To Scrap or Not to Scrap:  
A Dynamic Discrete Choice Model of Vehicle Scarrpage

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Abstract

The paper constructs a dynamic stochastic discrete choice model of vehicle ownership and scrappage at the household level, and examines the factors that determine vehicle scrappage, new vehicle sales and fuel efficiency of new vehicles. The model is capable of generating annual scrappage rates of vehicles by age and fuel efficiency under various aggregate economic conditions, and replicating stylized features which are broadly consistent with historical data, including procyclicality of new vehicle sales, the comovement between gasoline prices and fuel efficiency of new vehicles, and varying scrappage rates for vehicles of different fuel efficiency during gasoline price hikes. The next step is to estimate the model and use it to evaluate the economic effect of the “cash-for-clunkers”-type programs.

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1 Introduction

The paper constructs a dynamic stochastic discrete choice model of vehicle ownership and scrappage at the household level, and examines the factors that determine vehicle scrappage, new vehicle sales and fuel efficiency of new vehicles. We then use the model to evaluate the economic effect of the “cash-for-clunkers”-type program. By giving scrappage credits based on the fuel efficiency of old and new vehicles, the cash-for-clunkers program was promoted as providing economic incentives to purchase new vehicles and reducing gasoline consumption by substituting more fuel-efficient vehicles for clunkers.

In order to evaluate the merits of the cash-for-clunkers program, we construct a structural model where households make a discrete choice regarding whether to keep the vehicle, or scrap it and purchase a new one. If the household decides to purchase a new vehicle, it then decides on the fuel efficiency of new vehicles. These decisions are influenced by the scrappage policy, such as the “cash-for-clunkers” program, and other relevant variables, including the age and fuel efficiency of vehicles currently owned, household income, gasoline and new vehicle prices, etc.

The paper builds upon the framework of Adda and Cooper (2000, 2007) which also investigate the effects of scrappage subsidies on car purchases. In their model, vehicles differ only by age. In our model, however, vehicles differ by both age and fuel efficiency, and the latter affects the cost of driving through gasoline prices. This extension allows us to examine the sensitivity of vehicle scrappage, new vehicle sales, composition of vehicles by age and fuel efficiency to gasoline prices. It also allows us to examine the implications of scrappage subsidies on both new vehicle sales and total gasoline consumption. These implications are very important for evaluating policies such as the cash-for-clunkers program.

In the current version of the paper, we calibrate and solve the benchmark model. We are able to derive annual scrappage rates of vehicles by age and fuel efficiency under given aggregate economic conditions. These aggregate economic conditions are represented by four state variables: average household income, gasoline prices, average prices of new vehicles, and an aggregate taste shock which affects the contemporaneous marginal rate of substitution between auto services and nondurables. The model generates stylized features which are broadly consistent with historical data observations, including procyclicality of new vehicle sales, the comovement between
gasoline prices and fuel efficiency of new vehicles, and varying scrappage rates for vehicles of different fuel efficiency during gasoline price hikes.

We intend to estimate the model using both aggregate sales data and cross-sectional data on the distribution of vehicles by age and fuel efficiency, and then use the estimated model to evaluate the short- and long-term effects of the cash-for-clunkers program on the vehicle scrappage, purchase and fuel efficiency of vehicles in use. We also plan to compare the implemented cash-for-clunker program with alternative designs in terms of cost effectiveness and welfare implications.

The paper is organized as follows. Section 2 describes historical data observations. Section 3 details the discrete choice model and the optimization conditions. Section 4 calibrates the model. Section 5 conducts a quantitative analysis of model implications for scrappage rates and choices of fuel efficiency for new vehicles. Section 6 estimates the model and conducts policy experiments. Section 7 concludes.

2 Historical Data

This section presents historical data observations on new vehicle sales, vehicle survival rates and cross-sectional distributions of vehicles by age and fuel efficiency.

Figure 1 presents annual average household income (US census), average vehicle prices (WARDS Automotive Yearbook), gasoline prices in the U.S. from 1967 to 2008, together with new vehicle sales from 1975 to 2008. The first three variables are in 2008 dollars. They are aggregate state variables that affect household decisions in our dynamic model. While household income and vehicle prices have an upward trend, gasoline prices exhibit wide swings during the period. Total new vehicle sales are highly pro-cyclical: the correlation coefficient between household income and new vehicle sales is 0.783 from 1975 to 2008. As the average household income declines during the economic recessions in the early 1980s, 1990, 2001 and 2007, new vehicle sales

\footnote{The section on estimation will replace the section on calibration when estimation results are available.}

\footnote{We collect annual average household income from US census, average vehicle prices from WARDS Automotive Yearbook, total vehicle sales from the US EPA Light-Duty Vehicle Fuel Economy Technology Annual Report, which contains total number of new light-duty vehicles (passenger cars and light trucks) sold each year from 1975 to 2008 as well as the average MPG of these vehicles.}
sales decline as well. Since gasoline price shocks either precede or coincide with the economic recessions in the early 1980s, 1990 and 2007, it is not surprising that higher gasoline prices during those three episodes also coincided with declining vehicle sales. Average vehicle prices do not fluctuate much during the time period under study.

