $\text{IW}_K > 0$, more capital tightens the incentive compatibility constraint. Hence, the optimal tax system discourages agents from setting K_1 too high by increasing the after tax volatility of capital returns. Instead, for $\text{IW}_K < 0$, more capital relaxes the incentive compatibility constraint. The optimal tax system encourages entrepreneurs to hold capital by reducing the after tax volatility of capital returns. By result ii), the intertemporal wedge on the bond is equal to the aggregate intertemporal wedge IW, and hence is positive. Then, higher holdings of B_1 tighten the entrepreneurs' incentive compatibility constraints. This explains result iv), that marginal bond taxes are decreasing in entrepreneurial earnings. The optimal tax system discourages entrepreneurs from holding B_1 in excess of B_1^* by making bonds a bad hedge against idiosyncratic capital risk.

[16] Our definition of competitive equilibrium allows the government to issue bonds at time 0, denoted B_1^G. The government budget constraints at time 0 and at time 1 are, respectively, $B_0 - B_1^G \leq 0$ and $E_eT(K_1, B_1, x) - B_1^G(1 + r) \geq 0$, where e corresponds to the effort chosen by the entrepreneurs in equilibrium. Given that the government does not need to finance any expenditures, the amount of government bonds issued does not influence equilibrium consumption, capital and effort allocations, or the equilibrium interest rate. However, if the government did have an expenditure stream to finance, the choice of bond holdings would be consequential.

Finally, result v) states that capital is subsidized with respect to bonds in the bad state. This results stems from the fact that consumption and entrepreneurial earnings are positively correlated at the optimal allocation, which means that capital returns and the inverse of the stochastic discount factor, which pins down marginal taxes, are also positively correlated. By definition, there is no correlation between bond returns and the inverse of the stochastic discount factor.

To illustrate the properties of optimal marginal asset taxes, we plot them for the numerical examples analyzed in section 2.2 in figure 2, assuming $r = E_1(x)$. Each row corresponds to one of the examples, the left panels plot the marginal capital taxes, while the right panels plot the marginal bond taxes. The solid line plots the intertemporal wedge for the corresponding asset. The dashed-star line corresponds to marginal taxes in state $\underline{x}$, whereas the dashed-cross line corresponds to optimal marginal taxes in state $\bar{x}$. The vertical scale is in percentage points and is the same for all panels.

The first example is one in which the individual intertemporal wedge is always negative. The marginal tax on capital is negative in the low state and positive in the good state, while the opposite is true for the marginal tax on bonds. Hence, the marginal capital tax is increasing in earnings, while the marginal bond tax is decreasing in earnings. The second row corresponds to the example with lower spread in capital returns, which exhibits a positive individual intertemporal wedge for intermediate values of the coefficient of relative risk aversion σ. The third row reports the optimal marginal asset taxes for the third example, in which σ is fixed and we vary the spread in capital returns. In the second and third examples, when $\mathrm{IW}_K > 0$, the marginal tax on entrepreneurial capital is also decreasing in x, positive in the bad state and negative in the good state. However, for all examples, it is always the case that the marginal tax on capital is smaller than the one on bonds in the low earnings state, $\underline{x}$. In the third example, since the constrained-efficient allocation only depends on the expected value of capital returns (held constant here) and not on their spread, the marginal bond tax taxes are constant. Instead, as discussed, the intertemporal wedge on capital is decreasing in the spread of capital returns.

Despite the fact that wedges are everywhere quite small in percentage terms, the marginal asset taxes can be sizable. The capital tax ranges from 2 to 23% in absolute value, while the bond tax ranges from 0 to 30% in absolute value.

The main finding in the fiscal implementation for the market structure considered in this section is the *optimality of differential asset taxation.* The optimal tax system equalizes after tax returns on entrepreneurial capital and risk less bonds, thus it reduces the after tax spread in capital returns and it increases the after tax spread in the returns to the risk less bond. Consequently, entrepreneurial capital is subsidized relatively to a riskless asset in the bad state.

3.1 Risky Securities

We now generalize these results to a market structure that allows for an arbitrary set of risky securities. The returns to these securities can be correlated with idiosyncratic capital returns. Thus, we denote with $r^i(x)$, for $x = \underline{x}, \bar{x}$, the return to a security F_1^i for $i \in \{1, ..., M\}$ with $M \geq 1$. The previous section studies the special case in which $M = 1$ and $r(x) = r$ for $x = \underline{x}, \bar{x}$. Entrepreneurs can trade this security at price q^i at time 0.

Letting the candidate tax system be given by $T\left(\{F_1^i\}_{i=1,M}, K_1, x\right) = \tau_K(x) K_1 + \sum_{i=1,..M} \tau_F^i(x) F_1^i + \rho(x)$. A competitive equilibrium can be defined generalizing Definition 2. Without loss of generality, we will consider equilibria in which each security is in zero net supply, and in which each

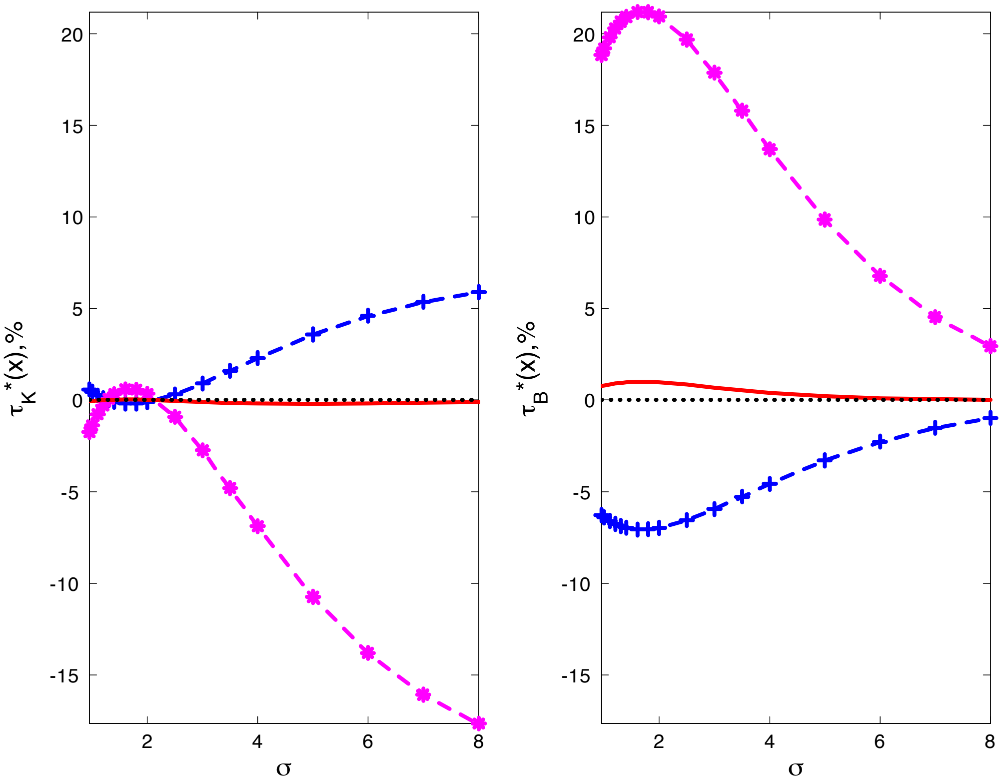

FIGURE 2: Optimal marginal taxes on entrepreneurial capital and bonds

entrepreneurs holds the same portfolio, so that $F_1^{i*} = 0$ for all i.[17] To implement the constrained-efficient allocation, set $\tau_K^*(x)$ and $\rho^*(x)$ as in (21) and (22). Set marginal taxes on each security according to:

$$1 + r^i(x) - \tau_F^{i*}(x) = \frac{q^i u'(c_0^*)}{\beta u'(c_1^*(x))}. \tag{24}$$

Following a proof strategy similar to that in Proposition 3, it is possible to show that the resulting tax system implements the constrained-efficient allocation.

The equilibrium price of each security is $q^i = \frac{E_1\left(1+r^i(x)-\tau_F^{i*}(x)\right)}{E_1\left(1+x-\tau_K^*(x)\right)}$.[18] Then, (24) implies $E_1\tilde{r}^i(x) = E_1 x$, where $\tilde{r^i}(x)$ is the equilibrium rate of return on each security $i = 1, 2, ..M$, that is $\tilde{r}^i(x) = \frac{1+r^i(x)}{q^i} - 1$. The intertemporal wedge on each risky security is:

$$IW^i = E_1 u'(c_1^*(x))\left(1 + \tilde{r}^i(x)\right) - u'(c_0^*),$$

Let $Corr_e$ denote the correlation conditional on $\pi(e)$. Then, the following result holds.

Proposition 5 *If $Cov_1(\tilde{r}^i(x), x) > 0$ and $V_1(x) > V_1(\tilde{r}^i(x))$, then:*

$$E_1 u'(c_1^*(x))\left(1 + \tilde{r}^i(x)\right) > E_1 u'(c_1^*(x))(1 + x),$$

[17]As previously discussed, the optimal marginal tax can implement any level of asset holdings, since it decouples the choice of action from the choice of assets. If financial securities were in positive net supply, this would affect only the lump-sum component of the optimal tax system, $\rho(x)$.

