The Evolving Relationships Between Agricultural and Energy Commodity Prices: A Shifting-Mean Vector Autoregressive Analysis

by

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Abstract:

We identify the key factors responsible for the general run-up of U.S. grain prices by extending Enders and Holt's (2012) analysis to a time-varying multiple equation setting. Given that the methodology for co-breaking is in its infancy, we utilize two very different methodologies to examine the underlying reasons for shifts in grain prices. A simple VAR indicates the important effects of mean shifts in real energy prices, exchange rates, and interest rates on grain prices. We go on to develop a parametric model of structural change that allows for smoothly shifting means. In addition to the general rise in real energy prices, the introduction of ethanol as an important fuel source has contributed to the run-up in grain prices. Economic growth in emerging economies such as China, India, and Brazil are also identified as a possible contributing factor.

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1. Introduction

That primary commodity prices have, in recent years, steadily moved higher into uncharted territory is unassailable. As illustrated by the plot of the World Bank's nominal monthly food price index shown in Figure 1, there was nearly an exponential increase in the overall price of food from the late 1990s through late 2008. Despite the so-called Great Recession, the absolute high for the food price index was 223.56 in February, 2011, indicating that food prices at this point were 224-percent higher than in 2005. Prices for other primary commodities, including those for many other field crops, many livestock and livestock products, as well as various energy products have followed similar patterns in recent years.

The overall goal of this chapter is to identify the key factors responsible for the general run-up of U.S. grain prices. We do so by building on Enders and Holt's (2012) analysis of the recent run-up of sixteen different commodity prices using univariate time series methods. Instead, we use a time-varying multiple equation model to focus on interactions among the prices for oil, maize, soybeans, ethanol, and ocean freight rates over the 1985-2011 period. In Section 2, we review some of the arguments that have been put forth to explain the recent price boom. We also discuss some of the modeling strategies that have been employed to measure the proposed explanations. In Section 3 we discuss our data set and the rationale for selecting the variables to include in the analysis. Given that the methodology for co-breaking is in its infancy, we utilize two very different methodologies to measure the effects of shifts in the underlying causal variables on grain prices. In Section 4 we use a simple unrestricted vector autoregression (VAR) to estimate some of the key relationships between grain prices and a number of macroeconomic variables. The nature of the model is such that mean shifts in any one variable are allowed to change the means of all other variables. Given some of the limitations of VAR analysis, in Section 5 we discuss some of the issues involved in estimating nonlinear models of shifting means. In order to determine whether the variables are stationary, in Section 6 we report results of nonlinear unit root tests. In particular, we perform unit root and stationary tests of all of the variables by employing a new testing procedure developed by Enders and Lee (2012). The advantage of their approach is that we can readily test for a unit root in the slowly evolving mean. In Section 7, we go on to develop a parametric model of structural change in the spirit of the shifting-mean vector autoregressive framework similar to that considered by Ng and Vogelsang (2002), but modified in a manner consistent with Holt and Teräsvirta (2012), to allow for the possibility of gradual or smooth shifts (as opposed to discrete breaks). The results are assessed by, among other things, decomposing the effects of the shifts of, in particular, oil prices on the prices for other commodities. The final section concludes.

2. The Recent Commodity Price Boom: A Brief Review

As detailed in Hamilton (2009), Wright (2011), Carter, Rausser, and Smith (2011) and Enders and Holt (2012), there are likely a variety of reasons underlying the recently observed boom-bust-boom pattern for many primary commodity prices. Clearly, the 2000s have generally been a period of significant income growth in many developing countries, and most notably in China, India, and parts of South America including Brazil. Zhang and Law (2010) and Hernderson (2011) show that this income growth has led the "BRIC" countries to incorporate larger quantities of grains, meat and other proteins in their diets.[1]

The second notable effect of increased purchasing power in developing countries has been a sharp increase in the demand for energy, and most notably for petroleum. Hamilton (2009) reviews many of the details surrounding recent shifts in energy consumption and, specifically, discusses the role of the BRICs. The recent situation is

[1] BRIC is an acronym that stands for the emerging economies of Brazil, India, China, and Russia.

summarized in Figure 2, which shows the percent of total world oil consumption from 1992 − 2011 by the BRIC nations. As illustrated there, in the mid 1990s consumption was stable at about 14-percent of global consumption. Beginning in the late 1990s and early 2000s, however, these countries share of total world consumption rose steadily to just slightly over 21-percent by 2011.

Of more than passing interest is that the prices for many coarse grains (and sugar) and crude oil are increasingly tied in new and evolving ways. Specifically, the rise of ethanol production and use in the U.S. and elsewhere has had a large impact on land use, commodity prices, and the relationship between prices for energy and non-energy commodities (Abbott, Hurt, and Tyner, 2008). In the United States ethanol production was first encouraged by the tax incentives included in the 1978 Energy Tax Act, providing for federal excise tax exemptions for gasoline blended with 10-percent ethanol. Over time other federal- and state-level subsidies were also created. As well, import tariffs were incorporated to limit the amount of ethanol coming into the United States from abroad. Furthermore, a so called Renewable Fuel Standard, which dictates that gasoline sold in the United States contains a certain volume of renewable fuels, was established as part of the Energy Policy Act of 2005. Of equal if not greater importance for the rise of ethanol were the state bans on Methyl Tertiary Butyl Ether (MTBE), as noted by Zhang et al. (2007) and Serra et al. (2011). MTBE is a widely used oxygenate in the gasoline production process, and is a known contaminant of water supplies. Ethanol is a reasonable substitute for MTBE in the refining process, with the switch from MTBE to ethanol gaining considerable traction in early 2006 (Serra et al., 2011).

Perhaps nowhere has the impact of increased ethanol use been more profound than in the market for maize, as illustrated in Figure 3. As the Figure shows, between 1986 and 2001 the total amount of maize used for ethanol in the United States never exceeded 10-percent of total maize production. A notable uptick in this pattern occurred in the early 2000s, with dramatic increases being observed starting in 2006. The result is that by 2011 over 40-percent of the total annual maize crop was being utilized in ethanol production. Because in the United States maize and soy in particular can be produced on much of the same land base, much of the increased maize acreage apparently came at the expense of area planted to soy.

Other factors have undoubtedly played a role in the most recent surge in commodity prices. Carter, Rausser, and Smith (2011) and Wright (2011), for example, discuss the importance of stockholding behavior, both for storable field crops as well as for nonrenewable energy resources, in price determination. For example, shortfalls in crop production will result in inventories being drawn down. Moreover, even seemingly small production shocks are capable, given the generally inelastic nature of short-run consumption demands, of causing rather large price swings (see, e.g., Roberts and Schlenker, 2010). Certainly there is considerable evidence of weather shocks during much of the period in question in various producing regions of the world. Wright (2011) argues that much of the recent increase in nominal prices for major field crops can be explained by a standard model of supply and demand with storage. Specifically, Wright (2011) notes that during much of the mid and late 2000s stock-to-use ratios for major grains were, on a global level, at or near the levels observed during the previous commodity price boom in the mid 1970s.

It is also likely that general macroeconomic conditions have also had an impact on commodity price behavior in recent times. As Frankel (2007) discusses, there is evidence of linkages via monetary policy between real interest rates, exchange rates, and the prices for agricultural and mineral commodities. The fact that real interest rates are at long term lows, has decreased storage costs and induced the stockpiling of commodities. Moreover, declines in the real value of the dollar have made U.S. grains relatively less expensive to foreigners. There is ample evidence that low interest rates and a weak dollar were at work in the most recent commodity price boom. For example, Chen et al. (2010) apply a factor model to prices for 51 traded commodities. They show that not only does the first, highly persistent component, mimic (nominal) exchange rate movements to a high degree, but the factor model also provides substantially improved forecasts of exchange rates relative to a random walk model. These macroeconomic factors, perhaps exacerbated by relatively loose monetary policy in the United States and elsewhere during the mid 2000s, where likely played an important role in the recent commodity price boom. Hamilton (2010), for example, has argued that the second round of quantitative easing (i.e., undertaken by the Federal Reserve in 2010 likely helped boost commodity prices in 2010 and 2011 even after their steep but temporary declines following the financial crises in 2008 and 2009.[2]

What is clear is that a variety of conditions likely contributed to the recent commodity price boom. The evolving and changing relationship between energy and food, and most notably, between energy and coarse grains, is likely a contributing factor. So, too, are the likely effects of macroeconomic conditions tied to real interest rates and, relatedly, real exchange rates. As well, inventory behavior in the face of increasing consumption demand and supply shocks also likely played a role. Identifying and isolating each of these effects in a comprehensive structural model, while perhaps desirable, is likely not feasible. For these reasons we follow Carter and Smith (2007), Serra et al. (2011), Enders and Holt (2012), and others, and focus here on a set of reduced form time series models. Specifically, we are interested in seeing how the time and nature of structural shifts or breaks in set of variables identified in some sense as being "causal" for commodity prices (including commodity prices themselves) affected

[2] Gilbert (2010), for example, argues that a driving force behind the recent run-up in commodity prices is speculation, either through physically holding (and withholding) stocks or indirectly by the influence of index-based investment funds on futures prices. We do not consider the role of speculation as a factor in the longer term movements in grain prices as Irwin and Sanders (2011) provide rather convincing evidence that there are no obvious empirical links between index fund trading and commodity futures price movements.

commodity price behavior. While Enders and Holt (2012) examined issues of this sort in a univariate setting, a central innovation of this paper is to extend their analyses to a multivariate framework.

3. Data

Given the large number of factors that have been identified with the recent run-up in commodity prices, we focus on two estimation strategies each with its own set of causal variables. The first uses an unrestricted vector autoregression (VAR) to analyze the relationship between grain prices and a number of macroeconomic variables including real exchange rates, interest rates, and energy prices. The second uses shifting-mean vector autoregression (SM-VAR) that focuses on a larger set of agricultural commodities and variables more directly influencing commodity prices such as transport costs, climate conditions. In both analyses, all commodity prices are converted to real terms by deflating by the producer price index (PPI). We then further transform the data by converting it to natural logarithmic form.

Data Used in the VAR

In the broad overview analysis, a standard VAR analysis is performed by focusing on relationships among real grain prices, real energy prices, the real exchange rate, and a measure of the real interest rate. The grain price measure is an index constructed by the World Bank as a composite of representative world prices for rice (weight of 30.2-percent), wheat (weight of 25.3-percent), maize and sorghum (weight of 40.8 percent) and barley (weight of 3.7-percent).[3] The energy price index is also constructed by the World Bank; it is a composite of the prices for coal (weight of 4.7-percent), crude oil (weight of 84.6-percent), and natural gas (weight of 10.8-percent). Both indices are normalized to average to 100 during 2005. The real exchange rate is

[3] A time series compilation of World Bank commodity price data may be downloaded from the url: http://blogs.worldbank.org/prospects/category/tags/historical-commodity-prices

the so called broad exchange trade-weighted exchange rate, which in turn is a weighted average of the foreign exchange values of the U.S. dollar against the currencies of a large group of major U.S. trading partners converted to real terms. The real exchange rate is constructed and reported by the Board of Governors of the Federal Reserve System.[4] Finally, the interest rate is the three-month Treasury bill secondary market rate adjusted for inflation. The inflation rate, in turn, is constructed as:

$$infl_t = 400\left(CCPI_t / CCPI_{t-3} - 1\right),$$

where $CCPI$ denotes the core consumer price index, that is, the CPI adjusted by deleting prices for food and energy. The real interest rate measure is constructed then by subtracting the inflation rate from the nominal three-month Treasury bill rate.[5]

Time series plots for these four monthly series, 1974-2011, are presented in Figure 4. There we see that the real grain price index declined from 1974 through the mid 1980s, leveled off until the mid 1990s, declined again until about until about 2000, and since then has generally increased. The real energy price index was generally stable from the mid 1980s through the mid-to-late 1990s, then declined sharply in 1999, and has since tended to increase rather steadily. The real exchange rate shows sharp increases in the eaerly-to-mid 1980s and again in the late 1990s and early 2000s, with a generally steep starting in about 2002. As expected, the real Treasury bill rate peaked in the early 1980s, and has since then has generally declined, although several plateau periods have also been observed.

Data Used in the SM-VAR

Turning to the data used in the SM-VAR analysis, we focus on interactions among a select set of specific commodity prices. Specifically, we focus on interactions

[4] The data may be obtained from the url:
http://www.federalreserve.gov/releases/h10/summary/default.htm

[5] Data for core CPI and the three-month Treasury bill rate were obtained from the St. Louis Federal Reserve's FRED database.

among monthly prices for maize, soy, crude oil (or more simply, oil), a measure of ocean freight transport costs, and the price of ethanol. As well, because the production and transport of agricultural commodities are subject to the vagaries of weather, we also consider a climate extremes index. The maize, soy, oil prices used in this analysis are reported by the World Bank. Maize prices are recorded in dollars per metric ton (dollars/mt), and represent U.S. number 2 yellow, f.o.b., Gulf prices. Likewise, soy prices are also reported in dollars per metric ton, and are U.S., c.i.f., Rotterdam prices. The crude oil price is recorded in dollars per barrel (dollars/bbl), and represent an average of spot market prices for Brent, Dubai and West Texas intermediate crude; crude oil prices are equally weighted in constructing the World Bank composite oil price measure. Additional details regarding these variables are reported in the Technical Appendix that accompanies Enders and Holt (2012).

Transport costs are a major factor in world trade of primary commodities. Moreover, because in the short run the fleet of transport vessels is essentially fixed, Kilian (2009) argues that variations in ocean freight transport costs can be viewed as a general measure of global economic activity, that is, as a composite measure of global demand for traded goods and commodities. The data were constructed by Lutz Kilian, and represent an average of dry bulk shipping freight rates for cargoes consisting of grain, oilseeds, coal, iron ore, fertilizer, and scrap metal as reported by *Drewry's Shipping Monthly*. A composite index is then constructed in a manner described in more detail in Kilian (2009). In the index the value for January, 1968 is normalized to one. These data were obtained directly from Lutz Kilian by private correspondence. Importantly, unlike the data used in Kilian's (2009) paper and reported on his website, the data we use for the dry bulk shipping freight rates have not been detrended.

Because markets for energy have evolved rapidly in recent years with the rise of ethanol production, there is reason to believe that prices for major field crops (and most notably, maize) and energy are now linked in new and more complex ways (Abbott, Hurt, and Tyner, 2008). In an attempt to examine these linkages in more detail, we also include a measure of ethanol price. Specifically, the ethanol price used here is the F.O.B. Omaha rack price, quoted in dollars per gallon, and collected and reported by the Nebraska State Government.[6] Ethanol price data are only available beginning in January, 1982.

A final measure of interest relates to climate anomalies that might affect the production, marketing, and transport of agricultural commodities. Although several alternatives are available, we use the National Oceanic and Atmospheric Administration's (NOAA) National Climatic Data Center's climate extreme index (CEI) for the Upper Midwest climate region.[7] The index, developed by Karl et al. (1996) and Gleason et al. (2008), incorporates information on monthly maximum and minimum temperature, daily precipitation, and the monthly PDSI measures.

Time series plots, 1974-2011, of the data used in the SM-VAR model are reported in Figure 5. For our purposes, it is important to note that the real prices for maize and soy generally declined until the early 2000s, at which point they started to trend upward. A somewhat similar pattern is evident for the price of crude oil, although the upturn since the early 2000s has been more pronounced. The real price of ocean freight generally trended down from the early 1970s through the early 2000s, and experienced a notable upturn until the most recent recession beginning in late 2007. Since then real ocean freight rates have generally remained low relative to historical norms. The real price of ethanol also tended to trend downward from 1982 through the early 2000s, and then trended upward rather sharply, again, until the onset of the most recent recession.

[6] The data were obtained from the url: http://www.neo.ne.gov/statshtml/66.html. Similar data for ethanol were employed by, for example, Serra et al. (2011).
[7] For example, Fox, Fishback, and Rhode (2009) explored the impacts of a well-known drought measure, the Palmer Drought Severity Index (PDSI), along with other measures, on the price of maize, 1895-1932. Likewise, Schmitz (1997) examined the role of the PDSI in explaining inventory adjustments in U.S. beef cow breeding herd inventory adjustments.

Finally, the climate extreme index is apparently rather volatile, although without any discernable trend. Even so, it may contain a cyclical component.

4. A VAR Analysis

In this section we employ a vector-autoregression (VAR) to analyze the dynamic interrelationships between real grain prices and the key macroeconomic variables that have been identified as affecting the agricultural sector. As indicated in Ng and Vogelsang (2002), a VAR containing variables with structural breaks is misspecified unless the breaks are properly modeled and included in the estimated VAR. Nevertheless, the cobreaking literature is still in its early stages and, as we explain in more detail in following sections, it is not always clear how to estimate a system with cobreaking variables. Moreover, given that we are working with the variables shown in Figure 4, the breaks are likely to be smooth so that the number of breaks, the functional form of the breaks, and the breakdates are unknown. As such, in this section, we utilize the results from a VAR without incorporating an explicit parametric model of the breaks. The benefit of our VAR analysis is that we can measure the extent to which shifts in the macroeconomic variables are transmitted to real grain prices without having to impose any particular structural assumptions on the data. We rely on Sims (1980) and Sims, Stock and Watson (1990) who indicate how to conduct inference is a regression (or a VAR) combining stationary and nonstationary variables. Subsequently, we develop a more disaggregated model in which we explicitly estimate the structural breaks and their transmission across sectors.

Since an unrestricted VAR is atheoretic, we need only select the relevant variables to include in the model, determine the lag length, and decide on an orthogonalization of the regression residuals. In addition to the real price of grain, we began with a block of three variables that have often been credited with influencing real agricultural prices: the real price of energy, the real interest rate, and the real multilateral exchange rate. When we used the sample period running from 1974:01 to 2011:12, the multivariate AIC selected a lag length of 7 months for our basic four-variable VAR. As shown by Sims, Stock and Watson (1990), it is not appropriate to apply Granger causality tests to nonstationary variables. Hence, we performed the standard block exogeneity test described in Enders (p. 318–19; 2010) but we let the AIC suggest which other variables we might want to add to the four-variable VAR. Even though the AIC is quite generous in this regard, we maintained the four-variable model as none of the following variables reduced the AIC: real ocean freight rates, the climate index, and various measures of real U.S. output including the cyclical portion of HP-filtered U.S. real disposable income.

In order to avoid performing our innovation accounting using an *ad hoc* Choleski decomposition, we used the following strategy to decompose the regression residuals into pure orthogonal shocks. Let the subscripts $i = 1, 2, 3$ and 4 denote real energy prices, the real exchange rate, the real T-bill rate, and the real grain price, respectively. Also, for each period t, let e_{it} denote the regression residual from the i–th equation of the VAR and let ε_{it} denote the pure orthogonal innovation (*i.e.*, the "own" shock) to variable i. In every period t, the relationship between the regression errors and the orthogonal innovations is:

$$(1) \qquad \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix}$$

so that in matrix form: $e_t = G\varepsilon_t$ where the g_{ij} are parameters such that the covariance matrix of the regression residuals, Ee_te_t', is $GE(\varepsilon_t\varepsilon_t')G'$ and G is the (4 x 4) matrix of the g_{ij}.

