Equilibrium Credit Ratings and Policy *

Francesco Sangiorgi† Chester Spatt‡

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Abstract

We develop a rational expectations model in which the issuer of a financial instrument purchases credit rating(s) in order to provide useful information for investors and attract investor demand. We examine the nature of a credit-rating equilibrium in a staged game in which an issuer first solicits ratings and then selects which ratings to purchase and publish. We examine the impact of the timing of when rating fees are charged (e.g., when solicited ratings are initially provided and/or when some are published) and the nature of transparency and disclosure at the solicited rating stage; we consider scenarios in which solicited ratings are disclosed, the contact with rating agencies is disclosed or the solicited rating stage is opaque. Our results suggest that in a market with transparent contacts, market competition drives the fees to the solicited stage, so that ratings bias and shopping do not arise in the resulting equilibrium. Absent disclosure requirements (the opaque case) ratings bias would arise whenever the equilibrium involves purchase and publication of fewer ratings than the number of solicited ratings obtained.

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†Stockholm School of Economics

‡Carnegie Mellon University and National Bureau of Economic Research
1 Introduction

In the aftermath of the financial market crisis there has been a considerable spotlight on the accuracy of credit ratings and the potential for upward bias in these, especially in light of the issuer paying for ratings, purchasing and selecting those that would be available for the marketplace to consider in evaluating complex financial instruments.\(^1\) This context raises fundamental questions about the nature of equilibrium. To what degree are the communications between the issuer and rating agency publicly available at the stage of obtaining a solicited rating? How are the costs incurred by the rating agencies reflected in the pricing for providing solicited ratings and for publishing ratings? The underlying broad economic question is under what conditions do the ratings purchased in equilibrium reflect selective disclosure and bias to which investors would like to adjust? This is central to understanding the nature of credit ratings that are purchased.

While ratings bias emerges in the literature under a variety of assumptions in which investors react myopically to ratings, it is helpful to address whether the incentives to shop for ratings and selectively disclose disappears under rationality.\(^2\) For example, is the rationality of investors sufficient to guarantee unbiased ratings? Would associating the costs of the rating process with the solicited rating stage eliminate ratings shopping and selective disclosure? Would the enforcement of mandatory disclosure of solicited ratings guarantee unbiased ratings?

A potential source of ratings bias is the ability of issuers to obtain information from rating agencies without being required to disclose that. Of course, disclosure of such ratings could take a variety of forms such as mandatory disclosure of the information provided by the rating agency, disclosure of the contact of a particular rating agency by the issuer in the particular context (e.g., the underlying information might be complex and for which it would be too difficult to mandate the disclosure of the fundamental information), or as has been historically the case, viewing the contact as private. Indeed, the economics of the context might point to the universality of such contacts as when the costs of soliciting ratings are sufficiently low, which often has mirrored the past practice. This discussion suggests a number of policy issues including what types of disclosure should be required, the extent to which the regulator should influence the structure of fees across stages and the incentives between the stages of getting a solicited rating and purchasing and publishing the rating. It also emphasizes the importance of the form of equilibrium. Under rational expectations disclosure of contacts with the rating

\(^1\) An interesting empirical analysis that documents the potential subjectivity in ratings is Griffin and Tang (2009). The potential for bias in ratings and more specifically, the apparent inverse relationship between ratings standards and the success of a rating organization is illustrated by Lucchetti (2007), who reports that Moody’s market share “dropped to 25% from 75% in rating commercial mortgage deals after it increased standards.”

\(^2\) One very interesting treatment of ratings under rational expectations is Opp, Opp and Harris (2010). They develop a model to examine information acquisition by a monopolist rating agency in the face of regulatory distortions and asset complexity.
agency is very powerful and can eliminate ratings bias.

We develop a rational expectations framework in which the issuer can help convey information to the market by using ratings agencies. We examine the impact of when ratings fees are charged (e.g., when solicited ratings are obtained or only if the issuer decides to purchase and publish the rating) and the transparency of the solicited ratings stage. In addition to mandatory disclosure of all solicited ratings we focus upon two disclosure alternatives—one in which all contacts for soliciting ratings are transparent (but not the ratings themselves) and an opaque alternative in which the contacts are not disclosed. Our results suggest that in a market with transparent contacts, market competition in fee setting leads the equilibrium structure of fees to be at the solicited stage rather than the ratings publication stage. In this equilibrium all potential ratings are solicited and disclosed, implying that ratings shopping and ratings bias do not arise, as it is optimal for the issuer to disclose all ratings obtained, if publication is costless. In the spirit of Akerlof (1970), when publication is costless investors would form an extremely adverse inference when solicited ratings were not disclosed. However, if fees are exogenously required to be paid at the ratings publication stage, then ratings shopping and ratings bias could arise. Furthermore, absent any disclosure requirement (the opaque case) ratings shopping and ratings bias would arise whenever the equilibrium entails purchase and publication of fewer ratings than the number of solicited ratings obtained—as the issuer would then selectively choose which ratings to publish (if publishing only one rating, the issuer would publish the higher rating), or indeed, whether to publish any ratings. Indeed, we show in the opaque case that discretionary disclosure arises in equilibrium, even without any disclosure cost.

While we are able to show that an equilibrium emerges without ratings bias under specific conditions (such as transparency of ratings contacts and endogenous allocation of ratings costs across stages), the absence of bias is not a robust or universal outcome of our model—even under the assumption of rational expectations. To the extent that ratings bias often emerges, this would appear to undercut the reliance on ratings by regulators.

The issue of allocation of cost has received important attention by policymakers. For example, in the New York State Attorney General’s 2008 settlement with the three major

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3 Also, see the related intuition in Grossman (1981) and Milgrom (1981). Furthermore, Verrecchia (1983) examines the case of costly disclosure, demonstrating that disclosure is optimal for realizations at or above a critical level (related ideas in a principal-agent context with costly state verification are developed in Townsend (1979)). The greater the cost the higher is the disclosure threshold and the more selective and discretionary is disclosure. Within Verrecchia’s framework public disclosure is optimal for all realizations when it is costless and disclosure is not discretionary (the Akerlof (1970) intuition in his context).

4 Indeed, much of the focus of earlier theoretical papers has been on situations in which ratings shopping and selective disclosure do arise in equilibrium (see, for example, Bolton, Freixas and Shapiro (2008), Mathis, McAndrews and Rochet (2009), Sangiorgi, Sokobin and Spatt (2009), and Skreta and Veldkamp (2009)). A notable exception is Opp, Opp and Harris (2010), who also use rational expectations and examine a context with a single credit-rating agency).
rating agencies mandated fees at the solicited stage (though not barring them at the publication stage) in an attempt to reduce ratings shopping (Office of New York State Attorney General, 2008). The greater the fee at the solicited stage the fewer the ratings that will be potentially be purchased and the more limited the scope for selective disclosure and ratings shopping. However, if the securities regulator requires mandatory disclosure of solicited ratings, then the difference between ex-ante and ex-post costs in not meaningful, as withholding a rating is not an option. However, even in the case of mandatory disclosure it is possible that informal discussion between the issuer and rating agency might arise that would be unobservable by investors, leading to the possibility of indirect selective disclosure in the equilibrium. This potentially ties back to the issue of model transparency—one of the potential reasons that a rating agency might choose to publish much of its model is to facilitate selective disclosure in equilibrium.

This draft is organized as follows. Section 2 describes the underlying specification of the model. In Section 3 we describe the issuer’s disclosure decision for a given number of solicited ratings. Sections 4 and 5 address the decision of how many ratings to solicit, given the extent of transparency of the market. To ease exposition, for each scenario we examine the cases of ex-ante and ex-post fees separately, and then we extend the analysis by endogenizing the cost as the result of competition in fees between the rating agencies. Section 6 describes mixed-strategy equilibria in the opaque market case. Section 7 describes the equilibrium in the mandatory disclosure regime. Section 8 concludes.

2 Setup

An issuer is endowed with one unit of an asset with random payoff

$$X \sim N(\mu_X, \sigma_X^2).$$

The issuer is risk neutral, and has an exogenous holding cost for the asset equal to $V$. The issuer can either hold the asset or sell it to the market, which is populated by a continuum (with mass equal to one) of risk-averse investors. Investors have constant absolute risk averse (CARA) preferences with common risk aversion coefficient $r$. As a consequence, investors’ valuation of the asset equals the expected value of the asset minus a risk adjustment. All players have rational expectations, so investors’ valuation of the asset will depend on the information available to them in equilibrium. Ex-ante, the issuer and the investors share the same prior information about the asset value. We assume $V > r\sigma_X^2$, so that the asset is worth more to investors than it is to the issuer, implying ex-ante gains from selling the asset to the market.
The issuer can influence investors’ valuation of the asset by conveying information to the market via rating agencies. We assume there are two such agencies, endowed with the same rating technology: each rating agency can produce an unbiased noisy signal, or rating

\[ S_i = X + \varepsilon_i, \]

for \( i = 1, 2 \), with \( \varepsilon_i \) uncorrelated with \( X \), i.i.d. and

\[ \varepsilon_i \sim N(0, \sigma^2_\varepsilon). \]

Hence, the two rating agencies have equivalent, but independent technologies.

The rating process is as follows. The issuer can approach rating agency \( i \) and solicit its rating; in which case the issuer observes \( S_i \). Once a rating is solicited, the issuer can either withhold the rating or publish it; only in the latter case would investors observe the rating. Ratings are costly: the issuer pays an ex ante fee \( c \) to solicit a rating, and pays an ex post publishing cost \( \chi \), whenever she decides to make the rating public. Moreover, the rating process can be either transparent or opaque, depending on whether or not the act of soliciting a rating is observable by investors.

The timing of the model is the following: first the issuer decides whether to hold the asset or to sell it to the market. Conditional on selling, the issuer decides whether to solicit zero, only one, or both ratings. After the solicited ratings are observed, the issuer decides which solicited ratings to disclose (if any). Then, the asset is sold to investors, the payoff \( X \) is realized and consumption takes place.

