TFP during a Credit Crunch*

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Abstract

The financial crisis of 2008 was followed by sharp contractions in aggregate output and employment and an unusual increase in aggregate TFP. This paper attempts to explain these facts by modeling the creation and destruction of jobs in the presence of heterogeneity in firm productivity and frictional credit and labor markets. After showing that job creation and destruction are functions of total financial costs in the economy, an aggregate production is derived in which the aggregate level of TFP depends on the underlying distribution of idiosyncratic shocks and the structures of both the credit and labor markets. The model implies that increased financial frictions: i) raise the level of aggregate TFP and unemployment, and reduce output; ii) raise the variance of the cross-sectional distribution of firm productivity.

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1 Introduction

The financial crisis that lead to the worst recession since the Great Depression began with an unexpected, drastic, shock to credit markets. Indeed, a salient feature of the crisis, emphasized in Caballero and Kurlat (2009), concerns the element of surprise in the event, not factored into expectations, that lead to sharp declines in bank lending and commercial paper issuance, along with spikes in interest rate spreads.\footnote{According to Caballero and Kurlat (2009), “the surprise was in the distress of many parts of the financial system, even those very distant from the sub-prime market itself, including all structured products, commercial paper, and interbank lending.” Certain asset markets simply froze, with few trades, while commercial banks doubled their holdings of cash assets. Bernanke and Lown (1991) use the term Credit Crunch to discuss the role of financial in the 1990-1991 recession and see large role in the slow down in credit market activity as the result of a drop in credit demand.} Another peculiar feature of this recession has been the increase in aggregate TFP concurrent with the strong contractions in output and employment. Fernald and Matoba (2009) report a 3.3\% in TFP increase over the period 2008:Q4 and 2009:Q3, compared to an average quarterly change of 1.4\% since 2001. This confounds real business cycles theory which is predicated on movements in TFP as driving the cyclical fluctuations in aggregate quantities and declining during recessions.

This paper provides a qualitatively and quantitatively consistent explanation of this set of observations on the Great Recession in which the sorting of individual firms following an unexpected shock to credit markets plays a central role. The theory models endogenous job creation and destruction in the presence of search frictions on credit and labor markets, and shows how both are functions of total financial costs in the economy. Firms are modeled as the joint venture between an entrepreneur and a creditor, and require a unit of labor to produce a good. The search duration for both sides of the credit market to meet determines the financial cost to creating a new firm, and the cost of hiring a worker, which requires resources and time, are born by the creditor. A first implication is that, by limiting firm entry on labor markets, financial imperfections restrict job creation.\footnote{As in Wasmer and Weil (2004) or Petrosky-Nadeau (2009).} Producing firms randomly draw new job productivities and jobs are destroyed when the worker-firm pair is no longer viable in the manner of Mortensen and Pissarides (1994), with the exception that the structure of the credit market now enters the determination of the job destruction threshold. Greater financial costs, by raising the opportunity cost of a match, raise the lowest viable job productivity and, hence, job destruction. By aggregating over active, in the sense of producing, micro units, the aggregate production function for this economy depends on the underlying distribution of shocks and the structure of both markets. In particular, the aggregate level of total factor

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productivity is increasing in the total costs of the financial sector as the latter determine the least productive active unit in the economy. However, as financial costs reduce job creation and increase job destruction, negative credit market shocks reduce aggregate employment and output.

The model is used to describe the effects of a breakdown in the system of financial intermediation on aggregate activity and the cross-sectional dispersion of firm productivity. This negative financial shock is defined as the unexpected breakdown in the credit matching technology, a freezing of the credit market making it difficult for lenders and borrowers to meet. Such an event leads to an increase in the costs on credit markets, sorting out the active units with the lowest productivity and generating an increase in aggregate TFP. At the same time, the financial shock reduces job creation, increases job destruction, and causes a sharp rise in the rate of unemployment. Modeling on events of the Fall of 2008, this credit crunch leads to declines in aggregate output and employment along with an increase in aggregate TFP of the magnitudes observed during the crisis. The model is also consistent with another peculiar feature of the Great Recession which is the moderate rise in the real wage between September 2008 and July 2009. Although the rise in aggregate unemployment, by weakening the outside option of workers in wage negotiations, puts downward pressure on individual wages, the destruction of the least productive jobs and their associated wages drives the increase in the average wage. Finally, it is important to stress that the model distinguishes active from idle firms, and that the variation in aggregate quantities arises from variations in the fraction of active firms and not the population of firms. In other words, the credit crunch caused variations in the use of the economy’s productive capacity. In an extension with endogenous firm exit, it is shown how a non-linear aggregate production function can result from micro units with linear production technologies.

The model also predicts an increase in the variance in the cross-sectional distribution of firm productivity during a credit crunch which, in the research of Bloom (2009), can be interpreted as an increase in uncertainty or risk. However, this paper argues that other shocks that do not affect the underlying distribution of productivity from which firms make their draws, or the true risk in the economy. This shock can be “perceived” as a “volatility” shock which is associated with declines in output and employment, and is the endogenous response of the economy to a credit crunch. Moreover, when we endogenize firm exit, there is an endogenous wedge between the Solow residual and the productivity of factors used in production. This Solow residual declines during a credit crunch, as the Solow residual in Bloom (2009) following a volatility shock.
In a broader sense, the model implies that movements in observed aggregate TFP can originate from shocks to credit markets, specifically, but in general from frictional markets that affect the sorting of active establishments. This calls into question how to correctly identify unanticipated movements in aggregate TFP as productivity shocks and their role in accounting for business cycle fluctuations, reminiscent of the distinction between productivity and technology in Basu and Fernald (2002) and the results in Lagos (2006) and Chari et al (2007), that disaggregate models with frictions can imply a different mapping between the parameters of the aggregate production technology and observables. This paper also contributes to recent research incorporating micro heterogeneity in macroeconomic models, and in particular in conjunction with financial frictions as in Kahn and Thomas (2010). In the latter, even temporary shocks to the financial sector cause lasting recessions as firms need time to accumulate assets to relax their collateral constraints. In the meantime, production shifts to larger, less productive firms, causing a drop, and not a rise, in aggregate TFP. In related research, Buera and Shin (2008) quantitatively evaluate the TFP costs of resource misallocation due to financial frictions that limit the ability of high productivity firms to borrow.\(^3\)

The rest of the paper is organized as follows. Section 2 develops the main model and discusses the relationship between financial costs and job creation and destruction. Section 3 derives the aggregate production function, examines its relationship with the state of the credit market and Section 4 explores the effects of a breakdown on financial markets. Section 5 allows for the endogenous the destruction of a firm and derives a Cobb-Douglas aggregate production function.

## 2 Main model

### 2.1 Credit markets and the creation of firms

Drawing on Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2010), there are three types of agents in this economy: entrepreneurs with no capital; banks with no ability to produce; workers with no capital and no ability to start a business. The timing of events is as follows: entrepreneurs initially need to find a “banker” in order to start a business. This search process costs \(e\) units of effort per unit of time. Search is successful with probability \(p\). Once a relationship on

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\(^3\)In Moll (2010) aggregate TFP is negatively linked to the degree of financial imperfection and the argument is that reducing financial friction allows more productive firms to rent more capital and increases output. In the limit, with perfect capital market, only the most productive firm produces, hiring all the capital. Although this paper is not concerned with the effects of financial frictions on growth, the model could be extended in future research. For implications of heterogeneous households in macroeconomic models, see Ayagari, 1994, and the survey by Heathcoate et al., 2008.
the credit market is established, a firm is created as the joint venture between the banker and the entrepreneur, the rents of which will be determined below. The banker finances vacancy posting or recruiting cost \( \gamma \) to attract a worker for the firm. This search process is successful with probability \( q \). The firm is then able to produce and sell in the goods market, which generates the flow profit to the entrepreneur \( y(x) - w(x) - \rho(x) \), where \( x \) is a random shock to the productivity of the firm, \( y(x) \) is the flow output assumed to be linear in \( x \), \( w(x) \) is the wage and \( \rho(x) \) the flow repayment to the bank. \( x \) is specific to each firm and drawn at Poisson rate \( \lambda \) from a distribution \( G(x) \), with density \( g(x) \). In addition, firms are subject to destruction shocks with Poisson parameter \( \delta \). The steady state asset values of the entrepreneurs are denoted by \( E_j \), with \( j = c, l \) or \( g \) for the state in which the entrepreneur finds him/herself, standing, respectively, for the credit, labor and goods markets. Free entry on the credit market is assumed, that is \( E_c \equiv 0 \). We therefore have the following Bellman equations:

\[
\begin{align*}
    rE_c &= 0 = -e + pE_l \quad (1) \\
    rE_l &= 0 + q \int \max [E_g(x) - E_l, 0] dG(x) \quad (2) \\
    rE_g(x) &= x - w(x) - \rho(x) + \lambda \int \max [E_g(z), E_l, 0] dG(z) - (\lambda + \delta)E_g(x) \quad (3)
\end{align*}
\]

Symmetrically, the bank’s asset values are denoted by \( B_j \), \( j = c, l \) or \( g \), for each of the stages. Free entry is also assumed on the banker’s side of the market, i.e.: \( B_c = 0 \). Let \( \kappa \) be the screening cost per unit of time of banks in the first stage, and by \( \hat{\rho} \) the Poisson rate at which a bank finds an entrepreneur to be financed. We have:

\[
\begin{align*}
    rB_c &= 0 = -\kappa + \hat{\rho}E_l \quad (4) \\
    rB_l &= -\gamma + q \int \max [B_g(x) - B_l, 0] dG(x) \quad (5) \\
    rB_g(x) &= \rho(x) + \lambda \int \max [B_g(z), B_l, 0] dG(z) - (\lambda + \delta)B_g(x) \quad (6)
\end{align*}
\]

