Bubbles in Prices of Exhaustible Resources*

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Abstract

This paper proposes a test for the presence of a bubble in the price of an exhaustible resource. A bubble is accompanied by a rise in the storage-to-consumption ratio: Consumption peters out, and a fraction of the original stock is held forever. The test suggests there is a bubble in the price oil and in the market for high-end Bordeaux wines, but other explanations are also possible. A bubble reduces welfare regardless of whether there are other stores of value, particularly fiat money.

1 Introduction

A non-reproducible durable good that is in fixed supply is a potential candidate for a bubble. Exhaustible resources are such durables; an inflating bubble on such goods cannot defeat itself by eliciting supply that exceeds what asset holders want to hold.

The paper proposes a test for bubbles designed for assets that are depleted via consumption. The test tracks the ratio of the resource stock relative to its consumption. The price of the resource contains a bubble if the ratio in question rises over time. When there is no bubble the ratio should remain constant or decline. The test suggests that the price of oil contains a bubble, as do the prices of some high-end wines.

The paper starts with Hotelling's (1931) model which has a continuum of bubble equilibria; in Hotelling's world the price of the resource must rise at the rate of interest even with*out* the bubble, and so one designate a fraction of the resource as

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destined for eternal storage. This raises the initial price of the resource, and the bubble is identified not by prices but by storage. The paper then shows that the storage-to-consumption ratio does not rise when there is no bubble.

Next, the paper considers oil and high-end wine in turn, and the test suggests that bubbles exist in both markets. Oil reserves relative to consumption have doubled over the last 30 years. As for vintage wine, there are no data on the amount stored, but after a few decades, very little wine sells at consumption outlets such as dealers, yet there is continued and active trading in the asset at auctions run by Christie's, Sotheby's, etc..

Finally in a general equilibrium overlapping-generations setting along the general lines of Samuelson (1958), Wallace (1980) and Tirole (1985), the paper looks at the welfare effects of a bubble on an exhaustible resource with and without the presence of outside money. A bubble reduces welfare because it needlessly retards the consumption of the resource; this remains true when other assets are available as stores of value, in particular outside money. An exception — real enough in the case of oil – arises if consuming the resource entails a negative external effect, in which case the bubble can *raise* welfare by correcting the externality.

Plan of paper.—Section 2 presents the partial equilibrium, one-capital "Hotelling" version which also contains the main argument. The strategy is to present the simplest case first, and then modify it as we go along. Section 3 looks at the counterfactual – what do we expect to see when there is no bubble. Sections 4 and 5 apply the model to the case of oil and vintage wine in turn. Section 6 studies an OG environment where a bubble does arise, with and without fiat money. Section 7 concludes the paper, and the Appendix contains several modifications of the model, and it describes the data in more detail.

2 Bubble equilibria

This section uses a partial equilibrium setting; it assumes that enough saving is forthcoming to absorb the rising value of the bubble. Section 6 will complete the discussion to general equilibrium and will draw out the welfare implications.

Consider a non-renewable resource, or "capital," that does not depreciate, and that cannot be augmented via investment or discovery. Hotelling's (1931) version of the problem goes as follows. Let the interest rate be r, and let the market demand for consuming the capital be $x = D(p)$. Capital can be delivered to consumers costlessly.[1] Suppose that $D(p) > 0$ for all $p < \infty$, implying an unbounded willingness to pay at small levels of consumption, which translates into an Inada condition on the utility function.[2] The capital must then be consumed at every date for, if at some date it were not consumed, its price would at such dates be infinite. But if supply is to be

[1] This assumption is dropped in Section 4 when we discuss oil prices.

[2] We relax this in Section 3.2.

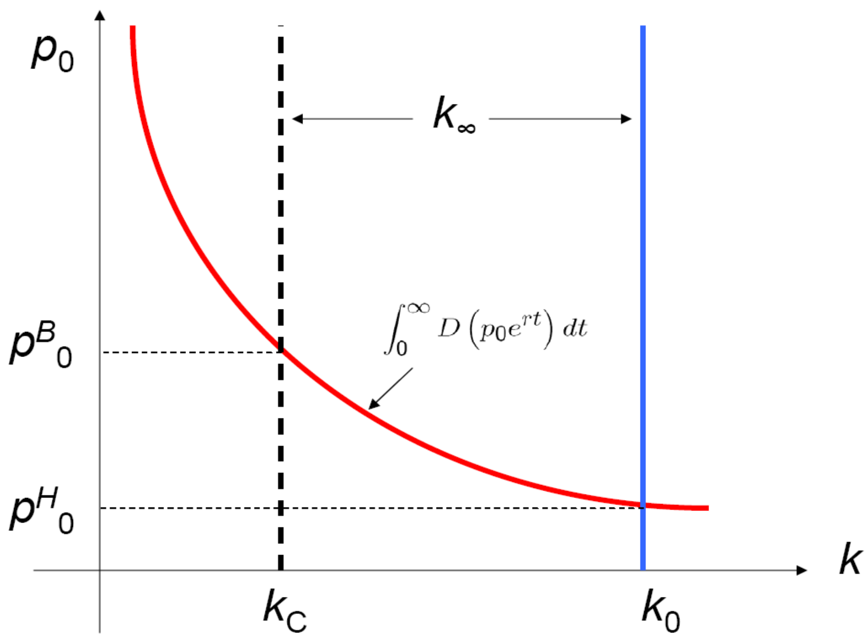

Figure 1: THE DETERMINATION OF THE EQUILIBRIA

positive at each date, we must have

$$p_t = p_0 e^{rt}$$

for some $p_0 > 0$.[3]

Hotelling's equilibrium.—To solve for p_0, Hotelling requires that the resource be fully exhausted:

$$k_0 = \int_0^\infty D\left(p_0 e^{rt}\right) dt. \tag{1}$$

Since D is strictly monotone, the solution for p_0 is unique and so, therefore, is equilibrium, and also the social optimum.[4] Moreover, at each date the price, p_t, of the asset equals the present value of the stream of dividends to which it is a claim.

Bubble equilibria.—We replace (1) by two conditions. The first states that k_0 is divided into a stock, k_c, that will at some point be consumed, and a stock, k_∞, that speculators hold for ever:

$$k_0 = k_c + k_\infty. \tag{2}$$

[3]We interpret r as net of any convenience yield or carrying costs of holding the asset. We analyze storage costs and convenience yield when we discuss wine prices in Section 5.

[4]The GE version of the Planner's problem is analyzed in Section 4.

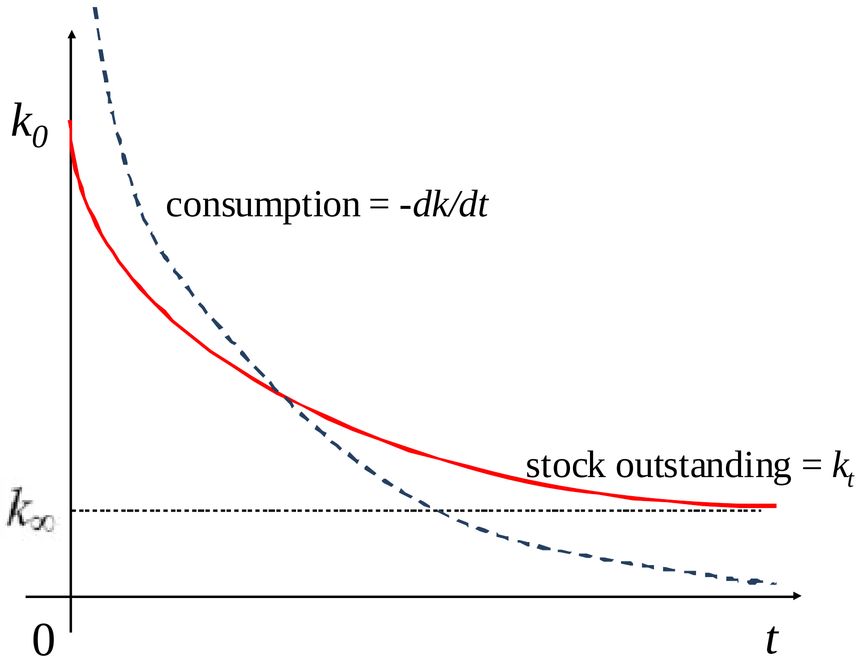

Figure 2: THE EVOLUTION OF k, AND OF CONSUMPTION

The second states that k_c is eventually exhausted:

$$k_c = \int_0^\infty D\left(p_0 e^{rt}\right) dt. \tag{3}$$

Hotelling's equilibrium is the one for which $k_c = k_0$. We say that a bubble exists if $k_\infty > 0$.

Figure 1 shows how the initial prices p_0 are determined – the Hotelling equilibrium, p_0^{H} is the lowest, and in a bubble equilibrium the date-zero prices, p_0^B, are higher. Any $k_\infty \in [0, k_0)$ is valid as long as the economy can absorb the bubble. Future sellers and speculators earn the same present value of revenues at each date, and there is no gains to arbitrage between the two markets. Figure 2 plots k_t in the left panel and the relation between consumption and k.

We conclude that if there is a bubble, k/c must rise and approach infinity. But the rise if need not be monotonic. In particular, $D(p)$ may, for some range of prices, have a highly *in*elastic portion. As p_t rises and passes through that range, c would remain constant, but k would continue to fall, so that the ratio k/c would temporarily decline.

2.1 Symmetric mixed strategy bubble equilibria

These connect our bubble test to statistical reliability theory, and to the usual test for bubbles. The owner of a unit of k can follow the mixed strategy "Sell a unit of k to consumers at date t with probability $\pi_t(p_0)\,dt$, where

$$\pi_t(p_0) \equiv \frac{D(p_0 e^{rt})}{k_0}, \tag{4}$$

and where the realizations are independent over agents so that there is no aggregate risk. Define the *consumption hazard*

$$h_t \equiv \frac{\pi_t}{1 - \int_0^t \pi_s ds} \tag{5}$$

Bubbles and defective waiting-time distributions.—A waiting-time distribution is "defective" if the probability that the event never occurs is positive, i.e., $\int_0^\infty \pi_t(p_0)\,dt < 1$ (Feller 1966, p. 115, note 13). We then have

Proposition 1 *If a bubble exists, the waiting-time distribution implied by the symmetric mixed-strategy equilibrium is defective. In particular,*

$$\int_0^\infty \pi_t dt = \frac{k_c}{k_c + k_\infty} \quad \textit{and} \quad k_\infty > 0 \Rightarrow h_t \to 0. \tag{6}$$

Proof. Integrating the numerator in (4) and applying (2) and (3) delivers the first claim, and the second is a property of any defective distribution. ■

The convergence of h can be non-monotonic: $\frac{dh}{dt} = \pi_t^2 \left(1 - \int_0^t \pi_s ds\right)^{-2} + \frac{D'}{k_0} r p_t = \left(\pi + \frac{D'}{D} r p\right) h \gtrless 0$.

2.2 Re-statement in terms of the standard test for bubbles

The standard test compares the asset's price to the stream of earnings to which it is a claim, its "fundamental." When the fundamental is mismeasured, one may infer that a bubble exists when in fact it does not, as Hamilton and Whitman (1985) stress. The probability of consuming the asset at date $s > t$ conditional on not having yet consumed it is $\left(1 - \int_0^t \pi_s ds\right)^{-1} \pi_s$. Then the fundamental at t is

$$F_t = \frac{\int_t^\infty e^{-r(s-t)} \pi_s p_s ds}{1 - \int_0^t \pi_s ds} = \frac{\int_t^\infty \pi_s ds}{1 - \int_0^t \pi_s ds} p_t, \tag{7}$$

Defined in the standard way (see LeRoy 2004), the bubble is

$$B_t \equiv p_t - F_t = \frac{1 - \int_0^\infty \pi_s ds}{1 - \int_0^t \pi_s ds} p_t, \tag{8}$$

We then have

Proposition 2 *The unconditional expectation of* B_t *grows at the rate of interest,*

$$E_0 B_t = B_0 e^{rt}, \tag{9}$$

whereas conditionally on not the asset not having been consumed before t*, the bubble grows faster:*

$$\frac{d}{dt} \ln B_t = r + h_t \to r \tag{10}$$

Proof. Differentiating the RHS of (8), $\frac{d}{dt} \ln B = \frac{d}{dt} \ln p_t + \pi_t \left(1 - \int_0^t \pi_s ds\right)^{-1}$ and using (5) yields (10). Now (10) implies that $E_0 B_t = b_0 \exp\left(rt + \int_0^t h_s ds\right) \int_t^\infty \pi_s ds$. But since $\int_t^\infty \pi_s ds = \exp\left(-\int_0^t h_s ds\right)$, (9) follows. ■

Stochastic-bubble equilibria.—Thus (9) and (10) describe the evolution of the bubble the surviving resource stock, and while each individual unit has its own random lifetime, aggregates are deterministic. Additionally, equilibria exist in which the aggregate bubble is random, but we defer their discussion to Section 4 when discussing the price of oil.

