Sharing the Burden: International Policy Cooperation in a World Liquidity Trap *

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Abstract

Negative demand shocks in one country may have international spillover effects which push optimal policy interest rates down to their zero bound in all countries. We explore the optimal policy response to these shocks, when governments cooperate on both fiscal and monetary responses. The key aspect of the model is to emphasize how both the transmission of shocks, and the optimal policy responses are affected by the degree of trade openness of each economy. When trade openness is full, both economies enter a liquidity trap simultaneously, and the optimal policy response is to have identical, expansionary fiscal packages. When trade is less than fully open, the economy suffering the shock is the worst hit. Then the optimal policy response is to have fiscal expansion in that economy, a much smaller expansion in the foreign economy, combined with monetary tightening in the foreign economy. Strikingly, the foreign economy may choose to raise interest rates even though its optimal policy rate should be zero, from the perspective of a closed economy.

Keywords:Liquidity Trap, Monetary Policy, Fiscal Policy, International Spillovers

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1 Introduction

The macroeconomic situation of the world economy has been profoundly altered by the experience of the Great Recession that began in 2008. By general consensus, the source of the shock was the US financial sector, but this subsequently led to a fall in world aggregate demand, and spilled over to the economies of many other countries. From a macroeconomic management perspective, the critical constraint was that policy interest rates were quickly pushed down close to zero, exhausting the ability of monetary policy to offset the full impact of the shock. The alternative was to use fiscal policy, and many countries followed significant expansions in goverment deficits, reducing taxes and/or increasing government spending. At the beginning of the downturn, there was a concerted effort to coordinate these fiscal expansion across countries, through the G20 process and other venues. But the ensuing fiscal responses were far from uniform across different countries. In addition, some countries have already begun to raise policy rates, while in the US, interest rates remain effectively at their zero bound.

While there has been a significant growth in research on the economics of liquidity traps in both open and closed economy settings, to date there has been little investigation of the global dimension of optimal policy responses to large macro shocks which push one or more countries towards the zero lower bound. In particular, a key question is how the 'burden of adjustment' to a global recession should be shared across countries that experience the downturn at different levels of severity.

This paper investigates the determination of optimal policy in response to a world liquidity trap in which two trading partners are well integrated through financial markets but less than perfectly integrated in goods markets. The international dimension to macroeconomic policy at the zero lower bound introduces some intriguing complications. For instance, as shown by Eggerston (2010) and Woodford (2010), fiscal policy may be highly expansionary in a liquidity trap via its effect on expected inflation and real interest rates. But in the open economy, the effect of fiscal policy on real interest rates will also impact the terms of trade through an interest parity linkage. Cook and Devereux (2010a) show that open economy multipliers on domestic fiscal policy will exceed the closed economy multiplier at the zero lower bound, regardless of the degree of trade integration. Expansionary fiscal policy which reduces the real interest rate facilitate real exchange rate *depreciation*, stimulating external trade. At the same time however, fiscal expansion will have negative spillovers on demand for goods produced by trading partners.

With highly open trade linkages however, the optimal cooperative fiscal response to a liquidity trap is essentially identical to the case of a closed economy. The reason is that when international trade is highly integrated, a demand slump in one country is felt equally in all other countries. Output and inflation in all countries will respond symmetrically to demand shocks, regardless of the source of the shock. The optimal fiscal policy will be symmetric and will be very close to optimal policy in a closed economy. At the same time, because both countries are equally affected by the 'zero lower bound', monetary policy is not effective[1].

However, the benchmark of fully open trade does not closely approximate the current situation of the world economy, where large, but relatively closed economies, such as Japan and the United States, are stuck in a liquidity trap. In these countries, exports make up substantially less than 20% of GDP. With home bias in consumption baskets, which acts so as to impart trade frictions between countries, both the propagation of demand shocks and the optimal response of policy to shocks takes on very different characteristics. Typically, a large negative demand shock in one country will push that country into a liquidity trap more quickly than its trading partners. Moreoever, the introduction of home bias complicates the analysis of optimal policy, since the international effects of a demand shock are not distributed equally across countries.

The paper conducts an analysis of the international propagation of a negative shock emanating from one country, in an environment where countries are only partially open to trade, and show how the optimal response of policy is extremely sensitive to the size of trade linkages.

Initially, we show analytically that, after a negative demand shock, the average level of the *world* output gap declines and the average *world* optimal fiscal policy response should be expansionary. In fact the decline in the average world output gap and the optimal world fiscal policy response are invariant to home bias and precisely equal to that observed in a closed economy.

However, the distribution of the effects of the shock among countries is less clear. As expected, the response of fiscal policy to the shock should be larger and more expansionary in the country that is the primary source of the shocks. If home bias is small and international trade is well integrated, a foreign trading partner might be strongly affected by a loss of demand in the source country and also need expansionary fiscal policy to offset the deflationary effects. But if the economies are less open to international trade, both the international tranmission of shocks, and the optimal policy response, may be very different. With substantially restricted trade, the decline in global interest rates that occur during the

[1]As discussed below, in this paper we are looking at discretionary macroeconomic policies only, so we ignore the potential use of announcement effects of future monetary expansion.

slump may actually be expansionary for global trading partners. In such a case, the optimal policy response will be to combine very slight fiscal expansion with monetary *contraction*. That is, the foreign economy should only minimally engage in a cooperative fiscal expansion, but should raise its policy rates. Strikingly, we find that the best policy (from a global cooperative perspective), is for the foreign country to tighten monetary policy, even though using the standard criterion from the closed economy logic, it should still be in a liquidity trap (where its 'natural' real interest rate is below zero). The logic behind this policy is that a foreign interest rate increase acts so as to dampen the (positive) response of the foreign economy to the home country savings shock, while at the same time facilitating a weakening of the home currency, and generating an expenditure switching of demand towards the worst hit (home) economy.

The message is that the open economy dimension has very substantial implications for both the occurance of a liquidity trap, in the sense that it predicts that policy is not restricted by the zero lower bound even when traditional indicators (which look at the value of the 'natural real interest rate') say that it should be, and for the way in which policy is designed when the world economy 'on average' is in a liquidity trap. More generally, the model predicts that the 'burden of adjustment' to a global liquidity trap may be spread quite unequally across countries, and imply some apparently counterintuitive policy responses.

The paper builds on a substantial recent literature on monetary and fiscal policy in a 'liquidity trap'. In particular, Krugman (1999), Eggertson and Woodford (2003, 2005), Jung et al. (2005), Svensson (2003), Auerbach and Obstfeld (2004) and many other writers explored how monetary and fiscal policy could be usefully employed even when the authorities have no further room to reduce short term nominal interest rates. Recently, a number of authors have revived this literature in light of the very similar problems now encountered by the economies of Western Europe and North America. Papers by Christiano et al (2009), Eggertson (2009), Taylor et al. (2008) have explored the possibility for using government spending expansions, tax cuts, and monetary policy when the economy is in a 'liquidity trap'. For the most part, these papers did not focus on the international dimension of liquidity traps. Some recent expections are Fujiwara et al. (2009, 2010), Erceg et al. (2009) and Jeanne (2009). Jeanne (2009) examines a 'global liquidity trap' in a model of one-period ahead pricing similar to that of Krugman (2009). Erceg. et al (2009) use a fully specific two country DSGE model to examine the international transmission of shocks when one country is in a liquidity trap, but do not focus on optimal monetary policy or fiscal policy choices. Fujiwara et al. (2009) examine the optimal monetary problem with commitment in a multi country situation, but do not examine the determination of fiscal policy, or the transmission of demand shocks across countries. Fujiwara et al. (2010) look at the impact of the international effects of fiscal policy in a liquidity trap, examing the sign and size of domestic and international fiscal multipliers. Our paper may be seen as complementary to theirs in that we extend the analysis to incorporate trade frictions, but more importantaly, investigate the determination of optimal policy[2].