We remark that the setup we consider is one in which:

i) There is no ex ante information asymmetry between the issuer and the investors.

ii) There is no agency problem on the part of the rating agencies: solicited ratings are unbiased and reported truthfully.

iii) Investors have rational expectations and understand the incentives behind the issuer’s actions. Consequently, the outcomes of the model are not driven by investors’ naiveté.

This setup represents a somewhat ideal situation in which many of the concerns regarding the rating industry in recent years have been resolved. Still, from a positive perspective, some questions remain. For example, is investors’ rationality sufficient to guarantee unbiased ratings? Will incentives to shop for ratings and to selectively disclose high ratings disappear
in this case? Also, from a normative perspective: do ex-ante costs eliminate ratings shopping? If enforced, does mandatory disclosure of solicited ratings guarantee unbiased ratings?

The solution concept implemented is the perfect Bayesian equilibrium (PBE). In this rating game, a PBE consists of a strategy profile for the issuer, a price function, and a belief system held by investors such that the strategies are sequentially rational given the belief system and the belief system is consistent given the strategy profile. In the rest of the analysis we informally refer to the equilibrium, avoiding the more formal definition.

3 Equilibrium price and disclosure policy

The model is solved backwards. In this Section we consider the equilibrium price and the issuer’s disclosure policy for the cases in which zero, one, or two ratings are solicited, under the assumption that number of solicited ratings is observed by investors.

3.1 No ratings solicited

In the case of no ratings solicited, the price is set by using only prior information, and therefore equal to

\[ p_0 = \mu_X - r\sigma_X^2. \]

3.2 One rating solicited

Lemma 1. (Verrecchia, 1983) Conditional on only one rating, \( S_i \), being solicited, the equilibrium has the following form: there exists a threshold \( \bar{S} \) and a price function \( p(\cdot) \) such that:

1) The issuer’s optimal strategy is to publish the rating if and only if \( S_i > \bar{S} \)

2) The price \( p(\cdot) \) is consistent with the strategy of the issuer. Let \( p(S_i) \) and \( p(\bar{S}) \) denote the asset price conditional on the rating being published or withheld, then

\[ p(S_i) = E(X|S_i) - r\text{Var}(X|S_i); \quad (1) \]

\[ p(\bar{S}) = E(X|S_i \leq \bar{S}) - r\text{Var}(X|S_i \leq \bar{S}). \quad (2) \]

As \( \chi \) tends to zero, \( \bar{S} \) tends to minus infinity, so that the rating is always disclosed.
3.3 Two ratings solicited

Given the two ratings, we use the following notation for first- and second-order statistics

\[ \hat{S}_1 := \max(S_1, S_2); \quad \hat{S}_2 := \min(S_1, S_2). \]

**Lemma 2.** Conditional on two ratings being solicited, the equilibrium has the following form: there exists a threshold $\hat{S}_0$, a function $s(\cdot)$ and a price function $p(\cdot, \cdot)$ such that:

1) The issuer’s optimal strategy is:

i) withhold both ratings, for $\hat{S}_1 \leq \hat{S}_0$

ii) publish $\hat{S}_1$ and withhold $\hat{S}_2$, for $\hat{S}_1 > \hat{S}_0$ and $\hat{S}_2 \leq s(\hat{S}_1)$

iii) publish both ratings, for $\hat{S}_1 > \hat{S}_0$ and $\hat{S}_2 > s(\hat{S}_1)$,

2) The price $p(\cdot, \cdot)$ is consistent with the strategy of the issuer. Let $p(\hat{S}_1, \hat{S}_2)$, $p(\hat{S}_1, \phi)$ and $p(\phi, \phi)$ denote the asset price conditional on two, one and no published ratings, then

\[
p(\hat{S}_1, \hat{S}_2) = E\left(X \mid S_1, S_2\right) - r\text{Var} \left(X \mid S_1, S_2\right); \quad (3)
\]

\[
p(\hat{S}_1, \phi) = E\left(X \mid \hat{S}_1, \hat{S}_2 \leq s(\hat{S}_1)\right) - r\text{Var} \left(X \mid \hat{S}_1, \hat{S}_2 \leq s(\hat{S}_1)\right); \quad (4)
\]

\[
p(\phi, \phi) = E\left(X \mid \hat{S}_1 < \hat{S}_0\right) - r\text{Var} \left(X \mid \hat{S}_1 < \hat{S}_0\right). \quad (5)
\]

As $\chi$ tends to zero, $\hat{S}_0$ and $s(\hat{S}_1)$ tend to minus infinity, so that both ratings are always disclosed.

Denote with $\Pi_2(S_1, S_2; c, \chi)$, $\Pi_1(S_i; c, \chi)$ and $\Pi_0$, issuer’s profits in the case of two, one and zero solicited ratings respectively. Lemma 1 and Lemma 2 imply that

\[
\Pi_2(\hat{S}_1, \hat{S}_2; c, \chi) = \max \left\{ p(\hat{S}_1, \hat{S}_2) - 2\chi, p(\hat{S}_1, \phi) - \chi, p(\phi, \phi) \right\} - 2c \quad (6)
\]

\[
\Pi_1(S_i; c, \chi) = \max \left\{ p(S_i) - \chi, p(\phi) \right\} - c \quad (7)
\]

\[
\Pi_0 = p_0 \quad (8)
\]

3.4 Shopping and ratings bias

Lemma 1 follows from Verrecchia (1983) discretionary disclosure model, in which sufficiently negative information is withheld in the presence of costly disclosure. Lemma 2 extends this intuition to the case of multiple sources of information and illustrates how cherry picking among multiple ratings is also fully consistent with a rational expectations equilibrium. Moreover, in
the presence of publishing costs, ratings are positively biased, where the bias is defined as the difference between the published ratings and the fundamental value of the asset. The bias in ratings originates from discretionary disclosure. As Lemma 2 illustrates, when two ratings are solicited and ex-post fees are strictly positive, in equilibrium sufficiently negative information is not disclosed, and in some states of the world only the highest of the two ratings is published.

The model considered here is one in which the market is dominated by sophisticated investors. As a consequence, the bias in the ratings is not reflected in the equilibrium price because of rational expectations, and biased ratings would not create allocative distortions. Nevertheless, there could be an infinite (but countable) number of less sophisticated investors in the model that take ratings at their face value. These investors would make suboptimal portfolio decisions because of the bias and suffer a welfare loss. Taking into account the welfare of such naive agents, biased ratings would be undesirable.

4 The soliciting decision: the transparent market case

This section determines the issuer’s choice of the number of solicited ratings, under the assumption that the rating process is transparent: investors are assumed to observe the number of solicited ratings. First we determine the issuer’s choice for exogenously given costs. To ease exposition, we initially deal separately with the cases of ex-ante and ex-post fees. Then we endogenize the costs as the result of competition in fees among rating agencies.

4.1 Exogenous fees

The next proposition deals with the case in which fees are charged only at the publishing stage.

Proposition 1. Let \( c = 0 \), and the market be transparent. For a given risk aversion \( r > 0 \), there exist values \( \chi_1, \chi_2 \) such that the issuer solicits both ratings for \( \chi < \chi_1 \), and is indifferent among soliciting zero, one or two ratings for \( \chi > \chi_2 \). For \( r = 0 \), the issuer never solicits any rating.

\[ b := E(S^p - X D), \]

where \( D = D_1 \cup D_2 \)

More formally, denote with \( D_1, D_1, D_2 \) the sets of states of the world in which, respectively, only \( S_1 \), only \( S_2 \), or both \( S_1 \) and \( S_2 \) are published. Let the variable \( S^p \) be defined as taking values \( S^p(\omega) = S_1 \) if \( \omega \in D_1 \), \( S^p(\omega) = S_2 \) if \( \omega \in D_2 \), and \( S^p(\omega) = (S_1 + S_2)/2 \) if \( \omega \in D_2 \). Then, the bias, \( b \), is defined as

For example, consider a sequence of \( n \) such agents distributed on the unit interval and spaced by a distance of \( 1/n \). Even for \( n \uparrow \infty \), the set of these agents has measure zero, would have no price impact and therefore no influence on the equilibrium.
For a given number of solicited ratings (zero, one or two), the issuer’s expected profits equal

\[ E(p) = E[E(X|I) - r \text{Var}(X|I)] - E(\text{cost}) \]

where, informally, \( I \) denotes the information set available to investors in equilibrium and \( E(\text{cost}) \) the expected costs paid by the issuer. A key feature of the model is that, with rational expectations, prices are efficient in a semi-strong sense, which allows us to use the law of iterated expectations and write

\[ E(p) = \mu - rE[\text{Var}(X|I)] - E(\text{cost}) \]

From an ex-ante perspective, the trade-off is between the costs of disclosing information and the benefits in terms of a lower risk discount. Therefore, the key parameters that shape this trade-off are the cost \( \chi \) and risk aversion \( r \). According to the Proposition, if costs are low enough, the issuer will minimize the risk discount by soliciting both ratings. If costs are extremely high, solicited ratings will never be published, which drives the irrelevance between the different options. In the risk neutral case, information is not valuable ex ante, so no rating will be solicited. Numerical analysis reveals that for high values of \( r \) relative to \( \chi \), the issuer will minimize the risk discount by soliciting both ratings; for intermediate values the issuer solicits only one rating.

For the case in which fees are charged only at the soliciting stage, we have the following Proposition.

**Proposition 2.** Let \( \chi = 0 \), and the market be transparent. Denote

\[ c_1 = r[\text{Var}(X|S_i) - \text{Var}(X|S_i,S_j)] ; \quad c_2 = r[\sigma_X^2 - \text{Var}(X|S_i)] \]

For \( c \leq c_1 \) the issuer solicits both ratings, for \( c_1 < c \leq c_2 \) the issuer solicits only one rating, and for \( c > c_2 \) no rating is solicited.

The Proposition shows how the value of ratings is related to the value of information: \( c_1 \) and \( c_2 \) are increasing in risk aversion and the variance of the fundamental. As for the case of ex-post fees, in the risk neutral case costly ratings would never be solicited.

