Fiscal Rules and Discretion under Persistent Shocks*

PRELIMINARY AND INCOMPLETE†

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Abstract

This paper studies the optimal level of discretion in policymaking. We consider a dynamic fiscal policy model in which the government has time-inconsistent preferences with a present-bias towards public spending. The government chooses a fiscal rule to trade off its desire to commit to not overspend against its desire to have flexibility to react to privately observed shocks to the value of spending. We allow the shocks to be persistent over time, and analyze the optimal structure of fiscal rules in this mechanism design problem. If shocks are i.i.d., the ex-ante optimal rule, chosen at the beginning of time, coincides with the sequentially optimal rule, chosen period by period. We show that when shocks are persistent, the ex-ante optimal rule is no longer sequentially optimal, as dynamic incentives are now provided. Furthermore, under persistent shocks, the ex-ante optimal rule exhibits history dependence, with high shocks leading to an erosion of future fiscal discipline compared to low shocks, which lead to the reinstatement of fiscal discipline. In contrast, the sequentially optimal rule is independent of history, and takes the simple form of a state-dependent debt limit.

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1 Introduction

Governments often impose rules on themselves to constrain their behavior in the future. One of the most prevalent forms of such rules are fiscal rules. In 2009, 80 countries had fiscal rules in place, a dramatic increase from 1990 when only seven countries had them. Of those 80 countries, 60 imposed a deficit limit, 60 imposed a debt limit, and 25 imposed a spending limit.\(^1\) Fiscal rules are also present within countries; in the United States, for example, 44 states have balanced budget rules, and 23 states have spending limits.\(^2\)

Despite their prevalence, little is known about the optimal structure of fiscal rules. Any theory of fiscal rules must take into account a fundamental tradeoff between commitment and flexibility: on the one hand, rules provide valuable commitment as they can fix distorted incentives in policymaking which result in a spending bias and excessive deficits; on the other hand, rules come at a cost of reduced flexibility as fiscal constitutions cannot spell out policy prescriptions for every single shock or contingency, and some discretion may be optimal.\(^3\)

This paper studies the tradeoff between commitment and flexibility in a dynamic self-control setting using mechanism design. In our model, a present-biased government privately observes a shock to the economy in each period, and a fiscal rule is a mechanism which assigns a policy as a function of the government’s reported shock. We follow a similar approach to that used in Amador, Werning, and Angeletos (2006), but we depart from their work by considering an environment in which shocks are persistent over time.\(^4\) We are motivated by the fact that shocks underlying fiscal policy are likely to be autocorrelated, consistent with the observation that fiscal policy variables are persistent in the data.\(^5\) Nonetheless, the problem of mechanism design under persistent private information is complicated, as single-crossing conditions which are generally used for the analysis may fail and a recursive representation of the problem becomes more difficult. A recent literature addresses the issue of persistence in a principal-agent setting, but this analysis does not apply to our self-control setting where transfers across players are not possible.\(^6\)

\(^1\)See International Monetary Fund (2009). 53 of these countries have national rules whereas the rest have rules connected to international treaties. The imposed limits on fiscal aggregates vary in their specification and the extent to which they adjust to levels of GDP and to the business cycle. Moreover, these limits also vary in the degree to which they are enforced.

\(^2\)See National Conference of State Legislatures (2008, 2010) for a discussion of state balanced budget provisions and state expenditure limits. These rules vary both in their exact specification and in the stages at which they are imposed throughout the budgetary process.


\(^4\)See also Amador, Werning, and Angeletos (2003), which includes additional results, as well as Amador and Bagwell (2011), Ambrus and Egorov (2012), and Athey, Atkeson, and Kehoe (2006) for related analyses in settings with i.i.d. shocks.

\(^5\)For example, see Barro (1990) for evidence.

\(^6\)Pavan, Segal, and Taikka (2010) show that under some conditions, the first order approach is valid in the analysis of these relationships. Farhi and Werning (2010) and Golosov, Tsyvinski, and Troshkin (2011) consider the
In this paper, we analyze and explicitly characterize the optimal structure of fiscal rules under persistent shocks. Our environment is a small open economy in which the government makes repeated spending and borrowing decisions. In each period, a shock to the marginal value of deficit-financed government spending is realized, where this shock follows a first-order Markov process. We consider two shocks in our benchmark setting, and a continuum of shocks in an extension. The government is benevolent ex ante, prior to the realization of the shock, but present-biased ex post, when it is time to choose fiscal policy. The shock in each period is privately observed by the government, capturing the fact that not all contingencies in fiscal policy are contractible or observable. Fiscal rules are then defined as a mechanism in which the government reports the shock in each period and is assigned a policy for every reported shock. Note that in the absence of private information, the first-best policy could be implemented with full commitment; i.e., by giving the government no discretion and committing it to the efficient path of spending. Similarly, in the absence of a present bias, the first-best policy could be implemented with full flexibility; i.e., by giving the government full discretion to choose spending in each period. When both private information and a present bias are present, however, a tradeoff between commitment and flexibility arises, and the optimal design of fiscal rules is then not trivial.

We study the ex-ante optimal fiscal rule and the sequentially optimal fiscal rule. The ex-ante optimal rule corresponds to the optimal dynamic mechanism that the government chooses at the beginning of time. This is a sequence of spending and borrowing levels as a function of the history of the government’s reports, which maximizes ex-ante social welfare subject to a sequence of period by period truth-telling constraints. The sequentially optimal rule, on the other hand, corresponds to a static mechanism that is chosen in every period $t$ by the government, which maximizes social welfare from date $t$ onward taking into account that future governments do the same. Our motivation for studying sequentially optimal rules is the observation that, in practice, fiscal rules often have a bite in the short term for the current fiscal year, but can be renegotiated and changed by the government in advance for the following fiscal year. While the sequentially optimal rule naturally yields (weakly) lower ex-ante welfare than the ex-ante optimal rule, it has the property that, by construction, the government would not want to alter the mechanism ex post; this may not necessarily be the case for the ex-ante optimal rule.

A result from previous literature is that the ex-ante optimal and sequentially optimal fiscal rules coincide when shocks are i.i.d. That is, under the ex-ante optimal mechanism, at any given date, prior to the realization of the shock, the government would not want to change the continuation mechanism. The reason is that no dynamic incentives are provided, and the mechanism at any date depends only on the payoff relevant state (the level of debt) and not on the history. Notably, this result is in contrast to that of the principal-agent literature, where, role of persistence in a dynamic Mirrleesian environment by assuming the first order approach and quantitatively verifying that it holds.

7The preference structure is identical to that of the quasi-hyperbolic consumption model; see Laibson (1997).
even under i.i.d. shocks, dynamic incentives are provided to an agent with private information along the equilibrium path. The difference is due to the fact that there is a single player in a self-control setting, and dynamic incentives cannot be provided by increasing one player’s welfare at the expense of another, as is possible in a principal-agent setting. If dynamic incentives were provided here, they would affect the government’s welfare on the equilibrium path (i.e., when reporting the shock truthfully) and off the equilibrium path (i.e., when misreporting the shock) in the same fashion, and therefore result in no welfare gains.

Our first main result is that, unlike under i.i.d. shocks, when shocks are persistent, the ex-ante optimal fiscal rule does not coincide with the sequentially optimal fiscal rule. The ex-ante optimal rule takes into account that the government in every period learns about its current and future spending needs, and now provides dynamic incentives for the government not to overspend and overborrow. For example, consider the problem of providing incentives to a government which is tempted to overspend today when its current needs are low. Dynamic incentives can be provided by introducing excessively lax and ex-post suboptimal rules tomorrow if the government chooses high levels of spending today. The expected cost to the government of such lax rules tomorrow is greater if spending needs are actually low today, as spending needs are then more likely to be low tomorrow. Thus, lax rules tomorrow affect welfare on and off the equilibrium path differently, and the threat of no discipline in the future ironically imposes discipline today.

Our second main result is that, in contrast to the case of i.i.d. shocks, when shocks are persistent, the ex-ante optimal fiscal rule is history dependent: the continuation mechanism at a given date is a function of not only the payoff relevant states, but also the entire history of shocks. This follows from the fact that the mechanism provides dynamic incentives. Specifically, because the shock at date \( t - 2 \) predicts the realization of the shock at \( t - 1 \) (as shocks follow a first-order Markov process), the shock at \( t - 2 \) affects the relative tightness of incentive compatibility constraints at \( t - 1 \), and this in turn affects the policies that are chosen at \( t \) in providing dynamic incentives at \( t - 1 \). For instance, in the example above, the shock at date \( t - 2 \) affects the costs and benefits of choosing excessively lax rules at \( t \) in providing dynamic incentives at \( t - 1 \). We explicitly characterize the dynamics implied by the ex-ante optimal fiscal rule in an infinite horizon economy using a recursive technique similar to that developed by Fernandes and Phelan (2000) for a principal-agent setting. We show that high shocks to the value of spending lead to an erosion of fiscal discipline compared to low shocks, which lead to the reinstatement of fiscal discipline. Fiscal rules thus continue to be history dependent even in the long run.

Our final result concerns the characterization of the sequentially optimal fiscal rule. We show that the sequentially optimal rule is history independent and simple, as dynamic incentives are not provided to the government. Moreover, we show that in the case of two shocks as well as in the extension with a continuum of shocks, the sequentially optimal mechanism takes the form of a debt limit that is a function of the payoff relevant states—the current level of debt and the previous period’s shock which forecasts the current shock. We characterize this debt limit and
show how it varies with these states.

The paper is related to several literatures. First, as previously discussed, it is related to the mechanism design literature on the tradeoff between commitment and flexibility. In contrast to this literature, we study the optimal dynamic mechanism in a setting with persistent shocks. Second, the paper is related to the literature on the political economy of fiscal policy. Most closely related is Azzimonti, Battaglini, and Coate (2011), which considers the quantitative welfare implications of a balanced budget rule in an i.i.d. setting where the government is present-biased towards pork-barrel spending. Our main departure is that we study optimal fiscal rules in a private information economy, and do not assume the structure of rules but use mechanism design tools to derive the optimal structure. Third, our work is related to the literature on dynamic principal-agent contracts under persistent shocks, although, because our application is a self-control environment, the methods used in that literature do not directly apply here. Within this literature, Strulovici (2011) provides a definition of renegotiation-proofness that is close in spirit to the sequentially optimal mechanism that we define in our context. Finally, our paper is related to work on hyperbolic discounting and the benefits of commitment devices. We contribute to this literature by providing a framework for modeling the tradeoff between commitment and flexibility in an environment with persistent shocks.