The rest of the paper is organized as follows. The next section develops the basic model. Section 3 examines the solution under sticky prices. Then in section 4 we analyze the impact of fiscal policies at the zero lower bound, and the role of international spillovers of policies. Section 5 examines the optimal policy making problem in a global cooperative agreement, including the possibility of using both monetary and fiscal policy for the least affected countries. Some conclusions are then offered.

2 A two country model of interacting monetary and fiscal policy

We construct a model in which there are two countries in the world economy. In each country, households consume both private and government goods, and supply labour. Denote the countries as 'home' and 'foreign', with foreign variables denoted with an asterisk superscript. The population of each country is normalized to unity. Each country produces a range of differentiated goods. Complete asset markets allow full insurance of consumption risk across countries. Households also hold their own country's nominal government bonds. Firms produce private goods, while governments produce government goods which are distributed uniformly across households. Firms production and supply is constrained by sticky prices. Governments have access to lump sum taxation.

2.1 Households

Utility of a representative infinitely lived home household evaluated from date 0 is:

$$U_t = E_0 \sum_{t=0}^{\infty} (\beta)^t (U(C_t, \xi_t) - V(N_t) + J(G_t)) \tag{1}$$

where U, V, and J represent the utility of the composite home consumption bundle C_t, disutility of labour supply N_t, and utility of the government supplied public good G_t,respectively.

[2]In addition, a previous paper (Cook and Devereux, 2010a) examines the linkages of natural real interest rates, the determination of fiscal multipliers and optimal fiscal policy in a simpler version of the model of the present paper, but does not allow for the endogenous response of monetary policy.

The variable ξ_t represents a shock to preferences or 'demand' . We assume that $U_{12} > 0$..

Composite consumption is defined as

$$C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}, \quad v \geq 1$$

where $\Phi = \left(\frac{v}{2}\right)^{\frac{v}{2}} \left(1 - \left(\frac{v}{2}\right)\right)^{\frac{v}{2}}$, C_H is the consumption of the home country composite good by the home household, and C_F is consumption of the foreign composite good. If $v > 1$ then there is a home preference bias for domestic goods. We will focus on the home bias case, where v is strictly greater than 1. This case will be most realistic for thinking about policy in large open economies.

Consumption aggregates, C_H and C_F are composites, defined over a range of home and foreign differentiated goods, with elasticity of substitution θ between goods, so that:

$$C_H = \left[\int_0^1 C_H(i)^{1-\frac{1}{\theta}} di \right]^{\frac{1}{1-\frac{1}{\theta}}}, \qquad C_F = \left[\int_0^1 C_F(i)^{1-\frac{1}{\theta}} di \right]^{\frac{1}{1-\frac{1}{\theta}}}, \quad \theta > 1.$$

Price indices for home and foreign consumption are:

$$P_H = \left[\int_0^1 P_H(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \qquad P_F = \left[\int_0^1 P_F(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

while the aggregate (CPI) price index for the home country is $P = P_H^{\frac{v}{2}} P_F^{1-\frac{v}{2}}$ and for the foreign is $P_t^* = P_F^{*v/2} P_H^{*1-v/2}$

Demand for each differentiated good ($j = H, F$) is

$$\frac{C_j(i)}{C_j} = \left(\frac{P_j(i)}{P_j} \right)^{-\theta}$$

The law of one price holds for each good so $P_j(i) = SP_j^*(i)$.where S_t is the nominal exchange rate (home price of foreign currency). Relative demand for the composites is:

$$\frac{C_H}{C_F} = \frac{P_F}{P_H} = \frac{SP_F^*}{P_H}$$

Home government spending falls on the home composite good and foreign government spending on the foreign composite good. Thus, government spending is assumed to have full 'home bias'. I addition, we assume that government spending demand for each variety of home goods has price elasticity θ, the same as that for private spending.

The household's implicit labour supply at nominal wage W_t is:

$$U_C(C_t,\xi_t)W_t = P_t V'(N_t). \tag{2}$$

Optimal risk sharing implies

$$U_C(C_t,\xi_t) = U_C(C_t^*,\xi_t^*)\frac{S_t P_t^*}{P_t} = U_C(C_t^*,\xi_t^*)T_t^{v-1}, \tag{3}$$

.Nominal bonds pay interest, R_t. Then the Euler equation is:

$$\frac{U_C(C_t,\xi_t)}{P_t} = \beta R_t E_t \frac{U_C(C_{t+1},\xi_{t+1})}{P_{t+1}}. \tag{4}$$

Foreign household preferences and choices can be defined exactly symmetrically. The foreign representative household has weight $v/2$, $(1-v/2)$ on the foreign (home) compositive good in preferences.

2.2 Firms

Each firm i employs labor to produce a differentiated good.

$$Y_t(i) = N_t(i),$$

Profits are $\Pi_t(i) = P_{Ht}(i)Y_t(i) - W_t H_t(i)\frac{\theta-1}{\theta}$ indicating a subsidy financed by lump-sum taxation to eliminate steady state first order inefficiencies. Each firm re-sets its price according to Calvo pricing with probability of adjusting prices equal to $1-\kappa$. Firms that adjust their price set new price given by $\widetilde{P}_{Ht}(i)$:

$$\widetilde{P}_{Ht}(i) = \frac{E_t \sum_{j=0} m_{t+j}\kappa^j \frac{W_{t+j}}{A_{t+j}} Y_{t+j}(i)}{E_t \sum_{j=0} m_{t+j}\kappa^j Y_{t+j}(i)}. \tag{5}$$

where stochastic discount factor $m_{t+j} = \frac{P_t}{U_C(C_t,\varepsilon_t)}\frac{U_C(C_{t+j},\xi_{t+j})}{P_{t+j}}$. In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht} = [(1-\kappa)\widetilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{6}$$

The behaviour of foreign firms and the foreign good price index may be described analogously.

2.3 Market Clearing

Equilibrium in the market for good i as

$$Y_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} \left[\frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^* + G_t\right],$$

where G_t represents total home government spending. Aggregate market clearing in the home good is:

$$Y_{Ht} = \frac{v}{2}\frac{P_t}{P_{Ht}}C_t + (1-\frac{v}{2})\frac{S_t P_t^*}{P_{Ht}}C_t^* + G_t. \tag{7}$$

Here $Y_{Ht} = V_t^{-1}\int_0^1 Y_{Ht}(i)di$ is aggregate home country output, where we have defined $V_t = \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} di$. It follows that home country employment (employment for the representative home household) is given by $N_t = \int_0^1 N(i)di = A^{-1}Y_{Ht}V_t$.

The aggregate market clearing condition for the foreign good is

$$Y_{Ft} = \frac{v}{2}\frac{P_t^*}{P_{Ft}^*}C_t^* + (1-\frac{v}{2})\frac{P_t}{S_t P_{Ft}^*}C_t^* + G_t^*, \tag{8}$$

where: $N_t^* = \int_0^1 N_t^*(i)di = A^{*-1}Y_{Ft}V_t^*$, where $V_t^* = \int_0^1 \left(\frac{P_{Ft}^*(i)}{P_{Ft}^*}\right)^{-\theta} di$.

An equilibrium in the world economy with positive nominal interest rates may be described by the equations (3), and (2), (4), (5) (6) and (**??**) for the home and foreign economy, as well as (7) and (8). For given values of V_t and V_t^*, and given government spending policies, these equations determine an equilibrium sequence for the variables $C_{t,}C_t^*, W_t, W_t^*, S_t, P_{Ht}$, $P_{Ft}^*, \widetilde{P}_{Ht}, \widetilde{P}_{Ft}^*$, R_t, R_t^*, and N_t, N_t^*.

3 New Keynesian Open Economy Model

3.1 Demand Shocks and Natural Interest Rates

Define $\sigma \equiv -\frac{U_{CC}\overline{C}}{U_C}$ as the inverse of the elasticity of intertemporal substitution in consumption, $\phi \equiv -\frac{V''\overline{H}}{V'}$ as the elasticity of the marginal disutility of hours worked and $\sigma_g \equiv -\frac{J''\overline{G}}{J'}$ as the elasticity of marginal utility of public goods. In addition, we assume that $\sigma_g = \sigma > 1$. Finally, $\varepsilon_t = \frac{U_{C\xi}}{U_C}\ln(\xi_t)$ is the measure of a positive demand shock in the home country, with an equivalent definition for the foreign country. Define $c_y = \frac{C}{Y}$ is the steady state share of consumption in output.We examine the effects of once and for all demand shocks which have a probability $1-\mu$ of reverting to mean.