[18]As in the case with risk-free bonds, the equilibrium expected return on this security is not separately pinned down from $E_1\tau_s^{i*}(x)$.

$$\tau_F^{i*}(\bar{x}) - \tau_K^*(\bar{x}) < 0 \textit{ and } \tau_F^{i*}(\underline{x}) - \tau_K^*(\underline{x}) > 0.$$

If $Cov_1(\tilde{r}^i(x), x) > 0$ and $V_1(x) > V_1(\tilde{r}^i(x))$, $Corr_1(\tilde{r}^i(x), x) \in (0,1)$ for $i = 1, 2, ..M$ The proposition states that a security positively correlated with capital with lower variance of returns has a higher intertemporal wedge than capital. An entrepreneur would be willing to hold such a security instead of capital, since it is associated with lower earnings risk. However, this has an adverse effect on incentives. This motivates the higher intertemporal wedge and the fact that $\tau_F^{i*}(x) - \tau_K^*(x)$ is decreasing in x, which implies that capital is subsidized with respect to the risky security in the bad state.

The compact notation adopted in this section allows each security to depend on the realization of exogenous random variables that have no effects on preferences and technologies and may be correlated with the idiosyncratic shocks. In this case, the constrained-efficient allocation is unaltered, and the only relevant characteristic for these securities is their in terms of their correlation with x. A similar reasoning can be applied to (non-idiosyncratic) shocks that affect resource constrains and preferences. For example, assume that in period 1 there is an exogenous aggregate endowment shock, so that the feasibility constraint (3) is now given by: $E_e c_1(x) \leq K_1 E_e(1+x) + \omega$, where $\omega \in \Omega \subseteq \mathbb{R}_+$ and $Pr(\omega = \tilde{\omega})$ denotes the probability distribution of this aggregate state, for $\tilde{\omega} \in \Omega$. Then, constrained-efficient consumption levels at time 1 will depend on the realization of ω, and can be denoted with $c_1^*(\omega, x)$.[19] Consider a market structure in which agents can trade a security with pay-off contingent on the realization of ω. Extending the previous logic, the optimal marginal tax, $\tau_S^{\omega *}$, on this security will be:

$$1 + r^{\omega}(\omega, x) - \tau_F^{\omega *}(\omega, x) = \frac{q^i u'(c_0^*)}{\beta u'(c_1^*(\omega, x))},$$

where $r^{\omega}(\omega, x)$ denotes the equilibrium return to the security. Thus, as in Proposition 5, the optimal marginal tax on a security contingent on the aggregate state depends on the hedging value of this security for the entrepreneur. Also, comparing the marginal tax on a risky security with the optimal marginal tax on entrepreneurial capital in (21), entrepreneurial capital is always subsidized relative to the risky security in the bad (idiosyncratic) state.

These results point to a general principle. The optimal marginal tax on an asset depends on the correlation of an asset's returns with the idiosyncratic risk, which determines the asset's effects on the entrepreneurs' incentives to exert effort. Entrepreneurial capital is subsidized relative to other assets in the bad idiosyncratic state.[20]

In this implementation, we considered an arbitrary and exogenous set of financial securities. In the next section, we consider an implementation in which entrepreneurs can sell shares of their own capital to external investors, thus giving rise to an endogenous equity market with a positive net supply of securities.

[19] Consumption at time 0 and investment will depend on the probability distribution of ω.

[20] Sheuer (2010) examines a moral hazard model which is a special case of this environment, with $K_1 = 1$ exogenously, which implies no capital risk. He considers an implementation with within period risky securities and finds that they need not be taxed if their returns are uncorrelated with the idiosyncratic shocks. This result is due to the fact that trade in these securities does not affect the agents' average consumption in the second period, and thus, can only influence incentives via their correlation with idiosyncratic shocks.

4 Optimal Double Taxation of Capital

We now allow entrepreneurs to sell shares of their capital and buy shares of other entrepreneurs' capital. Each entrepreneur can be interpreted as a firm, so that this arrangement introduces an equity market. The amount of capital invested by an entrepreneur can be interpreted as the size of their firm.

An entrepreneur's budget constraint in each period is :

$$c_0 = K_0 - K_1 - \int_{i\in[0,1]} S_1(i)\,di + sK_1, \tag{25}$$

$$c_1(x) = K_1(1+x) - sK_1(1+d(x)) + \int_{i\in[0,1]} (1+D(i))\,S_1(i)\,di - T\left(K_1, s, \{S_1\}_i, x\right), \tag{26}$$

where $s \in [0,1]$ is the fraction of capital sold to outside investors, $d(x)$ denotes dividends distributed to shareholders, $S_1(i)$ is the value of shares in company i in an entrepreneur's portfolio and $D(i,\tilde{x})$ denotes dividends earned from each share of company i if the realized returns are $\tilde{x}$ for $\tilde{x} \in X$. Let $D(i) = E_{\hat{e}}D(i,\tilde{x})$ denote expected returns for stocks in firm i. Gross stock earnings for an entrepreneur with equity portfolio $\{S_1(i)\}_i$ are given by $\int_{i\in[0,1]}(1+D(i))\,S_1(i)\,di$, where $D(i)$ denotes expected dividends from firm i. Since $D(i,\tilde{x}) = d(\tilde{x})$ for all i and $\tilde{x}$ is i.i.d., $D(i) = \bar{D}$ for all $i = [0,1]$. The dividend distribution policy is taken as given by the entrepreneurs and the shareholders. This arrangement should be interpreted as part of the share issuing agreement. Entrepreneurs choose K_1, $\{S_1(i)\}_i$ as well as effort at time 0, taking as given the distribution policy, dividends and taxes. At time 1, x is realized, dividends are distributed, the government collects taxes and the entrepreneurs consume. The variables K_1, x, $S_1(i)$, s and $d(x)$ are public information.

We consider candidate tax systems of the form:

$$T\left(K_1, \{S_1\}_i, x\right) \;=\; \tau_P(x)(1+x)K_1 + \tau_s(x)\int_i S_1(i)\,di + \rho(x). \tag{27}$$

Here, $\tau_P(x)$ can be interpreted as a marginal tax on entrepreneurial earnings. The marginal tax on stock returns, $\tau_S(x)$, depends only the realization of x for the agent holding the stock and is the same for all stocks since stock returns are i.i.d.

The entrepreneurs' problem is:

$$\left\{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i\right\}(K_0, T) = \arg\max_{\hat{e},\hat{K}_1,\hat{s},\left\{\hat{S}_1(i)\right\}_i} u(c_0) + E_e u(c_1) - v(e), \tag{Problem 4}$$

subject to (25), (26) and $\int_{i\in[0,1]} S_1(i)\,di \geq \bar{B} = \frac{K_1(1+\underline{x})(1-\tau_P(\underline{x}))-\rho(\underline{x})}{(1-\tau_S(\underline{x}))\int_{i\in[0,1]}(1+D(i))di}$, where $\bar{B}$ is the natural borrowing limit.

The Euler equations for this problem are:

$$\begin{aligned}&-(1-s)\left\{u'(c_0) - \beta E_{\hat{e}}\left[(1+x)(1-\tau_P(x))\,u'(c_1(x))\right]\right\} \\ &+\beta s E_{\hat{e}}\left[(1+x)(1-\tau_P(x)) - (1+d(x))\right]u'(c_1(x)) \begin{cases} = 0 \text{ for } K_1 > 0 \\ \leq 0 \text{ for } K_1 = 0\end{cases},\end{aligned} \tag{28}$$

$$-u'(c_0) + \beta E_{\hat{e}}(1 + D(i) - \tau_S(x))\,u'(c_1(x)) = 0, \tag{29}$$

$$
\left[u'\left(c_0\right)-\beta E_{\hat{e}}\left(1+d\left(x\right)\right)u'\left(c_1\left(x\right)\right)\right]K_1\left\{\begin{array}{l} =0 \text{ for } s\in(0,1) \\ \leq 0 \text{ for } s=0 \\ >0 \text{ for } s=1. \end{array}\right. . \tag{30}
$$

We define a competitive equilibrium for this trading structure and then consider how to implement the constrained-efficient allocation.

Definition 4 *A competitive equilibrium is an allocation* $\left\{c_0,\ \hat{e},\ \hat{K}_1,\ \hat{s},\ \left\{\hat{S}_1\left(i\right)\right\}_i,\ \hat{c}_1\left(x\right)\right\}$ *with* $\hat{s}\in[0,1]$, *a distribution policy* $\hat{d}\left(x\right)$ *and a dividend process* $\hat{D}\left(i,x\right)$ *for* $i\in[0,1]$, $x\in X$, *and a tax system* $T\left(K_1,\left\{S_1\right\}_i,x\right)$, *such that:*

i) the allocation $\left\{\hat{e},\hat{K}_1,\hat{s},\left\{\hat{S}_1\left(i\right)\right\}_i\right\}$ *solves the entrepreneurs' problem, for given* $\hat{d}\left(x\right)$, $\hat{D}\left(i,\tilde{x}\right)$, *and* T;

ii) the dividend process is consistent with the distribution policy, $\hat{d}\left(x\right)=\hat{D}\left(i,x\right)$ *for all* i *and* $x\in X$;

ii) the stock market clears, $\hat{s}\hat{K}_1=\hat{S}_1\left(i\right)$ *for* $i=[0,1]$;

iii) the resource constraint is satisfied in each period.