The paper is organized as follows. Section 2 describes our benchmark environment. Section 3 defines the ex-ante optimal and sequentially optimal fiscal rules. Section 4 illustrates the main insights from our model using a simple three-period example. Section 5 characterizes optimal fiscal rules in the infinite horizon economy. Section 6 extends our analysis to an economy with a continuum of shocks. Section 7 concludes. The Appendix contains the proofs and additional material excluded from the text.

2 The Model

We consider a simple model of fiscal policy in which a government makes repeated spending and borrowing decisions. At the beginning of each period, \( t \in \{0, 1, \ldots \} \), the government observes a shock to the economy, which is the government’s private information or type. The government’s type can be low or high, \( \theta_t \in \{\theta^L, \theta^H\} \), where \( \theta^H > \theta^L > 0 \). This type follows a first-order Markov process, with \( p(\theta_{t+1}|\theta_t) \) corresponding to the probability of type \( \theta_{t+1} \) at date \( t + 1 \) conditional on type \( \theta_t \) at date \( t \). We consider \( p(\theta^L|\theta^L) = p(\theta^H|\theta^H) \geq 0.5 \), and we compare

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8 This work is also related more generally to the literature on delegation, including Alonso and Matoushek (2008), Ambrus and Egorov (2009), and Holmstrom (1977, 1984). In contrast to this work, our focus is a self-control problem in which there is a single player. One can show for example that even under i.i.d. shocks, ex-post suboptimality emerges in a delegation setting, while this does not happen in our self-control framework.

9 In addition to the work cited in Footnote 3, some other recent examples are the work of Acemoglu, Golosov, and Tsyvinski (2008), Azzimonti (2010), and Song, Storesletten, and Zilibotti (2012).

10 In addition to the work cited in Footnote 6, see also Golosov and Tsyvinski (2006), Halac (2012), Kapicka (2010), and Williams (2010).

11 For example, see Barro (1999), Krusell, Kuruscu, and Smith (2010), Krusell and Smith (2003), Laibson (1997), and Phelps and Pollack (1968).
the case where types are i.i.d., i.e., \( p(\theta|\theta^i) = 0.5 \) for \( \theta^i \in \{\theta_L, \theta_H\} \), to the case where types are persistent over time, i.e., \( p(\theta|\theta^i) > 0.5 \) for \( \theta^i \in \{\theta_L, \theta_H\} \).\(^{12,13}\)

In every period \( t \), following the realization of \( \theta_t \), the government chooses public spending \( g_t \geq 0 \) and debt \( b_{t+1} \) subject to a budget constraint:

\[
g_t = \tau + b_{t+1}/(1+r) - b_t, \tag{1}
\]

for some \( \tau > 0 \) representing the exogenous fixed tax revenue collected by the government in each period.\(^{14}\) \( b_t \geq 0 \) is the level of debt with which the government enters the period, and \( r \) is the exogenous interest rate. \( b_0 \) is exogenous and

\[
\lim_{t \to \infty} b_{t+1}/(1+r)^t = 0,
\]

so that all debts must be repaid and all assets must be consumed. Constraint (1) can be rewritten as a weak inequality constraint to allow for money burning without affecting any of our results. Such a weak inequality constraint would take into account the possibility of introducing fines in this setting; we ignore this possibility here to simplify the exposition.\(^{15}\)

The government’s welfare at date \( t \), prior to the realization of its type \( \theta_t \), is

\[
\sum_{k=0}^{\infty} \beta^k \mathbb{E}[\theta_{t+k} U(g_{t+k})|\theta_{t-1}], \tag{2}
\]

where \( \theta_t U(g_t) \) is the social utility from public spending at date \( t \). The government’s welfare after the realization of its type \( \theta_t \) at date \( t \), when choosing spending \( g_t \), is

\[
\theta_t U(g_t) + \delta \sum_{k=1}^{\infty} \beta^k \mathbb{E}[\theta_{t+k} U(g_{t+k})|\theta_t], \tag{3}
\]

where \( \delta \in (0,1) \).

There are two important features of this environment. First, the government’s objective (3) following the realization of \( \theta_t \) does not coincide with its objective (2) prior to the realization of \( \theta_t \). In particular, the government’s \textit{ex-post} welfare outweighs the importance of current public spending compared to its \textit{ex-ante} welfare. This formulation captures a friction which is common in various models of political economy interactions featuring preferences analogous to (2) and

\(^{12}\)Our main results regarding the effects of persistence are robust to considering \( p(\theta|\theta^i) < 0.5 \), although the intuition is different. Given that fiscal policy variables are positively autocorrelated in the data, we focus our attention on \( p(\theta^i|\theta^i) \geq 0.5 \).

\(^{13}\)Section 6 extends our analysis to an economy with a continuum of types.

\(^{14}\)Our analysis also applies if instead of having an exogenous tax revenue, social welfare is an increasing function of the budget deficit.

\(^{15}\)Amador, Werning, and Angeletos (2006) allow (1) to be a weak inequality constraint in an i.i.d. setting and prove that it must bind in equilibrium. Similar arguments to theirs can be used in our setting with two persistent shocks to prove the same result. Details are available upon request.
For instance, preferences such as these may emerge naturally in settings with political uncertainty where policymakers place a higher value on public spending when they hold power and can make spending decisions. In such settings, policymakers are biased towards present public spending relative to future public spending and incur excessively high debts.

The second feature of this environment is that the realization of $\theta_t$—which affects the marginal social utility of public spending—is privately observed by the government. One possible interpretation is that $\theta_t$ is not verifiable ex-post by a rule-making body; therefore, even if $\theta_t$ is observable, fiscal rules cannot explicitly depend on the value of $\theta_t$. An alternative interpretation is that the exact cost of public goods is only observable to the policymaker, who may be inclined to overspend on these goods. A third possible interpretation is that citizens have either heterogeneous preferences or heterogeneous information regarding the optimal level of public spending, and the government sees an aggregate that the citizens do not see (see Sleet, 2004).

To simplify the analysis, we make the following assumption.\footnote{See Section 5.2 of Amador, Werning, and Angeletos (2006) for an analogous interpretation of these preferences.}

**Assumption 1.** $U(g_t) = \log(g_t)$.

Assumption 1 implies that welfare is separable with respect to the level of debt. To see this, define

$$\tilde{\theta}^i = \sum_{k=1}^{\infty} \beta^k \mathbb{E}\left[\theta_{t+k}|\theta_t = \theta^i\right],$$

for $i = \{L, H\}$, so that at any date $t$, $\tilde{\theta}_t = \tilde{\theta}^i$ if $\theta_t = \theta^i$. Let the savings rate at $t$ be $s_t \in [0, 1]$, corresponding to the fraction of lifetime resources which is not spent at $t$:

$$g_t = (1 - s_t)[(1 + r)\tau/r - b_t].$$

Using this notation, under Assumption 1, welfare in (2), at date $t$ but prior to the realization of the type $\theta_t$, can be rewritten as

$$\sum_{k=0}^{\infty} \beta^k \mathbb{E}[\theta_{t+k}U(1 - s_{t+k}) + \tilde{\theta}_{t+k}U(s_{t+k})|\theta_{t-1}] + \chi(b_t),$$

for a constant $\chi(b_t)$ which depends on $b_t$.\footnote{This assumption is made in previous work studying economies with hyperbolic preferences, such as Barro (1999).}

Analogously, welfare in (3), at date $t$ following the realization of $\theta_t$, can be rewritten as

$$\theta_t U(1 - s_t) + \tilde{\theta}_t U(s_t) + \delta \sum_{k=1}^{\infty} \beta^k \mathbb{E}[\theta_{t+k}U(1 - s_{t+k}) + \tilde{\theta}_{t+k}U(s_{t+k})|\theta_t] + \chi(b_t).$$

\footnote{This constant is equal to $\sum_{k=0}^{\infty} \beta^k \mathbb{E}\left[\theta_{t+k}U((1 + r)^k[\tau(1 + r)/r - b_t)]|\theta_{t-1}\right]$.}
Given the representation in (6) and (7), hereafter we consider the problem of a government which chooses a savings rate \( s_t \) in every period \( t \).

3 Equilibrium Definition

We define a fiscal rule as a mechanism in which the government reports the shock in every period and is assigned a policy as a function of the report. We distinguish between the ex-ante optimal fiscal rule and the sequentially optimal fiscal rule. The ex-ante optimal fiscal rule is a dynamic mechanism chosen by the government at the beginning of time. In contrast, the sequentially optimal fiscal rule is a static mechanism chosen in every period by the current government, taking into account the future static mechanisms chosen by future governments.

We define \( \theta^t = (\theta_0, \theta_1, ..., \theta_t) \in \Theta^t \) as the history of shocks through time \( t \), and let \( p(\theta^{t+k} | \theta^t) \) be the probability of a history \( \theta^{t+k} \) conditional on history \( \theta^t \).

3.1 Ex-ante Optimal Rule

To define the ex-ante optimal fiscal rule, let \( h_{t-1} = (\hat{\theta}_0, \hat{\theta}_1, ..., \hat{\theta}_{t-1}) \in \Theta^{t-1} \) be the history of reported types through time \( t-1 \). A mechanism is a sequence of savings rates \( s_t(h_{t-1}, \hat{\theta}_t) \) for all \( \{(h_{t-1}, \hat{\theta}_t)\}_{t=0}^{\infty} \), which effectively specify levels of public spending, \( g_t(h_{t-1}, \hat{\theta}_t) \), and debt, \( b_{t+1}(h_{t-1}, \hat{\theta}_t) \), as a function of the history of past reports and the current report.