### 4.2 Endogenous fees

We endogenize the fees as the result of competition between the two rating agencies. We assume that (ex-ante and ex-post) fees are announced simultaneously by the rating agencies at the beginning of the game, and that renegotiation is not possible.
**Proposition 3.** In a transparent market, rating agencies set the ex-ante fee equal to \( c_i = c_j = c_1 \) and the ex-post fee equal to zero.

The proof of the Proposition also shows how, in this case, the issuer’s participation constraint is never binding. Intuitively, a deviation from the fee structure in the Proposition can be profitable only if the rating is expected to be more costly, but at \( c = c_1 \), the issuer is indifferent between soliciting one or two ratings. Therefore, the issuer would react to such a deviation by soliciting only one (the cheapest) rating. With endogenous costs, Propositions 2, 3 and Lemma 2, imply that in a transparent market both ratings are always solicited and always disclosed. Therefore, shopping and ratings bias do not arise in equilibrium in this case.

From the definition of \( c_1 \) it follows that the level of noise in the rating technology that maximizes rating agencies’ profits is proportional to the variance of the payoff: \( \sqrt{2\sigma^2_X} \). This is intuitive: on the one hand, ratings that are too noisy have no value; on the other hand ratings that are very precise are also more homogenous, which reduces ratings agencies’ profits as a result of Bertrand competition.

5 The soliciting decision: the opaque market case

This Section analyzes the case in which investors cannot observe the number of solicited ratings, and highlights the different properties of the equilibrium.

5.1 Exogenous fees

**Proposition 4.** Let \( c = 0 \), and the market be opaque. The issuer always solicits both ratings.

The intuition behind the surprising result in Proposition 4 is simple: in an opaque market, independently of the value of risk aversion coefficient, soliciting one or no ratings cannot be an equilibrium. By contradiction, assume an equilibrium in which less than two ratings are solicited. The issuer would always have an incentive to deviate and solicit both ratings. The deviation is not detectable by investors and with positive probability ratings are high enough that publishing both of them is more profitable than publishing either one or none.

Consider now the case of ex-ante fees. The issuer could profitably deviate from the strategy proposed in Proposition 2: for example, assume \( c_1 < c \leq c_2 \). From Proposition 2 it follows that soliciting one rating dominates soliciting two ratings, ex-ante. Nevertheless, the issuer could solicit a second rating and then publish the highest among the two. Such a deviation would not be observable by investors in an opaque market, and, for \( c \) close enough to \( c_1 \), there are states in which the issuer expects such a deviation to be profitable. Next Proposition 5
describes the optimal strategy for the issuer, in which all possible deviations are taken care of. A description of the relevant belief system which supports the equilibrium is relegated to the Appendix. Denote with $\Phi(y)$ the CDF and with $f(y)$ the PDF of a Standard Normal valued at $y$, and define the function

$$g(y) := y\Phi(y) + f(y).$$

**Proposition 5.** Let $\chi = 0$, and the market be opaque. Denote

$$\hat{c}_1 = \sqrt{\text{Var}(X|S_i) - \text{Var}(X|S_i,S_j)}g\left(r\sqrt{\text{Var}(X|S_i) - \text{Var}(X|S_i,S_j)}\right);$$

$$\hat{c}_2 = \sqrt{\sigma_X^2 - \text{Var}(X|S_i)}g\left(r\sqrt{\sigma_X^2 - \text{Var}(X|S_i)}\right).$$

For $c < \hat{c}_1$ the issuer solicits both ratings, for $\hat{c}_1 \leq c < \hat{c}_2$ the issuer solicits only one rating, and for $c \geq \hat{c}_2$ no rating is solicited.

Differently from the transparent market counterpart, the thresholds $\hat{c}_1$ and $\hat{c}_2$ are strictly positive even in the risk neutral case $r = 0$. In an opaque market, Proposition 5 implies that ratings retain positive value in equilibrium even if information has no social value, due to the strategic aspect of ratings shopping.

### 5.2 Endogenous fees

The following Proposition deals with the endogenous determination of fees in the opaque market case, in which rating agencies are restricted to set fees at the ex-ante stage only.

**Proposition 6.** Let $\chi = 0$. For $V < 2\hat{c}_1 + r\text{Var}(X|S_i,S_j)$, the equilibrium fee is $c^* < \hat{c}_1$, where

$$c^* = \frac{1}{2}[V - r\text{Var}(X|S_i,S_j)].$$

For $V \geq 2\hat{c}_1 + r\text{Var}(X|S_i,S_j)$, there is no equilibrium in ex-ante fees.

An equilibrium with endogenous ex-ante fees exists, under the condition that ex-post fees are exogenously set to zero, only if the issuer’s holding costs are not too high. What drives the result is that, at the value of the cost for which the issuer is indifferent between soliciting one or two ratings, $c_1$, soliciting only one rating is not credible, as $\hat{c}_1 > c_1$. Notice that Proposition 6 does not imply that $c^*$ is an equilibrium in the general case in which rating agencies can set positive ex-post fees. More generally, Proposition 6 implies the following Corollary:

**Corollary 1.** In an opaque market, an equilibrium in fee-setting must involve discretionary disclosure for all $V \geq 2\hat{c}_1 + r\text{Var}(X|S_i,S_j)$. 

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If the market is opaque, therefore, ex-ante fees are generally not the outcome of competition in fee setting, and discretionary disclosure (and biased ratings) would arise whenever the equilibrium implies positive costs at the publication stage. Notice that in this Section we have restricted the analysis to the case of pure strategy equilibria. We relax this restriction in the following Section.

6 Opaque market and mixed strategy equilibria

In this Section we prove the existence and characterize mixed strategy equilibria in which the issuer is indifferent among two of the three alternatives (soliciting zero, one or two ratings). For ease of exposition, we focus here on the case of positive ex-ante fees and zero ex-post fees, and relegate to the Appendix the description of the more general case with strictly positive ex-ante and ex-post fees (see the proof of Proposition 7). The reason for focusing our discussion on ex-ante costs is that, for this case, the mixed strategy equilibria presented in this Section exhibit a qualitative difference with respect to the equilibria in pure strategy described in the previous Sections: discretionary disclosure is an equilibrium phenomenon, even in absence of any disclosure cost.

For each of the following Propositions, the specific value of $q$ is defined in the Appendix.

**Proposition 7.** For all $c > \hat{c}_2$, there exists an equilibrium in which the issuer solicits one rating with probability $q \in (0, 1)$ and no rating with probability $1 - q$. If a rating is solicited, it is disclosed if and only if above some value $\tilde{S}_m$, where $\tilde{S}_m$ solves

$$c = \sqrt{\sigma_X^2 - \sigma_{X|S}^2} g\left(\frac{\mu_X - \tilde{S}_m}{\sigma_S}\right).$$

**Proposition 8.** For all $c > \frac{1}{2} \frac{\sqrt{\sigma_X^2 - \sigma_{X|S_1,S_2}^2}}{\sqrt{\sigma_X^2 - \sigma_{X|S_1,S_2}^2}} g\left(\sqrt{\frac{\sigma_X^2 - \sigma_{X|S_1,S_2}^2}{\sigma_X^2 + \sigma_{X|S_1,S_2}^2}}\right)$, there exists an equilibrium in which the issuer solicits two ratings with probability $q \in (0, 1)$ and no rating with probability $1 - q$. If ratings are solicited, both ratings are disclosed if and only if the average rating is greater than some value $\tilde{S}$, where $\tilde{S}$ solves

$$c = \frac{1}{2} \sqrt{\sigma_X^2 - \sigma_{X|S_1,S_2}^2} g\left(\frac{\mu_X - \tilde{S}}{\sqrt{\sigma_X^2 + \sigma_{X|S_1,S_2}^2}}\right).$$

**Proposition 9.** For all $c > E \left[\max \left\{p(S_1), p(S_1, S_2)\right\} - p(S_1)\right]$, there exists an equilibrium in which the issuer solicits two ratings with probability $q \in (0, 1)$ and one rating with probability $1 - q$. If one ratings is solicited, it is always disclosed. If both ratings are solicited, both ratings are disclosed if $\tilde{S}_2 > s_m(\tilde{S}_1)$, for some function $s_m(\cdot)$ described in the Appendix, and only $\tilde{S}_1$
is disclosed otherwise.

The following Corollary is immediate.

**Corollary 2.** In all mixed strategy equilibria, ratings are selectively disclosed with strictly positive probability.

### 7 Mandatory disclosure regime

Assume now that the regulators impose the following rule on the game: every solicited rating must be disclosed. We refer to this situation as to the *mandatory disclosure* regime. Clearly, under mandatory disclosure, the difference between ex-ante and ex-post costs is meaningless because withholding the rating is not an option. Also, the difference between opaque and transparent market seems not to have much bite, because, as soon as the issuer decides to solicit a rating, and assuming the regulation is effective, the solicited ratings is eventually published and therefore the decision of soliciting the rating, trivially, is indirectly observed by investors.

