Country Solidarity, Private Sector Involvement and the Contagion of Sovereign Crises*

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Preliminary

Abstract

Classic analyses of sovereign debt make no predictions concerning the allocation of risk between the market and the official sector or among official sector creditors. To open the black box of the composition of a sovereign’s foreign liabilities, this paper develops a new framework and distinguishes between “ex-post solidarity”, aimed at avoiding collateral damages inflicted by a distressed country’s default, and “contractual solidarity”, illustrated by joint-and-several liability or lines of credit, that creates formal modes of insurance.

When countries differ substantially in their probability of distress, the optimal mechanism takes the form of a debt brake together with mixed public-private financing for the weaker country; no joint liability emerges. By contrast, in a more symmetrical, mutual-insurance context, contractual solidarity in the form of joint liability is optimal provided that country shocks are sufficiently independent and spillovers costs sufficiently large relative to default costs. Joint liability increases both borrowing capability and the risk of contagion. Spillovers, when endogenized, are larger under mutual insurance than under one-way insurance.

Finally, the paper considers the possibility of debt monetization, comparing the outcomes under a currency union and an own currency. It studies whether a currency area is more conducive to bailouts and whether bailouts are optimally denominated in domestic or foreign currency.

Keywords: Sovereign debt, joint liability, debt brakes, bailouts, contagion, private sector involvement, debt monetization.

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1 Motivation

The ongoing Eurozone crisis has reignited the old debate on the respective role of the official sector’s contribution to, and private sector involvement in restoring troubled countries’ access to refinancing. A further twist in the Eurozone case is the vivid controversy on country solidarity: Should Eurozone countries informally stand by to secure their peers’ access to borrowing, as was widely anticipated by markets until Greece’s debt on July 21, 2011? Should Europeans more formally issue Eurobonds, with full joint-and-several liability, or combine some such solidarity with a market mechanism for complementary financing? Variants of Eurobonds have been advocated by most leading European politicians, multi-lateral organizations (e.g., the IMF), the media (e.g., The Economist), and in several economists’ proposals that have attracted wide attention in policy circles. Assessing the relative merits of such policies requires addressing two key issues:

Solidarity area: The policy debate, negotiations and current bailout policies all take it for granted that Eurozone countries are the natural providers of insurance to each other; even non-Eurozone European countries are exempted from contributing to bailouts. This assumption is at first sight puzzling. After all, insurance economics points at the desirability of spreading risk broadly, rather than allocating it to a small group of countries, which moreover may well face correlated risk. Indeed, alternative cross-insurance mechanisms, such as the IMF’s Flexible Credit Line, the Chiang Mai Initiative, or credit lines offered to countries by consortia of banks, already exist, that do not involve insurance among countries within a monetary zone.

Market vs. official sector borrowing: While the solidarity area conundrum refers to the allocation of risk within the official sector, another topic for investigation relates to the relative weight of official vs. market borrowing. This allocation of risk between

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1See in particular Delpla-von Weizsacker (2010), Euro-nomics group (2011), and Hellwig-Philippon (2011). Related proposals include the European Commission’s green paper on “stability bonds” (2011), the Tremonti-Juncker proposal (2010), and the German Council of Economic Experts’ “European Redemption Pact” (2011). Most of these proposals advocate coupling Eurobonds with a debt brake mechanism. For example, Olivier Blanchard, IMF’s chief economist, argues in the Financial Times Deutschland (April 23, 2012) that: “When there was no fiscal treaty nor budgetary discipline instruments, the Germans had good reason to reject bearing the brunt of irresponsible policies by other states. But now we have a fiscal treaty. The Germans should accept that the Eurozone is going by way of Eurobonds.” The European Financial Stability Facility created in 2010 already can issue bonds backed by guarantees given by the Euro area member states.

2To be certain, the IMF has large programs in the Eurozone; the brunt of the risk however is borne by Eurozone countries.

3Philippon (2012) argues that shocks have recently been more asymmetric within the United States than in the Eurozone.

4The official sector comprises governments and their agencies, central banks, government controlled institutions and international institutions. For the purpose of an economic analysis, banks that are likely to be bailed out by their government can also be considered as part of the official sector.
the official and private sectors is particularly relevant for countries that are perceived as risky; a recent case in point is Spain, whose private and public sectors’ access to market financing has dwindled and which must rely on the ECB and the European Financial Stability Fund for refinancing.

These observations suggest that an analysis of solidarity must account for both the limited solidarity area (or tax base) and the endogenous allocation of risk between markets and governments. Economic studies of sovereign borrowing link a country’s incentive to repay, and therefore its ability to borrow abroad, to either its desire to keep its reputation and access to international financial markets or to the threat of sanctions. While these theories have proved very valuable to study the allocation of risk between a country and its foreign debtors, they make no predictions as to the allocation of risk between private creditors and the official sector; they also do not distinguish among official sector creditors. The purpose of this paper is to start opening the black box of the composition of a Sovereign’s foreign liabilities.

The idea is to apply the same economic analysis to both the debtor country and its potential guarantors: The latter show solidarity only if it is in their interest to do so. We distinguish between two forms of solidarity: ex post (spontaneous and thus involuntary) and ex ante (contractual). Ex post, the guarantors may stand by the troubled country because they want to avoid the externality or collateral damage inflicted by the latter’s default. Ex ante, the guarantors may agree on a joint-and-several liability. Spontaneous and contractual bailouts, which correspond roughly to the European approach to date and the various Eurobonds proposals, respectively, are not equivalent. Borrowing capabilities are larger under the joint-and-several liability approach, since a failure to stand by the failing country implies a cost of own default on top of the collateral damage incurred when the failing country defaults. However, joint-and-several liability creates a risk of domino effects and increases default costs.

The benchmark model (section 2) involves joint lending by the market and the official sector to a country. We follow the literature by assuming that the country’s income realization (or alternatively its liquidity needs realization) is random, and so there are states of nature in which the country cannot or does not want to repay. The twist here is that the official sector is willing to forgive some of its own claim on the country and to reduce private sector involvement (PSI) in order to avoid the collateral damage. Thus, in this “soft budget constraint model”, the narrowness of the tax base is rationalized by the heterogeneity in countries’ willingness to stand by the failing country: Countries that have a larger stake in avoiding a country’s default are more likely to bail out that country. Consequently, a borrowing country’s collateral is provided by the collateral damage its default creates onto peer countries.

The collateral damage cost admits both economic and political considerations. Eco-
nomic spillovers include reduced trade, banking exposures and the fear of a run on other countries. The end of the European construction can be viewed as a political cost; non-Eurozone political costs are evidenced by various countries’ access to cash through their nuisance power (collapse of USSR and fear of nuclear weapons proliferation, current assistance to North Korea, US support to Saudi Arabia, Pakistan or Israel) or conversely bailouts motivated by the desire to gain geopolitical influence.

Yet another non-economic motivation for bailing out another country is empathy, be it driven by ethnic, religious, vicinity or other considerations.

We first study what happens when the debtor country borrows only from foreign private creditors. Unlike the official sector, individual market investors do not internalize any default cost beyond the monetary loss on their own claim on the country. Unregulated borrowing generates two types of inefficiency, depending on the circumstances: default or limited access to the capital market. When the country is particularly eager to borrow or when the probability of a bad income realization is low, the country over-borrows in the private market. The size of private debt discourages the official sector from coming to the rescue of a distressed country. By contrast, when it is less eager to borrow or when its income is quite risky, the country chooses a low-enough debt level so as to always fall under the umbrella of the official sector’s implicit guarantee; it thereby does not maximize its debt capacity.

In general, a Pareto improvement can be obtained through a contract between the country and the official sector. The optimal contract then specifies a cap on private sector borrowing so as to protect the seniority of the official sector’s claim on the country. Furthermore, and a central result of our analysis, the optimal contract involves no joint-and-several liability. The intuition is that joint liability allows the debtor country to borrow more by making it more credible that it will be bailed out. But it has no ability to compensate the guarantor for the extra involvement. Thus, “asymmetric situations” in which the guarantor is unlikely to enter distress independently of the insuree lead to an implicit form of solidarity, ex post bailouts, but no explicit solidarity.

By contrast, in the “symmetrical environment” studied in section 3, debtor countries have a currency with which to pay for the formal insurance they receive through joint-and-several liability: they can reciprocate by offering guarantors some insurance.

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5 As Roubini (2004) notes, “Even before the September 11 events, but more so afterwards, the U.S. tendency to support financial aid to countries that are considered as friends, allies or otherwise strategically or systemically important (Turkey, Pakistan, Indonesia, and possibly Brazil) has clearly emerged, more strongly than during the previous administration. Even in the case of Argentina, where IMF support was eventually cut off leading to the sovereign default of this country, political considerations have been dominant: the August 2001 augmented package was pushed for political rather than economic reasons.”

6 This conclusion is in line with standard models of sovereign borrowing, which predict that countries will spontaneously cap their borrowing so as to make their repayment credible; the borrowing cap is here justified by the co-existence of private and official creditors.
in a situation in which the fortunes are reversed. We show that joint-and-several liability (cum joint monitoring of countries’ indebtedness) then may emerge as part of the optimal arrangement. More precisely, joint liability (in contrast with currency areas) is optimal provided that country shocks are sufficiently independent and spillovers costs sufficiently large relative to default costs. This contractual solidarity up to a level boosts access to the private capital market, but also enhances the overall instability by generating contagion.

While trade and political disruptions are by and large unavoidable, counterparty risk is in part determined by domestic prudential supervision as well as other mechanisms (such as the ECB’s recent LTRO facility that led to some “running for home”). Section 4 accordingly endogenizes spillovers. Under one-way insurance, the principal generally, although not always, chooses to minimize his exposure to the risky country. By contrast, mutual insurance often leads countries to contractually maximize their cross-exposures.

The basic model assumes that countries can only reduce the value of Sovereign claims through default; for instance they belong to a monetary union and cannot inflate their debt away. Section 5 introduces the possibility of debt monetization (“soft default”), a self-insurance mechanism and a substitute for hard default. This section first demonstrates that a hard currency union, relative to an autonomous monetary policy, a) generates more hard default by ruling out soft default, and b) is optimal for a debtor country facing no resource uncertainty, but may not be so when the country faces resource uncertainty.

For given country income and debt level, a currency union raises incentives for bailouts by making hard default more likely; but it also facilitates free riding by private investors when there is a rescue and so rules out private sector involvement. By contrast, debt monetization generates (involuntary) private sector involvement. Furthermore, the optimal bailout (support toward private debt relief) should be denominated in foreign currency so as to raise the country’s incentive to monetize debt and therefore to increase private sector involvement.

Section 6 concludes with some alleys for future research.

Relationship to the literature: The literature on sovereign borrowing has two (complementary) strands. One (e.g., Sachs 1983; Krugman 1985; Eaton et al 1986; Bulow-Rogoff 1989b; Fernandez-Rosenthal 1990) stresses the deterring effect of exogenous default costs, such as trade embargoes, seizure of assets or military interventions. Rose (2005) shows that debt renegotiations imply a substantial and long-lasting decline in trade.

7See e.g., chapter 6 of Obstfeld-Rogoff (1996) and Sturzenegger-Zettelmeyer (2007) for reviews of this literature. The following obviously does not do justice to this very rich literature.

8In Fernandez-Rosenthal (1990), the debtor, when repaying in full, receives a “bonus”, not paid by the
Another line, starting with Eaton-Gersovitz (1981) emphasizes that default tarnishes the country’s reputation and limits its future access to international financial markets. On the theory side, Bulow-Rogoff (1989a) argued that reputational concerns may not create access to international finance: a country cannot borrow if it can still save at going rates of interest after default. Some of the subsequent literature revisited Bulow and Rogoff’s provocative analysis. Hellwig-Lorenzoni (2009) showed that borrowing is feasible under maintained access to savings if the Bulow-Rogoff assumption that the rate of interest exceeds the rate of growth is relaxed. Cole-Kehoe (1995), Eaton (1996) and Kletzer-Wright (2000) stress that commitment is two-sided, as lenders may not comply with the punishment required to maintain discipline. Wright (2002) formalizes banks’ tacit collusion to punish a country in default. Cole-Kehoe (1998) argue that opportunistic behavior in the financial market may tarnish the sovereign’s overall reputation and create a collateral loss in the relationship with third parties (e.g. domestic constituencies). On the empirical front, a number of scholars have documented that defaulting countries recoup unexpectedly quickly access to international capital markets; Cruces-Trebesch (2011) however indicate that large haircuts are associated with high subsequent bond yield spreads and long periods of capital market exclusion.