Given this mechanism, the government chooses a reporting strategy \( m_t(h_{t-1}, \theta_t) \) for all \( \{(h_{t-1}, \theta_t)\}_{t=0}^{\infty} \), where \( \theta_t \) is the government’s type at date \( t \) and \( m_t(h_{t-1}, \theta_t) \in \{\theta_L, \theta_H\} \) is the government’s report of its type at \( t \). We restrict attention to public strategies, that is, strategies that depend only on the public history—reports and policies—and on the government’s current private information, but not on privately observed history.\(^\text{19}\) From the Revelation Principle, we can restrict attention to truth-telling equilibria in which \( m_t(h_{t-1}, \theta_t) = \theta_t \) for all \( h_{t-1} \) and \( \theta_t \).

A perfect Bayesian equilibrium of this revelation game is a mechanism and a reporting strategy such that the budget constraint (1) is satisfied in every period following every history, and the policy under the mechanism is incentive compatible, meaning that following every history and type realization, the government prefers to report \( m_t(h_{t-1}, \theta_t) = \theta_t \) rather than \( m_t(h_{t-1}, \theta_t) = \hat{\theta}_t \neq \theta_t \). An ex-ante optimal rule in this framework is one that selects a mechanism and a reporting strategy that maximize the ex-ante welfare (6) in period 0.

We formulate the ex-ante optimal rule as a solution to a sequence program. Given history \( \theta^{t-1} \), let \( W_{t+1}(\theta^{t-1}, \theta_t) \) be the expected continuation value normalized by \( b_t(\theta^{t-1}) \) for a type \( \theta_t \).

\(^{\text{19}}\) It follows by standard arguments that if all future governments choose public strategies, and if the mechanism is a function of the public history, then the current government’s best response is a public strategy.
who truthfully reports $\hat{\theta}_t = \theta_t$:

$$W_{t+1}(\theta^{t-1}, \theta_t) = \sum_{\theta^{t+1} \in \Theta^{t+1}} p(\theta^{t+1}|\theta^t) \left[ \theta_{t+1} U(1 - s_{t+1}(\theta^{t-1}, \theta_t, \theta_{t+1})) + \tilde{\theta}_{t+1} U(s_{t+1}(\theta^{t-1}, \theta_t, \theta_{t+1})) + \beta W_{t+2}(\theta^{t-1}, \theta_t, \theta_{t+1}) \right].$$

In contrast, given history $\theta^{t-1}$, let $V_{t+1}(\theta^{t-1}, \hat{\theta}_t)$ be the expected continuation value normalized by $b_t(\theta^{t-1})$ for a type $\theta_t$ who lies and reports $\hat{\theta}_t \neq \theta_t$:

$$V_{t+1}(\theta^{t-1}, \hat{\theta}_t) = \sum_{\theta^{t+1} \in \Theta^{t+1}} p(\theta^{t+1}|\theta^t) \left[ \theta_{t+1} U(1 - s_{t+1}(\theta^{t-1}, \hat{\theta}_t, \theta_{t+1})) + \tilde{\theta}_{t+1} U(s_{t+1}(\theta^{t-1}, \hat{\theta}_t, \theta_{t+1})) + \beta W_{t+2}(\theta^{t-1}, \hat{\theta}_t, \theta_{t+1}) \right].$$

Note that in the special case of i.i.d. shocks, expectations over future shocks do not depend on the current shock $\theta_t$, implying that $W_{t+1}(\theta^{t-1}, \hat{\theta}_t) = V_{t+1}(\theta^{t-1}, \hat{\theta}_t)$.

Using (7), the incentive compatibility constraint of the government following the realization of $\theta_t$ is then

$$\theta_t U(1 - s_t(\theta^{t-1}, \theta_t)) + \tilde{\theta}_t U(s_t(\theta^{t-1}, \theta_t)) + \beta W_{t+1}(\theta^{t-1}, \theta_t) (8)$$

$$\geq \theta_t U(1 - s_t(\theta^{t-1}, \hat{\theta}_t)) + \tilde{\theta}_t U(s_t(\theta^{t-1}, \hat{\theta}_t)) + \beta V_{t+1}(\theta^{t-1}, \hat{\theta}_t)$$

$$\text{for } \hat{\theta}_t \neq \theta_t \text{ and } \forall \theta^{t-1}. \quad (10)$$

Condition (10) says that the government prefers to report its true type $\theta_t$ rather than to lie and report $\hat{\theta}_t \neq \theta_t$. To understand this constraint, note that the true type $\theta_t$ not only directly affects the government’s immediate payoff by determining the marginal cost and benefit of current savings, but, if shocks are persistent, it can also affect the government’s continuation payoff by changing the expectations over the realizations of future types.\(^2\)

Let a stochastic sequence of savings rates and continuation values be denoted by

$$\rho = \left\{ \left\{ s_t(\theta^t), W_{t+1}(\theta^t), V_{t+1}(\theta^t) \right\} \theta^t \in \Theta \right\}_{t=0}^\infty.$$ 

The ex-ante optimal rule thus solves the following sequence problem:

$$\max_{\rho} \sum_{\theta_0 \in \{ \theta^L, \theta^H \}} p(\theta_0|\theta^L) \left[ \theta_0 U(1 - s_0(\theta^0)) + \tilde{\theta}_0 U(s_0(\theta^0)) + \beta W_1(\theta^0) \right]$$

$$\text{s.t. } (8), (9), \text{ and } (10).$$

It is clear that the solution to this program is invariant to the initial level of debt $b_0$.\(^3\)

\(^2\)Because there are only two types, we only need to define one such expected continuation value.

\(^3\)Because $\theta_t$ follows a first-order Markov process, the single period deviation principle holds.
3.2 Sequentially Optimal Rule

The sequentially optimal fiscal rule is the one that results if, at every history, the government chooses a static mechanism that maximizes social welfare given that future governments will do the same. Formally, given \( \theta_{t-1} \) and \( b_t \), the government chooses a mechanism \( \{ g_t(\hat{\theta}_t), b_{t+1}(\hat{\theta}_t) \} \), assigning a level of spending and debt conditional on the report \( \hat{\theta}_t \), which maximizes social welfare taking the actions of future governments as given. The future government knows the true value of \( \theta_t \) and, given \( \theta_{t-1} \) and \( b_{t+1}(\hat{\theta}_t) \), it analogously chooses an optimal static mechanism taking the actions of future governments as given.

The sequentially optimal rule thus solves the following problem:

\[
J(\theta_{t-1}, b_t) = \max_{\{ g_t(\theta_t), b_{t+1}(\theta_t) \}_{\theta_t \in \{\theta^L, \theta^H\}}} \sum_{\theta_t \in \{\theta^L, \theta^H\}} p(\theta_t|\theta_{t-1})(\theta_t U(g_t(\theta_t)) + \beta J(\theta_t, b_{t+1}(\theta_t)))
\tag{12}
\]

s.t.
\[
g_t(\theta_t) = \tau + b_{t+1}(\hat{\theta}_t)/(1 + r) - b_t \quad \text{and} \quad \theta_t U(g_t(\theta_t)) + \delta \beta J(\theta_t, b_{t+1}(\theta_t)) \geq \theta_t U(g_t(\hat{\theta}_t)) + \delta \beta J(\theta_t, b_{t+1}(\hat{\theta}_t)) \quad \text{for} \quad \hat{\theta}_t \neq \theta_t. \tag{13}
\]

\( J(\theta_{t-1}, b_t) \) is the value at \( t \) under the payoff relevant states \( \theta_{t-1} \) and \( b_t \) if the current government chooses an optimal static mechanism given that future governments will do the same. In a sense, \( J(\theta_{t-1}, b_t) \) thus corresponds to the solution to a two period mechanism design problem. In choosing its report \( \hat{\theta}_t \), the government’s flow welfare is \( \theta_t U(g_t(\hat{\theta}_t)) \) and its continuation welfare is \( J(\theta_t, b_{t+1}(\hat{\theta}_t)) \). Condition (14) corresponds to a truth-telling constraint where the government knows that if it lies and reports \( \hat{\theta}_t \), this affects its payoff from tomorrow onward only through the implied level of debt \( b_{t+1}(\hat{\theta}_t) \), since, given \( b_{t+1}(\hat{\theta}_t) \), the government which knows the true value of \( \theta_t \) will choose an optimal static mechanism going forward. We interpret the sequentially optimal fiscal rule that emerges from this recursion as a rule which has a bite in the short term, but can be renegotiated for the future.\(^{22}\)

In principle, in the recursive program defined above, \( J(\cdot) \) may have multiple solutions. We select a unique solution by considering the limit of a finite horizon economy with end date \( T \) as \( T \) approaches \( \infty \). As such, \( J(\theta_{t-1}, b_t) \) is characterized recursively via backward induction. In the Appendix, we use Assumption 1 to show that this implies that the solution to (12) – (14) admits a savings rate \( s_{t}(\theta_t) \) for each \( \theta_t \) which is invariant to the level of debt \( b_t \) and only depends on the previous shock \( \theta_{t-1} \). This observation together with the fact that welfare can be represented

\(^{22}\)This notion of sequential optimality is related to the notion of reconsideration-proofness in Kocherlakota (1996) and renegotiation-proofness in Farrell and Maskin (1989) and Strulovici (2011).
by (6) and (7) implies that, given \( \theta_{t-1} \), (12) – (14) can be rewritten as:

\[
\max_{\{s_t(\theta_t)\}_{\theta_t \in \{\theta^L, \theta^H\}}} \sum_{\theta_t \in \{\theta^L, \theta^H\}} p(\theta_t | \theta_{t-1}) (\theta_t U(1 - s_t(\theta_t)) + \tilde{\theta}_t U(s_t(\theta_t)))
\]

\[\text{s.t.}\]
\[
\theta_t U(1 - s_t(\theta_t)) + \tilde{\theta}_t U(s_t(\theta_t)) \geq \theta_{t+1} U(1 - s_t(\tilde{\theta}_t)) + \frac{\beta}{(1-\delta)} U(s_t(\tilde{\theta}_t)) \text{ for } \tilde{\theta}_t \neq \theta_t.
\]

This program makes clear that the sequentially optimal mechanism must satisfy the constraints of the problem defined by the ex-ante optimal rule in (11). This is because the incentive compatibility constraint (16) is more strict than the constraint (10), as by definition, the solution to (15) – (16) must admit a sequence of savings rates which satisfy \( W_{t+1}(\theta^{t-1}, \theta_t) \geq V_{t+1}(\theta^{t-1}, \theta_t) \). Thus, naturally, the sequentially optimal rule provides weakly lower welfare than the ex-ante optimal rule.