For any variable x_t, define the world average and world relative level, $x_t^W = \frac{x_t + x_t^*}{2}$ and $x_t^R = \frac{x_t - x_t^*}{2}$. Solving a first order apprixmation of the flexible price, zero inflation version of the above economy, we can derive the Wicksellian (or 'natural') interest rate of the home and foreign economy, $\widetilde{r}_t$ as a function of the demand shock.

$$\widetilde{r}_t = \overline{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W + \frac{\phi c_y (v-1)}{\Delta} \varepsilon_t^R \right) (1 - \mu) \tag{9}$$

The foreign efficient nominal interest rate is:

$$\widetilde{r}_t^* = \overline{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W - \frac{\phi c_y (v-1)}{\Delta} \varepsilon_t^R \right) (1 - \mu) \tag{10}$$

where $\Delta \equiv \phi c_y D + \phi(1-c_y) + \sigma$ and $\sigma > D \equiv (\sigma v(2-v) + (1-v)^2) > 1$. The way to interpret (9) and (10) is as the Fisherian, consumption based real interest rates that would obtain in an economy with fully flexible prices and no other distortions. These are critical variables for our analysis, since they govern the degree to which monetary policy can be efficiently employed to stabilize the economy. In particular, our model has the characteristic that when (9) and (10) are both positive, then monetary policy can perfectly achieve the joint target of zero inflation and zero output gaps, since home and foreign policy rates can simply be set to equal (9) and (10), respectively.

Note that in the no home bias case, when $v = 1$, then the natural interest rate for both economies should be the same. $\widetilde{r}_t = \widetilde{r}_t^* = \overline{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W \right) (1 - \mu)$. This is an example where the real exchange rate is constant, and given integrated financial markets, consumption-based real interest rates are equated across countries. But in fact, the case $v = 1$ is not particularly realistic. For most economies, and particlarly for large open economies, the lion's share of demand will come from the domestic economy, making the home bias case most relevant. We will therefore focus on the more general case where $v > 1$.

For concreteness, we also look at the case where the home country is the source of the shocks. In particular, we will assume that home consumers are affected by preference shocks which affect their propensity to save, whereas consumers in the foreign economy is not directly affected by these shocks. Of course foreign consumers will be indirectly affected by the shock, since integrated financial markets lead to linkages between interest rates. Thus, a saving shock that emanates from the home economy, pushing the monetary authority into a liquidity trap, may have similar effects on the foreign economy, even though the foreign consumers are not directly affected by the shock.

Making this assumption, we have in this case, $\varepsilon_t^* = 0$ and $\varepsilon_t^W = \varepsilon_t^R = \frac{\varepsilon_t}{2}$ and we can write the home country natural rate as

$$\widetilde{r_t} = \overline{r} + \left(\frac{\Delta + (\phi + \sigma)(v-1)}{(\phi+\sigma)\Delta}\right)(1-\mu)\phi c_y \frac{\varepsilon_t}{2} \tag{11}$$

Then the foreign natural rate is:

$$\widetilde{r_t^*} = \overline{r} + \left(\frac{\Delta - (\phi + \sigma)(v-1)}{(\phi+\sigma)\Delta}\right)(1-\mu)\phi c_y \frac{\varepsilon_t}{2} \tag{12}$$

It is clear that, when there is home bias, $(v > 1)$, and then $\Delta + (\phi+\sigma)(v-1) > \Delta - (\phi+\sigma)(v-1) > 0$[3]. A home preference shock that is sufficiently large can push the natural interest rate of both countries below zero, regardless of the degree of home bias.[4]

3.2 First Order Approximation

We derive a sticky price log-linear approximation of the model in terms of inflation and output gaps in a similar manner to Clarida et al. (2002) and Engel (2010). Let $\widehat{x}_t$ be the percentage deviation of a given variable x_t from the efficient zero flexible price equilibrium. As defined before, $D \equiv \sigma v(2-v)+(1-v)^2 > 1$. In addition, let $s \equiv \frac{\sigma}{c_y}$, and $s > s_D \equiv \frac{s}{D} > 1$. In order to explore the implications of the zero lower bound constraint, we begin with the standard forward looking inflation equations and open economy IS relationships for the home and foreign economy.

The home and foreign forward looking inflation equations are

$$\pi_{Ht} = k(\phi\widehat{n}_t + \frac{s_D}{2}\left[\widehat{ng}_t(1+D) + \widehat{ng}_t^*(D-1)\right]) + \beta E_t \pi_{Ht+1} \tag{13}$$

.

$$\pi_{Ft}^* = k(\phi\widehat{n}_t^* + \frac{s_D}{2}\left[\widehat{ng}_t^*(1+D) + \widehat{ng}_t(D-1)\right]) + \beta E_t \pi_{Ft+1}^* \tag{14}$$

Likewise, the home and foreign 'dynamic IS equations' are

$$\begin{aligned} & s_D E_t(\widehat{ng}_{t+1} - \widehat{ng}_t)(D+1) + s_D E_t(\widehat{ng}_{t+1}^* - \widehat{ng}_t^*)(D-1) \\ = \; & 2E_t\left(r_t - \widetilde{r}_t - \pi_{Ht+1}\right) \end{aligned} \tag{15}$$

[3]Note that $\Delta - (\phi+\sigma)(v-1) = \phi c_y(D-1) + (\phi+\sigma)(2-\nu)$

[4]However, it is also possible, in the case of strong home bias or in the case of a sufficiently small preference shock, that the natural interest rate of the home economy will be pushed below zero, while the foreign economy will retain a natural interest rate that will be positive.

$$s_D E_t(\widehat{ng}^*_{t+1} - \widehat{ng}^*_t)(D+1) + s_D E_t(\widehat{ng}_{t+1} - \widehat{ng}_t)(D-1) \tag{16}$$
$$= 2E_t\left(r^*_t - \widetilde{r}^*_t - \pi_{Ft+1}\right)$$

where π_{Ht} and π^*_{Ft} are the inflation rates of the domestic and foreign composite goods, $\widehat{ng}_t = (\widehat{n}_t - (1-c_y)\widehat{g}_t)$ and $\widehat{ng}^*_t = (\widehat{n}^*_t - (1-c_y)\widehat{g}^*_t)$, and the coefficient k depends on the degree of price rigidity. Note that, approximated around the steady state, $\widehat{n}_t \approx \widehat{y}_t$, $\widehat{n}^*_t \approx \widehat{y}^*_t$, so the labor gap will stand in for the output gap.

If the natural interest rates of both economies are always above zero, then the monetary and fiscal authorities can retain perfect price and output stability by setting the nominal interest rate equal to the natural real interest rate and keeping the fiscal gaps, $\widehat{g}_t$ and $\widehat{g}^*_t$ closed. However, a world wide liquidity trap, where desired real interest rates are both negative, will prevent the monetary authorities from achieving this outcome. As a result, there is a role for fiscal policy in stabilizing inflation and output gaps.