The matching rates \( p \) and \( \hat{\rho} \) are made mutually consistent by the existence of a matching function \( M_C(\mathcal{B}, \mathcal{E}) \) where \( \mathcal{B} \) and \( \mathcal{E} \) are, respectively, the number of bankers and of entrepreneurs in stage \( c \). This function is assumed to have constant returns to scale. Hence, denoting by \( \phi \) the ratio \( \mathcal{E}/\mathcal{B} \), which is a reflection of the tension on the credit market and that will be called credit market
tightness, we have

\[ p = \frac{M_c(\mathcal{B}, \mathcal{E})}{\mathcal{E}} = p(\phi) \text{ with } p'(\phi) < 0 \]
\[ \hat{p} = \frac{M_c(\mathcal{B}, \mathcal{E})}{\mathcal{B}} = \phi p(\phi) \text{ with } \hat{p}'(\phi) > 0 \]

After contact, the bank and the entrepreneur engage in bargaining over the rents generated by the creation of the firm \( F_t = E_t + B_t \) and determine \( \rho(x) \) as \( \arg\max(E_t)^{1-\beta}(B_t)^{\beta} \), where \( \beta \in (0,1) \) is the bargaining power of the bank relative to the entrepreneur. The first order condition yields a sharing rule \( (1-\beta)B_t = \beta E_t \) which, in combination with (1) and (4), yields an equilibrium value of \( \phi \) denoted by \( \phi^* \):

\[ \phi^* = \frac{1 - \beta}{\beta} \frac{\kappa}{e} \]

Matching in the labor market is denoted by \( M_l(\mathcal{Y}, u) \) where \( u \) is the rate of unemployment and the total number of unemployed workers since the labor force is normalized to 1. \( \mathcal{Y} \) is the number of “vacancies”, that is the number of firms at stage \( l \). The function is also assumed to be constant returns to scale and the rate at which firms fill vacancies is a function of the ratio \( \mathcal{Y}/u \equiv \theta \), that is the tightness of the labor market. We have

\[ q(\theta) = \frac{M_l(\mathcal{Y}, u)}{\mathcal{Y}} \text{ with } q'(\theta) < 0 \]

Once an entrepreneur-bank pair is formed, they jointly operate as a firm on the labor and goods markets. We therefore describe the asset value of being on either market for a firm before turning to workers:

\[ r_{F_t} = -\gamma + q(\theta) \int \max[F_g(x) - F_t, 0] dG(x) \]
\[ r_{F_g}(x) = x - w(x) + \lambda \int \max[F_g(z), F_t, 0] dG(z) - (\lambda + \delta)F_g(x) \]

Free entry on credit markets implies that the total financial costs related to creating a firm can be summarized as \( \frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} \equiv C(\phi) \), a convex function of credit market tightness and, by (1) and (4), the value of a firm in the recruiting stage is equal to total financial costs, i.e. \( F_t = C(\phi) \). In the canonical Mortensen-Pissarides model, entry by firms on the labor market is unhampered by frictional credit markets and the value of the recruiting stage is driven to 0. The presence of financial
imperfections implies $C(\phi) > 0$, a positive lower bound to the value of a vacancy to a firm that will restrict firm entry on labor markets and job creation. Petrosky-Nadeau and Wasmer (2010) derive a Hosios condition for the credit market in which financial costs $C(\phi)$ are minimized when the creditor’s bargaining weight $\beta$ and the elasticity of the credit matching function with respect to bankers, $\varepsilon$, are equalized. Away from this condition the distortions caused by credit markets increase, as does the sensitivity of the economy to perturbations. To be precise, Petrosky-Nadeau and Wasmer (2010) show that the elasticity of labor market tightness to productivity shocks is increasing in total financial costs, $C(\phi)$, and that this elasticity is minimized at the credit market Hosios condition.

A rise in these cost can originate from several sources: an increase in the prospecting costs $\kappa$ and $\epsilon$, a breakdown in the efficiency of the matching function $M_{c}(\mathscr{R}, \mathscr{E})$, or deviations from the credit market Hosios condition.

Workers are either unemployed, earning the flow value of non-employment $b$, or employed at the wage $w(x)$. Unemployed workers face a job finding hazard $\theta q(\theta)$ and employed workers are separated from employment either exogenously following the destruction of the firm at rate $\delta$ or endogenously, when the firm draws a sufficiently low idiosyncratic productivity that the relationship cannot be sustained. The Bellman equations for the worker are:

$$
\begin{align*}
\quad rU &= b + \theta q(\theta) \int \max\{W(x) - U, 0\} dG(x) \quad (9) \\
rW(x) &= w(x) + \lambda \int \max\{W(z) - U, 0\} dG(z) - (\lambda + \delta) [W(x) - U] \quad (10)
\end{align*}
$$

where $U$ is the value of unemployment and $W(x)$ the value of employment at a firm with productivity $x$. Workers and firms split the surplus of their relationship, defined as $S(x) = F_{g}(x) - F_{l} + W(x) - U$, under a generalized Nash Bargaining process. This involves choosing a wage $w(x)$ that satisfies $\arg\max (F_{g}(x) - F_{l})^{1-\alpha} (W(x) - U)^{\alpha}$, where $\alpha \in (0, 1)$ is the worker’s bargaining weight. The first order condition for this problem implies the sharing rule $(1 - \alpha) (W(x) - U) = \alpha (F_{g}(x) - F_{l})$.

### 2.2 Job creation and job destruction under frictional credit markets

Combine (7), (8), (9) and (10) to obtain a Bellman equation for the surplus to a worker-firm pair:

$$(r + \lambda + \delta) S(x) = x + \lambda \int \max\{S(z), 0\} dG(z) - (r + \delta) F_{l} - rU$$

\footnote{It will be useful to write this sharing rule as $(1 - \alpha) S(x) = F_{g}(x) - F_{l}$ and $\alpha S(x) = W(x) - U$.}
Since $S'(x) = \frac{1}{r + \lambda + \delta} > 0$, there exists a reservation strategy such that if $x < \bar{x}$, where $\bar{x}$ is such that $S(\bar{x}) = 0$, the match is dissolved. That is to say, if the job draws $x < \bar{x}$ there is no value to the relationship to either party and thus $\bar{x}$ defines a job destruction threshold. Using this result, the Bellman for the worker-firm pair’s surplus is

$$(r + \delta)S(x) + rU - \lambda \int_{\bar{x}} S(z) dG(z)$$

which, evaluated at $\bar{x}$, yields what can be called a job destruction (JD) condition:

$$(r + \delta)C(\phi) + rU - \lambda \int_{\bar{x}} S(z) dG(z)$$

where we have used the fact that $F_1 = C(\phi)$. For a job to remain viable, its expected future value must at least equal the values of the firm and worker’s outside options net of the match’s current production. Recall that in canonical Mortensen-Pissarides model $C(\phi) = 0$, such that financial costs raise the lowest viable job productivity relative to that benchmark. Inserting a solution $S(x) = \frac{x - \bar{x}}{r + \lambda + \delta}$ into the previous equation yields

$$\lambda \int_{\bar{x}} (z - \bar{x}) dG(z) = (r + \delta)C(\phi) + rU - \lambda \bar{x}.$$ 

Finally, using the existence of the job destruction threshold, a job creation (JC) condition is obtained by rearranging (7) as

$$(1 - \alpha)\int_{\bar{x}} (z - \bar{x}) dG(z) = \frac{rC(\phi) + \gamma}{q(\theta)}$$

where the use of the sharing rule $(1 - \alpha)S(x) = F_k(x) - F_1$ has been made. This condition states that the expected benefit from a job to the firm must equal the average creation cost, which not only depends on the unit recruiting cost $\gamma$ and the average duration of the recruiting spell, but also total financial costs $C(\phi)$. Thus frictional credit markets limit firm entry onto labor markets and job creation.

**Proposition 1** - Under the condition that $\bar{x}g(\bar{x}) + \int dG(x) \leq \frac{\lambda + r + \delta}{\lambda}$, there exists a unique equilibrium for this economy defined by a pair $(\bar{x}, \theta)$ that solve the job creation and job destruction conditions.
conditions:  

\[
\frac{(1 - \alpha)}{r + \lambda + \delta} \int (z - x)dG(z) - \frac{\gamma + rC(\phi)}{q(\theta)} = 0 \quad (11)
\]

\[
\pi - \left( b + \frac{\alpha}{1 - \alpha} \theta (\gamma + rC(\phi)) + (r + \delta)C(\phi) \right) + \frac{\lambda}{r + \lambda + \delta} \int (z - \pi)dG(z) = 0 \quad (12)
\]

Figure 1 plots the job creation and destruction conditions in \((\pi, \theta)\) space, along with the equilibrium effects of an increase in total financial costs \(C(\phi)\). A rise in \(C(\phi)\) causes a downward shift and decrease in the slope of the job creation condition. For a given separation productivity, the increase in the cost of creating a firm means that the average total cost to creating a job is no longer covered by the product of the job relationship. Fewer firms thus enter the labor market, reducing the vacancy to unemployment ratio such that the job filling hazard \(q(\theta)\) increases. This, in turn, lowers the average search duration and costs on labor markets. Finally, the reduction in average job creation costs pushes the lowest viable job productivity down along the job destruction curve.