4 Three Period Example

In order to provide some intuition for our main results, we start by considering a three-period economy with \( t \in \{0, 1, 2\} \). The purpose of this example is threefold. First, we show that in contrast with the special case of i.i.d. shocks, when shocks are persistent, the ex-ante optimal rule does not coincide with the sequentially optimal rule. Second, we show that, also unlike under i.i.d shocks, the ex-ante optimal rule under persistent shocks exhibits history dependence, that is, the continuation mechanism at \( t = 1 \) depends on more than the payoff relevant variables at \( t = 1 \). Finally, we describe properties of the solution which provide some insight into the dynamics that emerge in the infinite horizon economy discussed in Section 5.

We contrast the date 1 policies in the ex-ante optimal rule and the sequentially optimal rule. Given Assumption 1 and the arguments of Section 3, the problem can be stated as that of choosing a savings rate \( s_0(\theta_{-1}, \theta_0) \) at date 0 and a savings rate \( s_1(\theta_{-1}, \theta_0, \theta_1) \) at date 1, where the savings rate is defined as in (5).

We begin by considering the sequentially optimal rule, which can be solved for by backward induction. At date 1, the government solves an analogous program to (15) – (16), where \( \tilde{\theta}_1 \) is given by \( \beta E[\theta_2 | \theta_1] \). To simplify the analysis, and consistent with the extension to a continuum of types in Section 6, we assume here and in Section 5 that \( \theta^L \) and \( \theta^H \) are close enough.

\[\text{Assumption 2. Types are sufficiently close to one another:}\]
\[
\frac{\theta^H}{\theta^L} < \sqrt{1 + \frac{(1 - \beta)(1 - \delta)}{\delta}}.
\]

\[23\text{This assumption is required in Section 5 and is stronger than required for the results of the current section. The reason why the assumption depends on } \beta \text{ is that } \tilde{\theta}_1 \text{ in (4) depends on } \beta; \text{ in particular, the higher } \beta, \text{ the larger the difference between “normalized” types, } \theta^L / \tilde{\theta}^L \text{ and } \theta^H / \tilde{\theta}^H.\]
Assumption 2 implies that the first-best savings rate at date 1, defined by
\[
\theta^i U'(1 - s_1^{fb}(\theta^i)) = \beta \mathbb{E}[\theta_2|\theta_1 = \theta^i]U'(s_1^{fb}(\theta^i))
\]  (17)
for \(\theta^i \in \{\theta^L, \theta^H\}\), is not incentive compatible at date 1 for the low type, who would want to pretend to be a high type so as to spend and borrow more.\(^{24}\) Therefore, Assumption 2 guarantees that some incentive compatibility constraint binds in equilibrium at date 1.

An additional implication of Assumption 2 is that the optimal static mechanism at date 1 features pooling.\(^{25}\) In other words, following the realization of \(\theta_0\), the optimal mechanism assigns a fixed savings rate, independent of the reported type at date 1. This pooled savings rate is chosen optimally given the probability that the government is a high or low type at date 1. Thus, the savings rate at date 1 satisfies
\[
\mathbb{E}[\theta_1|\theta_0 = \theta^i]U'(1 - s_1(\theta_{-1}, \theta^i)) - \beta \mathbb{E}[\theta_2|\theta_0 = \theta^i]U'(s_1(\theta_{-1}, \theta^i)) = 0
\]  (18)
for \(\theta^i \in \{\theta^L, \theta^H\}\), where, with some abuse of notation, we have written \(s_1(\cdot)\) as a function of \(\theta_{-1}\) and \(\theta_0\) only given that the savings rate at date 1 does not depend on the realization of \(\theta_1\) under pooling.

Consider next the ex-ante optimal rule. Suppose the ex-ante optimum and the sequential optimum coincided at date 1. We show that if this were the case, it would be possible to perturb the ex-ante optimal mechanism in a way that reduces welfare from the perspective of date 0, thus increasing ex-ante welfare and contradicting the fact that the ex-ante optimum and the sequential optimum coincide.

To fix ideas, assume that at date 0 the incentive compatibility constraint binds for the low type and is slack for the high type. This means that the low type is under-saving relative to the efficient level, and he cannot be induced to save more as he would then want to pretend to be a high type to save less. Formally, the low type is indifferent between reporting the truth and receiving a payoff given by
\[
\theta^L U(1 - s_0(\theta_{-1}, \theta^L)) + \delta \beta \mathbb{E}[\theta_1 + \beta \theta_2|\theta_0 = \theta^L]U(s_0(\theta_{-1}, \theta^L))
\]
\[+ \delta \beta (\mathbb{E}[\theta_2|\theta_0 = \theta^L]U(1 - s_1(\theta_{-1}, \theta^L)) + \beta \mathbb{E}[\theta_2|\theta_0 = \theta^L]U(s_1(\theta_{-1}, \theta^L)))
\]  (19)

\(^{24}\)Formally, combining Assumption 2 and (17) together with the strict concavity of \(U(\cdot)\) implies that
\[
\delta \leq \frac{\theta^L}{\beta \mathbb{E}[\theta_2|\theta_1 = \theta^L]} \frac{U'(1 - s_1^{fb}(\theta^H))(s_1^{fb}(\theta^L) - s_1^{fb}(\theta^H))}{U'(s_1^{fb}(\theta^H))(s_1^{fb}(\theta^L) - s_1^{fb}(\theta^H))} < \frac{\theta^L}{\beta \mathbb{E}[\theta_2|\theta_1 = \theta^L]} \frac{U(1 - s_1^{fb}(\theta^H)) - U(1 - s_1^{fb}(\theta^L))}{U(s_1^{fb}(\theta^L)) - U(s_1^{fb}(\theta^H))},
\]
so that the incentive compatibility constraint (16) at date 1 is violated for the low type under the first-best allocation.

\(^{25}\)The results of this section also hold when the static optimal allocation in period 1 induces separation of types. Details are available upon request.
and lying and receiving a payoff given by

\[ \theta^L U(1 - s_0(\theta_{-1}, \theta^H)) + \delta \beta \mathbb{E}[\theta_1 + \beta \theta_2 | \theta_0 = \theta^L] U(s_0(\theta_{-1}, \theta^H)) \]

\[ + \delta \beta \left( \mathbb{E}[\theta_1 | \theta_0 = \theta^L] U(1 - s_1(\theta_{-1}, \theta^H)) + \beta \mathbb{E}[\theta_2 | \theta_0 = \theta^L] U(s_1(\theta_{-1}, \theta^H)) \right), \]

for \( s_1(\theta_{-1}, \theta^H) \) determined by (18), and where we have used Assumption 1 to derive welfare (normalized by the initial debt \( b_0 \)) as a function of the savings rates \( s_0(\theta_{-1}, \theta^H) \) and \( s_1(\theta_{-1}, \theta^H) \).

Clearly, if the low type’s payoff from pretending to be a high type (20) could be reduced, the low type could be induced to save more at date 0. Note that this payoff can be reduced by changing either the savings rate that is assigned at date 0 given a high reported type at date 0, or the savings rate that is assigned at date 1 given a high reported type at date 0. Moreover, the effect on welfare of changing the savings rate at date 1 depends on the probabilities of a low type and a high type at date 1, which under persistent shocks depend on the realized type at date 0.

With this observation in mind, consider the following perturbation. We reduce the savings rate at date 1 given a high reported type at date 0, \( s_1(\theta_{-1}, \theta^H) \), by \( \varepsilon > 0 \) arbitrarily small (where recall that \( s_1(\theta_{-1}, \theta^H) \) does not depend on \( \theta_1 \) because we are assuming that the mechanism is sequentially optimal at date 1 and thus pools the two types at date 1). This perturbation induces overspending at date 1 given a high type at date 0, which is clearly suboptimal ex post. We show however that this may be optimal ex ante.

To understand the effect of the perturbation on ex ante welfare, consider first the direct effect on equilibrium welfare from date 1 onward, that is, the effect on social welfare at date 1 given a truthful report of \( \hat{\theta}_0 = \theta^H \) at date 0. As just mentioned, welfare at date 1 goes down, but, from the envelope condition in (18), this is a second order loss, and thus this effect approaches 0 as \( \varepsilon \) approaches 0. In contrast, consider next the direct effect on off-equilibrium welfare from date 1 onward, that is, the effect on social welfare at date 1 given a non-truthful report of \( \hat{\theta}_0 = \theta^H \) at date 0. As \( \varepsilon \) approaches 0, this effect takes the same sign as

\[ \delta \beta \left( \mathbb{E}[\theta_1 | \theta_0 = \theta^L] U'(1 - s_1(\theta_{-1}, \theta^H)) - \beta \mathbb{E}[\theta_2 | \theta_0 = \theta^L] U'(s_1(\theta_{-1}, \theta^H)) \right). \] (21)

If shocks are i.i.d., \( \mathbb{E}[\theta_1 | \theta_0 = \theta^L] = \mathbb{E}[\theta_1 | \theta_0 = \theta^H] \), and, given (18), (21) must equal zero, implying that the perturbation affects continuation welfare on and off the equilibrium path equally, and hence cannot improve ex ante welfare. In contrast, when shocks are persistent, \( \mathbb{E}[\theta_1 | \theta_0 = \theta^L] < \mathbb{E}[\theta_1 | \theta_0 = \theta^H] \), and (21) is necessarily negative, implying that the perturbation reduces continuation welfare off the equilibrium path without affecting continuation welfare on path. Importantly, this means that the low type’s incentive compatibility constraint at date 0 is relaxed at no social cost, and thus ex ante welfare can be increased.

The economic intuition behind this perturbation is simple. In every period, the government learns about current spending and borrowing needs. If shocks are persistent, the current shock
also informs the government about its future spending and borrowing needs. The government always has a temptation to overspend today even if its needs are low. Optimal fiscal rules in this context have the perverse feature that they become excessively lax in the future if needs are high today. The reason is that the expected cost of excessively lax rules in the future is greater if spending needs are low today, as low spending needs today imply that spending needs are likely to be low in the future. As a result, the threat of no discipline in the future ironically imposes discipline today.