As it stands, equation (1) indicates that each variable is contemporaneously affected by the innovations in every other variable. However, it is far more likely that some variables are causally prior to others in the sense that they are affected by others only with a lag. For example, since a grain price shock is unlikely to have a contemporaneous effect on the macroeconomic variables, the macroeconomic block should be causally prior to real grain prices. Moreover, without imposing some structural relations on the on the G matrix, the ε_{it} shocks are unidentified. As described by Enders (*pp.* 325–9, 2010), exact identification of the orthogonal innovations from the covariance matrix requires six restrictions. The assumption that the 3 x 3 block of macroeconomic variables is causally prior to each other requires nine restrictions--$g_{ij} = 0$ ($i \neq j$ for $i < 4$)--whereas the exact identification requires only six restrictions. However, imposing these nine restrictions (so that the system is overidentified) results in a sample value of χ^2 equal to 11.88; with three degrees of freedom, the prob–value for the restriction 0.0078 level. The reason for the rejection of the restriction is that the contemporaneous correlation between the residuals of the real exchange rate and real T-bill equations (i.e., e_{2t} and e_{3t}) is 0.55. However, when we do not force $g_{23} = 0$, the following set of eight restrictions results in a χ^2 value of 0.975 which is insignificant at any conventional level:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 & 0 \\ 0 & g_{22} & g_{23} & 0 \\ 0 & 0 & g_{33} & 0 \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix} \tag{2}$$

As such, our decomposition allows real energy, real exchange rate, and real T-bill shocks to contemporaneously affect grain prices and allows real interest rate shocks to contemporaneously affect the real exchange rate. Otherwise, the contemporaneous innovations in each variable are due to "own" shocks.

Figure 6 shows the impulse responses of grain to a +1-standard deviation shock in each of the innovations given the set of shocks identified by equation (2). In order to make the comparisons meaningful, the magnitudes of the responses have been normalized by the standard deviation of the grain shock. Interestingly, the initial effect of a grain price innovation continues to build for three periods and, although it begins to decay, is quite persistent. A positive energy price shock has a positive effect on grain prices; by month 5, a +1-standard deviation shock in energy prices induces a 0.5 standard deviation increase in the real price of grain. Not surprisingly, higher interest rates and a stronger dollar both act to decrease the real price of grain. After all, higher interest rates increase grain holding costs and a stronger dollar increases the price of U.S. grain to importers. Note that after 6 months, +1-standard deviation shocks to the real exchange rate and the real interest rate depress real grain prices by about 0.50 and 0.35 standard deviations, respectively.

The variance decompositions suggest a modest degree of interaction among the macroeconomic variables and real grain prices. As shown below, almost all of the six-month ahead forecast error variance of the real price of grain is due to its own innovations (92.68%). However, after one year, real energy prices, the real exchange rate and the real T-bill rate account for 5.64%, 9.20% and 3.87% of the forecast error variance, respectively. After two years, these percentages grow to 8.87%, 14.51% and 5.93%, respectively.

Percentage of the Forecast Error Variance for Grain

Steps Ahead	Std. Error	Energy	Exchange Rate	T-bill	Grains
1	0.035	0.00	0.36	1.25	98.39
6	0.114	1.42	3.60	2.30	92.68
12	0.152	5.64	9.20	3.87	81.30
18	0.179	7.35	12.15	4.81	75.69
24	0.198	8.87	14.51	5.93	70.69

Nevertheless, these percentages can be misleading since there are subperiods during which the influence of the macroeconomic variables on grain prices was substantial. In order to show this, we decomposed the actual movements in real grain prices into the portions contributed by each of the four innovations. If we abstract from the deterministic portion of the VAR, each of the four variables can be written in the form:

(3) $$y_{iT+j} = A_{i1}(L)\varepsilon_{1T+j} + A_{i2}(L)\varepsilon_{2T+j} + A_{i3}(L)\varepsilon_{3T+j} + A_{i4}(L)\varepsilon_{4T+j} + y_{iT}$$

where the A_{ik} ($k = 1, 2, 3, 4$) are j–th order polynomials in the lag operator L. As such, the $A_{ik}(L)\varepsilon_{kT+j}$ are the part of variable i attributable to innovations in variable k over the period T+1 to T+j. As such, a time-series plot of $A_{4k}(L)\varepsilon_{kT+j}$ shows how movements in variable k affected the real price of grain. In essence, the plots show the counterfactual analysis of how variable real grain prices would have evolved had there been only k-type shocks.

The top portion of Figure 7 shows how real interest rate and real grain price innovations (*i.e.,* own innovations) affected the real price of grains. The solid line in the figure shows the actual movement in grain prices so that it is possible to see the influence of each of the two variables on actual grain price movements. As can be seen by the short-dotted line in the figure, real interest rate movements have a small positive effect in the mid-1990s and a small negative effect from 1998 through most of the remaining sample period. Nevertheless, the downward movement in real interest rates (see Panel D of Figure 6) has caused the absolute value of this negative effect to steadily diminish. As such, it can be argued that the decline of real interest rates has exerted pressure for grain prices to rise relative to per-2000 levels. Notice how shocks to the price of grain (i.e., "own" shocks) accounted for the sharp movements in real grain prices in 1987–1988, 1995, and 2007–2009.[8]

The lower portion of Figure 7 shows the effects of energy and exchange rate innovations on the price of grain over the 1986:1–2011:12 period. It appears that the effects of energy prices and the real exchange rate on the real price of grain were generally offsetting. From 1986 through 1997, the real exchange rate acted to boost the price of grain. After all, during the period when the dollar was relatively weak, the foreign demand for U.S. grains is anticipated to be relatively high. Since the prices are in logarithms, it should be clear that in the early 1990s, the exchange rate acted to increase real grain prices by as much as 25%. After all, as the weak dollar stimulated the foreign demand for U.S. grain, the dollar price of grain was bolstered. Subsequently, the steady appreciation of the real value of the dollar from 1995 through 2002, induced a decline in real grain prices. By 1996, the overall effect of exchange rate movements on grain prices was negative. In contrast, high energy process had a depressing effect on real grain prices through 1999. However, the run-up in energy prices beginning in 1999 acted to increase grain prices—by mid-2000, the overall effect of energy price innovations on grains became positive. By 2006, the effect was to increase grain prices by almost 20%.

5. Modeling Time Series Variables with Shifting Means

Although the VAR results are informative, it is useful to develop a complementary parametric model that allows us to explicitly estimate the shifting means. To begin, consider a stationary series y_t, $t = 1, \ldots, T$, that in the present case represents a

[8] Note that the term "own" shocks for grain can be misleading since all excluded variables actually affecting grain prices influence ε_{4t}.

particular commodity price. A simple shifting-mean (SM) autoregressive model of order p for y_t, that is, an SM-AR$\,(p)$, is given by:

$$y_t = \tilde{\delta}\,(t) + \sum_{j=1}^{p} \theta_j y_{t-j} + \varepsilon_t, \tag{4}$$

where $\varepsilon \sim \text{iid}\,(0, \sigma^2)$, and where under stationarity the condition all roots of the lag polynomial $1 - \sum_{j=1}^{p} \theta_j L^j$ lie outside the unit circle. In (4) $\tilde{\delta}\,(t)$ is the deterministic, nonlinear shift function. Following Dickey and Fuller (1979), it is standard to assume that $\tilde{\delta}\,(t)$ contains a time-invariant intercept and, perhaps, a deterministic linear trend (or quadratic trend) term. In this case y_t would be said to be "trend stationary."

In recent years economists have focused on more detailed specifications for the time-varying intercept, $\tilde{\delta}\,(t)$. For example, one approach, popularized by Bai and Perron (1998, 2003), is to assume that shifts over time in the intercept happen in a discrete manner. That is, we may write $\tilde{\delta}\,(t)$ as:

$$\tilde{\delta}\,(t) = \delta_0 + \sum_{i=1}^{k} \delta_i 1\,(t > \tau_i)\,, \tag{5}$$

where $1\,(.)$ is a Heaviside indicator function such that $I\,(.) = 1$ for $t > \tau_i$ and is zero otherwise. In equation (5) τ_i, $i = 1, \ldots, k$, denotes the discrete break dates. For our purposes, there are several problems with the specification in (5). First, the number of breaks or the timing of breaks are known *a priori*, and therefore these additional parameters must also be estimated as part of the modeling process. More importantly, the nature of the breaks in (5) is assumed to be sharp in that each break fully manifests itself at the date τ_i. However, suppose there is at least one relatively long, gradual shift in the evolution of y_t, which in turn must be accounted for by $\tilde{\delta}\,(t)$. In this instance it is likely that the Bai-Perron procedure would require multiple "breaks" in order to accurately account for what is otherwise one gradual shift. As an alternative to (5), then, Lin and Teräsvirta (1994) and González and Teräsvirta (2008) proposed the following nonlinear specification:

$$\tilde{\delta}(t) = \delta_0 + \sum_{i=1}^{k} \delta_i G(t^*; \eta_i, c_i), \tag{6}$$

where $G(.)$ is the so called transition function, where $t^* = t/T$. For example, $G(.)$ is often be given by:

$$G(t^*; \eta_i, c_i) = \left[1 + \exp\left\{-\exp(\eta_i)(t^* - c_i)/\sigma_{t^*}\right\}\right]^{-1}, \tag{7}$$

where σ_{t^*} denotes the "standard deviation" of t^*. [9] In other words, (7) is a standard two-parameter logistic function in the re-scaled time trend index, t^*, where by construction $G(.)$ is strictly bounded on the unit interval. The speed with which the logistic function transitions from zero to one is determined by the magnitude of $\gamma = \exp(\eta_i)$. For large values of γ, that is, as $\gamma \to \infty$, it follows that $G(.)$ will effectively become a step function with properties identical to those of the Heaviside indicator functions in (5), where the switch date or break date is associated with $t^* = c_i$. Alternatively, for considerably smaller values of γ the transition from zero to unity will be smooth or gradual, and in the extreme as $\gamma \to 0$ the shift effectively disappears. Lin and Teräsvirta (1994) refer to the combination of (4), (6), and (7) as the time-varying autoregressive model, or TVAR. [10] The TVAR model represents a generalization of the methods considered by Bai and Perrron (1998, 2003) in that both smooth shifts and sharp breaks are accommodated.

[9] Normalizing $\exp(\eta_i)$ by σ_{t^*} effectively renders this parameter unit free, which in turn is desirable for numerical reasons during estimation.
[10] More generally, Lin and Teräsvirta (1994) consider a situation where all parameters in (1) can change in a manner defined by the transition function, $G(.)$. As in González and Teräsvirta (2008) and Enders and Holt (2012), we restrict attention here to the case where only the intercept term varies over time.

Of course (7) is not the only transition function that might be considered. Others include the quadratic logistic function (see, e.g., van Dijk, Teräsvirta, and Franses, 2002) and the generalized exponential introduced by Goodwin, Holt, and Prestemon (2011). Considering the later, the transition function may be defined as:

$$(8) \qquad G\left(t^*;\eta_i,c_i,\kappa_i\right)=1-\exp\left\{-\exp\left(\eta_i\right)\left[\left(t^*-c_i\right)/\sigma_{t^*}\right]^{2\kappa_i}\right\},\ \kappa_i=1,2,\ldots,\kappa_{max}.$$

In (8) when $\kappa_i = 1$ the standard two-parameter exponential transition function obtains, which results in something analogous to a V-shaped transition function that is symmetric around the centrality parameter, c_i. When $\kappa_i \geq 2$ in (8) the generalized exponential function obtains, which generates a U-shaped time-path for the transition function, also symmetric around c_i. Indeed, as k_i becomes large, say, typically, 4 or 5, the generalized exponential function approximates a pair of Heaviside indicator functions that are offsetting.[11] Depending on the underlying properties of the data, combinations of logistic functions and/or the generalized exponential function provide considerable flexibility when modeling a combination of smooth shifts and discrete breaks in a univariate series.

Estimation of the SM-AR can be done by using nonlinear least squares (van Dijk, Teräsvirta, and Franses, 2002) or by using a grid search (Enders and Holt, 2012). Additional details regarding estimation of SM-AR models are provided by Teräsvirta, Tjøstheim, and Granger (2010).

A third alternative to modeling the intercept term, $\tilde{\delta}\,(t)$, in (4) was introduced by Becker, Enders, and Hurn (2004). Specifically, they propose approximating the time-varying intercept in (4) by using low-frequency terms from a Fourier approximation of $\tilde{\delta}\,(t)$ in t. For example,

[11] As well, a pair of logistic functions could also be used to approximate either V-shaped or U-shaped shifts, albeit at the expense of estimating more (nonlinear and correlated) parameters.

$$\tilde{\delta}(t) = \delta_0 + \delta_1 t + \sum_{k=1}^{n}\{\alpha_k \sin(2\pi kt/T) + \beta_k \cos(2\pi kt/T)\},\ q \le T/2. \tag{9}$$

As illustrated by Enders and Lee (2012), the combination of (9) with (4) provides considerable flexibility in modeling a wide array of smoothly shifting intercepts in univariate autoregressive models.

Irrespective of the method used to model the time-varying intercept in (4), the unconditional (shifting) mean of the series, y_t, may be obtained by taking the unconditional expectation of (4) and solving, to obtain:

$$\mathsf{E}_t y_t = \left(\sum_{j=1}^{p}\theta_j L^j\right)^{-1}\tilde{\delta}(t) = \sum_{j=0}^{\infty}\varphi_j\tilde{\delta}(t-j), \tag{10}$$

where $\varphi_0 = 1$. According to (10) the shifting mean of y_t will depend on the precise way for which $\tilde{\delta}(t)$ is specified, as well as the model's autoregressive parameters.

Shifting Means: Multivariate Methods

In principle the above specifications can be extended to a multivariate setting in a straightforward manner. For example, let i, $i = 1, \ldots, n$, index the particular commodity prices considered in the system. We may therefore define $\boldsymbol{y}_t = (y_{1t}, \ldots, y_{nt})'$ as an $(n \times 1)$ vector of observations on commodity prices at time t.[12] The multivariate counterpart to (4), that is, the shifting-mean vector autoregression (SM-VAR), is given by:

$$\boldsymbol{y}_t = \tilde{\boldsymbol{\delta}}(t) + \sum_{j=1}^{p}\boldsymbol{\Theta}_j \boldsymbol{y}_{t-j} + \boldsymbol{\varepsilon}_t, \tag{11}$$

[12] Henceforth bolded variables are used to denote appropriately defined vectors or arrays.

where $\boldsymbol{\Theta}_j$ is a $(n \times n)$ parameter matrix, $j = 1, \ldots, p$, and where $\boldsymbol{\varepsilon}_t \sim \text{iid}\,(\mathbf{0}, \boldsymbol{\Sigma})$, where $\mathsf{E}_t\,(\boldsymbol{\varepsilon}_t) = \mathbf{0}$, and where $\boldsymbol{\Sigma}$ is a $(n \times n)$ positive definite covariance matrix. Assuming the vector autoregressive structure of the system is dynamically stable, the roots of $|\mathrm{I} - \sum_{j=1}^{p} \boldsymbol{\Theta}_j L^j|$ are assumed to lie outside the unit circle. In (11) $\tilde{\boldsymbol{\delta}}\,(t) = (\delta_1\,(t^*)\,, \ldots, \delta_n\,(t^*))'$ is a $(n \times 1)$ time-varying intercept vector, where a typical element might be given by:

$$\tilde{\delta}_j\,(t) = \delta_{0j} + \sum_{i=1}^{k_j} \delta_{ji} G\,(t^*; \eta_{ji}, c_{ji})\,,\ j = 1, \ldots, n. \tag{12}$$

In (12) the $G\,(.)$ transition functions could, as in the univariate case, be given by some combination of (7) and/or (8). In a manner analogous to the univariate case, the system in (11) may be written as:

$$\left(\mathbf{I} - \sum_{j=1}^{p} \boldsymbol{\Theta}_j L^j\right) \boldsymbol{y}_t \;=\; \tilde{\boldsymbol{\delta}}\,(t) + \boldsymbol{\varepsilon}_t. \tag{13}$$

so as in (10), the vector-valued shifting-mean for $\boldsymbol{y}_t$ can be generalized such that:

$$\mathsf{E}_t \boldsymbol{y}_t \;=\; \left(\mathbf{I} - \sum_{j=1}^{p} \boldsymbol{\Theta}_j L^j\right)^{-1} \tilde{\boldsymbol{\delta}}\,(t) \;=\; \sum_{j=0}^{\infty} \boldsymbol{\Phi}_j \tilde{\boldsymbol{\delta}}\,(t - j)\,, \tag{14}$$

where $\boldsymbol{\Phi}_0 = \mathbf{I}$, an $(n \times n)$ identity matrix. Note that (13) implies that a shift in the series for, say, y_{it}, will necessarily cause a shift in, say, y_{jt} (Ng and Vogelsang, 2002). Indeed, the only way this is not true is either if (1) the coefficients on lagged y_{jt} in the equation for y_{it} sum to zero or, alternatively, if (2) if prior observations on y_{jt} do not Granger cause y_{it}.

Again, depending on the nature of causal relationships amongst the variables in (11), shifts in the mean of one series will necessarily result in mean-shifts not only for the variable in question, but also for the remaining variables in the system. To underscore this issue, a simple example will suffice. Consider a "near-VAR" with data generating process (DGP) specified as:

$$\begin{aligned} y_{1t} &= -0.10 + 1.50G\left(t/T; \eta_1 = 1.40, c_1 = 0.90\right) + 1.50G\left(t/T; \eta_2 = 3.91, c_1 = 0.45\right) \\ &\quad + \Theta_{11} y_{1t-1} + \Theta_{12} y_{2t-1} + \varepsilon_{1t} \end{aligned}$$

$$\begin{aligned} y_{2t} &= -0.10 + 1.25G\left(t/T; \eta_1 = 1.40, c_1 = 0.90\right) - 1.50G\left(t/T; \eta_3 = 3.91, c_3 = 0.25\right) \\ &\quad + \Theta_{22} y_{2t-1} + \varepsilon_{2t}, \end{aligned}$$

where presently $\Theta_{11} = \Theta_{22} = 0.5$ and $\Theta_{12} = 0.2$, and where

$$\boldsymbol{\varepsilon}_t = \left(\varepsilon_{1t}, \varepsilon_{2t}\right)' \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}\right), \ \boldsymbol{\Sigma} = \begin{bmatrix} 0.5 & \rho 0.5 \\ \rho 0.5 & 0.5 \end{bmatrix}, \quad \rho = 0.5.$$

As specified in the DGP, the first mean shift, which is centered rather late in the sample, is shared by both equations. Even so, both equations have independent mean shifts that occur during the first half of the sample. Moreover, the DGP represents a near-VAR in that y_{2t} is exogenous. For the above bivariate DGP, it follows that the shifting means are defined as:

$$\mathsf{E}_t y_{1t} = \frac{\left(1 - \Theta_{22}\right) \tilde{\delta}_1\left(t\right) - \left(1 - \Theta_{12}\right) \tilde{\delta}_2\left(t\right)}{\left(1 - \Theta_{11}\right)\left(1 - \Theta_{22}\right)} = 2\tilde{\delta}_1\left(t\right) - 3.2\tilde{\delta}_2\left(t\right),$$

$$\mathsf{E}_t y_{2t} = \frac{\tilde{\delta}_2\left(t\right)}{\left(1 - \Theta_{22}\right)} = 2\tilde{\delta}_2\left(t\right),$$

where

$$\tilde{\delta}_1\left(t\right) = -0.10 + 1.50G\left(t/T; \eta_1 = 1.40, c_1 = 0.90\right) + 1.50G\left(t/T; \eta_2 = 3.91, c_1 = 0.45\right),$$

$$\tilde{\delta}_2\left(t\right) = -0.10 + 1.25G\left(t/T; \eta_1 = 1.40, c_1 = 0.90\right) - 1.50G\left(t/T; \eta_3 = 3.91, c_3 = 0.25\right).$$

The time paths for the shifting means for y_{1t} and y_{2t}, that is, $\mathsf{E}_t y_{1t}$ and $\mathsf{E}_t y_{2t}$, along with a single realization where the sample size T, is fixed at 350, are plotted in Figure 8. Note that even though the DGP includes two shifts in the equation for y_{1t}, by virtue of the model specification the second shift in y_{2t} also appears in $\mathsf{E}_t y_{1t}$. This result is illustrated in the upper panel of Figure 8, where the first down-shift in $\mathsf{E}_t y_{1t}$ is actually due to the second shift in the intercept fory_{2t}. As illustrated by simulation results in the upper panel of Figure 8, the first shift for $\mathsf{E}_t y_{1t}$, occurring between observations 80 and 100, is actually due to the corresponding shift in $\mathsf{E}_t y_{2t}$. It is difficult to know if this shift would be detected by usual univariate shift (break) identification and testing strategies. But as reported by Holt and Teräsvirta (2012), simulation evidence indicates it certainly would be identified as such in certain circumstances with reasonable frequency. For these reasons the methods used to test for, identify, and include mean shifts (breaks) in a univariate setting are generally not appropriate in a multivariate framework.