The same increase in financial costs \(C(\phi)\) causes a downward shift and a slight decrease in the slope of the job destruction curve. For a given labor market tightness, this increases the level of the lowest viable job productivity as the latter must compensate for the rise in the value of the firm’s outside option. By the same token, this leads to a decline in labor market tightness along the job creation curve. The net effect is, first, an unambiguous decline in equilibrium labor market tightness. Second, movements of the JD curve dominate the effect from the JC curve for the job destruction threshold when the condition \(\frac{\lambda}{\lambda + r + \delta} \left[ \pi g(\pi) + \int dG(x) \right] \leq \frac{\lambda q'(\theta)}{\lambda q'(\pi) + \alpha q(\theta)} < 1\) holds (see appendix), such that we have the following proposition:

**Proposition 2** - If \(\frac{\lambda}{\lambda + r + \delta} \left[ \pi g(\pi) + \int dG(x) \right] \leq \frac{\lambda \eta}{\lambda \eta + \alpha \theta}\), where \(\eta\) is the elasticity of the job filling rate with respect to labor market tightness, increases total financial costs \(C(\phi)\) cause a rise in the job destruction rate and a decline in labor market tightness.

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5The derivation of the job destruction condition involved, as intermediary steps, combining \(\int S(z)dG(z) = \frac{\gamma + rC(\phi)}{\lambda - \alpha q(\theta)}\) with (9) to write \(rU = b + \frac{\alpha}{1 - \alpha} \theta (\gamma + rC(\phi))\). The job creation condition has negative slope, and the job destruction condition positive slope under the condition that \(\pi g(\pi) + \int dG(x) \leq \frac{\lambda \eta \alpha \theta}{\lambda}\) (see the appendix for details).
2.3 A note on the financial relationship and the efficiency of job separation

The job destruction condition was written so as to be efficient from the point of view of the worker and the firm. The latter, however, is comprised of two agents and it may be the case that the job separation threshold is not efficient for either the entrepreneur or the creditor individually. That is to say, although \( F_g(\bar{x}) - B_l = 0 \), it is not necessarily the case that \( B_g(\bar{x}) - B_l = 0 \) and \( E_g(\bar{x}) - E_l = 0 \).

The asset value of the production stage for the entrepreneur (3) has slope \( E'_g(x) = \frac{(1-\beta)(1-\alpha)}{\lambda + r + \delta} \) from the fact that a fraction \((1-\beta)\) of the surplus from the creation of the firm accrues to the entrepreneur, and that a producing firm retains a fraction \((1-\alpha)\) of the profit flow from labor. Recall that there is free entry on credit markets, such that \( E_l = \frac{e}{p(\phi^*)} \), and call \( x^* \) a candidate job separation threshold efficient for an entrepreneur defined as \( E_g(x^*) - E_l = x^* \frac{(1-\beta)(1-\alpha)}{\lambda + r + \delta} - \frac{e}{p(\phi^*)} = 0 \).

Turning to the banker, since the slope of the latter’s asset value of the production stage is given by \( B'_g(x) = \frac{\beta(1-\alpha)}{\lambda + r + \delta} \), following the same arguments a candidate job separation threshold for the banker \( \tilde{x} \) is defined by \( B_g(\tilde{x}) - B_l = \tilde{x} \frac{\beta(1-\alpha)}{\lambda + r + \delta} - \frac{\kappa}{\phi^* p(\phi^*)} = 0 \). We now need to determine under what conditions \( x^* = \tilde{x} \). By combining the two candidate thresholds conditions, \( \frac{x^*}{\tilde{x}} = \frac{\kappa}{\phi^* e} \frac{1-\beta}{\beta} \). Since free entry on the credit market yielded an equilibrium credit market tightness \( \phi^* = \frac{1-\beta}{\beta} \frac{\kappa}{p^* e} \), it will always be the case that \( \frac{x^*}{\tilde{x}} = 1 \). It is straightforward to show in a final step that \( \bar{x} = \tilde{x} \), which leads to the following proposition:

**Proposition 3** - The job separation threshold \( \bar{x} \) is efficient not only from the firm’s but also the entrepreneur and banker’s points of view.
3 Financial markets, unemployment and aggregate productivity

The equilibrium rate of unemployment is determined by equating the steady state flows in and out of unemployment. Unemployed workers find a job at the rate \( \theta q(\theta) [1 - G(\bar{x})] \), as those meetings drawing a productivity below \( \bar{x} \) do not form a match, and workers can exit employment either because the firm is destroyed, at rate \( \delta \), or the job draws a productivity below the reservation \( \bar{x} \). The latter implies an endogenous exit rate \( \lambda \int_{\bar{x}}^{\infty} dG(x) \) such that the equilibrium rate of unemployment is given by

\[
    u = \frac{\delta + \lambda G(\bar{x})}{\theta q(\theta) [1 - G(\bar{x})] + \delta + \lambda G(\bar{x})}
\]  

(13)

By proposition 2, flows out of unemployment are reduced when total financial cost \( C(\phi) \) increase as the lower equilibrium market tightness reduces the meeting hazard rate \( \theta q(\theta) \), and the increased threshold \( \bar{x} \) reduces the fraction \( [1 - G(\bar{x})] \) of viable initial contacts. The rise in the lowest viable job productivity itself increases the equilibrium rate of endogenous job destruction \( \lambda G(\bar{x}) \). Thus greater financial intermediation costs reduce job creation and increase job destruction, resulting in an unambiguous increase in the equilibrium rate of unemployment.

Figure 2 provides a Beveridge curve representation of (13). The equilibrium effects of a rise in total financial costs appear as a change in the slope of the ray representing equilibrium labor market tightness in the \((\nu', u)\) plane and a change in the slope of the Beveridge curve. Greater financial costs are represented by an array with lesser slope \( \theta_1 < \theta_0 \), leading to a movement down the Beveridge curve with fewer vacancies and greater unemployment. As credit markets also affect job destruction, the Beveridge curve tilts upwards, further increasing unemployment, but also increasing equilibrium job vacancies. This raises the possibility that, under some configurations of the parameter space, credit market shocks lead to a positive co-movement of unemployment and vacancies. We will return to this point below.

We have determined that only firms with productivity greater than some threshold \( \bar{x} \), determined by frictions on labor and credit markets, remain in the goods market. That implies that the average of productivity of firms actually producing is given by the expected productivity conditional on surviving: \( \frac{\bar{x}}{1 - G(\bar{x})} \). Before going further, it is useful to express the cross-sectional dispersion in productivity of active firms, as in Lagos (2006), with the distribution \( H(x) \equiv \frac{G(x) - G(\bar{x})}{1 - G(\bar{x})} \). The latter
has the property \( \int_{x} x dH(x) = \frac{\int_{x} x dG(x)}{1 - G(x)} \) which we use to express aggregate output:

\[
Y = (1 - u) \int_{x} x dH(x)
\]

\[
\Rightarrow Y = AN
\]

where \( N = (1 - u) \) and \( A \equiv \int_{x} x dH(x) \). It appears clearly from this aggregate production function that frictions on credit markets affect both level of aggregate TFP and employment, and hence output. In other words, financial frictions, and possibly financial shocks, have real effects in this model by altering the flow in and out of aggregate employment and, through the sorting of firms, affecting the aggregate level of productivity. Moreover, increases in financial cost in the economy, \( C(\phi) \), lead to an increase in measured aggregate TFP as \( \frac{\partial A}{\partial C(\phi)} = \frac{\partial A}{\partial x} \frac{\partial x}{\partial C(\phi)} > 0 \). The intuition for this result is quite straightforward: increased total financial costs push the lowest productivity firms out of production, the sorting resulting in an increase in aggregate TFP. Summarizing, we have:

**Proposition 4** - An increase in total financial costs in the economy \( C(\phi) \) leads to an increase in aggregate unemployment and an increase in aggregate TFP.

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This follows from \( \frac{\partial A}{\partial C(\phi)} = \int_{x} x dG(x) \frac{f(x - x^*) dG(x)}{(1 - G(x))^2} > 0 \) and \( \frac{\partial A}{\partial C(\phi)} > 0 \). Aggregate TFP will depend on the labor market in the same manner as in Lagos (2006).
The effect on aggregate output depends on whether the productivity or employment change dominates. In the example below the latter will dominate and the equilibrium effect of a rise to total costs on financial markets will be to reduce aggregate output.

4  TFP during a Credit Crunch

The recent financial crisis presents many features that correspond to a sudden, unexpected, a breakdown in the efficiency of the credit matching technology. We study the aggregate effects of such a “credit crunch,” describing first the choice of the form for the underlying distribution of idiosyncratic shocks and calibrating the model on a set of observations for the US economy prior to the crisis.

4.1 Choosing a functional form for the underlying distribution of productivity shocks

Several papers which model firm heterogeneity in productivity argue that the data on the cross-section of firms present a fat right tail, and are well describe by a Pareto. Examples include Melitz (2003) in the international trade and Jones (2005) in the aggregation literature. We follow this tradition for the distribution of idiosyncratic productivity shocks $G(x)$ and assume:

$$G(x) = \begin{cases} 0 & \text{if } x < m \\ 1 - \left( \frac{m}{x} \right)^\mu & \text{if } x \geq m \end{cases}$$

where $m$ is a scale, and $\mu$ a curvature parameter. Pareto distributions have convenient expressions for the mean, $E(x) = \frac{\mu m}{\mu - 1}$ for $\mu > 1$, and variance, $Var(x) = \frac{m^2 \mu}{(\mu - 1)(\mu - 2)}$ for $\mu > 2$, and imply that the cross-sectional distribution of idiosyncratic productivity shocks $H(x)$ is also Pareto:

$$H(x) = \begin{cases} 0 & \text{if } x < \bar{x} \\ 1 - \left( \frac{x}{\bar{x}} \right)^\mu & \text{if } x \geq \bar{x} \end{cases}$$

with the same curvature parameter $\mu$ and where the job destruction threshold $\bar{x}$ determines the scale of the distribution.