We illustrate the properties of the ex-ante optimum and the sequential optimum with a numerical example. Figure 1 shows the savings rate at date 1, $s_1(\theta_{-1}, \theta^i)$, in the ex-ante optimal ("EO") and sequentially optimal ("SO") rules for $\theta_0 = \theta^L$ ("S1L") and $\theta_0 = \theta^H$ ("S1H"), given $\theta_{-1} = \theta^H$, as a function of the persistence of types, denoted by $\alpha \equiv p(\theta^i|\theta^i)$ in the figure. It can be shown that under Assumption 2, the ex-ante optimal rule also features pooling at date 1; thus, under the two rules, $s_1(\theta_{-1}, \theta^i)$ is independent of the realized type at date 1.

Three points are evident in Figure 1. First, consistent with the perturbation just described, the figure shows that following $\theta_0 = \theta^H$, at date 1, the ex-ante optimal savings rate is below the sequentially optimal savings rate. As explained above, inducing overspending ex post is efficient ex ante as it allows to relax the low type’s incentive compatibility constraint and curb

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**Figure 1:** Savings rate at $t = 1$ in the ex-ante and sequentially optimal rules given $\theta_{-1} = \theta^H$.

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\textsuperscript{26}We consider $\beta = 0.9$, $\delta = 0.6$, $\theta^L = 2$, and $\theta^H = 3$.

\textsuperscript{27}An analogous figure can be drawn for $\theta_{-1} = \theta^L$. 

13
his spending at date 0. Second, the figure shows that following $\theta_0 = \theta^L$, at date 1, the ex-ante optimal savings rate coincides with the sequentially optimal savings rate. The logic is also related to the low type’s incentive compatibility constraint at date 0—to relax this constraint, the ex-ante optimal rule induces the highest continuation welfare for the low type given a truthful report at date 0, which corresponds to assigning the ex-post optimal savings rate at date 1. Finally, the figure shows how the savings rate at date 1 depends on the persistence of types, and in particular that the ex-ante optimal and sequentially optimal rules coincide when types are i.i.d. ($\alpha = 0.5$) and when they are fully persistent ($\alpha = 1$). Intuitively, in these cases, $E[\theta_1|\theta_0 = \theta^i] = E[\theta_2|\theta_0 = \theta^i]$, so the relative marginal benefit of spending at dates 1 and 2 is the same. Given (18), this means that the sequentially optimal savings rate at date 1 is independent of $\theta_0$, and hence any perturbation in this rate affects welfare on the equilibrium path (given a truthful report at date 0) and off the equilibrium path (given a non-truthful report at date 0) equally. Consequently, in these cases, any perturbation that is ex-post suboptimal cannot improve ex-ante welfare.

Figure 2 explores the extent to which the ex-ante optimal mechanism exhibits history dependence. The figure shows the ex-ante optimal savings rate at date 1 conditional on $\theta_0$ and $\theta_{-1}$. The main result here is that, given $\theta_0 = \theta^H$, this savings rate depends on $\theta_{-1}$. Specifically, the ex-ante optimal savings rate at date 1 following $\theta_0 = \theta^H$ is lower if $\theta_{-1} = \theta^L$ than if $\theta_{-1} = \theta^H$, so fiscal rules become more excessively lax after a high shock at date 0 if $\theta_{-1} = \theta^L$. The intu-
ition is that $\theta_{-1}$ affects the distribution of types at date 0, and thus the benefits and costs of perturbing the savings rate at date 1. More precisely, the benefit of lowering the savings rate below the sequentially optimal rate at date 1 following $\theta_0 = \theta^H$ is that the low type’s incentive compatibility constraint at date 0 is relaxed and savings given $\theta_0 = \theta^L$ are increased. The cost of this distortion, on the other hand, is that it induces an ex-post suboptimal savings rate given $\theta_0 = \theta^H$. If $\theta_{-1} = \theta^L$, the probability of $\theta_0 = \theta^L$ is higher, and hence the relative benefits of perturbing the savings rate at date 1 are larger. Regarding the savings rate at date 1 following $\theta_0 = \theta^L$, Figure 2 shows that this rate is independent of $\theta_{-1}$—because this rate is sequentially optimal, it only depends on payoff relevant states.

In sum, this three period example shows that following the realization of a high shock at date 0, the ex-ante optimal fiscal rule at date 1 becomes excessively lax. The extent to which this occurs is a function of how important it is for social welfare to curb spending under the low shock at date 0, which is a function of how likely this shock is and thus of initial conditions. Moreover, if the low shock occurs at date 0, there is no benefit from ex-post suboptimality at date 1, and therefore the ex-ante optimal rule at date 1 coincides with the sequentially optimal rule. In the next section, we further study and formalize these results in a dynamic infinite horizon economy.

5 Optimal Fiscal Rules

In this section, we explicitly characterize optimal fiscal rules in an infinite horizon economy. Because the sequentially optimal fiscal rule is simple, we begin by characterizing this rule and then use it as a benchmark in describing the ex-ante optimal fiscal rule.

5.1 Sequentially Optimal Rule

Consider the program in (15) – (16) which defines the sequentially optimal fiscal rule. Given Assumption 2, it can be shown that the optimal mechanism features pooling: the savings rate $s_t(\theta^t)$ is independent of the realization of $\theta_t$ and depends only on $\theta_{t-1}$, which is used in predicting the value of $\theta_t$. This is stated formally in the proposition below.

**Proposition 1 (sequential optimum).** $\forall \theta^t$ and $\forall \theta^k$, the sequential optimum features

$$s_t(\theta^t) = s_k(\theta^k) \text{ if and only if } \theta_{t-1} = \theta_{k-1}.$$ 

Moreover, $s_t(\theta^t)$ satisfies

$$\mathbb{E}[\theta_t|\theta_{t-1}]U'(1 - s_t(\theta^t)) - \mathbb{E}[\tilde{\theta}_t|\theta_{t-1}]U'(s_t(\theta^t)) = 0. \quad (22)$$

Given the budget constraint (1), this proposition implies that the sequentially optimal rule at $t$ prescribes a level of debt $b_{t+1}(\theta^t)$ as a function of $\theta_{t-1}$ and $b_t(\theta^{t-1})$ only. If $\theta_{t-1} = \theta^H$, this rule is...
the prescribed level of debt at \( t \) is higher than if \( \theta_{t-1} = \theta^L \), as a high shock is then more likely and thus the sequentially optimal level of deficit-financed spending is higher. Furthermore, if \( b_t(\theta^{t-1}) \) is relatively high, then \( b_{t+1}(\theta^t) \) must also be relatively high to facilitate the servicing of the debt while simultaneously providing public goods. A useful implication is that the sequential optimum can be implemented with a renegotiated debt limit, as we next state formally.

**Corollary 1.** The sequentially optimal rule at any date \( t \) can be implemented with a debt limit, \( b(\theta_{t-1}, b_t(\theta^{t-1})) \).

Given Assumption 2 and equation (22), the sequentially optimal fiscal rule is such that both the low and high types would like to spend and borrow more than they are allowed to. Thus, the sequentially optimal rule takes the form of a renegotiated debt limit, where conditional on \( \theta_{t-1} \) and \( b_t(\theta^{t-1}) \), both types choose the maximum allowable debt. In Section 6, we show that these results extend to an economy in which the support for \( \theta_t \) is continuous, but, in that case, some types choose to borrow below the debt limit.

### 5.2 Ex-ante Optimal Rule

We now consider the ex-ante optimal fiscal rule defined in (11). We first develop a recursive representation of the problem and then characterize the solution.

#### 5.2.1 Recursive Representation

Our recursive representation is similar in spirit to that of Fernandes and Phelan (2000), who develop a recursive technique to study a principal-agent problem under persistent shocks. The idea behind our technique is to solve program (11) by choosing \( s_t(\theta^t) \) and \( V_{t+1}(\theta^t) \) defined in (9) sequentially for each history \( \theta^{t-1} \), where associated with each choice of \( V_{t+1}(\theta^t) \) is some continuation welfare \( W_{t+1}(\theta^t) \) which is a function of the realized type \( \theta_t \) and the chosen \( V_{t+1}(\theta^t) \).

To characterize this value of \( W_{t+1}(\theta^t) \), let \( W(\theta^i, V) \) for \( \theta^i \in \{\theta^L, \theta^H\} \) correspond to the solution to (11) given \( \theta_{t-1} = \theta^i \) and subject to the additional constraint that

\[
V = \sum_{\theta_0 \in \{\theta^L, \theta^H\}} p(\theta_0 | \theta_{t-1} = \theta^{-i}) \left[\theta_0 U(1 - s_{\theta_0}(\theta^0)) + \tilde{\theta}_0 U(s_{\theta_0}(\theta^0)) + \beta W_1(\theta^0)\right].
\]

Constraint (23), which is often referred to as a threat-keeping constraint, implies that if \( \theta_{t-1} \) is actually equal to \( \theta^{-i} \) as opposed to \( \theta^i \), the expected welfare under the savings rate sequence that solves the program is equal to \( V \). Using this formulation, we rewrite program (11) recursively.
as follows:

\[
W(\theta^i, V) = \max_{\{s_L, s_H, V^L, V^H\}} \left\{ \begin{array}{c}
p(\theta^L|\theta^i)(\theta^L U(1 - s^L) + \theta^L U(s^L) + \beta W(\theta^L, V^L)) \\
+ p(\theta^H|\theta^i)(\theta^H U(1 - s^H) + \theta^H U(s^H) + \beta W(\theta^H, V^H)) \end{array} \right\} \tag{24}
\]

\[
V = \left\{ \begin{array}{c}
p(\theta^L|\theta^{-i})(\theta^L U(1 - s^L) + \theta^L U(s^L) + \beta W(\theta^L, V^L)) \\
+ p(\theta^H|\theta^{-i})(\theta^H U(1 - s^H) + \theta^H U(s^H) + \beta W(\theta^H, V^H)) \end{array} \right\}, \tag{25}
\]

\[
\theta^L U(1 - s^L) + \delta \theta^L U(s^L) + \delta \beta W(\theta^L, V^L) \geq \theta^L U(1 - s^H) + \delta \theta^L U(s^H) + \delta \beta V^H, \tag{26}
\]

\[
\theta^H U(1 - s^H) + \delta \theta^H U(s^H) + \delta \beta W(\theta^H, V^H) \geq \theta^H U(1 - s^L) + \delta \theta^H U(s^L) + \delta \beta V^L, \tag{27}
\]

\[
V^L \leq V^L \leq V^L, \quad \text{and} \quad V^H \leq V^H \leq V^H. \tag{28}
\]