Shifting Means: A Testing Framework

An automatic question is how might the presence of shifting means be tested for, especially in a multivariate framework? And how many shifts, k_j, might be required for each equation? Prior research has focused almost exclusively on testing in a univariate autoregressive (AR) context. We review the general univariate testing approach and then discuss how such tests can be adapted for use in a SM-VAR setting. We focus on the shifting-mean model where either the logistic function in (7) or the generalized exponential function in (8) are used to characterize mean shifts.

Univariate Models

Consider the following univariate AR model or order p, that is, an AR(p):

$$y_t = \delta_0 + \boldsymbol{\alpha}' \boldsymbol{z}_t + \varepsilon_t \tag{15}$$

where $\boldsymbol{z}_t = (y_{t-1}, \ldots, y_{t-p})'$ is a $(p \times 1)$ vector, and where $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_p)'$, a $(p \times 1)$ parameter vector. Of course (15) is just a special case of the SM-AR where, in particular, no mean (intercept) shifts occur. The alternative to (12) might simply be

$$(16) \qquad y_t \; = \; \delta_0 + \delta_1 G\left(t^*; \gamma_1, c_1\right) + \boldsymbol{\alpha}' \boldsymbol{z}_t + \varepsilon_t,$$

where $G\left(t^*; \eta_1, c_1\right)$ is the transition function, presumably associated with either the logistic function in (7) or the generalized exponential function in (8). At this point it would seem that (15) could be estimated and the results used to simply test the hypothesis $\mathrm{H}_0 : \delta_1 = 0$. Such an approach would be invalid, however, in that (15) can be obtained from (16) either by restricting $\delta_1 = 0$ or by setting $\gamma_1 = 0$ (so that the logistic function degenerates into a constant). The point is, when $\delta_1 = 0$ there are unidentified nuisance parameters under the null, namely, γ_1 and c_1. The result is that the estimator for δ_1 (and likewise, for γ_1) will be associated with a non-standard distribution, even as $T \to \infty$. This general result is due to a series of papers by Davies (1977, 1987), and is typically referred to simply as the "Davies problem" in the literature. To circumvent the problem, Lukkonen, Saikkonen, and Teräsvirta (1988) proposed that the $G\left(.\right)$ function in (16) could be replaced with a reasonable Taylor series approximation, taken at $\gamma = 0$. For example, if a third-order Taylor approximation is used, (16) may be rewritten as:

$$(17) \qquad y_t \; = \; \beta_0 + \beta_1 t^* + \beta_2 t^{*2} + \beta_3 t^{*3} + \boldsymbol{\pi}' \boldsymbol{z}_t + \xi_t,$$

where $\boldsymbol{\pi}$ is a $(p \times 1)$ parameter vector, and where ξ_t equals the original error term, ε_t, plus approximation error. The LM test for a constant mean can be conducted by regressing the residuals from (15) on the regressors in (17) and using the standard sample F-statistic for the null hypothesis:

$$(18) \qquad \mathrm{H}_0 \; = \; \beta_1 \; = \; \beta_2 \; = \; \beta_3 \; = \; 0.$$

Assuming that the null hypothesis of a constant mean in (18) is rejected, Lin and Teräsvirta (1988) go on to describe a sequence of tests that may be used in an attempt to identify the nature of the mean shift, that is, whether it is more likely to be of the logistic function of generalized exponential function variety. Specifically, if (18) is rejected we may take (17) as the maintained model, and then test:

$$
\begin{aligned}
&\mathrm{H}_{03} : \beta_3 = 0, \\
&\mathrm{H}_{02} : \beta_2 = 0 \,|\beta_3 = 0 \;, \\
&\mathrm{H}_{01} : \beta_1 = 0 \,|\beta_2 = \beta_3 = 0 \;.
\end{aligned}
\tag{19}
$$

The idea is that if either H_{03} or H_{01} is associated with the smallest p–value that the corresponding mean-shift is more likely with a logisitic function as identified in (7). And of course in this case the possibility of a sharp break in the model's intercept is not precluded. Alternatively, if H_{02} has the smallest p–value, then the mean-shift is more likely to have occurred in a manner consistent with the generalized exponential function in (8).

Escribano and Jordà (1999) consider a modification to the testing sequence outlined above. Specifically, they extend the testing equation in (17) to include a fourth-order term, t^{*4}. That is, the testing equation they propose is:

$$
y_t \;=\; \beta_0 + \beta_1 t^* + \beta_2 t^{*2} + \beta_3 t^{*3} + \beta_4 t^{*4} + \boldsymbol{\pi}' \boldsymbol{z}_t + \xi_t. \tag{20}
$$

The LM test for null hypothesis of no mean shifts in (20) is the sample F-value for the restriction:

$$
\mathrm{H}_0' \;=\; \beta_1 \;=\; \beta_2 \;=\; \beta_3 \;=\; \beta_4 \;=\; 0, \tag{21}
$$

Escribano and Jordà (1999) propose the following testing sequence:

$$
\begin{aligned}
&\mathrm{H}_{0E} : \beta_2 = \beta_4 = 0, \\
&\mathrm{H}_{0L} : \beta_1 = \beta_3 = 0,
\end{aligned} \tag{22}
$$

as an aid in identifying the form of the underlying shift (transition) function. Specifically, if H_{0E} has the smallest p–value, then the underlying mean-shift is most likely of the generalized exponential form in (8). Otherwise, H_{0L} is associated with the smallest p–value, then the underlying mean shift is likely associated with the logistic transition function in (7).

After completion of the testing sequence, a provisional SM-AR model may be specified as:

$$
y_t = \delta_0 + \delta_1 G_1\left(t^*; \eta_1, c_1\right) + \sum_{j=1}^{p} \theta_j y_{t-j} + \varepsilon_t,
$$

where $G_1\left(.\right)$ is given by either (7) or (8). And once the parameters of the provisional SM-AR model have been estimated, it is desirable to perform additional diagnostic tests or checks. For example, it is useful to know if there is any evidence of remaining autocorrelation or, most importantly, if there is evidence of remaining intercept shifts. As described by Eitrheim and Teräsvirta (1996), the provisional SM-AR model may be used to perform a series of LM tests designed to address these questions. Specifically, define the skeleton of the SM-VAR as:

$$
F\left(\boldsymbol{x}_t, \boldsymbol{\psi}\right) = \delta_0 + \delta_1 G_1\left(t^*; \boldsymbol{\theta}\right) + \boldsymbol{\alpha}' \boldsymbol{z}_t,
$$

where $\boldsymbol{x}_t = \left(1, \boldsymbol{z}_t\right)'$ and $\boldsymbol{\varphi} = \left(\delta_0, \delta_1, \boldsymbol{\alpha}', \boldsymbol{\theta}\right)$, where $\boldsymbol{\theta} = \left(\eta_1, c_1\right)$. Let $\hat{\varepsilon}_t$ denote the estimated residuals from the estimated SM-AR. And let $\nabla F\left(\boldsymbol{x}_t, \boldsymbol{\psi}\right)$ denote the gradient of the skeleton of the SM-AR with respect to its parameters, that is, define $\nabla F\left(\boldsymbol{x}_t, \hat{\boldsymbol{\psi}}\right) = \partial F\left(.\right) / \partial \boldsymbol{\psi}|_{\boldsymbol{\psi} = \hat{\boldsymbol{\psi}}}$.

In order to test for remaining autocorrelation, an auxiliary regression of the form:

$$\hat{\varepsilon}_t \;=\; \boldsymbol{\omega}'\nabla F\left(\boldsymbol{x}_t, \hat{\boldsymbol{\psi}}\right)' + \sum_{j=1}^{q} \varrho_j \hat{\varepsilon}_{t-j} + \xi_t \tag{23}$$

may be performed as an LM-type F test of the null hypothesis $\mathrm{H}_0 : \varrho_1 = \ldots = \varrho_q = 0$. Doing so constitutes a test for remaining serial correlation at lag q. To test for remaining mean the auxiliary regression from (21) may be modified as follows:

$$\hat{\varepsilon}_t \;=\; \boldsymbol{\omega}'\nabla F\left(\boldsymbol{x}_t, \hat{\boldsymbol{\psi}}\right)' + \sum_{j=1}^{\tau_{max}} \vartheta_j t^{*j} + \xi_t, \tag{24}$$

where τ_{max} typically equals either three or four. The null hypothesis of no remaining intercept shifts is $\mathrm{H}_0 : \vartheta_1 = \ldots \vartheta_{\tau_{max}} = 0$. Again, this LM-test for remaining intercept (mean) shifts may be performed as an F test as previously described. As well, if necessary the testing sequence in either (19) or (22) may also be used to help identify the nature of the underlying transition function for any remaining mean shifts.

Multivariate Models

To date relatively limited research has been conducted on the general topic on of SM-VAR models or, similarly, shifting-mean near vector autoregressive (SM-NVAR) models. Unlike the approaches of Anderson and Vahid (1998), Rothman, van Dijk, and Franses (2001) and Camacho (2004), we use the scaled time variable $t^* = t/T,\ t = 1,\ldots,T$, and do not wish to impose *a priori* the same transition function across equations. Furthermore, we want to consider the possibility that a mix of logistic and generalized exponential transition functions might be used in the modeling exercise. Conducting systems tests in cases like this can quickly become unwieldy, especially when n, the number of equations in the system, is large. For these reasons we follow Holt and Teräsvirta (2012) and proceed by employing univariate tests on an equation-by-equation basis. Provisional models for each equation may be estimated by using nonlinear least squares, and model assessments performed. Once provisional models have been satisfactorily estimated, it is then possible to use these as starting values to jointly estimate the parameters in a SM-VAR of SM-NVAR.

A final caveat is in order. As illustrated by the previous simulation example and in Figure 8, it is likely not desirable to use univariate methods to identify shifting means if additional explanatory variables should be included in the regression. Specifically, using (15) where $\boldsymbol{z}_t = (y_{t-1}, \ldots, y_{t-p})'$ will generally not yield the correct number of shifts if, in fact, additional explanatory variables should be included in the model. This assertion has been verified by simulation exercises reported by Holt and Teräsvirta (2012). Fortunately, the solution in this case is relatively straightforward. Suppose, for example, that the focus is on modeling y_{it} and, moreover, that y_{jt} apparently Granger causes y_{it}. In this case $\boldsymbol{z}_t$ can be redefined as $\boldsymbol{z}_t = (y_{it-1}, \ldots, y_{it-p}, y_{jt-1}, \ldots, y_{jt-p})'$, in which case the models in (15), (16), and (17) directly apply. In other words, by including appropriate conditioning variables in $\boldsymbol{z}_t$ the univariate testing and evaluation procedures defined previously may be readily applied. Simulation results reported by Holt and Teräsvirta (2012) indicate this approach tends to pick the correct number of shifts, k_i, with reasonable accuracy. Moreover, this basic framework is exactly that described originally by Lin and Teräsvirta (1994) when considering the specification and estimation of TVAR models.

6. Unit Root Tests with Shifting Mean Alternatives

Before beginning to estimate our SM-VAR of SM-NVAR, it is necessary do determine whether or not the variables used in the analysis contain unit roots. As demonstrated by Perron (1989, 1997), in the presence of neglected structural change, standard unit root tests are misspecified and suffer from serious size distortions. If the breaks are sharp, it is possible to use dummy variables to construct a modified unit root test with good size and reasonable power. Nevertheless, as shown by Prodan (2008), if there are offsetting or U-shaped breaks, the dummy variable approach performs poorly when estimating the number of breaks and the break dates. Moreover, Becker, Enders and Hurn (2004) show that the dummy variable approach loses power in the presence of the types of smooth shifts displayed by real commodity prices. In essence, to mimic a gradual structural break, it is necessary to combine a number of dummy variables into a single step-function.

In order to control for smooth structural change, Enders and Lee (2012) augment the standard Dickey-Fuller test with a Fourier approximation for the deterministic terms. Consider:

$$\Delta y_t = a_0 + \gamma t + d(t) + \rho y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + \varepsilon_t \tag{25}$$

where the structural breaks are approximated by the deterministic Fourier expression $d(t)$,

$$d(t) = \sum_{k=1}^{n} a_k \sin(2\pi kt/T) + \sum_{k=1}^{n} b_k \cos(2\pi kt/T) + e(n) \tag{26}$$

In equation (26), n is the number of frequencies used in the approximation, the a_k and b_k are parameters, and $e(n)$ is approximation error. The notation is designed to highlight the fact that $e(n)$ is a decreasing function of n such that $e(n) = 0$ when $n = T/2$. In the absence of structural change, $d(t) = 0$, so that the linear model is nested in (25) and (26).

Note that the specification in equation (26) has a number of desirable econometric properties. Unlike a Taylor series approximation in the powers of t (i.e., $t, t^2, t^3, \ldots$), the trigonometric components are all bounded. Moreover, since a Fourier approximation is an orthogonal basis, hypothesis testing is facilitated in that each term in the approximation is orthogonal to every other term. Perhaps most important, unlike a Taylor series expansion, a Fourier approximation is a global (not a local) approximation that need not be evaluated at a particular point in the sample space. Least squares and maximum likelihood estimation methods force the evaluation of a Taylor series expansion to occur at the mean of the series. However, this is undesirable in a model of structural change because the behavior of a series near its midpoint can be quite different from that elsewhere in the sample.

In order to avoid overfitting and to preserve degrees of freedom, Enders and Lee (2011, 2012) recommend using only a few low frequency components in the estimation. Since structural breaks shift the spectral density function towards zero, they are able to demonstrate that the low frequency components of a Fourier approximation can often capture the behavior of a series containing multiple structural breaks. Although the approximation works best with smooth breaks, it is also the case that the approximation with only a few low frequency components is able to detect and control for many types of sharp breaks.

The critical values for the null hypothesis of a unit root (i.e., $\rho = 0$) depend on whether t is included as a regressor in (25) and on the value of n used in (26). The value of n can be pre-specified or selected by using a standard model selection criterion such as the AIC or SBC.

Instead of using cumulative frequencies, it is possible to reduce the number of parameters estimated by performing a grid search over the low-order frequencies ($k = 1, 2, 3, \ldots$) and then conducting the unit root test using the single best-fitting frequency k^*. Another variant of the test relies on the well-known fact that the trend coefficient in (22) is poorly estimated in highly persistent data. In order to produce a test with enhanced power, Enders and Lee (2011) develop a testing procedure based on the Lagrange Multiplier (LM) methodology. The idea is to estimate the coefficients of the deterministic terms using first-differences and then to detrend the series using these coefficients. The third variant is Becker, Enders, and Lee's (2006) introduction of Fourier terms into the Kwiatkowski, Phillips, Schmidt, and Shin (1992) stationary test. As such, it is possible to test the null of a stationary series fluctuating around a slowly changing mean against the alternative of a unit root. Since all unit root tests suffer from low power, it often makes sense to confirm unit root tests with a procedure using the null of stationarity.

Table 1 reports the results of the standard Dickey-Fuller test and the four different Fourier tests applied to the seven series used in our analysis. Notice that the start of the sample period is 1974:01 for all variables save ethanol. For each series, the first row of the table shows the estimated value of ρ assuming linearity (i.e., setting $d(t) = 0$) and the second row shows the associated t-statistic for the null hypothesis $\rho = 0$. Given that the time trend is insignificant in each equation, the 5% critical value for the null hypothesis is -2.87. Notice that it is possible to reject the null hypothesis of a unit root for maize, soybeans, ocean freight, and the climate index, but not for oil, ethanol, and the real exchange rate.

The next three rows of the table show the results when we augment (26) with cumulative frequencies and use the AIC to select the value of n from the subset of possibilities: $n = 1$, 2, or 3. For example, for oil, the AIC selects a value of $n = 3$ and the estimate of ρ (called $\rho(n)$, to denote the use of cumulative frequencies) is -0.096. The t-statistic for the null hypothesis $\rho(n) = 0$ is equal to -5.50 whereas the 5% critical value is -5.03. Notice that the Fourier unit root suggests that every series, except the real exchange rate, is stationary around a slowly evolving mean. We reach the same conclusion with the variant of the test using the single best-fitting frequency k^* and with the LM version of the test. However, when we use the Fourier-augmented KPSS test, at conventional significance levels, we cannot reject the null hypothesis of stationarity for any of the series. Nevertheless, given the preponderance of the evidence, we proceed assuming that only the real exchange rate is nonstationary. As such, it is excluded from our SM-NVAR. Moreover, we exclude the real interest rate as our unrestricted VAR indicated that it has only limited effects on real grain prices.

7. Empirical Results: SM-NVAR Model

The discussion in previous sections serves as an important guide to determining which variables to include in the SM-VAR analysis of linkages among the (real) prices for: (1) maize. $ln\,(pc_t/ppi_t)$; (2) soy, $ln\,(ps_t/ppi_t)$, (3) crude oil, $ln\,(po_t/ppi_t)$, (4) ocean freight rates, $ln\,(pf_t/ppi_t)$, and (5) ethanol, $ln\,(pe_t/ppi_t)$.[13] As well, because of the role that weather conditions and climate shocks play in the production and transportation of maize and soy, we also consider (6), the climate extreme index, cei_t as well. Due to data limitations for ethanol, the period we investigate, after reserving the first 13 months for lag-length tests, runs from February, 1985 through December, 2011, a total of 323 observations.

Basic Model Specification

The testing and estimation framework described above for univariate shifting-mean models are used here to investigate intercept shifts (breaks) in a select group of commodity prices. The approach requires that we first fit a separate transfer-type function (without shifts) to each variable considered. Following Zhang (2008), the lag length for each equation is determined by using the Hannan-Quinn (1979) criterion, which in turn is something of a compromise between the more liberal Akaike Information Criterion (AIC) and the more conservative Schwarz Crtierion (SBC). A series of Granger non-causality tests are performed in order to determine which

[13] More specifically, prior to estimation all real prices are normalized to a unit value for January, 1996 and then multiplied by 100. The natural logarithm is then applied to this transformed series.

variables should be included in each transfer function (equation). The variables included in each equation also with optimal lag lengths are reported in Table 2.

As indicated in Table 2, the base (i.e., linear model with no shifts) model for maize contains two lags of its own price, as well as the prices for soy, oil, ocean freight. In addition, two lags of the climate extreme index are also included. Preliminary results indicate that ethanol price does not Granger cause maize price, a result that, moreover, was confirmed by using similar data by Elmarzougui and Larue (2011). As well, Rapsomanikis and Hallam (2006) and Balcombe and Raspomanikis (2008) reached a similar conclusion regarding the relationship between ethanol and sugar prices in Brazil. As also indicated in Table 2, the base model for soy prices contains two lags of its own price as well as two lags of the ocean freight rate. Of interest is that corn prices apparently do not Granger cause soy prices. Preliminary results indicated that oil price is apparently strongly exogenous; the linear model for oil includes only two lags of its own price. Again, similar results were reported by Rapsomanikis and Hallam (2006), Balcombe and Raspomanikis (2008), and Elmarzougui and Larue (2011). Over a somewhat different time period Kilian (2009) did, however, find evidence of the ocean freight rate, as a measure of real economic activity, having significant feedbacks to oil prices. The ocean freight index is associated with three lags of its own values and the price of oil. Likewise, ethanol price is also specified with three lags, and is a function of its own lagged values, the lagged price of maize, and the lagged price of oil. This result is also consistent with prior findings. Lastly, the climate extreme index is determined to be best explained by only one lag of its own value.