Figure 2 plots the average fuel efficiency (MPG) of new light-duty vehicles (passenger cars and light trucks) sold each year from 1975 to 2008 versus the real gasoline prices (in 2008 dollars). The average MPG reached its peak in 1987 after the Corporate Average Fuel Economy (CAFE) standard was increased to 27.5 for passenger cars in 1985, which has been kept constant ever since. The slight decrease in average MPG from the peak level is largely due to the increasing market share of light trucks which have lower average MPG and lower CAFE standards. The market share of light trucks increased from about 20 percent to about 50 percent during the period. The declining average MPG coincided with low and stable gasoline prices from mid-1980s to 2003. After 2003, as gasoline prices take an upward swing, the average MPG of light-duty vehicles increases as well. Historical data suggest that the average fuel efficiency of new vehicles are responsive to gasoline price changes.

Figure 3 shows the average survival rates for vehicles of various ages.\(^4\) The survival rate describes survivability by age based on 1977-2003 NVPP registration database compiled by R.L.Polk and Company. The figure shows that the average survival rate varies inversely with the vehicle age, with passenger cars ten years and older displaying a steeper decline in the survival rate as compared to light trucks in the same age group. Only 10% of passenger cars survive to 20 years of age, while 25% of light trucks survive to the same age.

The NVPP (National Vehicle Population Profile) registration database also contains the number of vehicles registered by model and vintage. We are in the process of matching this data set with the fuel economy database by the Environmental Protection Agency (EPA) to generate the cross-sectional distribution of age and average MPG in 2000 and 2005. Figure 4 plots the number of vehicles by age in 2000 and 2005. The downward slope reflects varying vehicle scrappage rates as vehicles age. The fluctuations around the

decreasing trend reflect sales variations when the vehicles were sold as new. For example, the large drop in vehicles of age 5 in 2000 is due to the decrease in new vehicles sales in 1996: the total new vehicle sales was 13.1 million in 2006, compared with 15.1 and 14.5 in 1995 and 1997.

Figure 5 plots the average MPG by age in 2000 and 2005. The variations in the average MPG across age groups mainly come from two sources. The first is the differences in the average MPG when those vehicles were new and the second is the differences in scrappage rate across vehicles with different MPGs. Li, Timmins, and von Haefen (2009) show that the scrappage rate for vehicles over 10 years old decreases with MPG, all else equal and that the relation is stronger when gasoline prices are high. Knittel and Sandler (2010) confirm these results and find stronger relationships for California.

We can summarize the following observations from the time series and cross sectional distribution data described above:

1. Total new vehicle sales are highly procyclical.

2. Episodes of gasoline price spikes coincide with plunges in new vehicle sales.

3. Fuel Efficiency (MPG) of new vehicles declines while gasoline prices are low and stable, but increases when gasoline prices go up.

4. Average survival rates of vehicles decline by age.

5. Cross-sectional distribution of vehicles by age and MPG is affected by new vehicle sales, MPGs of new vehicles, and scrappage rates of vehicles by age and fuel efficiency. There is empirical evidence of an inverse relationship between fuel efficiency and scrappage rates, which tend to be stronger in times of high gasoline prices.

These MPGs are not consistent with the the MPGs of new vehicle in Figure 2. This is because when matching the NVPP registration data at the model level with MPG data from EPA, we used the MPG of base trim level (likely highest) for a given vehicle model. As a result, the average MPG in Figure 5 are on average higher than that in Figure 2. There are two ways to deal with this. One is to rematch the data and use the average MPG of all trim levels for a given model as the MPG of that particular model. The second way is to generate MPG using the trim level even for new vehicle in Figure 2. The consistency between MPGs of new and aged vehicles is important for model estimation. For this reason, we are postponing the estimation of our model until a rematching of the two data sets is completed.
In the section below, we construct a dynamic discrete choice model to capture the stylized features of the historical data. We focus in particular on households’ scrappage decisions on vehicles of given characteristics under various aggregate economic conditions.

3 The Model

The aggregate state of the economy at time $t$ is characterized by four state variables: the average household income $Y_t$, the real gasoline price $O_t$, the aggregate (real) price of new vehicles $P_t$, and an aggregate taste shock $\xi_t$, which affects the agents’ preference for the service flow of vehicles. We use $S_t \equiv \{Y_t, O_t, P_t, \xi_t\}$ to denote the vector of aggregate state variables.