These papers focus on the allocation of risk between the country and foreign creditors. So does the work of Gennaioli et al (2011) and Mengus (2012), which stresses the role of domestic banking exposures in the sovereign’s decision to default. Arteta-Hale (2008), Borensztein-Panizza (2009) and Gennaioli et al (2011) provide empirical evidence on the internal cost of default. Jeske (2006) and Wright (2006) analyze the impact of the allocation of country liabilities between private and public borrowing. The innovation in these papers is the introduction of resident default on international borrowing (associated with a lack of enforcement of foreign claims on domestic residents by domestic enforcement institutions), on top of standard default on public debt.

By contrast, this paper takes a shot at analyzing the equilibrium allocation of claims on the sovereign between the private and official sectors as well as the split within the official sector; to this purpose it introduces two features that are traditionally absent in the literature: collateral damage costs and the possibility of cross-insurance among countries. This holds even if the sovereign can engage in bailouts of domestic banks, provided that it has incomplete information on the quality of balance sheets: see Mengus (2012). Models of moral hazard (e.g., Tirole 2003) often stress the benefits of a home bias in savings on the government’s incentive to behave.

In the banking context, Rochet-Tirole (1996a) derives optimal cross-exposures as the outcome of a
Corsetti et al. (2006) develop a model of mixed private-public financing, in which international institutions serve as a lender of last resort and prevent self-fulfilling liquidity runs. They emphasize the role of the precision of the international institution’s information, and show that official lending may not increase moral hazard. Persson-Tabellini (1996) studies cross-country fiscal externalities when political institutions are not integrated but (a varying degree of) fiscal integration is in place. Bolton-Jeanne (2011) shows how monetary integration may create a premium on a healthy country’s debt through the collateral demand by banks in weaker ones, and that joint liability destroys this premium. Our paper has a different focus relative to these papers, such as the conditions of emergence of joint liability, PSI and contagion.

Bulow-Rogoff (1988) builds an infinite-horizon framework of a recurrent debt renegotiation among three players: the debtor country, creditor banks, and consumers in creditor countries, who benefit from the debtor country’s exports and therefore are willing to contribute in order to avoid the debtor country’s default and concomitant trade sanctions. The anticipation of future side-payments by consumers implies that bank lenders (the “market” in my model) are willing to lend more, which benefits the borrowing country. Bulow-Rogoff (1988)’s interesting analysis of repeated negotiation and private sector involvement (trade beneficiaries rather than investors in their model) does not address some of the main themes of this paper such as debt brakes, joint liability, contagion and debt monetization.

Finally, the paper offers some similarities with the literature on “cross-pledging”: cross pledging of the revenues in several activities by a single agent (Diamond, 1984) and among agents (literature on group lending and microfinance). It has been shown in the latter literature that group lending can increase entrepreneurs’ access to capital either by mobilizing social capital or by inducing mutual monitoring. Relative to this literature, the paper adds bailouts (the group lending literature assumes that joint liability is the only vector of solidarity) and the requirement that the exercise of even contractual solidarity must respect the guarantor’s willingness to pay constraint.

### 2 Collateral damage creates collateral

#### 2.1 Baseline model

This section develops a model of joint lending by the private sector (M, the market) and the official sector (P, the principal) to a country (A, the agent). All parties are risk neutral. The official sector can be thought of as a deep pocket country or the
international community. The private financial market is competitive. Both the agent and the principal honor their obligations if and only if they find it privately optimal to do so. In this “willingness-to-pay” model of sovereign borrowing, the agent’s incentive to repay is provided by the cost $\Phi_A \geq 0$ that it incurs in case of default.

For the principal to be willing to rescue the agent, we assume that the principal incurs cost $\phi_P$ whenever the agent defaults. Importantly, we distinguish between the “collateral damage cost” or “externality cost” $\phi_P \geq 0$ incurred by the principal when $A$ defaults and the larger cost $\phi_P + \Phi_P > \phi_P$ born by the principal when it also defaults on its obligations. Because the principal does not borrow in the basic model, it can only default by accepting to be jointly liable for the agent’s debt and by not honoring the associated commitment.

The default costs $\Phi_A$ and $\Phi_P$ admit a wide range of interpretations. Default costs may be associated with interruptions in the trade patterns, denial of trade credit, seizure of assets or other retaliatory moves, damages that default imposes on the domestic sector, alliance shifts, FDI interruptions and so forth. As discussed in the introduction, the collateral damage cost $\phi_P$ arises from the economic linkages studied in the contagion literature (reduced trade, impact of a foreign default on domestic banks, ...) and from political costs.

This modeling is a succinct way of capturing the idea that countries may want to prevent other countries’ default because of the concomitant collateral damage. We assume that the collateral damage is smaller than the country’s own damage: $\Phi_A > \phi_P$.

There are two periods, $t = 1, 2$. To simplify notation, there is no discounting between the two periods. The timing is described as in Figure 1.

Date 1: borrowing. At date 1, the agent borrows $b = b_M + b_P$ from the market and the principal for $A$.

Date 2: RESOLUTION. At date 2, the agent decides whether to pay back $\hat{d}_P$ and $\hat{d}_M$ and default otherwise. The principal decides whether to forgive some of the debt $d_P$ to $\hat{d}_P$, and proposes to share some of the burden $d_M - \hat{d}_M$ if $A$ reimburses its debt.

Figure 1: Timing

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12 In this basic model, $\Phi_A$ is the agent’s default cost when only the agent defaults. If the principal also defaulted (as will be the case in Section 3), the cost incurred by the agent would be higher; however multiple defaults do not occur in equilibrium in the basic model and so we do not introduce additional notation for the moment.

Also, we assume that the agent bears no cost when he is bailed out. The agent could incur some (reputation) cost without this altering the basic insights.

13 As in Gennaioli et al. (2011) and Mengus (2012) for instance.
principal, respectively, and values this borrowing at $Rb$. The parameter $R$ measures the intensity of the agent’s liquidity needs: current consumption needs or quality of his investment opportunities. The borrowing contract specifies debts $d_M$ and $d_P$ to be repaid at date 2 to private investors and to the principal, respectively. The amount of borrowing is observable. $Rb$ is to be interpreted as a private benefit for the agent; when $R$ stands for the value of investment opportunities, one must be careful to distinguish for comparative statics purposes investments in non-tradables (which are indeed private benefits) and investments in tradables (that are likely to raise date-2 income available for repayment).

**Date 2: income realization.** At date 2, the agent receives a random income, equal to $y$ with probability $\alpha$ (good state of nature, $G$) and 0 with probability $1 - \alpha$ (bad state of nature, $B$), where $y > \Phi_A$. Income “0” is to be interpreted more generally as some incompressible, minimum level of consumption below which the agent is not disposed to go. As is customary in willingness-to-pay\textsuperscript{14} and insurance\textsuperscript{15} models, the financial market does not observe the realization; equivalently, the market is uncertain as to whether $A$ is willing to make sacrifices to reimburse the debt (i.e., as to the level of the incompressible level of consumption). The agent and the principal do observe the realization, although the assumption that the principal observes the realization only serves to simplify expressions: Appendix 2 shows that qualitatively similar insights hold when only the agent observes the income shock. We assume that the principal and the agent form a coalition at date 2 when deciding whether to reimburse the agent’s debt\textsuperscript{16}.

**Debt forgiveness and bailout.** Following $A$’s income realization, the principal can forgive some of its debt and bring it down to $\hat{d}_P \leq d_P$. Similarly, the principal can offer to bring conditional support $d_M - \hat{d}_M$ provided that the agent reimburses the private investors\textsuperscript{17}. The remaining private debt burden on $A$ is then $\hat{d}_M$ and the agent’s effective debt burden is $\hat{d} = \hat{d}_P + \hat{d}_M$.

\textsuperscript{14}As well as in a number of standard corporate finance models (e.g., Bolton-Scharfstein 1990; Gale-Hellwig 1983; Townsend 1979).

\textsuperscript{15}In the tradition of Holmström (1979). Were the state of nature verifiable, then contingent debt contracts could be written, that deliver a higher utility to the agent. The latter would then be tempted to renege in the good state of nature, as optimal insurance would call for debt forgiveness in the bad state and a high repayment in the good state. See Grossman-van Huyck (1988) for the view that if states of nature are verifiable, the Sovereign’s ability to default partially or fully can, under some conditions, mimic an optimal state-contingent debt contract. Trebesch (2009) finds that domestic firms suffer more in their access to credit when the government has employed coercive actions instead of good faith debt renegotiations.

\textsuperscript{16}This rules out Nash implementation schemes in this model, so the contract studied in Section 2.3 will indeed be optimal.

\textsuperscript{17}Another form of conditionality (along the lines of the IMF programs) could be easily added to this framework: The principal could condition its support on the agent’s undertaking some costly action that will generate some pledgeable income at some later date 3.
Repayment decision. Finally, the agent decides whether to repay its renegotiated liabilities $\hat{d}_P$ and $\hat{d}_M$ or to default.

No-principal benchmark.

Suppose that there is no principal. Equivalently (as will be shown later), the principal incurs no spillover cost ($\phi_P = 0$). In this case, which is a special case of sections 2.2 and 2.3, the agent can borrow $b_M = \alpha \Phi_A$ from the market, and reimburse $d_M = \Phi_A$, the highest credible reimbursement, in the good state, at the cost of default in the bad state. The agent then receives utility

$$U_A = R(\alpha \Phi_A) + \alpha (y - \Phi_A) - (1 - \alpha) \Phi_A = (\alpha R - 1) \Phi_A + \alpha y.$$ 

The agent alternatively can refrain from borrowing ($b_M = 0$) and receive utility $\alpha y$. Thus the agent borrows from the market if and only if

$$\alpha R \geq 1.$$  

Debt forgiveness and bailouts.

Let us return to the case of interest ($\phi_P > 0$). In the last stage the agent reimburses its debts if and only if $\hat{d}_P + \hat{d}_M \leq \Phi_A$, and defaults otherwise.

In the state of nature in which the agent receives no income, the principal without loss of generality forgives official debt: $\hat{d}_P = 0$. The question is whether the principal is willing to foot the bill for the debt owed to the private sector. The principal bails out the agent if and only if

$$d_M \leq \phi_P$$

since the absence of agent income requires the principal to foot the bill for the entire private debt $d_M$. By contrast, when the private debt exceeds the collateral damage, the principal no longer stands by the agent.

In the state of nature in which the agent receives income $y$ and $d \leq \Phi_A$, the principal knows that debt will be repaid and therefore does not intervene.

Consider now the case in which the agent defaults when the principal remains passive: $d > \Phi_A$.

(a) If $d_M \leq \Phi_A$, then the principal just forgives $d - \Phi_A$. There is no default.

(b) If $\Phi_A < d_M \leq \Phi_A + \phi_P$, the principal forgives the entire official debt and further
offers conditional support \( d_M - \Phi_A \) so as to prevent default. The principal’s cost is, as in case (a), \( d - \Phi_A \).

(c) Finally, if \( d_M > \Phi_A + \phi_P \), then leaving aside debt forgiveness, the support, \( d_M - \Phi_A \), needed to rescue the agent exceeds \( \phi_P \) and the principal prefers to incur the collateral damage from the agent’s default. There is no bail out.

Figure 2 summarizes the outcome in the good state.

**Figure 2:** Repayment and default behavior in the good state when \( d > \Phi_A \)

**Lemma 1 (repayment and default).**

(i) In the bad state, the principal bails out the agent if \( d_M \leq \phi_P \) and lets the agent default otherwise.

(ii) In the good state, the agent defaults if and only if the private debt exceeds \( \Phi_A + \phi_P \). If \( d_M \leq \Phi_A + \phi_P \), the principal prevents default by forgiving debt and possibly by bringing further support to repay private debt.