3.3 The World and Relative Economy

We can simplify the equations by writing them in terms of world average and relative levels. The world average for global inflation is written as:

$$\pi^W_t = k(\phi + s)\widehat{n}^W_t - ks \cdot \widehat{cg}^W_t + \beta E_t \pi^W_{t+1} \tag{17}$$

$$sE_t(\widehat{n}^W_{t+1} - \widehat{n}^W_t) - sE_t(\widehat{cg}^W_{t+1} - \widehat{cg}^W_t) = E_t\left(r^W_t - \widetilde{r}^W_t - \pi^W_{t+1}\right) \tag{18}$$

The world 'relative' variables are written as:

$$\pi^R_t = k(\phi + s_D)\widehat{n}^R_t - ks_D \widehat{cg}^R_t + \beta E_t \pi^R_{t+1} \tag{19}$$

$$s_D E_t(\widehat{n}^R_{t+1} - \widehat{n}^R_t) - s_D E_t(\widehat{cg}^R_{t+1} - \widehat{cg}^R_t) = E_t\left(r^R_t - \widetilde{r}^R_t - \pi^R_{t+1}\right) \tag{20}$$

where $\widehat{cg}^W_t \equiv (1-c_y)\widehat{g}^W_t$ and $\widehat{cg}^R_t \equiv (1-c_y)\widehat{g}^R_t$. A requirement for stability and determinacy of this set of equations is that $\Delta^D_3 \equiv s_D(1-\beta\mu)(1-\mu) - \mu k\,(\phi + s_D) > 0$, an assumption that we will maintain throughout.

If we abstract from fiscal gaps, (i..e $\widehat{cg}_t^W = \widehat{cg}_t^R = 0$)we can rewrite these equations so as to most clearly understand them:

$$\pi_t^W = k(\phi + s)\widehat{n}_t^W + \beta E_t \pi_{t+1}^W \tag{21}$$

$$sE_t(\widehat{n}_{t+1}^W - \widehat{n}_t^W) = E_t\left(r_t^W - \widetilde{r}_t^W - \pi_{t+1}^W\right) \tag{22}$$

$$\pi_t^R = k(\phi + s_D)\widehat{n}_t^R + \beta E_t \pi_{t+1}^R \tag{23}$$

$$s_D E_t(\widehat{n}_{t+1}^R - \widehat{n}_t^R) = E_t\left(r_t^R - \widetilde{r}_t^R - \pi_{t+1}^R\right) \tag{24}$$

We can clearly see that both systems of equations (for the world average and the world relative economies) are in the canonical form of the New Keynesian closed economy equations. The only difference comes in the parameterization of the inverse elasticity of consumption: s , in the case of the average economy; and, s_D, in the case of the relative world economy. Note that $s_D < s$, so the world average level of demand is less sensitive to the average interest rate than the relative level of demand is sensitive to the relative interest rate. This reflects the expenditure switching effect of terms of trade changes. When worldwide interest rates are relatively low, then for intertemporal reasons, world demand will be relatively high. But when there is a large gap in interest rates across country, demand will be relatively high in the low interest rate country for the same intertemporal reasons, but on top of that, a relatively low interest rate country will be characterized by a current deterioration in its terms of trade (or a real exchange rate depreciation), implying relatively high current demand for their goods, through the expenditure switching channel.

4 Global Liquidity Traps

We now focus on the policy dilemma when there is a global liquidity trap. We assume that the liquidity trap is created by a negative shock to home country preferences, pushing up home country savings, and reducing both home and foreign natural real interest rates below zero. As a result countries are pushed to the zero lower bound of the policy rate, so that

$r_t^R = r_t^W = 0$[5]. As we noted before, it is assumed that the preference shock reverts back to zero with probability $1-\mu$ per period. As a result of this, all other variables in the world economy will inherit the same persistence, in expectation, so that for any variable x_t is $E_t[x_{t+1}] = \mu x_t$ within the liquidity trap[6]. Consider first the response of output and inflation to a negative demand shock from the home economy when the govenment spending gap is equal to zero. Using (17)-(20), we obtain:

$$\begin{aligned} \Delta_3 \widehat{n}_t^W &= (1-\beta\mu)\widetilde{r}_t^W \\ \Delta_3^D \widehat{n}_t^R &= (1-\beta\mu)\widetilde{r}_t^R \end{aligned}$$

where $\Delta_3 \equiv s(1-\beta\mu)(1-\mu) - \mu k(\phi + s) > \Delta_3^D > 0$. When both economies are in a liquidity trap, driven by a *home* preference shock, it must be that $\widetilde{r}_t^W < 0$ and $\widetilde{r}_t^R < 0$. So therefore, both $\widehat{n}_t^W < 0$, and $\widehat{n}_t^R < 0$. Since $\widehat{n}_t = \widehat{n}_t^W + \widehat{n}_t^R$, it must be that home country output falls. However, foreign output, defined as $\widehat{n}_t = \widehat{n}_t^W - \widehat{n}_t^R$, may or may not fall. As discussed in Cook and Devereux (2010b), the international transmission of a 'liquidity trap shock' may be positive or negative. In particular, if the preference shock is not too large, so that the foreign economy is only 'slightly' in the liquidity trap, then the fall in foreign policy rates to zero stimulates foreign spending, offseting the fall in demand coming from the home country, so that the foreign output gap may rise.

4.1 Fiscal Multiplier

In the absence of further monetary stimulus when both countries are in a liquidity trap, fiscal spending policies can be used to reduce output gaps and eliminate deflationary pressures. Consider the world fiscal multiplier in a global liquidity trap. For convenience define $\Delta_4 = s(1-\beta\mu)(1-\mu) - \mu k(s) > \Delta_4^D = s_D(1-\beta\mu)(1-\mu) - \mu k(s_D)$. Also note that $\Delta_4 > \Delta_3$, $\Delta_4^D > \Delta_3^D$ and $\Delta_4 = D\Delta_4^D$.

We can then combine equations (17) and (19) to get:

$$\Delta_3 \widehat{n}_t^W - \Delta_4 \widehat{cg}_t^W = (1-\beta\mu)\widetilde{r}_t^W,$$

[5]In fact, as we see below, the fact that natural interest rates in both countries are below zero does not guarantee that the optimal monetary response is to have zero interest rates in both countries. For now however, we focus on a situation where this is the case, so that there is a global liquidity trap. The conditions where this situation applies will be described later.

[6]Note that the implicit assumption is that, when the shock disappears, monetary policy will immediately close all gaps thereafter, and any fiscal gaps that are created by policy during the liquidity trap are also closed off.

so that:

$$\widehat{n}_t^W = \frac{\Delta_4}{\Delta_3}\widehat{cg}_t^W + \frac{(1-\beta\mu)}{\Delta_3}\widetilde{r}_t^W \tag{25}$$

From (25) , we have (noting that output and employment gaps are equivalent in this model) $\frac{d\widehat{y}^W}{d\widehat{g}^W} \approx \frac{\Delta_4}{\Delta_3}(1-c_y)$. But we can define $\frac{d\widehat{y}^W}{d\widehat{g}^W} = \frac{\frac{dY^W}{Y}}{\frac{dG^W}{G}} = \frac{dY^W}{dG^W}\frac{G^W}{Y^W} = \frac{dY^W}{dG^W}(1-c_y)$. Thus, the world government spending multiplier is given by $\frac{dY^W}{dG^W} = \frac{\Delta_4}{\Delta_3}$. Note that by the definition of $\Delta_3$and Δ_4, the world multiplier is independent of the degree of home bias, ν. In the extreme of $\nu = 2$, with complete home bias we would be dealing with two closed economies. However, equation (25) indicates that we can also view $\frac{\Delta_4}{\Delta_3}$ as equal to the multiplier in the closed economy . By definition $\Delta_4 > \Delta_3$ so the world, or closed economy multiplier is greater than one. From the forward looking inflation equation, we see that an increase in government spending will have a positive effect on inflation. Thus, a persistent increase in government spending will have a positive effect on expected inflation. At the zero lower bound, this will reduce the real interest rate. If an expansion in world government spending reduces world real interest rates, this will stimulate demand beyond the direct spending of government.