3.1 The Household’s Optimization Problem

We constrain each household to own at most a single vehicle for any given period. A typical household is characterized by a vector of idiosyncratic state variables, denoted by $s_t = \{n, x, z\}$, where $n$ denotes the age of the vehicle owned, $x$ denotes the fuel efficiency of the vehicle which is measured by miles per gallon (henceforth MPG), and $z$ is a pair of household-specific idiosyncratic taste shocks which affect its decisions to keep or scrap its vehicle. We assume that each component of $z$ has an extreme value distribution.

In state $\{s, S\}$, the household decides whether to retain a car of age $n$ and fuel efficiency $x$, or scrap it. If the household decides to scrap the vehicle, it receives a scrap value and purchases a new car. The representative household can make decisions on the fuel efficiency of new vehicles which are available immediately. Depending upon particular scrappage subsidy policies, the scrap value $\pi$ can be a function of the age and fuel efficiency of both new and replaced vehicles. If the household decides to retain the vehicle, it takes utility in the flow of services from that vehicle and also enjoys a constant utility gain, $\alpha$, from the option of keeping the car. The vector of household specific taste shocks, $z = \{z_k, z_r\}$, are independent and identical realizations of utility gains from the option of keeping or replacing the vehicle.

In the benchmark model, we follow Adda and Cooper (2007) in making two simplified assumptions. First, there is no second-hand market for vehicles. Second, vehicle purchases must be paid out of current income. We
consider relaxing these two restrictions later on.

Let \( V(n, x, z, S) \) represent the value function of a representative household in state \((s, S)\). Given the aggregate and individual state variables, the household decides whether to keep or scrap the vehicle it owns.

The value function, \( V(n, x, z, S) \), is given by,

\[
V(n, x, z, S) = \max \{ V^k(n, x, S) + \alpha + z_k, V^r(n, x, S) + z_r \},
\]

where

\[
\begin{align*}
V^k(n, x, S) &= u(n, c_k) + \beta (1 - \delta) E(z', S'|z, S)V(n + 1, x, z', S') + \beta \delta E(z', S'|z, S)V^a(z', S'), \\
V^r(n, x, S) &= \max_{x' \in X} \{ u(1, c_r) + \beta (1 - \delta) E(z', S'|z, S)V(2, x', z', S') + \beta \delta E(z', S'|z, S)V^a(z', S') \}, \\
V^a(z, S) &= \max_{x' \in X} \{ u(1, c_a) + \beta (1 - \delta) E(z', S'|z, S)V(2, x', z', S') + \beta \delta E(z', S'|z, S)V^a(z', S') \},
\end{align*}
\]

\[
\begin{align*}
c_k &= Y - b \left( \frac{O}{x} \right)^{\eta + 1}, \\
c_r &= Y - P_{x'} - b \left( \frac{O}{x'} \right)^{\eta + 1} + \pi(n, x, x') \\
c_a &= Y - P_{x'} - b \left( \frac{O}{x'} \right)^{\eta + 1} + \pi_a
\end{align*}
\]

Here \( V^k(n, x, S) + \alpha + z_k \) and \( V^r(n, x, S) + z_r \) represent the values from keeping and scrapping a vehicle in state \((s, S)\) respectively. If the household decides to keep the vehicle, it enjoys utility from consumption of nondurable goods \( c_k \), and auto services which depend upon the vehicle age. The total amount of nondurable goods consumption, \( c_k \), is equal to the household income minus total gasoline expenses on driving the vehicle. We assume that the vehicle miles of travel in a given period is an exponential function of the gasoline cost per mile, that is,

\[
VMT = b \left( \frac{O}{x} \right)^{\eta},
\]

where \( \frac{O}{x} \) represents the gasoline cost per mile, and the exponent \( \eta \) represents the elasticity of mileage with respect to fuel cost of per unit of travel, i.e., the rebound effect. As a result, the total gasoline expenses to drive a vehicle with fuel efficiency \( x \) are given by \( b \left( \frac{O}{x} \right)^{\eta + 1} \).
We assume that each vehicle has an exogenous probability \( \delta \) of being destroyed by accidents each period. In this case, the household’s value function becomes \( V^a(z, S) \), which does not depend upon the characteristics of the destroyed vehicle. If the household decides to keep the vehicle, and the vehicle is not destroyed by accidents, the expected value for next period becomes \( E_{(x', S'|z, S)} V(n + 1, x', z', S') \).