(24) – (28) corresponds to a recursive representation of (11) starting from some history \(\theta^{t-1}\). The program selects the optimal savings rates \(s^i\) for \(i \in \{L, H\}\), which effectively represent the values of \(s_t(\theta^i)\), as well as the optimal levels of threats \(V^i\) for \(i \in \{L, H\}\), which represent the values of \(V_{t+1}(\theta^i)\) defined in (9). (24) takes into account that social welfare from tomorrow onward conditional on \(\theta_t = \theta^i\) equals \(W(\theta^i, V^i)\). The program maximizes social welfare subject to a set of constraints, (25) – (28). Constraint (25) is a recursive representation of the threat-keeping constraint stating that if type \(\theta^{-i}\) in period \(t - 1\) deviates and pretends to be type \(\theta^i\), his expected continuation welfare at \(t\) is equal to \(V\). As such, \(V^i\), which is chosen in the current period \(t\), represents the continuation welfare at \(t + 1\) to a type \(\theta^{-i}\) who deviates and pretends to be type \(\theta^i\) at \(t\). Constraints (26) and (27) correspond to the recursive representations of the incentive compatibility constraints in (10). Finally, constraint (28) guarantees that the values of \(V^i\) are chosen within a feasible range.\(^{28}\)

We next characterize \(W(\theta^i, V)\). Clearly, if shocks are i.i.d., it must be that \(W(\theta^i, V) = V\). If shocks are persistent, the following lemma holds.

**Lemma 1.** If shocks are persistent, \(W(\theta^i, V)\) is strictly increasing, strictly concave, and continuously differentiable in \(V\) over the range \((\bar{V}^i, \overline{V}^i)\).

The range \((\bar{V}^i, \overline{V}^i)\) corresponds to the lowest and highest values of \(V_{t+1}(\theta^i)\) that can be attained in the solution to (11). The value of \(\bar{V}^i\) corresponds to the value of \(V\) that results from the sequence problem (11), given \(\theta_{-1} = \theta^i\) and subject to (23), when the constraint (23) does not bind, so that the solution effectively corresponds to the ex-ante optimum. What is important to note is that values of \(V_{t+1}(\theta^i)\) that exceed \(\bar{V}^i\) are never attained along the equilibrium path; such high values of \(V_{t+1}(\theta^i)\) tighten the incentive compatibility constraints (26) and (27) while simultaneously reducing the continuation welfare below \(W(\theta^i, \overline{V}^i)\), and are thus ex-ante inefficient. Hence, only threats such that \(V^i < \bar{V}^i\) are used, and the optimal level of threats

\(^{28}\)In principle, this range need not represent a convex set. See the Appendix for details.
depends on the benefits of relaxing incentive compatibility constraints relative to the costs of reducing continuation welfare \( W(\theta^i, V^i) \).

To understand the role of the threat-keeping constraint \( (25) \), let \( \lambda \) correspond to the Lagrange multiplier on this constraint. The Envelope condition implies that \( W_V(\theta^i, V) = -\lambda \), so that given the strict concavity of \( W(\cdot) \), lower values of \( V \) are associated with more negative values of \( \lambda \). It thus follows that the solution to \((24) - (28)\) is equivalent to the solution to the following problem:

\[
\max_{\{s^L, s^H, V^L, V^H\}} \left\{ \begin{array}{l}
(p(\theta^L|\theta^i) + \lambda p(\theta^L|\theta^{-i}))(\theta^LU(1 - s^L) + \beta W(\theta^L, V^L)) \\
+(p(\theta^H|\theta^i) + \lambda p(\theta^H|\theta^{-i}))(\theta^HU(1 - s^H) + \beta W(\theta^H, V^H))
\end{array} \right\}
\]

\( (29) \)

s.t. \( (26) - (28) \).

Under persistent types, the role of the threat-keeping constraint is then to effectively “twist” the probabilities assigned to each type. If, for example, \( \theta^i = \theta^H \), then the objective in \( (29) \) effectively under-weighs welfare conditional on the low type relative to the high type, and this is done more severely the lower is \( V \). The opposite is true if \( \theta^i = \theta^L \). Hence, to satisfy the threat-keeping constraint, the program over-weighs (under-weighs) welfare at \( t \) conditional on the type that is less (more) likely to occur from the perspective of a deviating type \( \theta^{-i} \) at \( t - 1 \). This has important implications for the characterization of the solution.

5.2.2 Characterization

We now characterize the ex-ante optimal fiscal rule. We begin by considering the i.i.d. benchmark.

**Proposition 2 (ex-ante optimum under i.i.d. shocks).** If shocks are i.i.d., \( \forall \theta^t \) and \( \forall \theta^k \), the ex-ante optimum features

\[
s_t(\theta^t) = s_k(\theta^k).
\]

Moreover, \( s_t(\theta^t) \) satisfies

\[
U'(1 - s_t(\theta^t)) - \beta U'(s_t(\theta^t)) = 0,
\]

so the ex-ante optimum coincides with the sequential optimum.

When shocks are i.i.d., the ex-ante optimal rule prescribes a constant savings rate in every period, which is the same as the sequentially optimal rule, and which can thus be implemented with a renegotiated debt limit. This rule is then history independent and does not provide dynamic incentives for truth-telling. The reason why providing dynamic incentives is inefficient in the i.i.d. case is as discussed in the three period example: any perturbation in the ex-post optimal rule would affect the continuation welfare on the equilibrium path (following a truthful report) and off the equilibrium path (following a non-truthful report) equally, and thus fail to
improve ex ante efficiency. This result is analogous to that of Amador, Werning, and Angeletos (2006), who study the tradeoff between commitment and flexibility under i.i.d. shocks.

Consider now the case of persistent shocks. Let the solution to (24) – (28) be denoted by

$$\{s^{L*}(\theta^i, V), s^{H*}(\theta^i, V), V^{L*}(\theta^i, V), V^{H*}(\theta^i, V)\}.$$  

**Lemma 2.** If shocks are persistent, the solution to (24) – (28) has the following properties for all $V \in [\underline{V}, \overline{V}]$:

1. $V^{L*}(\theta^i, V) = \overline{V}$ for $\theta^i \in \{\theta^L, \theta^H\}$,
2. $V^{H*}(\theta^H, \overline{V}) > V^{H*}(\theta^L, \overline{V})$ and $V^{H*}(\theta^H, V)$ is strictly decreasing in $V$, and
3. $s^{i*}(\theta^H, \overline{V}) < s^{i*}(\theta^L, \overline{V})$ and $s^{i*}(\theta^H, V)$ is strictly increasing in $V$ for $i \in \{L, H\}$.

Lemma 2 describes the solution to (24) – (28) given $\theta^i$ and $V$. The first part of the lemma states that the equilibrium at $t+1$ effectively resets whenever $\theta_t = \theta^L$. This result is analogous to the result in the three period example that the savings rate at date 1 is sequentially optimal if $\theta_0 = \theta^L$. The intuition is also as in that example: setting $V^{L*}(\theta^i, V) = \overline{V}$ maximizes the continuation payoff of the low type given a truthful report, and thus maximally relaxes the incentive compatibility constraint of the low type (26) while maximizing social welfare.

The second part of Lemma 2 concerns the magnitude of $V^{H*}(\theta^i, V)$ across different values of $V$. Comparing $V^{H*}(\theta^H, \overline{V})$ and $V^{H*}(\theta^L, \overline{V})$, the lemma states that the threat that is used in the ex-ante optimum to induce the low type to report truthfully is larger if $\theta^i = \theta^L$ than if $\theta^i = \theta^H$. This result is analogous to the result in the three period example that the savings rate at date 1 following $\theta_0 = \theta^H$ is lower if $\theta_{-1} = \theta^L$ than if $\theta_{-1} = \theta^H$. As in that example, the intuition is that when $\theta^i = \theta^L$, the low shock is more likely, and thus the benefits of using the threat—relaxing the low type’s incentive constraint and curbing his spending—are larger relative to the costs—reducing the continuation welfare of the high type. The lemma also compares $V^{H*}(\theta^H, V)$ across different values of $V$. As described in the previous section, the lower is $V$, the more “twisted” the effective maximization problem in (29) is in favor of welfare conditional on a high type. This means that using threats is more costly, and thus, the lower is $V$, the higher is $V^{H*}(\theta^H, V)$.

The third part of Lemma 2 compares savings rates across different values of $V$. Comparing $s^{i*}(\theta^H, \overline{V})$ and $s^{i*}(\theta^L, \overline{V})$, the lemma states that if $\theta^i = \theta^L$, the savings rate conditional on the realization of either shock is lower than if $\theta^i = \theta^H$. The intuition is straightforward. Because of Assumption 2, the savings rate of the low (high) type is always below (above) the first best rate defined in (17). Thus, if the objective function places a higher weight on welfare conditional on a high type, which is the case if $\theta^i = \theta^H$, then the savings rate of the high type must be lower and closer to first best; but then for the incentive constraint of the low type (26) to be satisfied, it must be that the savings rate of the low type is also lower. This explains why
\( s^i_s(\theta^H, V^H) < s^i_s(\theta^L, V^L) \). The lemma also compares \( s^i_s(\theta^H, V) \) across different values of \( V \). As discussed, the lower is \( V \), the higher the weight assigned in (29) in favor of welfare conditional on the realization of a high type, and, consequently, the lower is \( s^i_s(\theta^H, V) \).