We can also construct a parallel equation for the relative levels of the economies by combining (18) and (20), using

$$\Delta_3^D \widehat{n}_t^R - \Delta_4^D \widehat{cg}_t^R = (1-\beta\mu)\widetilde{r}_t^R,$$

so that:

$$\widehat{n}_t^R = \frac{\Delta_4^D}{\Delta_3^D}\widehat{cg}_t^R + \frac{(1-\beta\mu)}{\Delta_3^R}\widetilde{r}_t^R \tag{26}$$

Note that the *relative* government spending multiplier is also greater than one, $\frac{\Delta_4^D}{\Delta_3^D} > 1$, by the definition of $\Delta_4^D > \Delta_3^D$. That is, a rise in home relative to foreign government spending increases home relative to the foreign output gap more than one for one. The relative multiplier may be worth further examination. Noting that $Ds_D = s$, we have:

$$\begin{aligned}\frac{\Delta_4^D}{\Delta_3^D} &= \frac{D\Delta_4^D}{D\Delta_3^D} = \frac{Ds_D(1-\beta\mu)(1-\mu) - \mu k\left(Ds_D\right)}{Ds_D(1-\beta\mu)(1-\mu) - \mu k\left(D\phi + Ds_D\right)} \\ &= \frac{\Delta_4}{\Delta_3 - (D-1)\mu k\phi}\end{aligned}$$

Notice that the home bias term only enters through D. Remember $D = (\sigma v(2-v) + (1-v)^2) > 1$ so $\frac{dD}{dv} = -2(\sigma-1)(v-1) < 0$. Therefore, the greater is home bias, the smaller will be the relative multiplier. We can also see that the relative multiplier is larger than the world multiplier. A country that is doing a relatively large amount of government spending will have a relatively low interest rate, stimulating demand intertemporally. But the relatively low interest rate will also translate into a terms of trade depreciation, concentrating demand on the output of the country with high government spending. Then the high spending country will receive even higher demand through the expenditure switching effect, thus explaining why the relative multiplier is stronger than the world multiplier.

Now, we can examine how government spending in one country will affect both its own own output gap, and the output gap of its trading partner. Consider the effect of spending by the home economy alone, so that $\widehat{cg}_t^W = \widehat{cg}_t^R = \frac{\widehat{cg}_t}{2}$.

Proposition 1 *In a persistent global liquidity trap ($\mu > 0$), the multiplier of government spending on domestic output will be greater than one. The domestic output multiplier is a decreasing function of home bias; for $v < 2$, the domestic output multiplier is larger than the closed economy multiplier. The multiplier of government spending on foreign output is negative. This foreign multiplier is approaches zero as home bias increases.*

Proof. . $\widehat{n}_t = \widehat{n}_t^R + \widehat{n}_t^W$. Add both sides of (25) and (26) to get

$$\begin{aligned}\widehat{n}_t^R + \widehat{n}_t^W &= \widehat{n}_t = \frac{\Delta_4}{\Delta_3}\frac{\widehat{cg}_t}{2} + \frac{\Delta_4^D}{\Delta_3^D}\frac{\widehat{cg}_t}{2} + ... \\ &= \left[\frac{1}{2}\frac{\Delta_4}{\Delta_3} + \frac{1}{2}\frac{\Delta_4^D}{\Delta_3^D}\right]\widehat{cg}_t\end{aligned}$$

The multiplier on domestic government spending is the average of the closed economy multipier and the relative multiplier. As the relative multiplier is larger than the closed economy multiplier and both are greater than 1, the multiplier is also greater than one. Also, since the relative multiplier is a declining function of home bias, so is the multiplier on domestic government spending.

Also the foreign output gap is defined as: $\widehat{n}_t^* = \widehat{n}_t^W - \widehat{n}_t^R$. Subtract (26) from (25) to get

$$\begin{aligned}\widehat{n}_t^W - \widehat{n}_t^R &= \widehat{n}_t^* = \frac{\Delta_4}{\Delta_3}\frac{\widehat{cg}_t}{2} - \frac{\Delta_4^D}{\Delta_3^D}\frac{\widehat{cg}_t}{2} + ... \\ &= \frac{1}{2}\left[\frac{\Delta_4}{\Delta_3} - \frac{\Delta_4^D}{\Delta_3^D}\right]\widehat{cg}_t\end{aligned}$$

The multiplier on foreign goverment spending is half the difference between the closed economy multiplier and the relative multiplier. Since the relative multiplier is larger, the multiplier on foreign government spending is negative. Also, since the relative multiplier gets smaller as v gets larger, the multiplier on foreign government spending gets closer to zero as home bias increases. ■

In addition to the standard intertemporal effects, in a liquidity trap, expansionary government spending will weaken the domestic currency, leading to enhanced demand beyond the closed economy multiplier. By the same token, this tends to reduce demand for foreign output through the expenditure switching effect. But the greater is home bias, the less important is the expenditure switching effect. As home bias increases, the domestic multiplier goes to the closed economy multiplier and the foreign multiplier will go to zero.

One important factor in the calculation of the size of government spending multipliers is the degree of persistence of the liquidity trap shock itself. This is made clear in the following proposition.

Proposition 2 *In a transitory global liquidity trap ($\mu = 0$), the multiplier of government spending on domestic output will be equal to one regardless of home bias. The multiplier of government spending on foreign output is zero regardless of home bias.*

Proof. If $\mu = 0$ then $\Delta_3^D = \Delta_4^D = s_D$ and $\Delta_3 = \Delta_4 = s$ so the closed economy multiplier equals the relative multiplier $\frac{\Delta_4}{\Delta_3} = \frac{\Delta_4^D}{\Delta_3^D} = 1$. ■

If the initial shock is purely transitory, so that the government spending response is also transitory, there will be no impact of government spending on *expected* inflation. Thus, there will be no effects on real interest rates, and therefore no intertemporal or expenditure switching effects. An increase in government demand in the home economy will translate one-for-one into an increase in domestic output but will have no impact on the foreign economy. More generally, the size of the government spending multiplier will be much smaller, the less persistent is the initial savings shock.

5 Optimal Discretionary Policy

We now turn to the key part of the paper, which is the analysis of the optimal policy response to a liquidity trap shock. We explore optimal *cooperative* fiscal policy responses, where governments in each country cooperate on policies. In addition, we will show that in some cases, even when natural real interest rates in both countries are pushed below zero, monetary policy may be effectively used in the foreign country (which is not directly affected by the preference shock).

While a complete analysis of the determination of fiscal and monetary policy in a global liquidity trap would also require an exploration of the strategic interaction between non-cooperative policy authorities, this raises difficult technical issues (see Benigno and Benigno 2005), and so is left as a topic for future research. Focusing on the cooperative problem is a desirable first approach. since it sets out a benchmark for choosing a policy so as to maximize world welfare in response to a negative demand shock that undermines the normal mechanism of monetary policy[7].

In order to analyze optimal policy, we first need to define an objective function. As shown in Cook and Devereux (2010a), a second order approximation to an equally weighted world social welfare can also be constructed in world averages and world differences.

$$
\begin{aligned}
V_t &= -(\widehat{n}_t^R)^2 \cdot \frac{A}{2} - (\widehat{n}_t^W)^2 \frac{B}{2} - (\widehat{cg}_t^R)^2 \cdot \frac{F}{2} - (\widehat{cg}_t^W)^2 \cdot \frac{H}{2} - J(\widehat{n}_t^R)(\widehat{cg}_t^R) \\
&\quad -L(\widehat{n}_t^W)(\widehat{cg}_t^W) - \frac{\theta}{4k}(\pi_t^W + \pi_t^R)^2 - \frac{\theta}{4k}(\pi_t^W - \pi_t^R)^2
\end{aligned} \tag{27}
$$

where

$$
\begin{aligned}
A &\equiv \left\{ \frac{(1+\phi c_y)}{c_y^2} + \frac{(\sigma - D)}{D}(1 + \frac{(1-c_y^2)}{c_y^2 D}) \right\} \\
B &\equiv \frac{(\sigma + \phi c_y)}{c_y^2} = \frac{(s+\phi)}{c_y}, \\
H &\equiv \frac{1}{(1-c_y)} \frac{\sigma}{c_y^2} = \frac{1}{(1-c_y)} \frac{s}{c_y} \quad L \equiv \frac{-\sigma}{c_y^2} = \frac{s}{c_y}
\end{aligned}
$$

$$
\begin{aligned}
J &\equiv \left[-\frac{1}{c_y^2} - \frac{(\sigma - D)}{c_y^2 D^2}(1 + (v-1)(D-1)c_y^2) \right] \\
F &\equiv \frac{((1-c_y) + c_y \sigma)}{(1-c_y)c_y^2} + \frac{(\sigma - D)}{c_y^2 D^2}(1 + (v-1)(D-1)c_y^2)
\end{aligned}
$$

In the analysis here, we will concentrate exclusively on optimal discretionary fiscal policy responses. This implies that governments cannot commit themselves to future policy actions, so all policies become history independent. As a result, when natural real interest rates are negative, and both countries have current policy rates at the zero bound, monetary policy becomes entirely ineffective, since policy makers cannot commit to expansionary monetary

[7] The cooperative approach to fiscal policy in a global liquidity trap is not necessarily unrealistic. In the immediate aftermath of the financial crash of 2008, the G20 group agreed on a joint policy response to the crisis which assigned target levels of fiscal stimulus to each member country.

policy in the future, after natural real interest rates have risen again. In some cases, however, the foreign policy-maker may not be in a liquidity trap, because it can choose to set its policy rate higher than the natural real interest rate in the foreign country. Thus, we allow for the possibility that the foreign policy interest rate is not constrained by the zero lower bound.