Intercept Nonconstancy Test Results

As explained in the methodology section, the LM testing framework for shifting intercepts may be applied to each equation. Specifically, the basic (linear, no-shift) model specifications outlined in Table 2 are used to examine the presence of intercept shifts, and hence, shifting means. The results of these tests, obtained by using both third- and fourth-order Taylor approximations in time under the alternative, are summarized in Table A1 of the unpublished Appendix to this paper.

The result of testing intercept constancy for maize with a third-order approximation, that is, a test of H_0 in (18), indicates that the null of no intercept shifts cannot be rejected at the 5-percent significance level, but can be rejected at the 10-percent level. The results of the test based on a fourth-order approximation, that is, a test of H_0' in (19) are more conclusive, with the null in this case being rejected at the 5-percent level. The results of the testing sequence in this case, that is, tests of H_{0E} and H_{0L} in (22), provide support for an intercept shift in the maize price equation that is U-shaped, that is, a shift that belongs to the family of generalized exponential transition functions in (8). Results in Table A1 also suggest the presence of an intercept shift for soy. In this case, however, the testing sequence applied to the fourth-order approximation is indeterminate. Alternatively, when the testing sequence in (19) is applied to the soy equation, the evidence points toward an intercept shift consistent with the logistic transition function in (7).

Test results for a shifting intercept in the oil price equation strongly reject the null of no shift when either the third- and fourth-order approximations are used. Even so, the testing sequence in (19) based on the third-order approximation points to a U-shaped intercept shift, while the testing sequence in (20) points to an intercept shift of consistent with a logistic function specification. In the case of oil the correct specification will be determined by fitting both versions and then comparing results for overall explanatory power as well as model diagnostic test results.

Turning to the model for the ocean freight index, results in Table A1 indicate no evidence of an intercept shift when a third-order approximation is used. Alternatively, the null of no intercept shifts is clearly rejected when a fourth-order approximation is used. Moreover, the testing sequence in (22) suggests that the shift may be consistent with a generalized exponential transition function, although the evidence in favor of a logistic-type shift is also strong.

Ethanol is similar to soy in that null hypothesis of no intercept shifts is resoundingly rejected irrespective of whether a third-order or fourth-order approximation is used. Even so, the testing sequence in (22) applied to the fourth-order approximation is non informative. Alternatively, the testing sequence in (19) applied to the third-order approximation strongly suggest that the intercept break in the price of ethanol is consistent with a logistic function shift.

Finally, and perhaps not surprisingly given a visual inspection of the data plotted in Figure 5, there was no evidence of an intercept shift, and hence no evidence of a shifting mean, for the climate extreme index. Alternatively, Gleason et al. (2008) report notable trends in regional U.S. climate extreme indices during the summer and warm seasons since the mid 1970s. To further investigate this possibility, we employed the bootstrap testing framework based on a Fourier approximation to the shifting mean as outlined by Becker, Enders, an Hurn (2004). Applying this test we obtain an empirical p-value of 0.20, further confirming the results for the climate extremes index reported in Table A1.[14]

Single Equation Shifting Mean Results

The pre-tests for intercept constancy test results are used as a guide to fit provisional univariate shifting-mean model for each equation. In the case where a shifting mean consistent with the generalized exponential transition function in (8) is called for, a simple grid search over plausible values for the κ parameter are employed, namely, $\kappa = 1, \ldots, 8$. The diagnostic testing framework outlined in the methodology section, namely, testing for remaining autocorrelation and for remaining intercept shifts, is also

[14] Indeed, the sample employed here, that is, effectively from 1985 to 2011, may be too short to identify any meaningful shifts in climate extremes.

applied. Summary results for the preferred univariate shifting mean models are summarized in Table A2.

As reported in Table A2, with the exception of soy a single transition function (shift function) adequately captures the corresponding intercept shifts; in the case of soy two logistic transition functions are required to summarize its idiosyncratic shifts. Of course these results do not necessarily imply that only one or two mean shifts in the relevant price occurs. For example, maize has one idiosyncratic intercept shift, but in turn is a function of lagged prices for soy, ocean freight, oil, and climate extremes. By virtue of the algebraic result in (14) (and as illustrated by the simulation results in Figure 8), the shifting mean for maize will necessarily be a function of any (all) mean shifts in the right-hand-side variables as well. Alternatively, oil price, which is a function only of its own lagged values, will necessarily be identified as having one and only one mean shift.[15]

Returning to the results in Table A2, there is no strong evidence of remaining residual autocorrelation in any of the provisional shifting mean models. As well, tests for remaining intercept shifts indicate in all cases that the null hypothesis cannot be rejected at conventional significance levels.

Additional diagnostic test results for the provisional shifting mean models are reported in Table A3. Specifically, p-values for LM tests for omitted variables in each equation are reported in the Table. The results of these tests effectively confirm the basic model structure for each equation in the SM-NVAR outlined in Table 2. Taken as a whole, the results reported in Tables A2 and A3 suggest that the provisional shifting mean models are legitimate for further investigation in the form of a shifting-mean near vector autoregressive model. We now turn to these results.

[15] With respect to oil, both a logistic function shift and an generalized exponential function shift were fitted to the data. All model fit and diagnostic test results pointed toward the model with a single logistic function shift.

Shifting Mean Near Vector Autoregression Results

As described by Holt and Teräsvirta (2012), the parameter estimates for the single-equation shifting-mean models described previously may be used as starting values to estimate the parameters of an SM-NVAR by using full information maximum likelihood methods (FIML). Also, following van Dijk, Strikholm, and Teräsvirta (2003) and Teräsvirta, Tjøstheim, Granger (2010), we constrain the speed-of-adjustment parameters, that is, the η_i's, in the respective transition functions when performing the FMIL estimations. Specifically, we constrain each η_i so that $\exp(\eta_i) \in [0.75, 50]$. As well, we follow Enders and Holt (2012) and restrict the values for c_i in each transition function so that $c_i \in [0.05, 0.95]$, which in turn is akin to the so called "trimming condition" typically applied in the estimation of threshold models. Employing these restrictions helps alleviate numerical problems within the iterations of the FIML estimation framework.

Results for the estimated equations in the SM-NVAR are reported in Table A4, while summary statistics for the estimated SM-NVAR, including the estimated error correlation matrix, are presented in Table A5. Estimated transition functions along with the implied shifting means for each variable in the system are shown in Figure 9.

As indicated in Tables A4 and A5, the estimated SM-NVAR fits the data reasonably well, and it results in an improvement in fit relative to the standard NVAR—the system AIC and HQC measures for the SM-NVAR are lower than their counterparts for the corresponding NVAR that does not include mean shifts. Based on the system R^2 advocated by Magee (1990), the SM-NVAR with intercept shifts apparently results in substantial improvement in explanatory power relative to the NVAR without shifts. Finally, as reported in Table A5 estimated residual correlations are generally small with two exceptions: (1) between maize and soy (0.527); and (2) between oil and ethanol (0.305). There is also modest correlations between the residuals for oil and ocean freight (0.119).

Of interest here are the estimated mean-shift (transition) functions for each price equation. Results in Table A4 indicate that the idiosyncratic intercept shift for maize, a generalized exponential transition function, is centered around October, 2005, with the shift starting in late 1999 and ending in 2011. The two idiosyncratic intercept shifts for soy are fitted as logistic functions, with the first one being rather sharp, and centered at at March, 2007. In contrast the second shift for soy is evolving rather slowly (i.e., is close to linear), and is centered around August, 2008. The single logistic function intercept shift for crude oil is quite smooth, and is centered around March, 2004, with 10-percent of the adjustment taking place by June, 2006 and 90-percent of the adjustment occurring by December, 2007. Regarding ocean freight rates, the estimated idiosyncratic intercept shift also belongs to the family of generalized exponential functions. This shift is centered around September, 2005, which very nearly coincides with the center of the idiosyncratic shift for maize. The shift for ocean freight begins in 2002, and is complete by late 2010. Finally, the idiosyncratic shift for ethanol is also of the logistic function variety, and is centered around August, 2010. As with soy, this shift is also rather gradual throughout the sample period.

Shifting Means

As already noted, the algebraic solution for the SM-NVAR shifting means in (14) will, in principle, incorporate the intercept shifts of several, and perhaps all, equations in the system. In the present case it is possible to solve for the reduced form for these intercept shifts and, moreover, to obtain their approximate standard errors by using a standard delta method approximation. The estimated shifting means for each commodity price, including their constituent shifts and approximate standard errors, are reported in Table 3.[16]

Turning first to the shifting mean for maize, with the exception of the shifts for soy, that is, those for $G_2(.)$ and $G_3(.)$, the estimated mean shifts are apparently statistically significant at usual levels. The effect of the idiosyncratic shift for maize on its own mean price is positive. But recall that $G_1(.)$ is U-shaped, assuming unit values only between 1985 and 2000, and again starting in 2011. As well, the shift in crude oil price had a positive effect on the unconditional mean for maize and, moreover, was nearly equal in magnitude to the idiosyncratic shift for maize. The shift in ocean freight has a negative effect on the mean for maize, but recall this shift is also U-shaped. In other words, during the period when $G_5(.)$ was less than one, approximately between 2002 and 2010, the effect of the ocean freight shift on maize was mitigated. What is clear is the idiosyncratic shift in oil, occurring approximately between 2004 and 2007, had a direct effect on the unconditional mean for maize. Of course this does not mean that a structural shift in the real price of oil "caused" a corresponding shift in the real price of maize. In other words, the possibility that a common but otherwise excluded third factor could be the underlying driver cannot be ruled out. For example, expansionary monetary policy and, correspondingly, a devaluation of the U.S. dollar relative to other major currencies could be the underlying causal factor in this instance. Even so, whatever the reason, it seems that structural shifts in real prices for maize and oil during the 2004-2007 period coincided.

Turning next to the shifting mean for soy, results in Table 3 reveal that only the idiosyncratic shifts for soy had any statistically significant effect on the unconditional mean for soy price. Specifically, the shifts in both crude oil price and ocean freight rates

[16] Standard errors for the shift parameters are approximate for all of the usual reasons that standard errors derived by using the delta method are approximate. In addition, the Davies (1977, 1987) problem applies equally here as well, which only further contributes to the "approximate" nature of these measures.

appear to have only a negligible (and insignificant) impact on the shifting mean for soy. In this sense while movements in oil price and ocean freight rates apparently contributed to short- and intermediate-run movements in soy prices, their respective shifts had no lasting effect on the long-run mean price for soy.

The results in Table 3 indicate that the effect of the shift in crude oil price on ocean freight rates, while negative, was not statistically significant. It therefore seems for all practical purposes that the shifting mean for ocean freight rates, like those for crude oil and soy prices, really depends only on its own idiosyncratic shift. Finally, turning to the shifting mean for ethanol, results in Table 3 suggest that, in addition to the idiosyncratic shift in ethanol price, the only other factor that has a statistically significant effect on ethanol's underlying mean is the price of oil. Of interest is that the ethanol's own-shift, $G_6(.)$, is: (1) slowly evolving, and (2) has a negative effect on ethanol's underlying shifting mean. Even so, the effect of the shift in the price of oil on the unconditional mean for ethanol is quantitatively and qualitatively large, and from approximately 2000 on more than offsets the otherwise negative shift in the price of ethanol. The effect of the shift in crude oil price on the mean price of ethanol becomes qualitatively large starting in 2003, with the effect peaking in late 2008 with the onset of the financial crises. As already noted, a number of policy changes occurred during this period of time, including the U.S. renewable fuel standard put into place in the 2005 and the phasing out of MTBE in the gasoline production in 2006. Even so, it is likely that without the underlying recent shift in crude oil price that ethanol price (and presumably production) would be nowhere near the levels observed in recent years.

As a final exercise, it is also possible to the delta approximation method to obtain point-wise approximate standard errors, and therefore, say, 90-percent confidence intervals, for the shifting means themselves. The results of this exercise for each commodity price in the estimated SM-NVAR are reported in Figure 10. As illustrated in

Panel A of the Figure, the shifting mean for real maize price generally drifted down from the mid 1980s through about 2000, at which point it dipped significantly between early 2000 and the middle of 2002. This trend then reversed from 2002 until the fall of 2006. From late 2006 through late 2007 the upward trend was even more accelerated. From early 2008 through the middle of 2009, that is, during a period coinciding largely with the financial crises, the shifting mean for maize then reverse direction, drifting somewhat lower. Beginning in the middle of 2009 the upward trend in the mean real price for maize resumed. Aside from these general patterns, it is also interesting to note that beginning in early 2000, the approximate 90-percent confidence band for maize price began to widen. Moreover, the widening of this band accelerated dramatically starting in late 2006. The implication is that the recent shifts in the underlying unconditional mean for maize, while notable for both their direction and magnitude, were also associated with greater uncertainty.

Panel B in Figure 10 illustrates comparable results for soy. As illustrated there, the shifting mean for soy price generally drifted lower from the mid 1980s until late 2006. From the fall of 2006 through late 2007, the shifting mean for real soy prices increased dramatically. According to model results, the general downward trend in the mean for soy prices resumed at that time. But again, it is noteworthy that the approximate 90-percent confidence bands for soy's shifting mean started to widen in 2002, and widened dramatically starting in late 2008. Again, while the shifts in the underlying mean for real soy prices have been dramatic in recent years, they have apparently also been associated with a greater degree of overall uncertainty.

Regarding the shifting mean for the price of crude oil, the plots in Panel C of Figure 10 reveal nothing surprising—the shifting mean started to move steadily upward in early 2000, rose rather dramatically from 2001 through 2008, and has increased at a decreasing rate since then. The width of the 90-percent confidence bands also remained rather stable, although they widened slightly in the early 2000s and again since 2008.

Regarding the shifting mean for the ocean freight index, the plot in Panel D of Figure 10 shows that no discernable shifts occurred from the mid 1980s through the early 2000s. Beginning in early 2002 the mean for ocean freight started to move higher, and continued to do so through the middle of 2004. At that point the trend in ocean freight's mean started to edge lower, with the downward trend accelerating between early 2007 and late 2009. In the last several years in the sample it seems that the shifting mean for ocean freight rates has leveled off at a new, somewhat lower level. Of almost greater interest are the corresponding shifts in the 90-percent confidence bands for ocean freight's shifting mean. The confidence bands widened somewhat between early 2003 and late 2007, and then increased dramatically in magnitude between late 2007 and late 2009, a period that almost exactly coincides with the NBER dates for the most recent economic downturn (i.e., December, 2007 through June, 2009).

The shifting mean for real ethanol price is plotted in panel E of Figure 10. As indicated there, the underlying mean for real ethanol price drifted lower from the mid 1980s through late 2001. At that point ethanol's mean started moving higher, peaking in late 2007. Since that time the underlying mean for real ethanol price has resumed a gradual downward trend. Also, while there was some widening in the confidence bands for this mean starting 2000s, the increase has not been dramatic.

Effects of Shifts on Agricultural Prices

In Figures 11 − 13, we perform a counterfactual analysis to ascertain the effects of the various shifts on the mean prices of maize, soy and ethanol. Similar to our VAR results, we plot the estimated means of the various commodity prices along with the hypothetical paths obtained by zeroing-out each estimated shift. By comparing the two paths (and recalling that are variables are in logarithms), it is possible to directly show the influence of each shift. Regarding maize, it is not surprising to note that Panel A of Figure 11 shows that the idiosyncratic, or own, shift was especially important. Had the shift not occurred, the estimated mean price of maize in at the end of 2011 would have been about 30% less than the actual mean estimate. This is similar in magnitude to the results from the VAR analysis that was shown in the top Panel of Figure 7. Recall that the estimated "own" shift in maize can include shifts resulting from the real exchange rate and interest rate changes analyzed in the VAR portion of our analysis. The effects of the independent shifts in soy are mixed: the first mean shift for soy acted to increase the price of maize whereas the second acted to lower the price. What is clear (see Panel D) is that the recent run-up in oil prices has served to increase the price of maize by more than 20%. Moreover, as shown in Panel E of the Figure, the effect of the recent decline in the mean of ocean freight rates has had a depressing effect on maize prices of approximately 12%.

From Figure 12, it should be clear that the "own" shifts for soy were of primary importance in determining its time path. As with maize, the decline in ocean freight rates has acted to keep the mean price of soy about 11% lower than otherwise. Somewhat inexplicably, the effect of the run-up in oil prices had a small but negative effect on soy prices. As shown in Panel C of Figure 12, the estimated mean price of soy would have been about 10% higher had the mean price of oil not shifted. Even so, recall from Table 3 that the oil shift is not statistically significant in the soy price equation.

Panel F of Figure 13 indicates that the "own" shift had only very large effect on the price of ethanol. By the end of the sample, the magnitude of the effect was approximately 70%. Note that the shift in maize and the two shifts in soy had only minor effects on ethanol prices. The key result, shown in Panel D of the Figure, is that the run-up in oil prices had a pronounced effect on ethanol prices. We estimate than the mean price of ethanol would have actually declined had the mean shift in the price of oil not occurred. Instead, the run-up in oil prices added approximately 60% to the mean price of ethanol; instead of falling by almost 50%, the mean of ethanol prices rose by approximately 10%.

8. Conclusions

Increases in energy prices, income growth in China, Brazil and India, new uses for ethanol, changes in storage costs, and macroeconomic factors such as exchange rate and interest rate changes have all been blamed for the unprecedented high levels of grain. Since the cobreaking literature is just emerging, we perform two polar opposite methodologies in order to understand the contribution of these various factors in the run-up of grain prices. A simple VAR indicates that mean shifts in real energy prices, exchange rates and interest rates have all contributed to the higher grain prices. Ideosyncratic shocks have also played an important role. The second methodology entends Enders and Holt's (2012) univariate analysis to a time-varying multiple equation setting that allows for smoothly evolving mean shifts. In addition to the general rise in real energy prices, the introduction of ethanol as an important fuel source is found to be a causal factor in the run-up of grain prices. Economic growth in emerging economies such as China, India, and Brazil is also identified as a contributing factor.

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Table 1: Unit Root Test Results.

	Maize	Soybeans	Oil	Freight	Rexrate	Climate	Ethanol
ρ	-0.022	-0.037	-0.015	-0.048	-0.010	-0.605	-0.040
	(-2.92)	**(-3.54)**	(-1.88)	**(-4.27)**	(-1.70)	**(−8.22)**	(-0.901)
n	3	3	3	3	3	1	1
$\rho\,(n)$	-0.092	-0.099	-0.096	-0.074	-0.038	−0.610	-0.120
	(-5.43)	**(-5.77)**	**(-5.50)**	**(-5.28)**	(-3.491)	**(−8.24)**	**(-4.86)**
k^*	1	1	2	3	2	3	1
$\rho\,(k^*)$	-0.053	-0.071	-0.060	-0.073	-0.029	−0.626	-0.119
	(-4.14)	**(-4.93)**	**(-4.18)**	**(-5.17)**	(-3.15)	**(−8.36)**	**(-4.86)**
τ_{LM}	−0.088	−0.099	−0.095	−0.079	−0.047	−0.589	−0.125
	(-5.30)	**(-5.74)**	**(-5.34)**	(-4.66)	(-3.79)	**(−8.07)**	**(-4.97)**
τ_{KPSS}	0.0106	0.0122	0.0140	0.0127	0.0302	0.028	0.0085
lags	2	2	4	3	3	4	3
Start	1974:01	1974:01	1974:01	1974:01	1974:01	1974:01	1983:01

Note: No series contains a deterministic trend: the null that the coefficient on a trend term equals zero could never be rejected at conventional significance levels. ρ is the estimated parameter for the augmented Dickey Fuller test. The critical value is −2.87 at the 5% level. Bold figures are significant at the 5% level. n is the number of cumulative frequencies used in the estimation of the Fourier version of the ADF test and ρ^* is the coefficient on the lagged level term. The 5% critical value is −3.76 for n = 1, −4.45 for n = 2 and −5.03 for n = 3. Bold figures are significant at the 5% level. k^* is the best fitting frequency and $\rho\,(k^*)$ is the coefficient on the lagged level term. The 5% critical values are −3.76, −3.26 and – 3.06, for k^* = 1, 2, and 3, respectively. Bold figures are significant at the 5% level. τ_{LM} is the sample value of τ test for the LM version of the Fourier unit root test. The value of n is the same as that for the DF-version of the test. The critical values for n = 1, 2 and 3 are −4.05, −4.79, and −5.42, respectively. τ_{KPSS} is the sample for of the variance ratios for the stationary version of the Fourier test. Hence, the null hypothesis is that the series is stationary. The 5% critical values are 0.169, 0.102, and 0.072, for n = 1, 2, and 3, respectively. The null of stationarity cannot be rejected for any of the series. For the Climate series, the value of n selected by the Fourier KPSS test was 1. Lags denotes the number of lags in the model; lags−1 is the number of lags used in the ADF versions of the Dickey-Fuller type tests. Start is the starting date of the estimation (accounting for lags).