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7 The empirical literature on firm size finds support for either a Pareto or concludes that it is difficult to distinguish between a Pareto and a log-normal. See Axtell, 2001.
Given this assumption on \( G(x) \), the equilibrium conditions for \((\bar{x}, \theta)\) can be expressed as

\[
\pi - \left( b + \frac{\alpha}{1 - \alpha} \right) \left( \frac{(1 - \alpha)}{r + \lambda + \delta} - \frac{\gamma + rC(\phi)}{q(\theta)} \right) = 0
\]

where we have replace the term \( \int (z - \bar{x})dG(z) = \frac{m^\mu \mu^{1-\mu}}{\mu - 1} \). Moreover, aggregate TFP is then given by

\[
A = \frac{\mu^\pi}{\mu - 1}
\]

and the variance of the cross-sectional of firm productivity is

\[
Var_H(x) = \frac{\pi^2 \mu}{(\mu - 1)^2 (\mu - 2)}
\]

Thus under a Pareto distribution for the underlying idiosyncratic productivity shocks, an increase in total financial costs \( C(\phi) \) increases the level of aggregate TFP and the dispersion in firm productivity. Thus in this model, changes in the dispersion in the variance of the cross-section of firm productivity is the endogenous response to a credit market shock.

### 4.2 Parametrization

The baseline parametrization of the underlying distribution of productivity sets \( \mu = 3 \) and \( m = \frac{2}{3} \), such that \( E(x) = 1 \) and \( Var(x) = 1/3 \). The choice of the curvature parameter will also imply, when firm exit is endogenized, an elasticity of the aggregate production function to capital of \( 1/3 \), as is typically assumed for Cobb-Douglas specifications. As the model is calibrated to quarterly data, the risk free rate is set to \( r = 0.01 \), consistent with an annualized return on a Treasury bill of 4%.

With regards to the labor market, the matching technology is assumed to be a Cobb-Douglas

\[
M_L(V, u) = \chi_L V^{1-\eta} u^\eta
\]

The elasticity \( \eta = 0.72 \) is as provided by the estimates in Shimer (2005) and we use the labor market Hosios condition to set the bargaining power of the worker in the wage determination to \( \alpha = 0.72 \). The value of non-employment activities, \( b \), is set to 0.4 as suggested by Shimer (2005), the unit vacancy cost \( \gamma = 0.25 \) so as to be consistent labor cost of recruiting found in the studies of Barron (1987) and Baron et al. (1997). The rate of exogenous firm destruction is set to \( \delta = 0.01 \), consistent with the average quarterly business failure rate for the United States. We set \( \lambda = 0.25 \), the Poisson arrival rate of new idiosyncratic productivity draws, such that, given the equilibrium value of the threshold \( \bar{x} \), the endogenous rate of job destruction \( \lambda G(\bar{x}) = 0.05 \). As a result, total job separation at a quarterly frequency are 6%, as reported in Davis et al. (2007). The level parameter in the labor matching function \( \chi_L \) is to 0.75 such the rate of unemployment equals 6.2%, as it stood at the beginning of the Fall of 2008. Note the the baseline calibration implies a
Table 1: Summary parameter values

| Scale $m$  | 3          | Poisson arrival $\lambda$ | 0.25 |
| Curv. $\mu$ | 2/3        | Exog. exit shock $\delta$ | 0.01 |
| Credit matching: |           | Labor matching: |      |
| level $\chi_C$ | 0.6        | level $\chi_L$ | 0.75 |
| elasticity $\varepsilon$ | 0.5        | elasticity $\eta$ | 0.72 |
| bank barg. weight $\beta$ | 0.5        | worker barg. weight $\alpha$ | 0.72 |
| Credit market search: |           | Labor market search: |      |
| Entrep. cost $e$ | 0.05       | vacancy cost $\gamma$ | 0.25 |
| Bank cost $\kappa$ | 0.05       | unemp. value $b$ | 0.4  |

quarterly job finding rate of 0.55.

The calibration of the credit market requires choosing parameters of the credit matching function, assumed to be of the form $M_c(\mathcal{B}, \mathcal{E}) = \chi_C \mathcal{E}^{1-\varepsilon} \mathcal{B}^{\varepsilon}$, the costs of prospecting on credit markets and the bargaining weight $\beta$. The baseline calibration adopts a "balanced" credit matching function, in the sense of having equal elasticities to both inputs, and the credit market Hosios condition. This minimizes the distortions caused by financial imperfections, i.e., we minimize $C(\phi)$ by setting $\beta = \varepsilon = 0.5$.

The average flow repayment to banks in this economy is $\int_{\tau} \rho(x) dG(x)$. We call the average return the interest rate $R$ that equates the expected present discounted average repayment to a unit consumption good, or such that $\frac{\int_{\tau} \rho(x) dG(x)}{R + \delta + \lambda G(\bar{x})} = 1$. Thus we can express the an average excess return over the economy’s risky free rate as

$$R - r = \int_{\tau} \rho(x) dG(x) - (r + \delta + \lambda G(\bar{x}))$$

We assume a symmetry in the cost of prospecting on the credit market, $\kappa = e$, and set this cost and the level parameter $\chi_C$ in the credit matching function jointly such that the average excess return, $R - r$, equals an annualized 3.6% corresponding to the average spread between the return on a BAA corporate bond and a Treasury bill.\(^8\) All parameter values are reported in Table 1.

\(^8\)The average repayment is defined as: $\int_{\tau} \rho(x) dG(x) = \beta \int_{\tau} [x - w(x)] dG(x) + (1 - \beta) \frac{1}{q(\tau)} [r + \delta + \lambda G(\bar{x})]$, where the wage is $w(x) = \alpha [x + \theta \gamma - r(1 - \theta) + \delta |C(\phi)| + (1 - \alpha) b]$. The derivation of the flow repayment $\rho(x)$ and the wage $w(x)$ are presented in the appendix.
4.3 Quantitative results: aggregate effects of a credit crunch

The effects of the credit crunch are measured as the equilibrium effects of a drop in the efficiency of the credit matching function $\chi_C$ that provokes a doubling in the average spread $R - r$ from an annualized 3.6% to 7.2%, a jump similar to that of corporate bond spreads during the Fall of 2008. Table 2 reports the ensuing change in aggregate output, TFP and unemployment for the baseline calibration.

Table 2 helps to make sense of the order of magnitude of the changes in aggregate variables caused by the credit crunch. It reports the change in measured aggregate output, TFP and unemployment between the onset of the crisis during the third quarter of 2008 and the second quarter of 2009, the first record of a positive quarter to quarter change in aggregate output. The table reveals that during this period, output contracted by 3.39%, TFP rose by 3.35% and the rate of unemployment increased by 66.3%

The next rows of Table 2 report the effect of the credit crunch in the model. The baseline calibration yields a 2.79% decline in output that results from a 4.66% increase in aggregate TFP and 68.89% increase in the rate of unemployment. These values are very similar to those reported for the U.S. economy. If we depart from the Hosios condition on the credit market, the effects of the same shock to the efficiency of matching on credit markets are amplified. When bargaining in strongly favorable to the bank ($\beta = 0.8$), then the contraction in aggregate output can reach 4.15%, with a 6.58% increase in TFP and doubling in the rate of unemployment.

<table>
<thead>
<tr>
<th>% change in aggregate:</th>
<th>Output</th>
<th>TFP</th>
<th>Unemployment</th>
<th>Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008:Q3 to 2009:Q2</td>
<td>-3.39</td>
<td>3.35</td>
<td>66.3</td>
<td>1</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline with $\beta = \epsilon = 0.5$</td>
<td>-2.79</td>
<td>4.66</td>
<td>68.89</td>
<td>1.23</td>
</tr>
<tr>
<td>$\beta = 0.8, \epsilon = 0.5$</td>
<td>-4.15</td>
<td>6.58</td>
<td>96.95</td>
<td>1.44</td>
</tr>
</tbody>
</table>

Sources: B.E.A., B.L.S. and Fernald (2009)

The individual wage rules $w(x)$ imply that individual wages should decline during a credit market induced downturn, principally due to the deterioration of the labor market which weakens a worker’s outside option in wage negotiations. As a credit crunch destroys the least productive jobs it is possible, nonetheless, for the average wage in the economy $W_t = \int w(x)dH(x)$ to rise. This is
indeed what has occurred during the Great Recession: real average hourly earnings, obtained from the B.L.S., increased by 1% between September 2008 and July 2009. As reported in last column of Table 2, the model is consistent with this fact.

Finally, it is important to stress that variations in aggregate activity in this model arise from changes in the mass of active firms, and not the population of firm through entry and exit, which remains constant at the exogenous rate $\delta$\textsuperscript{9}. The population of firms is composed, at any point in time, of a mass $N$ active firms and $\mathcal{V}$ recruiting/idle firms, for a total mass $\mathcal{F} = N + \mathcal{V}$ of firms in existence. The aggregate production function derived only considered active firms, such that the model’s measure of aggregate TFP corresponds to a capacity utilization corrected measure similar that constructed by Basu et al. (2006) and used in Fernald and Matoba (2009). If we call $\tau \equiv \frac{N}{\mathcal{F}}$ a measure of capacity utilization, then utilization is decreasing in financial costs, i.e. $\frac{\partial \tau}{\partial C(\phi)} < 0$. We will discuss this issue further in Section 5 when endogenizing the destruction of the firm.