**Remark:** The extensive form depicted in Figure 1 creates a “soft budget constraint”, as the principal forgives its own claim and further bails out the agent if the latter’s market liability is not too large. An alternative extensive form would have the agent announce at date 2 its intended debt repayments, and then give the principal an opportunity to forgive its claim and reimburse the private investors’ shortfall, before default is pronounced. This would put the principal in an even weaker position. The qualitative results would be quite similar to those obtained below.

### 2.2 Borrowing from the private sector only (laissez faire)

Let us now investigate the agent’s date-1 borrowing behavior, starting with the case in which \( A \) does not enter an agreement with \( P \) (and so \( b_P = d_P = 0 \)). Given the principal’s behavior at date 2, the agent’s optimal indebtedness is either \( d_M = \Phi_A + \phi_P \), so as to benefit from the maximal bailout in the good state of nature, or \( d_M = \phi_P \).
so as to benefit from the maximal bailout in the bad state of nature. That is, the agent chooses between a risky, high-debt policy and a safe, low-debt one. The risky policy allows $A$ to borrow $b_M = \alpha (\Phi_A + \phi_P)$ but leads to a default in the bad state, while the safe policy raises $b_M = \phi_P$ and generates no default. The agent’s utility is

$$U^*_A = \max \left\{ R\alpha (\Phi_A + \phi_P) + \alpha (y - \Phi_A) - (1 - \alpha)\Phi_A; R\phi_P + \alpha (y - \phi_P) \right\}$$  \hspace{1cm} (1)$$

We assume that

$$R \geq R_0 = \min \left\{ \Phi_A / \alpha (\Phi_A + \phi_P), \alpha \right\},$$

so that the agent prefers borrowing to not borrowing, which yields utility $\alpha y$. Relative to this benchmark utility $\alpha y$, safe borrowing yields $R\phi_P$ at date 1 and leads to an effective debt burden for the agent $\phi_P$ with probability $\alpha$; so the agent prefers safe borrowing to no borrowing if $R \geq \alpha$. Risky borrowing implies a sure cost $\Phi_A$ (either in reimbursement or in default cost); but it allows immediate consumption $R\alpha (\Phi_A + \phi_P)$. Hence, if $R\alpha (\Phi_A + \phi_P) \geq \Phi_A$, the agent prefers risky borrowing to no borrowing. Note that $R_0 < 1$; the agent may select a negative NPV borrowing strategy so as to benefit from the soft-budget constraint.

The agent chooses the risky policy if and only if

$$R[\alpha \Phi_A - (1 - \alpha)\phi_P] \geq \Phi_A - \alpha \phi_P.$$  \hspace{1cm} (2)$$

In general, a higher nominal debt may make it less credible that it will be reimbursed; and so an increase in liabilities may not bring in more money for the agent. Indeed, the high-debt policy is also a high-borrowing one if and only if

$$\alpha \Phi_A > (1 - \alpha)\phi_P.$$  \hspace{1cm} (3)$$

If condition (3) is violated, then the safe policy both raises more income and generates less default cost, and so is always picked. If condition (3) is satisfied, then the high-debt policy is chosen whenever

$$R \geq \frac{\Phi_A - \alpha \phi_P}{\alpha \Phi_A - (1 - \alpha)\phi_P} \equiv R^*$$  \hspace{1cm} (4)$$

We will use the convention that $R^* = +\infty$ if (3) is violated. Note also that in the no-principal benchmark ($\phi_P = 0$), $R^*$ is indeed equal to $1/\alpha$. The principal’ s welfare is:

$$U^*_P = \begin{cases} -\phi_P & \text{if } R \geq R^* \\ -(1 - \alpha)\phi_P & \text{if } R < R^* \end{cases}$$  \hspace{1cm} (5)$$
The principal is always hurt in the bad state: either \( P \) shows solidarity with \( A \) and bails \( A \) out at level \( \phi_P \), or \( P \) lets \( A \) default and then also incurs cost \( \phi_P \). The principal is also hurt in the good state of nature under the risky, high-debt policy as \( \phi_P \) must be contributed by \( P \) to prevent default.

**Proposition 1 (laissez faire).** Assume that \( R \geq R_0 \). When the agent borrows from the market and does not contract with the principal, the agent’s optimal strategy is either a high-debt policy (borrowing \( \alpha(\Phi_A + \phi_P) \) and defaulting in the bad state) or a low-debt one (borrowing \( \phi_P \) and never defaulting, thanks to the principal’s rescue in the bad state). The high-debt policy leads to a default in the bad state of nature and to a bailout in the good state of nature.

(i) The agent picks the high-debt policy if \( R \geq R^* \).
(ii) The high-debt policy is more likely, the greater the probability of a good state.

### 2.3 Optimal contract with the official sector

Suppose now that prior to borrowing at date 1 the agent makes a take-it-or-leave-it offer to the principal. If the principal turns down the offer, the outcome is the unregulated one studied in the previous section; in particular, the principal’s utility is given by (5). We adopt a mechanism design approach. The agent’s contract offer to the principal specifies:

- A borrowing level \( b \) and its allocation between the market and the principal: \( b = b_M + b_P \).
- A state-contingent reimbursement and its allocation. For \( \omega \in \{G, B\} \), the agent effectively pays back \( d^\omega = d_M^\omega + d_P^\omega \). \(^{18}\)

Intuitively, the laissez-faire outcome can be improved upon in two ways. First, the risky borrowing strategy generates inefficient default and a collateral damage cost for the principal. The agent should arrange financing so as to avoid default and in exchange receive a favorable treatment from the principal. Second, the safe borrowing strategy substantially constrains the agent’s borrowing; the agent can credibly commit to reimburse \( \Phi_A \) in the good state, but, when the safe strategy is optimal, does not want to take on this commitment by fear of defaulting in the bad state. Transferring at least part of the liability from the market to the principal creates more flexibility in the level of repayment and allows the agent to borrow more without risking default. Proposition 2 establishes the validity of these intuitions.

\(^{18}\) \( d_P^\omega \) can be negative (bailout). Note also that this notation refers to the actual repayments and does not imply that state-contingent debt can be issued.
Proposition 2 (optimal contract). When the agent contracts with the principal at date 1 and \( R \geq 1 \),

(i) an upper bound on the agent’s utility is

\[
\hat{U}_A = R(\alpha \Phi_A - U^*_P) + \alpha(y - \Phi_A);
\]

(ii) this upper bound is reached through the following mix of public and private financing:

✓ the agent borrows \( b_M = d^G_M = d^B_M = \phi_P \) from the market; the principal monitors this cap on market financing (debt brake) and spontaneously bails out the agent in the bad state of nature;

✓ the agent borrows \( b_P = \alpha \Phi_A - \phi_P - U^*_P \) from the principal, repays the principal \( d^P_G = \Phi_A - \phi_P \) in the good state of nature, and receives bailout money \( -d^P_B = \phi_P \) in the bad state of nature from the principal to repay its private creditors.

The agent never defaults.

Proof:

(i) Consider the following program, consisting in maximizing the agent’s default-free utility subject to incentive and participation constraints:

\[
\max \left\{ U_A = Rb + \alpha(y - d^G) + (1 - \alpha)(-d^B) \right\}, \quad (I)
\]

where

\[
b = b_M + b_P,
\]

the participation constraints are satisfied:

\[
-b_P + \alpha d^G_P + (1 - \alpha)d^B_P \geq U^*_P
\]

\[
-b_M + \alpha d^G_M + (1 - \alpha)d^B_M \geq 0,
\]

and the incentive constraints are satisfied:

\[
d^G \leq \Phi_A
\]

\[
d^B \leq 0
\]

\[
-d^B_{\omega} \leq \phi_P + \Phi_P \quad \text{for} \quad \omega \in \{G, B\}.
\]

The latter constraints represent the incentive constraints for the principal. We here allow joint-and-several liability and so the principal’s cost for not complying can be as
high as \( \phi_P + \Phi_P \). Adding up the participation constraints and replacing in \( U_A \) yields

\[
U_A \leq R\left[\alpha d^G + (1 - \alpha)d^B - U^*_P\right] + \alpha(y - d^G) + (1 - \alpha)(-d^B).
\]

The upper bound \( \hat{U}_A \) is reached when \( d^G \) and \( d^B \) take their highest values, \( \Phi_A \) and 0, respectively.

(ii) Computations are straightforward.

Two points are worth noticing, though. First, in the proposed implementation of the upper bound, \( d^G_P \) is positive. By contrast, \(-d^B_P = \phi_P\). Joint-and-several liability (which would relax the potential commitment of the principal to \( \Phi_P + \phi_P > \phi_P \)) is not used and the principal spontaneously (that is, in the absence of contractual commitment) contributes \( \phi_P \) in the bad state of nature to rescue the agent.

Second, a control over private borrowing is in general required. Otherwise, the agent might well overborrow, preventing the optimum from being reached. To see this, suppose that the principal does not monitor that the agent borrows no more than the cap. Any increase in private debt leads to default in the bad state. So, conditional on increasing \( d_M \) beyond \( \phi_P \), the agent might as well borrow as much as is consistent with the absence of default in the good state. So suppose that the agent issues an extra claim \( \Phi_A \) on the market, bringing total private debt to \( d_M = \Phi_A + \phi_P \). In the good state, the principal forgives his own debt and further brings support \( \phi_P \) to enable the reimbursement of private debt. The agent thereby collects \( b_M = \alpha\Phi_A \) from private creditors since the extra borrowing generates default in the bad state. This strategy delivers utility

\[
U_A = R\left[2\alpha\Phi_A - \phi_P - U^*_P\right] + \alpha(y - \Phi_A) - (1 - \alpha)\Phi_A
\]

\[
= \hat{U}_A + R\left[\alpha\Phi_A - \phi_P\right] - (1 - \alpha)\Phi_A.
\]

The term \( R\left[\alpha\Phi_A - \phi_P\right] \) represents the benefit from overborrowing in the market, while \( (1 - \alpha)\Phi_A \) corresponds to the expected cost of default associated with this strategy. And so, for \( \alpha \) large enough, the agent overborrows from the market.

Note that making official sector debt senior (Hellwig-Philippon 2011; Delpla-von Weizsacker 2010) is not sufficient to prevent overborrowing: In the absence of an explicit constraint, the agent may try to activate a bailout in the good state of nature.\(^{19}\)

Finally, let us discuss the implementation of the optimal contract. First, because the principal’s incentive constraints are not binding, the implementation of the optimal contract is

\(^{19}\)This argument is a variant of the classic dilution problem (e.g., Bizer-de Marzo, 1992; Segal 1999), but with a twist: Overborrowing is here motivated by the desire to trigger an uncontracted-for bailout.
allocation developed in Proposition 2(ii) does not require an explicit joint-and-several liability. In the bad state of nature, \( P \) bails out \( A \), but \( P \) need not be in default if he does not do so. Put differently, \( A \)'s unpaid debt does not become \( P \)'s debt necessarily. By contrast, we will see that joint-and-several liability can be Pareto improving in the context of mutual insurance.

**Corollary 1 (no need for joint-and-several liability).** While laissez-faire is dominated by a contractual relationship between the principal and the agent, the optimal contract can be implemented without the principal’s being held legally liable for the agent’s liabilities.

Second, the implementation of the optimal allocation in Proposition 2(ii) involves mixed financing by the market and the official sector. Could the market be short-circuited and the entire loan be provided by either the official sector or the market (together with a debt ceiling) in an exclusive contract? The answer is often “no”, even if we ignore the possibility that the “official sector” may have limited cash itself (this will indeed be the case in the next section, in which countries are both borrowers that co-insure each other). Suppose exclusive lending by either the principal or the market. From Proposition 2, the agent must borrow \( b = \alpha \Phi_A - U^*_P \) and reimburse \( d^G = \Phi_A \) and \( d^B = 0 \) in either case.