3 The behavior of k/c when there is no bubble

According to Figure 2, a bubble causes $k_t/c_t \to \infty$. Since we shall later show evidence that this happens in fact with oil and with wine, it is reasonable to ask whether, given reasonable utility functions, such behavior can arise even without bubbles. The latter is easily done, because the assumed absence of a bubble allows us to have the standard infinitely lived agents whose savings behavior satisfies transversality. Therefore, let us study the behavior of k/c in an infinite-horizon, representative-agent economy in which a bubble cannot happen. The answer turns out to be that with standard CRRA preferences, k/c is a constant (see [14]), and with CARA preferences it declines (see [18]).

Consider the cake-eating problem that we shall treat this as the baseline case:

$$\max_{(c)_t} \int_0^\infty e^{-\rho t} U(c_t)\, dt \qquad \text{s.t.} \qquad \int_0^\infty c_t dt \le k_0.$$

The Lagrangian is

$$\int_0^\infty e^{-\rho t} U(c_t)\, dt + \lambda \left(k_0 - \int_0^\infty c_t dt\right). \tag{11}$$

The optimality conditions are

$$e^{-\rho t} U'(c) = \lambda \quad \Rightarrow \quad \frac{dU'}{dt} = \rho U', \tag{12}$$

i.e., that the marginal utility grows at the rate ρ. We show, by example, that without a bubble, k/c is constant in the CRRA-utility case or linearly declining in the CARA case.

3.1 CRRA utility

When there is no bubble and when the utility function is homothetic, the fraction of the resource that is optimally consumed is constant. We shall show this for the CRRA case for which $U(c) = \frac{1}{1-\gamma}c^{1-\gamma}$. Then (12) implies

$$\frac{\dot{c}}{c} = -\frac{\rho}{\gamma} \qquad \Rightarrow c_t = c_0 e^{-\frac{\rho}{\gamma}t} \quad \text{and} \quad k_t = k_0 - \int_0^t c_s ds = k_0 - c_0 \frac{\gamma}{\rho}\left(1 - e^{-\frac{\rho}{\gamma}t}\right) \to 0 \quad (13)$$

Eventual exhaustion of k_0 implies that $c_0 = \frac{\rho}{\gamma}k_0$, and therefore, the reserve-consumption ratio is

$$\frac{k_t}{c_t} = \frac{1}{c_0 e^{-\frac{\rho}{\gamma}t}}\left[k_0 - c_0\frac{\gamma}{\rho}\left(1 - e^{-\frac{\rho}{\gamma}t}\right)\right] = \frac{\gamma}{\rho} \approx \frac{2}{0.05} = 40, \tag{14}$$

a constant. Exhaustion takes for ever. At the standard values for ρ and γ,

$$\frac{k_t}{c_t} = \frac{\gamma}{\rho} \approx \frac{2}{0.05} = 40,$$

which is not a bad prediction of the average of this ratio that we shall plot in Figure 3 below.

3.2 CARA utility

Let $U(c) = 1 - e^{-\gamma c}$. This utility function is not homothetic. We no longer have the Inada condition and we therefore must impose the non-negativity condition on c_t because exhaustion will occur in finite time, at date T.[5] The criterion becomes

$$\max_{T,(c)_t^T} \int_0^T e^{-\rho t}U(c_t)\,dt \qquad \text{s.t.} \qquad \int_0^T c_t dt \leq k_0 \quad \text{and} \quad c_t \geq 0.$$

The Lagrangian is

$$\int_0^T e^{-\rho t}U(c_t)\,dt + \lambda\left(k_0 - \int_0^T c_t dt\right) - \left(\int_0^T \mu_t c_t\right). \tag{15}$$

The FOC w.r.t. T says that $e^{-\rho T}(1 - e^{-\gamma c_T}) = (\lambda + \mu_T)c_T$, so that

$$e^{-\gamma c_T} = 1 - (\lambda + \mu_T)e^{\rho T}. \tag{16}$$

[5] Because T is a variable, we need that the utility of zero consumption for $t \in [0, t]$ is the same as the post exhaustion utility. Hence, in a utility function of the form $A - e^{-\gamma c}$, we must have $A = 1$. Koopmans (1974) studies a related problem where consumption was constrained not by zero but by a positive subsistence level $c_t \geq \bar{c} > 0$, for $t \in [0, T]$.

While (16) is consistent with $c_T = 0$. Now, c_t must decline along the optimal path, and so $\mu_t = 0$ for all $t \in [0, T)$. The FOC w.r.t. c_t then says that for all $t \in [0, T]$, $\gamma e^{-\rho t - \gamma c_t} = \lambda \quad \Rightarrow$

$$\begin{aligned} c_t &= C - \frac{\rho}{\gamma} t, \quad \text{and} \\ T &= \frac{\gamma}{\rho} C \end{aligned}$$

Finally, to solve for λ (i.e., for C), we use the constraint

$$k_0 = \int_0^T c_t dt = CT - \frac{\rho}{2\gamma} T^2 = \frac{\gamma}{\rho} C^2 - \frac{\rho}{2\gamma} \left(\frac{\gamma}{\rho} \right)^2 C^2 = \frac{\gamma}{2\rho} C^2 \Rightarrow C = \sqrt{2\rho k_0} \tag{17}$$

Therefore

$$k_t = k_0 - \int_0^t c_s ds = k_0 - \sqrt{2\rho k_0} t + \frac{\rho}{2} t^2$$

and so[6]

$$\frac{k_t}{c_t} = \sqrt{k_0} - \left(1 - \sqrt{\frac{\rho}{2}} \right) t \tag{18}$$

which declines linearly in t as long as $\rho \leq 2$, i.e., for all reasonable discount rates. Appendix 2 shows how Hotelling's model is modified to handle bounded willingness to pay.

4 Oil

According to Figure 2, a bubble exists if consumption converges to zero while storage remains positive so that the ratio k/c rises without bound. Figure 3 shows world reserves per capita, world consumption per capita (both normalized to 100 in 1980) and the ratio of the two (read off the right axis), the empirical counterpart of k/c, has risen steadily, nearly doubling over the past 30 years.[7]

Can we therefore conclude that there is a bubble in oil prices? There have, however, been discoveries of oil over time and one wonders if, thanks to the flow of discovery such a steady rise in k/c could have occurred even if there was no bubble. We study this question next.

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[6]

$$\begin{aligned} \frac{k_t}{c_t} &= \frac{k_0 - \sqrt{2\rho k_0} t + \rho t^2 - \frac{\rho}{2} t^2}{\sqrt{2\rho k_0} - \rho t} = \frac{k_0 - \frac{\rho}{2} t^2}{\sqrt{2\rho k_0} - \rho t} - t = \frac{\left(\sqrt{k_0} + \frac{\sqrt{\rho}}{\sqrt{2}} t \right) \left(\sqrt{k_0} - \frac{\sqrt{\rho}}{\sqrt{2}} t \right)}{\sqrt{2\rho k_0} - \rho t} - t \\ &= \frac{\frac{1}{\sqrt{2\rho}} \left(\sqrt{2\rho k_0} + \rho t \right) \frac{1}{\sqrt{2\rho}} \left(\sqrt{2\rho k_0} - \rho t \right)}{\sqrt{2\rho k_0} - \rho t} - t = \frac{1}{\sqrt{2\rho}} \left(\sqrt{2\rho k_0} + \rho t \right) - t, \text{ i.e. (18)} \end{aligned}$$

[7] Oil Proved world reserves and world consumption data are taken from BP (2012) and Population data from the United Nations Statistical Division.

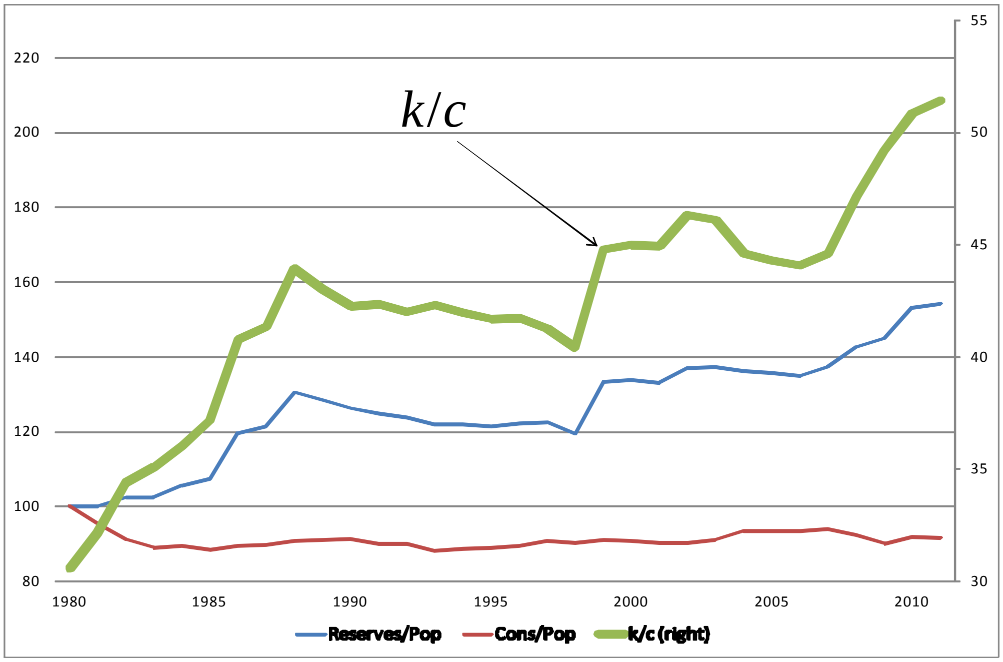

Figure 3: Reserves, Consumption and their ratio, 1981-2011.

4.1 Discovery and the time-path of k/c

Suppose a discovery flow $(\chi_t)_0^\infty$ is perfectly foreseen at date zero.[8] The law of motion for k changes to

$$\frac{dk}{dt} = \chi_t - c_t.$$

Let $\chi^* \equiv \int_0^\infty \chi_t dt < \infty$ denote the sum of all discoveries, assumed finite since the resource is exhaustible. Then (11) becomes

$$\int_0^\infty e^{-\rho t} U(c_t)\, dt + \lambda \left(k_0 + \chi^* - \int_0^\infty c_t dt \right) + \int_0^\infty \mu_t \left(k_0 + \int_0^t \chi_s ds - \int_0^t c_s ds \right) dt. \tag{19}$$

This Lagrangian now includes the continuum of constraints stating that $k_t \geq 0$ for each t, with the multiplier μ_t.

Let us first assume the μ_t are all zero, compute the solution, and then check that $k_t \geq 0$ *ex post*. This works when most of the discovery comes early; the agent then behaves as if he had the stock $k_0 + \chi^*$ and does not violate any constraints.

Then (19) becomes the same as (11) except that in place of k_0, we have $k_0 + \chi^*$. Then (12) applies except that λ is smaller. Since for the case where $U(c) =$

[8] A one-time discovery at an unknown Poisson future date but known size could be analyzed along lines similar to Salant and Henderson (1978). When future discoveries are unknown, the problem becomes similar to the one Loury (1978) studies, in which the size of the reserve is unknown.

$c^{1-\gamma}/(1-\gamma)$,

$$k_t = k_0 + \int_0^t \chi_t dt - c_0 \frac{\gamma}{\rho}\left(1 - e^{-\frac{\rho}{\gamma}t}\right).$$

and since $k_t \to 0$, $c_0 = \frac{\rho}{\gamma}(k_0 + \chi^*)$, which means that

$$k_t = k_0 + \int_0^t \chi_\tau d\tau - (k_0 + \chi^*)\left(1 - e^{-\frac{\rho}{\gamma}t}\right) = (k_0 + \chi^*)\, e^{-\frac{\rho}{\gamma}t} - \int_t^\infty \chi_\tau d\tau,$$

i.e., that

$$\frac{k_t}{c_t} = \frac{1}{c_0 e^{-\frac{\rho}{\gamma}t}}\left[k_0 + \int_0^t \chi_\tau d\tau - c_0 \frac{\gamma}{\rho}\left(1 - e^{-\frac{\rho}{\gamma}t}\right)\right] = \frac{\gamma}{\rho}\left(1 - \frac{e^{\frac{\rho}{\gamma}t}\int_t^\infty \chi_\tau d\tau}{\frac{\rho}{\gamma}(k_0 + \chi^*)}\right). \quad (20)$$

Then

$$\frac{d}{dt}\left(\frac{k}{c}\right) > 0 \quad \Longleftrightarrow \frac{d}{dt} e^{\frac{\rho}{\gamma}t}\int_t^\infty \chi_\tau d\tau < 0, \quad (21)$$

but this conclusion hinges on $\mu_t = 0$ for all t, and we need to verify this, and the latter requires that for all t

$$e^{\frac{\rho}{\gamma}t}\int_t^\infty \chi_\tau d\tau \le \frac{\rho}{\gamma}(k_0 + \chi^*). \quad (22)$$

The task will be easier if we specialize U and χ further, as we shall do next.