Nevertheless, assume informal discussions between the issuer and the rating agencies can take place before the soliciting stage. Such discussions, being informal, are not observable by investors. Assume that, during an informal discussion, the rating agency reveals to the issuer an indicative rating, $S_i^I = S_i + \delta_i$, with $\delta_i \sim N(0, \sigma_\delta^2)$ and uncorrelated with the other random variables in the model. Assume further that, in order to have such an informal discussion, rating agencies charge a cost $\tilde{c}$, and, if the issuer decides to go ahead in the process and solicit the rating, the rating agency charges a cost $c$. After solicitation, under the mandatory disclosure regime, the rating is automatically published. Our previous results can be reinterpreted to describe the outcome of this game exactly in these two cases: $\sigma_\delta^2 = \infty$, and $\sigma_\delta^2 = 0$. If $\sigma_\delta^2 = \infty$, then the informal discussion gives no information to the issuer, and the game has the same outcome of the transparent market case in which ex-ante costs equal to $\tilde{c} + c$, and ex-post costs equal zero. In fact, by Lemma 2, any solicited rating is disclosed in the transparent market case with no publishing costs, as it is in the mandatory disclosure regime. Therefore, the outcome of this game is described by Proposition 2. The description of this case is for comparison only, clearly, as totally uninformative informative discussions are meaningless. The more interesting case is $\sigma_\delta^2 = 0$. In this case the indicative rating reveals the rating perfectly, and the game is formally equivalent to the opaque market case of Sections 5 and 6, in which ex-ante costs equal $\tilde{c}$ and ex-post costs equal $c$. In light of the spirit of the regulation, one might think that the most relevant case is when rating agencies cannot (formally) charge issuers for such informal discussions, that is, $\tilde{c} = 0$. In this case, the outcome of the game is described by Proposition 4: in equilibrium the issuer has informal discussions with all rating agencies, and the soliciting
decision is formally equivalent to the publishing decision described by Lemma 2. Therefore, the outcome of the rating process with mandatory disclosure and costless perfectly revealing indicative ratings is observationally equivalent to the discretionary disclosure equilibrium in an opaque market with zero ex-ante fees. In essence, even if discretionary disclosure is ruled out directly in the mandatory disclosure regime, the possibility of having informal discussions with the rating agencies allows for selective disclosure indirectly, in equilibrium, via selective solicitation. The resulting ratings, conditional on having been solicited (and therefore published), would therefore be positively biased.

Clearly, one might consider the realistic case in which indicative ratings are informative but not fully revealing of the final rating, $\sigma_0^2 \in (0, \infty)$. A reasonable conjecture is that the resulting equilibrium would be continuous in $\sigma_0^2$, so that, qualitatively, the properties of the equilibrium with $\sigma_0^2 = 0$ would be robust, as well as the prediction of selective biased ratings.

8 Concluding Comments

Our paper uses a model based upon rational expectations to examine conditions under which selective disclosure and ratings bias emerge in equilibrium. We highlight the role of the structure of ratings costs and regulatory policy about disclosure and costs. For example, under some conditions requiring the disclosure of the existence of soliciting ratings may be equivalent to mandatory disclosure and eliminating ratings bias in equilibrium. However, mandatory disclosure may not be fully effective in eliminating bias and selective disclosure in a setting in which informal discussions between the issuer and the rating agencies take place prior to the soliciting stage.

The type of analysis we undertake in this paper also is relevant for understanding empirical aspects of credit ratings—especially multiple and split ratings. The information content in ratings reflects not only the ratings selected (those purchased and published), but also solicited ratings (even though unobservable) that are not selected (also discussed in Sangiorgi, Sokobin and Spatt (2009)). For example, at the most favorable rating obtained, the larger the number of these ratings the more favorable the information content as it implies the absence of fewer ratings at lower levels. A similar analysis can apply to split ratings. This also suggest the adverse nature of the absence of ratings (e.g., unrated securities), especially when the costs of ratings are very low.\footnote{There is considerable evidence with respect to both multiple ratings and split ratings (e.g., see Bongaerts, Cremers and Goetzmann (2009), Livingston, Naranjo and Zhou (2005) and Mattarocci (2005)).} For example, unless the costs are especially high, unrated instruments are likely to reflect especially adverse information. More generally, these types of models highlight the information content of published ratings at various levels.

Our formal analysis does highlight two reasons why issuers might want to publish multiple
ratings, even absent regulatory requirements. Because investors are risk averse, additional ratings reduce the required risk premium, offering more precision about the underlying signal. Additionally, to the extent that investors expect the issuer to solicit multiple ratings, absence of publication suggests adverse information, and implies an information discount relative to the issues with public ratings. For example, in the context of an opaque market with no ex-ante costs and positive publication costs (proposition 4), in equilibrium, investors expect the issuer to solicit all ratings, and, therefore, multiple ratings are published with positive probability. In fact, the second motive for multiple ratings is valid even in a risk-neutral setting. These motives tie closely to the “information production hypothesis” and “shopping hypothesis” in Bongaerts, Cremers and Goetzmann (2009).

9 Appendix A: notation and preliminary results

This Appendix contains the formulas used in the proof of the equilibrium, as well as in the numerical analysis of the model. The proofs of the results contained in this Section are omitted.

Notation

Denote with \( \Phi (y; \mu, \sigma^2) \) the CDF and with \( f (y; \mu, \sigma^2) \) the PDF of a Normal variable \( y \) with mean \( \mu \) and variance \( \sigma^2 \), and let

\[
\Phi (y) := \Phi (y; 0, 1); \quad f (y) = f(y; 0, 1).
\]

For generic random variables \( x \) and \( z \) denote: with \( p_x(\cdot) \) the PDF; with \( P_x(\cdot) \) the CDF, with \( p_{y|x}(\cdot | \cdot) \) and \( p_{y,z}(\cdot, \cdot) \) the conditional and joint distribution functions; with \( \mu_{y|x} \) and \( \sigma_{y|x}^2 \) the first and second moments of \( y \) conditional on the realization of \( x \). Furthermore define the functions \( h(\cdot) \) and \( g(\cdot) \) by

\[
\begin{align*}
  h(y) & := f(y)/\Phi(y) \\
  g(y) & := y\Phi(y) + f(y).
\end{align*}
\]

Let \( X \sim N(\mu_X, \sigma_X^2) \) and \( S_i = X + \varepsilon_i \), with \( \varepsilon_i \sim N(0, \sigma_{\varepsilon}^2) \) for \( i = 1, 2 \). Let \( X, \varepsilon_1 \) and \( \varepsilon_2 \) be uncorrelated. Then, we have the following standard results

\[
\begin{align*}
  \mu_{X|S} & = \mu_X + \frac{\sigma_{\varepsilon}^2}{\sigma_X^2 + \sigma_{\varepsilon}^2} (S - \mu_X); & \sigma_{X|S}^2 & = \frac{1}{\sigma_X^2 + 2\sigma_{\varepsilon}^2}, \quad (9) \\
  \mu_{X|S_1,S_2} & = \mu_X + \frac{\sigma_{\varepsilon}^2}{\sigma_X^2 + 2\sigma_{\varepsilon}^2} [(S_1 - \mu_X) + (S_2 - \mu_X)]; & \sigma_{X|S_1,S_2}^2 & = \frac{1}{\sigma_X^2 + 2\sigma_{\varepsilon}^2}. \quad (10)
\end{align*}
\]

\(^8\) Also, see Skreta and Veldkamp (2009).
where

\[ \sigma_S^2 = \sigma_x^2 + \sigma_e^2. \]

Denoting

\[ \mu(y) := \mu_X + \frac{\sigma_x^2}{\sigma_S^2}(y - \mu_X), \]

then

\[ \mu_{S_i | S_\Delta} = \mu(S_\Delta); \quad \sigma^2_{S_i | S_\Delta} = \sigma_x^2 + \sigma_e^2. \]

**Conditional distributions**

Using the following notation for first and second order statistics

\[ \hat{S}_1 := \max(S_1, S_2); \quad \hat{S}_2 := \min(S_1, S_2), \]

the following results can be shown:

\[ p_{\hat{S}_1}(y) = \frac{2}{\sigma_S} f \left( \frac{y - \mu}{\sigma_S} \right) \Phi \left( \frac{y - \mu}{\sigma_{S_\Delta}} \right) \] (11)

\[ p_{\hat{S}_1, \hat{S}_2}(x, y) = 2p_{S_1, S_2}(x, y) \mathbf{1}_{y < x} \] (12)

\[ p_{\hat{S}_2 | \hat{S}_1}(y | x) = \frac{1}{\sigma_{S_\Delta}} \frac{f \left( \frac{y - \mu_x}{\sigma_{S_\Delta}} \right)}{\Phi \left( \frac{y - \mu_x}{\sigma_{S_\Delta}} \right)} \mathbf{1}_{y < x} \] (13)

\[ p_{\hat{S}_1 | X}(y | x) = \frac{2}{\sigma_e} f \left( \frac{y - x}{\sigma_e} \right) \Phi \left( \frac{y - x}{\sigma_e} \right) \] (14)

\[ p_{X | \hat{S}_1}(x | y) = p_{X | S_1}(x | y) \frac{\Phi \left( \frac{y - \mu_x}{\sigma_e} \right)}{\Phi \left( \frac{y - \mu_x(\sigma_S^2)}{\sigma_S^2 \sigma_{S_\Delta}} \right)} \] (15)

**Conditional moments**

For a given function \( s(\cdot) \), define the functions \( d(\cdot), \delta(\cdot), t(\cdot) \) and \( \tau(\cdot) \) by

\[ d(y) := \frac{y - \mu}{\sigma_S}; \quad \delta(y) := \frac{h[d(y)]g[d(y)]}{\Phi(d(y))} \] (16)

\[ t(y) := \frac{s(y) - \mu(y)}{\sigma_{S_\Delta}^2}; \quad \tau(y) := \frac{h[t(y)]g[t(y)]}{\Phi(t(y))}. \] (17)

Notice that, by definition of \( g, h \) and by the properties of the normal distribution, the functions
\( \delta(\cdot) \) and \( \tau(\cdot) \) are bounded in the interval \([0, 1]\). Using (9)-(15), the following can be shown:

\[
E(X|S_1, S_2 < s(S_1)) = \mu(S_1) - \sqrt{\sigma^2_{X|S} - \sigma^2_{X|S_1, S_2}} h[t(S_1)] \tag{18}
\]

\[
\text{Var}(X|S_1, S_2 < s(S_1)) = [1 - \tau(S_1)] \sigma^2_{X|S} + \tau(S_1) \sigma^2_{X|S_1, S_2}, \tag{19}
\]

Notice that, if we take \( s(\cdot) \) to be the identity function, (18) and (19) give the first and second moments of \( X \) conditional on the realization of highest order statistic \( \hat{S}_1 \). Finally, for some constant value \( K \), we find

\[
E(X|S_i < K) = \mu_X - \sqrt{\sigma^2_X - \sigma^2_{X|S}} h[d(K)] \tag{20}
\]

\[
\text{Var}(X|S_i < K) = [1 - \delta(K)] \sigma^2_X + \delta(K) \sigma^2_{X|S_i}, \tag{21}
\]

and

\[
E(X|\hat{S}_1 < K) = \mu_X - \sigma^2_X \frac{p_{\hat{S}_1}(K)}{p_{\hat{S}_1}(K)} \tag{22}
\]

\[
\text{Var}(X|\hat{S}_1 < K) = \sigma^2_X \left\{1 - \sigma^2_X \frac{p_{\hat{S}_1}(K)}{p_{\hat{S}_1}(K)} \left[\frac{p_{\hat{S}_1}(K)}{p_{\hat{S}_1}(K)} + \frac{K - \mu}{\sigma_S} - \frac{\sigma^2_S}{\sigma^2_S \sigma_{S_i|S_1}} h\left(\frac{K - \mu}{\sigma_S} \frac{\sigma^2_S}{\sigma^2_S \sigma_{S_i|S_1}}\right)\right]\right\} \tag{23}
\]

10 Appendix B: proofs of Propositions

With some abuse of notation, in this section (as in the main text) the letter \( p \) denotes prices, as opposed to PDF as in the previous section. Notice that, for a given number of solicited ratings, the objective function of the issuer is maximize the price at which the asset is sold, net of the disclosure costs.