Since all entrepreneurs are ex ante identical, we restrict attention to symmetric equilibria in which s, K_1 and effort are constant for all entrepreneurs. The entrepreneurs face a portfolio problem in the selection of stocks. Given that all stocks have the same expected return net of taxes under the family of tax systems defined by $T\left(K_1,\left\{S_1\right\}_i,x\right)$, entrepreneurs are indifferent over which stocks to hold. However, they will always hold a continuum of stocks, since this ensures that their portfolio has zero variance. To break the entrepreneurs' indifference over portfolio selection, we assume, without loss of generality, that all entrepreneurs hold a perfectly differentiated portfolio. Hence, $\bar{D}$ corresponds to gross portfolio returns in equilibrium and we can restrict attention to the case $S_1\left(i\right)=S_1$ for all i.

We now construct a tax system that implements the constrained-efficient allocation. Set the marginal profit tax is $\tau_P^*\left(x\right)$ as follows:

$$
\left(1+x\right)\left(1-\tau_P^*\left(x\right)\right)=\frac{u'\left(c_0^*\right)}{\beta u'\left(c_1^*\left(x\right)\right)}. \tag{31}
$$

Let $d^*\left(x\right)=\left(1+x\right)\left(1-\tau_P\left(x\right)\right)-1$, so that dividends per share are simply given by after tax profits. This implies: $\bar{D}^*=E_1\left(1+x\right)\left(1-\tau_P^*\left(x\right)\right)-1$. Set $\tau_S^*\left(x\right)$ so that:

$$
1+\bar{D}^*-\tau_S^*\left(x\right)=\frac{u'\left(c_0^*\right)}{\beta u'\left(c_1^*\left(x\right)\right)}. \tag{32}
$$

Lastly, we choose $\rho^*\left(x\right)$ to satisfy:

$$
\begin{aligned} c_1^*\left(x\right) &= K_1^*\left(1+x\right)\left(1-\tau_P^*\left(x\right)\right)-s^*K_1^*\left(1+d^*\left(x\right)\right) \\ &\quad +\left(1+\bar{D}^*-\tau_S^*\left(x\right)\right)S_1^*-\rho^*\left(x\right), \end{aligned} \tag{33}
$$

for some $s^*\in[0,1)$, with $S_1^*=s^*K_1^*$.

We now prove that the tax system $T^*\left(K_1,\left\{S_1\left(i\right)\right\}_i,x\right)$ implements the constrained-efficient allocation.

Proposition 6 *The tax system* $T^*(K_1, \{S_1(i)\}_i, x) = \tau_P^*(x)(1+x)K_1 + \tau_s^*(x)\int_i S_1(i)\,di + \rho^*(x)$, *where* $\tau_P^*(x)$, $\tau_S^*(x)$ *and* $\rho^*(x)$ *satisfy (30), (31) and (32), respectively, implements the constrained-efficient allocation with distribution policy* $1 + d^*(x) = (1+x)(1-\tau_P^*(x))$ *and dividend process* $D^*(i)$ *for all* i. *The allocation* $\{K_1^*, s^*, \{S_1^*(i)\}_i, 1, c_1^*(x)\}$ *with* $s^* K_1^* = S_1^*(i)$ *for all* i *and* $s^* \in (0,1)$, *the tax system* $T^*(K_1, \{S_1(i)\}_i, x)$, *the distribution policy* $d^*(x)$ *and the dividend process* $D^*(i,x)$ *constitute a competitive equilibrium.*

The proof proceeds as the one for proposition 3. The values of S_1^* and s^* are not pinned down by the implementation. The setting of marginal taxes ensures that the entrepreneurs' Euler equations (27)-(29) are satisfied as an equality at any $s^* \in (0,1)$ for distribution policy $d^*(x)$, and that local second order sufficient conditions are also satisfied. It follows that the only interior solution to the entrepreneurs' optimization problem is $\{1, K_1^*, S_1^*(i)\}$ for any $s^* \in (0,1)$. In addition, it ensures that the allocation is globally optimal because it rules out any corner solutions to the entrepreneurs' investment and portfolio problems, irrespective of the level of effort. Lastly, the setting of $\rho^*(x)$ ensures high effort is optimal at the appropriate level of capital and portfolio choices. The optimal tax system does not pin down the equilibrium value of s^*. By (27), for $s^* \in (0,1)$, the tax system ensures that entrepreneurs find it optimal to choose K_1^*.

The properties of the optimal tax system can be derived from (30)-(32). First:

$$E_1 \tau_P^*(x) = 1 - E_1\left[\frac{u'(c_0^*)}{\beta(1+x)u'(c_1^*(x))}\right], \tag{34}$$

so that $E_1\tau_P^*(x) > 0$ if $IW_K > 0$ and $E_1\tau_P^*(x) < 0$ if $IW_K < 0$. However, using the planner's Euler equation delivers $E_1(1+x)\tau_P^*(x) = 0$, since:

$$E_1(1+x)(1-\tau_P^*(x)) = E_1\left[\frac{u'(c_0^*)}{\beta u'(c_1^*(x))}\right].$$

This implied that the expected tax paid is zero.[21] In addition, $\tau_P^*(\bar{x}) - \tau_P^*(\underline{x}) < 0$ when $IW_K > 0$ and $\tau_P^*(\bar{x}) - \tau_P^*(\underline{x}) > 0$ when $\mathrm{IW}_K < 0$ from:

$$\frac{u'(c_0^*)}{\beta(1+\underline{x})u'(c_1^*(\underline{x}))} - \frac{u'(c_0^*)}{\beta(1+\bar{x})u'(c_1^*(\bar{x}))} = \tau_P^*(\bar{x}) - \tau_P^*(\underline{x}),$$

since $\mathrm{IW}_K\,\tilde{}\,(1+\underline{x})u'(c_1^*(\underline{x})) - (1+\bar{x})u'(c_1^*(\bar{x}))$. Lastly, by (31):

$$1 + E_1 x - E_1\tau_P^*(x) - E_1 x\tau_P^*(x) - E_1\tau_S^*(x) = E_1\frac{u'(c_0^*)}{\beta u'(c_1^*(x))}. \tag{35}$$

This implies $E_1\tau_S^*(x) = -E_1\tau_P^*(x) - E_1 x\tau_P^*(x) = -E_1\tau_P^*(x)E_1(1+x) - Cov_1(x, \tau_P^*(x))$. If $\mathrm{IW}_K \gtreqless 0$, $Cov_1(x, \tau_P^*(x)) \lesseqgtr 0$ and $E_1\tau_P^*(x) \gtreqless 0$, as discussed above. Hence, the sign of $E_1\tau_S^*(x)$ is typically ambiguous.

Figure 3 plots the optimal marginal asset taxes for this implementation in the three numerical examples analyzed in section 2.2. The left panels correspond to the marginal tax on entrepreneurial earnings, while the right panels correspond to the marginal tax on stocks. The dashed-star line correspond to marginal taxes in the bad state, while the dashed-cross lines correspond to marginal taxes in the good state. The pattern of optimal marginal taxes is consistent with the previous discussion.

[21] The constrained-efficient allocation can equivalently be implemented with a marginal tax on tax on capital $\tau_K(x)$ that satisfies (20) and with distribution policy: $1 + d(x) = 1 + x - \tau_K(x)$ and dividend process $1 + D(i, \tilde{x}) = 1 + \tilde{x} - \tau_K^*(\tilde{x})$, so that $1 + D(i) = 1 + E_1(x)$, since $E_1\tau_K^*(x) = 0$. All other results can be derived with a similar reasoning.

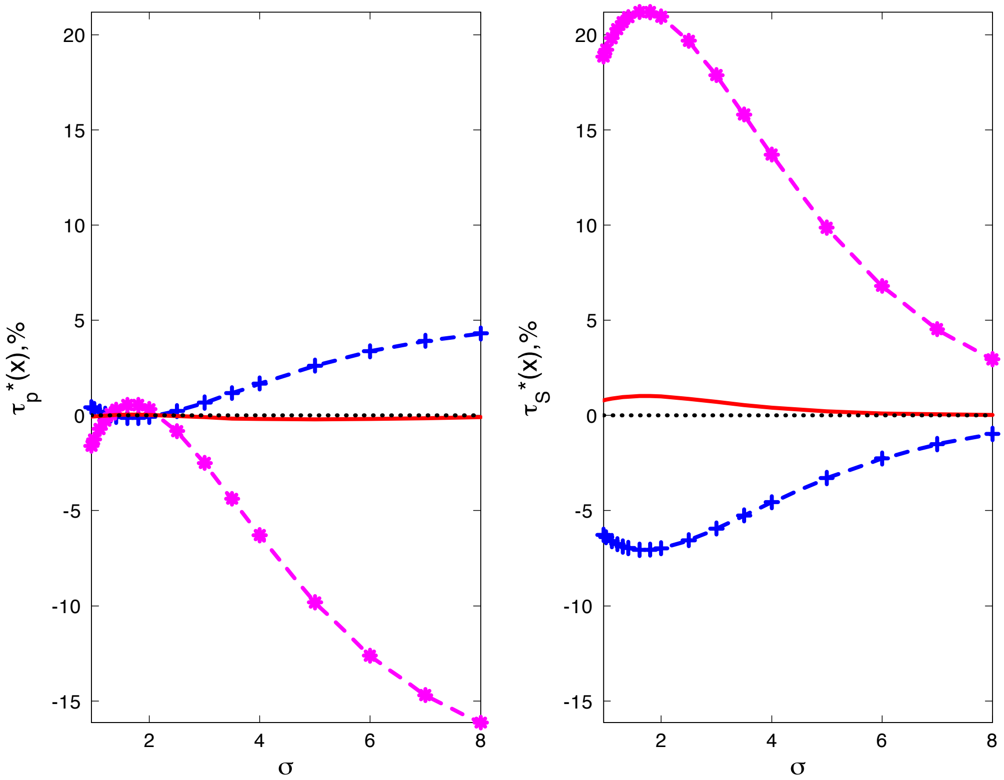

FIGURE 3: Optimal marginal taxes on entrepreneurial earnings and on stocks

4.1 Double Taxation of Entrepreneurial Capital

The tax system described in proposition 6 embeds a prescription for *double taxation of income from entrepreneurial capital*: at the firm level thought τ_P^*, and at the level of external investors, through τ_S^*. This property holds for any equilibrium in which $s^* \in (0, 1)$ and is jointly determined by the distribution policy and the tax system, since external investors receive a share of earnings *after* tax. We now show that this feature of the tax system necessary to implement the constrained-efficient allocation.