If the household decides to scrap the vehicle, it makes an optimal decision on the fuel efficiency of the new vehicle to purchase and enjoys its service in the same period. We use \( X \) to denote the set of fuel efficiency levels the household can choose from. The amount of nondurable goods consumption in this case is equal to the household income subtracted by both the purchase price of the new vehicle and gasoline expenses spent on driving it. The nondurable goods consumption may be augmented by scrappage subsidy, \( \pi(n, x, x') \), with the total subsidy amount depending upon the age and fuel efficiency of the new and replaced vehicle. In order to capture the differences in prices of new vehicles with different fuel efficiency levels, we assume that

\[
P_{x,t} = P_t \left( \frac{x}{\bar{x}} \right)^{\theta},
\]

where \( P_{x,t} \) denotes the price of a new vehicle with fuel efficiency \( x \) and the exponent \( \theta \) captures the positive relationship between fuel efficiency and vehicle prices after controlling for other variables.

In the case the vehicle is destroyed by an accident, the household’s problem is similar to the case when the household decides to scrap the vehicle, except that the scrap value of the vehicle is fixed at \( \pi_a \).

We define the utility function to be additively separable between nondurable goods consumption \( c \), and the service flow from vehicles. We assume that the service flow from vehicles is negatively related to its age. Specifically, the utility function is given by

\[
U(n, x, c) = -\exp \left[ -\exp (\varphi_0 + \varphi_1 n) \right] + \xi \frac{c^{1-\gamma}}{1-\gamma}, \varphi_1 < 0,
\]

where the first term represents the household’s utility from auto service, which declines over time as the vehicle ages. The parameters \( \varphi_0 \) and \( \varphi_1 \) governs the relationship between the utility and vehicle age. The variable \( c \) represents nondurable consumption, which can be either \( c_k, c_r \) or \( c_a \) depending upon the household’s vehicle scrappage decision. The parameter \( \gamma \) indexes
the curvature of the utility function in nondurable consumption c. The taste shock \( \xi \) affects the intratemporal marginal rate of substitution between auto services and nondurable consumption.

### 3.2 The Household’s Optimization Decisions

In this section, we describe optimal decisions on fuel efficiency of new vehicles and the optimal decisions on scrapping a vehicle.

#### 3.2.1 Fuel Efficiency Decisions

Conditional on the decision to scrap the vehicle it currently owns, the household needs to make an optimal decision on the fuel efficiency of new vehicles to be purchased. The first order condition with respect to the fuel efficiency is

\[
U_c (1, c_{r,t}) \left( \frac{\partial P_{x',t}}{\partial x'} - \frac{\partial \pi (n, x, x')}{\partial x'} \right) = U_c (1, c_{r,t}) b (\eta + 1) \left( \frac{O_t (\mu(x'))^{\eta+1}}{x'} \right) + \sum_{j=2}^{N} E_t \left\{ [\beta (1 - \delta)]^{j-1} \prod_{m=2}^{j} [\mu (m, x', S_{t+m-1})] U_c (j, c_{k,t+j-1}) b (\eta + 1) \left( \frac{O_{t+j-1} (\mu(x'))^{\eta+1}}{x'} \right) \right. \\
+ \sum_{j=2}^{N} E_t \left\{ [\beta (1 - \delta)]^{j-1} \prod_{m=3}^{j} [\mu (m, x', S_{t+m-1})] [1 - \mu (j, x', S_{t+j-1})] \\
\times U_c (j, c_{r,t+j-1}) \left. \frac{\partial \pi (j, x', x'')}{\partial x'} \right\} .
\]

Here the left hand side represents the marginal cost of fuel efficiency of new vehicles in the form of the marginal increase in the purchase price of new vehicles minus the marginal increase in the scrappage subsidy due to higher fuel efficiency. Since \( \frac{\partial P_{x',t}}{\partial x'} \) depends positively on the average price of new vehicles, \( P_t \), an increase in \( P_t \) is likely to increase the marginal cost of fuel efficiency. An increase in the aggregate taste shock, \( \xi_t \), may increase the marginal utility of consumption from nondurable goods and potentially increase the marginal cost as well. Scrappage subsidies which encourage higher fuel efficiency of new vehicles would reduce the marginal cost.
The right hand side represents the marginal benefit from an additional unit of fuel efficiency. The first term represents the marginal savings in gasoline expenses in the period of vehicle replacement. The higher the gasoline prices, the higher marginal savings of gasoline expenses for any given $x'$. The second term represents the marginal savings in gasoline expenses if the vehicle is kept and also not destroyed by accidents. Here $N$ represents the maximum life span of vehicles and $\mu(m, x', S_{t+m-1})$ represents the probability of keeping the vehicle as a function of vehicle age, fuel efficiency and aggregate state variables. The third term represents the marginal impact of a particular choice of $x'$ on the scrappage subsidy in the future. Since typically scrappage subsidies are one-time unexpected promotion, the third term is often equal to zero in such policy experiments.

In the case when the vehicle is destroyed by accidents, the first order condition on the fuel efficiency of new vehicles is essentially the same as the case when the vehicle is intentionally scrapped, except that the derivative of scrappage subsidy with respect to vehicle characteristics is now zero due to constant scrap value after an accident.