**Assumption 0:** assume the risk aversion coefficient \( r \) is such that the functions

\[
e_1(K) = E(X|S_i < K) - r\text{Var}(X|S_i < K)
\]

\[
e_2(K) = E(X|\hat{S}_1 < K) - r\text{Var}(X|\hat{S}_1 < K)
\]

are strictly decreasing in \( K \), and the function

\[
e_3(\hat{S}_1) = E(X|\hat{S}_1) - r\text{Var}(X|\hat{S}_1)
\]

is strictly increasing in \( \hat{S}_1 \). The required parametric restriction is satisfied for all \( r \in [0, r^*] \), with the specific value of \( r^* \) depending on \( \sigma_X \) and \( \sigma_2 \). The economic content of Assumption 0 is to disregard values of the risk aversion coefficient that are so high that the direct first moment effect on prices of negative information (lower values of \( K \) in \( e_1 \) and \( e_2 \) and lower values of \( \hat{S}_1 \)
in $e_3$) is dominated by indirect variance effects. In all proofs we assume Assumption 0 to be satisfied.

**Proof of Lemma 1.**

The value $\bar{S}$ must be such that, given the corresponding price function (1)-(2), the conjectured strategy is optimal for the issuer. This is achieved by $\bar{S}$ being defined by

$$p(\bar{S}) - \chi = p(\bar{\omega}).$$

Clearly, then, publishing the rating is optimal if and only if $S > \bar{S}$. Existence, uniqueness and the fact that $\bar{S}$ is a strictly increasing function of $\chi$, with $\lim_{\chi \to 0}\bar{S} = -\infty$ follow from Verrecchia (1983) and the proof is omitted here.

**Proof of Lemma 2**

The value $\bar{S}_0$ and the function $s(\cdot)$ must be such that, given the corresponding price function (3)-(5), the conjectured strategy is indeed optimal for the issuer. The Lemma is proven in three steps.

1) Define the threshold value $\bar{S}_1$ as the real number such that

$$p(\bar{S}_1, \bar{S}_1) - \chi = p(\bar{S}_1, \bar{\omega}),$$

where $s(\cdot)$ in the right hand side of (24) is taken to be the identity function. From the definitions (3)-(4) and (18) and (19), it follows that $\frac{p(y, \omega)}{p(y, y)} < 1$ for all $y > -\infty$, and $\lim_{y \to -\infty} \frac{p(y, \omega)}{p(y, y)} = 1$, where the limit follows applying l'Hôpital’s rule twice and using properties of the normal distribution. As a consequence, $\bar{S}_1$ exists and is finite for all $\chi \in (0, \infty)$. Moreover, by taking derivatives, one can verify that a sufficient condition for $\frac{d}{dy}(p(y, \omega) - p(y, y)) < 0$ is that $h'(\cdot) > -1$, which follows for all finite $y$ by the properties of the normal distribution. Therefore $\bar{S}_1$ is unique, and by applying the implicit function Theorem to (24), it follows that $\bar{S}_1$ is a strictly increasing function of $\chi$, with $\lim_{\chi \to 0}\bar{S}_1 = -\infty$.

2) Next, $s(\cdot)$ is conjectured to have the following form:

$$s(y) = \begin{cases} y, & \text{for } \bar{S}_1 \leq \bar{S}_1 \\
\mu(y) + (\bar{S}_1 - \mu)\frac{\sigma_y^2}{\sigma^2}, & \text{for } \bar{S}_1 > \bar{S}_1 \end{cases}$$

Notice that that $\mu(\bar{S}_1) + (\bar{S}_1 - \mu)\frac{\sigma_y^2}{\sigma^2} = \bar{S}_1$ and $\mu(y) + (\bar{S}_1 - \mu)\frac{\sigma_y^2}{\sigma^2} < y$ for $y > \bar{S}_1$. Therefore the conjectured $s(\cdot)$ function is continuous and $s(y) \leq y$. By the previous step,
whenever \( \hat{S}_1 \leq \bar{S} \) publishing \( \hat{S}_1 \) dominates publishing both ratings, for all values of \( \hat{S}_2 \). Therefore \( s(\hat{S}_1) = \bar{S} \) for \( \hat{S}_1 \leq \bar{S} \). By the same logic, it follows that, if \( \hat{S}_1 > \bar{S}_1 \) and \( \hat{S}_2 \) is sufficiently close to \( \bar{S}_1 \), publishing both ratings dominates publishing only \( \hat{S}_1 \). The \( s(\cdot) \) function must pin down, for all \( \hat{S}_1 > \bar{S}_1 \), the interval of values for \( \hat{S}_2 \) such that this is true, that is, the function \( s(\cdot) \) must solve the functional equation

\[
p(y,s(y)) - \chi = p(y,\varnothing), \quad \text{for all } y > \bar{S}_1. \tag{26}
\]

Since \( p(y,\varnothing) \) is increasing in \( x \), it follows that, for \( \hat{S}_1 > \bar{S}_1 \), publishing both ratings dominates publishing \( \hat{S}_1 \) if and only if \( \hat{S}_2 > s(\hat{S}_1) \). By substituting \( s(y) = \mu(y) + (\hat{S}_1 - \mu) \frac{\sigma^2_{\tilde{X}|S}}{\sigma_{\tilde{X}}} \) into (26), and using the definition of \( \bar{S}_1 \), it can be verified that second line in (25) solves (26) for all \( \hat{S}_1 > \bar{S}_1 \).

3) Finally, assume there exists some value \( \bar{S}_0 < \bar{S}_1 \) that satisfies the following condition

\[
p(\bar{S}_0,\varnothing) - \chi = p(\varnothing,\varnothing). \tag{27}
\]

Since the r.h.s. of (27) is constant, while \( p(y,\varnothing) \) is increasing in \( y \), it follows that publishing \( \bar{S}_1 \) dominates withholding both ratings if and only if \( \hat{S}_1 > \bar{S}_0 \). Existence and uniqueness of \( \bar{S}_0 \), as well as the fact that \( \bar{S}_0 \) is strictly increasing in \( \chi \) with \( \lim_{\chi \to 0} \bar{S}_0 = -\infty \), can be shown using similar arguments to those used in step 1 for \( \bar{S}_1 \). Numerical analysis over a wide range of parameter values reveals that \( \bar{S}_0 < \bar{S}_1 \) is indeed always satisfied. For the other disclosure alternatives (publishing both ratings if \( \hat{S}_1 \leq \bar{S}_0 \) or publishing only \( \hat{S}_2 \)) suboptimality follows trivially.

Some proofs will make use of the following variation of Lemma 2.

**Lemma 3.** Let \( \chi_i = 0, \chi_j > 0 \). Conditional on two ratings being solicited, the equilibrium has the following form: a function \( z(\cdot) \) and a price function \( p(\cdot,\cdot) \) such that:

1) the issuer’s optimal strategy is to always publish \( S_i \) and publish \( S_j \) if and only if \( S_{-i} > z(S_i) \)

2) the price \( p(\cdot,\cdot) \) is consistent with the strategy of the issuer. Let \( p(S_i,S,j), p(S_i,\varnothing) \) denote the asset price conditional on both ratings and conditional on rating \( S_i \) alone, then

\[
p(S_i,S,j) = E ( X| S_i, S,j ) - r \text{Var} ( X| S_i, S,j );
\]

\[
p(S_i,\varnothing) = E ( X| S_i, S,j \leq z(S_i)) - r \text{Var} ( X| S_i, S,j \leq z(S_i));
\]

**Proof of Lemma 3.**
Let the function $z(\cdot)$ be
\[
z(y) = \mu(y) + (\bar{S}_1 - \mu) \frac{\sigma^2_X|S|}{\sigma^2_X},
\] (28)
with $\bar{S}_1$ defined as in (24). Then, we have $z(\bar{S}_1) = \bar{S}_1$ and $z(y) > y \iff y > \bar{S}_1$, implying $p(s_i,s_{-i}) - \chi_{-i} > p(s_i,s_{-i}) \iff S_j > z(S_i)$.

**Proof of Proposition 1**

From Lemma 1 and Lemma 2 follows that, in the limit as $\chi \downarrow 0$, all ratings are disclosed, and therefore,
\[
\lim_{\chi \downarrow 0} E \left[ \Pi_2(\hat{S}_1, \hat{S}_2; 0, \chi) \right] = \mu_X - r \sigma^2_X|S_1, S_2|
\]
\[
\lim_{\chi \downarrow 0} E \left[ \Pi_1(S_i; 0, \chi) \right] = \mu_X - r \sigma^2_X|\chi|
\]

Given that $\mu_X - r \sigma^2_X|S_1, S_2 > \mu_X - r \sigma^2_X > \mu_X - r \sigma^2_X$, it follows that there exists some $\chi_1 > 0$, such that $E \left[ \Pi_2(\hat{S}_1, \hat{S}_2; 0, \chi) \right] > E \left[ \Pi_1(S_i; 0, \chi) \right] > \Pi_0$ for all $\chi < \chi_1$. Also, for $r = 0$ and all $\chi > 0$, denoting with $P_{\hat{S}_1}(\cdot)$ the CDF of $\hat{S}_1$, we have
\[
E \left[ \Pi_2(\hat{S}_1, \hat{S}_2; 0, \chi) \right]_{r=0} < \mu_X - \chi \left[ 1 - P_{\hat{S}_1}(\bar{S}_0) \right]
\]
\[
E \left[ \Pi_1(S_i; 0, \chi) \right]_{r=0} = \mu_X - \chi \left[ 1 - \Phi \left( \frac{\bar{S} - \mu_X}{\sigma_S} \right) \right]
\]
\[
\Pi_0|_{r=0} = \mu_X,
\]
form which it follows that, if $r = 0$, no rating is solicited for all $\chi > 0$.