Following the reasoning in section 2, taxation of entrepreneurial earnings and stock portfolios is *required* to ensure that entrepreneurs choose K_1^* and S_1^*, respectively. Absent a marginal tax on capital, entrepreneurs would have an incentive to increase/reduce holdings of K_1 relative to K_1^* and reduce effort. Similarly, given that stock returns are uncorrelated with entrepreneurs' idiosyncratic risk, the wedge on stock portfolios is positive and, by (33) and (34), equal to the aggregate intertemporal wedge IW:

$$\beta E_1 \left(1 + \bar{D}\right) E_1 u'\left(c_1^*\left(x\right)\right) - u'\left(c_0^*\right) = \text{IW} > 0.$$

Hence, absent a tax on stock holdings, entrepreneurs would have the incentive to increase their holdings of stocks and reduce effort.

We now show that it is necessary to tax distributed earnings to ensure that $0 < s^* < 1$ is chosen. To do this, we allow the marginal tax on distributed earnings, $\tau_d\left(x\right)$ to differ from the marginal tax on retained earnings, $\tau_P\left(x\right)$. Then, the distribution policy can be written as: $1 + d\left(x\right) = \left(1 + x\right)\left(1 - \tau_d\left(x\right)\right)$. Setting $\tau_d\left(x\right) = 0$ for $x \in X$ avoids double taxation of entrepreneurial earnings. By (29), $1 + d\left(x\right) = \left(1 + x\right)$ implies: $u'\left(c_0^*\right) - \beta E_1\left(1 + x\right) u'\left(c_1^*\left(x\right)\right) < 0$ and $s = 0$, if the individual intertemporal wedge is positive. Hence, when $\mathrm{IW}_K > 0$ and there is no tax on distributed earnings, $s^* = 0$ is the only implementable value of s. If the individual intertemporal wedge is negative and $\tau_d(x) = 0$, (29) implies: $u'(c_0^*) - \beta E_1 (1+x) u'(c_1^*(x)) > 0$ and $s = 1$. But equation (27) evaluated at $s = 1$ reduces to:

$$-\beta E_1 \left[(1+x) \tau_P^*(x) u'(c_1^*(x))\right] \leq 0.$$

This reasoning clearly entails a contradiction since when $\mathrm{IW}_K < 0$, $(1+x)\tau_P^*(x)$ is increasing in x and $Cov_1\left[(1+x)\tau_P^*(x), u'(c_1^*(x))\right] < 0$, which implies $E_1\left[(1+x)\tau_P^*(x) u'(c_1^*(x))\right] < 0$, since $E_1 (1+x)\tau_P^*(x) = 0$. Hence, there is no interior solution to the entrepreneur's choice of K_1. It follows that the constrained-efficient allocation cannot be implemented, since there is no way to set taxes to ensure that a particular value of K_1 will be chosen by the entrepreneur. Even if for any value of K_1 (and S_1), $\rho(x)$ can be set to ensure that high effort will be chosen, there is no way to guarantee that the corresponding value of K_1 will indeed arise.

To understand this property, note that an entrepreneur has three intertemporal margins in this market structure, corresponding to the Euler equation for K_1, the one for S_1 and the one for s. Therefore, there are three potential deviations in her asset position and three intertemporal fiscal instruments are needed to implement the constrained-efficient outcomes. For IW_K positive, an entrepreneur has an incentive to increase her holdings of K_1 relative to K_1^* and therefore would optimally not sell any equity under a tax system in which distributed earnings are not taxed. This simply implies that the only equilibrium is one in which $s^* = 0$. It is still possible to implement K_1^* and e^*. Instead, the case in which the individual intertemporal wedge is negative is particularly problematic. When $\mathrm{IW}_K < 0$, an entrepreneur would optimally reduce her holdings of capital when she reduces effort. This can be achieved directly or by increasing the fraction s sold to external investors. As shown above, if distributed earnings are not taxed, the optimal deviation is $s = 1$, which implies K_1^* and e^* cannot be implemented.

We now characterize the class of tax systems $T(K_1, s, \{S_1(i)\}_i)$ that rules out $s = 1$ as a possible solution to the entrepreneurs' problem.

Proposition 7 *In any competitive equilibrium under a tax system,* $T(K_1, s, \{S_1(i)\}_i) = \tau_P(x)(1-s)K_1 + \tau_d(x) s K_1 + \tau_s(x)\int_i S_1(i)\,di + \rho(x)$, *and distribution policy* $1 + d(x) = (1+x)(1-\tau_d(x))$, $s \in [0,1)$ *if and only if:*

$$E_{\hat{e}}(1+d(x)) u'(c_1(x)) \geq E_{\hat{e}}(1+x)(1-\tau_P(x)) u'(c_1(x)). \tag{36}$$

This proposition states that the expected discounted value of distributed earnings must be greater than the expected discounted value of retained earnings after tax to ensure that $s < 1$ in a competitive equilibrium under a tax system $T(K_1, s, \{S_1(i)\}_i)$. For this condition to be verified at the constrained-efficient allocation, it must be that $\tau_P(x) \geq \tau_d(x)$ if $u'(c_1^*(x))(1+x) > u'(c_1^*(x'))(1+x')$ for $x, x' \in \{\underline{x}, \bar{x}\}$. Then, since $u'(c_1^*(\underline{x}))(1+\underline{x}) \gtrless u'(c_1^*(\bar{x}))(1+\bar{x})$ for $\mathrm{IW}_K \gtrless 0$, this implies $\tau_P(\bar{x}) \leq \tau_d(\bar{x})$ and $\tau_P^*(\underline{x}) \geq \tau_d(\underline{x})$ for $\mathrm{IW}_K > 0$ and $\tau_P(\bar{x}) \geq \tau_d(\bar{x})$ and $\tau_P(\underline{x}) \leq \tau_d(\underline{x})$ for $\mathrm{IW}_K < 0$.

The rationale for this result is simple. When $\mathrm{IW}_K > 0$, entrepreneurs have an incentive to increase holdings of their own capital and reduce effort at the constrained-efficient allocation. A way to discourage this is to make external capital a good hedge. This is achieved by making dividend payouts greater in the good state and smaller in the bad state after tax. Conversely, when $\mathrm{IW}_K < 0$, entrepreneurs have an incentive to reduce holdings of their own capital and effort. To avoid an outcome in which entrepreneurs retain too little ownership, the tax system reduces the hedging value of external capital, by making dividend payments higher in the bad state and lower in the good state.

Obviously, under the optimal tax system $T^*(K_1, S_1, x)$, distributed and retained earnings are taxed at the same marginal rate $\tau_P^*(x)$ defined by (30). Thus, it satisfies (35), which ensures that $s^* < 1$. Moreover, by (29), $s^* > 0$. In general, the first order necessary conditions for K_1 can be rewritten as:

$$\begin{aligned} 0 &= -(1-s)\{u'(c_0^*) - \beta E_1[(1+x)(1-\tau_P^*(x))u'(c_1^*(x))]\} \\ &\quad + \beta s E_1[(1+x)(\tau_d(x) - \tau_P^*(x))u'(c_1(x))]. \end{aligned}$$

Then, for equilibria with $s^* \in (0,1)$, it must be that $E_1[(1+x)(\tau_d(x) - \tau_P^*(x))u'(c_1(x))] = 0$, if $\tau_P^*(x)$ satisfies (30), to ensure that K_1^* is chosen.

This argument implies that it is indeed *necessary* for distributed earnings, as well as retained earnings, to be taxed at the firm level to implement the constrained-efficient allocation. Hence, entrepreneurial capital is subject to *double taxation* in the optimal tax system.