Suppose first that the agent contracts liabilities solely with the market \( (d_P = 0) \); unlike in Section 2.2 we allow the principal to contract with the agent on other aspects such as the debt level or some date-1 lump-sum payment. If the country’s liability \( d_M \) is strictly smaller than \( \Phi_A \), then \( d^G < \Phi_A \) and so the scheme does not mobilize enough country collateral to implement the optimum. By contrast, if \( d_M \geq \Phi_A \), then in the absence of joint liability, the principal does not rescue the agent in the bad state and so the latter defaults. So pure market debt cannot implement the optimum in the absence of joint liability. If \( \Phi_P + \phi_P \geq \Phi_A \), then joint liability prevents a default in the bad state provided that \( d_M \leq \Phi_P + \phi_P \). Taking (without loss of generality) \( d_M = \Phi_A \), then the market lends \( b_M = \Phi_A \) at date 1. To implement the optimum, the agent must compensate the principal at level \((1 - \alpha)\Phi_A + U^*_P \), so that the agent’s date-1 consumption is indeed \( \alpha \Phi_A - U^*_P \).

Consider now the possibility of borrowing exclusively from the official sector. Under liability \( d_P = \Phi_A \), the principal forgives the debt in the bad state. If, as we have assumed until now, the principal is committed to force default in the absence of repayment, the agent pays back \( \Phi_A \) in the good state, and so the optimum can be implemented through lending only by the official sector. This conclusion, however, is not robust to our strong commitment assumption. To see this, note that the principal’s policy of putting the agent in default when the agent does not repay is not time consistent, as the principal incurs cost \( \phi_P \) of doing so. Repayment is jeopardized by official sector’s ex-
clusivity in lending. The threat of putting the country in default is less credible for the official sector (which incurs cost $\phi_P$ in case of default) than for the market (as a private investor bears no direct spillover externality from the country’s default). This may jeopardize repayment, as shown by the following example. Suppose that when the agent refuses to pay back $\hat{d}_p$, the principal enforced default sanctions only with probability $z < 1$ (instead of $z = 1$ in the model). The mixed-financing implementation of Proposition 2 still operates as long as $z \Phi_A \geq \phi_P$, that is as long as the attempt to pay only $\phi_P$ to private creditors proves too costly to the agent. By contrast, with pure public financing, the agent never repays the principal as long as $z < 1$.

**Corollary 2 (optimality of mixed financing).**

(i) In the absence of joint liability, the optimal allocation cannot be achieved if the agent takes on only private liabilities under the control of the principal. Purely private liabilities by contrast can deliver the optimal allocation under joint liability provided that $\Phi_p + \phi_P \geq \Phi_A$.

(ii) Pure official-sector financing cannot implement the optimum unless the principal’s probability of enforcing default when its claim is not repaid is exactly 1.

### 2.4 Discussion and some simple extensions

**Spreads.** Unsurprisingly a spread on the agent’s sovereign debt appears in the absence of contractual agreement when the pressing liquidity needs (a high $R$) induces the agent to opt for the risky strategy. Because in this model there is no shortage of stores of value, the agent’s choice and/or the institutional arrangement has no impact on the principal’s borrowing conditions: there is just no spread there. By contrast, if there were a shortage of safe financial instruments in the principal’s economy, safe instruments’ premium would increase due to a flight to quality, as in [Bolton-Jeanne (2011)].

**Eurobonds.** As mentioned in the introduction, a number of recent policy proposals by economists, think-tanks and politicians have proposed introducing limited solidarity through a two-tier borrowing structure: blue bonds, for which the Eurozone would be jointly liable, and red bonds, for which no such solidarity would operate. Blue bond issues would be capped at a fraction of GDP (say 60 %). These proposals all insist on a number of features: budgetary supervision (a policy that in our model would be akin to controlling moral hazard on the choice of $\alpha$, as we discuss shortly), joint liability on the blue bonds, no bail-out clause on the red bonds, and seniority of blue bonds over red bonds.

---

20The particular terminology is due to Delpla-von Weizsacker (2010). See also the closely related Eurobill proposal of Hellwig-Philippon (2011).
While we noted that joint liability is not required in order to implement the optimal contract, we may wonder whether a Eurobond-style arrangement could not achieve the same outcome. A first observation is that the no bail-out clause for red bonds together with the absence of default in the optimal contract imply in our model that no red bonds should be issued. Thus it must be the case that the agent issues only blue bonds; so all issuing is in Eurobonds. Could Eurobonds achieve by themselves the optimal outcome? Let $d^B$ denote the amount of Eurobonds. Because they are guaranteed, the agent can borrow $b = d^B$. The following corollary is proved in Appendix 1:

**Corollary 3 (joint liability).** The optimal contract (which from Corollary 1 can always be implemented without joint liability) is also implementable through a system of Eurobonds if and only if two conditions are both satisfied:

\[
R \geq R^* \quad \text{and} \quad (1 - \alpha)\Phi_A \leq \phi_P.
\]

Intuitively, the principal is willing to accept joint liability only if it is in a very weak bargaining position. This is indeed the case if $\phi_P$ is large and if $R \geq R^*$, so that the agent’s threat of overleveraging is credible.

**Ex-post moral hazard.** The model is easily generalized to accommodate ex-post moral hazard. Suppose that after borrowing takes place, the agent chooses the probability $\alpha$ of a good state at cost $g(\alpha)$ where $g$ is increasing and convex. This moral hazard adds one constraint to Program (I):

\[
g'(\alpha) = y - d^G + d^B.
\]

The optimal allocation still involves no transfer in the bad state: $d^B = 0$, but the repayment constraint $d^G \leq \Phi_A$ may no longer be binding so as to provide the agent with stronger incentives to avoid distress. Put differently, ex-post moral hazard unsurprisingly is likely to call for a tighter debt brake.

**Ex-ante moral hazard.** Suppose now that, “at date 0”, i.e., before borrowing, the agent incurs effort cost $\psi(e)$ in order to generate date-1 income $e$ (to which borrowing will be added to yield date-1 consumption). Suppose further that the date-1 utility from consumption $R(c_1)$ is concave in consumption $c_1$ rather than linear. Straightforward computations yield the following condition for the risky borrowing strategy to be optimal under laissez-faire:

\[
R(\alpha(\Phi_A + \phi_P) + e) - R(\phi_P + e) \geq \Phi_A - \alpha\phi_P.
\]

(2')
An increase in $e$ makes it less likely that the agent chooses the risky borrowing strategy.\footnote{Note that $a(\Phi_A + \phi_P) > \phi_P$ is a necessary condition for (2') to hold.} Thus, the agent may want to choose a low $e$ at date 0 in order to make it more credible that it will choose the risky borrowing strategy and (from Proposition 2) thereby extract better terms from the principal.

3 Contractual solidarity

Consider now the two-country symmetric version of the model of Section 2. Both countries borrow at date 1. Country $i$ values cash $b_i$ available at date 1 at $Rb_i$ where $R > 1$. At date 2, each country either has income $y$ (is “intact” or “healthy”) or has no income (is “troubled”). The probability that $k$ countries have income $y$ is $p_k$ (with $\sum_{k=0}^{2} p_k = 1$). By keeping these probabilities general, we allow arbitrary patterns of correlation between income shocks. $y$ is assumed large enough, so that a country’s willingness to pay rather than ability to pay is binding.

As earlier, we distinguish between a country’s own cost of defaulting, $\Phi$, and the collateral damage this default imposes on the other country, $\phi$. Let $\hat{\Phi} \equiv \Phi + \phi$ denote the total cost of a default.

More generally, “hats” will stand for total costs. Let $\hat{\Phi}_0$ and $\hat{\Phi}_2$ denote the per-country total cost of default (own default plus spillover) when $k = 0$ and $k = 2$, respectively. For example, $\hat{\Phi}_k = 0$ if no country defaults and $\hat{\Phi}_k = \hat{\Phi}$ if both countries default.

When $k = 1$, we will distinguish between the pain, $\hat{\Phi}^y_1$, inflicted upon the country that has income $y$, and that, $\hat{\Phi}^0_1$, inflicted upon the country with zero income. Let $x_1 \in [0, 2]$ denote the expected number of defaults when $k = 1$ (we do not restrict $x_1$ to $\{0, 1, 2\}$, as a priori there could be stochastic defaults). Note that

$$\hat{\Phi}^y_1 + \hat{\Phi}^0_1 = x_1 \hat{\Phi}, \quad \text{and} \quad \left| \hat{\Phi}^0_1 - \hat{\Phi}^y_1 \right| \leq \Phi - \phi. \quad (6)$$

Let $\hat{\Phi}_1 \equiv (\hat{\Phi}^0_1 + \hat{\Phi}^y_1) / 2$ denote the per-country average pain when $k = 1$; and let $d_k$ denote the expected, per-country reimbursement to private creditors in state of nature $k$. Obviously, $d_0 = 0$.

We assume as earlier that the state of nature is known to sovereigns (joint observability of a country’s income realization is not essential) but not observed by markets. The countries form a coalition in their report to the market. As usual, the strategy for
finding the optimal arrangement will consist in considering a subconstrained program and checking that its solution can indeed be implemented.

Thus, consider the following program:

\[
\begin{align*}
\max & \left\{ R \left[ \sum_{k=0}^{2} p_k d_k \right] - \sum_{k=0}^{2} p_k (d_k + \hat{\Phi}_k) \right\} \\
\text{s.t.} & \quad 0 \leq \hat{\Phi}_k \leq \hat{\Phi} \quad \text{for all } k \\
& \quad d_2 + \hat{\Phi}_2 \leq \hat{\Phi}_0 \\
& \quad d_2 + \hat{\Phi}_2 \leq d_1 + \hat{\Phi}_1 \\
& \quad 2d_1 + \hat{\Phi}_1^y \leq \hat{\Phi} \\
\end{align*}
\]

(II)

The objective function is the difference between a country’s date-1 benefit derived from borrowing \( b = \sum_{k=0}^{2} p_k d_k \), and the date-2 cost, which includes monetary reimbursement and the pain associated with defaults and spillovers. Constraint (7) simply states that the pain inflicted upon a country cannot exceed \( \hat{\Phi} \). Constraints (8) and (9) are coalition incentive constraints when both countries have income. Constraint (8) prevents the countries from claiming they have no income (remember that by necessity \( d_0 = 0 \)). Constraint (9) similarly prevents the countries from letting one claim not to have income (and possibly compensating the other, who has to foot the bill \( 2d_1 \)).

Constraint (10) says that when \( k = 1 \), the healthy country can always refuse to contribute. Its date-2 utility is then at least \(-\hat{\Phi}\), since \( \hat{\Phi} \) is the worse pain that can be inflicted on the country, while compliance by definition means paying the entire debt \( 2d_1 \) and incurring default cost \( \hat{\Phi}_1^y \).

Constraint (11) can be understood as follows: condition (11a) is equivalent to the absence of a collusion gain for the two countries of declaring that both are distressed when only one actually is. If condition (11a) is violated, and so there is a surplus from colluding and misrepresenting the state of nature, then condition (11b) states that the healthy country must be compensated to participate in the misrepresentation, which is infeasible as the other country has no income.

The analysis of this program can be found in Appendix 2. We here content ourselves with an informal account. Let us assume that borrowing is desirable, which is actually the case if and only if \( R > (1 + p_0) / (1 - p_0) \geq 1 \). Because any borrowing
leads to some default (unless, perhaps, \( p_0 = 0 \)), the return on borrowing must strictly exceed 1 to be worthwhile.

Second, the program’s linearity implies a number of simplifications. There is no default cost when both are intact: \( \Phi_2 = 0 \); conversely, the default costs are maximal when both are distressed: \( \Phi_0 = \Phi \). Furthermore, the analysis can focus on only two binding constraints:

\[
d_2 \leq d_1 + \frac{x_1\Phi}{2} \tag{9'}
\]

and

\[
2d_1 + \Phi_1^\prime \leq \Phi \tag{11b'}
\]

where, in the asymmetric state, the cost to the intact country when the other is distressed is minimal conditional on the number \( x_1 \) of defaults:

\[
\Phi_1^\prime = \begin{cases} 
  x_1\phi & \text{for } x_1 \leq 1 \\
  \phi + (x_1 - 1)\Phi & \text{for } x_1 \geq 1.
\end{cases}
\]

Increasing \( x_1 \) facilitates repayment when both countries are intact (condition (9')), but reduces the maximand and also makes it more difficult to obtain repayment in the asymmetric state (condition (11b')). At the optimum \( x_1 \) is an integer: \( x_1^* \in \{0, 1, 2\} \).