Example—Let $\chi_t = \phi e^{-\phi t}$, so that $\chi^* = 1$ and $\int_t^\infty \chi_\tau d\tau = e^{-\phi t}$. Discovery is more frontloaded when ϕ is high, and that is when we expect (20) to be valid. Substituting into (20) yields

$$\frac{k_t}{c_t} = \frac{\gamma}{\rho}\left(1 - \frac{\exp\left(\left[\frac{\rho}{\gamma} - \phi\right] t\right)}{\frac{\rho}{\gamma}(1 + k_0)}\right)$$

and (22) reads

$$e^{\left(\frac{\rho}{\gamma} - \phi\right)t} \le \frac{\rho}{\gamma}(k_0 + 1)$$

for all t. This is possible only if

$$\phi > \frac{\rho}{\gamma} \quad (23)$$

so that discovery is sufficiently front loaded, but this also implies $\frac{d}{dt}\left(\frac{k}{c}\right) > 0$. Thus we conclude that if the optimal path entails $k_t > 0$ for all t, the rise in k/c that Figure 3 shows could have been the result of discovery if (23) holds.

More generally, there will be dates at which at some dates $\mu_t > 0 \Longleftrightarrow k_t = 0$ in which case we still would expect a discovery to raise in k/c. We also would expect k/c to fall in periods when there is no discovery, and Figure 3 suggests that this may have happened once or twice. Just prior to Sec. 3.1, however, we noted that such temporary declines can happen in the Hotelling model even without discovery and, in particular, that they can happen if there is a bubble. We have also omitted cyclical reasons why c may fall; in a recession, fewer people drive to work, and the demand for gasoline declines; thus we would expect k/c to rise during recessions.

4.2 Explaining oil prices since 1861

Whether oil prices contain a bubble or not, Hotelling's model explains these prices pretty well. We simply need to add extraction costs that decline over time due to technological progress. This yields a U-shaped equilibrium price path for the resource. Let the extraction cost per barrel be z_t and let there be exogenous technological progress at the rate g so that

$$z_t = z_0 e^{-gt}.$$

Profits per barrel extracted then are $\pi_t = p_t - z_t$, and intertemporal arbitrage now states that

$$\pi_t = \pi_0 e^{rt}. \tag{24}$$

Since $\pi_0 = p_0 - z_0$, this gives the equilibrium price path

$$\hat{p}_t = (p_0 - z_0)\, e^{r(t-T_0)} + z_0 e^{-g(t-T_0)} \tag{25}$$

where T_0 is the "initial date".

We have 160 years of oil-price data since $1861 \equiv T_0$.[9] We interpret z_t as the combined cost of drilling, extraction, and delivery, each of which has declined over the past 160 years. We assume perfect foresight, and use the series $(p_t)_{t=1861}^{2011}$ to estimate the parameters p_0, z_0, r and g by non-linear least squares:

p_0	z_0	r	g
63.2	61.43	0.024	0.033

$$= \arg \min_{p_0, z_0, r, g} \sum_{t=1861}^{2011} (p_t - \hat{p}_t)^2. \tag{26}$$

Slade (1982) shows that most other commodities have a similar U-shaped price evolution; Aluminum, copper, iron, nickel, silver, and natural gas. See Krautkramer (1998) and Hamilton (2011) for more discussion.

Figure 4 gives the best-fitting price path. But the shape of the price path looks the same whether a bubble exists or not. We plot three additional equilibrium possibilities indexed by $p_0 = 62, 65$ and 67. All these time paths look alike and it is impossible to tell whether the path that fits the best (the thickest of the four lines that starts at $p_0 = 63.2$) is a bubble path or not.

[9] Prices for1861-1944 are US average; for 1945-1983 Arabian Light posted at Ras Tanura, and for 1984-2011 Brent dated. Source: BP Annual Statistical Review 2012.

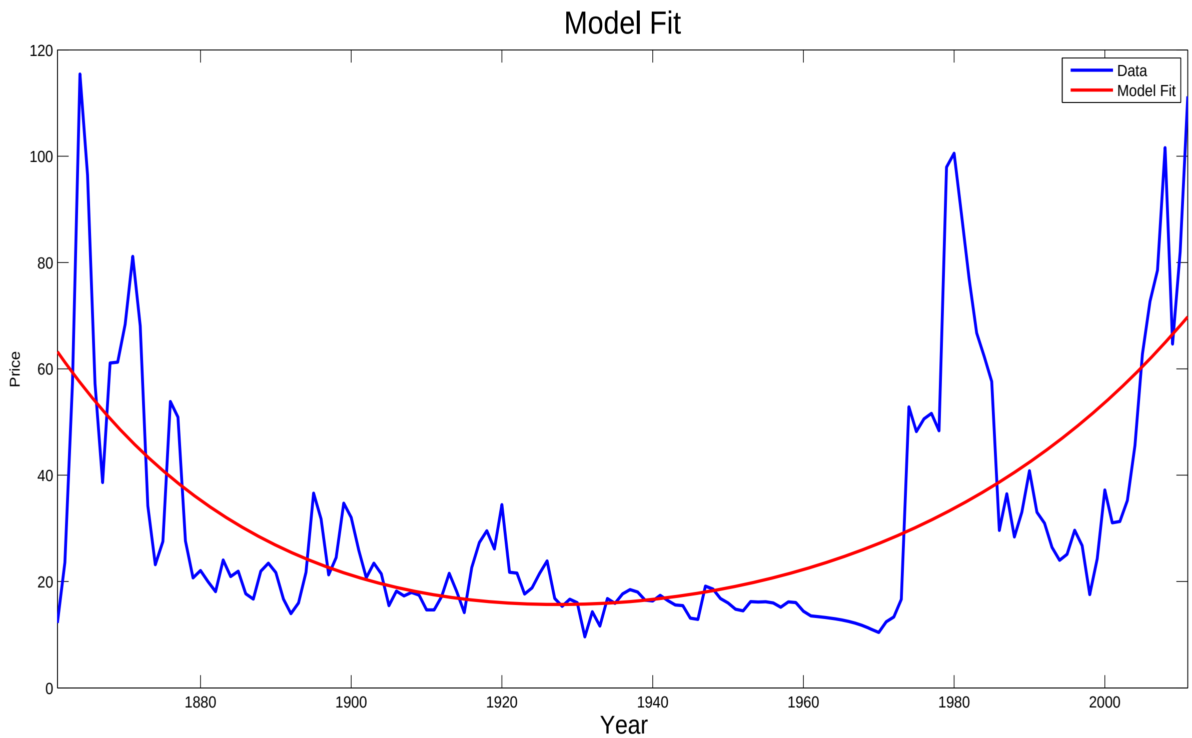

Figure 4: Oil prices and model fit at $p_0 = 63.2$.

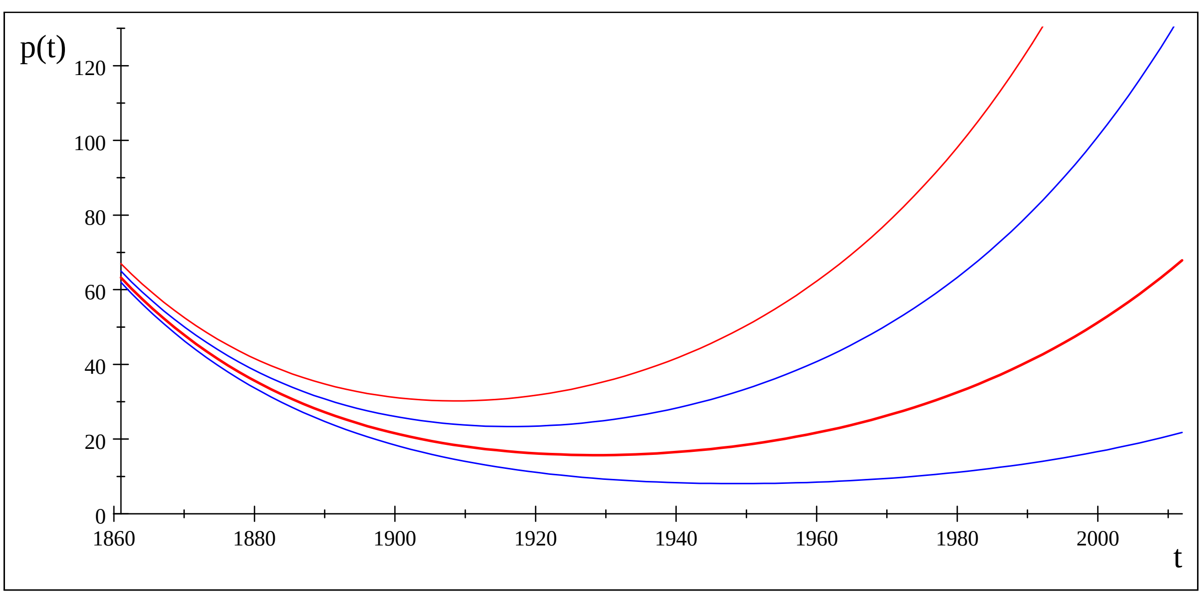

Figure 5: Equilibrium prices for $p_0 \in \{62, \mathbf{63.2}, 65, 67\}$

5 Wine

Let us now apply the model to vintage wines. We shall assume that wine from a given chateau-(i.e., label-)vintage pair is homogeneous and distinct from other chateau-vintage pairs. Thus we interpret k_0 as, say, the total amount of the 1870 Lafite bottled in 1870. The stock is not renewable – different vintages of a given wine are imperfect substitutes, judging by the vastly different prices at which they sell. Since each chateau-vintage type of wine faces competition from close substitutes that appear each year, we extend Hotelling's model to many capital goods in Appendix 1 and show that bubbles can exist on some subset of the menu of vintages available to the consumer. A chateau has a monopoly on its wine which is regarded as distinct from other wines, but each vintage soon passes out of its hands[10] and into the cellars of many dealers, restaurants and private individuals, so that the chateau can focus on producing its next vintage. Thereafter, the competitive model seems to be appropriate.

According to Figure 2, a bubble exists if k/c rises over time. One virtue of wine over oil is that aside from the appearance of counterfeits (a negligible inflow by all accounts), there is no discovery of new quantities of a given vintage and therefore a rise in k/c does signal a bubble and not discovery. In contrast to oil, however, we do not have data on k. We shall therefore try to infer these magnitudes indirectly from the frequencies with which a wine is offered for sale in three different modes – by auctions, by dealers, and by restaurants. A wine sold by a dealer or by a restaurant is usually consumed. By contrast, the sale of a wine at auction is likely to be stored. We can thus hope to learn how much of a particular wine is consumed and how much of it is stored, by comparing the frequency with which the wine is offered for sale at these three venues.

Age distributions.—Figure 6 shows the age distribution of wine offered for sale by dealers and restaurants, and wine actually sold at auction (we have transactions only for auctions). Until a few years ago, vintage wines were sold mainly at auction and not by dealers.[11] Not surprisingly, therefore, the wines offered for sale by dealers are considerably younger than those sold at auctions. On the other hand, the wines offered for sale at restaurants are significantly older than the ones sold at auction.[12]

We cannot take the restaurant numbers at face value, however. First, while auc-

[10] except for a stock that a chateau may keep to re-top old bottles, although this practice is in decline because re-topped bottles look more like counterfeits.

[11] Market structure has been changing recently and dealers have started to hold auctions. Dealers now offer wines that they do not necessarily store themselves. The oldness of the vintages offered for sale today by the Antique Wine Company and described in Figure 8 is a new phenomenon. For most of the 20th Century, one of the world's most prestigeous dalers, Berry Brothers & Rudd, offered wines that were at most 40 years old.

[12] At auction, a bottle of wine need not actually change hands physically; in models of commodity money the point has been raised that if k (the commodity) is a store of value, it does not have to circulate. Claims to k can circulate.

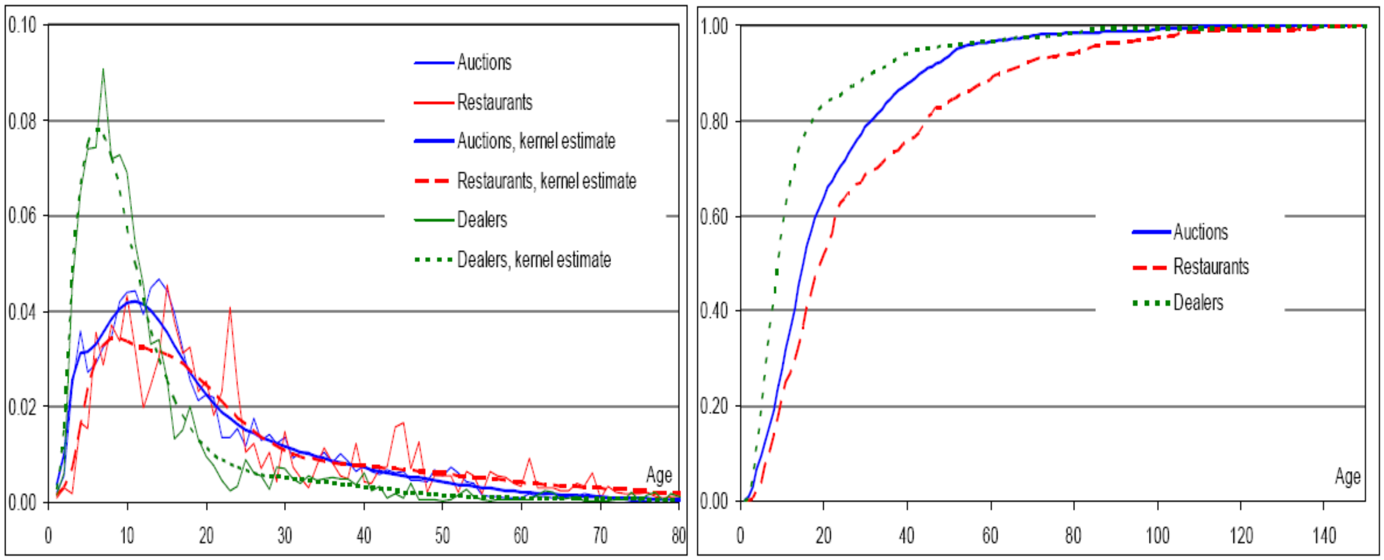

Figure 6: Ages of wines offered by auctions, dealers, and restaurants.

tions prices are transactions prices, the dealer and restaurant prices are list prices. A vintage wine will often appear on a restaurant's wine list without ever being sold. Therefore neither the restaurant nor the dealer age distributions pertain to the distribution of ages of wine actually consumed. Second, even as a distribution of listed prices, the restaurant data are biased towards the older vintages because (in contrast to a dealer's list) a restaurant wine list typically does not provide a wine's vintage for the young wines. The unidentified vintages were excluded from the data which, therefore, heavily oversample the older vintages. Therefore, while the restaurant age distributions lie to the right of those for auction sales, this does not prove that the wines consumed in restaurants are older than those traded at auction. The reverse is highly probable.