5 Concluding Remarks

This paper analyzes optimal taxation of entrepreneurial capital in a dynamic moral hazard model with idiosyncratic capital risk. First, we characterize the properties of constrained-efficient allocations and show that the intertemporal wedge on entrepreneurial capital can be positive or negative. A negative intertemporal wedge signals that more capital relaxes the incentive compatibility constraint. This can occur since the returns from effort are increasing in capital. The main contribution of the paper is to characterize the optimal tax systems that implement the constrained-efficient allocation in different market structures with multiple assets. We derive three results. First, marginal asset taxes depend on the *correlation* of their returns with the entrepreneurs' idiosyncratic capital risk. We also consider whether entrepreneurial capital earnings distributed to outside investors should be taxed at the firm level. We find that entrepreneurial capital should be taxed at the firm level and again when it accrues to outside investors in the form of stock returns. This generates a theory of optimal differential asset taxation and provides a foundation for the double taxation of capital earnings.

The empirical public finance literature has documented substantial differences in the tax treatment of different forms of capital income. Specifically, interest income is taxed at a higher rate than stock returns, as discussed in Gordon (2003), while dividends are taxed at a higher rate than realized capital gains. As documented by Gordon and Slemrod (1988), the higher marginal tax rate on interest income is a stable property of empirical tax systems in many industrialized economies. These studies focus mainly on differences in average taxes. Instead, the theory developed in this paper generates predictions on the correlation of marginal asset taxes with individual earnings, and average taxes do not play an important role. Poterba (2002) documents a strong response of household portfolio composition to this differential tax treatment. Auerbach (2002) finds that form's investment decisions appear to be sensitive to the taxation of dividend income at the personal level and their choice of organization form is responsive to the differential between corporate and personal tax rates. In the economy studied in this paper, the optimal tax system implements the constrained-efficient allocation by influencing portfolio choice and sales of private equity by entrepreneurs. Differential tax treatment of different asset classes is essential to achieve this goal.

The linearity in asset levels of the optimal tax system is an important property in the two implementations we consider, since it implies that optimal marginal taxes are independent from the individual level of asset holdings. Then, the government does not need to observe entrepreneurs' portfolios to administer the optimal tax system, if financial securities are traded via intermediaries and taxes on holding of those securities are collected *at the source*. This arrangement is similar to the one in place for consumption taxes in the US, where merchants observe individual units of consumption and apply a mandated consumption tax schedule. They then transfer total tax revenues to the relevant tax authority (the city, county or state for consumption taxes). Similarly, financial intermediaries clearing trades on bonds B_1 could levy marginal tax $\tau_B^*(x)$ on an entrepreneur with observable returns x. This suggests that observability of individual portfolio positions is not required for fiscal implementation, as long as financial trades are intermediated.

The incentive problem that arises with entrepreneurial capital arguably also applies to top executives who hold company stock and other assets. Hence, this analysis could be adapted to such a setting. A quantitative version of the model can be used to provide an assessment of empirical tax systems. Lastly, this model does not consider entrepreneurial entry. By introducing ex ante heterogeneity in private entrepreneurial abilities, it would be possible to analyze optimal selection into entrepreneurship and optimal income and capital taxation in a model with workers and entrepreneurs. We leave these extensions for future work.

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A Proofs

Proof. [Proof of Proposition 1] Letting μ be the multiplier on the incentive compatibility constraint and λ the one on the resource constraint, the first order necessary conditions for the planning problem at $e = 1$ are:

$$-u'(K_0 - K_1) + \lambda E_1(1+x) = 0,$$
$$(1-\pi(1))\beta u'(c_1(\underline{x})) - \mu(\pi(1) - \pi(0))\beta u'(c_1(\underline{x})) - \lambda(1-\pi(1)) = 0,$$
$$\pi(1)\beta u'(c_1(\bar{x})) - \mu(\pi(0) - \pi(1))\beta u'(c_1(\bar{x})) - \lambda\pi(1) = 0.$$

At $e = 0$, the same first order necessary conditions hold with $\mu = 0$. If $e^* = 1$ is optimal, the first order conditions can be simplified to yield (5) and (6). ■

To prove Proposition 2, we first establish that the variance of consumption is always smaller than the variance of earnings at the constrained-efficient allocation.

Lemma 2 *If* $\{e^*, K_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ *solve Problem 1 and* $e^* = 1$*, then* $(1+\bar{x})K_1^* \geq c_1^*(\bar{x}) > c_1^*(\underline{x}) \geq (1+\underline{x})K_1^*$.

Proof. [Proof of Lemma 2] Suppose instead that $(1+\bar{x})K_1^* < c_1^*(\bar{x})$ and $c_1^*(\underline{x}) < (1+\underline{x})K_1^*$. Consider a class of perturbations to the optimal allocation that increase consumption in the bad state by $\Delta\underline{c}_1$ and reduce consumption in the good state by $\Delta\bar{c}_1$, and preserve incentive compatibility and feasibility. Such perturbations must satisfy:

$$(\pi(1) - \pi(0))\left[-u'(c_1^*(\bar{x}))\Delta\bar{c}_1 + u'(c_1^*(\underline{x}))\Delta\underline{c}_1\right] = 0,$$

$$-\pi(1)\Delta\bar{c}_1 + (1-\pi(1))\Delta\underline{c}_1 = \Delta\underline{c}_1\left[1 - \pi(1)\left(1 + \frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))}\right)\right] \leq 0.$$

These conditions imply $\Delta\bar{c}_1 = \Delta\underline{c}_1 \frac{u'\left(c_1^*(\underline{x})\right)}{u'\left(c_1^*(\bar{x})\right)} > \Delta\underline{c}_1$, and $\left[\frac{1}{\pi(1)} - \left(1 + \frac{u'\left(c_1^*(\underline{x})\right)}{u'\left(c_1^*(\bar{x})\right)}\right)\right] \leq 0$, where the latter is always satisfied for $\pi(1) \geq 1/2$. Now let $c_1^*(\underline{x}) + \Delta\underline{c}_1 \geq K_1^*(1+\underline{x})$. Then:

$$\begin{aligned}
c_1^*(\bar{x}) - \Delta\bar{c}_1 &= c_1^*(\bar{x}) - \Delta\underline{c}_1 \frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))} \\
&\leq c_1^*(\bar{x}) - \left[K_1^*(1+\underline{x}) - c_1^*(\underline{x})\right] \frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))} \\
&= \frac{E_1 K_1^*(1+x) - (1-\pi(1))\, c_1^*(\underline{x})}{\pi(1)} - \left[K_1^*(1+\underline{x}) - c_1^*(\underline{x})\right] \frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))} \\
&= K_1^*(1+\bar{x}) + \left[\frac{1}{\pi(1)} - \left(1 + \frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))}\right)\right]\left[K_1^*(1+\underline{x}) - c_1^*(\underline{x})\right] \\
&\leq K_1^*(1+\bar{x}),
\end{aligned}$$

by $\left[\frac{1}{\pi(1)} - \left(1 + \frac{u'\left(c_1^*(\underline{x})\right)}{u'\left(c_1^*(\bar{x})\right)}\right)\right] \leq 0$. Hence, this perturbation is incentive compatible, uses fewer resources than the optimal allocation at time 1 and implies that consumption in the good/bad state is smaller/greater than earnings. Since the perturbed allocation uses fewer resources at time 1, it is feasible to increase consumption at time 0 by reducing the level of K_1 by the amount:

$$\Delta K_1 = \frac{\Delta\underline{c}_1}{E_1(1+x)}\left[1 - \pi(1)\left(1 + \frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))}\right)\right] \leq 0.$$

The resulting effect on welfare is:

$$\begin{aligned}
&-u'(c_0^*)\,\Delta K_1 + \pi(1)\left[-u'(c_1^*(\bar{x}))\,\Delta\bar{c}_1 - u'(c_1^*(\underline{x}))\,\Delta\underline{c}_1\right] + u'(c_1^*(\underline{x}))\,\Delta\underline{c}_1 \\
= \;& \Delta\underline{c}_1 \left\{-\frac{u'(c_0^*)}{E_1(1+x)}\left[1 - \pi(1)\left(1 + \frac{u'(c_1^*(\underline{x}))}{u'(c_1^*(\bar{x}))}\right)\right] + (1 - 2\pi(1))\, u'(c_1^*(\underline{x}))\right\} \\
= \;& \Delta\underline{c}_1 u'(c_1^*(\underline{x}))\left\{-\frac{u'(c_0^*)}{E_1(1+x)}\left[\frac{1-\pi(1)}{u'(c_1^*(\underline{x}))} - \frac{\pi(1)}{u'(c_1^*(\bar{x}))}\right] + 1 - 2\pi(1)\right\} \\
= \;& \Delta\underline{c}_1 u'(c_1^*(\underline{x}))\left\{-1 + \frac{u'(c_0^*)}{E_1(1+x)}\frac{2\pi(1)}{u'(c_1^*(\bar{x}))} + 1 - 2\pi(1)\right\} \\
= \;& \Delta\underline{c}_1 u'(c_1^*(\underline{x}))\, 2\pi(1)(1-\pi(1))\frac{u'(c_0^*)}{E_1(1+x)}\left\{\frac{1}{u'(c_1^*(\bar{x}))} - \frac{1}{u'(c_1^*(\underline{x}))}\right\} > 0,
\end{aligned}$$

where the third and fourth equality use the inverted Euler equation. Hence, the class perturbations that make consumption in the good/bad state is smaller/greater than earnings is incentive compatible, requires fewer resources and increases welfare. This violates the assumption that $\{K_1^*, c_1^*(x)\}$ was optimal and satisfies $(1+\overline{x})K_1^* < c_1^*(\overline{x})$ and $c_1^*(\underline{x}) < (1+\underline{x})K_1^*$. ■