The next proposition uses Lemma 2 to describe the ex-ante optimal fiscal rule. Define \( \eta_t(\theta^t) \) as the number of periods since the last time that \( \theta^t = \theta^L \):

\[
\eta_t(\theta^t) = \begin{cases} k & \text{if } \theta^t - k = \theta^L \text{ and } \theta^t - l = \theta^H \forall l \in [0, k] \\
 & \text{otherwise}
\end{cases}
\]

**Proposition 3 (ex-ante optimum under persistent shocks).** If shocks are persistent, the ex-ante optimum has the following features:

1. \( \forall \theta_t \text{ and } \forall \theta_k \text{ with } \theta_t = \theta_k, s_t(\theta^t) = s_k(\theta^k) \text{ if } \eta_t(\theta^{t-1}) = \eta_k(\theta^{k-1}), \text{ and} \)
2. \( \exists \theta_t, \theta_k \text{ with } \theta_t = \theta_k \text{ and } \theta_{t-1} = \theta_{k-1} \text{ for which } s_t(\theta^t) \neq s_k(\theta^k). \)

Thus, the ex-ante optimum does not coincide with the sequential optimum, and it exhibits history dependence.

Unlike in the sequential optimum and in the i.i.d. case, in the ex-ante optimum under persistent shocks, the savings rate at history \( \theta^t \) conditional on \( \theta_t \) depends not only on \( \theta_{t-1} \), but also on when the low shock occurred for the last time. Hence, the ex-ante optimal mechanism exhibits history dependence: the prescribed mechanism depends not only on the payoff relevant variables, \( \theta_{t-1} \) and \( b_t(\theta^{t-1}) \), but on the entire history of shocks. The idea here is that the mechanism provides dynamic incentives, and past shocks affect the relative tightness of current incentive compatibility constraints. The resetting property described in Lemma 2 implies in fact that the tightness of current incentive constraints depends on when resetting began, which explains why prescribed policies depend on the time that passed since a low shock was realized.\(^{30}\)

To understand the resetting property, consider the equilibrium starting from date 0, given \( \theta_{-1} = \theta^H \). If \( \theta_0 = \theta^L \) is realized, the equilibrium transitions to the ex-ante optimum associated with \( \theta_{-1} = \theta^L \); this is efficient because it maximally relaxes the incentive compatibility constraint of the low type at date 0. If instead \( \theta_0 = \theta^H \) is realized, the fiscal rule at date 1 seeks to punish the low type who would have lied at date 0, while at the same time not harming too much the high type who told the truth. There are two ways in which this can be done. On the one hand, spending at date 1 given a low type at date 1 can be made higher and further away from first best. On the other hand, the expected continuation welfare from date 2 onward at date 1, given a low type at date 1, can be made lower. Because the low type at date 0 has a higher probability of being a low type at date 1, either of these changes hurt the deviating low type at date 0 more than the truthful high type at date 0. However, while the first option slackens the incentive compatibility constraint of the low type at date 1, the second option tightens it.

\(^{29}\)Note that this variable is not defined if the low shock has never been realized.

\(^{30}\)It is straightforward to show that for histories for which the low shock has never realized, policies are history dependent in the sense that they are a function of the date \( t \).
For this reason, it is cheaper from a date 0 perspective to provide incentives to the low type by increasing spending for a single period following a high report, while resetting the equilibrium thereafter given a low report.

A natural question regards the equilibrium dynamics for a sequence of consecutive high shocks. These can be described using the second and third parts of Lemma 2, as shown in the following proposition.

**Proposition 4 (dynamics under persistent shocks).** \( \exists \hat{V} \) s.t. \( \forall \theta_t, \) if \( \theta_{t-1} = \theta_t = \theta^H, \) then \( V_{t+1}(\theta^t) > (\leq) \hat{V} \) if \( V_t(\theta^{t-1}) < (\geq) \hat{V}. \)

Combined with Lemma 2, Proposition 4 states that under a sequence of consecutive high shocks, the equilibrium oscillates between periods of high spending and periods of low spending, around some fiscal rule associated with a threat \( \hat{V} \) such that \( V^H(\theta^H, \hat{V}) = \hat{V}. \) In all of these periods, there is a lack of fiscal discipline in that the level of spending exceeds the level under \( \theta^t = \theta^H, V = V^H. \)

To understand the oscillatory dynamics, consider again the equilibrium starting from date 0, given \( \theta_{-1} = \theta^H. \) As explained above, if \( \theta_0 = \theta^H \) is realized at date 0, the ex-ante optimal rule induces high spending at date 1 given a low type at date 1, where this spending is further away from first best compared to spending at date 0 given a low type at date 0. Now this means that at date 1, the low type’s incentives to lie and pretend to be a high type are relatively lower, and, in turn, a smaller threat at date 2 following a high report at date 1 is sufficient to provide incentives at date 1. Yet, note that the equilibrium does not reset to the ex-ante optimum at date 2; the reason is simply that if spending at date 2 corresponded to the ex-ante optimal spending level given \( \theta_{-1} = \theta^H, \) incentive compatibility constraints at dates 0 and 1 could be relaxed by increasing such spending, implying a first-order gain and causing only a second order loss at date 2.

The economics behind the oscillatory dynamics in Proposition 4 emerge from the self-control nature of the problem at hand. The absence of discipline tomorrow is used to induce discipline today. In turn, the absence of discipline tomorrow allows to reinstitute discipline the day after while preserving incentives tomorrow.\(^{31}\)

The analysis of this section shows that persistence has important effects on the optimal structure of fiscal rules. When shocks to the economy are i.i.d., the ex-ante optimal rule is sequentially optimal. Dynamic incentives are not provided and the mechanism is history independent, taking the simple form of a renegotiated debt limit. In contrast, when shocks are persistent, the ex-ante optimal rule is no longer sequentially optimal. Dynamic incentives are now provided and the mechanism exhibits history dependence even in the long run. The ex-ante optimal mechanism features rich dynamics, with high shocks being followed by an erosion of fiscal discipline compared to low shocks, which are followed by the reinstatement of fiscal

\(^{31}\)These oscillatory dynamics could imply convergence towards the fiscal rule associated with \( V = \hat{V}, \) although this depends on the slope of the policy function and may not be the case.
6 Extension to Continuum of Shocks

In this section, we extend our analysis to a setting with a continuum of shocks. We consider this setting not only to explore the robustness of our main results, but also because economies with a continuum of shocks are the main focus of the mechanism design literature that studies the tradeoff between commitment and flexibility.\(^{32}\)

The main complication that emerges in this extension is that, under multiple shocks, one must ensure that not only local but also global incentive compatibility constraints are satisfied. This does not complicate the analysis of the sequential optimum, as under Assumption 1 that problem reduces to a two period problem. However, this does complicate the analysis of the ex-ante optimum, as the recursive method described in Section 5.2.1 no longer applies. Nonetheless, we show in this section that the main economic insights from the economy with two shocks continue to hold under a continuum of shocks.

6.1 Environment

Consider the benchmark environment described in Section 2 but with the government’s type, \(\theta_t > 0\), now being drawn from a continuous support \(\Theta \equiv [\underline{\theta}, \overline{\theta}]\). Let \(p(\theta_t|\theta_{t-1})\) and \(\tilde{\theta}_t\) be as previously defined, and assume that \(p(\theta_t|\theta_{t-1})\) is strictly positive for all \(\theta_t\) and \(\theta_{t-1}\) and continuously differentiable with respect to \(\theta_t\) and \(\theta_{t-1}\). Assumption 3 below, which holds in the two shock economy, ensures that \(\theta_t\) is mean reverting.

**Assumption 3.** \(\theta_t/\tilde{\theta}_t\) is strictly increasing in \(\theta_t\).

We also make a technical assumption regarding the distribution of shocks. Define \(\theta_p(\theta_{t-1}) = \max\{\overline{\theta}, \theta'\}\) where \(\theta'\) is the lowest \(\theta \in \Theta\) such that \(\forall \theta'' \geq \theta,\)

\[
\frac{\tilde{\theta}''}{\mathbb{E}[\tilde{\theta}_t|\theta_t \geq \theta'', \theta_{t-1}]} \leq \frac{\mathbb{E}[\theta_t|\theta_t \geq \theta'', \theta_{t-1}]}{\theta''} \leq \frac{1}{\delta},
\]

where \(\tilde{\theta}''\) is the value of (4) associated with \(\theta_t = \theta''\). Note that if shocks are i.i.d., \(\theta_p(\theta_{t-1}) = \theta_p\), independent of \(\theta_{t-1}\). Given our assumptions, \(\theta_p(\theta_{t-1})\) is a continuously differentiable function of \(\theta_{t-1}\). Using this definition, we assume:

**Assumption 4.** \(\forall \theta_t \leq \theta_p(\theta_{t-1}),\)

\[
\frac{d \log p(\theta_t|\theta_{t-1})}{d \log \theta_t} \geq -\frac{2 - \delta - d \log \tilde{\theta}_t/d \log \theta_t}{1 - \delta}.
\]

\(^{32}\)See references in the Introduction.
Assumption 4 is satisfied if the severity of the time-inconsistency problem is sufficiently low (i.e., $\delta$ is sufficiently high) and the elasticity of the density function is not too negative (i.e., $d\log p(\theta_t|\theta_{t-1})/d\log \theta_t$ is bounded from below). This assumption is isomorphic to Assumption A in Amador, Werning, and Angeletos (2006)’s study of an economy with i.i.d. shocks, where the main difference is that our condition incorporates the potential persistence of shocks through $d\log \tilde{\theta}_t/d\log \theta_t$.

Definitions for the ex-ante optimal and sequentially optimal fiscal rules analogous to those provided for the two-shock economy apply in this setting. We thus use the analysis of Section 3 to characterize the equilibrium.

6.2 Sequentially Optimal Rule

An analogous program to (15) – (16) defines the sequentially optimal fiscal rule. The incentive compatibility constraints (16) effectively imply that the problem is static and that global incentive constraints can be ignored. As such, analogous techniques to those used in the analysis of the two-period problem of Amador, Werning, and Angeletos (2006) apply here and can be used to characterize the sequential optimum. To this end, define $s^f(\theta)$ as the flexible optimum of a government of type $\theta$ who is awarded full discretion:

$$\theta_t U'(1 - s^f(\theta)) = \delta_t U'(s^f(\theta)).$$

The next proposition characterizes the sequential optimum.