If we fix a chateau-vintage pair, we still reach a similar conclusion. Figure 7 shows the history of prices for a bottle of the 1870 Lafite, prices at auction, prices in restaurants, and prices offered by dealers. The data for this wine are incomplete as they are for all the wines in my sample, but the Figure describes fairly well what the entire sample shows: Consumption occurs early, and later transactions mainly reallocate assets. Consumption demand is typically met by dealers and restaurants, and not by purchases at auction where the buyers are restaurants, dealers and private collectors. The wine's average rate of price increase is 5.29 percent (auctions), 5.15 percent (restaurants) and 4.54 percent (dealers). The point of the graph is that the red and green squares predominate in the early years of the wine's life, whereas the blue dots are spread out more evenly and predominate in the more recent period. Dealers offered the 1870 Lafite for sale in the first few decades of its life, and more

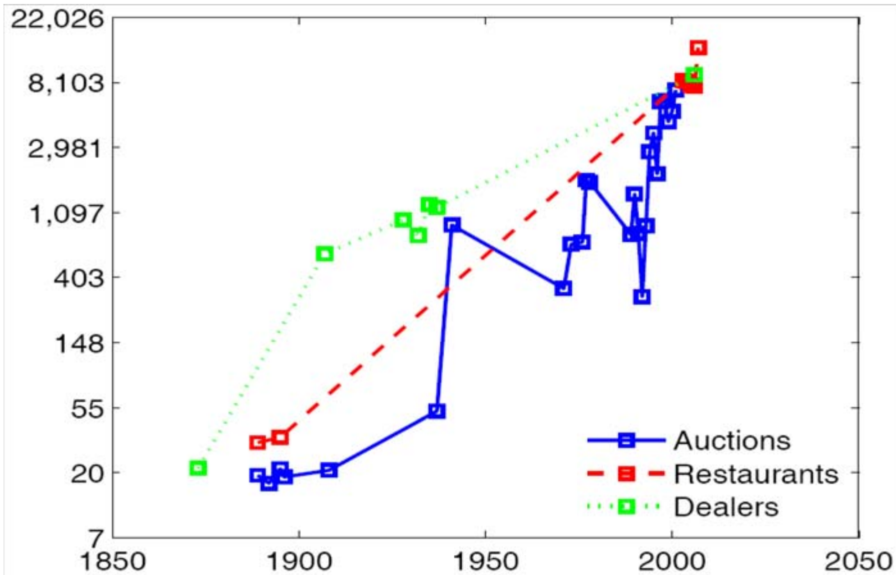

Figure 7: The time series: The price of a bottle of the 1870 Lafite, in year-2000 dollars

recently it has shown up at many auctions. Moreover, the dots connected by the solid blue line represent actual transactions, whereas the other dots are list prices; they indicate that the wine was offered for sale on a wine list, but not necessarily sold.[13] This pattern is typical of the wines in the sample, and similar graphs for nine other popular wines are reported in the Appendix.

Evidently the solid line wiggles a lot and this reflects within-chateau-vintage heterogeneity. Second, there are counterfeit versions of at least some famous vintages. But in fact, even within a vintage-chateau pair there is significant heterogeneity that can be detected by inspection and that therefore affects prices at which the bottles sell. The buyer has two main concerns: Is the bottle authentic, and has it been properly stored.[14] (Although the quantities are thought to be negligible, if the counterfeits are indistinguishable from the real versions and if their inflow is known, the effect on k/c would be the same as that of discovery, as analyzed in Section 3.1). Thus the series in Figure 7, or in the Figures in the Appendix, do not all represent the movement in the prices of a claim to a *given* bottle, although as the vintage becomes old, it is ever more likely that the same bottle appears on a restaurant's wine list or

[13]In particular, the cluster of red dots in the years 2003-7 represents the sale price at the same (Chicago's Charlie Trotter's) restaurant where the bottle has been offered for sale (but presumably has not sold). See the Appendix table for an account of all the data plotted in Figure 7.

[14]Some bottles were stored improperly which affects the level of the wine in the bottle and the sedimentation, some bottles are stored by reputable dealers and some not, some have a reputable distributors and some not, some have been re-corked or "reconditioned" and some not, etc.. Counterfeiting is on the rise for the old, valuable vintages.

a dealer's list, and ever more likely that the same bottles will be traded again and again at auction..

To sum up, in line with Figure 2, evidence shows that it is high priced wines like the Bordeaux wines in my sample that survive a long time and continue to be traded. Low-priced wines disappear rather quickly. This indicates that these wines acquire the properties of an asset to be held as an investment rather than as a consumable item, i.e., that there is a bubble on the price of these old wines.

5.1 Convenience yield

An alternative explanation that may apply to wine and oil alike is that the owner of the asset may derive pleasure from holding it (or showing it off to friends, e.g., in the case of wine) or may draw a convenience yield from holding it (as a deterrent for wartime purposes, e.g., in the case of oil). Wine is also used as a collateral,[15] Can this explain why an asset asymptotically would cease to be consumed with significant reserves held in perpetuity? Let utility depend on both consumption, x, storage, k. That is, let utility be $U(x,k)$, with U increasing, differentiable, and concave in both of its arguments, and let r be the discount rate.[16] For now, assume that $\lim_{x\to 0} U_x = +\infty$, and to simplify further, consider a representative agent setup in which every agent chooses the same (x,k) pair.

The price of capital and the marginal utility of consuming another unit of it must, at each date, equal the marginal utility of lifetime storage:

$$p_t = U_x(x_t, k_t) = \int_t^\infty e^{-r(s-t)} U_k(x_s, k_s)\, ds. \tag{27}$$

The RHS of (27) insists that there is no bubble and that the value of the asset is, at each date, equal to the discounted flow of its fundamentals. Differentiating the RHS of (27) and applying (27) to the result, we have the ODE

$$\frac{dp}{dt} = rp - U_k(x,k). \tag{28}$$

Therefore p grows more slowly than at the rate r, and may even decline.

Now, convenience yield for old wines is indisputably higher than for young wines. Therefore, whether a bubble exists or not, Hotelling's model implies that price should be appreciating more slowly for the older vintages. This implication is refuted by

Figure 8 which shows prices per bottle at which the Antique Wine Co., a wine dealer, offered various vintages of six Bordeaux wines for sale in 2007. Average real appreciation as a function of vintage is 2.2 percent per year and the oldest vintages

[15] See David Andolfatto's blog http://andolfatto.blogspot.com/2012/03/turning-wine-into-liquidity.html

[16] which, in an economy with no growth, would equal the rate of interest.

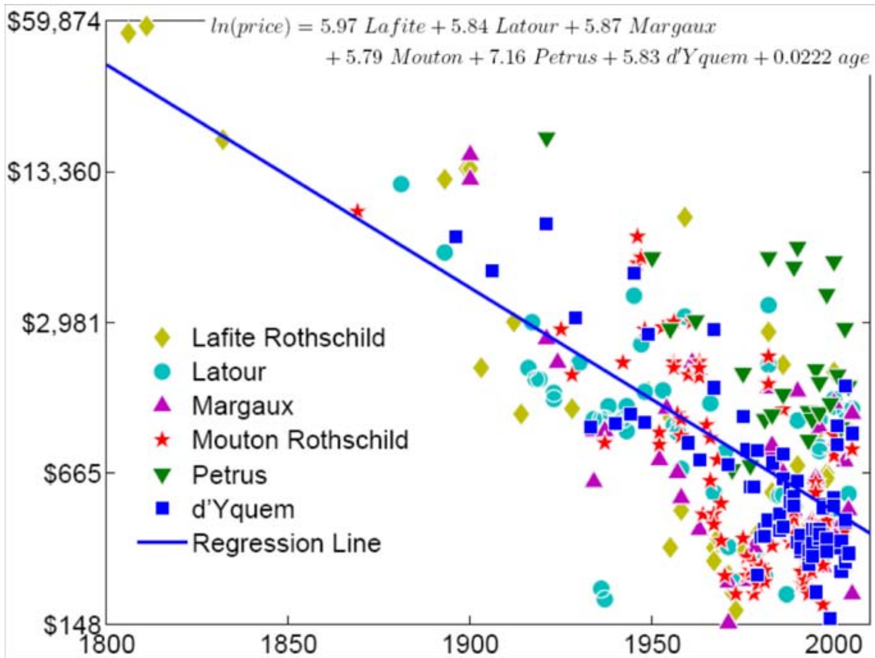

Figure 8: THE CROSS SECTION: APRIL 2007 DEALER PRICES PER BOTTLE FOR VARIOUS VINTAGES

are in fact above the regression line, indicting a faster appreciation in that age range. [17] Adding a quadratic term confirms this claim, as shown in Figure 9, casting doubt on convenience yield as an explanation for why k/c appears to rise.

All that notwithstanding, as it ages, wine undoubtedly acquires the status of a collectible, of an antique. This convenience yield is almost surely highest among restaurants. The sommelier of a famous New York restaurant said this about the most expensive wines on his wine list: "*I don't want to sell this wine. It makes the list look better.*" No doubt this helps explain why Figure 6 shows restaurants as listing older wines. Indeed Figure 10 below shows some evidence that among restaurants, older bottles entail a convenience yield.

Dealers and most private individuals probably do not have as large a convenience yield, which explains why, until quite recently, dealers did not hold wines older than 30 or 40 years. Instead dealers would leave the market for older wines altogether, and sell it to those agents who derive pleasure or other forms of gain from simply holding them. This is another way to interpret the data in Figure 6. Restaurants are holding

[17] The data were collected in 2007. Not in the data is the 1787 Lafite for which the record price was set at 1985 at a Christies auction by Malcolm Forbes, the late publisher, when he paid US$156,450 for it. Analysis then showed that the bottle was at least half full of the 1962 vintage of the same wine. It was, in other words, later discovered to be a counterfeit. Including that data point would strengthen the conclusion that older vintages appreciate faster, contrary to the convenience-yield hypothesis.

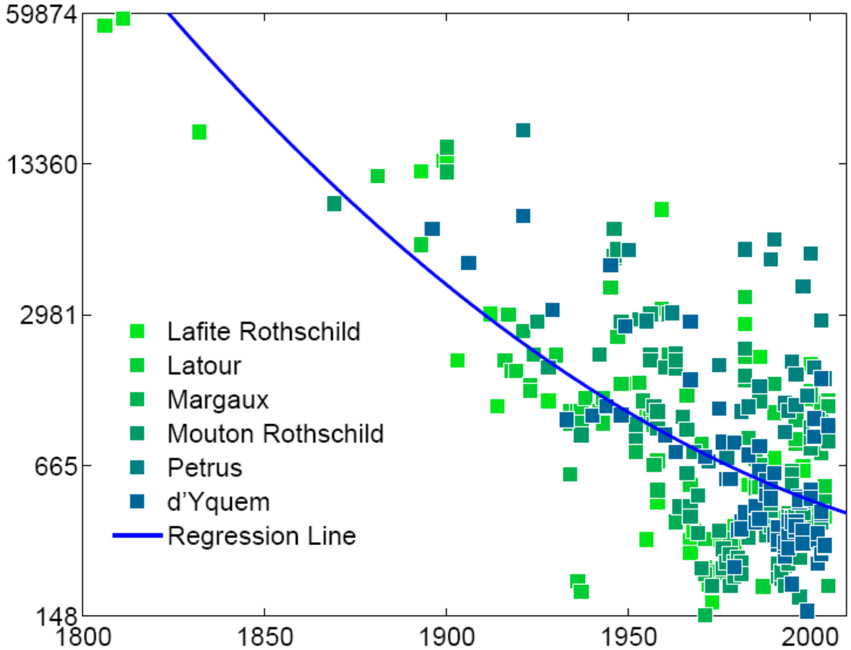

Figure 9: OLDER WINES APPRECIATE FASTER

on to older wine, and not selling it. This only reinforces the general impression that old wine is simply being stored and not consumed.