From the definitions of $\bar{S}, \bar{S}_0$ and $\bar{S}_1$, follows that, as $\chi \uparrow \infty$, solicited ratings are never disclosed, implying
\[
\lim_{\chi \uparrow \infty} E \left[ \Pi_2(\hat{S}_1, \hat{S}_2; 0, \chi) \right] = \lim_{\chi \uparrow \infty} E \left[ \Pi_1(S_i; 0, \chi) \right] = \mu_X - r \sigma^2_X,
\]
from which it follows that there exists some $\chi_2$ such that $E \left[ \Pi_2(\hat{S}_1, \hat{S}_2; 0, \chi) \right] = E \left[ \Pi_1(S_i; 0, \chi) \right] = \Pi_0$ for all $\chi > \chi_2$.

**Proof of Proposition 2**

From Lemma 2, as $\chi = 0$ all solicited ratings are always disclosed, from which if follows
that
\[
E \left[ \Pi_2(\tilde{S}_1, \tilde{S}_2; c, 0) \right] \geq E \left[ \Pi_1(S_i; c, 0) \right] \iff c \leq r \left( \sigma_{X|S}^2 - \sigma_{X,S_1,S_2}^2 \right)
\]
\[
E \left[ \Pi_1(S_i; c, 0) \right] \geq \Pi_0 \iff c \leq r \left( \sigma_X^2 - \sigma_{X|S}^2 \right)
\]
\[
E \left[ \Pi_2(\tilde{S}_1, \tilde{S}_2; c, 0) \right] \geq \Pi_0 \iff c \leq \frac{1}{2} r \left( \sigma_X^2 - \sigma_{X|S_1,S_2}^2 \right)
\]
As, for all values of \( \sigma_X \) and \( \sigma_{e} \),
\[
\sigma_X^2 - \sigma_{X|S_1,S_2}^2 < \frac{1}{2} \left( \sigma_X^2 - \sigma_{X|S_1,S_2}^2 \right) < \sigma_X^2 - \sigma_{X|S}^2,
\]
under the convention that, when indifferent, the issuer solicits more ratings, the statement in the Proposition follows immediately.

**Proof of Proposition 3**

First, we derive explicit expressions for the issuer’s expected payoffs from the options of soliciting different ratings. Let \( \Pi_2(S_i, S_j; c_i, \chi_i, c_j, \chi_j) \) denote issuer’s payoff from soliciting both ratings when rating agencies charge different costs. From Lemma 3, we have:

\[
E \left[ \Pi_2(S_i, S_j; c_i, c_1, \chi_i, 0) \right] = \mu_X - \left\{ r E \left[ \text{Var} \left( X | S_j, S_i \leq z(S_j) \right) | S_j \leq z(S_j) \right] \times \text{Prob} \left( S_i \leq z(S_j) \right) \right. \\
+ \text{Var} \left( X | S_i, S_j \right) \times \left[ 1 - \text{Prob} \left( S_i \leq z(S_j) \right) \right] \right\} \\
- \left\{ c_i + \chi_i \times \left[ 1 - \text{Prob} \left( S_i \leq z(S_j) \right) \right] \right\} - c_1,
\]

Defining \( \alpha \) as the constant value
\[
\alpha := \frac{z(S_j) - \mu(S_j)}{\sigma_{S_i|S_j}} = \frac{(\tilde{S}_1 - \mu)}{\sigma_S} \sqrt{\sigma_X^2 - \sigma_{X|S_1,S_2}^2},
\]
using (17), (19), we can write

\[
\text{Var} \left( X | S_j, S_i \leq z(S_1) \right) = \left[ 1 - \frac{h(\alpha) g(\alpha)}{\Phi(\alpha)} \right] \sigma_X^2 + \frac{h(\alpha) g(\alpha)}{\Phi(\alpha)} \sigma_{X|S_1,S_2}^2,
\]
and,

\[
\text{prob} \left( S_i > z(S_j) \right) = \int_{-\infty}^{\infty} \frac{1}{\sigma_S} f \left( \frac{S_j - \mu_X}{\sigma_S} \right) \left( \int_{z(S_j)}^{\infty} \frac{1}{\sigma_{S_i|S_j}} f \left( \frac{S_i - \mu(S_j)}{\sigma_{S_i|S_j}} \right) dS_i \right) dS_j = [1 - \Phi(\alpha)].
\]
Using (31) and (32) and simplifying, (29) can be written as

\[
E [\Pi_2(S_i, S_j; c_i, c_1, \chi_i, 0)] = \mu_X - r \left\{ \left( \sigma_X^2 - \sigma_X^2 | S \right) \Phi (\alpha) [1 - h (\alpha) [\alpha + h(\alpha)]] + \sigma_X^2 | S_1, S_2 \right\} \\
- \left\{ c_i + \chi_i [1 - \Phi (\alpha)] \right\} - c_1.
\] (33)

Expected payoff from soliciting only rating \( i \) is

\[
E [\Pi_1(S_i; c_i, 0)] = \mu_X - r \left\{ \text{Var} (X | S_i < \bar{S}) \times \text{Prob}(S_i \leq \bar{S}) + \text{Var} (X | S_i) \times \left[ 1 - \text{Prob}(S_i \leq \bar{S}) \right] \right\} \\
- \left\{ c_i + \chi_i \times \left[ 1 - \text{Prob}(S_i \leq \bar{S}) \right] \right\}
\]

Denoting \( \beta = \frac{\bar{S} - \mu_X}{\sigma_S} \), using (16) and (21) and simplifying, the previous expression can be written as

\[
E [\Pi_1(S_i; c_i, 0)] = \mu_X - r \left\{ \left( \sigma_X^2 - \sigma_X^2 | S \right) \Phi (\beta) [1 - h (\beta) [\beta + h(\beta)]] + \sigma_X^2 | S \right\} \\
- \left\{ c_i + \chi_i [1 - \Phi (\beta)] \right\},
\] (34)

In the case in which only rating \( i \) is solicited, from the proof of Proposition 2 we have, equivalently,

\[
E [\Pi_1(S_i; c_i, 0)] = \mu_X - \sigma_X^2 | S - c_1
\]

Next, we prove that \( c_i = c_{i,j} = c_1 \) and \( \chi_i = \chi_{i,j} = 0 \) is a Nash equilibrium in the fee setting game. First of all, any deviation in ex-ante fees only cannot be profitable because, by Proposition 2, if rating agency \( i \) deviates to \( c_i > c_1 \), it’s rating will not be solicited. Consider now rating agency \( i \) deviating simultaneously in both costs to \( c_i \geq 0, \chi_i > 0 \). For the deviation to be profitable, it is necessary that rating \( i \) is solicited and rating agency \( i \) earns expected profits strictly higher than \( c_1 \). Formally, we need to consider two cases. The first case relates to the possibility that both ratings are solicited. In this case, for the deviation to be profitable, it has to be

\[
\begin{align*}
E [\Pi_2(S_i, S_j; c_i, c_1, \chi_i, 0)] &\geq E [\Pi_1(S_j; c_1, 0)] \\
&\geq E [\Pi_1(S_i; c_i, 0)] \\
c_i + \chi_i [1 - \Phi (\alpha)] &> c_1
\end{align*}
\] (37)

Using (33) and (36), the system (37) implies

\[-r \left( \sigma_X^2 | S - \sigma_X^2 | S_1, S_2 \right) \Phi (\alpha) [1 - h (\alpha) [\alpha + h(\alpha)]] > 0.\]

Since the l.h.s. of the last inequality is non-positive for all \( \alpha \), we have a contradiction. The second case relates to the possibility that only rating \( i \) is solicited, in which case, the system
of necessary conditions for a profitable deviation is

\[
\begin{cases}
E [\Pi_1(S_i; c_i, \chi_i)] \geq E [\Pi_1(S_j; c_1, 0)] \\
c_i + \chi_i [1 - \Phi(\beta)] > c_1
\end{cases}
\]  

(38)

Using (34) and (35), the system (38) requires

\[-r \left( \sigma_X^2 - \sigma_{X|S_i}^2 \right) \Phi(\beta) \left[ 1 - h(\beta) [\beta + h(\beta)] \right] > 0.\]

Since the l.h.s. of the last inequality is non-positive for all \(\beta\), we have a contradiction.

Finally, assume there exists a (symmetric) Nash equilibrium in which rating agencies set ex ante fees equal to \(c' \geq 0\) ex-post fees \(\chi' > 0\) and earn expected profits (per rating agency) equal to \(c_1 + \delta\), with \(\delta \geq 0\). Denote \(E[\text{Var}(X|I)]\) the expected risk discount in the resulting equilibrium. Notice that, by lemma 2, \(E[\text{Var}(X|I)] > \text{Var}(X|S_i, S_j)\) if \(\chi > 0\). Assume rating agency \(i\) deviates to \(c_i = c_1 + \eta, \chi_i = 0\), where

\[\eta = \frac{r}{2} \left\{ E[\text{Var}(X|I)] - \sigma_{X|S_i,S_j}^2 \right\} + 2\delta > \delta.\]

It easily verified that

\[E [\Pi_1(S_i; c_1 + \eta, 0)] > E [\Pi_2(S_i, S_j; c', \chi')],\]

and, given the fact that we assumed the initial situation to be an equilibrium in which both ratings are solicited,

\[E [\Pi_2(S_i, S_j; c', \chi')] \geq E [\Pi_1(S_j; c', \chi')]\]

and

\[E [\Pi_2(S_i, S_j; c', \chi')] \geq \Pi_0.\]

The last three inequalities imply that rating \(i\) is solicited. As \(c_i > c_1 + \delta\) by construction, the deviation is profitable, contradicting the original equilibrium.

