

# Insuring Non-Verifiable Losses and the Role of Intermediaries<sup>1</sup>

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## **Abstract**

We analyze optimal risk sharing arrangements when losses are observable by policyholders and insurers but not verifiable. The optimal contract to insure individual losses can be implemented through a standard insurance contract with a deductible where the policyholder bears all losses lower than the deductible and an upper limit that restricts the maximum payment to the policyholder. For a group of policyholders it is optimal to choose contracts with individual deductibles and a joint upper limit. Insurance brokers can play an important role in implementing these contracts.

**Key Words** deductible insurance, upper limit, implicit insurance contracts, insurance brokers

**JEL Classification** D86, G22, L14

# 1 Introduction

Without market frictions, it is optimal for a risk-averse policyholder to purchase full insurance coverage from risk-neutral insurers. The only friction that we introduce is the non-verifiability of losses. Thereby we capture the idea that individuals and firms face many risks for which it is difficult to write explicit contingent insurance contracts. The optimal self-enforcing contract for the policyholder to insure observable but non-verifiable losses can be implemented through an insurance contract with a deductible where the policyholder bears all losses lower than the deductible and an upper limit that restricts the maximum payment to the policyholder.

We consider a model with infinite periods where, in each period, risk-averse policyholders incur a random loss. The magnitude of a policyholder's loss is observable by the insurer but not verifiable. At the beginning of each period, the policyholder and the insurer write a contract that specifies the level of the indemnity payment as a function of the realized loss. If the insurer honors the contract, the policyholder renews the coverage for the next period; if not, he will choose another insurer. In equilibrium, the policyholder pays an insurance premium in excess of the expected loss so that the insurer earns a rent when signing the contract. The expected future rent from continued business induces the insurer to make a payment despite the non-verifiability of losses. The maximum indemnity payment determines the level of the required rent; losses below the maximum claim can be insured at a fair premium that equals the expected payment. An upper limit reduces the required rent. At the same time, reducing coverage in high-loss states increases the marginal utility in these states and a deductible becomes optimal.

When the indemnity payment implied by the contract is lower than the upper limit, the insurer would be willing to pay more for the continued renewal of the contract than required by the contract. However, it is not possible for the policyholder to take advantage of this situation by blackmailing the insurer and requiring a higher payment for future continuation. The reason is that blackmailing reduces the insurer's willingness to pay in the future, which reduces the level of future risk sharing. For this reason, it would be optimal for the blackmailing policyholder to switch the insurer after the current insurer made the

payment. Anticipating non-renewal incentives, the insurer would require the policyholder to commit to future renewal. But with such a commitment the insurer would not make any payments in the future so that commitment is not feasible.

Although the “excess willingness to pay” cannot be exploited through blackmailing, policyholders can benefit from it by writing contracts with a joint upper limit. To provide intuition, assume that there are two individuals with two separate contracts. Each contract has a deductible and an upper limit on each individual’s loss. Compare this situation with a joint contract where each individual still has the same deductible on the individual loss. But the new contract has a joint upper limit on the total payment to the two policyholders, where the joint upper limit is equal to the sum of the two individual upper limits. This contract involves the same rent, but results in improved risk sharing: if one policyholder has a loss below the individual upper limit and the other policyholder has a loss in excess of the upper limit, a joint upper limit allows for a higher total payment. Of course, the joint contract involves a higher premium, but the increased premium merely reflects the increased expected insurance coverage. We show that a joint contract with individual deductibles and a joint upper limit on the total loss is optimal for policyholders.

Implementing a contract with a joint upper limit requires that the policyholders observe each others’ losses and that they collectively switch to another insurer if the incumbent insurer shirks. Thus, the coordination cost of directly writing an explicit joint contract with the insurer would be high and such contracts are not observed in practice. However, an insurance broker may step in as an intermediary to implement the implicit joint contract. By bundling the policies of multiple policyholders, the broker has a higher bargaining power than individual policyholders: the broker can threaten to leave with all clients if the insurer shirks on one. Thereby, the broker increases the expected payments to policyholders for a given level of rent paid to the insurer. Moreover, by coordinating claims settlement, the broker oversees the claims and insurance payments of its clients.

The role for the insurance broker portrayed in our model is reflected in the contractual arrangement with insurers. It is normal for brokers to “own the renewal rights” on the book of business they place with the insurer. That is, the broker is free to recommend to its clients that they renew with the current insurer or switch to a rival. Accordingly, the

insurer revokes any right to directly solicit business placed through the broker. This provision vests the broker with considerable bargaining power and enables the broker to elicit higher transfers for non-verifiable losses than would be possible (at the same rent) with individual contracting.

The design of insurance contracts has received considerable attention in the insurance literature. An important result is that, with proportional loading, the insurer optimally covers claims in excess of some threshold (deductible) (see, e.g., Arrow, 1963; Raviv, 1979; Gollier and Schlesinger, 1995). The optimality of deductible insurance is also derived by Cummins and Mahul (2004) assuming an exogenous upper policy limit. In this literature it is assumed that losses are observable and verifiable so that enforceable contracts can be written contingent on the realized loss. An interesting result of our paper is that a contract with an (endogenous) upper limit and a standard deductible is optimal with observable but non-verifiable losses.

The costly state verification literature analyzes situations in which the realized payoff is observable and verifiable only at a cost (Townsend, 1979; Gale and Hellwig, 1985; Bond and Crocker, 1997). Since losses are verifiable after auditing, the insurer can write an enforceable contract where the policyholder is punished for false reports. It is therefore possible to implement a risk-sharing contract where the policyholder truthfully reports the realized loss. Focusing on the financing motive and assuming risk-neutral agents, a standard debt contract is optimal with costly state verification. But with risk-averse agents, the optimal risk-transfer contract does not resemble a standard insurance contract with a deductible or an upper limit. We assume losses are non-verifiable.

In the insurance literature non-verifiable losses are interpreted as uninsurable background risk. The focus is on how background risk affects the insurance demand for verifiable losses (see, e.g., Doherty and Schlesinger, 1983; Gollier, 1996). In contrast, we assume that the insurer can observe the non-verifiable loss. Thus, reputation makes it possible to insure this risk and we focus on the optimal contract to insure non-verifiable losses.

There is a large literature that analyzes lending and risk sharing relations when profits and losses are observable but non-verifiable. One strand of the literature focuses on the threat of liquidation as a means to induce borrowers to make payments to lenders (Bolton

and Scharfstein, 1990; Hart and Moore 1998). Our paper is related to a second strand, where, in a model of repeated interaction, the threat of exclusion is the only punishment available (Kimball, 1988; Kehoe and Levine, 1993; Kocherlakota, 1996; Allen and Gale, 1999; Bond and Krishnamurthy, 2003). Kimball (1988) and Kocherlakota (1996) analyze mutual implicit risk sharing between two risk-averse agents where the threat of exclusion from future risk sharing provides a risk-averse agent with incentives to make payments to another agent. In our setting, insurance is offered by risk-neutral insurers that earn a rent when selling insurance contracts and a policyholder can switch the insurer. We contribute to the literature by deriving the optimal insurance contract in this setting.<sup>1</sup>

We also analyze how risk sharing can be improved through joint contracts and discuss the role of brokers. Brokers act as agents who match trading partners and generate information (e.g., Rubinstein and Wolinsky, 1987; Biglaiser, 1993; Cummins and Doherty, 2006). We argue that brokers can also serve as a clearing house of the reputation of insurers and, in doing so, risk sharing is more efficient than with bilateral trading. The role of the broker in our model is related to the role of intermediaries in Kingston (2006) where individuals can defect on trades. In a repeated game, traders who shirk can be excluded from future trades and intermediaries can act as a clearing house for information. In our model information is also important, but in addition, the broker improves efficiency through the implementation of joint contracts.