Proof. [Proof of Proposition 2] By equation (11), $\mathrm{IW}_K \,\tilde{}\, u'(c_1^*(\underline{x}))(1+\underline{x}) - u'(c_1^*(\bar{x}))(1+\bar{x})$. By Lemma 2, $(1+\overline{x})K_1^* \geq c_1^*(\overline{x}) > c_1^*(\underline{x}) \geq (1+\underline{x})K_1^*$ with at least one inequality strict. Then:

$$\begin{aligned}
\mathrm{IW}_K &\;\tilde{}\; \left[u'(c_1^*(\underline{x}))(1+\underline{x}) - u'(c_1^*(\overline{x}))(1+\overline{x})\right] K_1^* \\
&< u'(c_1^*(\underline{x}))c_1^*(\underline{x}) - u'(c_1^*(\overline{x}))c_1^*(\overline{x}) \leq 0
\end{aligned}$$

if $u\prime(c)\,c$ is weakly increasing in c. Since

$$\frac{\partial u\prime(c)\,c}{\partial c} = u''(c)\,c + u'(c) > 0,$$

for $\sigma(c) \leq 1$, the result follows. ■

Proof. [Proof of Lemma 1] We first show that under $\bar{T}$, $\hat{B}_1$ and $\hat{K}_1$ cannot both be interior. Suppose not. If optimal bond and capital holdings under $\bar{T}$ are interior:

$$\begin{aligned} u'(\hat{c}_0) &= E_0 u'(c_1^*(x))(1 + r - \bar{\tau}_B), \\ u'(\hat{c}_0) &= E_0 u'(c_1^*(x))(1 + x - \bar{\tau}_K). \end{aligned}$$

But:

$$\begin{aligned} & u'(\hat{c}_0) - E_0 u'(c_1^*(x))(1 + x - \bar{\tau}_K) \\ = \;& \beta\left[\frac{E_1 u'(c_1^*(x))(1+x)}{E_1 u'(c_1^*(x))} - \frac{E_0 u'(c_1^*(x))(1+x)}{E_0 u'(c_1^*(x))}\right] \gtrless 0 \text{ if } IW_K \lesseqgtr 0, \end{aligned} \tag{37}$$

since $\frac{E_e u'\left(c_1^*(x)\right)(1+x)}{E_e u'\left(c_1^*(x)\right)}$ is increasing in e if $\mathrm{IW}_K < 0$ and decreasing for $\mathrm{IW}_K > 0$. Contradiction. Then, if $\mathrm{IW}_K < 0$, and $\hat{B}_1$ is interioir, (36) implies $\hat{K}_1 = 0$. If instead $\mathrm{IW}_K > 0$, assume that $\hat{B}_1$ is interior and $\hat{K}_1 = 0$. Then, (36) implies that an entrepreneur would like to increase capital holdings further. Since this can always be achieved by reducing bond holdings, an interior value of B_1 cannot be optimal. Hence, if $\mathrm{IW}_K > 0$ it must be that $\hat{B}_1 = \underline{B}$ and $\hat{K}_1 > 0$ under $\bar{T}$. Moreover, by (18), $\hat{K}_1 > K_1^*$. The binding Incentive compatibility constraint implies that such a deviation obtains higher lifetime utility for the agent. ■

Proof. [Proof of Proposition 3] We want to show that

$$\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}(B_0^*, K_0, T^*) = (1, K_1^*, B_1^*),$$

for some $B_1^* \geq \bar{B}$ and for given r. Suppose that $\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}(B_0^*, K_0, T^*) \neq (1, K_1^*, B_1^*)$. If $\left\{\hat{e}, \hat{K}_1, \hat{B}_1\right\}$ is interior in $\hat{K}_1$ and $\hat{B}_1$, at T^* :

$$1 = E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}.$$

It follows that for any interior $\hat{K}_1, \hat{B}_1$ such that $\hat{K}_1 + \hat{B}_1 \gtrless K_1^* + B_1^*$, then (20) and (21) imply $\frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \lesseqgtr \frac{u'\left(c_1^*(x)\right)}{u'\left(c_0^*\right)}$ irrespective of the value of $\hat{e}$, a contradiction. Since after tax returns are equated for capital and bonds, agents are indifferent between any portfolio allocation such that total wealth at the beginning of period 1 is $K_1^* + B_1^*$. Moreover, at T^*, the local sufficient conditions for optimality are also satisfied irrespective of the value of $\hat{e}$. To see this, consider the sub-optimization problem associated with the choice of B_1 and K_1 for given e. The elements of the Hessian, H_U, for this problem are:

$$U_{BB}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x))(1 + r - \tau_B^*(x))^2 \leq 0,$$

$$U_{KK}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x))(1 + x - \tau_K^*(x))^2 \leq 0,$$

$$U_{BK}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x))(1 + r - \tau_B^*(x))(1 + x - \tau_K^*(x)),$$

where U_{xy} denotes a cross-partial derivative with respect to the variables x, y.Under (20)-(21), $|H_U| = 0$. Hence, the Hessian is negative semi-definite irrespective of the value of $\hat{e}$. We now consider values of $\hat{K}_1, \hat{B}_1$ that are not interior. The Inada conditions exclude non-interior solutions that result from the non-negativity constraint on time 0 consumption being binding. Hence, there are two candidate non-interior solutions: $\hat{K}_1 = 0$ and $\hat{B}_1 > 0$, and $\hat{K}_1 > 0$ and $\hat{B}_1 = \bar{B}$. In both cases, one of the Euler equations must hold with equality and the other as a strict inequality. Under T^*:

$$1 = E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}, \tag{38}$$

$$1 > E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}. \tag{39}$$

This is a contradiction, since (37) and (38) clearly cannot hold at the same time. Then, K_1^*, B_1^* are globally optimal irrespective of the value of $\hat{e}$. At at K_1^*, B_1^*, $\rho^*(x)$ implies $\hat{e} = 1$ since the constrained-efficient allocation is incentive compatible. Hence, the allocation $\{1, K_1^*, B_1^*\}$ is optimal for the agent given the initial endowments B_0^*, K_0, the tax system T^*, and the interest rate r. ■

Proof. [Proof of Corollary 1] By Proposition 3, for any $r \geq 0$ and $B_1^* \geq \bar{B}$, the allocation $\{c_0^*, 1, K_1^*, B_1^*, c_1^*(\underline{x}), c_1^*(\bar{x})\}$ solves the agents' optimization problem in the market economy for initial endowments B_0^* and K_0. In addition, at $B_0^* = B_1^* = B_1^G$ the bond market clears and the resource constraint is satisfied at time 0. The resource constraint at time 1 is satisfied by construction. Hence, by (22), $E_1 c_1^*(x) = K_1 E_1(1+x) + B_1^*(1+r) - E_1 T^*(K_1^*, B_1^*, x)$, so that the government budget constraint is satisfied at time 1. ■

Proof. [Proof of Proposition 4] By (20):

$$E_1\left[1 + x - \frac{u'(c_0^*)}{u'(c_1^*(x))}\right] = E_1 \tau_K^*(x),$$

which from (6) implies i). ii) follows from the planner's Euler equation, since:

$$E_1 \tau_B^*(x) = 1 + r - E_1\left(\frac{u'(c_0^*)}{\beta u'(c_1^*(x))}\right).$$

(20) also implies:

$$u'(c_1^*(\overline{x}))\tau_K^*(\overline{x}) - u'(c_1^*(\underline{x}))\tau_K^*(\underline{x}) = u'(c_1^*(\overline{x}))(1+\overline{x}) - u'(c_1^*(\underline{x}))(1+\underline{x}).$$

Since:

$$sign\left[u'(c_1^*(\overline{x}))(1+\overline{x}) - u'(c_1^*(\underline{x}))(1+\underline{x})\right] = sign\,(\text{-IW}_K)$$

and $u'(c_1^*(\overline{x})) < u'(c_1^*(\underline{x}))$, iii) follows. iv) follows directly from (20) and $u'(c_1^*(\bar{x})) < u'(c_1^*(\underline{x}))$. To show v) note that (20) and (21) imply $\tau_B^*(x) - \tau_K^*(x) = E_1 x - x$. ■

Proof. [Proof of Proposition 5] Assume $M = 1$ and let $\tilde{r}(x)$ denote the return to the risky security. Then, the result follows from:

$$\begin{aligned} E_1 u'(c_1^*(x))(1+\tilde{r}(x)) - E_1 u'(c_1^*(x))(1+x) &= Cov_1(u'(c_1^*(x)), \tilde{r}(x)) - Cov_1(u'(c_1^*(x)), x) \\ &= Cov_1(u'(c_1^*(x)), \tilde{r}(x) - x). \end{aligned}$$

$Cov_1(u'(c_1^*(x)), \tilde{r}(x) - x) > 0$ if $\tilde{r}(x) - x$ is decreasing in x, or $Cov_1(\tilde{r}(x) - x, x) < 0$. By the definition of covariance and by the fact that $E_1 x = E_1 \tilde{r}(x)$:

$$Cov_1(\tilde{r}(x) - x, x) = E_1 \tilde{r}(x) x - E_1 x^2 = Cov_1(\tilde{r}(x), x) - V_1(x). \tag{40}$$