Finally, notice that issuer’s expected profits when soliciting both ratings at \(c = c_1\) equal

\[\mu_X - r \sigma_{X|S_i, S_j}^2 - 2c_1 < \mu_X - r \sigma_X^2.\]

Since \(V > r \sigma_X^2\), the issuer’s participation constraint is satisfied.

**Proof of Proposition 4**

Assume, by contradiction, an equilibrium in which the issuer solicits only one rating. Assume now that the issuer deviates and soliciting both ratings, publishing only the highest among the two whenever \(S_1 > S_0\). Clearly, such a deviation is not detectable by investors.
Then, expected profits are

\[ E \left[ \max \left\{ p(S_i) - \chi ; p(\varnothing) \right\} \right] > E \left[ \max \left\{ p(S_i) - \chi ; p(\varnothing) \right\} \right] = E \left[ \Pi_1(S_i; 0, \chi) \right], \]

implying the deviation is profitable. A similar argument can be made for the equilibrium in which the issuer is expected to solicit no ratings. Finally, in the case in which the issuer is expected to solicit all ratings, no deviation can be profitable, as

\[ E \left[ \Pi_2(S_1, S_2; 0, \chi) \right] = E \left[ \max \left\{ p(S_1, S_2) - 2 \chi ; p(S_1, \varnothing) - \chi ; p(\varnothing, \varnothing) \right\} \right] > E \left[ \max \left\{ p(S_i, \varnothing) - \chi ; p(\varnothing, \varnothing) \right\} \right] > p(\varnothing, \varnothing). \]

**Proof of Proposition 5**

The proof is divided in three parts. First, we rule out the equilibrium in which no rating is solicited for all \( c < \hat{c}_2 \), and we rule out the equilibrium in which only one rating is solicited for all \( c < \hat{c}_1 \). In equilibrium, if investors expect \( N \) ratings to be solicited, given no publishing costs, they also expect \( N \) ratings to be published. Off equilibrium, we assume:

- if \( N \) ratings are expected and \( J > N \) ratings are observed, investors believe that \( J \) ratings were solicited;
- if investors expect only rating \( i \) to be published, and observe instead only rating \( \neq i \), then investors cannot distinguish if only rating \( i \) was solicited or if both were solicited and rating \( i \) was withheld. We assume investors attach strictly positive probability to the path in which the issuer solicited both ratings and withheld rating \( i \), and believe rating \( i \) is of the worst type, that is, minus infinity.

Assume an equilibrium in which the issuer solicits zero ratings. In an opaque market, the issuer could deviate and solicit one rating. For the deviation not to be profitable, it has to be

\[ E \left[ \max \left\{ p(S_1) ; p_0 \right\} \right] - c \leq p_0. \tag{39} \]

By direct computation of the expectation and rearranging, we can rewrite the inequality (39) as

\[ c \geq \sqrt{\sigma_X^2 - \text{Var}(X \mid S_i)} g \left( r \sqrt{\sigma_X^2 - \text{Var}(X \mid S_i)} \right), \tag{40} \]

the r.h.s of (40) provides the expression for \( \hat{c}_2 \) in the text. Instead of soliciting only one, the issuer could, upon deviation, solicit two ratings. In this case, it can be shown that the condition that guarantees that such a deviation is not profitable is weaker than (40). Next, assume an equilibrium in which the issuer solicits only one rating, which investors anticipate to be rating \( i \). The issuer could deviate and solicit two ratings; in which case, given the off-equilibrium beliefs, it can never be optimal to deviate and publish \( S_i \), unless \( S_i \) is also published. Hence,
for the deviation not to be profitable, we must have

$$E \left[ \max \left\{ p(S_i, S_j), p(S_k) \right\} \right] - (c_i + c_j) \leq E \left[ \Pi_1(S_i; c_k, 0) \right] = \mu_X - r \sigma^2_{X|S} - c_i,$$

or equivalently, by direct computation of the expectation and rearranging,

$$c_j \geq \sqrt{\sigma^2_{X|S} - \sigma^2_{X|S_1, S_2} g \left( r \sqrt{\sigma^2_{X|S} - \sigma^2_{X|S_1, S_2}} \right)}.$$

the r.h.s of (41) being defined as $\hat{c}_1$ in the text. From the respective definitions it follows that:

$$\hat{c}_1 > c_1; \quad \hat{c}_2 > c_2; \quad \hat{c}_2 > \hat{c}_1.$$

Second, we prove that soliciting two ratings is not an equilibrium for all $c \geq \hat{c}_1$ and that soliciting one rating is not an equilibrium for all $c \geq \hat{c}_2$. If investors in equilibrium expect to observe $N$ ratings, and instead the off equilibrium information set is reached in which investors observe only $M < N$ ratings published, investors cannot distinguish if they observed $M$ ratings because the issuer solicited $M$ ratings and published all of them, or because the issuer solicited $H$ ratings, where $M < H \leq N$, and selectively published only $M$ out of the $H$ that were solicited. For this case, the off-equilibrium beliefs are specified as follows: either

a) the primitives of the model are such that the issuer would have no incentive to solicit more than $M$ ratings if investors had anticipated to observe exactly $M$ ratings; in which case investors attach probability one to the path in which the issuer solicited exactly $M$ ratings; or,

b) condition a) is not satisfied, in which case investors attach strictly positive probability to the path in which the issuer solicited $H > M$ ratings and therefore withheld $H - M \geq 1$ ratings. Then, they would believe that the non published rating is of the worst type (minus infinity).

Let $c \geq \hat{c}_1$ and assume soliciting two ratings is an equilibrium. The equilibrium expected profits for the issuer equal $\mu_X - r \sigma^2_{X|S_1, S_2} - 2c$. The issuer could deviate from the strategy of soliciting two ratings and solicit (and publish) only one rating. Since $c \geq \hat{c}_1$, condition a) applies to $M = 1$, and upon observing only one rating, investors would interpret the deviation as a deviation in the number of solicited ratings (from two to one) as opposed to a deviation in the disclosure rule (solicited two and disclosed only one). As a consequence, expected payoff upon deviating equal $\mu_X - r \sigma^2_{X|S} - c$. Since $\hat{c}_1 > c_1$, the deviation would be profitable, as $\mu_X - r \sigma^2_{X|S_1, S_2} - 2c < \mu_X - r \sigma^2_{X|S} - c$ for all $c > c_1$, contradicting the equilibrium. With a similar argument, let $c \geq \hat{c}_2$ and assume soliciting one rating is an equilibrium. The issuer could deviate and solicit no ratings. Upon observing no ratings, since $c \geq \hat{c}_2$, condition a)
applies to $M = 0$, and investors would interpret the deviation as a deviation in the number of solicited ratings (from one to zero) as opposed to a deviation in the disclosure rule (solicited one but disclosed none). Since $\hat{c}_2 > c_2$, such a deviation would be profitable, contradicting the equilibrium.

Third, we prove that soliciting two ratings is an equilibrium for all $c < \hat{c}_1$ and that soliciting one rating is an equilibrium for all $\hat{c}_1 \leq c < \hat{c}_2$. Assume two ratings are solicited and $c < \hat{c}_1$. If the issuer published less than two ratings, since condition a) does not hold neither for $M = 1$ not for $M = 0$, investors would interpret the deviation as a deviation from the disclosure rule, and believe that the withheld information is extremely negative (minus infinity), which would result in a price that is so low that it is never profitable for the issuer to deviate. Next, assume $\hat{c}_1 \leq c < \hat{c}_2$ and that investors expect the issuer to solicit and publish one rating, rating $i$. Since condition a) does not hold for $M = 0$, it is never profitable to publish less than one rating. By the first part of the proof and the off equilibrium beliefs specified there, it is never optimal to solicit rating $j$, nor to solicit two ratings, as $\hat{c}_1 \leq c$. Therefore, soliciting and publishing one rating is an equilibrium for $\hat{c}_1 \leq c < \hat{c}_2$. Finally, by the first part of the proof it follows that soliciting zero ratings is an equilibrium for $c \geq \hat{c}_2$. This completes the proof.

**Proof of Proposition 6**

Assume first

$$\mu_X - V > \mu_X - r\sigma^2_{X|S_1,S_2} - 2\hat{c}_1,$$

(42)

and let $c^* < \hat{c}_1$ be such that

$$\mu_X - V = \mu_X - r\sigma^2_{X|S_1,S_2} - 2c^*.$$

For $c = c^*$, by Proposition 5, both ratings are solicited. Moreover this is a Nash equilibrium because, upon deviating to $c_i > c^*$, soliciting rating $i$ is either suboptimal (if only rating $i$ is solicited) or violates the issuer’s participation constraint (if both are solicited). Rearranging (42) gives the expression in the text of the Proposition. Assume instead that (42) is violated, and let $c < \hat{c}_1$. By proposition 5 both ratings are solicited. Nevertheless, a rating agency could profitably deviate to $c_i = c + \frac{\hat{c}_1 - c}{2}$, as it’s rating will still be solicited. Let $\hat{c}_1 \leq c < \hat{c}_2$ and assume $V$ is such that

$$\mu_X - V \geq \mu_X - r\sigma^2_{X|S_1} - c$$

(43)

By proposition 5, only the rating of agency $i$ will be solicited, which cannot be an equilibrium as agency $i$ can profitably deviate to $c_i = c$ and it’s rating will be solicited. Finally, let $c \geq \hat{c}_2$. By Proposition 5, no rating is solicited, which cannot be an equilibrium because a rating agency could profitably deviate to $c_i < \hat{c}_2$ such that (43) holds.