The paper proceeds as follows. In the next section we discuss non-verifiability in insurance contracts. We introduce the model in Section 3 and derive the optimal incentive compatible contract for a single policyholder in Section 4. In Section 5, we examine the optimal joint contract for a group of agents. We discuss the role of brokers in Section 6 and conclude in Section 7 by discussing the implications of our results. All proofs are in the appendix.

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<sup>1</sup>Excluding an agent from purchasing insurance is difficult in an anonymous insurance market where insurers compete for business. If exclusion were possible in our setting, it would be possible to punish the policyholder for false reports and the insurer would be willing to commit to a transfer mechanism that satisfies the policyholder's truth telling constraint. Such a mechanism does not require the policyholder to pay a rent to the insurer, but the truth telling constraint may still be binding if the possible loss is large and the policyholder not very risk averse. In this case, it is necessary to limit the maximum indemnity payment to satisfy the policyholder's truth telling constraint and, as in our setting, the optimal insurance contract again contains a deductible and upper limit.

## 2 Non-verifiability in insurance relations

We assume that insured losses are non-verifiable. This assumption is introduced to model the complexity and incompleteness of insurance contracts. Insurance, and in particular, corporate insurance contracts are not a straightforward mapping from realized losses to claims payments. Instead, losses have to be adjusted for particular circumstances and events. Some of these events (terms and conditions) are included in the contract, but difficult to verify or enforce; other events are simply too complex to be included in the contract, but the parties may still know that they should apply; yet other events have not been anticipated, but the parties know what they would have agreed on ex ante if they had anticipated the event. In the following we provide examples for such contingencies and events that cause complexity and incompleteness of insurance contracts and are underlying the non-verifiability of insured losses.

Because of moral hazard, contracts require certain precautionary actions that are difficult to verify and claims may be denied when these actions were not taken. Another important aspect of insurance is the speed with which claims are settled. This aspect of quality is also difficult to enforce, in particular, if the contracting parties can haggle over the terms of the contract. Also, an insurance policy might be specific about how a claim is to be settled (damage to an office building or its contents might be limited to the repair cost or the cost of replacement with something of similar condition), but the exact terms and conditions are difficult to write down in an enforceable way as the ex ante optimal claim settlement also depends on idiosyncratic circumstances. Thus, to enforce claims in a court it is usually important to not only verify the incurred loss but also the cause.

As an illustration consider the World Trade Center losses. Policies normally exclude war damage. The question is whether the World Trade Center losses resulted from an act of war. Certainly the response of the Bush administration was couched in the language of war and the subsequent military operations in Iraq and Afghanistan are clearly considered to be wars. But were the initiating event of 9/11 acts of “war”? There is clearly some ambiguity here. However, the interesting point is that the domestic insurance industry never even entertained the notion of appealing to the war exclusion. One can, of course argue, that this cooperative

stance by US insurers was adopted in the spirit of national unity that followed 9/11 and had little to do with the contracting problem we are presenting here. However, if we look at the international reinsurers (presumably not caught up in nationalistic fervor) on whom much of the ultimate loss fell, they also readily accepted to indemnify the primary companies for 9/11 losses. Indeed, there seems to be a complete meeting of the minds between primary insurer and reinsurers, and importantly also brokers. It may be supposed that any dissenting reinsurers might have subsequently had a tough time in attracting business in this mostly brokered market.

Some insurers, such as Chubb Insurance company, have made and protected a reputation for going the “extra mile” to ensure that policyholders are happy with their claims settlements. The strategy is to resolve ambiguity over the amount or coverage in the policyholder’s favor to ensure that the policyholder is adequately compensated. This flexibility cannot be written in an enforceable contract.

Non-contractible events can arise because of unanticipated losses. Although it is clear that unanticipated losses arise on a regular basis, individual unforeseen risks cannot be specified (excluded or included) in a formal contract. Even if they could be specified, they might be unsuited to insurance perhaps because they would incite severe ex post moral hazard, or because they are undiversifiable. For example, consider toxic mold, which burst onto the insurance scene as an unanticipated loss. Its coverage carries significant moral hazard since insurance may be seen as a substitute for proper repair and maintenance of property. It may not be practical to write into contracts enforceable exclusions based, not only on the peril which is unanticipated, but on the moral hazard it might engender. These are examples where enforceable contracts cannot be written but where a self-enforcing insurance contract might provide a significant degree of insurance.<sup>2</sup>

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<sup>2</sup>Allen and Gale (1999) also discuss the role of self-enforcing (implicit) contracts to insure against unforeseen contingencies and nonspecific risks. They focus on the difference between financial transactions that are carried out through intermediaries and direct market transactions.

### 3 The Model

There are risk-averse policyholders (households or firms) and risk-neutral insurers in an infinite-period economy. All policyholders are infinitely lived and identical with strictly increasing and concave utility function  $u$ . In each period, each policyholder  $j$  is endowed with an initial wealth  $w_0$  and a loss of random size  $L_j$ . Losses are identically and continuously distributed on the interval  $[0, \bar{l}]$ .

The sequence of events in each period is as follows. In a first step, insurers simultaneously quote an insurance premium  $P$  and a non-negative coverage schedule  $I = I(l)$  for all loss realizations  $l \in [0, \bar{l}]$ , which we discuss in greater detail below. Each policyholder chooses the insurance contract that maximizes the expected utility. Then, losses are realized. Each policyholder and the insurer observe the realized loss. However, this loss is not verifiable so that the promised coverage schedule is not enforceable in a court. Instead, insurers can choose the level of transfer to their policyholders after the loss has been realized. In particular, they may choose to deviate from the coverage schedule which they initially offered. Each insurer has sufficient wealth to honor all claims if it wishes to do so. Finally, all policyholders consume their end of period net wealth, which consists of the initial wealth and the insurer's transfer net of the premium and the realized loss. This assumption implies that there is no intertemporal borrowing or lending by policyholders. Moreover, insurance contracts are one-period contracts. Maximizing the present value of the expected utility in all future periods is therefore equivalent to maximizing the one-period expected utility.

The risk-free rate of return used to discount periods is  $r$ ; there is no discounting within periods.

### 4 Individual contracting

We first consider the case where each policyholder chooses a coverage schedule that depends on the individual loss only. Because all policyholders are identical, we suppress the index  $j$  and the promised compensation after a loss realization  $l \in [0, \bar{l}]$  is  $I(l)$ . Since the loss is not verifiable, the compensation has to be incentive compatible. The only mechanism that

can provide the insurer with incentives to honor the promise is the policyholder's threat to choose a different insurer in the future if the current insurer shirks on  $I(l)$ .