By $V_1(x) > V_1(\tilde{r}(x))$ and $Cov_1(\tilde{r}(x), x) > 0$, $0 < Corr_1(\tilde{r}(x), x) < 1$. Then:

$$Cov_1(\tilde{r}(x), x) - V_1(x) = SD_1(x)[Corr_1(\tilde{r}(x), x) SD_1(\tilde{r}(x)) - SD_1(x)] < 0.$$

In addition, $\tau_F^*(x) - \tau_K^*(x) = \tilde{r}(x) - x$. Since $\tilde{r}(x) - x$ is decreasing in x and $E_1\tilde{r}(x) = E_1 x$, $\tau_F^*(\bar{x}) - \tau_K^*(\bar{x}) < 0$ and $\tau_F^*(\underline{x}) - \tau_K^*(\underline{x}) > 0$. This derivation can be applied to any additional security available in the decentralization. ■

Proof. [Proof of Proposition 6] Suppose that $\left\{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i\right\}(K_0, T) \neq \{1, K_1^*, s^*, \{s^* K_1^*\}_i\}$ for some $s^* \in [0,1)$. If $\left\{\hat{e}, \hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i\right\}$ is interior, by (30), (31) and (32), (27)-(29) simplify to:

$$1 = E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}.$$

Then, $\hat{K}_1(1-\hat{s}) + \int_{i\in[0,1]} \hat{S}_1(i)\,di \gtreqless K_1^*(1-s^*) + \int_{i\in[0,1]} S_1^*(i)\,di$, with $\hat{s} \in (0,1)$, implies $\frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \lesseqgtr \frac{u'(c_0^*)}{u'(c_1^*(x))}$, irrespective of the value of $\hat{e}$. Contradiction. Hence, the only interior solution to (27)-(29) is $\{K_1^*, s^*, \{s^* K_1^*\}_i\}$ for $s^* \in (0,1)$. In addition, at T^* the local second order sufficient conditions are satisfied. To see this, consider the sub-optimization problem associated with the choice of $\{S_1(i)\}_i$ and K_1 for given e. In the symmetric equilibria we are considering, expected returns are the same for all stocks and we can restrict attention to the choice of S_1, where $S_1(i) = S_1$ for all $i = [0,1]$. The elements of the Hessian, H_U, for this problem are:

$$U_{BB}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x)) \left(1 + \bar{D}^* - \tau_S^*(x)\right)^2 \leq 0,$$

$$U_{KK}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x)) (1+x)^2 (1 - \tau_P^*(x))^2 \leq 0,$$

$$U_{BK}(\hat{e}) = u''(c_0^*) + E_{\hat{e}} u''(c_1^*(x)) \left(1 + \bar{D}^* - \tau_S^*(x)\right)(1+x)(1 - \tau_P^*(x)),$$

where U_{xy} denotes a cross-partial derivative with respect to the variables x, y.Under (30)-(31), $|H_U| = 0$. Hence, the Hessian is negative semi-definite irrespective of the value of $\hat{e}$. We now consider values of $\hat{K}_1, \hat{S}_1$ that are not interior. The Inada conditions exclude non-interior solutions that result from the non-negativity constraint on time 0 consumption being binding. Hence, there are two candidate non-interior solutions: $\hat{K}_1 = 0$ and $\hat{S}_1 > 0$, and $\hat{K}_1 > 0$ and $\hat{S}_1 = \bar{B}$. In both cases, of the two Euler equations for K_1 and S_1, one holds with equality and the other as a strict inequality. Under T^*:

$$1 = E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}, \tag{41}$$

$$1 > E_{\hat{e}} \frac{u'(\hat{c}_1(x))}{u'(\hat{c}_0)} \frac{u'(c_0^*)}{u'(c_1^*(x))}. \tag{42}$$

Moreover, (29) implies $\hat{s} = 0$. This is a contradiction, since (40) and (41) clearly cannot hold at the same time. Then, K_1^*, S_1^* are globally optimal irrespective of the value of $\hat{e}$ for some $s^* \in [0,1)$. Moreover, at K_1^*, S_1^*, $\rho^*(x)$ implies $\hat{e} = 1$ since the constrained-efficient allocation is incentive compatible. Hence, $\{1, K_1^*, s^*, \{s^* K_1^*\}_i\}$ is optimal for the agent given the initial endowment K_0, the tax system T^*, and the distribution policy $d(x)^*$, which implies expected return process $\bar{D}^*$. It follows that the resulting allocation, $\{c_0^*, 1, K_1^*, s^*, \{s^* K_1^*\}_i, c_1^*(x)\}$, jointly with the distribution policy $d^*(x)$ and the resulting expected return process $\bar{D}^*$ constitute a competitive equilibrium, according to definition 4. ■

Proof. [Proof of Proposition 7] Suppose that the distribution policy is $\hat{d}(x)$ and that

$$E_{\hat{e}}\left(1+\hat{d}(x)\right) u'(c_1(x)) \neq E_{\hat{e}}(1+x)(1-\tau_P(x)) u'(c_1(x)),$$

for some tax system where (27) holds with equality at $\hat{\tau}_P(x)$. Denote the corresponding competitive equilibrium allocation with $\left\{\hat{K}_1, \hat{s}, \left\{\hat{S}_1(i)\right\}_i, \hat{e}, \hat{c}_1(x)\right\}$, with $\hat{K}_1 > 0$. If $E_{\hat{e}}\left(1+\hat{d}(x)\right) u'(c_1(x)) > E_{\hat{e}}(1+x)(1-\tau_P(x)) u'(c_1(x))$, for some $0 < \hat{s} < 1$, we can write:

$$\begin{aligned} 0 &= -u'(\hat{c}_0)(1-\hat{s}) + \beta E_{\hat{e}}\left[(1+x)(1-\hat{\tau}_P(x)) - \left(1+\hat{d}(x)\right)\hat{s}\right] u'(\hat{c}_1(x)) \\ &< -(1-\hat{s})\left[u'(\hat{c}_0) - \beta E_{\hat{e}}\left(1+\hat{d}(x)\right) u'(\hat{c}_1(x))\right], \end{aligned}$$

which implies $0 > u'(\hat{c}_0) - \beta E_1\left(1+\hat{d}(x)\right) u'(\hat{c}_1(x))$. But by (29), $\hat{s} = 0$. Contradiction. Similarly, if $E_{\hat{e}}\left(1+\hat{d}(x)\right) u'(c_1(x)) < E_{\hat{e}}(1+x)(1-\tau_P(x)) u'(c_1(x))$ for some $0 < \hat{s} < 1$:

$$\begin{aligned} 0 &= -u'(\hat{c}_0)(1-\hat{s}) + \beta E_{\hat{e}}\left[(1+x)(1-\hat{\tau}_P(x)) - \left(1+\hat{d}(x)\right)\hat{s}\right] u'(\hat{c}_1(x)) \\ &> -(1-\hat{s})\left[u'(\hat{c}_0) - \beta E_{\hat{e}}\left(1+\hat{d}(x)\right) u'(\hat{c}_1(x))\right]. \end{aligned}$$

Then, $u'(\hat{c}_0) - \beta E_1\left(1+\hat{d}(x)\right) u'(\hat{c}_1(x)) > 0$, which by (29) implies $\hat{s} = 1$. Contradiction. ■

B Extensions

B.1 Decreasing Returns and Multiple States

We generalize the investment technology by allowing decreasing returns to capital and more than two states in the stochastic process for capital returns. Preferences are the same as in the benchmark model. We assume that entrepreneurs are endowed with $k_0(i)$ units of the consumption good at time 0, for $i \in [0,1]$, so that the initial endowment of capital can be different across entrepreneurs. We let the aggregate endowment of capital be given by: $K_0 = \int_0^\infty k_0(i)\, di$. We assume that the investment technology can take on the general form:

$$R(k_1, x) = f(k_1)(1+x),$$

where x is the random net return on capital and f is a strictly increasing and concave function. We assume that $f'(k_1) \to \infty$ for $k_1 \to 0$ and that $f'(k_1) \to 0$ for $k_1 \to \infty$. The variable x takes on values $\{x_j\}_{j=1}^N$ with corresponding probability distribution $\pi_j(e)$, $\pi_j(e) \in (0,1)$ for all $j = 1, N$ and $\sum_j \pi_j(e) = 1$ for $e = 0, 1$. We assume that x_j is increasing in j and that the monotone likelihood ratio condition (MLRC) holds so that $\frac{\pi_k(1)}{\pi_k(0)}$ is increasing in k. This implies that the distribution of x under high effort stochastically dominates the distribution under low effort, so that $E_1(x) > E_0(x)$, where E_e denotes the expectation operator for probability distribution $\pi(e)$. Hence, the expected return on capital is increasing in effort.

We assume that effort is *private information*, while the realized value of x, as well as its distribution, and individual investment $k_1(i)$ are *public information*. This implies that entrepreneurial activity is subject to a dynamic moral hazard problem.