4.4 Dispersion and the choice for the underlying distribution

The assumption of a Pareto distribution implied a variance of the cross-sectional dispersion of in productivity of active firms, $Var_H(x) = \frac{\bar{x}^{\mu}}{(\mu-1)^2(\mu-2)}$, that is increasing in the cut-off productivity $\bar{x}$. The baseline calibration resulted in $\bar{x} = 0.72$ and, hence, a variance $var_H(x) = 0.38$. After the credit crunch, this variance increases by 11% to $var_H(x) = 0.42$, consistent with evidence that measures dispersion in firm productivity are counter-cyclical, e.g. Bloom (2009) or Bachmann and Bayer (2009). This increase in dispersion, however, does not arise from a change in the properties of the distribution from which firm specific productivity draws are made, $G(x)$, but to the cross-section of active firms $H(x)$. This bring a slight distinction between the risk from the underlying technology draws $G(x)$ and risk from the macroeconomic environment measured by the dispersion in $H(x)$. The latter evolves as the endogenous response of the economy to shocks. This suggests that measures of dispersion used for the purpose of quantifying risk and uncertainty based on estimates from observed firm production and cost should to be corrected to account for firm entry and exit over the business cycle to identifying changes to $G(x)$.

In a related issue, empirical studies have difficulty rejecting a log-normal for a Pareto distribution for measures of firm size or productivity. The choice of the form of the underlying distribution

\textsuperscript{9}Section 5 endogenizes the break up of the firm. Clementi and Palazzo (2010) present an industry model with endogenous entry of exit with aggregate fluctuations in which the mean of the cross-section of firm productivity decreases after an expansionary aggregate productivity shock due to selection.
of idiosyncratic productivity shocks, however, can be crucial for the effects of a credit crunch on the dispersion of the cross-section. Consider an alternative assumption in which firms draw from a normal distribution, centered around either 0 or 1. The cross-section of firm productivity under such assumptions are represented in Figure 3, along with the change in the equilibrium job destruction productivity that follows from an increase in financial costs. In the first panel, where $x \sim N(0, \sigma^2)$, the cut-off productivity is to the right of the mean and the measure cross-sectional distribution of firm productivity would be well approximated by a Pareto. Moreover, an increase in financial costs implies the same qualitative effects, namely an increase in the level of aggregate TFP and an increase in the cross-sectional variance of firm productivity. The second panel of Figure 3 represents the cross-section of active firms when $x \sim N(1, \sigma^2)$. The implications of an increase in financial costs for the variance in firm productivity are the opposite to previously if the threshold productivity is below the mean, that is, there is a reduction in the variance of firm productivity.

### 4.5 The nature of technology shocks

The analysis raises a more the fundamental question on the nature technology shocks and makes the important distinction between technology and productivity, as in Basu and Fernald (2002). Indeed, shocks to credit markets generate positive co-movement between employment and output characteristic of business cycles, without any change in the underlying distribution of productivity. However, a residual of output and employment would indicate an unanticipated movement in aggregate TFP that is counter-cyclical, raise two issues. First, movements in aggregate TFP can be unrelated to shocks to technology, here the properties and the distribution $G(x)$. That credit shocks can move TFP and generate business cycle co-movement in quantities reinforces the view that other shocks are important for business cycles. Their identification, however, is not trivial. Second, the model
provides a potential mechanism for understanding the low correlation in aggregate data between employment and labor productivity over the business cycle.

The most appropriate concept of productivity shocks in the current framework are events that impact the mean $E(x)$ of the underlying distribution of idiosyncratic productivity. Under the assumption of a Pareto distribution, these can be of two sorts: either to the scale ($m$) or curvature ($\mu$) parameters. Table 3 presents the results of subjecting the model to such shocks that lead to a 1% increase in $E(x)$ for measured aggregate output, TFP and unemployment.

<table>
<thead>
<tr>
<th>% change in:</th>
<th>Scale $m$</th>
<th>Curv. $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.08%</td>
<td>1%</td>
</tr>
<tr>
<td>TFP</td>
<td>0.85%</td>
<td>2.05%</td>
</tr>
<tr>
<td>Unemployment</td>
<td>-2.22%</td>
<td>9.88%</td>
</tr>
<tr>
<td>$var_H(x)$</td>
<td>0.3</td>
<td>0.43</td>
</tr>
</tbody>
</table>

A shock that increases the scale parameter ($m$) presents all the features of what is traditionally thought of as a technological shock: a 1% increase in $E(x)$ induces a 0.85% increase in TFP, a 1.08% increase in aggregate output and a 2.22% drop in unemployment. A change in the curvature $\mu$ that results in the same 1% increase in $E(x)$ results in increases of 1% and 2.05% in aggregate output and TFP, respectively and, however, a 9.88% increase in unemployment.

5 Endogenizing firm exit

In this section we endogenize the break up of a firm, or the exit rate $\delta$, by assuming that the cost of operating a firm, even when idle/recruiting, is a function $\tilde{y}(x)$ of the idiosyncratic productivity $x$ with the properties $\tilde{y}'(x) < 0$. For the sake of simplicity, this function is assumed to take the form $\tilde{y}(x) = \max[\gamma - \sigma x, 0]$ where $0 < \sigma < 1$. This may alternatively be interpreted as the consequence of persistence in idiosyncratic shocks: lower draws are more costly if they tend to stay low for a prolonged period of time. We will show that there exists a reservation strategy for the entrepreneur-banker pair such that a firm is endogenously dissolved if $x < \bar{x}$, where the threshold is defined by a condition on the value of the firm at the recruiting stage, i.e. by $F_i(\bar{x}) = 0$. This threshold productivity for firm destruction is also shown to be below that of job destruction.
We begin with the modified Bellman equations for both agents on credit markets, maintaining the assumption of free entry:

\[ rE_c = 0 = -e + p(\phi) \int \max [E_l(x),0] dG(x) \]  
\[ rB_c = 0 = -\kappa + \phi p(\phi) \int \max [B_l(x),0] dG(x) \]  

(15)  
(16)

Since the entrepreneur-bank pair share the surplus \( E_l(x) + B_l(x) = F_l(x) \), under Nash bargaining the sharing rule is \( (1 - \beta)B_l(x) = \beta E_l(x) \) and it remains the case that equilibrium credit market tightness is given by

\[ \phi^* = \frac{1 - \beta}{\beta} \frac{\kappa}{e} \]

Combining (15) and (16) links the expected value of a new firm to total financial costs,

\[ \frac{\phi e + \kappa}{\phi p(\phi)} \equiv C(\phi) = \int \max [F_l(x),0] dG(x) \]  

(17)

Finally, the values of the recruiting and production stage are now:

\[ rF_l(x) = -\tilde{\gamma}(x) + q(\theta) [\max [F_g(x) - F_l(x),0]] + \lambda \int \max [F_l(z),0] dG(z) - \lambda F_l(x) \]  
\[ rF_g(x) = x - w(x) + \lambda \int \max [F_g(z),F_l(z),0] dG(z) - \lambda F_g(x) \]  

(18)  
(19)

Equation (18) assumes that firms do not draw a new productivity when meeting a worker, but can draw new values during the recruiting/idle stage. The asset equation of the production stage (19) has the same interpretation as earlier.

### 5.1 A margin of firm destruction

From equation (18), the slope of of the value of the recruiting stage, assuming the existence of a job destruction threshold \( \bar{x} \) such that if \( x \geq \bar{x} \) then \( F_g(x) - F_l(x) \geq 0 \) (which will be shown below), is

\[ F_l'(x) = \begin{cases} \frac{-\tilde{\gamma}(x)}{r + \lambda} > 0 & \text{if } x < \bar{x} \\ \frac{-\tilde{\gamma}(x) + q(\theta) \frac{dF_l(x)}{dF_l(x)}}{r + \lambda + q(\theta)} > 0 & \text{if } x \geq \bar{x} \end{cases} \]  

(20)
where the slope of the value of the production stage is \( F'_g(x) = \frac{1-\alpha}{r+\lambda} > 0 \) for all values of \( x \). Then for some \( x \) we have that \( F_l(x) < 0 \), such that \( F_l(x) = 0 \) defines a threshold below which the relationship between creditor and borrower cannot be sustained and is dissolved.

To ensure the existence of an equilibrium with employment, it must be that the value of the production stage is greater than the value of the recruiting stage for values of productivity greater than the job destruction threshold, defined by the condition \( F_g(\bar{x}) - F_l(\bar{x}) = 0 \). Thus a necessary condition for \( F'_g(x) > F'_l(x) > 0 \) is \( (1-\alpha) > \omega \).10 The same condition ensures that \( F'_g(x) > F'_l(x) \) and \( F_g(x) > F_l(x) \) for \( x < \bar{x} \), and that \( x < \bar{x} \). Finally, the slope of the recruiting stage below the job destruction threshold is smaller than above the threshold if \( \omega < \frac{1-\alpha}{r+\lambda} \), which, for plausible values of \( r \) and \( \lambda \) and the previous condition will be the case.