In equilibrium, the policyholder chooses the insurance contract that maximizes the expected utility given the premium  $P$  and incentive-compatible compensation schedule  $I(l)$ . The policyholder forms rational expectations about an insurer's transfer for each possible loss  $l \in [0, \bar{l}]$  and is not fooled by compensation schedules that are not incentive compatible. Unless the insurer shirks or another insurer's contract yields a higher expected utility, the policyholder will continue to do business with the insurer. The contract is incentive compatible if, for all  $l \in [0, \bar{l}]$ , the present value of the future rent from continued business is at least as large as the required payment:

$$\frac{P - E[I(L)]}{r} \geq I(l). \quad (1)$$

(1) is satisfied for all indemnity payments if it is satisfied for the maximum promised payment  $I^{\max} = \max_{l \in [0, \bar{l}]} I(l)$ . Substituting  $I^{\max}$  and rearranging terms yields  $P \geq E[I(L)] + rI^{\max}$ . Therefore, in addition to the expected claims payment,  $E[I(L)]$ , the premium includes a rent,  $rI^{\max}$ , that provides sufficient incentives to the insurer to honor the claim payments.

With insurance, the policyholder's level of consumption at the end of period is  $w(l) = w_0 - l - P + I(l)$ , and the incentive-compatible insurance contract that maximizes the policyholder's expected utility from end of period consumption is determined by the following optimization problem

$$\max_{(P, I(\cdot))} E[u(w(L))] \quad (2)$$

$$\text{s.t. } P \geq E[I(L)] + rI^{\max}, \quad (3)$$

$$I^{\max} = \max_{l \in [0, \bar{l}]} I(l),$$

$$0 \leq I(l) \text{ for all } l \in [0, \bar{l}].$$

In the following proposition we derive the optimal structure of the implicit insurance contract.

**Proposition 1** *The optimal individual insurance contract  $I^*(\cdot)$ , which maximizes the poli-*

policyholder's expected utility and is incentive compatible, includes a strictly positive deductible,  $D^* > 0$ , an upper limit,  $I^{\max*} < \bar{l} - D^*$ , and full compensation of losses in excess of the deductible until the upper limit is reached:  $I^*(l) = \min \{(l - D^*)^+, I^{\max*}\}$  with  $D^* > 0$  and  $D^* + I^{\max*} < \bar{l}$ .

The contract is piece-wise linear and resembles a standard insurance contract with a deductible and an upper limit. The novel feature of this optimal contract is that we derive it for non-verifiable losses and that we obtain an endogenous upper limit. Arrow (1963), Raviv (1979), and Gollier and Schlesinger (1995) show that with verifiable losses a straight deductible insurance policy is optimal for risk-averse policyholders if insurance involves a frictional cost that is proportional to each indemnity payment. In that case, the marginal cost of providing an additional dollar of insurance coverage is constant. With non-verifiable losses, the friction stems from the incentive-compatibility constraint (3), which is binding and requires that the insurer earns a rent that is proportional to the maximum indemnity payment. In this case, the marginal frictional cost of providing an additional dollar of insurance coverage is zero below the maximum coverage. An upper limit reduces the cost of providing incentive-compatible insurance and full insurance of losses below the upper limit is possible at a fair premium. However, a straight upper limit policy implies that the policyholder's marginal utility for losses above the upper limit is higher than for losses below the upper limit. A deductible reduces the premium level and thereby allows the policyholder to transfer wealth from the states with low marginal utility to those with high marginal utility. This motive for a deductible is also discussed by Cummins and Mahul (2004) for verifiable losses and an exogenous upper limit; a related argument is made by Doherty and Schlesinger (1990) who show that non-performance of insurance contracts makes it optimal for policyholders to choose partial insurance.

**Verifiable and non-verifiable losses** We focus on non-verifiable losses where the policyholder optimally retains some risk. This risk constitutes background risk, which can have an effect on the insurance demand and structure for other verifiable losses. If verifiable losses are insurable at a fair premium and uncorrelated with the background risk, full insurance of the verifiable losses is optimal. It is difficult to draw general conclusions about the interactions

if the background risk is correlated with other risks (see, e.g., Schlesinger, 2000). However, it is interesting to note that the optimal insurance contract in Proposition 1 does not change even if all losses below  $D^* + I^{\max*}$  are verifiable. The reason is that the policyholder could have increased coverage for these losses at a fair premium, but voluntarily did not choose to do so.

**Patience, financial distress, and the level of insurance** The interest rate  $r$  plays a critical role in the level of rent that is required for an incentive-compatible maximum indemnity payment. The interest rate can be interpreted as a measure of patience. Alternatively, we could have assumed that there is an exogenous probability that the policyholder (firm) will stop purchasing insurance, e.g., because the firm goes bankrupt. A model with zero discounting but an exogenous probability of termination yields equivalent results as our model. A higher probability of termination is equivalent to a higher interest rate.

**Proposition 2** *There exists a level of interest rate  $\bar{r}$  such that no insurance is optimal for all  $r \geq \bar{r}$ , i.e.,  $D^* = \bar{l}$  and thus  $I^*(l) = 0$  for all  $l \in [0, \bar{l}]$ . If preferences exhibit constant absolute risk aversion (CARA),  $D^*$  is strictly increasing and  $D^* + I^{\max*}$  is strictly decreasing in  $r$  for all  $r < \bar{r}$ .*

For CARA preferences, the optimal level of insurance decreases in  $r$ . In particular, the optimal deductible  $D^*$  increases and the upper limit  $I^{\max*}$  decreases in  $r$  (faster than the deductible level increases). This observation is particularly interesting for an exogenous probability of termination. It implies that firms with a higher probability of distress, which have a higher probability of not buying insurance in the future, might optimally reduce the insurance coverage of non-verifiable losses. Thus, we provide an additional justification for why highly levered or distressed firms choose a lower level of insurance that does not rely on risk shifting or wealth effects. It is well known that with CARA, insurance demand decreases when the price for insurance increases. However, the interesting effect here is that the cause for the higher cost stems from the firms' higher probability of financial distress. Although the probability of distress is unrelated to the insured risk, it increases the cost of

providing incentives to the insurer as these incentives stem from future business with the insurer. Moreover, one reason for a policy limit is that it reduces the insurer's risk of financial distress. We show that it may be related to the policyholder's risk of financial distress.

**Blackmailing** If, for a given loss  $l$ , the required indemnity payment  $I(l)$  is lower than  $I^{\max}$ , the insurer would be willing to pay more for the continued renewal of the contract than required by the contract. Thus, the policyholder may be tempted to blackmail the insurer and require a total payment up to  $I^{\max}$ . Obviously, blackmailing is not a problem if the policyholder could commit not to engage in blackmailing. But blackmailing is not possible without commitment either. Assume that after having been blackmailed once, the insurer expects a level of blackmailing of  $E[x] > 0$  in every future period. Even if  $E[x]$  is very small, the insurer will not yield to blackmailing. The reason is that these beliefs reduce the insurer's incentives, which in turn reduces the policyholder's willingness to purchase insurance from this insurer in the future.

After blackmailing, the insurer's incentive-compatibility constraint becomes

$$\frac{P - E[I(L)] - E[x]}{r} \geq I(l).$$

Given that the incentive constraint was binding for  $I^{\max}$  without blackmailing, the insurer no longer has an incentive to pay  $I^{\max}$  when the policyholder incurs high losses. Thus, blackmailing today reduces the insurer's maximum willingness to pay in the future. Given this adverse effect of blackmailing, it is not optimal for the policyholder to renew the insurance contract after blackmailing. Instead, it is optimal for the policyholder to switch the insurer after the incumbent insurer made the payment. Anticipating non-renewal incentives, the insurer will not make a payment. We note that the policyholder cannot commit to purchasing insurance in the future because with such a commitment the insurer's incentive-compatibility constraint will be violated.