### 3.2.2 Scrappage Decisions: the Optimal Stopping Problem

The decision on scrapping the vehicle is essentially an optimal stopping problem. The household decides on whether to scrap the vehicle at this period, or delay the decision for another period. On the one hand, the household may intend to scrap an aged vehicle, on the other hand, there may be option value to wait for more favorable aggregate states in the next period.

As shown in Rust (1987), given the extreme value distribution of idiosyncratic taste shocks $z$, the aggregate probability of keeping a vehicle of age $n$ and fuel efficiency $x$ in a given period characterized by the aggregate state $S$, is given by

$$
\mu(n, x, S) = \frac{\exp[V^k(n, x, S) + \alpha]}{\exp[V^k(n, x, S) + \alpha] + \exp[V^r(n, x, S)]}.
$$

Accordingly, the scrappage rate among vehicles of age $n$ and fuel efficiency $x$ is $1 - \mu(n, x, S)$. As shown in the above equation, a higher $\alpha$ raise the probability of keeping vehicles across all ages. The scrappage rate depends upon aggregate state variables and the age and fuel efficiency of the vehicle under consideration.
3.2.3 The Effect of Scrappage Subsidies on Decision Making

In the model, we assume that a household gets a constant amount of scrap value of $\pi_a$ if the vehicle is destroyed by accidents. When vehicles are intentionally scrapped, the household is eligible for scrappage subsidies which may depend upon the age of the scrapped vehicle, and the differences in the fuel efficiency between the new and scrapped vehicles, just as in the cash-for-clunkers program.

Based on the optimal decision rules above, scrappage subsidies affect households’ decision making through both income and substitution effects. We first discuss the income effect. A higher amount of scrappage subsidies relax the household’s budget constraint. Such a relaxation of the budget constraint increases the relative value of scrapping the vehicle, and reduces the marginal cost of purchasing a new vehicle. Since vehicles with higher fuel efficiency are relatively more expensive, larger scrappage subsidies also makes high fuel efficiency vehicles more affordable. Now we turn to the substitution effect. When scrappage subsidies depend positively on the age of the scrapped vehicle, and also the differences between the fuel efficiency of new and scrapped vehicles, not only does the relative value of scrapping the vehicle increase, but the marginal cost of purchasing vehicles of higher fuel efficiency decreases as well compared to those vehicles with lower fuel efficiency.

3.3 Aggregate Implications

Given the scrappage rates across different ages and fuel efficiency levels, and the initial distribution of vehicles across these two dimensions, the model can generate aggregate sales of new vehicles, the average fuel efficiency of new vehicles and the distribution of vehicles across ages and fuel efficiency levels over time. Since we solve the optimization problem by discretization of the state space, for simplicity we treat $x$ as a discrete variable when describing the evolution of the distribution.

Let $G_t^-(n, x)$ be the period $t$ cross sectional distribution of $n$ and $x$ prior to households’ scrappage decisions, and $G_t^+(n, x)$ be the period $t$ cross sectional distribution across these two dimensions after scrappage decisions are made. Accidents that destroy vehicles occur at the end of period, and explain the difference between $G_t^+(n, x)$ and $G_{t+1}^-(n, x)$.
Specifically, the evolution of the cross-sectional distributions is given by:

\[
G_t^+ (n, x) = \mu(n, x, S_t)(1 - \delta) G_{t-1}^+ (n - 1, x) \text{ for } n \geq 2,
\]

\[
G_t^+ (1, x') = \sum_{n \geq 2} \sum_{x \in X} \left\{ f(n, x, S_t; x') [1 - \mu(n, x, S_t)] (1 - \delta) G_{t-1}^+ (n - 1, x) + \delta g(S_t; x') \right\}.
\]

(10)

where \( f(n, x, S_t; x') = 1 \) if the household decides to replace the scrapped vehicle with a new vehicle with fuel efficiency \( x' \) given the set of state variables, \( \{n, x, S_t\} \), and \( g(S_t; x') = 1 \) if the household decides to choose a new vehicle with fuel efficiency \( x' \) after the vehicle is destroyed by an accident. In the latter case, only aggregate state variables, \( S_t \), matter for the decision making. The average fuel efficiency of new vehicles is thus given by:

\[
\sum_{x \in X} \left[ x G_t^+ (1, x) \right].
\]

The age-specific survival rate represents the average fraction of surviving vehicles out of their age cohorts. We integrate the probability of keeping the vehicle, \( \mu(n, x, S_t) \), over the invariant distribution of \( S \) to compute the average survival rate for a vehicle of age \( n \) and fuel efficiency \( x \) for a given period. We then use this average survival rate per period to approximate the probability of a vehicle of fuel efficiency \( x \) surviving to age \( n \). Specifically, such a probability is approximated by:

\[
\omega(n, x) = \prod_{j=1}^{n} \left[1 - \delta \right] \int \mu(j, x, S) dF(S)
\]

(12)

The aggregate sales of new vehicles is given by

\[
A_t = \sum_{x \in X} G_t^+ (1, x) dx + \delta,
\]

(13)

which is the sum of new purchases to replace vehicles that are scrapped or destroyed by accidents.