Evaluating the value of the firm at the destruction margin \( \bar{x} \), we have

\[
0 = -\tilde{\gamma}^{(\bar{x})} + \lambda \int_{\bar{x}}^{x} F_l(z) dG(z)
\]

which, with the expected value of the firm \( \int_{\bar{x}}^{x} F_l(z) dG(z) = C(\phi) \) defined in (17), yields a firm destruction condition

\[
\tilde{\gamma}^{(\bar{x})} = \lambda C(\phi)
\] (21)

that ties the destruction threshold \( \bar{x} \) directly to conditions on credit markets. This threshold is decreasing in the financial costs involved in to setting-up a new firm as it raises the opportunity cost of the existing relationship between an entrepreneur and a bank.

We can now solve for the value function of the firm in each stage. From the destruction condition (21), we have a value for the firm destruction productivity \( \bar{x} = \frac{\gamma - \lambda C(\phi)}{\omega} \), and a value function of the firm during the recruiting stage for \( x < \bar{x} \) that takes the form

\[
F_l(x) = \frac{\lambda C(\phi) - \gamma + \omega x}{r + \lambda}
\]

Using the job destruction condition, the value of the production stage is

\[
F_g(x) = F_l(\bar{x}) + \frac{1-\alpha}{r+\lambda} (x - \bar{x})
\]

\[ ^{10} \text{Throughout we will assume that that the job destruction threshold is below that value of productivity for which } \tilde{\gamma}(x) = 0, \text{ that is, that } \bar{x} < \bar{x} \text{ where } \bar{x} = \frac{\lambda}{\omega}. \text{ The appendix offers more details on the derivations pertaining to this section.} \]
Figure 4: Firm asset values and the non-active region

or

\[ F_g(x) = \frac{\lambda C(\phi) - \gamma - (1 - \omega - \alpha)\bar{x} + (1 - \alpha)x}{r + \lambda} \]

Finally, the value of the recruiting stage for \( x \geq \bar{x} \)

\[ F_l(x) = \frac{\lambda C(\phi) - \gamma + \omega x + q(\theta)F_g(x)}{r + \lambda + q(\theta)} \]

Figure 4 plots the firm asset values for each stage as a function of idiosyncratic productivity. The region between the thresholds \( \underline{x} \) and \( \bar{x} \) holds the mass of non-producing/idle firms, which is an endogenous quantity of the degree of financial market imperfection.

### 5.2 Job creation, destruction and equilibrium

We can now turn to the determination of job creation, job destruction and the equilibrium. In order to derive the extended model’s job destruction condition, write the surplus of a worker-firm pair with productivity draw \( x \) as

\[ rS(x) = x + \bar{\gamma}(x) - rU + \lambda \int \max[S(z), 0] dG(z) - \lambda (1 - \alpha)S(x) - q(\theta)(1 - \alpha)\max[S(x), 0] \]

\[ F_l(x) = \frac{\lambda C(\phi) + q(\theta)F_g(x)}{r + \lambda + q(\theta)} \]

For any positive \( C(\phi) \), \( \bar{x} \) will always be greater than the firm destruction threshold \( \underline{x} \).

---

\( ^{11} \)Note that for \( x > \bar{x} = \frac{y}{\phi} \), \( \bar{\gamma}(x) = 0 \) and the recruiting stage is \( F_l(x) = \frac{\lambda C(\phi) + q(\theta)F_g(x)}{r + \lambda + q(\theta)} \). For any positive \( C(\phi) \), \( \bar{x} \) will always be greater than the firm destruction threshold \( \underline{x} \).
This surplus has slope

\[ S'(x) = \begin{cases} \frac{1+\gamma'(x)}{r+(1-\alpha)x} > 0 & \text{if } x < \bar{x} \\ \frac{1+\gamma'(x)}{r+(1-\alpha)(\lambda+q(\theta))} > 0 & \text{if } x \geq \bar{x} \end{cases} \]  

(22)

where \( \bar{x} \) is given as \( S(\bar{x}) = 0 \). Since the slope is everywhere positive, there exists an \( x < \bar{x} \) for which \( S(x) < 0 \) such that \( \bar{x} \) does indeed define a job destruction threshold. Given (22), a solution for the surplus is given by \( S(x) = \frac{(1-\sigma)x-\bar{x}}{r+(1-\alpha)x} \) over the region \( x < \bar{x} \), and by \( S(x) = \frac{(1-\sigma)x-\bar{x}}{r+(1-\alpha)(\lambda+q(\theta))} \) over the region \( x \geq \bar{x} \). Evaluating the job surplus at the job destruction threshold \( \bar{x} \), we obtain the job destruction condition:

\[ rU - \bar{x} - \tilde{y}(\bar{x}) = \lambda \int_{\bar{x}} S(z) dG(z) \]

The job creation condition is found by combining (17) and (18) with the definition of the job and firm destruction margins (see the appendix for details),

\[ (r+\lambda G(\bar{x})) C(\phi) + \int_{\bar{x}} \tilde{y}(x) dG(x) = q(\theta)(1-\alpha) \int_{\bar{x}} S(x) dG(x) \]

such that we have the following proposition:

**Proposition 5** - Under the conditions that \( \bar{x} g(\bar{x}) + \int_{\bar{x}} dG(x) \leq \frac{\bar{x}+\delta}{\bar{x}} \) and \( \omega < \frac{1-\alpha}{\bar{x}} \), there exists a unique equilibrium for this economy defined by the triplet \((\underline{x}, \bar{x}, \theta)\) that solves the firm destruction, job creation and job destruction conditions:

\[ \tilde{y}(\bar{x}) - \lambda C(\phi) = 0 \]

\[ \frac{(1-\alpha)}{r+(1-\alpha)(\lambda+q(\theta))} \int_{\bar{x}} [(1-\omega)z-\bar{x}] dG(z) - \frac{\tilde{r} C(\phi) + \Gamma}{q(\theta)} = 0 \]

\[ \bar{x} + \tilde{y}(\bar{x}) - \left( b + \frac{\alpha}{1-\alpha} \theta (\Gamma + \tilde{r} C(\phi)) \right) + \frac{\lambda}{r+(1-\alpha)(\lambda+q(\theta))} \int_{\bar{x}} [(1-\omega)z-\bar{x}] G(z) = 0 \]

where \( \tilde{r} \equiv r + \lambda G(\bar{x}) \) and \( \Gamma \equiv \int_{\bar{x}} \tilde{y}(x) dG(x) \).
5.3 The aggregate production function

If each firm operates with a unit of capital and a unit of labor, we show that when there is a margin of endogenous firm destruction and idiosyncratic productivity is drawn from a Pareto, there is a non-linear aggregate production function relating the aggregate stock of capital, employment and output of the Cobb-Douglas form.

Assuming each firm operates with a unit of labor and a unit of capital, and that the cost paid by the banker in the recruiting stage cover the rental of the equipment which must be in place before a worker is hired, then aggregate stock of capital is simply the number of firms in existence \( K \equiv \bar{Y} + N \). The cross-sectional dispersion in firm productivity, both idle and producing, is given by the distribution \( T(x) = \frac{G(x)-G(x)}{1-G(x)} \). Thus the fraction of active firms is \( \frac{N}{\bar{Y}+N} = 1 - T(\bar{x}) \) and idle firms \( \frac{\bar{Y}}{\bar{Y}+N} = T(\bar{x}) \). As a result, the relationship between the aggregate stock of capital and aggregate employment can be expressed as \( N = [1-T(\bar{x})]K \).

Assuming idiosyncratic productivity is drawn from the earlier Pareto distribution, the cross-section of all firm in existence is also Pareto with scale parameter \( \bar{x} \) and curvature \( \mu \), such that \( 1 - T(\bar{x}) = (\frac{\bar{x}}{\bar{Y}})^\mu \). Consequently, \( N = (\frac{\bar{x}}{\bar{Y}})^\mu K \) which, by inverting, relates the job destruction and the firm destruction thresholds through the aggregate capital-labor ratio and the curvature parameter \( \mu \) as \( \bar{x} = (\frac{\bar{Y}}{N})^{1/\mu} \).

Aggregate output remains given by \( Y = AN \), where \( A = \frac{\mu}{\mu-1} \). Inserting the expression for employment \( N = (\frac{\bar{x}}{\bar{Y}})^\mu K \), we have \( Y = \frac{\mu}{\mu-1} \bar{x}^{1-\mu} \bar{x}^\mu K \) which, with the previous expression for the threshold \( \bar{x} \), yields the aggregate production function

\[
Y = \tilde{A} N^{1-v} K^v
\]  

(23)

where \( \tilde{A} \equiv \frac{\mu}{\mu-1} \) and \( v = 1/\mu \), \( 0 < v < 1 \). This function is homogeneous of degree 1 in labor and capital and presents all the usual properties of the Cobb-Douglas production function.

Assuming one considers the aggregate production function to be given by (23), the Solow residual constructed with data on output, employment and capital would be underestimate the productivity of producing firms by a factor \( \frac{\tilde{A}}{A} = \frac{\bar{x}}{\bar{X}} > 0 \). Moreover, this mis-measurement is increasing in the degree of credit market imperfection. In addition, a researcher constructing TFP as the the Solow residual \( \tilde{A} \) would measure a decline in productivity during a credit crunch, while the utilization adjusted measure of TFP, \( A \), increases.
Note also, that the aggregate capital-labor ratio is increasing in the productivity difference between the least productive active unit and the firm destruction threshold $\bar{x}$. Thus in a downturn provoked by a credit crunch, there is an increase in the aggregate capital-labor ratio. However, the ratio of capital effectively used in production to labor remains 1 at all times.