Blackmailing is payoff-equivalent to requiring the insurer to reduce the price for insurance right before renewal of a contract. However, this cannot be optimal if such a request is likely to come again in the future. The price was chosen to maximize the policyholder's expected

utility subject to the insurer's incentive-compatibility constraint. Therefore, reducing the price cannot be in the interest of the policyholder, who will choose a different insurer if the insurer is willing to reduce the price.

## 5 Joint contracting

We now consider a group of  $n$  policyholders with individual loss exposures  $L_1, \dots, L_n$ . The sequence of events is equivalent to the one with individual insurance. The notable difference is that a joint insurance contract is possible where each policyholder's compensation is now given by  $I_j = I_j(l^n)$  for  $l^n = (l_1, \dots, l_n) \in [0, \bar{l}]^n$  and  $j = 1, \dots, n$ . Each policyholder in the group observes the entire realization of losses  $l^n = (l_1, \dots, l_n)$  of all group members and coordinates his or her action with the other group members in the following way. If the insurer pays  $I_j = I_j(l^n)$  to each policyholder, then each policyholder purchases insurance from the insurer again in the future. However, if the insurer shirks on one of the policyholders, then all policyholders switch to a rival insurer.

The incentive-compatibility constraints ensuring that the insurer honors the claim payments are then

$$nP \geq E \left[ \sum_{j=1}^n I_j(L^n) \right] + r \cdot \sum_{j=1}^n I_j(l^n)$$

for all  $l^n \in [0, \bar{l}]^n$ . The necessary and sufficient constraint is determined by the maximum aggregate claim payment to the group, i.e.,  $nP \geq E \left[ \sum_{j=1}^n I_j(L^n) \right] + r I_n^{\max}$ , where  $I_n^{\max} = \max_{l^n \in [0, \bar{l}]^n} \sum_{j=1}^n I_j(l^n)$ . The end of period consumption of policyholder  $j$  is  $w(l_j, l^n) = w_0 - l_j - P + I_j(l^n)$ , for all  $l^n \in [0, \bar{l}]^n$ . The optimal premium  $P$  and incentive-compatible compensation structure  $(I_j(\cdot))_{j=1, \dots, n}$  that maximize the policyholders' utilities are given by

the solution to the following optimization problem

$$\begin{aligned}
& \max_{(P, (I_j(\cdot))_{j=1, \dots, n})} \sum_{j=1}^n E[u(w(L_j, L^n))] & (4) \\
& \text{s.t. } nP \geq E\left[\sum_{j=1}^n I_j(L^n)\right] + rI_n^{\max}, \\
& I_n^{\max} = \max_{l^n \in [0, \bar{l}]^n} \sum_{j=1}^n I_j(l^n), \\
& 0 \leq I_j(l) \text{ for all } l_j \in [0, \bar{l}] \text{ and } j = 1, \dots, n.
\end{aligned}$$

In the following proposition we derive the optimal structure of the implicit insurance contracts.

**Proposition 3** *The incentive-compatible insurance contracts that maximizes the policyholders' expected utility are characterized as follows.*

1. Each contract  $I_j^*(\cdot)$  includes a strictly positive individual deductible  $D^* > 0$ .
2. The aggregate claim payments  $\sum_{j=1}^n I_j^*(\cdot)$  to the group of policyholders includes a joint upper limit  $I_n^{\max*} < n(\bar{l} - D^*)$ .
3. The optimal contract can be implemented by setting

$$I_j^*(l^n) = \begin{cases} (l_j - D^*)^+ & \text{if } \sum_{j=1}^n (l_j - D^*)^+ \leq I_n^{\max*} \\ (l_j - D(l^n))^+ & \text{otherwise} \end{cases}$$

where  $D(l^n)$  is given by  $\sum_{j=1}^n (l_j - D(l^n))^+ = I_n^{\max*}$ .

The joint contract has the following properties. First, if the joint upper limit is not binding, each policyholder receives the transfer  $I_j^*(l^n) = (l_j - D^*)^+$ , which only depends on a policyholder's own loss. Second, if  $\sum_{j=1}^n (l_j - D^*)^+ > I_n^{\max*}$ , the deductible  $D^*$  is increased to  $D(l^n)$  and  $I_j^*(l^n) = (l_j - D(l^n))^+$ , where  $D(l^n)$  is chosen so that the total claim payments equal the maximum indemnity, i.e.,  $\sum_{j=1}^n (l_j - D(l^n))^+ = I_n^{\max*}$ .

The joint contract is beneficial even when the joint upper limit is not binding. The reason is that a joint upper limit allows for a more efficient use of the rent that the policyholders have

to pay to the insurer. As an example, we assume that there are two policyholders ( $n = 2$ ) and that the joint upper limit equals the sum of the individual upper limits ( $2I^{\max*} = I_2^{\max*}$ ). Thus, the total rent under the two contracts is equal. Moreover, we assume that the loss of policyholder  $j$  exceeds the individual upper limit, but the loss of policyholder  $i$  is lower than the upper limit ( $l_i < D + I^{\max*} < l_j, i \neq j$ ). Thus, policyholder  $j$ 's maximum payment is  $I^{\max*}$  with separate contracting but can be increased to  $\min\{l_j - D, 2I^{\max*} - (l_i - D)^+\}$  under joint contracting without changing the transfer to policyholder  $i$ .

When the joint upper limit is binding, joint contracting allows for an improved allocation of the total transfer from the insurer to the policyholders. It is ex ante optimal for both policyholders to agree on an allocation where all policyholders' marginal utilities are equal (subject to the constraint that indemnity payments are not negative). The optimal contract for a binding upper limit resembles the contract proposed by Mahul and Wright (2004) for catastrophic risk sharing arrangements within a pool of policyholders when financial resources are limited.

## 6 The role of brokers

Implementing a joint contract requires that policyholders observe each others' losses and the insurer's transfers. Moreover, they jointly have to leave the insurer and switch to another insurer if the insurer shirks on any one of them. The cost of joint contracting between an insurer and  $n$  policyholders may therefore be very high and, indeed, we do not observe such contracts in practice.

However, implementing such contracts may be an important role of intermediaries such as brokers. A broker monitors losses and transfers to its clients and thereby has the required information readily available. If the insurer shirks on one of his clients, the broker can recommend all clients to switch to another insurer.

The brokers' ability to recommend to clients to switch insurers is a central element of brokerage. Typically, brokers "own the renewal rights" on the book of business they place with the insurer. That is, the broker is free to recommend to its clients that they renew with the current insurer or switch to a rival. Correspondingly, the insurer revokes any right to

direct solicitation of business placed through the broker.

The broker can monitor the execution of a joint upper limit. Even if the joint upper limit is not explicitly stated in insurance contracts, by coordinating his clients' choice of insurer, the broker has bargaining power that allows him to implement the same allocation. In this case the broker bargains on behalf of his clients for a more generous transfer. However, when total losses are very high, there are also policyholders who will receive less than what they would receive under individual contracting given their loss realizations. Thus, it is important that policyholders trust that their broker will act in their best interest. Trust is particularly important since the insurer (policyholders) may be tempted to collude with the broker to reduce (increase) transfers in its (their) interest. Of course, such a collusion with the insurer or individual policyholders is not allowed in practice and therefore difficult to enforce. However, if the broker receives side payments from the insurer or other parties, the broker's clients may not trust the broker to improve risk sharing compared to an individual direct contract.