Table 2: Structure of Individual Equations in the Shifting-Mean Near VAR.

Commodity	Lag Length	Maize (y_{1t})	Soybeans (y_{2t})	Oil (y_{3t})	Ocean Freight (y_{4t})	Ethanol (y_{5t})	Climate Extreme (y_{6t})
Maize (y_{1t})	2	$\surd$	$\surd$	$\surd$	$\surd$		$\surd$
Soybeans (y_{2t})	2		$\surd$		$\surd$		
Oil (y_{3t})	2			$\surd$			
Ocean Freight (y_{4t})	3			$\surd$	$\surd$		
Ethanol (y_{5t})	3	$\surd$		$\surd$		$\surd$	
Climate Extreme (y_{6t})	1						$\surd$

Note: Lag length is determined by using the Hannan-Quinn (HQC) criterion. A $\surd$ indicates that lags of the variable in the associated column are included in the respective equation.

Table 3: SM-VAR Shifting Means for Maize, Soy, Oil, Ocean Freight, and Ethanol..

Maize:
$$\mathsf{E}_t y_{1t} = \underset{(0.130)}{4.172} + \underset{(0.110)}{0.299}\, G_1\left(t^*;\hat{\eta}_1,\hat{c}_1\right) + \underset{(0.171)}{0.180}\, G_2\left(t^*;\hat{\eta}_2,\hat{c}_2\right) - \underset{(0.269)}{0.310}\, G_3\left(t^*;\hat{\eta}_3,\hat{c}_3\right) + \underset{(0.162)}{0.279}\, G_4\left(t^*;\hat{\eta}_4,\hat{c}_4\right) - \underset{(0.078)}{0.149}\, G_5\left(t^*;\hat{\eta}_5,\hat{c}_5\right)$$

Soy:
$$\mathsf{E}_t y_{2t} = \underset{(0.095)}{4.798} + \underset{(0.133)}{0.584}\, G_2\left(t^*;\hat{\eta}_2,\hat{c}_2\right) - \underset{(0.306)}{1.008}\, G_3\left(t^*;\hat{\eta}_3,\hat{c}_3\right) - \underset{(0.134)}{0.113}\, G_4\left(t^*;\hat{\eta}_4,\hat{c}_4\right) - \underset{(0.120)}{0.149}\, G_5\left(t^*;\hat{\eta}_5,\hat{c}_5\right)$$

Oil:
$$\mathsf{E}_t y_{3t} = \underset{(0.079)}{4.678} + \underset{(0.255)}{1.157}\, G_4\left(t^*;\hat{\eta}_4,\hat{c}_4\right)$$

Freight:
$$\mathsf{E}_t y_{4t} = \underset{(0.206)}{4.959} - \underset{(0.300)}{0.303}\, G_4\left(t^*;\hat{\eta}_4,\hat{c}_4\right) - \underset{(0.234)}{0.400}\, G_5\left(t^*;\hat{\eta}_5,\hat{c}_5\right)$$

Ethanol:
$$\mathsf{E}_t y_{5t} = \underset{(0.086)}{4.705} + \underset{(0.031)}{0.022}\, G_1\left(t^*;\hat{\eta}_1,\hat{c}_1\right) + \underset{(0.018)}{0.013}\, G_2\left(t^*;\hat{\eta}_2,\hat{c}_2\right) - \underset{(0.030)}{0.023}\, G_3\left(t^*;\hat{\eta}_3,\hat{c}_3\right) + \underset{(0.146)}{0.647}\, G_4\left(t^*;\hat{\eta}_4,\hat{c}_4\right) - \underset{(0.017)}{0.011}\, G_5\left(t^*;\hat{\eta}_5,\hat{c}_5\right) - \underset{(0.188)}{1.039}\, G_6\left(t^*;\hat{\eta}_6,\hat{c}_6\right)$$

Note: Approximate standard errors obtained by using the delta method are given below parameter estimates in parentheses. $G_1(.)$ is the idiosyncratic transition function for maize; $G_2(.)$ and $G_3(.)$ are similarly defined for soy; $G_4(.)$ is the idiosyncratic shift for oil; $G_5(.)$ is likewise defined for the ocean freight index; and $G_6(.)$ is the idiosyncratic shift for ethanol. Specifications for the transition functions along with their estimated parameters are reported in Table 6.

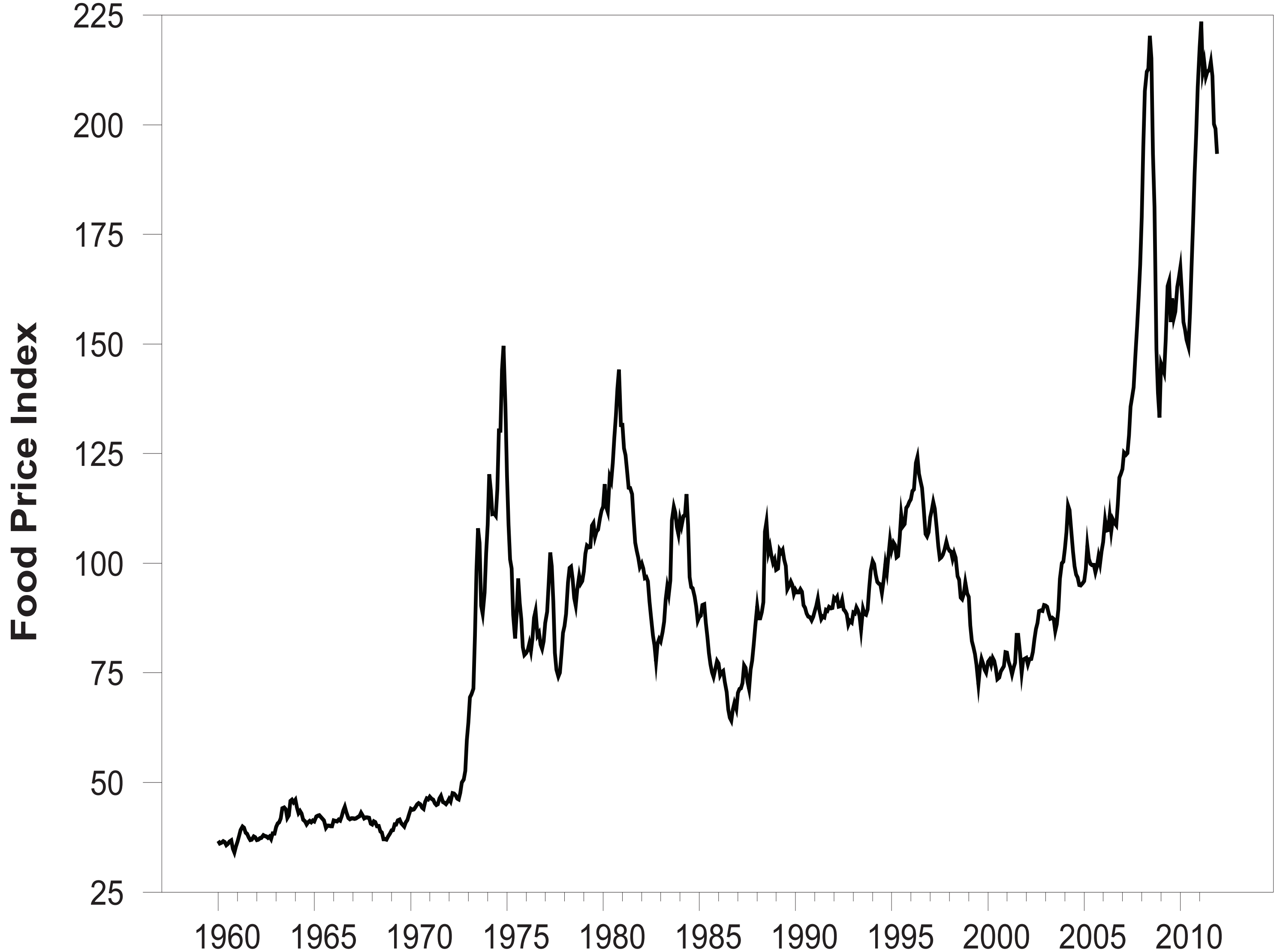


Figure 1: Monthly World Bank Food Price Index, 2005=100, 1946–2011.

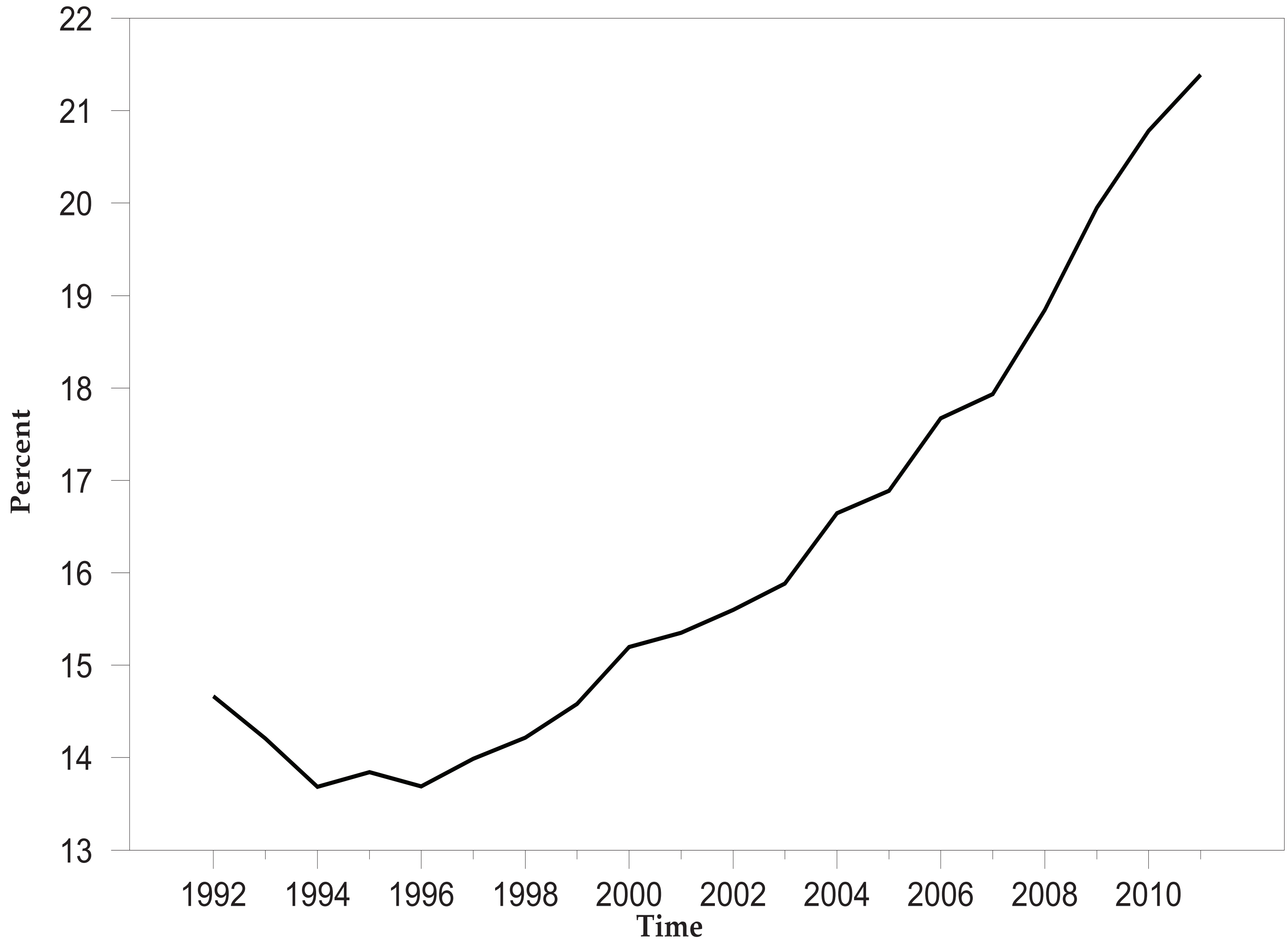


Figure 2: Percent of Total Global Oil Consumption by Brazil, China, India, and Russia, 1992–2011. (Source: U.S. Energy Information Administration, http://www.eia.gov/.)

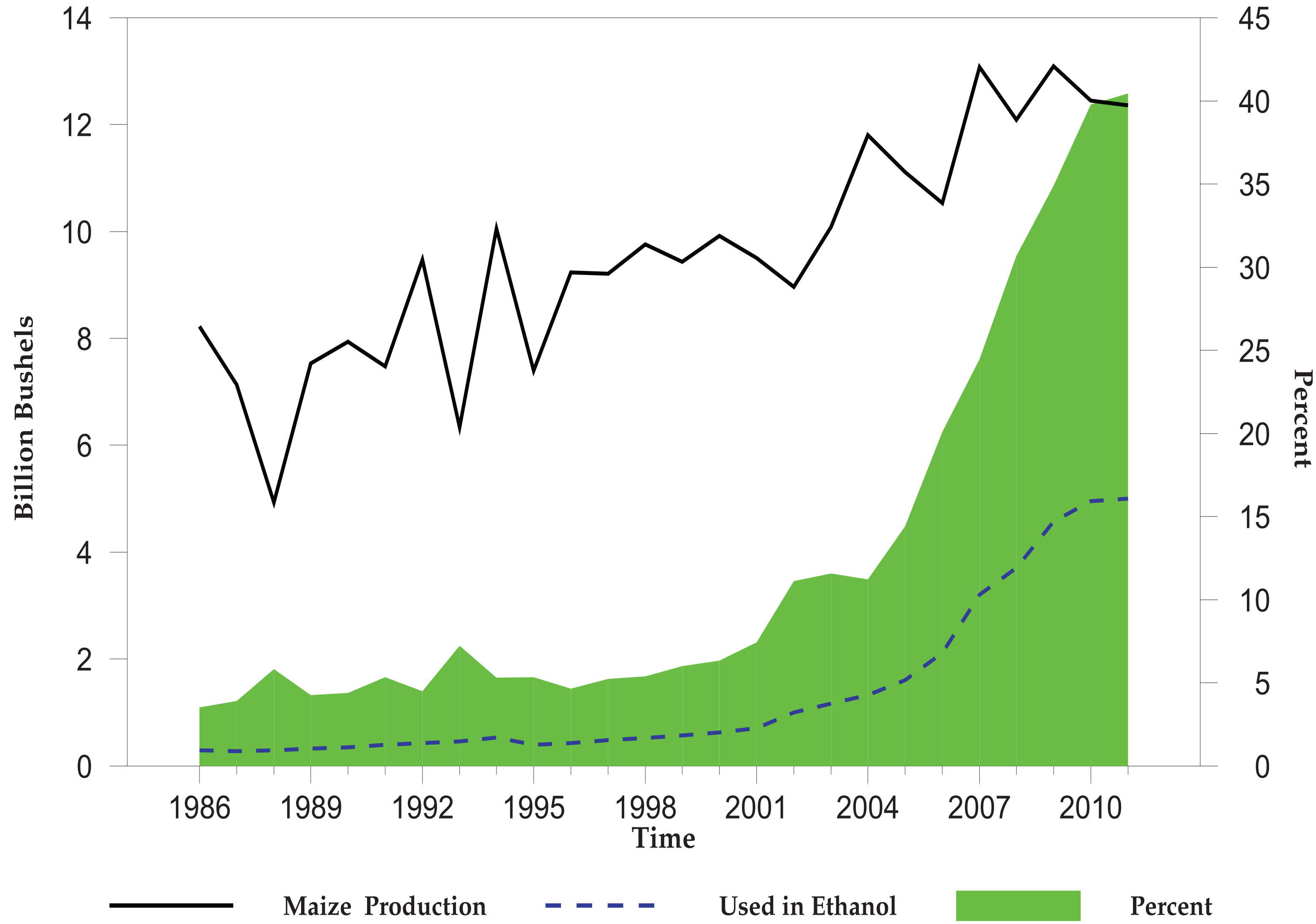


Figure 3: U.S. Maizew Production (solid line), Maize Used in Ethanol (dashed line), and Percent of U.S. Maize Production Used in Ethanol Production (shaded area), 1986–2011. (Source: U.S. Department of Energy, http://www.afdc.energy.gov/afdc/data/)

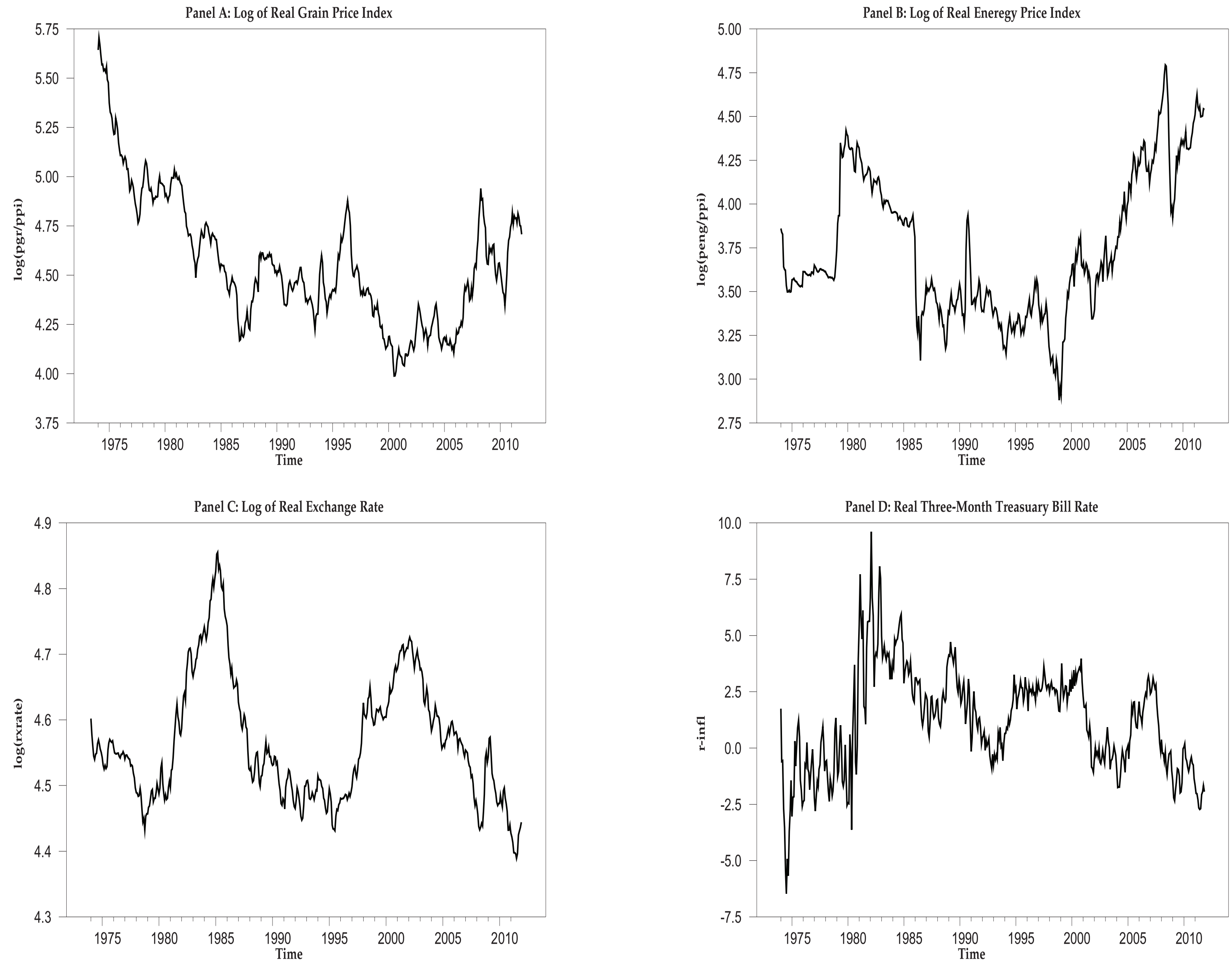


Figure 4: Monthly Data Used in Preliminary VAR Analysis, 1974–2011. Log of Real Grain Price Index (A), Log of Real Price of Energy Index (B), Log of Real Exchange Rate (C), and Real Three–Month Treasury Bill Rate.