4 Calibration and Specification

In this section, we provide specifications for exogenous aggregate state variables and calibrate the rest of the benchmark parameters.
4.0.1 Transition Dynamics of Aggregate State Variables

Before conducting the simulation, we first specify stochastic processes for the four aggregate state variables: income, gasoline price, vehicle price, and aggregate taste shock. We assume that these variables follow a VAR(1) process.

\[
\begin{bmatrix}
Y_t \\
O_t \\
P_t \\
\xi_t
\end{bmatrix}
= \begin{bmatrix}
\mu_Y \\
\mu_o \\
\mu_p \\
\mu_\xi
\end{bmatrix}
+ \begin{bmatrix}
\rho_{yy} & \rho_{yo} & \rho_{yp} \\
\rho_{oy} & \rho_{oo} & \rho_{op} \\
\rho_{py} & \rho_{po} & \rho_{pp} \\
\rho_{\xi y} & \rho_{\xi o} & \rho_{\xi p}
\end{bmatrix}
\begin{bmatrix}
Y_{t-1} \\
O_{t-1} \\
P_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
e_{Y_t} \\
e_{ot} \\
e_{pt}
\end{bmatrix},
\]

where the mean of the error terms are zero and the covariance matrix of the error terms has a block diagonal structure:

\[
\Omega = \begin{bmatrix}
\omega_{yy} & \omega_{yo} & \omega_{yp} & 0 \\
\omega_{oy} & \omega_{oo} & \omega_{op} & 0 \\
\omega_{py} & \omega_{po} & \omega_{pp} & 0 \\
0 & 0 & 0 & \omega_{\xi\xi}
\end{bmatrix}.
\]

Since these processes are exogenous to the household and the parameters for the first three state variables are assumed to not depend on the unobserved aggregate taste shocks, we estimate the parameters in the first three equations outside of the dynamic optimization problem in order to reduce the computation burden.

Based on data on average household income, gasoline prices, and new vehicle prices from 1967 to 2008, we estimate a VAR(1) process for these three variables after removing the trend using a quadratic function. The parameter estimates are presented in Table 1.

In the benchmark simulation, we assume the aggregate taste shock to be an independent and identical random variable with a unit mean.

4.1 Calibration of Benchmark Parameters

In addition to the parameters in the VAR process, the set of remaining parameters for the model contains \( \{\beta, \delta, \gamma, N, b, \eta, \theta, \alpha, \varphi_0, \varphi_1\} \). Each period in the model represents one year, we thus set \( \beta \) to 0.97. The probability of
Table 1: Parameters in the VAR(1) Process

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<tr>
<td>$\rho_{yy}$</td>
<td>0.588</td>
<td>0.111</td>
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<td>0.188</td>
<td>$\rho_{py}$</td>
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<td>0.068</td>
<td>$\rho_{oo}$</td>
<td>0.840</td>
<td>0.116</td>
<td>$\rho_{po}$</td>
<td>-0.016</td>
<td>0.032</td>
</tr>
<tr>
<td>$\rho_{yp}$</td>
<td>0.445</td>
<td>0.233</td>
<td>$\rho_{pp}$</td>
<td>-0.998</td>
<td>0.396</td>
<td>$\rho_{pp}$</td>
<td>0.858</td>
<td>0.111</td>
</tr>
</tbody>
</table>

vehicle being destroyed by accidents, $\delta$, is set to 0.01. The parameter, $\gamma$, which governs the curvature of the utility function, is set to 1.8. We set the maximum life span of vehicles, $N$, at 20.\textsuperscript{6}

The parameters $b$ and $\eta$ determine the relationship between vehicle miles of travel and gasoline cost per mile. Small and Van Dender (2007) estimate the short-run and long-run rebound effect to be 0.045 and 0.22, respectively using U.S. state-level data from 1966-2001. These estimates are broadly consistent with many previous studies. The difference between the short-run and long-run effects reflects that consumers can make more adjustments (e.g., in travel pattern and vehicle purchase decisions) over a longer period. When using the data from 1997 to 2001, they find that the short-run and long-run rebound effects to be 0.022 and 0.107. They attribute the declining rebound effect largely to the rise in real income. Give that our model is calibrated using annual data, we use 0.022 as the value for $\eta$ and 1.2 as the value of $b$.