Proposition 3 (contractual solidarity). Let \( r \equiv \frac{\Phi}{\Phi} \) (spillover-default cost ratio) and \( \ell \equiv \frac{p_1}{p_2} \) (likelihood ratio). If \( R < \frac{1}{1 + p_0} \), then it is optimal for the countries not to borrow. If \( R > \frac{1}{1 + p_0} \), then countries borrow; there is no default when both are intact and full default when both are distressed. Furthermore:

1. **Solidarity region**: If \( R(1 - \ell r) < 1 + \ell \), then there is no equilibrium default when one country is distressed: \( x_1^* = 0 \). Each country takes on debt \( d_2 = (\Phi + \phi)/2 \) and accepts to be jointly liable for the full amount of unpaid debt by the other country. Joint-and-several liability is required as \( d_2 > \phi \) and so an intact country would not spontaneously bail out a distressed one.

2. **PSI region**: If \( R(1 - \ell r) > 1 + \ell > R(1 - \ell r) \), then the distressed country, but not the intact one defaults: \( x_1^* = 1 \). Each country takes on debt \( d_2 = \Phi + (\phi/2) \), but is not jointly liable for the other country’s debt. There is private sector involvement, in the sense of a voluntary reduction in the debt owed to the intact country to \( \Phi < d_2 \) when the distressed country defaults. This debt forgiveness is meant to prevent the intact country, which already incurs spillovers from the other country’s default, from defaulting.

3. **Contagion region**: If \( R(1 - \ell r) > 1 + \ell \), then both countries default if at least one of them
is in distress: $x_1^* = 2$. Both countries take a high level of debt $d_2 = \Phi$; so they default unless they are both intact.

Minor liquidity needs (low $R$) and a high probability of a joint shock (high $p_0$) both make borrowing suboptimal. As liquidity needs increase, say, borrowing combined with joint liability becomes desirable. Debt remains limited so as to make credible the assumption of a troubled country’s debt by an intact one. As liquidity needs become more pressing, countries increase their indebtedness and abandon joint liability; the troubled country defaults; perhaps surprisingly, the intact country is granted some debt forgiveness by the private sector: Because the troubled country defaults anyway, the intact country’s incentives to repay must be re-established through debt reduction. Finally, for very large liquidity needs (or when shocks are highly correlated), countries choose a very risky strategy: they lever up a lot and default unless both are intact. Contagion occurs because a country’s debt is at its highest possible level, namely the level at which an intact country is indifferent between reimbursing and entering a default that brings down both countries; if the other country is troubled and defaults anyway, then incentives for debt repayment by the intact country are reduced and that country prefers to default as well.

The ratio $\ell$ can be interpreted as a measure of independence of country shocks, while $r$ measures the relative cost of spillovers of default to country cost of default. Figure 3 depicts the various regimes. The following corollary provides the comparative statics with respects to $\ell$ and $r$.

**Corollary 4 (comparative statics).**

(i) Solidarity in the form of joint liability is more likely to be optimal, the more independent the shocks (i.e., the larger $\ell$ is).

(ii) The solidarity and contagion regions expand when relative spillover costs ($r$) increase.

---

22If $\rho$ denotes the correlation and $\alpha$ is the marginal probability of a good state for a country, then $p_2 = \rho \alpha + (1 - \rho)\alpha^2$ and $p_1 = (1 - \rho)2\alpha(1 - \alpha)$. And so

$$\ell = \frac{2\alpha(1 - \alpha)}{\frac{\alpha^2 + \frac{p}{1 - \rho}}{\alpha}}.$$

Thus $\ell$ decreases with $\rho$. 22
4 Endogenous spillovers

Spillovers such as trade and political disruptions can be large and be assumed exogenous. By contrast, banking exposures to a potentially distressed country are in part endogenous if only because banking regulation still operates at the domestic level. This section extends the model of Sections 2 and 3, and asks whether countries would want to minimize spillovers as a first intuition would suggest. To study this question, we assume that country $i$ chooses a degree of exposure $z_i \in [0,1]$ that determines the spillover from country $j$’s default:

$$\phi_i = \phi_0 + z_i(\phi - \phi_0),$$

where $\phi_0 < \phi$ stands for the non-controllable spillovers.\(^23\)

**Asymmetric insurance**

Consider first the model of Section 2. From Proposition 2, the outcome under contracting depends solely on the principal’s utility in the absence of a contract ($U^*_P$); this is due to the fact that the principal’s incentive compatibility constraint is non-binding.

\(^{23}\)To illustrate this, suppose that the principal’s banking sector invests $b_{MP} (\leq b_M)$ in the agent’s debt. Let $d_{MP}$ denote the corresponding liability and $\beta \geq 0$ the weight put by the principal on his banking sector. Then $\phi_P = \phi_0 + \beta d_{MP}$. A control by the principal of the volume of agent liabilities purchased by its banks amounts to choosing the spillover cost for the principal. One can for example suppose that the principal’s banks have a slightly higher valuation for the agent’s debt than other private-sector investors (for example, they share the same currency in the case of the Eurozone). They will then invest up to the cap set by the principal.
In turn, the principal’s utility under laissez-faire is given by (5), which implies that in each region \( R \supseteq R^*(\phi) \) the principal is better off minimizing its exposure \( z_P = 0 \).

Furthermore \( \frac{\partial R^*}{\partial \phi} = \text{sign} (1 - \alpha - \alpha^2) \). Thus, if \( 1 < \alpha + \alpha^2 \), \( z_P = 0 \) is indeed the optimal policy, as minimizing the exposure both increases the principal’s utility for a given borrowing strategy of the agent, but also makes the risky strategy less appealing to the agent.

By contrast, if \( 1 > \alpha + \alpha^2 \), then the principal may not want to minimize exposure. It may be that being exposed to the agent \( (z_P > 0) \) encourages the safe strategy, offsetting the direct cost for the principal of being exposed.

**Symmetric insurance.**

Let us now turn to the model of contractual solidarity of Section 3. A change in spillovers moves the boundaries of the various regions as described in Corollary 4. More to the point, Appendix 2 studies the impact of an increase in \( \phi \) on the maximand.

In particular, in the solidarity region \( (x^* = 0) \), increasing spillovers is costless as default does not occur in equilibrium; furthermore, by increasing collateral damages it increases borrowing capacity and allows both countries to borrow more. Similarly increasing spillovers is optimal in the contagion region. By contrast, the impact of an increase in \( \phi \) is ambiguous in the PSI region.

**Proposition 4 (endogenous spillovers).** Suppose that the collateral damages can be chosen in an interval \( [\phi_0, \phi] \).

(i) **One-way insurance:** The optimal contract can be implemented by any choice in this range. However, this contract depends on the no-contract, status-quo outcome. In the absence of contract, and conditional on choosing either the risky, high-debt policy or the safe, low-debt policy, the principal minimizes its exposure: \( \phi_P = \phi_0 \). Yet, the principal may end up choosing \( \phi_P > \phi_0 \) in order to incentivize the agent to choose the safe policy.

(ii) **Two-way insurance:** In the optimal contract, the countries always maximize their cross-exposure in the solidarity and contagion regions: \( \phi_i = \phi \). By contrast, the impact on country welfare of an increase in spillovers in the PSI region is ambiguous.

**Remark:** We have focused on the affected country’s manipulation of the collateral damage. In practice, the borrower may also want to manipulate the collateral damage (for instance through a military program) by increasing it so as to benefit from the perception or the reality of implicit support.

\[24\] This happens for instance if \( R \) lies just above \( R^*(\phi_0) \). By minimizing exposure, the principal then induces the risky borrowing strategy. By increasing \( \phi_P \) a bit above \( \phi_0 \), the principal induces the safe strategy and incurs expected loss slightly higher than \( (1 - \alpha)\phi_0 \) instead of \( \phi_0 \).
5 Debt monetization: soft vs. hard default

5.1 Adding a possibility of currency debasement

Countries with an independent currency and a nominal debt labelled in their domestic currency can always resort to devaluation to escape formal (hard) default. This devaluation option is part of the reason why the US, the UK and Japan’s debts currently enjoy high ratings despite levels of public debt comparable to, or exceeding Eurozone levels. This section takes a first stab at looking at the implications of the possibility of implementing a “soft default” by debasing the currency. We make a series of simplifying assumptions in order to obtain some interesting insights on the relevant tradeoffs.

In the tradition of Calvo (1988) and Barro-Gordon (1983 a,b) we assume that a country with its own currency can choose a parameter \( \theta \leq 1 \) or equivalently a debasement factor \( 1 - \theta \). The value of nominal debt \( d \) is then \( \theta d \). The country then incurs cost \( C(\theta) \) where \( C(1) = 0, C' < 0 \) and \( C'' > 0 \). The cost \( C \) of debasing the currency is a reduced form for a variety of costs: costs of devaluation or inflation (reduced monetary balances, distorted relative prices, redistributive consequences, increase in price of imports) or reputation costs (say, higher borrowing costs in the future; note that in this case we assume that a stronger devaluation increases the reputation cost). Conversely, \( C(\theta) \) also embodies the country’s benefits from currency debasement (competitive devaluation). For convenience, we will assume that \( C(\theta) = -\log \theta \) for \( \theta \in [0, 1] \).

We make two more minor changes relative to the basic framework. First, we assume that the country is risk averse, with utility at date 2 from assumption \( c: u(c) = \log c \). The concavity of the utility function will make the devaluation decision state-contingent, with a higher temptation to devalue for low income realizations. Second, we assume that the income in the bad state of nature is positive, so as to allow for the possibility of devaluation to escape default in that state; so the country receives \( y^G \) with probability \( \alpha \) and \( y^B \) with probability \( 1 - \alpha \) where \( y^G > y^B > 0 \).

We first consider the case of a small country that can either have its own currency (and therefore choose \( \theta \) ex post) or become part of a strong currency union (which guarantees that \( \theta \equiv 1 \); this is indeed the case in this model if other countries in the union have low debts)).

(i) Ex post behavior:

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25 Consider a currency union among \( n \) countries with debts \( d_i \) and realized income \( y_i, i = 1, \ldots, n \). Suppose that the monetary authority maximizes the sum of the welfares \( \Sigma_{i=1}^n \log (y_i - \theta d_i) + \log \theta \). Then \( \theta = 1 \) is optimal provided that \( 1 \geq \left[ \Sigma_{i=1}^n \left[ \delta_i / (1 - \delta_i) \right] \right] / n \) where \( \delta_i \equiv d_i / y_i \) is the debt over GDP ratio. If only one country is highly indebted while others are not, then collectively the union prefers to let the former country default rather than inflate and hurt all countries.
Suppose that the country ex post has income $y$ and debt $d$.

**Currency union**

In a currency area, the country cannot devalue. So its decision boils down to:

$$\max \left\{ u(y - d), u(y) - \Phi_A \right\}.$$ 

Using the functional form for $u$, the country defaults if and only if:

$$d > k_1 y$$

where $k_1 \equiv \frac{\exp(\Phi_A) - 1}{\exp(\Phi_A)}$. We assume that $\Phi_A$ is “not too small” so that $k_1 > 1/2$.

**Own currency**

The country solves:

$$\max \left\{ \max_{\{\theta \leq 1\}} \left\{ u(y - \theta d) - C(\theta), \ u(y) - \Phi_A \right\} \right\}.$$ 

When not defaulting, $\theta$ maximizes

$$\log(y - \theta d) + \log \theta$$

and so

$$\theta = \begin{cases} 
1 & \text{if } d \leq \frac{y}{2} \\
\frac{y}{2d} & \text{if } d \geq \frac{y}{2} 
\end{cases}$$

The country defaults if and only if

$$d > k_2 y,$$

where $k_2 \equiv \frac{\exp(\Phi_A)}{4} > k_1$.

The country’s ex-post behavior is summarized in Figure 4.

The benefit of a currency union for the country is an increased commitment power ($\theta = 1$ for $d \leq k_1 y$), and the cost is a loss in flexibility (the possibility of devaluation when $k_1 y < d \leq k_2 y$), leading to more frequent default. Put simply, *a currency union precludes soft default and generates more hard default.*

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26If $k_1 \leq 1/2$, the outcome is the same under a currency union and own currency.
Remark: While highly stylized, this modelling captures many of the desirable features of the Mundell-Fleming analysis. For instance, it predicts that the spread of a risky country falls when joining a hard currency union, i.e. a union involving healthy countries. Similarly, countries facing correlated shocks (either naturally or by design through the creation of a fiscal union) are better suited for a currency union. Finally, the existence of a currency union creates a new incentive for debt surveillance, namely the avoidance of the pressure to inflate to rescue highly indebted countries. While developing these themes lies outside the limited scope of this paper, their compatibility with the modelling strategy is comforting.