In the case of policy discretion, the cooperative fiscal policy response to a liquidity trap involves maximizing (27) in each period, taking expectations of all future variables as given, subject to the inflation equations for world averages and differences, given by (17) and (19), and subject to the non-negativity constraints on nominal interest rates in each country. Since the non-negatively constraint on the home country policy rate will also bind for the duration of the shock, we only impose the non-negativity condition on the foreign interest rate.

Given this, we have the Lagrangean expression:

$$
\begin{aligned}
\max_{\widehat{n}_t^R,\widehat{n}_t^W,cg_t^R,cg_t^W,\pi_t^W,\pi_t^R,r_t^*} L_t \quad = \quad & -(\widehat{n}_t^R)^2 \cdot \frac{A}{2} - (\widehat{n}_t^W)^2 \frac{B}{2} - (c\widehat{g}_t^R)^2 \cdot \frac{F}{2} - (\widehat{cg}_t^W)^2 \cdot \frac{H}{2} \\
& -J(\widehat{n}_t^R)(\widehat{cg}_t^R) - L(\widehat{n}_t^W)(\widehat{cg}_t^W) - \frac{\theta}{4k}(\pi_t^W + \pi_t^R)^2 - \frac{\theta}{4k}(\pi_t^W - \pi_t^R)^2 \\
& +\lambda_{1t}\left[\pi_t^W - k(\phi+s)\widehat{n}_t^W + ks \cdot \widehat{cg}_t^W - \beta E_t \pi_{t+1}^W\right] \\
& +\lambda_{2t}\left[\pi_t^R - k(\phi+s_D)\widehat{n}_t^R + ks_D \widehat{cg}_t^R - \beta E_t \pi_{t+1}^R\right] \\
& +\psi_{1t}\left[sE_t(\widehat{n}_{t+1}^W - \widehat{n}_t^W) - sE_t(\widehat{cg}_{t+1}^W - \widehat{cg}_t^W) - E_t\left(\frac{r_t^*}{2} - \widetilde{r}_t^W - \pi_{t+1}^W\right)\right] \\
& +\psi_{2t}\left[s_D E_t(\widehat{n}_{t+1}^R - \widehat{n}_t^R) - s_D E_t(\widehat{cg}_{t+1}^R - \widehat{cg}_t^R) - E_t\left(-\frac{r_t^*}{2} - \frac{\widetilde{r}_t^R}{2} - \pi_{t+1}^R\right)\right] \\
& +\gamma_t\left[r_t^*\right]
\end{aligned}
$$

The first two constraints are the inflation equations in average and relative terms. The second two constraints are the average and relative 'IS' equations. The final constraint is the non-negativity constraint on the foreign policy interest rate. The policy optimum involves the choice of the output gaps, the government spending 'gaps', the inflation rates and the foreign interest rate to maximize this Lagrangean. The first order conditions of the maximization are:

$$-A\widehat{n}_t^R - J(\widehat{cg}_t^R) = \lambda_2 k(\phi + s_D) + s_D\psi_2 \equiv \Upsilon_2 \tag{28}$$

$$-B\widehat{n}_t^W - L(\widehat{cg}_t^W) = \lambda_1 k(\phi + s) + s\psi_1 \equiv \Upsilon_1 \tag{29}$$

$$F\widehat{cg}_t^R + J(\widehat{n}_t^R) = ks_D\lambda_2 + s_D\psi_2 = \Upsilon_2 - \phi(k\lambda_1) \tag{30}$$

$$H\widehat{cg}_t^W + L(\widehat{n}_t^W) = ks\lambda_1 + s\psi_1 = \Upsilon_2 - \phi(k\lambda_1) \tag{31}$$

$$k\lambda_1 = \theta\pi_t^W \tag{32}$$

$$k\lambda_2 = \theta\pi_t^R \tag{33}$$

$$\psi_{2t} - \psi_{1t} + \gamma_t = 0 \tag{34}$$

We analyze these first order conditions in the next series of propositions.

To begin with, assume that the zero bound constraint on r_t^* is binding. In this case, $\gamma_t > 0$, and we may solve (28)-(33) for output gaps, government spending gaps, and inflation rates. First insert (28) and (33) into (30) to get:

$$-(J+F)\widehat{cg}_t^R - (J+A)(\widehat{n}_t^R) = \phi\theta\pi_t^R \tag{35}$$

Then insert (29) and (32) into (31) to get:

$$-(H+L)\widehat{cg}_t^W - (B+L)\widehat{n}_t^W = \phi\theta\pi_t^W \tag{36}$$

Using these conditions, we establish the following:

Proposition 3 *When the zero bound constraint binds in both countries, the optimal discretionary average level of the fiscal gap, $\widehat{cg}_t^W$, will be positive, independent of the degree of home bias, and equal to the optimal closed economy fiscal gap*

Proof. See Appendix ■

To see the intuition behind the proposition, we first take equation (36). Then, using the condition that all expectations terms of variable x_t will satisfy $E_t(x_{t+1}) = \mu x_t$, we can solve (17) and (18) for a second relationship between $\widehat{cg}_t^W$ and $\widehat{n}_t^W$, given by:

$$-\Delta_3\widehat{n}_t^W + \Delta_4\widehat{cg}_t^W = -(1-\beta\mu)\widetilde{r}_t^W \tag{37}$$

Then, combining (36) and (37), we arrive at

$$[(\Delta_3 HL + \Delta_4 BL) + \phi f(1-\mu)s]\widehat{cg}_t^W = -[f(\phi+s) + (1-\beta\mu)BL]\,\widetilde{r}_t^W \tag{38}$$

where the expressions $HL>0$ and $BL > 0$ and $f > 0$ are defined in the Appendix. From this, it is clear that when the world average natural rate falls, the world average fiscal gap must increase.

Note that, outside a liquidity trap, it would never be desirable to have a non-zero fiscal gap. But when the policy rate is constrained by the zero lower bound, fiscal spending, by creating anticipated inflation, can reduce real interest rates, stimulate private demand, and reduce the current world output gap. Moreover, since all terms in (38) are independent of the degree of home bias v, the optimal fiscal gap for the world average is the same as that in a closed economy, despite the fact that the initial shock in this exercise emanates from the home country alone and the shock is distributed unequally across the world economy.

The key focus of interest however, is the breakdown of the fiscal spending policies between the home and foreign economy. On the surface, because opening up fiscal gaps is costly in welfare terms, it would seem that the optimal cooperative policy should involve sharing the burden of fiscal adjustment across the two countries. But the appropriate response for fiscal policy depends upon how the policy itself can impact upon expected inflation, and the output gap in each country, which in turn depends on the degree of home bias, and other features of the model, as discussed in section 3 above. Nevertheless, it is easy to establish that the home fiscal gap should always be positive, as stated in the following proposition.