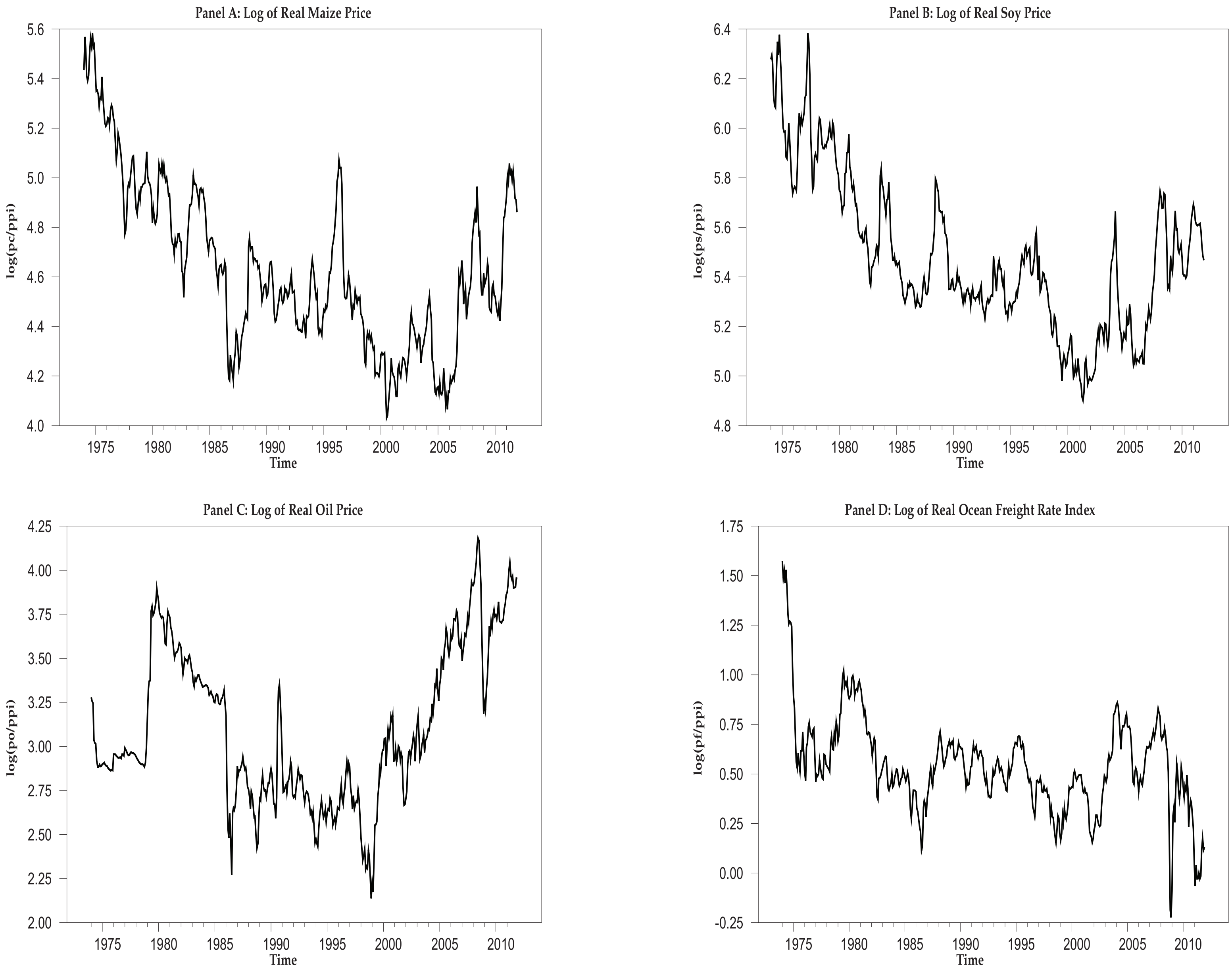


Figure 5: Monthly Data Used in SM–NVAR Analysis, 1974–2011. Log of Real Maize Price (A), Log of Real Soy Price (B), Log of Real Crude Oil Price (C), and Log of Real Ocean Freight Index (D).

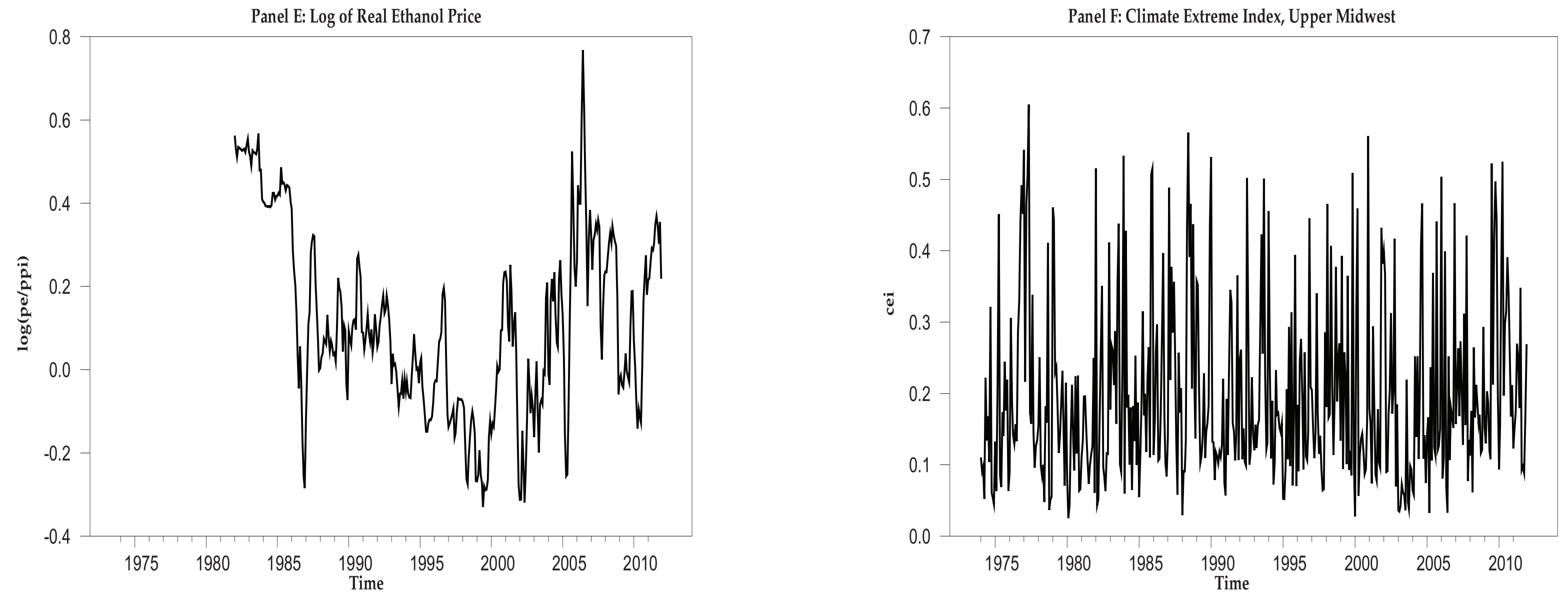


Figure 5: (Continued). Monthly Data Used in SM–NVAR Analysis, 1974–2011. Log of Real Ethanol Price (E) and Climate Extreme Index (F).

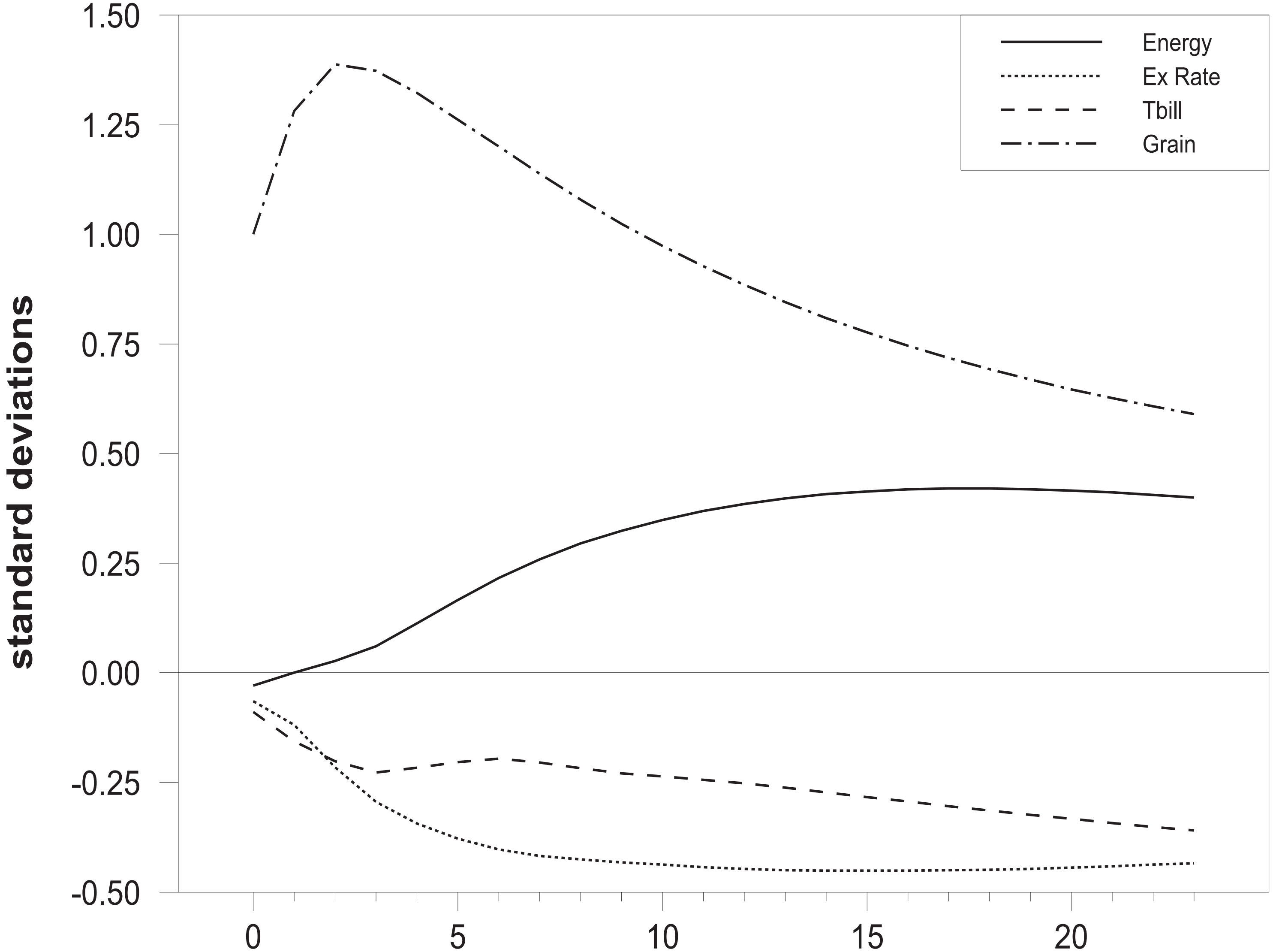


Figure 6: Impulse Response Functions for Grain with Respect to Energy Price (dotted line); the Real Exchange Rate (dotted line); the Real Treasury Bill Rate (dashed line); and Own Grain Price (dash-dot line). All Impulse Response Functions are Normalized by the Standard Deviation of Grain.

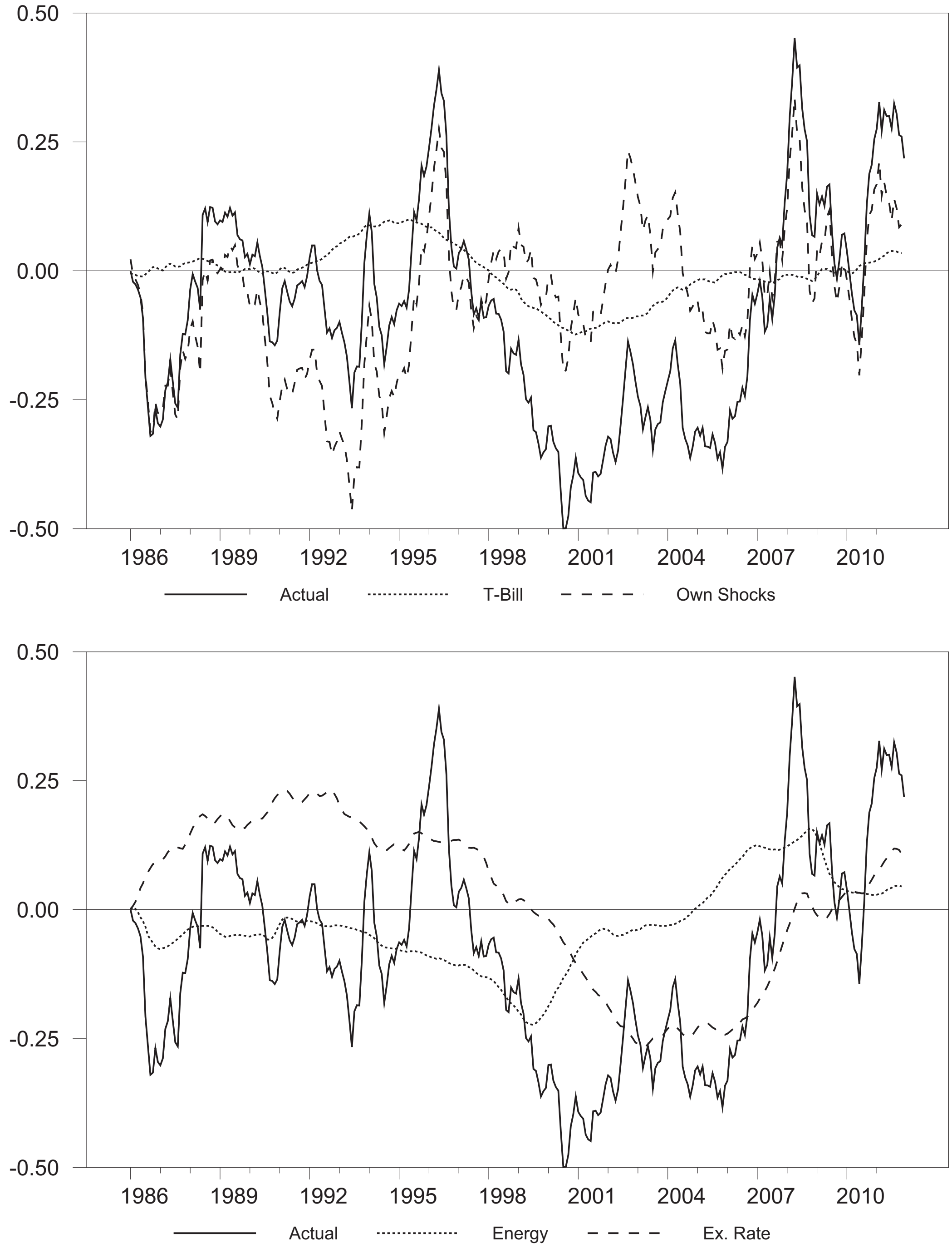


Figure 7: Historical Decompositions of Real Grain Prices with Respect to the Real Treasury Bill Rate and Own Shocks (Top Panel) and Real Energy Price and Real Exchange Rate (Bottom Panel).

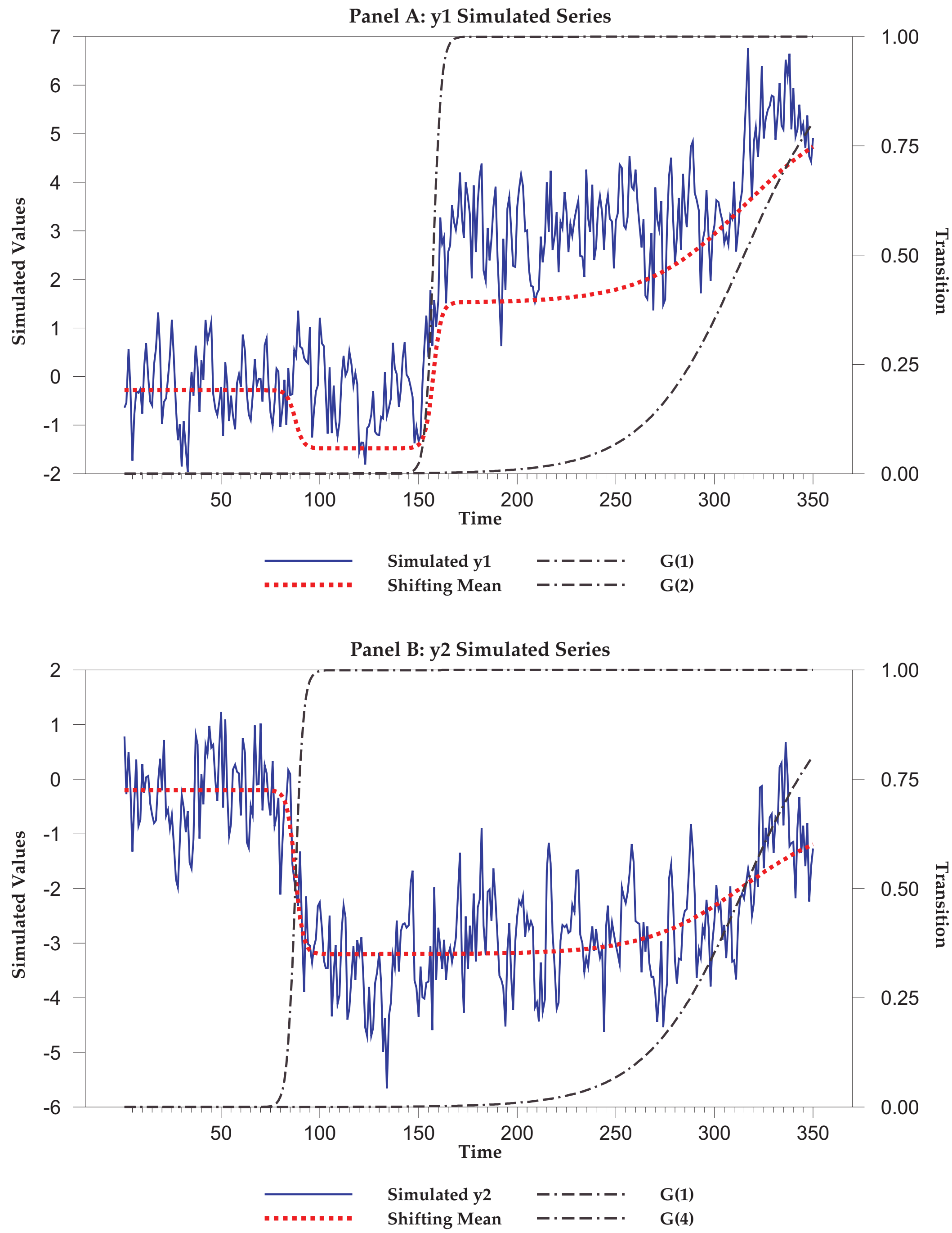


Figure 8: A Single Realization for the Bivariate Simulated System with $\rho = 0.5$: (Panel A) y_{1t} and (Panel B) y_{2t}. The dashed lines indicate the shifting means of the DGP and the dash–dot lines indicate the true transition functions.