5.2 No income uncertainty

**Proposition 5 (optimality of currency union under resource certainty).** In the absence of income uncertainty, it is optimal for the country to join the currency union, and strictly so if \( R_y > 2 \).

**Proof:** Under a currency union, the country can borrow at any level \( b \leq k_1y \). Its utility is then
\[
Rb + \log (y - b).
\]
Under own currency, it is suboptimal to choose \( d \) in the devaluation region \( d \in [y/2, k_2y] \); its utility is then
\[
R[\theta^d(d)d] + \log (y - \theta(d)d) + \log \theta(d),
\]
where \( \theta^d(d) \) is the market’s anticipation of \( \theta \) at the date of lending and \( \theta(d) \) the actual choice; of course in equilibrium \( \theta(d) = \theta^d(d) \). Because \( \theta(d)d = \frac{y}{2} \) in the devaluation...
region, this utility can be rewritten as:

$$R \left( \frac{y}{2} \right) + \log \left( \frac{y}{2} \right) + \log \left( \frac{y}{2d} \right) < R \left( \frac{y}{2} \right) + \log \left( \frac{y}{2} \right).$$

So the country is strictly better off with \( b = y/2 \) which rules out devaluation costs.

Finally, the condition \( Ry > 2 \) means that the country wants to borrow more than \( y/2 \) if it can commit not to devalue.

Intuitively, under rational expectations borrowing reflects the expected debasement and so the country can do better by committing not to debase, and thus avoiding the cost of inflation. This commitment is achieved through the currency union.

5.3 The commitment/flexibility trade off under income uncertainty

Suppose now that

$$y = \begin{cases} y_G & \text{with probability } \alpha \\ y_B & \text{with probability } 1 - \alpha \end{cases}$$

where \( y_G > y_B > 0 \).

Currency union (\( \theta = 1 \))

Without the ability to devaluate the currency, the agent chooses between a risky and a safe policy:

**Risky policy.** The agent defaults in the bad state of nature, i.e.,

$$k_1 y_G \geq d > k_1 y_B.$$

The expected utility is then

$$U'_A(d) \equiv Rd + \alpha \log (y_G - d) + (1 - \alpha) \left[ \log y_B - \Phi_A \right].$$

The agent’s utility is then

$$U'_A = \begin{cases} R [ak_1 y_G] + \alpha \log (1 - k_1) y_G + (1 - \alpha) \left[ \log y_B - \Phi_A \right] & \text{if } Ry_G \geq 1/(1 - k_1) \\ a(Ry_G - 1) - a \log R + (1 - \alpha) \left[ \log y_B - \Phi_A \right] & \text{otherwise}. \end{cases}$$

**Safe policy.** Default never occurs provided that \( d \leq k_1 y_B \). The agent’s utility is then:

$$U'_A(d) \equiv Rd + \alpha \log (y_G - d) + (1 - \alpha) \log (y_B - d).$$
Then \( d = \min \{ d^*, k_1 y_B \} \) where
\[
R \equiv \frac{\alpha}{y_G - d^*} + \frac{1 - \alpha}{y_B - d^*};
\]
because the agent is indifferent between repaying and defaulting in the bad state, the agent’s utility is
\[
U_A^s \equiv R[k_1 y_B] + \alpha \log (y_G - k_1 y_B) + (1 - \alpha)(\log y_B - \Phi_A)
\]
if \( k_1 y_B \leq d^* \). If \( k_1 y_B > d^* \), then the agent’s utility is
\[
U_A^s \equiv Rd^* + \alpha \log (y_G - d^*) + (1 - \alpha) \log (y_B - d^*).
\]

**Own currency.**

Suppose now that the agent can devalue and thereby escape default in the bad state of nature:
\[
\theta_B < \theta_G \equiv 1,
\]
or, equivalently
\[
k_2 y_B \geq d \geq \frac{y_B}{2} \quad \text{and} \quad d \leq \frac{y_G}{2}
\]
The agent’s utility is then
\[
V_A(d) = R\left[\alpha d + (1 - \alpha) \frac{y_B}{2}\right] + \alpha \log (y_G - d) + (1 - \alpha) \left[\log \frac{y_B}{2} + \log \frac{y_B}{2d}\right]
\]
Intuitively, the risky strategy is unappealing when the cost of default is high. And the safe strategy under a currency union seriously limits the amount that can be borrowed, which is especially costly if the bad state is unlikely. The next proposition, proved in Appendix 4, indeed shows that keeping one’s own currency may be desirable under resource uncertainty.

**Proposition 6 (trade-off between flexibility and commitment).** While the commitment effect always makes belonging to a currency union optimal in the absence of resource uncertainty, resource uncertainty may make it desirable for the country to keep its own currency.

\[\text{If } \theta_G < 1, \text{ then } d \text{ could be reduced so as to achieve } \theta_G = 1. \text{ To see this suppose that } k_2 y_B \geq d > \frac{y_G}{2} \text{ so that } \theta_B < \theta_G < 1. \text{ Then the agent’s utility is } \alpha U_G(d) + (1 - \alpha) U_B(d) \text{ where}
\]
\[
U_i(d) \equiv R \frac{y_i}{2} + \log \left( \frac{y_i}{2} \right) + \log \left( \frac{y_i}{2d} \right) \quad \text{with} \quad i \in \{B, G\},
\]
using \( \theta_i(d) \equiv y_i/2 \). Because \( U_i'(d) < 0 \) in this region, reducing \( d \) at the margin is desirable.
5.4 Bailouts and debt monetization

We now embed the model of Section 5.3 in a principal-agent context similar to that of Section 2. Our main objective is to analyze how bailout incentives are altered by the absence of a currency union.

We assume that the principal is risk neutral (it is a diversified group of healthy countries, say). We need to specify how spillovers are affected by a currency debasement. We posit that

\[ \phi_p = \begin{cases} 
\phi_0 + d_p \text{ if hard default} \\
(1 - \theta)d_p \text{ if soft default,}
\end{cases} \]

where \( d_p \) is the agent’s liability with the principal. That is, soft default creates a counterparty loss on the principal’s claim on the agent of \( (1 - \theta)d_p \) as opposed to \( d_p \) for hard default.

The timing is the same as in Figure 1 except that at the ultimate stage at date 2, the agent has a third option, which is to debase its currency and debt. As earlier, the principal can forgive some of his debt and also bring some support toward the reimbursement of the agent’s private debt; the principal however cannot contract on the value of \( \theta \) as monetary policy is the agent’s privilege.

(i) Bailout incentives under a currency union.

The treatment of bailouts under a currency union is the same as in Section 2 although it is adapted to accommodate the agent’s risk aversion. For a given income \( y \in \{y_B, y_G\} \) and private debt \( d_M \), the agent reimburses the debt if and only if \( d_M \leq k_1y \) in the absence of a bailout. And so, if \( d_M \leq k_1y \) the principal has no incentive to offer support to help the agent reimburse his debt. At most will the principal forgo some of his own claim if \( d_M + d_p > k_1y \), so as to bring total debt back to \( k_1y \).

By contrast, if \( d_M > k_1y \), the principal needs to rescue the agent if he wants to prevent default. The level of debt support must yield a net market debt burden \( \hat{d}_M \) satisfying:

\[ u(y - \hat{d}_M) \geq u(y) - \Phi_A \]

or

\[ \hat{d}_M \leq k_1y. \]

28 There may be other costs than a loss on his claim for the principal. For example, the principal might be affected by the agent’s competitive devaluation. The analysis is not affected if these costs decrease in \( \theta \). For example, if \( \phi_P = (1 - \theta)a \) for some \( a > 0 \), then \( \phi_P = (a + d_p)(1 - \theta) \) and similar derivations as those below can be performed.

29 The extension to the possibility of contracting on \( \theta \) is straightforward.
The principal bails out the agent if and only if $-\phi_0 - d_P \leq -(d - k_1 y)$, or

$$d_M - k_1 y \leq \phi_0.$$  \hspace{1cm} (11)

Note that the principal’s claim $d_P$ does not enter (11) since it is wiped out whether or not the principal rescues the agent.

(ii) Bailout incentives under own currency.

When the agent has his own currency, the principal can bring support denominated either in the agent’s currency or in the principal’s currency. Indeed, one of the interesting insights of this section relates to the currency choice for the support.

First, we show that, due a “leakage effect”, the principal has no incentive to forgive his liability or cover some of the private debt whenever the agent is unwilling to engage in hard default.

**Lemma 2.** Suppose that $d \leq k_2 y$. Then the principal neither forgives any liability nor offers to cover the reimbursement of some private debt. More generally, whenever private debt satisfies $d_M \leq k_2 y$, the principal never offers any contribution toward private debt relief.

**Proof:** The only reason why the principal might want to offer relief is to reduce currency debasement. Suppose that the principal reduces debt from $d \leq k_2 y$ to $\hat{d} = d - s$ such that $\theta(\hat{d})\hat{d} = y/2$ (there is no point reducing debt in the no-debasement region $\theta = 1$). The principal’s claim is then, assuming $s \leq d_P$ (otherwise the principal no longer has a claim):

$$\theta(d - s)(d_P - s) = \frac{y(d_P - s)}{2(d - s)},$$

which is decreasing in $s$. In effect, by increasing $s$, the principal increases the reimbursement rate not only for his own debt $d_P$, but also for the entire debt $d$, a leakage effect.

Suppose now that in the absence of support, the agent would default: $d > k_2 y$.

**Support toward private debt relief denominated in the agent’s currency.**

Suppose $d_M > k_2 y$. In this case, the principal must not only forgive his own claim $d_P$, but also offer support toward private debt reimbursement. Let $s$ now denote this support. The principal must ensure that the new net market debt for the agent is $\hat{d}_M = k_2 y$ (from Lemma 2, there is no point bringing further support). And so

$$s = d_M - k_2 y.$$

The reimbursement rate is then given by $\theta k_2 y = y/2$, or $\theta = 1/2k_2$. Thus, the
principal’s cost of the support, expressed in the principal’s currency, is \[ \frac{1}{2k_2} [d_M - k_2 y], \] where \( d_M \equiv d - d_p \).

The principal is willing to bail out the agent if and only if

\[
\left[ \frac{1}{2k_2} \right] [d_M - k_2 y] - d_p \geq -\phi_0 - d_p
\]

or,

\[
d_M - k_2 y \leq 2k_2 \phi_0. \tag{12}\]

Letting

\[
\hat{s} \equiv \frac{1}{2} \left[ \frac{d_M}{k_2 y} - 1 \right] y,
\]

then the principal is willing to bail out the agent if and only if

\[
\hat{s} \leq \phi_0.
\]

Comparing (11) and (12), we observe that:

(a) The private-debt ceiling under which there is no need for a bailout is smaller under a currency union (\( k_1 y \) instead of \( k_2 y \)). This corresponds to our earlier observation that a country is more prone to hard default under a currency union because it loses the ability to monetize its debt.

(b) The principal is willing to bail out the agent for higher levels of overindebtedness \( (d_M - k_2 y \leq 2k_2 \phi_0, \text{ with } k_2 > k_1 > 1/2, \text{ instead of } d_M - k_1 y \leq \phi_0) \) when the agent can monetize the debt. The reason for this is that the private sector does not fully free ride on the bailout, unlike in the case of a currency union: Devaluation will still eat some of the private debt away. Thus, a devaluation creates some automatic private sector involvement, which, ceteris paribus, increases the principal’s incentive to come to the agent’s rescue.

Support toward private debt relief denominated in the principal’s currency.