Proposition 4 *In a global liquidity trap, both the world relative fiscal gap and the optimal home fiscal gap will be positive,* $\widehat{cg}_t^R > 0$ *and* $\widehat{cg}_t > 0$

Proof. *See Appendix.* ■

The first part of the proposition is established along the same lines as Proposition (). Putting together the first order conditions for the world relative variables, and the conditions from the model equations (19) and (20), we may derive a relationship between the world relative shock to the natural interest rate and the optimal response of the world relative fiscal spending gap as follows:

$$\left[\Delta_3^D HL + \Delta_4^D JA + f\phi s_D(1-\mu)\right]\widehat{cg}_t^R = -\left[f(\phi + s_D) + (1-\beta\mu)JA\right]\widetilde{r}_t^R \qquad (39)$$

where the terms HL, and JA are positive, and defined in the Appendix. Since $\widetilde{r}_t^R > 0$ for a shock that emanates from the home economy, it must be that the relative fiscal gap $\widehat{cg}_t^R$ is positive also. But since from the definition of $\widehat{cg}_t^W$ and $\widehat{cg}_t^R$, we have $\widehat{cg}_t = \widehat{cg}_t^W + \widehat{cg}_t^R$, it then follows that the home fiscal spending gap is positive. Thus, an optimal response to the liquidity trap for the home country is to follow an expansionary fiscal policy.

What is not so clear, however, is the optimal fiscal response for the foreign economy, when both countries are in a liquidity trap. Continue to assume that the $r_t^* = 0$ constraint still binds. Note, that $\widehat{cg}_t^* = \widehat{cg}_t^W - \widehat{cg}_t^R$. Since both $\widehat{cg}_t^W$ and $\widehat{cg}_t^R$ are positive, we cannot obviously sign this expression. Using the solutions from the Appendix, we can write

$$\widehat{cg}_t^W - \widehat{cg}_t^R$$
$$= -\frac{[f(\phi+s)+(1-\beta\mu)BL]}{[(\Delta_3 HL+\Delta_4 BL)+\phi f(1-\mu)s]}\widetilde{r}_t^W + \frac{[f(\phi+s_D)+(1-\beta\mu)JA]}{[\Delta_3^D HL+\Delta_4^D JA+f\phi s_D(1-\mu)]}\widetilde{r}_t^R \quad (40)$$

The first term on the right hand side is positive, while the second term is negative, when the (negative) shock hits the home economy. So in principle, it would seem as if the foreign fiscal expansion (when both countries are in a liquidity trap) could go either way. Note, from (9) and (10) we have the world average and relative natural interest rates as:

$$\widetilde{r}_t^W = \overline{r} + \left(\frac{1}{(\phi+\sigma)}\right)(1-\mu)\phi c_y \frac{\varepsilon_t}{2} \quad (41)$$

$$\widetilde{r}_t^R = \left(\frac{(v-1)}{\Delta}\right)(1-\mu)\phi c_y \frac{\varepsilon_t}{2} \quad (42)$$

Insert (41) and (42) into (40) to get

$$\widehat{cg}_t^* = -c_r\overline{r} + c_\varepsilon \varepsilon_t \quad (43)$$

where $c_r > 0$ and $c_\varepsilon > 0$ are complicated functions of the underlying coefficients, defined in the Appendix.

The coefficient on $\overline{r}$ is negative. As the foreign economy approaches the zero lower bound, there will be a cut in the interest rate of size, $\overline{r}$, which will have an expansionary effect on foreign output. This will give the foreign fiscal authority an incentive to cut expenditure to stabilize output and inflation. Thus, if the foreign economy maintains a zero interest rate, in face of the home savings shock, the optimal foreign fiscal response may be *contractionary.*

This raises the possibility that if the shock to ε_t is sufficiently weak,foreign fiscal policy will be contractionary. Consider the extreme case where the home demand shock is sufficiently small that the possibility of a liquidity trap in the home economy is marginal and $\widetilde{r}_t^* = 0$.

Proposition 5 *If the natural interest rate in the foreign economy is $\widetilde{r}_t^* = 0$ and the foreign central bank sets r_t^* at the zero lower bound, then $\frac{\phi}{1-c_y} > 1$ is sufficient condition for the optimal to be negative, $\widehat{cg}_t^* < 0$.*

Proof. .See Appendix ■

If the foreign central bank acts in the orthodox manner of closing its interest rate gap, the direct expansionary effect of cutting interest rates will more than offset the weak negative impact of the foreign demand shock and the optimal foreign fiscal policy will be contractionary. Note however that the orthodox policy of keeping the foreign interest rate at the natural rate will not generally be the optimal cooperative monetary policy in this case. This will be illustrated more clearly below.

While a full analytical analysis of the jointly optimal cooperative choice of fiscal gaps and r_t^* is quite complicated, we can establish that, holding fiscal gaps equal to zero, the optimal monetary response for the foreign economy, if the home economy is in a liquidity trap, is to raise its policy rate above the natural foreign real interest rate.

Proposition 6 *Assume a passive fiscal policy, so that home and foreign fiscal gaps are closed* $\widehat{g}_t = \widehat{g}_t^* = 0$. *Then if the non-negativity constraint on the foreign interest rate is not binding, the optimal foreign policy rate is above the foreign natural interest rate* $r_t^* > \widetilde{r}_t^*$.

Proof. *See Appendix* ■

The intuitive explanation for this result is that the foreign economy tightens monetary policy in order to foster a home country exchange rate depreciation, so as to cushion the impact of the savings shock on the home country. The foreign policy maker will not close the interest rate gap, in its own economy, even if it does have the ability to do this. More strikingly, this proposition does not require that $\widetilde{r}_t^* > 0$. That is, the foreign country may choose to follow a positive policy rate even though, when looked at from the perspective of the foreign country alone, it would be in a liquidity trap. We illustrate this outcome in the numerical analysis below.

5.1 Numerical analysis of optimal cooperative monetary and fiscal policy

We now address the problem of jointly optimal cooperative monetary and fiscal policy. We tackle this problem by solving the model numerically. To evaluate the economy quantitatively, we adopt some parameters from Cook and Devereux (2010a). Let $\beta = 0.99$, so each period is a quarter, and this translates to a value of the steady state interest rate $\overline{r} = 0.01$. The Frisch labor supply elasticity is $\phi = 1$. Price stickiness is $\kappa = 0.85$, so that $k = 0.027$, as in Christiano et al. (2009). Let the share of government in output be 20 percent, $c_y = 0.8$. We assume the inverse of intertemporal demand elasticity σ, is equal to 2. The persistence of the demand shock is set at 0.8 ($\mu = 0.8$) implying an expected length of the slump to be 5 quarters. We set the elasticity of substitution between good varieties within a country, θ, equal to 5. Finally, we set the preference shock in the home country ε_t so that at $v = 1$ (the case without any home bias), the natural real interest rate at the quarterly frequency would fall from 1 percent to -1.7 percent, with persistence μ.

Figure 1 illustrates the response of home and foreign output gaps, home and foreign government spending gaps, home and foreign inflation, the foreign country optimal policy rate, as well as the foreign natural real interest rate, and the home country terms of trade, for different values of v, when the optimal fiscal and monetary policy response is chosen. The figure takes account of condition (34), so that, at each value of v, the non-negativity constraint on r_t^* is tested, and if it is not binding, the optimal foreign policy rate is chosen. The first thing to note is that at $v = 1$, then clearly the liquidity trap is binding in both countries, and all variables respond in the same way in the two countries. The output gap falls by over 7 percent in both countries, and this is coupled with a fall in the rate of inflation by equal amounts. Since both countries are affected equally, it must be that the zero lower bound is binding in both countries, and the foreign policy rate is set at zero. The response of policy is illustrated by the positive response of the fiscal gap in each country. Thus, fiscal policy should behave countercyclically, and equally so in each country for a world without home bias in preferences.