**Proposition 5** (sequential optimum under continuum of shocks). Let $\underline{s}(\theta_{t-1})$ be defined by $\underline{s}(\theta_{t-1}) = s^f(\theta_p(\theta_{t-1}))$ if $\theta_p(\theta_{t-1}) > \theta$, and

$$E[\theta_t|\theta_{t-1}]U'(1 - \underline{s}(\theta_{t-1})) - E[\tilde{\theta}_t|\theta_{t-1}]U'(\underline{s}(\theta_{t-1})) = 0$$

otherwise. $\forall \theta$, the sequential optimum features

$$s_t(\theta) = \max\{s^f(\theta), \underline{s}(\theta_{t-1})\}.$$

**Corollary 2.** The sequentially optimal rule at any date $t$ can be implemented with a debt limit, $b_t(\theta_{t-1}, b_t(\theta_{t-1})).$

This proposition and corollary state that if $\theta_p(\theta_{t-1}) > \theta$, all types $\theta$ below $\theta_p(\theta_{t-1})$ are awarded full discretion, so they can choose their flexible optimal savings rate, and all types above $\theta_p(\theta_{t-1})$ are awarded no discretion, so they must choose the same savings rate as type $\theta_p(\theta_{t-1})$. If instead $\theta_p(\theta_{t-1}) = \theta$, no type is given discretion, and all types are assigned a savings rate $\underline{s}(\theta_{t-1})$ satisfying (30).

The dependence of the minimum savings rate $\underline{s}(\theta_{t-1})$ on $\theta_{t-1}$ captures the fact that the shock at date $t - 1$ provides information regarding the tradeoff between commitment and flexibility.
at date $t$. Note that if shocks are i.i.d., $q(\theta_{t-1})$ and the associated debt limit $\bar{b}(\theta_{t-1}, b_t(\theta_{t-1}))$ are independent of $\theta_{t-1}$. Moreover, note that as $\delta$ approaches 1, so that the time-inconsistency problem vanishes, $\theta_p(\theta_{t-1})$ approaches $\bar{\theta}$, and thus the sequentially optimal rule provides full discretion to all types.

As in the two type case, the sequentially optimal rule does not provide dynamic incentives. Because of this, it takes the form of a set of allowable savings rates, from which the government chooses the one that is closest to its flexible optimum. To understand why very high types are given no discretion, note that because of the bounded distribution of shocks, allowing flexibility for very high types has no ex-ante welfare gain—such types would be overborrowing under any realized shock, so there is no tradeoff between commitment and flexibility for these types. $\theta_p(\theta_{t-1})$ can be interpreted as the type above which the value of flexibility is exceeded by the value of commitment.

To understand why all types below the cutoff $\theta_p(\theta_{t-1})$ are given full discretion, consider the alternative of having a mechanism that admits “holes,” namely, where some interior interval of savings rates are not allowed. Given such a hole, the types whose flexible optimum is inside the hole would choose savings rates at the boundaries of the hole. Thus, introducing the hole would cause those inside the hole whose type is relatively high to reduce their savings by choosing the lower boundary—which is socially costly—and those inside the hole whose type is relatively low to increase their savings by choosing the upper boundary—which is socially beneficial. The resulting total change in welfare then depends on the slope of the density function; under Assumption 4, which effectively puts a lower bound on the elasticity of $p(\theta_t|\theta_{t-1})$ with respect to $\theta_t$, the total welfare change of introducing a hole is always negative.

6.3 Ex-ante Optimal Rule

We next consider the ex-ante optimal fiscal rule. For this analysis, we assume that the mechanism at time $t$ admits savings rates which are piecewise continuously differentiable with respect to the history $\theta^t$. As in the two-type benchmark, we begin by showing that when shocks are i.i.d., the ex-ante optimal rule is sequentially optimal.

**Proposition 6 (ex-ante optimum under continuum of i.i.d. shocks).** Suppose $p(\theta_t|\theta_{t-1})$ is independent of $\theta_{t-1}$. Let $s$ be defined by $s = s^f(\theta_p)$ if $\theta_p > \bar{\theta}$, and

$$U'(1-s) - \beta U'(s) = 0$$

otherwise. $\forall \theta^t$, the ex-ante optimum features

$$s_t(\theta^t) = \max\{s^f(\theta_t), \bar{s}\},$$

33This assumption is without loss in our setting in the case of i.i.d. shocks, and it is used in other settings in related work such as Athey, Atkeson, and Kehoe (2006).
so the ex-ante optimum coincides with the sequential optimum.

The intuition for this result is isomorphic to that of the two type case. Under i.i.d. shocks, any ex-post suboptimality cannot enhance efficiency, as it affects welfare on the equilibrium path—following a truthful report—and off the equilibrium path—following a non-truthful report—equally. Therefore, dynamic incentives are not provided and the ex-ante optimum coincides with the sequential optimum. This of course also implies that, when shocks are i.i.d., the ex-ante optimum can be implemented with a renegotiated debt limit.\footnote{Note that in contrast with the two-type case, Proposition 6 requires Assumption 4 on the distribution function. The reason is that, if this assumption is not satisfied, the sequential optimum can admit a hole, in which case dynamic incentives could be provided to types who bunch at the lower boundary of the hole to induce them to borrow less.}

Consider now the case of persistent shocks, where \( p(\theta_t|\theta_{t-1}) \) depends on \( \theta_{t-1} \). We define persistence with the following condition.

**Condition 1 (mechanism relevance of past information).** There exist \( \theta_{t-1} \) and \( \theta_t \) of positive measure such that \( \theta_{p(\theta_{t-1})} > \theta_{\theta_t} \), \( s^f(\theta_t) > s^f(\theta_{p(\theta_{t-1})}) \), and \( \theta'_{p(\theta_t)} \neq 0 \).

Condition 1 concerns the sequence of minimum savings policies implied by the sequential optimum described in Proposition 5. It states that in the sequentially optimal fiscal rule, there exist two types \( \tilde{\theta} \) and \( \tilde{\theta} \) of positive measure with the property that, if \( \theta_{t-1} = \tilde{\theta} \) and \( \theta_t = \tilde{\theta} \), the government of type \( \tilde{\theta} \) at date \( t \) has full discretion, so it spends above first best level, and, moreover, such government has information which is locally relevant regarding the sequentially optimal mechanism at \( t+1 \) (i.e., \( \theta'_{p(\tilde{\theta})} \neq 0 \)). The analog of Condition 1 is trivially satisfied in a two-type economy, as the low type always spends above first best in the sequential optimum, and this type also has information about the future sequentially optimal mechanism. In the setting with a continuum of types, Condition 1 is always satisfied if \( \theta'_{p(\theta_t)} \neq 0 \) for some \( \theta_t \) with positive measure, so that current information is relevant to future mechanisms, and if \( \delta \) is sufficiently close to 1, so that such type \( \theta_t \) with mechanism-relevant information has full discretion at \( t \) starting from some \( \theta_{t-1} \).

While an explicit characterization of the ex-ante optimal fiscal rule under a continuum of persistent shocks is complicated, we show that under Condition 1, this rule does not coincide with the sequentially optimal rule, and it exhibits history dependence.

**Proposition 7 (ex-ante optimum under continuum of persistent shocks).** Suppose Condition 1 is satisfied. The ex-ante optimum has the following features:

1. It does not coincide with the sequential optimum, and
2. It exhibits history dependence: \( \exists \theta^t, \theta^k \) with \( \theta_t = \theta_k \) and \( \theta_{t-1} = \theta_{k-1} \) for which \( s_t(\theta^t) \neq s_k(\theta^k) \).

The intuition for Proposition 7 is familiar from the two-type economy. For the first part of the proposition, suppose by contradiction that the ex-ante optimum coincided with the sequential
optimum. We show that ex-ante welfare at date 0 could then be improved with a perturbation that induces ex-post suboptimality. For concreteness, suppose that $\theta_p(\theta_{-1}) > \theta$ for all $\theta_{-1}$ and $\theta_p'(\theta_{-1}) > 0$, so the sequentially optimal rule at date 0 becomes more relaxed the higher the shock $\theta_{-1}$. For all types $\theta_0 < \theta_p(\theta_{-1})$, consider perturbing the mechanism so that the fiscal rule becomes excessively slack at date 1; that is, consider assigning at date 1 the fiscal rule associated with a cutoff $\theta_p(\theta_0 + \mu(\theta_0))$, for some $\mu(\theta_0) > 0$ arbitrarily small. This relaxes incentive compatibility constraints at date 0, allowing to increase the savings rates assigned to all types $\theta_0 < \theta_p(\theta_{-1})$, from $s(\theta_0)$ to $s(\theta_0) + \epsilon(\theta_0)$, for some $\epsilon(\theta_0) > 0$ that satisfies incentive compatibility. Using Envelope arguments analogous to those of Section 4, it can be shown that this perturbation increases ex-ante welfare: the first order gain of bringing savings closer to first best at date 0 outweighs the second order loss of assigning suboptimal rules at date 1. Therefore, as in Section 4, the ex-ante optimal mechanism uses a threat of lack of fiscal discipline in the future to induce discipline today.

The logic for the second part of Proposition 7 is also analogous to the two-type case. Given persistent shocks, the shock at date $t - 2$ predicts the realization of the shock at $t - 1$. Thus, the shock at $t - 2$ affects the relative tightness of incentive compatibility constraints at $t - 1$, which in turn affect the policies that are chosen at $t$ to provide dynamic incentives at $t - 1$.

### 7 Concluding Remarks

This paper has studied the role of persistence in determining the optimal structure of fiscal rules. We have shown that while ex-ante optimal fiscal rules are sequentially optimal when shocks are independent over time, ex-ante optimal rules induce ex-post suboptimality when shocks are persistent. Sequentially optimal fiscal rules are simple and can be implemented with a renegotiated debt limit, which is in fact the type of rule currently in place in several countries, including the United States. In contrast, ex-ante optimal fiscal rules are more complicated, and exhibit history dependence and rich dynamics even in the long run.

We believe our paper leaves interesting questions for future research. First, the political structure of our model can be enriched to consider more general time-inconsistent preferences with hyperbolic discounting. The nature of the problem would change compared to our quasi-hyperbolic setting, as the preferences of the current government regarding future policies would no longer coincide with those of society. Second, the economic structure of our model can be enriched to consider more general social preferences for public spending, as well as a micro-founded economic environment taking into account the endogeneity of interest rates and the distortionary effects of taxation. A natural future direction would be to quantitatively assess the properties of optimal fiscal rules and how they depend on the economic structure.
Appendix

TO BE COMPLETED

References