Can the storage patterns be explained by convenience yield? The summary statistics concerning the distribution of annual growth rates are shown in Table 1:

Stat	Auctions	Restaurants	Dealers
Mean	8.8	5.07	13.7
Min	-96	-87	-99
Max	1630	669	5838
Std	48.9	16.6	138
Skew	12.8	19.9	25.8
Kurt	282	674	964

Table 1: *The distribution of annual growth rates of prices*

First off, we note the return to holding even the oldest wines (i.e., those from which the largest convenience yield would presumably flow) seems comparable to the stock market, and hence the non-pecuniary return is probably small. Nevertheless, there are rate-of-return differentials which may reflect a rising convenience yield and, hence, a falling equilibrium return, as a function of wine age. If dealers hold only young wines as Figure 6 shows, we could see the pattern in Table 1. When we hold the wines

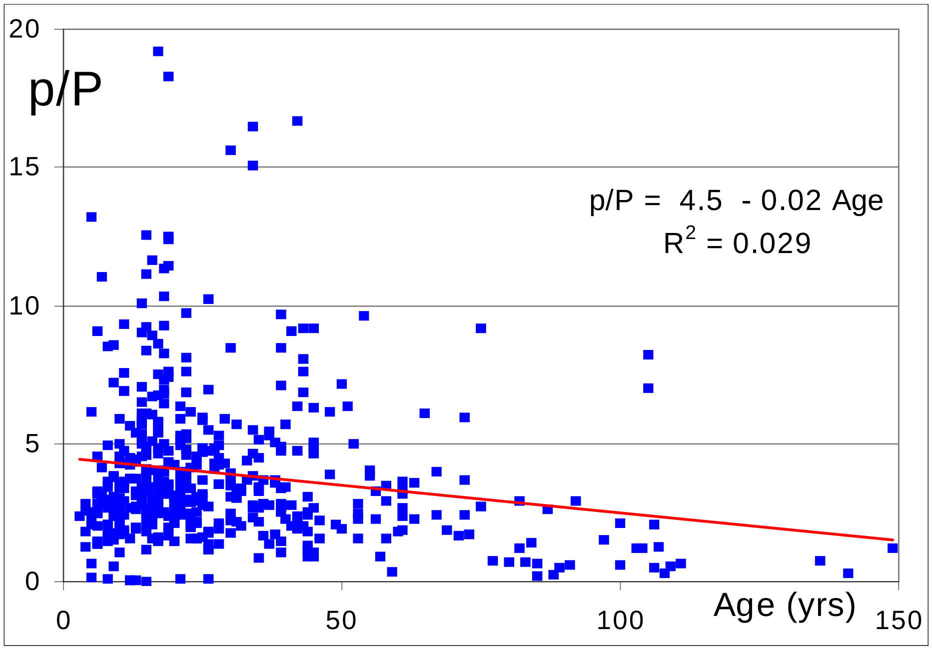

Figure 10: Ratio of restaurant prices to auction prices as a function of the age of the bottle.

constant and look at the prices charged in restaurants and at auction, we find the pattern portrayed in Figure 10. To be included in that plot, the bottle must have the *same label*, *year* and *vintage* at an auction and in a restaurant. There are 530 data points and they show that even for the same set of wines, restaurant prices do grow more slowly than auction prices, perhaps indicating a convenience yield. As eq. (28) would suggest, wines in the possession of restaurants appear to have appreciated more slowly.

6 General equilibrium and welfare

Except for Section 3 that dealt with an infinite-horizon representative-agent model in which no bubbles could arise, we have been working with various versions of the partial equilibrium model of Hotelling, and have so far been assuming that a rational bubble can exist in the economy at large. Aside from k, we now assume that there is a second perishable good, y, which can be produced at constant returns to scale using labor only, and which acts as the numeraire. We shall assume a constant population of two-period-lived agents. The only real asset and the only durable good is k, and its initial stock is held by the date-zero old generation. There are no bequests.

An agent has a unit labor endowment when young. Consumption of k occurs when old, and that of y in both periods of life. An agent born at date t has lifetime utility

$$c_t + \beta \left(c_{t+1} + U \left[x_{t+1} \right] \right), \tag{29}$$

where c_t and c_{t+1} denote his consumption of y in youth and old age, and x is his consumption of capital.

Production of the perishable good.—The technology is

$$y = a_0 A^t L, \tag{30}$$

where L is labor services employed. We assume $A > \beta^{-1}$ so that there is technological progress at a rate faster than the discount rate.

Evolution of capital.—Capital evolves as

$$k_{t+1} = k_t - x_t. \tag{31}$$

Resource constraint.—Consumption of y per old agent (there are n^t) of them must equal output per old agent

$$c_t^o + c_t^y = a_0 A^t. \tag{32}$$

At full employment $L = 1$, and therefore aggregate output is

$$y_t = a_0 A^t = w_t, \tag{33}$$

where w_t is the wage the young receive from the competitive firms that earn zero profits.

It is much simpler to proceed with an economy in which the only asset is k. After we solve for the equilibrium, adding money will be transparent.

6.1 Equilibrium with no outside money

With no money in the economy, we let the numeraire be y. In terms of y the gross rate of interest must be β^{-1}. Let p be the price of k in terms of y. Storing k must yield a gross return of β^{-1}, and therefore

$$p_t = p_0 \beta^{-t}. \tag{34}$$

Budget constraint of young:

$$p_t k_{t+1} + c_t^y = w_t. \tag{35}$$

Budget constraint of old:

$$p_t k_t = c_t^o + p_t x_t. \tag{36}$$

At an interior optimum,

$$p_t = U'(x_t) \quad \Rightarrow \quad x_t = (U')^{-1}(p_t)$$

which, combined with (31) and (34) yields

$$k_{t+1} = k_t - (U')^{-1}\left(p_0 \beta^{-t}\right) \tag{37}$$

with the one-parameter family of solutions for k_t

$$k_t = k_0 - \sum_{s=0}^{t} (U')^{-1}\left(p_0 \beta^{-s}\right). \tag{38}$$

The equilibrium selection parameter, p_0, is arbitrary, and indexes the size of the bubble. The bubble must not be larger than the young agents' willingness to save:

Bubble cannot be too large.—The young must hold k_{t+1}. Since $A > \beta^{-1}$, the bubble shrinks relative to output over time, and so if it is not too large initially, it is never too large. Since date zero output is a_0, it is necessary and sufficient that $p_0 k_1 \leq a_0$. Since k_1 is endogenous, we eliminate it to get the necessary and sufficient condition

$$p_0 \left(k_0 - (U')^{-1}(p_0)\right) \leq a_0, \tag{39}$$

There must initially be enough output, a_0, to draw forth the savings needed to absorb the capital.

A *non-monetary equilibrium* is a price sequence $\tilde{p}_t = \tilde{p}_0 \beta^{-t}$ – the price of k in terms of y – for which (34)-(39) hold for all t. Thus the equilibrium is fully described by just one number, $\tilde{p}_0$, which cannot be too large. Denote by $p_{\max}$ the largest that p_0 can be. That is, let $p_{\max}$ as the[18] solution to (39) when it holds with equality:

$$p_{\max} \left(k_0 - (U')^{-1}(p_{\max})\right) = a_0. \tag{40}$$

[18] We say "the" solution because The solution for $p_{\max}$ is unique because the RHS of (39) is strictly increasing in p_0:

$$\frac{\partial\,(\text{RHS})}{\partial p} = k_1 - p\left((U')^{-1}\right)'(p) > 0$$

because U' is a decreasing function and so is $(U')^{-1}$. E.g.,

$$U(x) = \frac{x^{1-1/\gamma} - 1}{1 - 1/\gamma} \Longrightarrow U'(x) = x^{-1/\gamma} \Longrightarrow (U')^{-1}(p) = p^{-\gamma}$$

where $\gamma > 0$. Then (39) reads $p_0\left(k_0 - p_0^{-\gamma}\right) \leq 1$.

Then the admissible solutions are all those for which $\tilde{p}_0 \in \left[p^{\mathrm{H}}, p_{\max}\right]$, where the Hotelling price, p^{H}, solves

$$k_0 = \sum_{t=0}^{\infty} \left(U'\right)^{-1} \left(p_0 \beta^{-t}\right).$$

Welfare.—A rise in p_0 raises the welfare of the current old while it lowers the welfare of all future generations. As p_0 varies between p_0^{H} and $p_{\max} = 100$, it traces out a utility frontier between the current old (who are the best off at $p_{\max}$) and all future generations (who are the best off at p^{H}). But the bubble equilibria are inefficient: A feasible Pareto improvement exists in that k_∞ could be consumed at some dates without reducing any generation's consumption of x and y. This conclusion echoes those in the commodity-money literature.

6.1.1 Example

In the following example, k_t will converge to its limit geometrically. For $\gamma > 0$, take

$$U(x) = \frac{x^{1-\gamma} - 1}{1-\gamma} \Longrightarrow U'(x) = x^{-\gamma} \Longrightarrow \left(U'\right)^{-1}(p) = p^{-1/\gamma}$$

Then (38) yields the one-parameter family of solutions

$$\begin{aligned} k_t &= k_0 - p_0^{-1/\gamma} \sum_{s=0}^{t-1} \beta^{s/\gamma} = k_0 - p_0^{-1/\gamma} \frac{1-\beta^{t/\gamma}}{1-\beta^{1/\gamma}} \\ &\rightarrow k_0 - \frac{p_0^{-1/\gamma}}{1-\beta^{1/\gamma}} \equiv k_\infty \end{aligned}$$

whereas (40) reads

$$p_{\max}\left(k_0 - p_{\max}^{-1/\gamma}\right) = a_0 \tag{41}$$

The Hotelling equilibrium, p_0^{H} has $k_\infty = 0 \Rightarrow$

$$p_0^{\mathrm{H}} = k_0^{-\gamma}\left(1-\beta^{1/\gamma}\right)^{-\gamma} \quad \text{and} \quad k_t^{\mathrm{H}} = k_0 \beta^{t/\gamma}$$

As for the bubble equilibria, the higher is k_∞, the higher is p_0:

$$p_0 = \left(k_0 - k_\infty\right)^{-\gamma}\left(1-\beta^{1/\gamma}\right)^{-\gamma}. \tag{42}$$

Simulated example.—Set $k_0 = \gamma = 1$, $\beta = 0.97$, and $a_0 = 99$ Then (40) implies[19]

$$p_{\max} = 1 + a_0 = 100, \text{ and } \quad p_0^{\mathrm{H}} = \frac{1}{1-\beta} = 33.3$$

[19] We had to assume that $1 + a_0 > (1-\beta)^{-1}$, otherwise we would have $p_0^{\mathrm{H}} > p_{\max}$ and no equilibrium would exist because even p_0^{H} would imply a greater carryover of capital than the young are willing to hold.

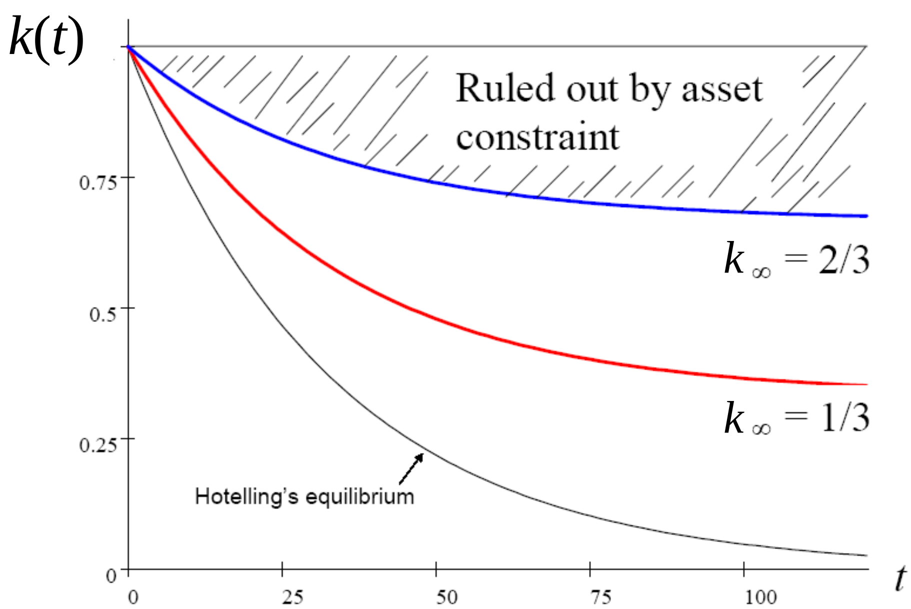

Figure 11: THE SET OF EQUILIBRIA

The bubble can be no larger than

$$k_\infty \leq k_0 \left(1 - \frac{1}{1-\beta}\frac{1}{1+a_0}\right) = \frac{2}{3}$$

Figure 11 plots the solution for $k_\infty = \frac{1}{3}$ (red line) and $k_\infty = \frac{2}{3}$ (blue line).

6.2 Equilibrium with outside money

We now add constant stock of money, $\hat{M}$, in the possession of the date-0 old. All prices are now denominated in terms of money. Let P_t denote the price of y_t and p_t the price of k_t. Zero profits now imply a nominal wage of

$$W_t \equiv P_t a_0 A^t$$

The decision problem of the young.—Let $k_{t+1} \equiv Z$ the number of units of k that you buy when young,[20] and M the number of dollars. The two budget constraints are now written in terms of dollars:

$$\begin{aligned} &\text{(youth)} & W_t &= P_t c_t^y + p_t Z + M, & (43)\\ &\text{(old age)} & p_{t+1} Z + M &= P_{t+1} c_{t+1}^o + p_{t+1} x_{t+1}. & (44) \end{aligned}$$

[20] An individual does not face the aggregate resource constraint $k' = k - x$ and so we use Z to keep that clear.