6 Conclusion

TO BE COMPLETED
References


Appendix to : TFP during a Credit Crunch

TO BE COMPLETED

A  A model of credit and job destruction

The first section of this appendix details the derivations in the main text and provides the proofs for the propositions. These include the existence of a unique equilibrium on the labor market and the necessary conditions on the equilibrium effects of a change in financial costs on the equilibrium job destruction threshold.

A.1 Worker-firm surplus:

Using equations (8) and (10), the steps to deriving the worker-firm surplus $S(x) = F_g(x) - F_l + W(x) - U$ are:

$$rS(x) = y(x) - w(x) + \lambda \int \max[F_g(z) - F_l, 0] dG(z) - \lambda [F_g(x) - F_l] - \delta F_g(x)$$

$$w(x) + \lambda \int \max[W(z) - U, 0] dG(z) - (\lambda + \delta) [W(x) - U]$$

$$-rF_l - rU$$

$$rS(x) = y(x) + \lambda \int \max[S(z), 0] dG(z) - (\lambda + \delta) S(x) - (r + \delta) F_l - rU$$

$$rS(x) = x + \lambda \int \max[S(z), 0] dG(z) - (r + \delta) F_l - rU$$

A.2 Restriction for a unique equilibrium

Differentiating equations (11) and (12), the slopes of the job creation (JC) and destruction (JD) curves are:

$$(JC): \frac{d\theta}{d\theta} = \frac{(1 - \alpha)}{r + \lambda + \delta} \left[ \frac{\lambda g(x) + \int dG(z)}{r C(\phi) + \gamma \frac{\partial q(\theta)}{\partial \theta}} \right] < 0$$
since \( q'(\theta) < 0 \), and

\[
(JD) : \quad \frac{d\theta}{dC} = \frac{1 - \frac{\lambda}{r + \lambda + \delta}}{\alpha p} \left[ \overline{x}_g(x) + \int dG(x) \right] < 0 \quad \text{if} \quad \overline{x}_g(x) + \int dG(x) \leq \frac{\lambda + r + \delta}{\lambda}
\]

A positive slope to the job destruction curve places restrictions on parameters, \( \overline{x}_g(x) + \int dG(x) \leq \frac{\lambda + r + \delta}{\lambda} \). To verify that a unique equilibrium exists, consider first that from the JC condition, labor market tightness tends to 0 as \( \overline{x} \) tends to the upper bound of its support. Second, from the JD condition, labor market tightness tends to infinity when as \( \overline{x} \) tends to the upper bound of its support guaranteeing, a unique equilibrium if the JC curve is above the JD at the lower bound of the support for \( x \). This will be the case for distributions such as the normal, and places a restriction on the scale parameter of a Pareto distribution.

### A.3 Equilibrium effect of financial costs \( C(\phi) \) on the job destruction threshold \( \overline{x} \)

Differentiate the job creation equation (11):

\[
\left( \frac{1 - \alpha}{r + \lambda + \delta} \right) \left[ \overline{x}_g(x) - \int dG(x) \right] \frac{\partial \overline{x}}{\partial C} = \frac{r}{q(\theta)} - \frac{\gamma + rC(\phi)}{q(\theta)^2 q'(\theta)} \frac{\partial \theta}{\partial C}
\]

\[
\frac{\partial \theta}{\partial C} = \frac{r}{\gamma + rC(\phi) q'(\theta)} + \frac{(1 - \alpha)q(\theta)^2}{(r + \lambda + \delta)(\gamma + rC(\phi) q'(\theta))} \left[ \overline{x}_g(x) + \int dG(x) \right] \frac{\partial \overline{x}}{\partial C}
\]

(24)

Now differentiate the job destruction equation (12)

\[
0 = \frac{\partial \overline{x}}{\partial C} - \frac{\alpha \theta r}{1 - \alpha} - (r + \delta) - \frac{\alpha}{1 - \alpha} (\gamma + rC(\phi)) \frac{\partial \theta}{\partial C} + \frac{\lambda}{r + \lambda + \delta} \left[ \overline{x}_g(x) - \int dG(x) \right] \frac{\partial \overline{x}}{\partial C}
\]

\[
0 = \frac{\partial \overline{x}}{\partial C} \left[ \frac{\lambda}{1 - \alpha} \left[ \overline{x}_g(x) + \int dG(x) \right] \right] - \frac{\alpha}{1 - \alpha} (\gamma + rC(\phi)) \frac{\partial \theta}{\partial C} - \frac{\alpha \theta r}{1 - \alpha} + r + \delta
\]

(25)
Substitute (24) into (25)

\[
\begin{align*}
\left[ \frac{\alpha \theta r}{1-\alpha} + r + \delta \right] &= \frac{\delta \bar{x}}{\partial C} \left[ 1 - \frac{\lambda \left[ \bar{x}g(\bar{x}) + \int dG(x) \right]}{r + \lambda + \delta} \right] - \frac{\alpha}{1-\alpha} \left[ \gamma + rK \right] \frac{\partial \theta}{\partial K} \\
\left[ \frac{\alpha \theta r}{1-\alpha} + r + \delta \right] + \frac{\alpha \cdot rq(\theta)}{1-\alpha \cdot q'(\theta)} &= \frac{\delta \bar{x}}{\partial C} \left[ 1 - \frac{\lambda \left[ \bar{x}g(\bar{x}) + \int dG(x) \right]}{r + \lambda + \delta} \right] - \frac{\alpha q(\theta)^2}{(r + \lambda + \delta)q'(\theta)} \frac{\partial \bar{x}}{\partial C} \\
\left[ \frac{\alpha \theta r}{1-\alpha} + r + \delta \right] + \frac{\alpha \cdot rq(\theta)}{1-\alpha \cdot q'(\theta)} &= \left[ 1 - \frac{\lambda q'(\theta) + \alpha q(\theta)^2}{(r + \lambda + \delta)q'(\theta)} \left[ \bar{x}g(\bar{x}) + \int dG(x) \right] \right] \frac{\partial \bar{x}}{\partial C}
\end{align*}
\]

The left hand side is strictly positive. The right hand side is of ambiguous sign, and hence \( \frac{\delta \bar{x}}{\partial K} > 0 \) if \( \frac{\lambda q'(\theta) + \alpha q(\theta)^2}{(r + \lambda + \delta)q'(\theta)} \left[ \bar{x}g(\bar{x}) + \int dG(x) \right] < 1 \). Using the condition for existence of a unique equilibrium \( \bar{x}g(\bar{x}) + \int dG(x) \leq \frac{\lambda + \alpha + \delta}{\lambda} \), call \( \Phi \equiv \frac{\lambda}{\lambda + r + \delta} \left[ \bar{x}g(\bar{x}) + \int dG(x) \right] \leq 1 \) and re-express the condition for \( \frac{\delta \bar{x}}{\partial K} > 0 \) as \( \Phi \left( 1 + \frac{\alpha q(\theta)^2}{\lambda q'(\theta)} \right) < 1 \). This places a further restriction as \( \Phi \leq \frac{\lambda q'(\theta) + \alpha q(\theta)^2}{\lambda q'(\theta) + \alpha q(\theta)^2} < 1 \) to guarantee that the equilibrium job destruction productivity is increasing in financial costs.

**A.4 Wage \( w(x) \)**

The wage is the outcome of Nash bargaining over the surplus of the worker-firm pair \( S(x) = F_g(x) - F_l + W(x) - U \), which yields the sharing rule \( (1 - \alpha) (W(x) - U) = \alpha (F_g(x) - F_l) \). To obtain the wage rule:

\[
(1 - \alpha) \left( w(x) + \lambda \int [W(z) - U] dG(z) - (\lambda + \delta) [W(x) - U] - rU \right) = \alpha \left( x - w(x) + \lambda \int [F_g(z) - F_l] dG(z) - \lambda [F_g(x) - F_l] - \delta F_g(x) - rF_l \right)
\]

Using the sharing rule \( (1 - \alpha) (W(x) - U) = \alpha (F_g(x) - F_l) \), this simplifies to

\[
(1 - \alpha) (w(x) - \delta [W(x) - U] - rU) = \alpha (x - w(x) - \delta F_g(x) - rF_l)
\]
which can be rearranged to have the wage on the left hand side as

\[
w(x) = \alpha (x - \delta F_g(x) - rF_l) + (1 - \alpha) (rU + \delta [W(x) - U])
\]

\[
w(x) = \alpha (x - \delta [F_g(x) - F_l] - (r + \delta)F_l) + (1 - \alpha) (rU + \delta [W(x) - U])
\]

Replacing \( rU \) with \( b + \frac{\alpha}{1-\alpha} \theta (\gamma + rC(\phi)) \) yields the wage rule

\[
w(x) = \alpha (x - (r + \delta)F_l) + (1 - \alpha) \left( b + \frac{\alpha}{1-\alpha} \theta (\gamma + rC(\phi)) \right)
\]

\[
w(x) = \alpha [x - (r + \delta)C(\phi) + \theta (\gamma + rC(\phi))] + (1 - \alpha)b
\]

### A.5 Repayment to creditors \( \rho(x) \)

The repayment to bankers is determined as \( \rho(x) = argmax (B_l - B_c)^{\beta} (E_l - E_c)^{1-\beta} \). The first order condition form a log transformation is

\[
\frac{\beta}{B_l-B_c} \frac{\partial B_l}{\partial \rho(x)} + \frac{1-\beta}{E_l-E_c} \frac{\partial E_l}{\partial \rho(x)} = 0.
\]