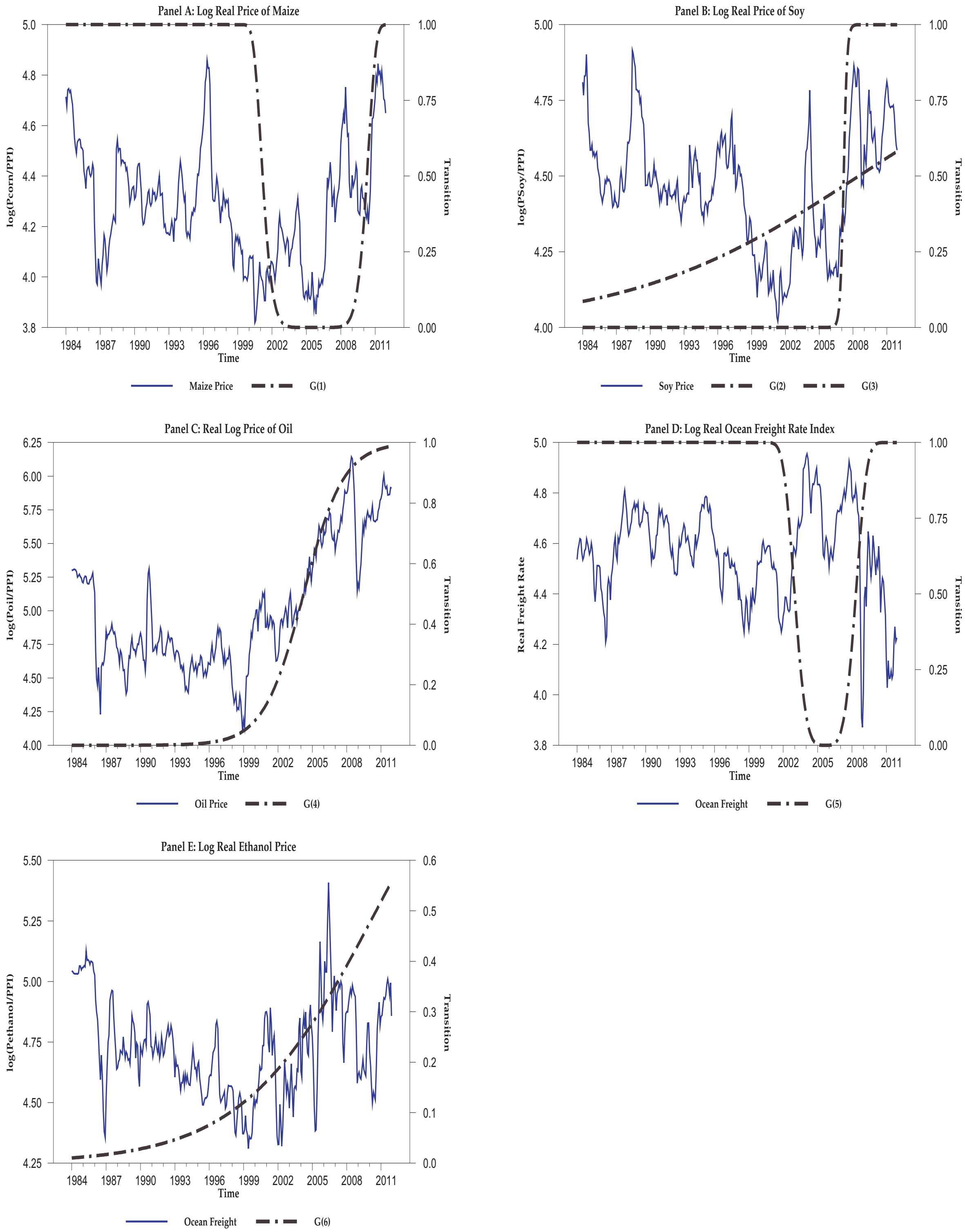


Figure 9: Data and Estimated Transition Functions, 1984–2011. Maize (A), Soy (B), Oil (C), Ocean Freight Rate (D), and Ethanol (E).

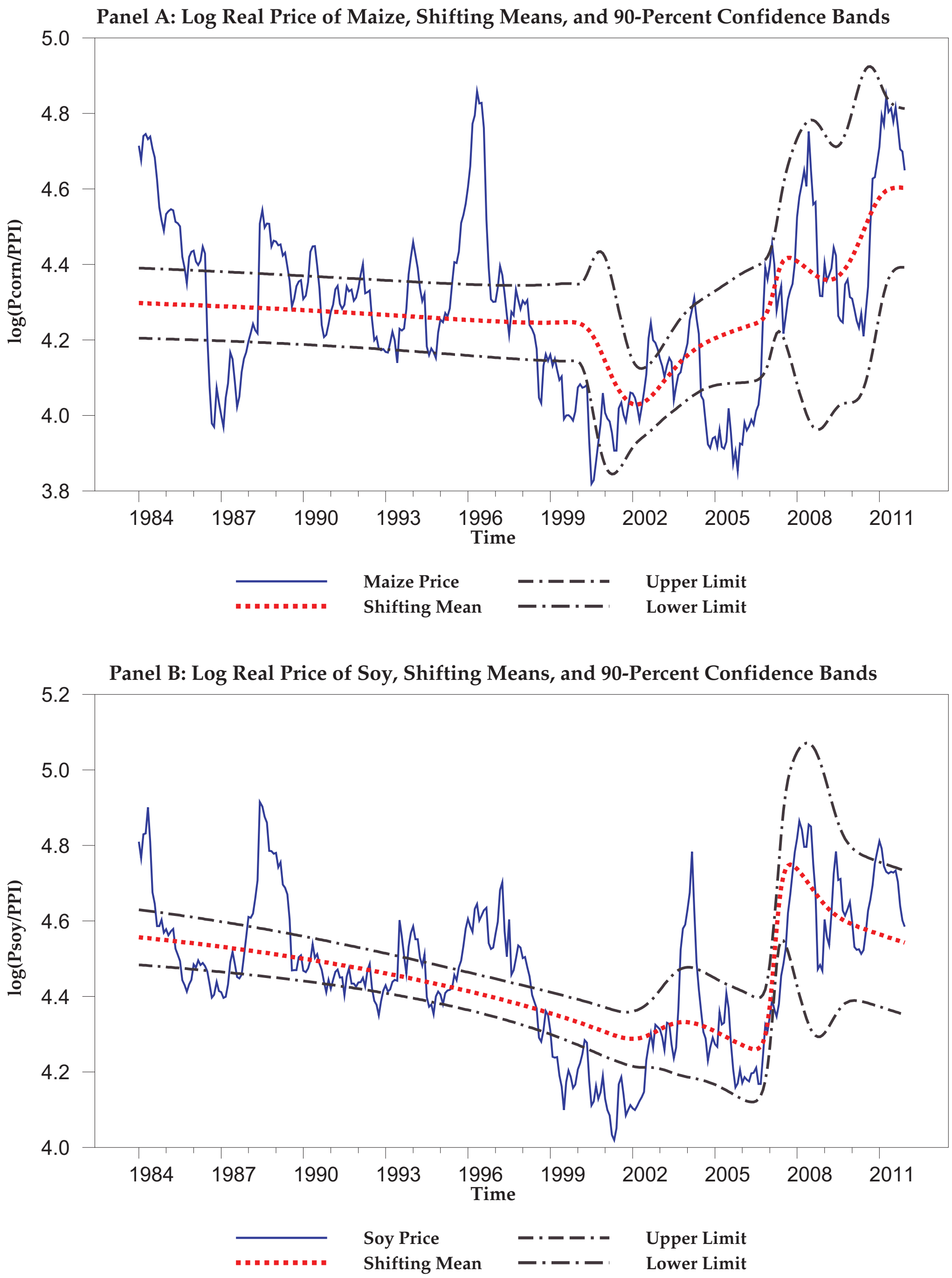
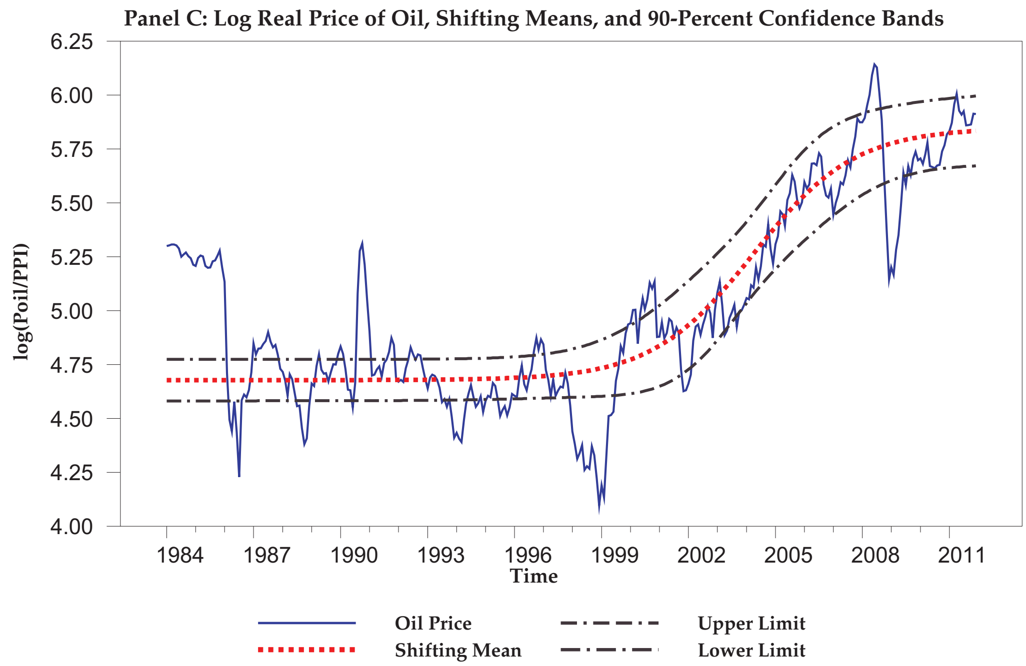


Figure 10: Observed log Real Prices, Shifting Means, and 90–percent Confidence Intervals: (Panel A) Maize,(Panel B) Soy, (Panel C) Oil, (Panel D) Ocean Freight, (Panel E) Ethanol.



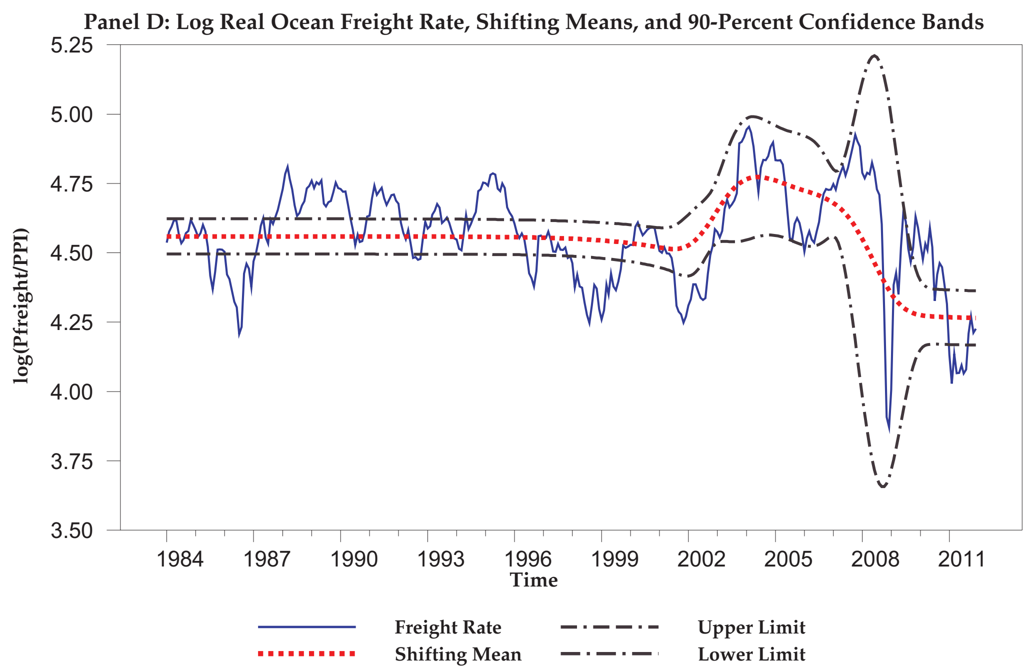


Figure 10: (Continued.)

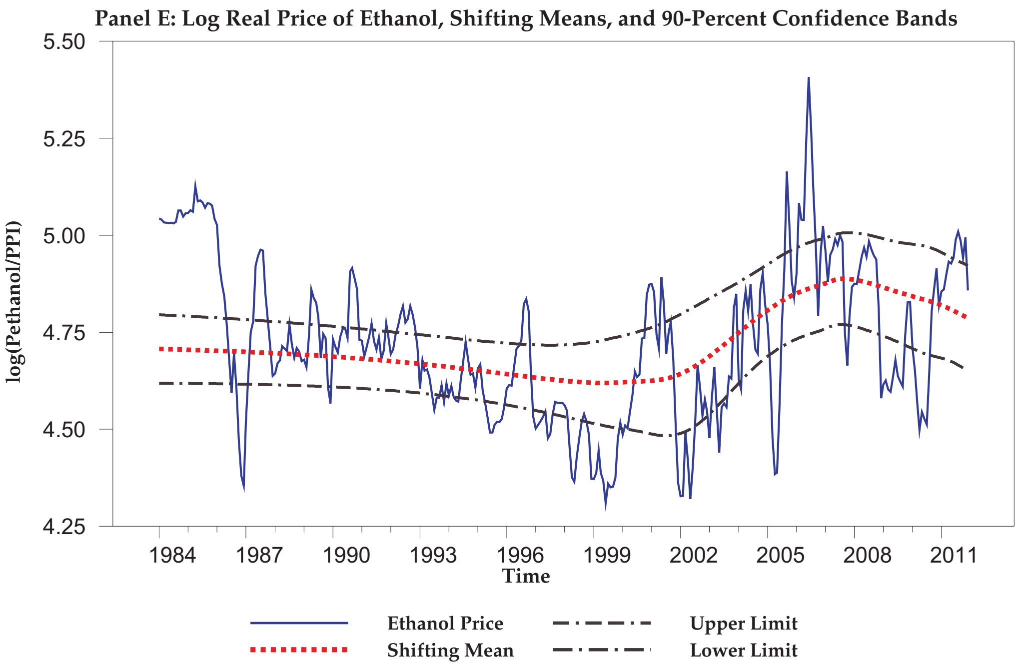


Figure 12: (Continued).

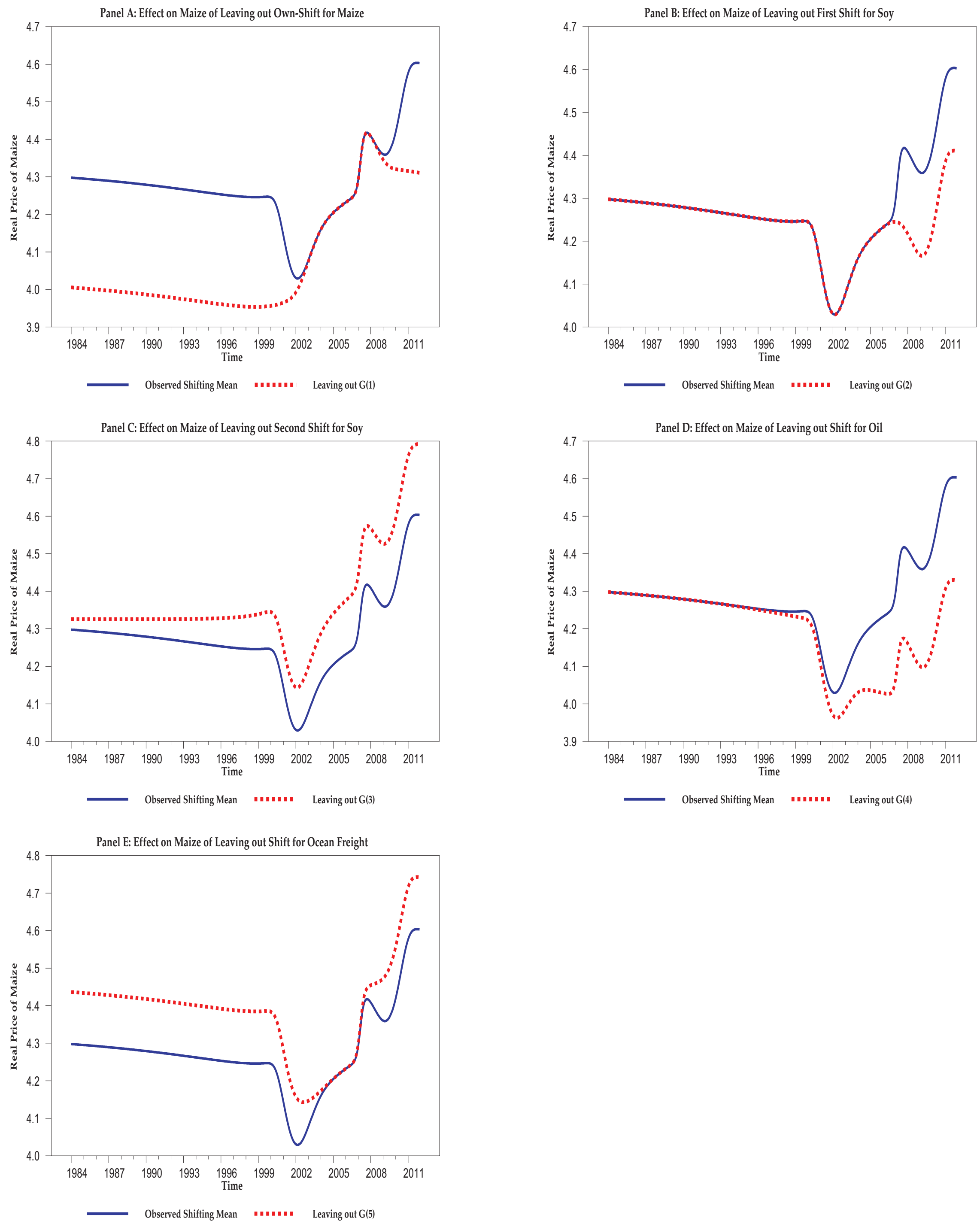


Figure 11: Comparative Dynamics of the Shifting Mean for Real Maize Price with Excluded Shifts, 1984–2011. Excludes Maize's Own Shift (A), Excludes First Soy Shift (B), Excludes Second Soy Shift (C), Excludes Oil Shift (D), and Excludes Ocean Freight Shift (E).

Figure 12: Comparative Dynamics of the Shifting Mean for Real Soy Price with Excluded Shifts, 1984–2011. Excludes Soy's First Own-Shift (A), Excludes Soy's Second Own-Shift (B), Excludes Oil Shift (C), and Excludes Ocean Freight Shift (D).