The Lagrangian now is

$$\mathcal{L} = c^y + \beta\left(c^o + U\left[x\right]\right) + \lambda_1\left(W_t - P_t c^y - p_t Z - M\right) + \lambda_2\left(M + p_{t+1} Z - P_{t+1} c^o - p_{t+1} x\right)$$

The FOCs are

$$\begin{aligned} c^y &: \quad 1 - \lambda_1 P_t = 0 && (45) \\ M &: \quad \lambda_2 - \lambda_1 = 0 && (46) \\ Z &: \quad \lambda_2 p_{t+1} - \lambda_1 p_t = 0 && (47) \\ c^o &: \quad \beta - \lambda_2 P_{t+1} = 0 && (48) \\ x &: \quad \beta U'\left(x_{t+1}\right) - \lambda_2 p_{t+1} = 0 && (49) \end{aligned}$$

From (46) and (45) we have $\lambda_1 = \lambda_2 = P_t^{-1}$. Then (47) implies that the nominal price of k is constant:

$$p_{t+1} = p_t \equiv p \tag{50}$$

Then (48) implies that P_t declines at the rate of discount:

$$P_{t+1} = \beta P_t \quad \Rightarrow \quad P_t = \beta^t P_0, \tag{51}$$

which ensures that agents are be willing to postpone consumption of y in order to hold money. The rate of deflation equals the rate of discount, and money offers the same real rate of return as does k. Thus the "Friedman rule"[21] emerges endogenously.

Finally (49) implies

$$U'\left(x_{t+1}\right) = \frac{p}{\beta P_{t+1}} \quad \Rightarrow \quad U'\left(x_t\right) = \frac{p}{\beta P_0}\beta^{-t} = \frac{p}{P_0}\beta^{-(t+1)}. \tag{52}$$

Equilibrium.—As before, the equilibrium conditions are

$$\begin{aligned} c_t^y + c_t^o &= a_0 A^t && (53) \\ c_t^o + \frac{p}{P_t} x_t &= \frac{p}{P_t} k_t + \frac{\hat{M}}{P_t} && (54) \end{aligned}$$

Comparison to the model without money

First we show that in the monetary economy, all the old equilibria except one survive as limiting cases when money is driven out. Then we show that money adds equilibria not attainable without it.

[21] which says that the rate of deflation should equal the rate of return on other assets, in which case the nominal interest rate should be zero.

6.2.1 Limiting case of $P_0 \to \infty$

Let $\tilde{p}_t \equiv p/P_t = \tilde{p}_0 \beta^{-t}$ denote the price of k in terms of y. The solution for x and, hence, for all the other variables become the same as before if

$$\frac{p}{P_0} = \tilde{p}_0 \quad \text{and} \quad P_0 \to \infty \tag{55}$$

As money loses value we get back to a world in which there is effectively only one asset

Bubble cannot exceed income condition.—At date t, the condition reads,

$$pk_{t+1} + \hat{M} \leq W_t \tag{56}$$

i.e., using (51),

$$\frac{p}{P_0}\beta^{-t}k_{t+1} + \beta^{-t}\frac{\hat{M}}{P_0} \leq a_0 A^t \tag{57}$$

Since $A > \beta$, the RHS of (57) rises faster than the LHS of (57) and therefore it is necessary and sufficient that

$$\frac{p}{P_0}k_1 + \frac{\hat{M}}{P_0} \leq a_0,$$

and since $k_1 = k_0 - (U')\left(\frac{p}{P_0}\right)$, it is necessary and sufficient that

$$\frac{p}{P_0}\left(k_0 - (U')^{-1}\left(\frac{p}{P_0}\right)\right) \leq a_0 - \frac{\hat{M}}{P_0}, \tag{58}$$

If we impose (55), this too becomes the same condition as before, except that the weak inequality becomes a strong one:

$$\frac{p}{P_0}\left(k_0 - (U')^{-1}\left(\frac{p}{P_0}\right)\right) < a_0 \tag{59}$$

Strictly speaking, since $M/P_0 > 0$ for any $P_0 > 0$ no matter how large, it means that $\hat{p}k_1 < a_0 - \varepsilon$ for every $\varepsilon > 0$, no matter how small. Thus this set of equilibria is open, i.e.,

$$\frac{p}{P_0} \in [p_0^{\mathrm{H}}, p_{\max}).$$

Equilibria when $P_0 < \infty$

Now we ask if when P_0, new possibilities are introduced for the equilibrium allocations of consumption: Let

$$s \equiv (c_t^y, c_t^o, x_t)_0^\infty,$$

and let

$$S \equiv \{s \mid s \text{ solves the no-money version (34)-(39)}\} \tag{60}$$

and let

$$S_{\mathrm{M}} \equiv \{s \mid s \text{ solves the monetary model (43)-(54)}\} \tag{61}$$

Keeping the notation $\tilde{p}_t$ for the equilibrium price of capital in the no-money case, let us use tilder notation for the no-money allocations $\tilde{s} \equiv (\tilde{c}_t^y, \tilde{c}_t^o, \tilde{x}_t)_0^\infty$.

(i) We already established that there is a point in S that is not in S_{M}, namely the least efficient set of allocations corresponding to $p_0 = p_{\max}$ at which the bubble is at its largest possible.

(ii) Are there points in S_{M} that are not in S? I.e., is there an $s \in S_{\mathrm{M}}$ but $s \notin S$? If so, at what prices $(p_t, P_t)_0^\infty$?

Yes. The allocations are different. Let us compare that point $\tilde{s} \in S$ with the corresponding point $s \in S_{\mathrm{M}}$ that has the same initial price of k in terms of the consumption good so that

$$\tilde{p}_0 = \frac{p}{P_0} \Rightarrow \tilde{p}_t = \frac{p}{P_t} \tag{62}$$

Then (i) $(x_t)_0^\infty$ is the same in the two equilibria, and therefore so is (k_t). But (ii) the young now save more so that they can absorb the money stock as well, and this means that

$$c_t^y = \tilde{c}_t^y - \beta^{-t}\frac{\hat{M}}{P_0}$$

because

$$c_t^y = a_0 A^t - \frac{1}{P_t}\left(pk_{t+1} + \hat{M}\right) < \tilde{c}_t^y = a_0 A^t - \tilde{p}_t k_{t+1} = a_0 A^t - \frac{p}{P_t} k_{t+1}$$

and consequently they spend more on c_{t+1} in old age:

$$c_{t+1}^o = \tilde{c}_{t+1}^o + \beta^{-(t+1)}\frac{\hat{M}}{P_0}$$

because

$$c_{t+1}^o + \frac{p}{P_{t+1}} x_{t+1} = \frac{P_t}{P_{t+1}}\left(pk_{t+1} + \hat{M}\right) > \tilde{c}_{t+1}^0 = \frac{P_t}{P_{t+1}} pk_{t+1}$$

Of course, in this way we have the current old over-consuming as much as the current young under-consume:

$$c_t^o - \tilde{c}_t^o = -\left(c_t^y - \tilde{c}_t^y\right).$$

The new equilibria of the monetary economy do not change welfare, they only redistribute consumption from youth into old age. If (62) holds, the welfare of each generation is the same; the difference is simply that the young get β^{-1} more consumption in old age than they give up in youth. The set of attainable welfare levels is therefore the same, except for the exclusion of the welfare pertaining to the worst equilibrium. The young do not care if they postpone consumption as long as this they get β^{-1} as much back next period. Therefore the welfare of generation born at t is

$$V_t(\tilde{p}_0) = a_0 A^t + \beta \max_x \left\{ U(x) - \tilde{p}_0 \beta^{-t} x \right\}$$

and the only welfare level not attainable in a monetary equilibrium is $V_t(\tilde{p}_{\max})$. If k was not available as a store of value, money would raise welfare since only the young are endowed with labor.

6.3 Money and welfare.

Money can remove the dynamic inefficiency entailed in the bubble. Without money, the old are made worse off if the bubble is removed. But in the monetary economy, they can be given a transfer of money equal to the size of the bubble. This would be compatible with (58) which depends only on the sum $\frac{pk_0}{P_0} + \frac{\hat{M}}{P_0}$, and which therefore would remain unchanged. In this sense we can say that removing the bubble entails a Pareto improvement.

We thus reinforce the conventional wisdom, first expressed by Friedman (1960), that commodity money wastes resources and that its displacement by fiat money should raise welfare by freeing up the resources for other uses. Keeping total saving fixed, if the introduction of fiat money displaces a fraction of the date-zero bubble, $\tilde{p}_0 - \tilde{p}_0^{\mathrm{H}}$ but keeps total saving the same, then the bubble at $t > 0$, i.e., $\tilde{p}_t - \tilde{p}_t^{\mathrm{H}}$ shrinks by the same proportion and welfare goes up because a larger flow of k can be consumed. This is true for any $\tilde{p}_0 > \tilde{p}_0^{\mathrm{H}}$, and corresponds to a shift of k_t from a path above the Hotelling path to another closer to it. In this sense, there is no conflict with received wisdom regarding welfare and commodity money: A bubble reduces welfare for the same reason that commodity money does.

6.4 Discussion

A feature of our OG model is that k and M are in fixed supply, not augmentable by private agents, and k is consumable. Most models of commodity money endow agents with a production function that allows them to convert labor or goods into the storable commodity. Thus, Kiyotaki and Wright (1988) Burdett, Trejos and Wright (2001) and Lagos and Rocheteau (08) all have reproducible commodity money and study only steady states in which the price of the commodity is constant.

Closest to the OG version of our model is Sargent and Wallace (1983) who, in Section 3.3, study the case in which the commodity money is not reproducible. Instead of technological progress they feature population growth that must be rapid enough to absorb the bubble. Their Prop. 6 asserts uniqueness for the gross return on storage but they do not raise the possibility of multiple solutions for the level of the price of gold, which would be the analog of the multiple bubble solutions that we have here.

Among other related models, Dasgupta and Heal (1979, Ch. 8) assume exogenous savings so that issues like transversality do not come up. Tirole (1985, sec. 7[b]) connects their argument to the existence of bubbles. The condition $\beta A > 1$ guarantees the existence of the bubble and it implies the Santos-Woodford (1997) condition that the present value of aggregate consumption must be infinite.

A bubble on the exhaustible resource can raise welfare if there is a negative external effect on its consumption, and this may be the case with oil. Friedman's claim about commodity money would also need modification in that case. Olivier (2000) has a positive welfare effect of a bubble on equity; in his model there is a positive knowledge spillover and a bubble raised the incentives to implement ideas via IPOs.

Finally, is our use of the term "bubble" conventional? Does the term "bubble" mean that the difference between price and fundamentals is positive in the GE version? Does this relate to money being "essential" in the sense defined by Wallace? And when an asset's higher price serves to relax borrowing constraints, should the benefits from such a relaxation ought to be included in the fundamental?

7 Conclusion

When it comes to bubbles on a consumable exhaustible resource, two things are special. First, it is easier for the bubble to form and, second, detecting the bubble is easier, requiring simply that the asset-to-consumption ratio rise over time. Using this simple test, we have found that it is possible that a bubble exists in the price of oil and in the prices on some vintage wines, but we also entertained some other hypotheses such as discovery and convenience yield.

The model may apply to certain other assets. Land is in fixed supply but is not consumed, and the same is true of art and other collectibles. Gold, and silver have a significant salvage value even after being converted into jewelry, so they are asset that appear to carry a large convenience yield that lower their equilibrium returns towards zero.

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8 Hotelling's model with many capital goods

The point of this section is to show that Hotelling's analysis and our extension of it to bubbles, new continue to hold when there are many goods. This is relevant to the application to wines where substitutes are born each year. Each vintage is then a different non-renewable capital type.[22] Let $v \in R$ denote the vintage of the capital. Write the demand for this vintage as

$$D^v(P),$$

where P is the infinite-dimensional price vector for all the other vintages, past, present and future. Once again, we assume an unbounded willingness to pay at small quantities, and so arbitrage across dates requires that under perfect foresight, the price of vintage v should satisfy for all t

$$p_{v,t} = p_v e^{r(t-v)}, \tag{63}$$

where p_v is the initial price of the vintage-v capital, so that we define $P : R^2 \rightarrow R_+ \cup \{\infty\}$ by

$$P = \begin{cases} p_v e^{r(t-v)} & \text{if } t \geq v \\ +\infty & \text{if } t < v \end{cases}.$$

Hotelling's equilibrium in many dimensions.—The initial stock of each vintage can be written as k_v. Instead of just one number, p_0, as we had above, we now have to solve for the vector $(p_v)_{v \in R}$ of the initial prices of each vintage. To solve for it, acting in the spirit of Hotelling we write the simultaneous equation system of resource-exhaustion conditions:

[22] Different vintages trade at vastly different prices. Some of the great vintages are 1865, 1870, 1900, 1929 and 1961. See Figure 2 of Jovanovic (2001) for estimates of vintage effects in wine prices.

$$k_v = \int_0^{\infty} D^v (P_t)\, dt, \qquad v \in R, \tag{64}$$

which is to be solved for the vector $(p_v)_{v \in R}$.