In practice, where insurance contracts are incomplete and difficult to enforce, the relationship often is brokered. For example, Chubb uses a set of independent agents and brokers. These intermediaries "own" the renewal rights and can advise clients to move business if they become unhappy with Chubb's claims performance. Chubb entrusts its reputation to these agents and brokers to commit to its promised claim settlement strategy. As another example, contracts in the reinsurance market are not very detailed. Contracts rarely specify the underwriting and claims settlement practices to be adopted by the primary insurer and often are not specific in defining coverage. This allows some ex post flexibility for the reinsurer to respond to losses that may not be covered. A feature of the reinsurance market is the ubiquity of brokers. If a reinsurer behaves badly to the primary insurer, the broker will know of it. The consequence for the reinsurer might be not only a loss of that contract, but a diversion of other business from the broker to other reinsurers. This leveraging of reputation enhances the bargaining power of the primary insurer.

## 7 Discussion

We derive the structure of the optimal insurance contract when losses are non-verifiable. The optimal contract to insure individual losses resembles a standard insurance contract with a deductible and an upper limit on the maximum indemnity payment. The optimal joint contract to insure the losses of a group of policyholders involves a deductible on individual losses and a joint upper limit on the total maximum indemnity payment to the group of policyholders.

Insurers honor policyholders' claims because they earn a rent from future business. To limit the rent, it is optimal for policyholders to introduce a deductible and an upper limit. With proportional loading, the frictional cost of insurance depends only on the expected claims payment and not whether the underlying risk is a high-severity-low-probability event or a low-severity-high-probability event. In our setting, where losses are non-verifiable, the rent and thus the frictional cost is proportional to the maximum indemnity payment. For high-severity-low-probability events, the maximum loss relative to the expected loss is particularly large and insuring non-verifiable high-severity-low-probability risk is particularly costly relative to the expected coverage. This is consistent with high prices in the reinsurance market for catastrophe risk. Froot (2001) and Froot and O'Connell (2008) show that reinsurance premiums are a multiple of expected losses; during some periods up to seven times. This multiple is particularly high for low-probability layers. Froot (2001) reviews several explanations and argues that shortage of risk-taking capital in the reinsurance market due to capital market imperfections is the most convincing reason. In our context, if claims due to catastrophic events are difficult to verify, high premiums in the reinsurance market for catastrophe risks result from the high rent which the insurer has to pay for providing the reinsurer with sufficient incentives not to dispute the validity of claims or to delay payments. Whereas Froot (2001) argues that it is costly for a reinsurer to hold sufficient capital to cover claims (ability to pay), we suggest that it is also costly to induce a reinsurer to honor claims (willingness to pay).

The required rent is also increasing if there is a positive probability that the insurance relation terminates for exogenous reasons, for example, because of the client's financial dis-

tress. If the client's probability of financial distress increases, covering non-verifiable losses will become more costly. Reinsurance and retrocession are based on long term relations and mutual trust. Large catastrophic losses in the reinsurance market tend to increase reinsurance premiums and reduce quantities through a reduction of reinsurance capacity and increase in counterparty risk (Gron, 1994; Winter, 1994; Cagle and Harrington, 1995; Cummins and Danzon, 1997; Froot, 2001; Froot and O'Connell, 2008). Both reasons originate from the protection seller. Our paper suggests another link related to the protection buyer. An increase in the probability of financial distress of the protection buyer due to a large catastrophic loss reduces the likelihood that a business relation is continued. Thus, premiums for the reinsurance of losses increase and demand for reinsurance decreases if some of the losses are non-verifiable.

# A Appendix: Proofs

## A.1 Proof of Proposition 1

Let  $F$  denote the cumulative distribution function of the loss  $L$  with support  $[0, \bar{l}]$  and probability density function  $f$ . The solution to the optimization problem (2) is identical to the solution of the following problem

$$\max_{(P(\cdot), I(\cdot), I^{\max}(\cdot))} E[u(w(L))] \quad (5)$$

$$\text{s.t. } E[P(L)] \geq E[I(L)] + rE[I^{\max}(L)], \quad (6)$$

$$P'(l) = 0 \text{ for all } l \in [0, \bar{l}], \quad (7)$$

$$I^{\max\prime}(l) = 0 \text{ for all } l \in [0, \bar{l}], \quad (8)$$

$$0 \leq I(l) \leq I^{\max}(l) \text{ for all } l \in [0, \bar{l}] \quad (9)$$

with  $w(L) = w_0 - L - P(L) + I(L)$ . The Lagrangian function to this function is then given by

$$\begin{aligned} \mathcal{L} = & u(w(l)) f(l) + \lambda(P(l) - I(l) - rI^{\max}(l)) f(l) \\ & - \mu(l) P'(l) - \nu(l) I^{\max\prime}(l) + \xi(l) I(l) + \zeta(l) (I^{\max}(l) - I_j(l)) \end{aligned}$$

where  $\lambda$ ,  $\mu(l)$ ,  $\nu(l)$ ,  $\xi(l)$ , and  $\zeta(l)$  are the Lagrange multipliers to the constraints (6), (7), (8), and (9), respectively, with

$$\xi(l) \begin{cases} = 0 & \text{if } I(l) > 0 \\ > 0 & \text{if } I(l) = 0 \end{cases}$$

and

$$\zeta(l) \begin{cases} = 0 & \text{if } I(l) < I^{\max}(l) \\ > 0 & \text{if } I(l) = I^{\max}(l) \end{cases}.$$

Defining the Quasi-Hamiltonian function as

$$\mathcal{H}(P(l), I(l), I^{\max}(l), l) = u(w(l)) f(l) + \lambda(P(l) - I(l) - rI^{\max}(l)) f(l)$$

we can write the optimization problem (5) as

$$\begin{aligned} \max_{(P(\cdot), I(\cdot), I^{\max}(\cdot))} \int_0^{\bar{l}} & (\mathcal{H}(P(l), I(l), I^{\max}(l)), l) + (\xi(l) - \zeta(l)) I(l) \\ & + \zeta(l) I^{\max}(l) - \mu(l) P'(l) - \nu(l) I^{\max\prime}(l)) dl. \end{aligned}$$

Variational calculus and partial integration implies

$$\begin{aligned} & \int_0^{\bar{l}} \left( \left( \frac{\partial \mathcal{H}}{\partial I} + \xi(l) - \zeta(l) \right) \delta I(l) + \left( \frac{\partial \mathcal{H}}{\partial P} + \mu'(l) \right) \delta P(l) + \left( \frac{\partial \mathcal{H}}{\partial I^{\max}} + \zeta(l) + \nu'(l) \right) \delta I^{\max}(l) \right) dl \\ & - (\mu(l) \delta P(l) + \nu(l) \delta I^{\max}(l)) \Big|_0^{\bar{l}} \\ & = 0 \end{aligned}$$