We characterize constrained-efficient allocations for this economy by deriving the solution to a particular planning problem. The planner maximizes agents' lifetime expected utility by choice of consumption, individual investment and effort allocation. The planning problem is[22]:

$$\left\{e^*, k_1^*(i), c_0^*, \{c_1^*(x_j)\}_{j=1}^N\right\} = \arg\max_{e\in\{0,1\},\ k_1(i),c_0,c_1(x)\geq 0} u(c_0) + \beta E_e u(c_1(x)) - v(e) \quad \text{(Problem 1)}$$

subject to

$$c_0 + K_1 \leq K_0,\ \ E_e c_1(x) \leq E_e(1+x)\int_0^\infty f(k_1(i))\,di, \tag{43}$$

$$\beta E_1 u(c_1(x)) - \beta E_0 u(c_1(x)) \geq v(1) - v(0), \tag{44}$$

where E_e denotes the expectation operator with respect to the probability distribution $\pi(e)$ and $K_1 = \int_0^\infty k_1(i)\,di$. The constraints in (2) stem from resource feasibility, while (43) is the incentive compatibility constraint, arising from the unobservability of effort.

This program assumes that since all agents have the same preferences and capital can be freely transferred across agents by the planner at time 0, and that the government will treat all agents identically. A social welfare function with equal weighting on all agents is necessary to generate this outcome, but it's not sufficient. It may be optimal under certain conditions to transfer all capital to one one agent so that the incidence of the incentive problem is minimized. To rule out this rather irrealistic case, one needs to assume $f'(k) \to \infty$ for $k_1 = 0$. To see this, we can inspect the first order necessary conditions:

$$-u'(c_0(i)) + \lambda f'(k_1(i))\,E_e(1+x) \leq 0,$$

$$\beta\pi_j(e)\,u'(c_{1,i}(x_j)) - \lambda\pi_j(e_i) - \mu\beta(\pi_j(0) - \pi_j(1))\,u'(c_{1,i}(x_j)) = 0.$$

Under this additional assumption, $K_1 = k_1(i)$ for all $i \in [0,1]$ and the consumption and effort allocation are the same for all agents. So we drop the i index henceforth. The optimal allocation only depends on the value of initial aggregate capital K_0. We will denote the value of the optimized objective for Problem 1 with $U^*(K_0)$.

The first order necessary conditions for the planning problem are:

$$u'(c_0) = \lambda f'(K_1)\,E_e(1+x),$$

$$\beta\pi_j(e)\,u'(c_1(x_j)) - \lambda\pi_j(e) - \mu\beta(\pi_j(0) - \pi_j(1))\,u'(c_1(x_j)) = 0,$$

[22]Given that the investment technology is linear in capital, the efficient distribution of capital is degenerate, with one entrepreneur operating the entire economywide capital stock. Since this result is not robust to the introduction of any degree of decreasing returns, and this in turn would not alter the structure of the incentive problem, we simply assume that the planner cannot transfer initial capital across agents.

plus the feasibility constraints with equality. Here, λ is the multiplier of the feasibility constraint at time 1 and μ is the multiplier on the incentive compatibility constraint. The first equation implies that k_1

We shall now assume that high effort is implemented and derive the basic properties of the optimal allocation. By the second condition:

$$\beta\left(1+\mu-\mu\frac{\pi_j(0)}{\pi_j(1)}\right)=\frac{\lambda}{u'(c_1^*(x_j))}.$$

By MLRC, the left hand side of this equation is increasing in j, which implies that the optimal consumption at time 1 is increasing in x. This is the familiar partial insurance condition.

Dividing the second condition by $u'(c_1(x_j))$, summing over j and combining with the first condition delivers the familiar inverted Euler equation:

$$1=\frac{u'(c_0^*)}{\beta f'(K_1^*)\,E_1(1+x)}E_1\left[\frac{1}{u'(c_1^*(x))}\right].$$

Multiplying the second condition by $(1+x_j)$, summing over j and combining with the first condition delivers:

$$\mathrm{IW}_K=\beta f'(K_1^*)\,E_1u'(c_1^*(x))(1+x)-u'(c_0^*)=\mu\beta f'(K_1^*)\left[E_0u'(c_1^*(x))(1+x)-E_1u'(c_1^*(x))(1+x)\right].$$

Multiplying the second condition by $f'(K_1^*)\,E_1(1+x)$, summing over j and combining with the first condition delivers:

$$\mathrm{IW}=\beta f'(K_1^*)\,E_1u'(c_1^*(x))\,E_1(1+x)-u'(c_0^*)=\mu\beta\left[E_0u'(c_1^*(x))-E_1u'(c_1^*(x))\right]f'(K_1^*)\,E_1(1+x).$$

This derivation completly parallels the special case studied in Section 2, with constant returns to capital and two values of x. A straightforward extension of Propositions Thus, the qualitative properties of the optimal allocation and optimal taxes do not depend on the presence of decreasing returns in the model. In fact, the solution resembles one with a representative entrepreneur. However, decreasing returns influence the optimal size of the entrepreneurial activity.

B.1.1 More Than Two Effort Levels

The analysis can further be extended to more than two effort levels: $\{e_k\}_{k=1}^M$, with e_k increasing in k. Also, assume that the MLRC holds so that:

$$\frac{\pi_j(e_i)}{\pi_l(e_i)}\geq\frac{\pi_j(e_k)}{\pi_l(e_k)}\text{ for all }k<i\text{ and for all }l<j.$$

If the planner wants to implement the most costly action, then the case with more than two effort levels does not differ from the previous case. If instead the planner wishes to implement an intermediate action, to insure that partial insurance holds and that consumption in increasing in x at time 1, we need in addition to assume the convexity of the distribution function condition (CDFC), which requires that the cumulative distribution function of x is convex in e. This can be interpreted as the returns to effort being stochastically decreasing.

Summing up, introducing decreasing returns and allowing for the planner to redistribute capital across agents does not change any of the results unless strong redistributional concerns are introduced.

C Alternative Interpretations of the Model

Following Demarzo and Fishman (2006), we now show that the model is equivalent to one in which the entrepreneur privately observes capital returns and can divert realized output to obtain a private benefit. Specifically, if the capital returns are high, that is equal to $\bar{x}$, the entrepreneur could report $\underline{x}$, and receive the private benefit $v\left(\left(\bar{x}-\underline{x}\right)K_1\right)$. The optimal allocation can be characterized by assuming that the entrepreneur transfers realized output to the principal and receives a consumption transfer contingent on the reported value of x. The state contingent consumption award, $c_1\left(x\right)$, must ensure that the entrepreneur will not under-report x. This gives rise to the following incentive compatibility constraint:

$$u\left(c_1\left(\bar{x}\right)\right)+v\left(0\right)\geq u\left(c_1\left(\underline{x}\right)\right)+v\left(\left(\bar{x}-\underline{x}\right)K_1\right). \tag{45}$$

Let the distribution of capital returns be be given by: $\Pr\left(\bar{x}\right)=\pi\in\left(0,1\right)$ and $\Pr\left(\underline{x}\right)=1-\pi$ and denote with $E\left(\cdot\right)$ the corresponding expectations operator. Then, the optimal capital and investment allocation solves Problem 1 with (44) replacing (43) and

$$c_0+\frac{Ec_1\left(x\right)}{E\left(1+x\right)}\leq K_0,$$

replacing (2).

The first order necessary conditions for the planning problem are:

$$-u'\left(K_0-K_1\right)+\lambda E\left(1+x\right)-\mu\beta\left(\bar{x}-\underline{x}\right)v'\left(\left(\bar{x}-\underline{x}\right)K_1\right)=0,$$
$$\left(1-\pi\right)\beta u'\left(c_1\left(\underline{x}\right)\right)-\mu\beta u'\left(c_1\left(\underline{x}\right)\right)-\lambda\left(1-\pi\right)=0,$$
$$\pi\beta u'\left(c_1\left(\bar{x}\right)\right)+\mu\beta u'\left(c_1\left(\bar{x}\right)\right)-\lambda\pi=0.$$

It is straighforward to derive the aggregate and individual intertemporal wedges for this version of the model:

$$IW_K=\mu\beta\left[(1+\underline{x})u'\left(c_1\left(\underline{x}\right)\right)-(1+\overline{x})u'\left(c_1\left(\bar{x}\right)\right)\right]+\mu\beta\left(\bar{x}-\underline{x}\right)v'\left(\left(\bar{x}-\underline{x}\right)K_1\right).$$

The individual intertemporal wedge depends on the gain in expected utility obtained by diverting output and on the spread in discounted capital returns, and it can be positive or negative. It will be negative if the spread in capital returns is sufficiently high. More capital increases the potential amount of output that the entrepreneur does not report to the principal, which tightens the incentive compatibility constraint. On the other hand, tf the entrepreneur has higher capital holdings, the marginal benefit of misreporting will be lower since the private benefit decreases with the magnitude of diverted output. This relaxes the incentive compatibility constraint.

The aggregate intertemporal wedge corresponds to:

$$IW=\beta\mu\left[E(1+x)(u'\left(c_1\left(\underline{x}\right)\right)-u'\left(c_1\left(\bar{x}\right)\right))+\left(\bar{x}-\underline{x}\right)v'\left(\left(\bar{x}-\underline{x}\right)K_1\right)\right],$$

and is always positive, by the usual logic. Then, in the version of the model with privately observed capital returns, the incentive effects of assets holdings are the same as in the dynamic moral hazard model.

It is straightforward to show that a version of the model in which x is publically observable but capital returns are not collectible delivers the incentive compatibility constraint (44), and thus delivers similar results for the intertemporal wedge.