Now, as v rises above unity, we know that the impact of the shock on the foreign natural interest rate becomes muted, while the opposite occurs for the home natural interest rate. The negative response of the foreign output gap is then reduced, while that of the home output gap is increased. As v rises more and more, we find, as discussed above, that the foreign output gap may actually increase. A similar dynamic occurs in the response of the inflation rates in the two countries - home inflation becomes more and more negative as v rises, while the negative response of foreign inflation becomes less and less. The optimal response of fiscal policy gaps is illustrated in panel b of the Figure. As v rises, home fiscal policy becomes more aggressive, while the foreign fiscal policy becomes more muted.

Panel d illustrates the optimal response of the foreign country policy rate, alongside the foreign country natural real interest rate. Note that at $v = 1$, the foreign policy rate is stuck at zero, while the natural real interest rate is at -0.017. As v rises, the response of the foreign natural interest rate becomes less and less, as is obvious from the formula (10). Eventually, as v rises to 2, the foreign country would be entirely unaffected by the shock, and the foreign natural interest rate would rise to 0.01, the steady state natural interest rate. But the key feature of panel d is the the foreign country will raise its policy rate above zero for values of $\widetilde{r}_t^* < 0$. That is, the foreign country will choose positive interest rates, as part of an optimal cooperative policy package, even though, by the usual closed economy logic, it should be still in a liquidity trap, since its natural rate of interest is below zero. Equivalently, and in consistency with Proposition 6 (which held only for zero fiscal gaps), the foreign country will not follow a policy of offsetting the movement in the foreign natural interest rate, to the greatest extent that it can, so long as the policy rate is above the zero bound. Rather, it chooses to raise policy rates, even though $\widetilde{r}_t^* < 0$. In fact, panel d makes clear that as v rises, it may be optimal for the foreign country to raise its policy rate *above the steady state natural rate of interest*. Thus, by any definition of the term, the optimal monetary stance for the foreign country, in face of the home liquidity trap, is to tighten its monetary policy.

Thus, surprisingly, an optimal cooperative policy response to a liquidity trap can be characterized by expansionary fiscal policy in all countries, but contractionary monetary policy in the least affected country. This seemingly paradoxical result is related to the results of section (4) above. As v rises, the home economy is significantly more affected by the negative demand shock. An optimal policy response is to raise world demand, and to re-orient world demand towards the home country. The raising of the foreign policy rate is associated with an appreciation of the foreign currency, which generates an additional expenditure switching of demand towards the home country. Since the impact of the home country shock on foreign output is positive in any case, when v is sufficiently greater than unity, the rise in the foreign policy rate has the additional benefit that it helps to minimize the response of the foreign output gap to the home country shock. The Figure shows that the tightening of the policy rate in the foreign country as v rises reduces the degree to which the home terms of trade appreciates in response to the initial savings shock, and for sufficiently high v the home country terms of trade will actually depreciate. Thus, the key benefit of the foreign monetary response is to shape the response of the terms of trade.

We note that, when an optimal foreign monetary policy is used, the foreign country has a very small fiscal gap. Since $\dot{g}_t^* > 0$, it is optimal for the foreign country to follow an expansionary fiscal policy. But quantitatively, the size of the fiscal expansion is much less than that of the home country.

Figure 2 contrasts the optimal policy to an alternative possibility for foreign monetary policy. Here we assume that the foreign country follows the same monetary strategy as the home country, setting the policy rate equal to zero when the natural real interest rate is negative, and adjusting the policy rate to the natural real interest rate when it is above zero. Thus, we assume that

$$r_t^* = \max(\widetilde{r}_t^*, 0).$$

The Figure shows that the response of fiscal policy is substantially different when $v > 1$ and the foreign economy follows this (non-optimal) monetary rule. The key feature of this policy is that it is excessively expansionary for the foreign economy, relative to the optimal rule. As v rises more and more, the foreign economy experiences a boom, which is countered by a *contractionary* fiscal policy. At the same time, the outcome of expansionary monetary and contractionary fiscal policy in the foreign country leads to an excessive contraction in the home economy, which then requires a much greater fiscal expansion than would take place under the optimal policy. The foreign economy experiences inflation, while the deflation in the home economy is greater than it would be under the optimal policy. In addition the terms of trade appreciates much more for the home economy that it would under the optimal policy.

6 Conclusions

To be added

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Figure 1: Optimal Policy

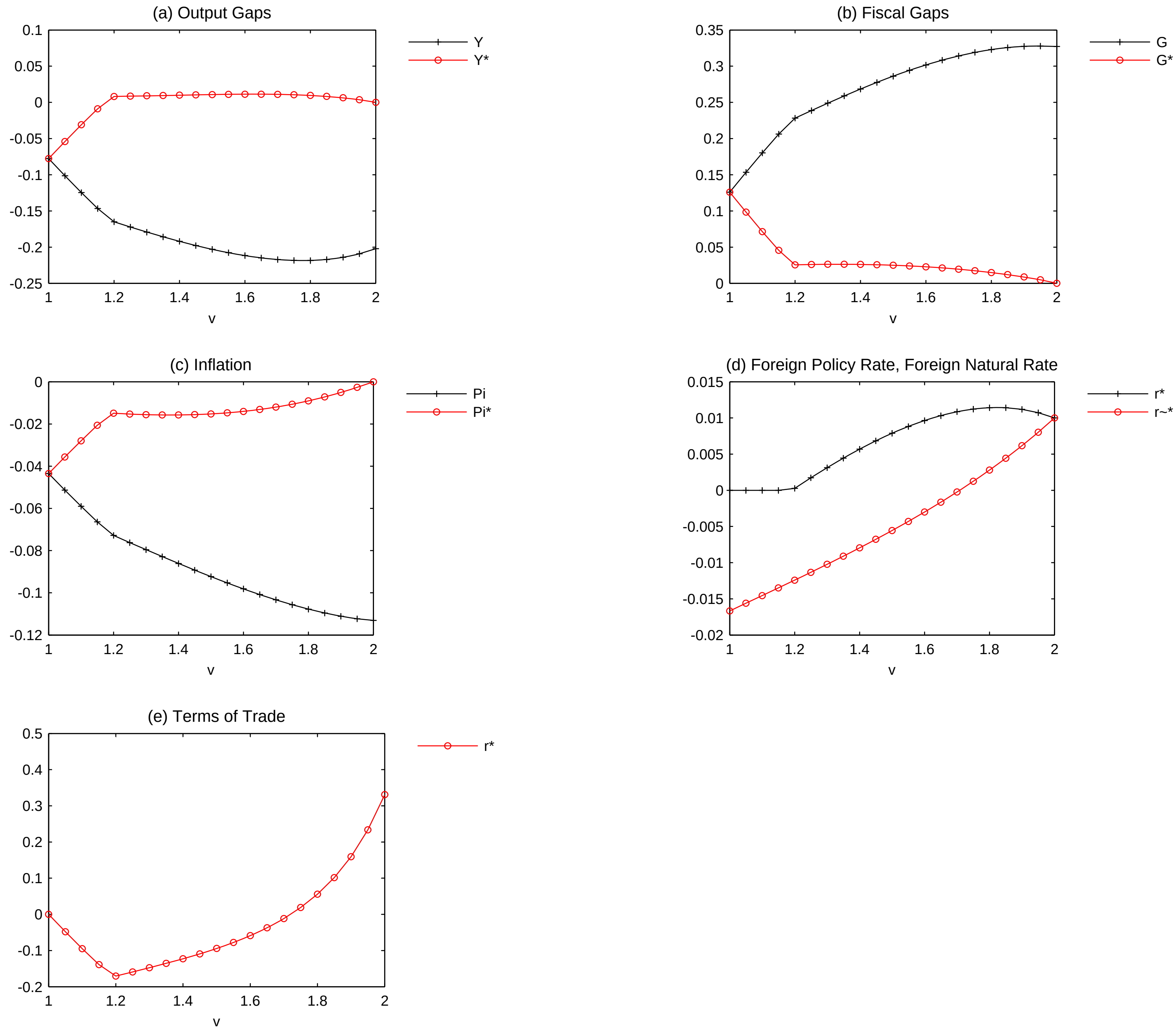

Figure 2: Optimal and Constrained Policy