The necessary conditions for optimality are

$$u'(w(l)) f(l) - \lambda f(l) + \xi(l) - \zeta(l) = 0 \quad (10)$$

$$-u'(w(l)) f(l) + \lambda f(l) + \mu'(l) = 0 \quad (11)$$

$$-\lambda r f(l) + \zeta(l) + \nu'(l) = 0 \quad (12)$$

$$\mu(\bar{l}) - \mu(l) = 0 \quad (13)$$

$$\nu(\bar{l}) - \nu(l) = 0 \quad (14)$$

The sufficient condition is satisfied since

$$\int_0^{\bar{l}} u(w(l)) f(l) dl$$

is concave in  $I$ . Conditions (11) and (13) imply

$$\int_0^{\bar{l}} u'(w(l)) f(l) dl = \lambda \quad (15)$$

and conditions (12) and (14) yield

$$\int_0^{\bar{l}} \zeta(l) dl = \lambda r. \quad (16)$$

Combining conditions (10), (15), and (16) implies

$$\int_0^{\bar{l}} \xi(l) dl = \lambda r. \quad (17)$$

For inner solutions  $0 < I^*(l) < I^{\max}(l)$  we have  $\xi(l) = \zeta(l) = 0$  and condition (10) simplifies to  $u'(w(l)) = \lambda$ . Marginal utility is thus constant which implies  $I^{*'}(l) = 1$  for all  $l$  such that  $0 < I^*(l) < I^{\max}(l)$ . Furthermore, because  $u$  is strictly increasing, condition (15) implies  $\lambda > 0$ . Since  $r > 0$ , condition (17) implies that there exists a subset in  $[0, \bar{l}]$  for which  $\xi(l) > 0$ , i.e., for which  $I^*(l) = 0$ . Since  $I^*(l)$  is strictly increasing for inner solutions, this implies that the subset is of the form  $[0, D]$  for some deductible level  $D > 0$ . Analogously, condition (16) implies that there exists a subset in  $[0, \bar{l}]$  for which  $\zeta(l) > 0$ , i.e., for which  $I^*(l) = I^{\max*}(l) = I^{\max*}$ . Again, since  $I^*(l)$  is strictly increasing for inner solutions, this subset is of the form  $[U, \bar{l}]$  for some  $U < \bar{l}$ . Combining these three conditions implies that  $U = D + I^{\max*}$  and  $I^*(l) = \min\{(l - D)^+, I^{\max*}\}$  with  $D > 0$  and  $D + I^{\max*} < \bar{l}$ .

## A.2 Proof of Proposition 2

In Proposition 1, we have shown that the optimal contract is of the form  $I^*(l) = \min\{(l - D^*)^+, I^{\max*}\}$  with  $D^* > 0$  and  $D + I^{\max*} < \bar{l}$ . The optimal deductible level  $D^*$  and maximum compensation  $I^{\max*}$  are given by the maximization of expected utility, that is

$$\begin{aligned} & \max_{(D, I^{\max})} E[u(w_0 - L - P^* + I^*(L))] \\ &= \max_{(D, I^{\max})} \int_0^D u(w_0 - l - P) f(l) dl + u(w_0 - D - P) (F(D + I^{\max}) - F(D)) \\ & \quad + \int_{D+I^{\max}}^{\bar{l}} u(w_0 - l - P + I^{\max}) f(l) dl \end{aligned}$$

with  $P^* = E[I^*(L)] + rI^{\max*}$ . For arbitrarily small  $r > 0$ , the rent included in the premium is close to zero and some strictly positive level of insurance is optimal. For  $r = 1$ , the premium is greater than the maximum payment of the insurance policy and it is thus optimal not to buy insurance, i.e.  $I^*(l) = 0$  for all  $l \in [0, \bar{l}]$ . Since the solutions  $D^* = D^*(r)$  and  $I^{\max*} = I^{\max*}(r)$  to the above maximization problem are continuous in  $r$  (see the first-order conditions below), there exists a level

$\bar{r}$  such that, for all  $r \geq \bar{r}$ ,  $I^*(l) = 0$  for all  $l \in [0, \bar{l}]$ . Note that

$$P = \int_D^{D+I^{\max}} (l - D) f(l) dl + I^{\max} (1 - F(D + I^{\max}) + r)$$

and thus  $\partial P / \partial D = -(F(D + I^{\max}) - F(D))$  and  $\partial P / \partial I^{\max} = 1 - F(D + I^{\max}) + r$ . The first-order conditions for inner solutions  $D^* = D^*(r)$  and  $I^{\max*} = I^{\max*}(r)$  of the maximization problem are

$$\begin{aligned} \frac{\partial E[u(w(L))]}{\partial D} &= (F(D^*(r) + I^{\max*}(r)) - F(D^*(r))) \\ &\quad \cdot (E[u'(w_0 - L - P^* + I^*(L))] - u'(w_0 - D^*(r) - P^*)) \\ &= 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial E[u(w(L))]}{\partial I^{\max}} &= -(1 - F(D^*(r) + I^{\max*}(r)) + r) E[u'(w_0 - L - P^* + I(L))] \\ &\quad + \int_{D^*(r)+I^{\max*}(r)}^{\bar{l}} u'(w_0 - l - P^* + I^{\max*}(r)) f(l) dl \\ &= 0. \end{aligned}$$

These two equations can be simplified to

$$E[u'(w_0 - L - P^* + I^*(L))] - u'(w_0 - D^*(r) - P^*) = 0 \quad (18)$$

and

$$\begin{aligned} (1 - F(D^*(r) + I^{\max*}(r)) + r) u'(w_0 - D^*(r) - P^*) \\ - \int_{D^*(r)+I^{\max*}(r)}^{\bar{l}} u'(w_0 - l - P^* + I^{\max*}(r)) f(l) dl = 0. \end{aligned} \quad (19)$$

Implicitly differentiating the first condition (18) with respect to  $r$  yields

$$\begin{aligned}
& \frac{d}{dr} (E [u' (w_0 - L - P^* + I^* (L))] - u' (w_0 - D^* (r) - P^*)) \\
&= -P^{*'} (r) E [u'' (w_0 - L - P^* + I^* (L))] \\
&\quad - D^{*'} (r) u'' (w_0 - D^* (r) - P^*) (F (D^* (r) + I^{\max * *} (r)) - F (D^* (r)) - 1) \\
&\quad + I^{\max * *} (r) \int_{D^*(r)+I^{\max *}(r)}^{\bar{l}} u'' (w_0 - l - P^* + I^{\max * *} (r)) f (l) dl \\
&= D^{*'} (r) (F (D^* (r) + I^{\max * *} (r)) - F (D^* (r))) (E [u'' (w_0 - L - P^* + I^* (L))] - u'' (w_0 - D^* (r) - P^*)) \\
&\quad - I^{\max * *} (r) \left( (1 - F (D^* (r) + I^{\max * *} (r)) + r) E [u'' (w_0 - L - P^* + I^* (L))] \right. \\
&\quad \quad \left. - \int_{D^*(r)+I^{\max *}(r)}^{\bar{l}} u'' (w_0 - l - P^* + I^{\max * *} (r)) f (l) dl \right) \\
&\quad - I^{\max * *} (r) E [u'' (w_0 - L - P^* + I^* (L))] + D^{*'} (r) u'' (w_0 - D^* (r) - P^*) \\
&= 0
\end{aligned}$$

Note that

$$\begin{aligned}
P^{*'} (r) &= -D^{*'} (r) (F (D^* (r) + I^{\max * *} (r)) - F (D^* (r))) \\
&\quad + I^{\max * *} (r) (1 - F (D^* (r) + I^{\max * *} (r)) + r) + I^{\max * *} (r).
\end{aligned}$$