It is straightforward to show that

\[-\frac{\partial E_l}{\partial \rho(x)} = \frac{\partial B_l}{\partial \rho(x)} \text{ such that, with the free entry condition on the credit market, } \left( \frac{\beta}{B_l-B_c} - \frac{1-\beta}{E_l-E_c} \right) \frac{\partial B_l}{\partial \rho(x)} = 0 \]

which yields the sharing rule \((1-\beta)B_l = \beta E_l\), or \( B_l = \beta F_l \). From equations (5) and (7) we then have that

\[
-\gamma + q(\theta) \int_{\tau} [B_g(x) - B_l] dG(x) = \beta \left( -\gamma + q(\theta) \int_{\tau} [F_g(x) - F_l] dG(x) \right)
\]

\[
\int_{\tau} B_g(x) dG(x) = (1-\beta) \frac{\gamma}{q(\theta)} + \beta \int_{\tau} F_g(x) dG(x) \tag{26}
\]

### A.5.1 Average repayment

Substitute the definitions of the production stage asset values into equation (26):

\[
\int_{\tau} \left( \rho(x) + \lambda \int_{\tau} max [B_g(z), B_l, 0] dG(z) - (\lambda + \delta)B_g(x) \right) dG(x) = (1 - \beta) \frac{\gamma q(\theta)}{\gamma q(\theta)}
\]

\[
+ \beta \int_{\tau} \left( x - w(x) + \lambda \int_{\tau} max [F_g(z), F_l, 0] dG(z) - (\lambda + \delta)F_g(x) \right) dG(x)
\]
Rearrange the term to isolate the average repayment:

\[
\int \rho(x) dG(x) = (1 - \beta) \frac{ry}{q(\theta)} + \beta \int (x - w(x)) dG(x) + (\lambda + \delta) \int (B_g(x) - \beta F_g(x)) dG(x) - \lambda \int \left( \int \max [B_g(z), B_f, 0] dG(z) - \beta \int \max [F_g(z), F_f, 0] dG(z) \right) dG(x)
\]

and use the fact that Nash bargaining implied that \( \int B_g(x) dG(x) - \beta \int F_g(x) dG(x) = (1 - \beta) \frac{\gamma}{q(\theta)} \) to obtain

\[
\int \rho(x) dG(x) = \beta \int (x - w(x)) dG(x) + (1 - \beta) \frac{ry}{q(\theta)} + (\lambda + \delta)(1 - \beta) \frac{\gamma}{q(\theta)} - \lambda \int \left( (1 - \beta) \frac{\gamma}{q(\theta)} \right) dG(x)
\]

which simplifies to

\[
\int \rho(x) dG(x) = \beta \int \left[ x - w(x) \right] dG(x) + (1 - \beta) \frac{\gamma}{q(\theta)} [r + \delta + \lambda G(\pi)]
\]

### A.5.2 Individual repayment

Note from equation (8) and the wage equation that the slope of the firm asset equation in the production stage is \( F_g'(x) = \frac{(1 - \alpha)}{\lambda + \delta + \beta} \) where \( \alpha \) is the worker’s weight in wage bargaining. At the job destruction margin, \( F_g(\bar{x}) = F_i = C(\phi) \), such that a solution for \( F_g(x) \) is \( F_g(x) = \frac{(1 - \alpha)(x - \bar{x})}{r + \lambda + \delta} + C(\phi) \).

The slope of the banker’s value of the production state is \( B_g'(x) = \frac{\beta(1 - \alpha)}{r + \lambda + \delta} \) and, at the job destruction margin, we had that \( B_g(\bar{x}) = B_f = \frac{\gamma}{\phi p(\phi)} \). Thus a solution is \( B_g(x) = \frac{(1 - \alpha)(x - \bar{x})}{r + \lambda + \delta} + \frac{\lambda}{\phi p(\phi)} \).

Recall the value of the production stage to the banker, \((r + \lambda + \delta)B_g(x) = \rho(x) + \lambda \int B_g(z) dG(z)\), which may be re-expressed as

\[
\rho(x) = (r + \lambda + \delta)B_g(x) - \lambda \int B_g(z) dG(z)
\]
Using solutions for the value functions, we have

\[
\rho(x) = \beta (1 - \alpha) (x - \bar{x}) + \frac{\kappa (r + \lambda + \delta)}{\phi p(\phi)} - \lambda \int \left[ \frac{\beta (1 - \alpha) (z - \bar{x})}{r + \lambda + \delta} + \frac{\kappa}{\phi p(\phi)} \right] dG(z)
\]

Since \( \frac{\kappa}{\phi p(\phi)} = \beta C(\phi) \), this expression further simplifies to

\[
\rho(x) = \beta (1 - \alpha) (x - \bar{x}) + \frac{\beta (1 - \alpha) \lambda}{r + \lambda + \delta} \int \frac{[(z - \bar{x})] dG(z)}{r + \lambda + \delta} - \beta C(\phi) \int dG(z)
\]

\[
= \beta (1 - \alpha) (x - \bar{x}) + (r + \lambda) \beta C(\phi) - \frac{\beta (1 - \alpha) \lambda}{r + \lambda + \delta} \int [(z - \bar{x})] dG(z)
\]

A.6 Some calculus for Pareto distributions

For a Pareto, \( g(x) = \frac{\mu}{\lambda} \left( \frac{a}{x} \right)^{\mu} \) such that \( \int x dG(x) = \int x g(x) dx = \int \mu m^{a-\mu} dx \). Thus \( \int x dG(x) = \mu m^{a-\mu} \frac{\lambda}{\mu - a} \). Applying this to \( \int (z - \bar{x}) dG(z) \), we have \( \int \frac{dG(z)}{x} = \int \frac{\mu m^{1-\mu}}{\mu - 1} \). Since \( \int (z - \bar{x}) dG(z) = (\frac{\mu}{\lambda})^{\mu} \) we have \( \frac{\mu m^{1-\mu}}{\mu - 1} = \frac{a}{\lambda} G(\bar{x}) \). Hence, as used in the text, \( \int (z - \bar{x}) dG(z) = \frac{a}{\lambda} G(\bar{x}) \).

B Endogenous firm exit

B.1 Worker-firm surplus under endogenous firm death

Deriving the job destruction surplus in the extension to endogenous credit destruction

\[
rS(x) = x - w(x) + \lambda \int \max [F_{\bar{g}}(z), F_{\bar{f}}(z), 0] dG(z) - \lambda F_{\bar{g}}(x)
\]

\[
+ \bar{g}(x) - q(\theta) [\max [F_{\bar{g}}(x) - F_{\bar{f}}(x), 0]] - \lambda \int \max [F_{\bar{f}}(z), 0] dG(z) + \lambda F_{\bar{f}}(x)
\]

\[
w(x) + \lambda \int \max [W(z) - U, 0] dG(z) - \lambda [W(x) - U] - rU
\]

\[
rS(x) = x + \bar{g}(x) + \lambda \int \max [F_{\bar{g}}(z) - F_{\bar{f}}(z) + W(z) - U, 0] dG(z) - \lambda [F_{\bar{g}}(x) - F_{\bar{f}}(x)]
\]

\[
- q(\theta) [\max [F_{\bar{g}}(x) - F_{\bar{f}}(x), 0]] - \lambda [W(x) - U] - rU
\]
B.2 Job creation condition in extension

\[ r C(\phi) = r \int \frac{F_i(x)}{x} dG(x) \]

\[ = \int \frac{-\bar{\gamma}(x) + \lambda C(\phi) - \lambda F_i(x)}{x} dG(x) + \int \frac{-\bar{\gamma}(x) + q(\theta) [F_\theta(x) - F_i(x)] + \lambda C(\phi) - \lambda F_i(x)}{x} dG(x) \]

\[ = \int \frac{-\bar{\gamma}(x) + \lambda C(\phi) - \lambda F_i(x)}{x} dG(x) + q(\theta) \int \frac{F_\theta(x) - F_i(x)}{x} dG(x) \]

\[ (r + \lambda) C(\phi) = \int \left[ -\bar{\gamma}(x) + \lambda C(\phi) \right] dG(x) + q(\theta)(1 - \alpha) \int S(x) dG(x) \]

\[ \bar{\gamma}(x) - \lambda C(\phi) = 0 \]

\[ [r + \lambda G(x)] C(\phi) + \int \frac{\bar{\gamma}(z) dG(z) - q(\theta)(1 - \alpha) \int S(z) dG(z)}{x} = 0 \]

\[ rU - \bar{\gamma}(x) - \lambda \int S(x) dG(x) = 0 \]

Then we have, from the job creation condition, \( \int S(z) dG(z) = \frac{RC(\phi) + \Gamma}{(1 - \alpha)q(\theta)} \) and, \( rU = b + \alpha \frac{\thetaRC(\phi) + \Gamma}{1 - \alpha} \)

B.3 Deriving the aggregate production function

\[ Y = \frac{\mu}{\mu - 1} \bar{x}^{1-\mu} \bar{N}^{\mu} K \]

\[ = \frac{\mu}{\mu - 1} \left( \bar{N}^{1/\mu} N^{-1/\mu} \bar{x}^{1-\mu} \right)^{1-\mu} \bar{x}^{\mu} K \]

\[ = \frac{\mu}{\mu - 1} \bar{N}^{1/\mu} N^{-1/\mu} \bar{x}^{1-\mu} \bar{x}^{\mu} K \]

\[ = \frac{\mu \bar{x}}{\mu - 1} N^{1-\mu} \bar{x}^{\mu} K \]

\[ Y = \bar{A} N^{1-\nu} K^{\nu} \]