Bubble equilibria in many dimensions.—As before, we replace (64) by the two conditions

$$k_v = k_{v,C} + k_{v,\infty}, \tag{65}$$

and

$$k_{v,C} = \int_0^{\infty} D^v (P_t)\, dt, \tag{66}$$

both holding for all $v \in R$. A no-bubble equilibrium is the one for which $k_{v,C} = k_v$ for all v. The rest are bubble equilibria on at least some of the vintages.

Example.—Consider the following static allocation problem of the consumer. His utility function depends on an array of capital goods $(x_v)_{v \le t}$ and on an outside good y in the following way:

$$U\left[(x_v)_{v \le t}, y\right] = y + X,$$

where $X = \left(\int_0^{\infty} a_v x_v^{\rho} dv\right)^{1/\rho}$ denotes the 'aggregate' capital good that, at date t, takes on the value

$$X_t = \left(\int_0^t a_v x_v^{\rho} dv\right)^{1/\rho}.$$

The consumer's date-t income is I_t and his budget constraint is

$$I_t = y + \int_0^t p_{v,t} x_v dv.$$

The price of vintage-v capital at date t is given by (63). The Lagrangian is

$$L = y + X_t - \lambda \left(I_t - y - \int_{-\infty}^t p_{v,t} x_v dv \right).$$

The first-order condition are $\lambda = 1$ (for an interior optimum w.r.t. y), and $a_v x_v^{\rho-1} X_t^{1-\rho} = p_{v,t}$, for each $v \in [0, t]$. Together with (63), the latter yield the demand functions

$$x_{v,t} = \left(\frac{p_v}{a_v}\right)^{1/(\rho-1)} X_t e^{r(t-v)/(\rho-1)}. \tag{67}$$

Suppose that the time path of X_t is determined. We now show that some vintages of capital can carry large bubbles while others need carry no bubbles. Suppose that vintage $t = 0$ is priced according to its fundamental alone, i.e., that

$$k_0 = \left(\frac{p_0}{a_0}\right)^{1/(\rho-1)} \int_0^{\infty} X_t e^{rt/(\rho-1)} dt,$$

whereas vintage ε has a bubble, so that

$$k_\varepsilon = k_{\varepsilon,\infty} + \left(\frac{p_\varepsilon}{a_\varepsilon}\right)^{1/(\rho-1)} \int_\varepsilon^\infty X_t e^{r(t-\varepsilon)/(\rho-1)} dt,$$

Let ε be small and suppose that the fundamentals of capital ε and capital 0 are the same, i.e., that $a_0 = a_\varepsilon$, and that $k_0 = k_\varepsilon$. As $\varepsilon \to 0$, however,

$$\frac{p_\varepsilon}{p_0} \to \left(1 - \frac{k_\infty}{k_0}\right)^{\rho-1},$$

which means that the prices can be quite different, depending on the magnitude of k_∞ – the price ratio is unbounded. The difference between p_0 and p_ε is due entirely to bubbles.

9 Bounded willingness to pay

Relating to the CARA utility treatment in Section 4, here we show how Hotelling's model extends to this case. Bounded willingness to pay for the resource removes the Inada condition at zero consumption, and this implies exhaustion in finite time. and Let $\bar{p}$ be maximal willingness to pay, so that $\bar{p}$ is the smallest p for which

$$D(p) = 0. \tag{68}$$

We continue to assume that D is continuous.[23] Hotelling's equilibrium now entails exhaustion of the resources in finite time, T. Thus his equilibrium is a price path $p_t = p_0^{\mathrm{H}} e^{rt}$ for $t \in \left[0, T^{\mathrm{H}}\right]$, where $\left(p_0^H, T^H\right)$ solves the pair of equations

$$k_0 = \int_0^T D\left(p_0 e^{rt}\right) dt, \tag{69}$$

and

$$p_0 e^{rT} = \bar{p} \tag{70}$$

for $\left(p_0^{\mathrm{H}}, T^{\mathrm{H}}\right)$.

Bubble equilibria.—A bubble equilibrium also entails the cessation of consumption in finite time, given by

$$T = \frac{1}{r} \ln\left(\frac{\bar{p}}{p_0}\right) \tag{71}$$

Formally, equilibrium is a pair (p_0, k_∞) where $p_0 \in \left[p_0^{\mathrm{H}}, \bar{p}\right]$ and $k_\infty \in [0, k_0)$ that solves (70)

$$k_0 - k_\infty = \int_0^T D\left(p_0 e^{rt}\right) dt, \tag{72}$$

in which T is given by (71). Once again there is a continuum of solutions for $p_0$for As $\bar{p} \to \infty$ we recover the original equilibrium set.

[23] The CARA utility function fits this case if we restrict $c \geq 0$. If $U(c) = 1 - e^{-\gamma c}$, willingness to pay for an additional unit is $U'(c) = \gamma e^{-\gamma c} \leq U'(0) = \gamma$, with $D(p) \equiv \frac{1}{\gamma} \ln(\gamma/p)$.

10 Hotelling (1931) with depreciation of k

Let k depreciate so that

$$\frac{dk}{dt} = -\delta k - x_t, \tag{73}$$

where x_t is consumption. Bubble equilibria remain, but now k_t must always converge to zero. Storage of wine now requires that price appreciate at $r + \delta$:

$$p_t = p_0 e^{(r+\delta)t}.$$

We now have $x_t = D\left(p_0 e^{(r+\delta)t}\right)$ for some unknown constant p_0. The solution to (73) for k_t is

$$k_t = e^{-\delta t} k_0 - \int_0^t e^{-\delta(t-s)} D\left(p_0 e^{(r+\delta)s}\right) ds. \tag{74}$$

A "Hotelling equilibrium", p_0^H should be the smallest p_0 for which $k_t \to 0$. Any smaller p_0 will cause k_t to eventually become negative. Before solving for p_0^H note that there is again a continuum of bubble equilibria indexed by $p_0 > p_0^H$, but that now they all entail $k_t \to 0$. The simple test of the time-path of consumption relative to that of trading such as is depicted on the right panel in Figure 2 will not work.

EXAMPLE: $D(p) = p^{-\beta}$ with $\beta > 1$ (the elastic demand case). Below, we shall show that Hotelling's equilibrium p_0 is

$$p_0^H = \left(\frac{1}{k_0}\right)^{1/\beta} \left(\frac{1}{\beta r + (\beta - 1)\delta}\right)^{1/\beta}. \tag{75}$$

and the Hotelling sequence for k_t is just

$$k_0 e^{-\beta(r+\delta)t}. \tag{76}$$

Because depreciation raises the growth rate of p_t and because demand is elastic, holding p_0 constant, a higher δ reduces consumption by more than δk, and the net effect is to lower p_0^H. For $k_0 = 1, \beta = 2$, and $r = \delta = 0.1$, Figure 12 plots the evolution of k in Hotelling's equilibrium and in a bubble equilibrium. It also plots an infeasible path for k_t, one that would be implied by a price lower than p_0^H.

Derivation of (75) *and* (76).—Let us analyze first the example in Section 2.2.1. The example was $D(p) = p^{-\beta}$ with $\beta > 1$. Then

$$\begin{aligned} \int_0^t e^{-\delta(t-s)} D\left(p_0 e^{(r+\delta)s}\right) ds &= p_0^{-\beta} \int_0^t e^{-\delta(t-s)-\beta(r+\delta)s} ds \\ &= p_0^{-\beta} e^{-\delta t} \int_0^t e^{-[\beta r + (\beta-1)\delta]s} ds \\ &= p_0^{-\beta} e^{-\delta t} \frac{1 - e^{-[\beta r + (\beta-1)\delta]t}}{\beta r + (\beta - 1)\delta}. \end{aligned}$$

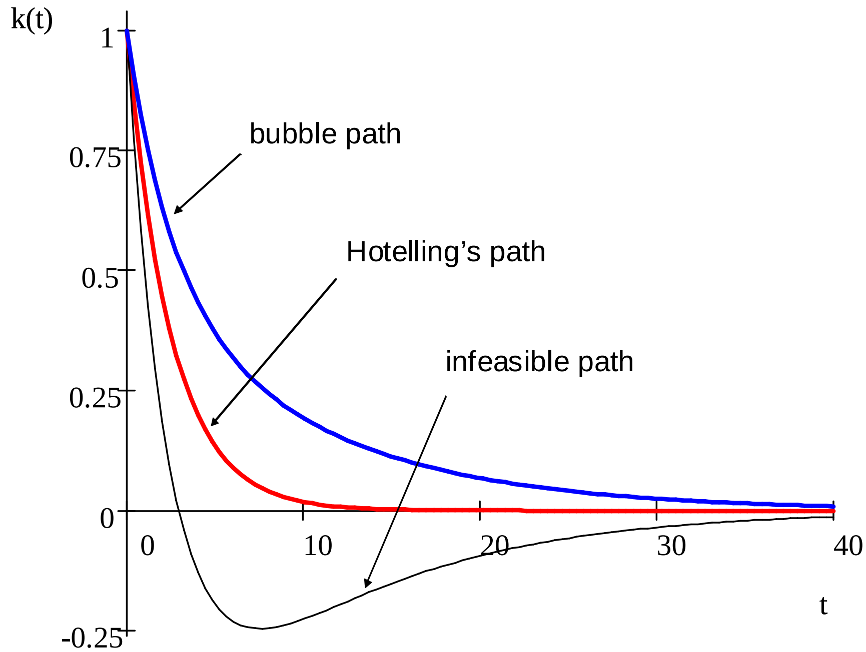


Figure 12: PATHS FOR k_t WHEN $\delta > 0$

Substituting into (74),

$$k_t = e^{-\delta t} \left(k_0 - p_0^{-\beta} \frac{1 - e^{-[\beta(r+\delta)-\delta]t}}{\beta r + (\beta - 1)\,\delta} \right), \tag{77}$$

whence we see that the smallest p_0 that keeps the RHS of this equation non-negative for all t is in (75). Substituting p_0^H for p_0 into (77), we get (76).

11 The data

The data include only incomplete histories of the various wines. Each data point includes: label, vintage, year offered for sale, quantity, size, price and currency. Three kinds of prices were collected:

1. *Auctions.*—1766-2007. About 100,000 observations. All are transactions prices.

2. *Dealers.*—Mid 1800s-2007. About 4,000 observations. For the 19th century, main source is the Guildhall Library, London. For most of the 20th Century, Berry Brothers and Rudd, London, and on-line sources. All are list prices.

3. *Restaurants.*—Mid 1800s-2007. About 5000 observations. For the pre-WW2 period, main source is the NY Historical Society. A handful from the U.S. Library of Congress and the NY Public Library. All are list prices.

Wines included.—Only 9 Chateau wines were selected: Haut Brion (1), Lafite Rothschild (2), Latour (3), Margaux (4), Mouton Rothschild (5), Ausone (6), Cheval Blanc (7), Petrus (8), D'Yquem (9). All are from the Bordeau region in France which, for the past 200 years has supplied most of the highest-priced wines.

No data are available on the stock of wine by vintage.

Main data sources and # observations

Dealer	Obs.
Berry Bros.	2702
FARR	1167
21 Club(?)	54
B&S	12
J.D.C	11
W.C&C	2
Day Watson	1

Restaurant	Obs.
21 Club	490
Berns Stk Hs, Tampa	490
Charlie Trotters, Chi.	318
Name unknown	283
Cru, NYC	223
Le Cirque, NYC	83
Morrell Bar, NYC	27
Antoine's, New Orl.	22
Harry Waugh D Rm	19
LF	17
Taillevent, Paris	12
Canlis, Seattle	11
Locke Ober	7
Simpson's, Edgbstn	4

Auctioneer	Obs.
Chicago Wine Co.	32962
Christie's, London	25600
Sotheby's, London	17904
Zachy's/Christie NY	7965
S. Lehman/Sthby NY	6604
Butterfield, SF	5202
David and Co., Chi.	3788
Morrell and Co. NY	3455
Christie's, Chi.	3357
Christie's, Amstrdm	1254
Christie's, LA	883
Christie's, Geneva	819
Sotheby's, Chicago	669
Acker Merril, New York	411
Sotheby's, New York	214
W.T. Restell, London	205
Christie's, Bordeaux	99
Christie's, NY	8

Conversion table.—All prices are per bottle and in year-2000$ U.S. The conversion between different-sized bottles is described in the following table:

Code	Conversion	Description
B	1.0	Bottle
M	2.0	Magnum
DM	4.0	Double magnum
IP	0.0	Imperial pint
MJ	3.0	Marie-Jeanne
TM	6.0	Triple magnum
QM	8.0	Quadruple magnum
J	6.0	Jeroboam
R	6.0	Rehaboam
I	8.0	Imperial
1/10	0.5	One-tenth (of a gallon)
H	0.5	Half bottle
1/5	1.0	One-fifth (of a gallon)
Pint	0.5	Pint

11.1 The history of the 1870 Lafite-Rothschild

A major concern with a wine that old is that it is undrinkable, that it has "turned into vinegar." But the evidence is that if properly stored, wines retain their quality even after they are 100 years old. Notes on some recent tastings are at http://www.vintagetastings.com/.