The parameter $\theta$ captures the price elasticity of fuel efficiency. We estimate $\theta$ to be 0.128 with a standard error of 0.048 in a hedonic price equation using data at vehicle model level from 1998 to 2005.\textsuperscript{7}

The parameters $\{\alpha, \varphi_0, \varphi_1\}$ are calibrated to approximate the survival rate of vehicles by age. The parameter $\alpha$ represents a constant utility gain from keeping the vehicle. As a result, its magnitude affects the level of the survival rate. The parameters $\varphi_0$ and $\varphi_1$ determine the rate at which the

\textsuperscript{6}We assume that the value to the owner of a vehicle in its 21st year is $V^a(z, S)$.

\textsuperscript{7}The dependent variable is the logarithm of average transaction prices at the model level while the explanatory variables include MPG, size, horsepower, weight (all in logs), as well as 36 dummies variables for division (e.g, Acura, Honda), 15 dummies for segment, and 7 year dummies. The coefficients on all four continuous variables are positive and statistically significant. The $R^2$ is 0.858.
survival rate declines by age. Specifically, we set $\varphi_0$ to 1.65 for all vehicle ages, and $\varphi_1$ to $-0.05$ for vehicles below ten years old and $-0.02$ for vehicles above ten years old to capture the age profile of the survival rate.

In the benchmark case, we set $\pi_o$, the scrap value in case of accidents, to $500$, and $\pi_r$, the autonomous component of the subsidy, to $1000$. For now we suppress the substitution effect of scrappage subsidies by assuming away their dependence on vehicle characteristics, and examine the policy implications of such dependence in the future.

5 Quantitative Analysis

Given the parameters, we can simulate the model. Since we are not yet estimating the model at this stage, the benchmark calibration mainly serves illustrative purposes. We first approximate the VAR process of aggregate state variables with a Markov chain process, where the total number and location of grids reflect the estimated parameters of the VAR processes. In our benchmark simulations, the number and range of grids for aggregate and household-specific state variables are as follows:

<table>
<thead>
<tr>
<th>Aggregate Gasoline Price</th>
<th>Vehicle Price</th>
<th>Household Income</th>
<th>Taste Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Grids</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Range</td>
<td>[1.87, 3.34]</td>
<td>[1.66, 1.91]</td>
<td>[5.08, 5.65]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Individual Vehicle age Fuel Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Grids</td>
</tr>
<tr>
<td>Range</td>
</tr>
</tbody>
</table>

Average vehicle prices and household income are in ten thousand dollars. In total, there are 1250 combinations of aggregate state variables. We solve the model for all the different combinations of aggregate and household-specific state variables.

5.1 Age-Specific Survival Rate

Figure 6 shows the age-specific survival rates for passenger cars and light trucks in the data, compared to the average survival rates simulated by the model. As shown in the figure, the survival rates of both passenger cars and light trucks diminish as vehicles age. According to the data, the average survival rate is around 50% for vehicles of 12 to 14 years of age. At age 25, the
survival rate for passenger cars and light trucks are respectively around 10% and 25%. The simulated average survival rates under benchmark calibrations follow those of light trucks very closely, except that the simulated survival rate declines drastically for vehicles at age 19 and 20, as compared to corresponding figures in the data. This discrepancy is partially explained by the mandatory maximum life span of 20 years specified in the model. The discrepancy will be reduced if we extend the maximum life span of vehicles.

5.2 Sensitivity of Scrappage Rates

Figure 7 shows the sensitivity of annual scrappage rates to gasoline prices, average vehicle prices, household income and aggregate taste shock, as simulated under benchmark calibrations. In order to focus on one particular aggregate state variable, we integrate over all the other dimensions of the aggregate state variables using the invariant distribution.

The first column of Figure 7 shows the average annual scrappage rates corresponding to a given range of gasoline prices. In the upper panel of the first column, we focus on 20-year-old vehicles with either the highest or the lowest fuel efficiency, and examine the changes in their annual scrappage rates as the gasoline price increases. As shown in the first column of Figure 7, as the gasoline price increases, the scrappage rates of 20-year-old vehicles increase. The scrappage rates of less fuel efficient vehicles are more sensitive to gasoline price changes. In the case where $n = 20$ and $x = 15$, the scrappage rate increases by around 0.3% in response to an increase in gasoline prices from $1.86 to $3.37.

The lower panel of the first column shows the change in the scrappage rates of 10-year-old vehicles with either the highest or the lowest fuel efficiency as the gasoline price increases. The pattern is different than for twenty-year-old vehicles. As the gasoline price increases, the scrappage rate of the 10-year-old vehicles with the lowest fuel efficiency increases, but for those with the highest fuel efficiency, the scrappage rate declines as the gasoline price increases. This is due to gasoline savings brought out by higher fuel efficiency of vehicles.