CARA implies  $u'' (w) = -\alpha u' (w)$  where  $\alpha$  is the coefficient of absolute risk aversion. The two conditions (18) and (19) then imply

$$E [u'' (w_0 - L - P^* + I^* (L))] - u'' (w_0 - D^* (r) - P^*) = 0$$

and

$$\begin{aligned}
& (1 - F (D^* (r) + I^{\max * *} (r)) + r) E [u'' (w_0 - L - P^* + I^* (L))] \\
&\quad - \int_{D^*(r)+I^{\max *}(r)}^{\bar{l}} u'' (w_0 - l - P^* + I^{\max * *} (r)) f (l) dl = 0.
\end{aligned}$$

Therefore

$$\begin{aligned}
& \frac{d}{dr} (E [u' (w_0 - L - P^* + I^* (L))] - u' (w_0 - D^* (r) - P^*)) \\
&= (-I^{\max*} (r) + D^{*'} (r)) u'' (w_0 - D^* (r) - P^*) \\
&= 0
\end{aligned}$$

and thus

$$D^{*'} (r) = I^{\max*} (r) > 0.$$

Implicitly differentiating the second condition (19) with respect to  $r$  yields

$$\begin{aligned}
& \frac{d}{dr} \left( (1 - F (D^* (r) + I^{\max*} (r)) + r) u' (w_0 - D^* (r) - P^*) \right. \\
& \quad \left. - \int_{D^*(r)+I^{\max*}(r)}^{\bar{l}} u' (w_0 - l - P^* + I^{\max*} (r)) f (l) dl \right) \\
&= -P' (r) \left( (1 - F (D^* (r) + I^{\max*} (r)) + r) u'' (w_0 - D^* (r) - P^*) \right. \\
& \quad \left. - \int_{D^*(r)+I^{\max*}(r)}^{\bar{l}} u'' (w_0 - l - P^* + I^{\max*} (r)) f (l) dl \right) \\
& \quad - I^{\max*' } (r) \int_{D^*(r)+I^{\max*}(r)}^{\bar{l}} u'' (w_0 - l - P^* + I^{\max*} (r)) f (l) dl \\
& \quad - D^{*'} (r) (1 - F (D^* (r) + I^{\max*} (r)) + r) u'' (w_0 - D^* (r) - P^*) + u' (w_0 - D^* (r) - P^*) \\
&= 0
\end{aligned}$$

CARA and condition (19) imply

$$\begin{aligned}
& \frac{d}{dr} \left( (1 - F (D^* (r) + I^{\max*} (r)) + r) u' (w_0 - D^* (r) - P^*) \right. \\
& \quad \left. - \int_{D^*(r)+I^{\max*}(r)}^{\bar{l}} u' (w_0 - l - P^* + I^{\max*} (r)) f (l) dl \right) \\
&= -I^{\max*' } (r) (1 - F (D^* (r) + I^{\max*} (r)) + r) u'' (w_0 - D^* (r) - P^*) \\
& \quad - D^{*'} (r) (1 - F (D^* (r) + I^{\max*} (r)) + r) u'' (w_0 - D^* (r) - P^*) + u' (w_0 - D^* (r) - P^*) \\
&= 0
\end{aligned}$$

and thus

$$\begin{aligned}
I^{\max'}(r) &= -D^{*'}(r) + \frac{u'(w_0 - D^*(r) - P^*)}{u''(w_0 - D^*(r) - P^*)} \cdot \frac{1}{1 - F(D^*(r) + I^{\max*}(r)) + r} \\
&= -D^{*'}(r) - \frac{1}{\alpha} \cdot \frac{1}{1 - F(D^*(r) + I^{\max*}(r)) + r} \\
&< 0.
\end{aligned}$$

Therefore

$$\frac{d}{dr} (I^{\max*}(r) + D^*(r)) = -\frac{1}{\alpha} \cdot \frac{1}{1 - F(D^*(r) + I^{\max*}(r)) + r} < 0.$$

### A.3 Proof of Proposition 3

The proof is analogous to the proof of Proposition 1. Let  $F^n$  denote the joint cumulative distribution function of  $L^n = (L_1, \dots, L_n)$  with support  $[0, \bar{l}]^n$  and joint probability density function  $f^n$ . The solution to the optimization problem (4) is identical to the solution of the following problem

$$\max_{(P(\cdot), (I_j(\cdot))_{j=1, \dots, n}, I_n^{\max}(\cdot))} \sum_{j=1}^n E[u(w(L_j, L^n))] \quad (20)$$

$$\text{s.t. } nE[P(L^n)] \geq E\left[\sum_{j=1}^n I_j(L^n)\right] + rE[I_n^{\max}(L^n)], \quad (21)$$

$$\frac{\partial P(l^n)}{\partial l_j} = 0 \text{ for all } l^n \in [0, \bar{l}]^n \text{ and } j = 1, \dots, n, \quad (22)$$

$$\frac{\partial I_n^{\max}(l^n)}{\partial l_j} = 0 \text{ for all } l^n \in [0, \bar{l}]^n \text{ and } j = 1, \dots, n, \quad (23)$$

$$0 \leq I_j(l^n) \text{ and } \sum_{j=1}^n I_j(l^n) \leq I_n^{\max}(l^n) \text{ for all } l^n \in [0, \bar{l}]^n \text{ and } j = 1, \dots, n. \quad (24)$$

with  $w(L_j, L^n) = w_0 - L_j - P(L^n) + I_j(L^n)$ . The Lagrangian function to this function is then given by

$$\begin{aligned}
\mathcal{L} &= \sum_{j=1}^n u(w(l_j, l^n)) f(l^n) + \lambda \left( nP(l^n) - \sum_{j=1}^n I_j(l^n) - rI_n^{\max}(l^n) \right) f(l^n) \\
&\quad - \sum_{j=1}^n \mu_j(l^n) \frac{\partial P(l^n)}{\partial l_j} - \sum_{j=1}^n \nu_j(l^n) \frac{\partial I_n^{\max}(l^n)}{\partial l_j} \\
&\quad + \sum_{j=1}^n \xi_j(l^n) I_j(l^n) + \zeta(l^n) \left( I_n^{\max}(l^n) - \sum_{j=1}^n I_j(l^n) \right)
\end{aligned}$$

where  $\lambda$ ,  $\mu_j(l^n)$ ,  $\nu_j(l^n)$ ,  $\xi_j(l^n)$ , and  $\zeta(l^n)$  are the Lagrange multipliers to the constraints (21), (22), (23), and (24), respectively, with

$$\xi_j(l^n) \begin{cases} = 0 & \text{if } I_j(l^n) > 0 \\ > 0 & \text{if } I_j(l^n) = 0 \end{cases}$$

and

$$\zeta(l^n) \begin{cases} = 0 & \text{if } \sum_{j=1}^n I_j(l^n) < I_n^{\max}(l^n) \\ > 0 & \text{if } \sum_{j=1}^n I_j(l^n) = I_n^{\max}(l^n) \end{cases}.$$