The last known (to me) tasting of the 1870 Lafite was in 1970, and was organized by Michael Broadbent, the then head of Christie's wine department. Describing his experience of tasting the 1870 Lafite at age 100, Broadbent said: "I am very often asked by journalists which is my favorite wine. This, I believe, is the most spectacular and memorable one." A detailed write-up of the event is at http://www.empireclubfoundation.com/details.asp?. A more recent, 2002 tasting of an 1870 Château Cos d'Estournel (not in my sample) showed that the flavor was still good.

The following three tables provide the details of each data point in Figure 7. For some years, more than one auction- and restaurant-price observation was available. In that case, the observations were averaged for the purpose of the plot.

Following the tables documenting the history of the 1870 Lafite, we shall display a collection of plots for certain other vintages and other labels. The Table and the plots should provide a fairly accurate feel for the kind of coverage that the data provide, and for the patterns that these data show.

The 1870 Château Lafite-Rothschild
AUCTIONS

Auction	Loc	Year	Age	Price
Christie's, London	UK	1889	19	22
Christie's, London	UK	1889	19	17
Christie's, London	UK	1892	22	17
Christie's, London	UK	1895	25	23
Christie's, London	UK	1895	25	22
Christie's, London	UK	1895	25	19
Christie's, London	UK	1896	26	19
Christie's, London	UK	1908	38	21
Christie's, London	UK	1937	67	52
Restell, London	UK	1941	71	905
Christie's, London	UK	1971	101	341
Christie's, London	UK	1973	103	675
Christie's, London	UK	1976	106	696
Christie's, London	UK	1977	107	1809
Christie's, London	UK	1978	108	1747
Butterfield and Butterfield	US	1989	119	660
Butterfield and Butterfield	US	1989	119	903
Sotheby's, London	UK	1990	120	643
Christie's, London	UK	1990	120	316
Christie's, London	UK	1990	120	1248
Christie's, London	UK	1990	120	3626
Christie's, London	UK	1991	121	580
Christie's, Chicago	US	1991	121	1011
Christie's, London	UK	1992	122	302
Christie's, London	UK	1993	123	894
Christie's, London	UK	1993	123	894
Christie's, London	UK	1993	123	894
Christie's, London	UK	1993	123	894
Christie's, London	UK	1993	123	894
David & Co.	US	1994	124	2789
David & Co.	US	1994	124	2789
David & Co.	US	1995	125	3616
Christie's, New York	US	1995	125	3955
David & Co.	US	1995	125	3616
The Chicago Wine Company	US	1996	126	1866
Christie's, London	UK	1996	126	2055
Christie's, London	UK	1996	126	2055
Sherry Lehman/Sotheby's	US	1997	127	2468

Auction	Loc	Year	Age	Price
Morrell & Co.	US	1997	127	9656
Zachy's/Christie's	US	1998	128	1336
Morrell & Co.	US	1998	128	11621
Zachy's/Christie's	US	1998	128	3645
Sherry Lehman/Sotheby's	US	1998	128	2219
Morrell & Co.	US	1998	128	11621
Christie's, London	UK	1999	129	8434
Christie's, London	UK	1999	129	1756
Zachy's/Christie's	US	1999	129	3101
Christie's, London	UK	1999	129	6689
Sherry Lehman/Sotheby's	US	1999	129	5685
Zachy's/Christie's	US	1999	129	3101
Sherry Lehman/Sotheby's	US	1999	129	2247
The Chicago Wine Company	US	2000	130	5200
The Chicago Wine Company	US	2001	131	7195
Zachy's/Christie's	US	2006	136	3611
Zachy's/Christie's	US	2006	136	20063
Christie's, London	UK	2006	136	7507

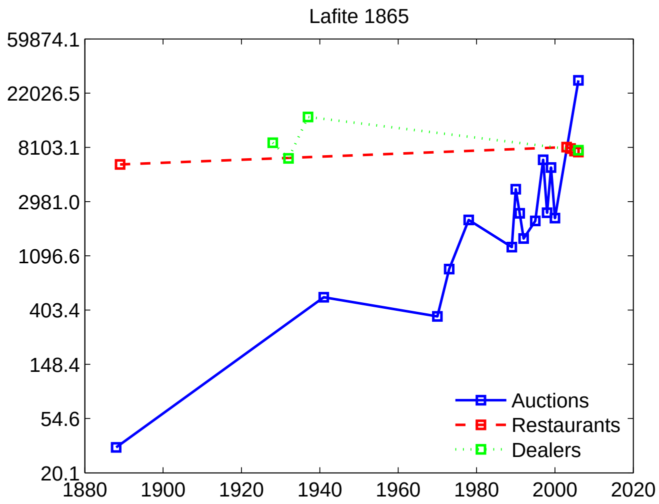
The 1870 Lafite – RESTAURANTS

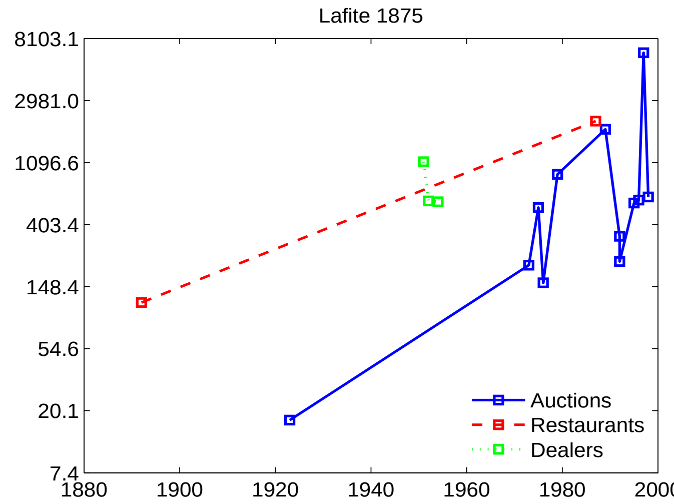
Restaurant	Loc	Year	Age	Price
Fest-Essen, Dusseldorf	GE	1889	19	32
CentralStelle, Dusseldorf	GE	1895	25	35
Charlie Trotters, Chicago	US	2003	133	7931
Charlie Trotters, Chicago	US	2003	133	8891
Charlie Trotters, Chicago	US	2004	134	8660
Charlie Trotters, Chicago	US	2004	134	7726
Charlie Trotters, Chicago	US	2005	135	7367
Charlie Trotters, Chicago	US	2005	135	8258
Charlie Trotters, Chicago	US	2006	136	8111
Charlie Trotters, Chicago	US	2006	136	7235
Charlie Trotters, Chicago	US	2006	136	8111
Charlie Trotters, Chicago	US	2007	137	12806
Charlie Trotters, Chicago	US	2007	137	14941

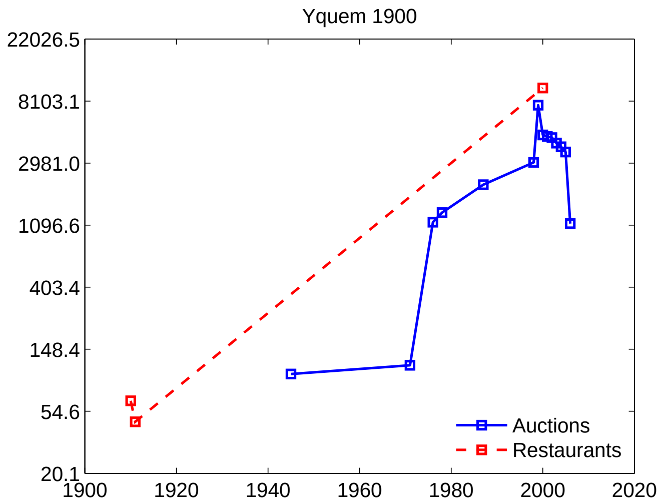
The 1870 Lafite – DEALERS

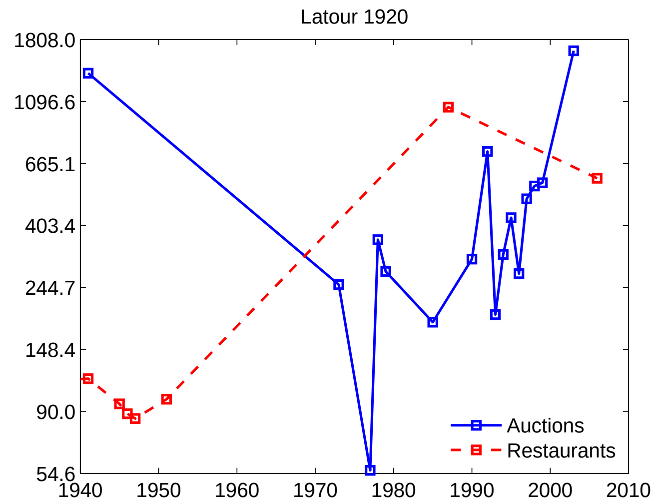
Dealer	Loc	Year	Age	Price
Day Watson	UK	1873	3	22
Berry Bros. & Rudd	UK	1907	37	583
Berry Bros. & Rudd	UK	1928	58	980
Berry Bros. & Rudd	UK	1932	62	771
Berry Bros. & Rudd	UK	1935	65	1232
Berry Bros. & Rudd	UK	1937	67	1182
CellarBrokers.com	US	2007	137	9074

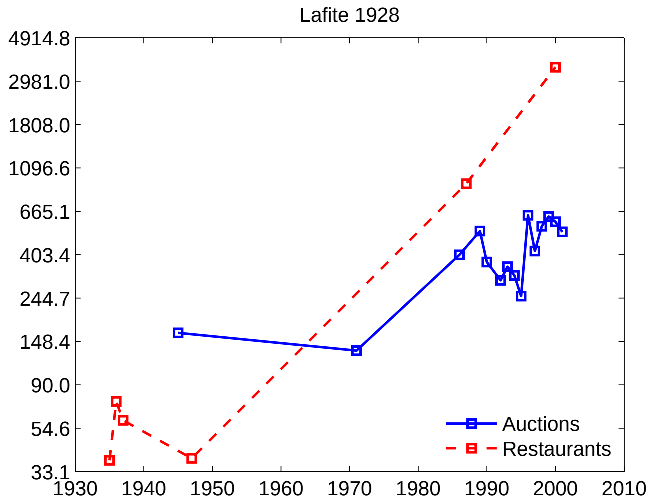
11.2 The histories of some other wines

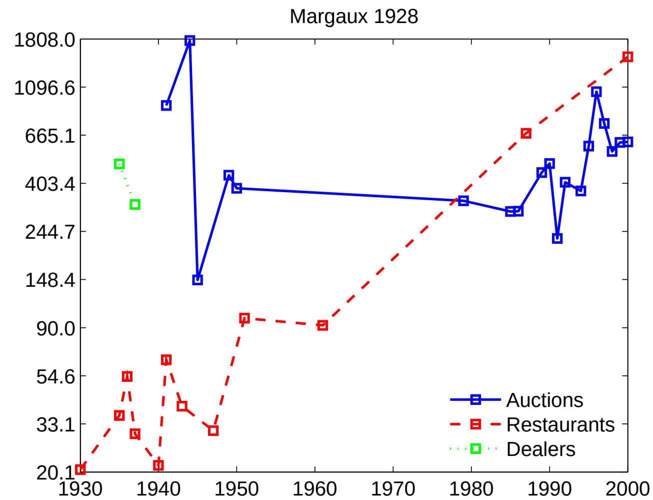


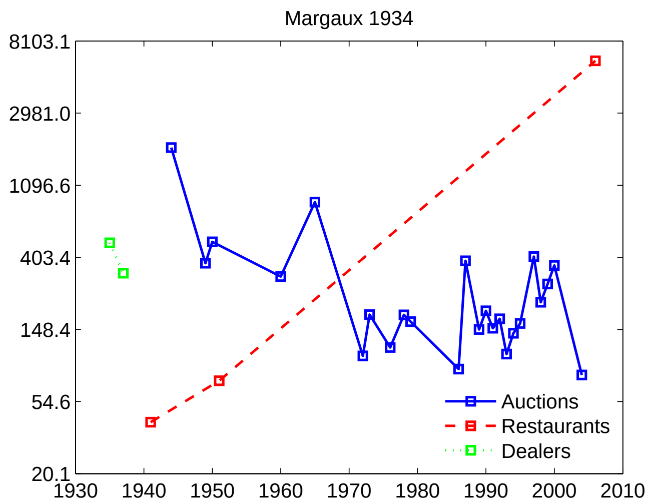







Lafite 1928
4914.8
2981.0
1808.0
1096.6
665.1
403.4
244.7
148.4
90.0
54.6
33.1
1930 1940 1950 1960 1970 1980 1990 2000 2010
Auctions
Restaurants


Margaux 1928
1808.0
1096.6
665.1
403.4
244.7
148.4
90.0
54.6
33.1
20.1
1930 1940 1950 1960 1970 1980 1990 2000
Auctions
Restaurants
Dealers


Margaux 1934
8103.1
2981.0
1096.6
403.4
148.4
54.6
20.1
1930 1940 1950 1960 1970 1980 1990 2000 2010
Auctions
Restaurants
Dealers