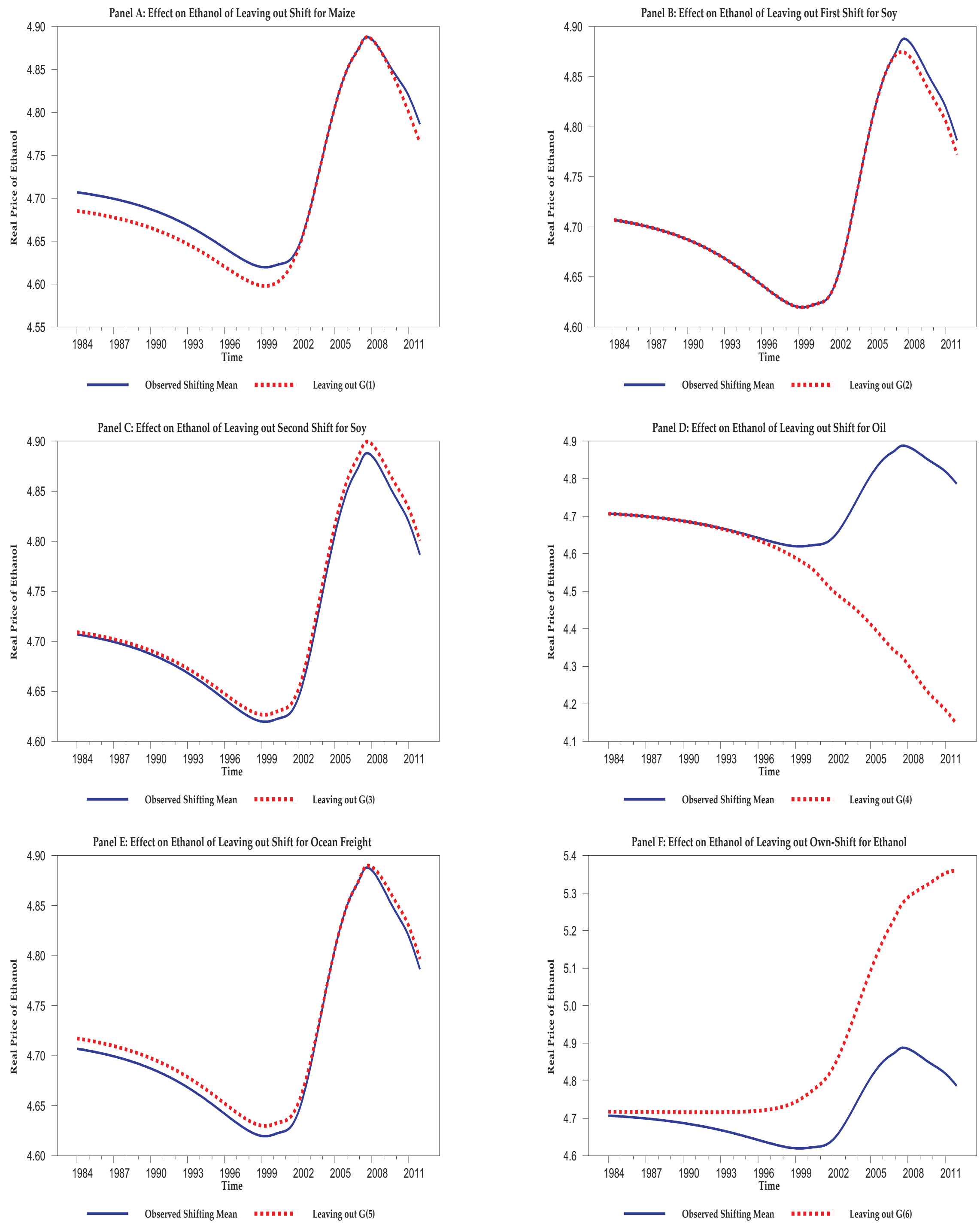


Figure 13: Comparative Dynamics of the Shifting Mean for Real Soy Price with Excluded Shifts, 1984–2011. Excludes Soy's First Own-Shift (A), Excludes Soy's Second Own-Shift (B), Excludes Oil Shift (C), and Excludes Ocean Freight Shift (D).

Appendix Table A1: Results of Intercept Constancy Tests for Select Commodity Prices.

Commodity	H_0	H_{03}	H_{02}	H_{01}	H_0'	H_{0E}	H_{0L}	Shift Type
Maize	0.056	0.006	0.835	0.995	**0.030**	0.008	0.050	Exponential
Soybeans	**0.021**	0.029	0.030	0.644	**0.045**	0.206	0.807	Logistic
Oil	$\mathbf{1.60 \times 10^{-4}}$	0.328	1.63×10^{-3}	2.33×10^{-3}	$\mathbf{1.08 \times 10^{-4}}$	0.047	0.024	Undetermined
Freight	0.792	0.332	0.764	0.955	**0.027**	4.89×10^{-3}	6.81×10^{-3}	Exponential
Ethanol	$\mathbf{1.25 \times 10^{-3}}$	0.369	0.176	2.82×10^{-4}	$\mathbf{3.10 \times 10^{-3}}$	0.781	0.718	Logistic
Climate Index	0.196	0.648	0.037	0.734	0.319	0.987	0.805	–

Note: The column headed H_0 includes approximate p-values for a test of the null hypothesis in (15) obtained by including third-order terms in the trend variable in testing equation (14). Columns headed H_{03}, H_{02}, and H_{03} record p-values for the testing sequence in (17), as proposed by Lin and Teräsvirta (1994). Similarly, the column headed H_0' includes approximate p-values for a test of the null hypothesis in (19) obtained by including fourth-order terms in the trend variable in testing equation (14). Columns headed H_{0E} and H_{0L} report p-values for the testing sequence in (21), as proposed by Lin and Teräsvirta (1994) Escribano and Jordà (1999). Bolded numbers in the H_0 and H_0' indicate that the null hypothesis of no intercept shifts is rejected at the 0.05 significance level. Underlined numbers in the columns headed H_{03}, H_{02}, and H_{03} and, likewise, H_{0E} and H_{0L}, indicate the minimal p-value in the testing sequence. The final column indicates the likely nature of the intercept shift as determined from the testing sequences.

Appendix Table A2: Single-Equation Model Assessment and Diagnostic Test Results.

Measure	Maize	Soybeans	Oil	Freight	Ethanol
No. Shifts	1	2	1	1	1
Shift Type	GEXP	LOGIT	LOGIT	GEXP	LOGIT
$\hat{\kappa}$	4	--	--	2	--
R^2	0.943	0.944	0.970	0.920	0.885
$\hat{\sigma}_{\varepsilon}$	0.054	0.046	0.079	0.054	0.067
$\hat{\sigma}_{\varepsilon,NL}/\hat{\sigma}_{\varepsilon,L}$	0.993	0.991	0.967	0.977	0.968
AIC	-2.960	-3.299	-2.224	-2.973	-2.528
HQC	-2.895	-3.248	-2.197	-2.926	-2.468
AR(4)	0.714	0.595	0.568	0.753	0.388
AR(6)	0.780	0.458	0.142	0.870	0.638
AR(12)	0.333	0.590	0.076	0.056	0.797
ARCH(6)	0.959	0.458	0.142	2.97×10^{-16}	5.45×10^{-4}
ARCH(12)	0.845	0.590	0.001	2.60×10^{-13}	4.55×10^{-5}
H_0	0.132	0.799	0.515	0.083	0.168
H_0'	0.073	0.469	0.675	0.078	0.251
LJB	105.46	121.20	181.53	324.07	16.75

Note: The effective sample size, T, is 323 observations. No. of Shifts indicates the number of intrinsic intercept shifts estimated for each equation. Shift Type indicates whether the intercept shift is of the generalized exponential (GEXP) or logistic (LOGIT) form. $\hat{\kappa}$ indicates the estimated value for the κ parameter in the generalized exponential shift function, determined by simple grid search. R^2 is the unadjusted R^2, and $\hat{\sigma}_{\varepsilon}$ is the residual standard error. $\hat{\sigma}_{\varepsilon,NL}/\hat{\sigma}_{\varepsilon,L}$ is the ratio of the respective standard error from the shifting-mean model relative to the constant intercept model. AIC is the Akaike Information Criterion, and HQC is the Hannan-Quinn Information Criterion. AR(j), j = 4, 6, 12, is the p-value from an F-version of the LM test for remaining autocorrelation up to lag j. Entries for ARCH(j), j = 6,12 are similarly defined for ARCH errors up to lag j. Entries for H_0 are p-values from an F-version of an LM test for remaining intercept shifts based on using third-order terms in t^*. Likewise, values for H_0' are p-values from an F-version of an LM test for remaining intercept shifts based on using fourth-order terms in t^*. LJB is the Lomnicki-Jarque-Bera test of normality of the residuals (critical value from the χ^2 (2) distribution is 13.82 at the 0.001 significance level).

Appendix Table A3: Single Equation Lagrange Multiplier Test Results for Excluded Variables.

Null Hypothesis	p-value
No Lagged Ethanol Price Effects in Maize Price Eqn.	0.073
No Lagged Maize Price Effects in Soy in Maize Price Eqn.	0.178
No Lagged Oil Price Effects in Soy Price Eqn.	0.165
No Lagged Ethanol Price Effects in Soy Price Eqn.	0.096
No Lagged Climate Extreme Effects in Soy Price Eqn.	0.832
No Lagged Maize Price Effects in Oil Price Eqn.	0.608
No Lagged Soy Price Effects in Oil Price Eqn.	0.858
No Lagged Ocean Freight Rate Effects in Oil Price Eqn.	0.490
No Lagged Ethanol Price Effects in Oil Price Eqn.	0.724
No Lagged Climate Extreme Effects in Oil Price Eqn.	0.160
No Lagged Maize Price Effects in Ocean Freight Rate Eqn.	0.409
No Lagged Soy Price Effects in Ocean Freight Rate Eqn.	0.072
No Lagged Ethanol Price Effects in Ocean Freight Rate Eqn.	0.074
No Lagged Climate Extreme Effects in Ocean Freight Rate Eqn.	0.070
No Lagged Soy Price Effects in Ethanol Price Eqn.	0.960
No Lagged Climate Extreme Effects in Ethanol Price Eqn.	0.250

Note: In all instances the null hypothesis is that lagged values of the variable indicated should be excluded from the equation indicated. Entries in the column headed p-values are approximate $p-$values from an F-version of an LM test of the indicated null hypothesis. All tests were performed in a manner consistent with the diagnostic testing framework for smooth transition models outlined by Eitrheim and Teräsvirta (1996).

Appendix Table A4: SM-VAR Estimation Results

Panel A: Maize Price, $y_{1t} = \ln\left(pc_t/ppi_t\right)$

$$y_{1t} = \Bigg(-\underset{(0.519)}{0.201} + \underset{(0.075)}{0.293}\; G_1\left(t^*; \hat{\eta}_1, \hat{c}_1\right)\Bigg)\underset{(0.008)}{0.085} + \underset{(0.053)}{1.081}\, y_{1t-1} + \Bigg(1 - \underset{(0.053)}{1.081} - \underset{(0.008)}{0.085}\Bigg) y_{1t-2} + \underset{(0.006)}{0.210}\, y_{2t-1} - \underset{(0.005)}{0.182}\, y_{2t-2} - \underset{(0.005)}{0.042}\, y_{3t-1}$$

$$+ \underset{(0.004)}{0.070}\, y_{3t-2} + \underset{(0.008)}{0.092}\, y_{4t-1} - \underset{(0.004)}{0.072}\, y_{4t-2} + \underset{(0.016)}{0.047}\, y_{6t-1} - \underset{(0.014)}{0.018}\, y_{6t-1} + \hat{\varepsilon}_{1t},$$

$$G_1\left(t^*; \hat{\eta}_1, \hat{c}_1\right) = 1 - \exp\left\{-\exp\left(\underset{(-)}{3.912}\right)\left[\left(t^* - \underset{(0.037)}{0.770}\right)\Big/\hat{\sigma}_{t^*}\right]^8\right\}, \quad \mathrm{R}^2 = 0.942$$

Panel B: Soybean Price, $y_{2t} = \ln\left(ps_t/ppi_t\right)$

$$y_{2t} = \Bigg(-\underset{(0.399)}{2.944} + \underset{(0.144)}{0.582}\; G_2\left(t^*; \hat{\eta}_2, \hat{c}_2\right) - \underset{(0.228)}{0.982}\; G_3\left(t^*; \hat{\eta}_3, \hat{c}_3\right)\Bigg)\underset{(0.020)}{0.090} + \underset{(0.097)}{1.150}\, y_{2t-1} + \Bigg(1 - \underset{(0.097)}{1.150} - \underset{(0.020)}{0.090}\Bigg) y_{2t-2} + \underset{(0.012)}{0.086}\, y_{4t-1}$$

$$- \underset{(0.013)}{0.052}\, y_{4t-2} + \hat{\varepsilon}_{2t},\; G_2\left(t^*; \hat{\eta}_2, \hat{c}_2\right) = \left[1 + \exp\left\{-\exp\left(\underset{(-)}{3.912}\right)\left(t^* - \underset{(0.003)}{0.824}\right)\Big/\hat{\sigma}_{t^*}\right\}\right]^{-1},$$

$$G_3\left(t^*; \hat{\eta}_3, \hat{c}_3\right) = \left[1 + \exp\left\{-\exp\left(\underset{(-)}{-0.288}\right)\left(t^* - \underset{(0.003)}{0.874}\right)\Big/\hat{\sigma}_{t^*}\right\}\right]^{-1}, \quad \mathrm{R}^2 = 0.944$$

Panel C: Oil Price, $y_{3t} = \ln\left(po_t/ppi_t\right)$

$$y_{3t} = \Bigg(\underset{(0.075)}{0.292} + \underset{(0.115)}{1.170}\; G_4\left(t^*; \eta_4, c_4\right)\Bigg)\underset{(0.025)}{0.105} + \underset{(0.072)}{1.193}\, y_{3t-1} + \Bigg(1 - \underset{(0.072)}{1.193} - \underset{(0.025)}{0.105}\Bigg) y_{3t-2} + \hat{\varepsilon}_{3t},$$

$$G_4\left(t^*; \eta_4, c_4\right) = \left[1 + \exp\left\{-\exp\left(\underset{(0.380)}{1.490}\right)\left(t^* - \underset{(0.037)}{0.770}\right)\Big/\hat{\sigma}_{t^*}\right\}\right]^{-1},\; \mathrm{R}^2 = 0.972$$

Table A4: Continued

Panel D: Ocean Freight Rate, $y_{4t} = \ln(pf_t/ppi_t)$

$$y_{4t} = \Big(\underset{(0.181)}{6.131} - \underset{(0.110)}{0.383}\, G_5(t^*;\hat{\eta}_5,\hat{c}_5)\Big)\underset{(0.030)}{0.086} + \underset{(0.002)}{0.111}\, y_{3t-1} - \underset{(0.002)}{0.138}\, y_{3t-2} + \underset{(0.003)}{0.005}\, y_{3t-3} + \underset{(0.099)}{1.312}\, y_{4t-1} - \underset{(0.138)}{0.595}\, y_{4t-2}$$

$$\Big(1 - \underset{(0.099)}{1.312} + \underset{(0.138)}{0.595} - \underset{(0.030)}{0.086}\Big) y_{4t-3} + \hat{\varepsilon}_{4t},\; G_5(t^*;\hat{\eta}_5,\hat{c}_5) = 1 - \exp\left\{-\exp\Big(\underset{(1.874)}{3.900}\Big)\left[\Big(t^* - \underset{(0.045)}{0.768}\Big)\Big/\hat{\sigma}_{t^*}\right]^4\right\},\; \mathrm{R}^2 = 0.920$$

Panel E: Ethanol, $y_{5t} = \ln(pe_t/ppi_t)$

$$y_{5t} = \Big(\underset{(0.114)}{1.889} - \underset{(0.102)}{1.029}\, G_6(t^*;\hat{\eta}_6,\hat{c}_6)\Big)\underset{(0.027)}{0.206} + \underset{(0.004)}{0.120}\, y_{1t-1} + \underset{(0.004)}{0.057}\, y_{1t-2} + \underset{(0.003)}{0.162}\, y_{1t-3} - \underset{(0.006)}{0.150}\, y_{3t-1} - \underset{(0.005)}{0.115}\, y_{3t-2} + \underset{(0.004)}{0.076}\, y_{3t-3}$$

$$+\ \underset{(0.099)}{1.120}\, y_{5t-1} - \underset{(0.070)}{0.451}\, y_{5t-2} + \Big(1 - \underset{(0.099)}{1.120} + \underset{(0.070)}{0.451} - \underset{(0.027)}{0.206}\Big) y_{5t-3} + \hat{\varepsilon}_{5t},$$

$$G_6(t^*;\hat{\eta}_6,\hat{c}_6) = \left[1 + \exp\left\{-\exp\Big(\underset{(0.278)}{0.290}\Big)\Big(t^* - \underset{(-)}{0.950}\Big)\Big/\hat{\sigma}_{t^*}\right\}\right]^{-1},\; \mathrm{R}^2 = 0.885$$

Panel F: Climate Extreme Index, $y_{6t} = cei_t$

$$y_{6t} = \Big(\underset{(0.014)}{0.201}\Big)\underset{(0.064)}{0.823} + \Big(1 - \underset{(0.064)}{0.823}\Big) y_{6t-1} + \hat{\varepsilon}_{6t},\; \mathrm{R}^2 = 0.029$$

Note: Asymptotic heteroskedasticity robust standard errors are given below parameter estimates in parentheses; R^2 is the squared correlation between actual and fitted values for each equation; $\hat{\varepsilon}_{jt}$ denotes the j'th equation's residual at time t, $j = 1, \ldots, 6$.

Appendix Table A5: SM-VAR Summary Statistics.

$$\ln L_{SM-VAR} = 2606.598,$$

$$\mathrm{AIC}_{SM-VAR} = -32.690,\ \mathrm{AIC}_{VAR} = -32.560,$$

$$\mathrm{HQC}_{SM-VAR} = -32.331,\ \mathrm{HQC}_{VAR} = -32.284$$

$$\tilde{\mathrm{R}}^2 = 0.999,\ \tilde{\mathrm{R}}^{*2} = 0.121$$

System Covariance Matrix:

$$\hat{\boldsymbol{\Sigma}} = \boldsymbol{\Upsilon P \Upsilon}, \text{ where}$$

$$\boldsymbol{P} = \{\rho_{ij}\} = \begin{array}{c} \\ y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \\ y_{5t} \\ y_{6t} \end{array} \begin{array}{c} \begin{array}{cccccc} y_{1t} & y_{2t} & y_{3t} & y_{4t} & y_{5t} & y_{6t} \end{array} \\ \begin{pmatrix} 1 & 0.527 & -0.054 & -0.024 & -0.012 & -0.038 \\ & 1 & -0.002 & 0.094 & -0.011 & -0.067 \\ & & 1 & 0.119 & 0.305 & -0.010 \\ & & & 1 & -0.003 & -0.018 \\ & & & & 1 & -0.021 \\ & & & & & 1 \end{pmatrix} \end{array}$$

$$\boldsymbol{\Upsilon} = \mathrm{diag}\{\hat{\sigma}_1, \ldots, \hat{\sigma}_6\} = \mathrm{diag}\{0.053, 0.045, 0.078, 0.053, 0.066, 0.122\}$$

Note: AIC is the system Akaike Information Criterion and HQC denotes the system Hannan-Quinn Criterion. A subscripted SM-VAR refers to the model estimated as a shifting-mean vector autoregression and a subscripted VAR refers to a standard VAR model without intercept shifts. $\tilde{\mathrm{R}}^2$ denotes the likelihood system R^2 defined by Magee (1990); while $\tilde{\mathrm{R}}^{*2}$ indicates the relative contribution to $\tilde{\mathrm{R}}^2$ of the intercept shifts. $\boldsymbol{P}$ indicates the estimated correlation matrix, and $\boldsymbol{\Upsilon}$ is a diagonal matrix with the square root of each equation's estimated error variance on the main diagonal. $i = 1 = \ln(pc_t/ppi_t),\ i = 2 = \ln(ps_t/ppi_t)$, $i = 3 = \ln(po_t/ppi_t)$, $i = 4 = \ln(pf_t/ppi_t)$, $i = 5 = \ln(pe_t/ppi_t)$, $i = 6 = cei_t$. $i = j = 1, \ldots, 6$.