Defining the Quasi-Hamiltonian function as

$$\begin{aligned} \mathcal{H} & \left( P(l^n), (I_j(l^n))_{j=1, \dots, n}, I_n^{\max}(l^n), l^n \right) \\ & = \sum_{j=1}^n u(w(l_j, l^n)) f(l^n) + \lambda \left( nP(l^n) - \sum_{j=1}^n I_j(l^n) - rI_n^{\max}(l^n) \right) f(l^n) \end{aligned}$$

we can write the optimization problem (20) as

$$\begin{aligned} \max_{(P(\cdot), (I_j(\cdot))_{j=1, \dots, n}, I_n^{\max}(\cdot))} & \int_0^{\bar{l}} \dots \int_0^{\bar{l}} \left( \mathcal{H} \left( P(l^n), (I_j(l^n))_{j=1, \dots, n}, I_n^{\max}(l^n), l^n \right) \right. \\ & + \sum_{j=1}^n (\xi_j(l^n) - \zeta(l^n)) I_j(l^n) + \zeta(l^n) I_n^{\max}(l^n) \\ & \left. - \sum_{j=1}^n \mu_j(l^n) \frac{\partial P(l^n)}{\partial l_j} - \sum_{j=1}^n \nu_j(l^n) \frac{\partial I_n^{\max}(l^n)}{\partial l_j} \right) dl^n. \end{aligned}$$

Variational calculus and partial integration implies

$$\begin{aligned} & \int_0^{\bar{l}} \dots \int_0^{\bar{l}} \left( \sum_{j=1}^n \left( \frac{\partial \mathcal{H}}{\partial I_j} + \xi_j(l^n) - \zeta(l^n) \right) \delta I_j(l^n) + \left( \frac{\partial \mathcal{H}}{\partial P} + \sum_{j=1}^n \frac{\partial \mu_j(l^n)}{\partial l_j} \right) \delta P(l^n) \right. \\ & \quad \left. + \left( \frac{\partial \mathcal{H}}{\partial I_n^{\max}} + \zeta(l^n) + \sum_{j=1}^n \frac{\partial \nu_j(l^n)}{\partial l_j} \right) \delta I_n^{\max}(l^n) \right) dl^n \\ & - \sum_{j=1}^n \underbrace{\int_0^{\bar{l}} \dots \int_0^{\bar{l}}}_{n-1} (\mu_j(l^n) \delta P(l^n) + \nu_j(l^n) \delta I_n^{\max}(l^n)) \Big|_{l_j=0}^{\bar{l}} dl_{-j}^n \\ & = 0 \end{aligned}$$

where  $l_{-j}^n = (l_1, \dots, l_{j-1}, l_{j+1}, \dots, l_n) \in [0, \bar{l}]^{n-1}$ . The necessary conditions for optimality are

$$u'(w(l_j, l^n)) f(l^n) - \lambda f(l^n) + \xi_j(l^n) - \zeta(l^n) = 0 \text{ for all } j = 1, \dots, n \quad (25)$$

$$\sum_{j=1}^n \left( -u'(w(l_j, l^n)) f(l^n) + \lambda f(l^n) + \frac{\partial \mu_j(l^n)}{\partial l_j} \right) = 0 \quad (26)$$

$$-\lambda r f(l^n) + \zeta(l^n) + \sum_{j=1}^n \frac{\partial \nu_j(l^n)}{\partial l_j} = 0 \quad (27)$$

$$\sum_{j=1}^n \underbrace{\int_0^{\bar{l}} \dots \int_0^{\bar{l}}}_{n-1} (\mu_j(l_1, \dots, l_{j-1}, \bar{l}, l_{j+1}, \dots, l_n) - \mu_j(l_1, \dots, l_{j-1}, 0, l_{j+1}, \dots, l_n)) dl_{-j}^n = 0 \quad (28)$$

$$\sum_{j=1}^n \underbrace{\int_0^{\bar{l}} \dots \int_0^{\bar{l}}}_{n-1} (\nu_j(l_1, \dots, l_{j-1}, \bar{l}, l_{j+1}, \dots, l_n) - \nu_j(l_1, \dots, l_{j-1}, 0, l_{j+1}, \dots, l_n)) dl_{-j}^n = 0 \quad (29)$$

The sufficient condition is satisfied since

$$\sum_{j=1}^n \int_0^{\bar{l}} \dots \int_0^{\bar{l}} u(w(l_j, l^n)) f(l^n) dl^n$$

is concave in  $I_j$ . Conditions (26) and (28) imply

$$\int_0^{\bar{l}} \dots \int_0^{\bar{l}} u'(w(l_j, l^n)) f(l^n) dl^n = \lambda \quad (30)$$

and conditions (27) and (29) yield

$$\int_0^{\bar{l}} \dots \int_0^{\bar{l}} \zeta(l^n) dl^n = \lambda r. \quad (31)$$

Combining conditions (25), (30), and (31) implies

$$\int_0^{\bar{l}} \dots \int_0^{\bar{l}} \xi_j(l^n) dl^n = \lambda r \text{ for all } j = 1, \dots, n. \quad (32)$$

We first show that  $I_j^*(l^n) = (l_j - D)^+$  for some  $D > 0$  and all  $l^n$  if  $\sum_{j=1}^n I_j(l^n) < I_n^{\max}(l^n)$ . For inner solutions  $I_j(l^n) > 0$  we have  $\xi_j(l^n) = \zeta(l^n) = 0$  for all  $j = 1, \dots, n$  and condition (25) simplifies to  $u'(w_0 - l_j - P + I_j(l^n)) = \lambda$ . This implies  $I_j(l^n) = l_j - D$  for some  $D \geq 0$ . Next we show that  $D > 0$ . Since  $u$  is strictly increasing, condition (30) implies  $\lambda > 0$  and condition (32)

thus yields that there exists a subset in  $[0, \bar{l}]^n$  for which  $\xi_j(l^n) > 0$  for all  $j = 1, \dots, n$ , i.e., for which  $I_j(l^n) = 0$ . Since  $I_j(l^n)$  is strictly increasing in  $l_j$  for inner solutions and does not depend on other policyholders' losses, this implies  $I_j(l^n) = 0$  for  $l_j \leq D$  and  $D > 0$ .

Next, if  $\sum_{j=1}^n (l_j - D)^+ \geq I_n^{\max}(l^n)$  we show that  $I_j^*(l^n) = (l_j - D(l^n))^+$  for some  $D(l^n) > 0$  where  $D(l^n)$  is defined by  $\sum_{j=1}^n (l_j - D(l^n))^+ = I_n^{\max}(l^n)$ . For inner solutions  $I_j(l^n) > 0$  we have  $\xi_j(l^n) = 0$  for all  $j = 1, \dots, n$  and condition (25) simplifies to  $u'(w_0 - l_j - P + I_j(l^n)) = \lambda + \zeta(l^n) / f(l^n)$ . This implies that  $l_j - I_j(l^n)$  is identical for all  $j = 1, \dots, n$  and  $I_j(l^n) = l_j - D(l^n)$ . As shown above, condition (32) implies  $I_j(l^n) = 0$  for  $l_j \leq D(l^n)$ .

Finally, condition (31) implies that there exists a subset in  $[0, n\bar{l}]$  for which  $\zeta(l^n) > 0$ , i.e., for which  $\sum_{j=1}^n I_j^*(l^n) = I_n^{\max*}(l^n) = I_n^{\max*}$ . Again, since  $\sum_{j=1}^n I_j^*(l^n)$  is strictly increasing in  $l_j$  for inner solutions, this subset is of the form  $[U, n\bar{l}]$  for some  $U < n\bar{l}$ . Since  $I_j^*(l^n) = (l_j - D)^+$ ,  $I_n^{\max*} = U - nD < n(\bar{l} - D)$ .

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