The sensitivity of scrappage rates to gasoline prices is consistent with the empirical findings by Li, Timmins and Von Haefen (2009). They find that the scrappage rate for vehicles over 10 years old decreases with MPG, all else equal and that the relation is stronger when gasoline prices are high.

The second column of Figure 7 shows the average annual scrappage rates
corresponding to a given range of average vehicle prices. The annual scrap-
page rate decreases as the average vehicle price increases, due to higher costs
of purchasing a new vehicle. The second column shows that for vehicles of
the same age, the scrappage rate of less fuel efficient vehicles is higher for
all the vehicle price levels and also declines more as average vehicle prices
increase. In the case where $n = 20$ and $x = 15$, the scrappage rate decreases
by around 035\% in response to a 15\% increase in average vehicle prices from
$16,635$ to $19,108$.

The third column of Figure 7 shows the average annual scrappage rates
corresponding to a given range of average household income. As shown in
the graph, as average household income varies between $50,819$ and $56,474
(around 11\%), the annual scrappage rate increases slightly. This pattern is
consistent with procyclicality of new vehicle sales.

The fourth column of Figure 7 shows that average annual scrappage rates
vary greatly for the given range of values for the aggregate taste shock. It
is evident that aggregate taste shocks may play a very important role in the
estimation of the model.

5.3 Choices of fuel efficiency of new vehicles

When households purchase a new vehicle, their choices of its fuel efficiency
depend upon both aggregate state variables and scrappage subsidies. If the
particular scrappage subsidy differs according to the age and fuel efficiency of
the scrapped vehicle, these characteristics affect the choices of fuel efficiency
as well.

In our benchmark case, we assume a type of scrappage subsidy policy
independent of the characteristics of vehicles. As a result, he scrappage sub-
sidy households receive only differs depending on whether the vehicle was
scrapped on purpose or destroyed in an accident. Figure 8 demonstrates
the choices of fuel efficiency of new vehicles corresponding to a particular
dimension of aggregate state variables for the cases of intentional and acci-
dental scrappage. Interestingly, households consistently choose a higher fuel
efficiency for their new vehicles when they intentionally scrap the vehicle
as compared to the case of accidental scrappage. Since the fuel efficiency
decisions in our benchmark case do not depend upon the characteristics of
current vehicles, the differences in the choices of fuel efficiency in these two
cases reflect the impact of different absolute amount of scrappage subsidies.
Since we assume that intentional scrappage is granted $500 more subsidy,
the income effect of the subsidies leads to the choice of higher fuel efficiency of new vehicles.

Now we focus on the fuel efficiency choices following optimal scrappage decisions. Figure 8 shows that as the gasoline price increases from $1.86 to $3.36, the fuel efficiency of new vehicles increases by about 3.5 miles per gallon on average. As average vehicle prices increase from $16,635 to $19,108, fuel efficiency of new vehicles decreases by around 4 miles per gallon on average. This pattern reflects the positive relationship between vehicle fuel efficiency and vehicle prices after controlling for other characteristics. As average vehicle price increases, the prices of vehicles with high fuel efficiency increase as well. The increase in vehicle prices leads to a decline in the fuel efficiency of new vehicles. As household income increases, fuel efficiency of new vehicles increases slightly. Since average household income does not vary much over time, its impact on the fuel efficiency of new vehicles is limited. The fuel efficiency of new vehicles varies greatly given the range of aggregate taste shocks, again underscoring the importance of aggregate taste shocks in estimation of the model.

6 Estimation and Policy Experiments

Given the cross-sectional distribution of vehicles by age and fuel efficiency in 2000, and conditional on data observations on household income, gasoline prices, and average vehicles prices, we will be able to simulate the cross-sectional distribution of vehicles in 2005 for given parameters. We intend to make use of the data on cross-sectional distribution of vehicles in 2005 to estimate the structural parameters in the model. Given the estimated structural parameters, we will proceed to evaluate the impact of the cash-for-clunkers program under different scenarios.

7 Conclusion
References


Figure 1: Income, Vehicle Prices, Gasoline Prices and New Vehicle Sales
Figure 2: Average MPG of New Vehicles versus Gasoline Prices (1975-2008)
Figure 3: Average Survival Rates of Vehicles by Age

![Graph showing average survival rates of vehicles by age. Two lines represent passenger cars and light trucks, with passenger cars having a slightly higher survival rate than light trucks at all ages. The y-axis represents the average survival rate, ranging from 0 to 1, while the x-axis represents vehicle age, ranging from 0 to 20.](Image)
Figure 4: Number of Vehicles by Age in 2000 and 2005
Figure 5: Average MPG by Age in 2000 and 2005
Figure 6: Average Survival Rates: Data and Simulation

Figure 7: Sensitivity of Scrappage Rates
Figure 8: Choices of Fuel Efficiency for New Vehicles