Suppose now that the principal forgives debt \( d_p \) and further offers a support \( s \) in the principal’s currency toward private debt relief. The agent refrains from defaulting if

\[
\max_{\theta} \left\{ \log (y + s - \theta d_M) + \log \theta \right\} \geq \log y - \Phi_A
\]

or

\[
d_M \leq k_2 \frac{(y + s)^2}{y}.
\]

The optimal support \( s^* \) makes the agent indifferent between defaulting and monetizing.
the debt:

\[ s^* = \left[ \sqrt{\frac{d_M}{k_2 y}} - 1 \right] y. \]

Finally, the principal is willing to bail out the agent if and only if:

\[ s^* \leq \phi_0. \]

A key observation is that

\[ s^* < \bar{s} \]

given that \( d_M > k_2 y \). It is cheaper to offer support in the principal’s currency. The reason for this is that such a denomination increases the agent’s incentive to monetize the debt so as to “make the most” of the principal’s support. In turn, the increased monetization increases private sector involvement and thereby burden sharing between the principal and the private sector.

We summarize these results in the following proposition.

**Proposition 7 (bailouts and debt monetization).** For given country debt \( d \) and income \( y \), the principal bails the agent out if and only if

- \( d_M - k_1 y \leq \phi_0 \) under a currency union,
- \( d_M - k_2 y \leq 2k_2 \phi_0 \) under own currency, and support denominated in the agent’s currency,
- \( \left[ \sqrt{\frac{d_M}{k_2 y}} - 1 \right] y \leq \phi_0 \) under own currency and support denominated in the principal’s currency,

where \( k_2 > k_1 \) and \( 2k_2 > 1 \). Thus

(i) The principal intervenes for lower levels of debt under a currency union as soft default is then not an option.

(ii) Due to (involuntary) private sector involvement under a soft default, the principal is willing to rescue for larger shortfalls of required income \( k_i y \) relative to market debt \( d_M \).

(iii) When the agent has his own currency, the principal elects to bring support in the principal’s rather than the agent’s currency, so as to increase the magnitude of private sector involvement:

\[ s^* = \left[ \sqrt{\frac{d_M}{k_2 y}} - 1 \right] y < \bar{s} = \frac{1}{2} \left[ \frac{d_M}{k_2 y} - 1 \right] y \]
6 Conclusion

Bailouts are driven by the fear that spillovers from the distressed country’s default negatively affect the rescuer. This paper’s first contribution was to provide formal content to the intuitive notion that collateral damages of a country’s default are de facto collateral for the country.

The paper’s second contribution was to unveil the conditions under which joint-and-several liability may emerge. Standard liquidity provision or risk sharing models presume that accord is reached behind the veil of ignorance. Once the veil of ignorance is lifted (as is currently the case in the Eurozone), healthy countries have no incentive to accept obligations beyond the implicit ones that arise from spillover externalities. Put differently, it is not in the self-interest of healthy countries to accept joint-and-several liability and assume the concomitant risk of a domino effect, even though they realize that they will be hurt by a default and thus will want to ex post offer some solidarity in order to prevent spillovers; an ex-ante transfer from distressed countries to healthy ones to compensate them for, and make them accept the future liability is ruled out as it would just add to the distressed countries’ indebtedness and thus the compensation would be in funny money.

Third, this paper endogenized spillovers and provided conditions under which a country deliberately chooses to be exposed to another country’s default. Fourth, it obtained a few first insights on the interaction between debt monetization and solidarity. A currency union precludes soft default and generates more hard default. The choice between a currency union and an own currency exhibits a familiar trade-off between commitment and flexibility. A country’s having its own currency creates some automatic private sector involvement and makes other countries more willing to bail it out for a given shortfall between debt and willingness to pay; furthermore, rescuing countries optimally denominate their support in foreign rather than domestic currency.

On the theoretical front, the paper is only a first attempt at understanding the fundamentals of country solidarity, whether reluctantly provided or more pro-actively contracted for. There are many interesting alleys for future research in the area. For instance, one might extend the analysis of Section 3 to consider extended solidarity; first losses could be covered by an inner circle of countries within a solidarity area and macro shocks within this area might be partly insured by an outer solidarity area (rest of the world, IMF). Another fascinating topic for future analysis would result from asymmetries of information about collateral damages and the concomitant posturing behaviors in the international community.

On the policy front, one should investigate the likelihood of emergence of alternative solidarity zones. Recall for instance the puzzle stated in the introduction: both
the bailout contributions and the policy debate about Eurobonds mostly concern a very limited insurance pool, namely the Eurozone, while basic principles of insurance economics would call for a much broader solidarity area. Although the following suggestions are no substitute for a careful analysis, the model arguably sheds light on the puzzle. First, the monetary union has drastically increased the degree of financial integration among Eurozone countries. Financial integration implies increased spillovers from default. And indeed, France and Germany have much larger exposure to, say, Italy than the UK, let alone the US and China. Second, the establishment of the monetary union in large part was driven by a political project. In this sense, it reflects the presence of strong spillover effects; and abandoning the Euro, or letting some Eurozone countries default would have a substantial symbolic impact. These two factors are likely explanations for the otherwise peculiar risk-sharing arrangement.

Similarly, one may build on this paper to investigate the impact of fiscal unions. A fiscal union creates an automatic risk sharing mechanism and thus correlates income realizations; it further generates some extent of joint liability through the issuance of federal debt. And, as is well-known (and consistent with the modelling in Section 5), the increase in correlation facilitates the conduct of monetary policy as well. Nonetheless, states still enjoy some degree of subsidiarity; the implications of fiscal federalism for solidarity are definitely worth investigating.

Finally, the paper’s modeling and implications focused on its initial, international finance motivation. Its potential scope of applications however is much broader. A corporation may choose to guarantee or not a key supplier’s debts by integrating it as its division or by keeping it independent and not promising to cover its liabilities. Banks may enter various kinds of contractual agreements, including credit lines, which imply varying degrees of solidarity. Individuals choose between giving a helping hand to members of their family (children) or friends facing financial straits and more formally standing surety for them, thereby facilitating their access to credit or housing. Integrating the specificities of these other contexts would be of much interest.

I leave these and the many related topics on solidarity to future research.

\[ \Phi_A + \phi_P \]
References


Appendix 1: Proof of Corollary 3

Suppose first that \( R < R^* \). The principal’s participation constraint requires that \(-(1 - \alpha) d^B \geq U_p = -(1 - \alpha) \phi_p\) and so \( d^B \leq \phi_p < b^* = \alpha \Phi_A + (1 - \alpha) \phi_p \), where \( b^* \) is the optimal borrowing level as given by Proposition 2. The agent underborrows relative to the optimum.

Suppose next that \( R \geq R^* \). The principal’s participation constraint is looser: \( d^B \leq \phi_p / (1 - \alpha) \), while optimal borrowing is now \( b^* = \alpha \Phi_A + \phi_p \). Requiring \( d^B = b^* \) yields a necessary condition for Eurobonds to implement the optimum: \( (1 - \alpha) \Phi_A \leq \phi_p \), which is equivalent to \( d^B = b^* \geq \Phi_A \). So let us assume that \( (1 - \alpha) \Phi_A \leq \phi_p \). Then Eurobonds do implement the optimum; for, the principal ex post offers financial support \( r_p \equiv d^B - \Phi_A \) to help the agent reimburse its debt in the good state: \(-r_p \geq \max \{-d^B, -\Phi_p - \phi_p\}\) where the first option corresponds to honoring the agent’s debt, and the second to not honoring any debt. That \(-r_p \geq -d^B\) follows from \( d^B \geq r_p \). Similarly \(-r_p \geq -\Phi_p - \phi_p\). Finally, the principal’s participation constraint is satisfied: \(-\alpha r_p - (1 - \alpha) d^B = -d^B + \alpha \Phi_A = -\phi_p\).

Appendix 2: Proof of Propositions 3 and 4

To solve Program (II), let us first note that at its optimum there is no point punishing countries that announce truthfully that they both are healthy:

\( \hat{\Phi}_2 = 0 \).

Another preliminary result that comes from inspecting this program is that for a given \( x_1 \), i.e., for a given total punishment \( \hat{\Phi}_1^y + \hat{\Phi}_1^0 = x_1 \hat{\Phi} = 2\hat{\Phi}_1 \) when only one of the countries is healthy, it is optimal to minimize \( \hat{\Phi}_1^y \) as this relaxes constraints (10) and (11b) without altering the rest. So we can without loss of generality assume that

\[ \hat{\Phi}_1^y \equiv \Phi_1^y(x_1) \equiv \begin{cases} x_1 \phi & \text{for } x_1 \leq 1 \\ \phi + (x_1 - 1)\Phi & \text{for } x_1 \geq 1. \end{cases} \]

(a) Let us first assume that (11b) is binding, and so ignore (11a). We further ignore condition (8) and verify ex post that it is indeed verified. From (7), we can also ignore (10), which is implied by (11b). It is then clear that (9) and (11b) must be binding, otherwise \( d_1 \) or \( d_2 \) or both would tend to infinity, which would violate some of the constraints, e.g., (8). And so

\[ d_1 = \frac{\hat{\Phi}_0 - \Phi_1^y(x_1)}{2} \]
and

\[ d_2 = \frac{\hat{\Phi}_0 - \Phi_1^y(x_1)}{2} + \frac{x_1 \hat{\Phi}}{2} \]

The maximand can then be written as:

\[
U^* = \max_{\{x_1, \hat{\Phi}_0\}} \left\{ (R - 1) \left[ \left( p_1 + p_2 \right) \frac{\hat{\Phi}_0 - \Phi_1^y(x_1)}{2} + p_2 \frac{x_1 \hat{\Phi}}{2} \right] - p_1 \frac{x_1 \hat{\Phi}}{2} - p_0 \hat{\Phi}_0 \right\}
\] (12)

Either the optimal punishment when both are distressed is equal to 0 and then the maximand is also equal to 0. Or

\[ (R - 1) \left( \frac{p_1 + p_2}{2} \right) > p_0 \] (13)

and then the optimal punishment when both are distressed is maximal:

\[ \hat{\Phi}_0 = \hat{\Phi} \]

Maximizing with respect to \( x_1 \) yields:

\[ x_1 = 0 \text{ if } (R - 1) \left[ \Phi - \frac{p_1 \phi}{p_2} \right] < \frac{p_1 \hat{\Phi}}{p_2} \]

\[ x_1 = 1 \text{ if } (R - 1) \left[ \Phi - \frac{p_1 \phi}{p_2} \right] > \frac{p_1 \hat{\Phi}}{p_2} > (R - 1) \left[ \phi - \frac{p_1 \phi}{p_2} \right] \]

\[ x_1 = 2 \text{ if } (R - 1) \left[ \phi - \frac{p_1 \phi}{p_2} \right] > \frac{p_1 \hat{\Phi}}{p_2} \] (14)

Finally, we verify that constraints (8) and (11b) are satisfied. They can be rewritten as:

\[ \frac{\hat{\Phi} - \Phi_1^y(x_1)}{2} + \frac{x_1 \hat{\Phi}}{2} \leq \hat{\Phi} \text{ for (8)} \]

and

\[ \hat{\Phi} - \Phi_1^y(x_1) + x_1 \hat{\Phi} \leq 2\hat{\Phi} \text{ for (11b)} \]

So we need only to check that

\[ x_1 \hat{\Phi} - \Phi_1^y(x_1) \leq \hat{\Phi} \text{ for all } x_1, \]

which is straightforward (the left-hand side is an increasing function of \( x_1 \) and takes value \( \hat{\Phi} \) for \( x_1 = 2 \)).

*Implementation*
When \( x^*_1 = 0 \), then 
\[
d_1 = d_2 = \frac{\Phi}{2}.
\]
The intact country takes on the entire debt of the distressed country (reimburses \( 2d_1 = 2d_2 \)).

Joint-and-several liability is required as 
\[
d_1 = \frac{\Phi + \phi}{2} > \phi,
\]
and so there is no spontaneous bailout in state 1. Finally, note that, again in state of nature 1, the intact country does not want to default and receive \( -\Phi = -2d_1 \).

When \( x^*_1 = 1 \), then 
\[
d_1 = \frac{\Phi}{2} \quad \text{and} \quad d_2 = \Phi + \frac{\phi}{2}.
\]

In state of nature 1, the intact country owes \( \Phi \) and suffers spillover \( \phi \). It therefore cannot improve its welfare by defaulting itself (it would then have utility \( -(\Phi + \phi) \)).

Would the intact country want to rescue the distressed country by paying \( 2d_2 \)? Its utility would then be \( -2d_2 = -2\Phi - \phi < -\Phi - \phi \), and so there is indeed no bailout (\( x_1 = 1 \)).

When \( x^*_1 = 2 \), then 
\[
d_1 = 0 \quad \text{and} \quad d_2 = \hat{\Phi} = \Phi + \phi.
\]

Again, in state 1, the intact country does not want to bail out the distressed country as 
\[
-2d_2 = -2\hat{\Phi} < -\hat{\Phi}.
\]

(b) Second, let us investigate the possibility that (11a) is binding. We can then ignore (8), which is implied by (9) and (11a). Condition (9) must be binding, otherwise \( d_2 \), and thereby the maximand, could be increased:
\[
d_2 = d_1 + \hat{\Phi}_1.
\]

Similarly (11a) must be satisfied with equality. Otherwise \( \hat{\Phi}_0 \) could be reduced (if \( \hat{\Phi}_0 = 0 \), the maximand is equal to 0 anyway):
\[
d_1 + \Phi_1 = \hat{\Phi}_0.
\]

Substituting \( d_1 \) and \( d_2 \), we can rewrite the program as
\[
\max \left\{ \left[ (R - 1)(p_2 + p_1) - p_0 \right] \Phi_0 - p_1 R \Phi_1 \right\}
\]
subject to constraint (10), rewritten as:

\[ 2\hat{\Phi}_0 \leq \hat{\Phi} + \hat{\Phi}_1. \]  

(10')

Assume \((R - 1)(p_2 + p_1) - p_0 > 0\) (otherwise the value of the program is 0). Then (10') must be binding, and so the maximand becomes, up to a constant,

\[
\max_{\{x_1\}} \left\{ \left[ (R - 1)(p_2 + p_1) - p_0 \right] \frac{\Phi_0(x_1)}{2} - p_1 Rx_1 \hat{\Phi} \right\}.
\]

Again the optimal \(x_1\) is either 1 or 2, if it is not equal to 0. For \(x_1 = 1\), then \(d_1 = \Phi/2\) and \(d_2 = \Phi + (\phi/2)\), which is the same solution as in case (a).

For \(x_1 = 2\), then \(d_1 = 0\) and \(d_2 = \hat{\Phi}\), again as in case (a). So the solutions considered in case (a) cover the entire range of solutions. ■

**Endogenous spillovers**

From (12), and assuming that (13) (which is independent of \(\phi\)) is satisfied, the change in the maximand \(U^*\) is given by:

\[
\frac{\partial U^*}{\partial \phi} = \begin{cases} 
(R - 1) \left( \frac{p_1 + p_2}{2} \right) - p_0 & \text{in the solidarity region} \\
(R - 1) \left( \frac{p_2}{2} \right) - \frac{p_1}{2} - p_0 & \text{in the PSI region} \\
(R - 1) \left( p_2 \right) - p_1 - p_0 & \text{in the contagion region}
\end{cases}
\]

From (13), \(\frac{\partial U^*}{\partial \phi} > 0\) in the solidarity region. In that region, \(x_1^* = 0\), and so an increase in \(\phi\) has no impact on the two constraints (9') and (11b'). Hence increasing \(\phi\) is optimal whenever borrowing is.

The impact of an increase in \(\phi\) is ambiguous in the PSI and contagion regions. Because \(x_1^* > 0\) in these regions, increasing \(\phi\) is costly when \(k = 1\) or 2, while it has no impact on constraint (11b') and relaxes constraint (9').

For example, in the PSI region,

\[
\frac{\partial U^*}{\partial \phi} \begin{cases} 
> 0 & \text{for } p_0 \text{ small} \\
< 0 & \text{for } p_0 \text{ large (close to } R - 1 \div R + 1\text{), } r \text{ small and } R \text{ close to } 1 + \ell.
\end{cases}
\]

In the contagion region, \(\partial U^*/\partial \phi < 0\) requires that \(p_2 R < 1\). Furthermore, to be in the contagion region, it must be the case that investment be desirable:

\[
R \geq \frac{1 + p_0}{1 - p_0}
\]

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and that
\[ R\left(1 - \frac{\ell}{r}\right) \geq 1 + \ell. \]
Because \( r \leq 1 \), the latter condition requires that
\[ R\left(2p_2 + p_0 - 1\right) \geq 1 - p_0. \]
Putting the three conditions together, \( \partial U^* / \partial \phi < 0 \) requires that
\[ 1 < \max \left\{ \frac{1 + p_0}{1 - p_0}, \frac{1 - p_0}{2p_2 + p_0 - 1}\right\} < \frac{1}{p_2}, \]
which can be checked is impossible. ■

Appendix 3: Asymmetric information within the official sector

We have assumed homogeneous information between the official sector and debtor countries. More generally the official sector could receive a noisy signal regarding the debtor country’s willingness to pay. This Appendix checks the robustness of our results in the polar case of complete absence of a signal: The principal of Section 2 has no better information than the market about the debtor country’s willingness to pay. The timing is the same as in Figure 1 except that only \( A \) observes the realized income.

Laissez-faire.

Suppose first that the agent borrows from the market without contracting with the principal. Suppose that the agent has borrowed \( d_M \leq \Phi_A \) in the international financial market. The principal is willing to cover this debt as long as
\[ d_M \leq (1 - \alpha)\phi_P \]
because failure to guarantee the country’s liabilities results in a default with probability \( 1 - \alpha \).

Similarly, suppose that \( d_M \in (\Phi_A, \Phi_A + \phi_P] \). Then the principal is willing to cover \( d_M - \Phi_A \) (conditionally on the agent’s repaying its private liabilities) in order to bring probability of default from 1 to \( (1 - \alpha) \): the principal obtains \(-\phi_P\) (certain default) by not intervening and \(-\alpha(d_M - \Phi_A) - (1 - \alpha)\phi_P\) by offering a conditional guarantee.

As in section 2.2 the agent optimally chooses between:
- a safe debt policy at level \( d_M = (1 - \alpha)\phi_P \), which never leads to default, and
✓ a risky debt policy at level $\Phi_A + \phi_P$, which results in default with probability $1 - \alpha$.

The low-debt policy is less attractive than under symmetric information between $P$ and $A$, while the attractiveness of the high-debt policy is unchanged. The agent’s utility is thus

$$U_A^{**} = \max \left\{ R(1 - \alpha)\phi_P + \alpha y; \right.$$ 

$$Ra \left( \Phi_A + \phi_P \right) + \alpha(y - \Phi_A) - (1 - \alpha)\Phi_A \right\}$$

and so the agent chooses the risky policy if and only if

$$R \geq R^{**} = \frac{\Phi_A}{\alpha \Phi_A - (1 - 2\alpha)\phi_P}$$

(with the convention that $R^{**} = +\infty$ if $\alpha \Phi_A \leq (1 - 2\alpha)\phi_P$). Note that $R^* \geq R^{**}$ if and only if $(1 - \alpha)\Phi_A - (2\alpha - 1)\phi_P \geq 0$.

The principal’s utility is

$$U_P^{**} = \begin{cases} -\phi_P & \text{if } R \geq R^{**} \\ -(1 - \alpha)\phi_P & \text{if } R < R^{**} \end{cases}$$

Optimal contract.

Like in the case of laissez faire, the analysis is remarkably similar to that of symmetric information between the principal and the agent. We obtain an upper bound on the agent’s expected utility and show that this upper bound can indeed be implemented.

The twist with respect to the analysis of Section 2.3 is that some default must be induced in the bad state in order to induce repayment in the good state (Gale-Hellwig 1985; Townsend 1979). Let $x$ denote the probability of default in the bad state. Then incentive compatibility requires that

$$y - d^G \geq y - x\Phi_A \quad \text{or} \quad d^G \leq x\Phi_A.$$

The modified program is then (with the notation of Section 2.3)

$$U_A = Rb + \alpha(y - d^G) + (1 - \alpha)(-d^B - x\Phi_A)$$

where

$$b = b_M + b_P,$$

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the participation constraints are satisfied:

$$- b_P + \alpha d_P^G + (1 - \alpha) \left( d_B^P - x\phi_P \right) \geq U_P^{**}$$

$$- b_M + \alpha d_M^G + (1 - \alpha) d_M^B \geq 0$$

and the incentive constraints are satisfied:

$$d^G \leq x\Phi_A$$

$$d^B \leq 0.$$

We ignore the principal’s incentive constraints (they will be satisfied in the implementation).

Substituting, one obtains:

$$U_A \leq \max \{ d^G \leq x\Phi_A, d^B \leq 0 \} \left\{ R \left[ \alpha d^G + (1 - \alpha) \left( d^B - x\phi_P \right) - U_P^{**} \right] 
+ \alpha (y - d^G) + (1 - \alpha) \left( -d^B - x\Phi_A \right) \right\}.$$

So, provided that $R \geq 1$, $d^B = 0$ and $d^G = x\Phi_A$. Furthermore, letting

$$R^{**} \equiv \frac{\Phi_A}{\alpha\Phi_A - (1 - \alpha)\phi_P} > R^{**},$$

$$x^* = \begin{cases} 0 & \text{if } R < R^{***} \\ 1 & \text{if } R \geq R^{***}. \end{cases}$$

The implementation of the optimal allocation is described in the following proposition.

**Proposition 8 (asymmetric information).** When the principal does not observe the agent’s income realization and $R > 1$,

(i) **Laissez faire.** The agent picks a high-debt policy ($d_M = \Phi_A + \phi_P$) when $R \geq R^{**}$ and a low-debt one ($d_M = (1 - \alpha)\phi_P$) if $R < R^{**}$. Default occurs in the bad state under the high-debt policy. The principal’s expected utility is $U_P^{**} = -\phi_P$ if $R \geq R^{**}$ and $-(1 - \alpha)\phi_P$ if $R < R^{**}$.

(ii) **Optimal contracting.** The optimal contract between the principal and the agent can be implemented as follows:

✓ If $R < R^{***}$, the principal pays the agent $b_P = -U_P^{**}$ for not taking on any liability.
If $R \geq R^{**}$, the agent takes on liability $d_M = \Phi_A$ and defaults in the bad state. The principal enforces the debt ceiling $d_M$, transfers to the agent $b_P = \alpha \phi_P$ at date 1, and takes no debt claim in exchange.

Appendix 4: Proof of Proposition 6

To build an example in which a currency union is suboptimal, let us assume that

$$y_G \geq 2y_B$$

and

$$Ry_G \geq 2.$$  \hfill (16)

Conditions (15) and (16) imply two other conditions that will be useful for what follows:

$$R(y_G - y_B) > 1$$

and

$$R \left( \frac{y_G}{2} - y_B \right) > \log \left( \frac{2(y_G - y_B)}{y_G} \right).$$

And let us consider a sequence $(\alpha, \Phi_A)$ going to $(1, \infty)$ such that $(1 - \alpha)\Phi_A$ tends to infinity. Note that

$$\lim_{\alpha \to 1} \left\{ \max_{d} V_A(d) \right\} = \frac{Ry_G}{2} + \log \frac{y_G}{2}$$

since condition (18) implies that the constraint $d \leq y_G/2$ is binding for $\alpha$ large, and the fact that $\Phi_A$ is large implies that the constraint $k_2y_B \geq d$ is not binding.

Next, consider a currency union. $d^* \leq k_1y_B$ is equivalent to

$$\left( R - \frac{1}{y_G - y_B} \right)y_B \leq (1 - \alpha)e^{\Phi_A},$$

which is satisfied for the sequence $(\alpha, \Phi_A)$ considered. Because $R(y_G - y_B) > 1$ and $d^* \leq k_1y_B$,

$$d^* \simeq y_B$$

and

$$U_A^{\delta} \simeq Ry_B + \log (y_G - y_B) + \lim_{\alpha \to 1} \left\{ (1 - \alpha) \log \left( \frac{R - \frac{1}{y_G - y_B}}{y_G - y_B} \right) \right\}$$

$$= Ry_B + \log (y_G - y_B).$$

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Condition (18) further ensures that

\[ V_A > U_A^S \]

Finally, we need to ensure that the risky strategy is unappealing. For \( \Phi_A \) large, \( R_y G < 1 / (1 - k_1) \), and so given that \( (1 - \alpha) \Phi_A \) goes to infinity, \( U_A^r \) tends to \(-\infty\). ■