**Proof of Proposition 7**
We start by considering the case in which there is a single rating agency. We look for an equilibrium in which the issuer solicits one rating with probability \( q \in (0, 1) \) and zero ratings with probability \( 1 - q \). If one rating is solicited, we conjecture there exists a threshold value \( S_m \) such that the solicited rating is published if and only if \( S_i > S_m \). Hence, conditional on observing zero ratings, investors price the asset according to such beliefs, implying:

\[
p_m(\varnothing) = (1 - q)p_0 + q \left[ E \left( X \mid S_i \leq S_m \right) - r \text{Var} \left( X \mid S_i \leq S_m \right) \right], \tag{44}
\]

where \( p_0 \) is given in (2). As \( q < 1 \), the event that no ratings are published is consistent with the possibility that no rating was actually solicited. As no additional information is transmitted in equilibrium, the posterior probability that the rating was not solicited equals the prior probability \( (1 - q) \), as reflected in (44). If, instead, one rating is published, the asset is priced according to the belief that one rating was solicited, that is

\[
p_m(s_i) = p(s_i),
\]

where \( p(s_i) \) is given in (1). In equilibrium, the conjectured disclosure rule has to be optimal given the beliefs, that is, \( S_m \), is such that

\[
p(s_m) - \chi = p_m(\varnothing). \tag{45}
\]

Existence and uniqueness of such an \( S_m \) follows indirectly by Lemma 1. Clearly, since the r.h.s. in the last equation is a constant and \( p(\cdot) \) is an increasing function, disclosure is profitable if and only if \( S_i > S_m \). From (44) we must have \( p_m(\varnothing) \in \left( E \left( X \mid S_i \leq S_m \right) - r \text{Var} \left( X \mid S_i \leq S_m \right), p_0 \right) \), which can be represented equivalently in terms of the following restriction on \( S_m \):

\[
S_m \in \left( S, \mu_X - \sigma_S \left( r \sqrt{\sigma_X^2 - \sigma_X^2|S} - \frac{\chi}{\sqrt{\sigma_X^2 - \sigma_X^2|S}} \right) \right), \tag{46}
\]

The upper limit in (46) follows trivially by imposing \( p(s_m) - \chi < p_0 \), while the lower limit follows by imposing \( p(s_m) - \chi > E \left( X \mid S_i \leq S_m \right) - r \text{Var} \left( X \mid S_i \leq S_m \right) \), and letting \( \hat{S} \) denote the value that solves \( p(s) - \chi = p(\varnothing) \). Notice that (46) guarantees that \( q \in (0, 1) \), and \( q \) can be obtained by solving (44) and (45) for \( q \) giving

\[
q = \frac{p_0 - (p(s_m) - \chi)}{p_0 - \left[ E \left( X \mid S_i \leq S_m \right) - r \text{Var} \left( X \mid S_i \leq S_m \right) \right]} \tag{47}
\]

Ex ante, the issuer must be indeed indifferent between soliciting zero or one rating, that is,

\[
p_m(\varnothing) = -c + E \left[ \max \{p(s_i) - \chi, p_m(\varnothing)\} \right], \tag{48}
\]
which, computing explicitly the expectation and solving for \( c \) can be written as

\[
c = \sqrt{\sigma_X^2 - \sigma_X^2 S} g \left( \frac{\mu_X - \bar{S}_m}{\sigma_S} \right). \tag{49}
\]

The last equation, combined with (46), imply the range of ex-ante costs \( c \) such that, for a given disclosure cost \( \chi \), the equilibrium exists:

\[
c \in \left( \sqrt{\sigma_X^2 - \sigma_X^2 S} g \left( r \sqrt{\sigma_X^2 - \sigma_X^2 S} - \frac{\chi}{\sqrt{\sigma_X^2 - \sigma_X^2 S}} \right), \sqrt{\sigma_X^2 - \sigma_X^2 S} g \left( \frac{\mu_X - \bar{S}}{\sigma_S} \right) \right). \tag{50}
\]

Therefore, for a given \( \chi \), and any \( c \) satisfying (50), this equilibrium is fully characterized by \( \bar{S}_m \) and \( q \) as determined, respectively, by (49) and (47).

Now, consider the same equilibrium but with two rating agencies. The only difference in this case is that, in equilibrium, the option of deviating from this equilibrium and soliciting two ratings must not be profitable, that is, for a given \( \chi \), and \( c \) satisfying (50) we must have

\[
-c + E \left[ \max \{p(\hat{S}_1, \hat{S}_2) - 2\chi, p(\hat{S}_i) - \chi, p_m(\varnothing)\} \right] \leq E \left[ \max \{p(\hat{S}_i) - \chi, p_m(\varnothing)\} \right]. \tag{51}
\]

Notice that, in the l.h.s. of (51), the expectation of future profits from soliciting two ratings takes into account that, if the highest of the two ratings is published, the deviation is not detected by investors and therefore the asset is priced as if only one rating was solicited. Numerical analysis over a wide range of parameter values reveals that (51) is indeed always satisfied.

**Proof of Proposition 8**

We look for an equilibrium in which the issuer solicits two ratings with probability \( q \in (0, 1) \) and zero ratings with probability \( 1 - q \). If two ratings are solicited, we conjecture there exists a threshold value \( \bar{S} \) such that, conditional on both ratings being solicited, both are published if and only if \( \bar{S} > \bar{S} \), where \( \bar{S} \) is defined to be the average rating, \( \bar{S} := (S_1 + S_2)/2 \). Therefore, the price of the asset, conditional on observing zero ratings, is set according to such beliefs, that is:

\[
p_{m(\varnothing, \varnothing)} = (1-q)p_0 + q \left[ E \left( X \mid \bar{S} \leq \bar{S} \right) - r \text{Var} \left( X \mid \bar{S} \leq \bar{S} \right) \right], \tag{52}
\]

where \( p_0 \) is given in (2). As for the previous mixed strategy equilibrium, the same logic implies that, if no rating is published, the posterior probability that the ratings were not solicited equals the prior probability \( (1-q) \). If, instead, both ratings are published, the price equals \( p(\hat{S}_1, \hat{S}_2) \), or, equivalently, from (3) and (10), \( p(\hat{S}, \hat{S}) \). In equilibrium, the conjectured disclosure
rule has to be optimal given the beliefs, that is, \( \hat{S} \) must solve

\[
p_{(\hat{S}, \hat{S})} = p_m(\varnothing, \varnothing). \tag{53}
\]

Existence and uniqueness of such an \( \hat{S} \) follows indirectly by Lemma 1. Ex ante indifference among the two alternatives requires

\[
p_m(\varnothing, \varnothing) = -2c + E\left[\max\{p_{(\hat{S}, \hat{S})}, p_m(\varnothing, \varnothing)\}\right],
\]

which, computing explicitly the expectation solving for \( c \) can be written as

\[
c = \frac{1}{2} \sqrt{\sigma_X^2 - \sigma_{X|S_1,S_2}^2} \left( \frac{\mu_X - \hat{S}}{\sqrt{\sigma_X^2 + \sigma_\varepsilon^2/2}} \right). \tag{54}
\]

Equations (52) and (53) imply \( p_{(\hat{S}, \hat{S})} < p_0 \), or, equivalently \( \hat{S} < p_0 \), which imposes the following restriction on ex-ante costs \( c \) such that the equilibrium exists:

\[
c > \frac{1}{2} \sqrt{\sigma_X^2 - \sigma_{X|S_1,S_2}^2} \left( r \sqrt{\sigma_X^2 - \sigma_{X|S_1,S_2}^2} \right).
\]

For any value \( c \) satisfying the last inequality the equilibrium is fully characterized by \( \hat{S} \) defined in (54) and \( q \), obtained from (52) and (53) as

\[
q = \frac{p_0 - p_{(\hat{S}, \hat{S})}}{p_0 - E(X|\hat{S} \leq \hat{S}) - rVar(X|\hat{S} \leq \hat{S})}.
\]

This equilibrium is supported by off-equilibrium beliefs that, upon observing only one rating, the issuer deviated from the disclosing rule (that is, the issuer solicited two ratings but published only the highest rating) as opposed to the soliciting rule (the issuer solicited only one rating). Soliciting one rating is therefore never profitable.

**Proof of Proposition 9**

We look for an equilibrium in which the issuer solicits two ratings with probability \( q \in (0, 1) \) and one rating with probability \( 1 - q \). In equilibrium at least one rating is always published, and, upon soliciting two ratings, the issuer publishes only the highest of the two if \( \hat{S}_2 < s_m(\hat{S}_1) \), for some function \( s_m(\cdot) \), and publishes both ratings otherwise. Accordingly, the price function, conditional on observing only one rating is

\[
p_{(S_i, \varnothing)} = (1 - q(S_i))p_{(S_i)} + q(S_i)\left[ E(X|S_i, S_j \leq s_m(S_i)) - rVar(X|S_i, S_j \leq s_m(S_i)) \right],
\]

where \( p_{(S_i)} \) is given in (1), and \( q(S_i) \) is the posterior probability that two ratings were solicited,
obtained applying Bayes’ rule consistently with equilibrium beliefs:

\[
q(S_i) = \frac{q \times \text{prob}(S_i | \text{two ratings solicited})}{q \times \text{prob}(S_i | \text{two ratings solicited}) + (1 - q) \times \text{prob}(S_i | \text{one rating solicited})}
\]

\[
= \frac{q \frac{2}{\sigma S} f \left( \frac{S_i - \mu}{\sigma S} \right) \Phi \left( \frac{s_m(S_i) - \mu(S_i)}{\sigma S | S_i} \right)}{q \frac{2}{\sigma S} f \left( \frac{S_i - \mu}{\sigma S} \right) \Phi \left( \frac{s_m(S_i) - \mu(S_i)}{\sigma S | S_i} \right) + (1 - q) \frac{1}{\sigma S} f \left( \frac{S_i - \mu}{\sigma S} \right)}.
\]

Conditional on two ratings being published, the price function is simply \(p(\hat{S}_1, \hat{S}_2)\). The definition of the function \(s_m(\cdot)\) follows the same logic of the proof of Lemma 2. We proceed in two steps: first we define the threshold value \(\hat{S}_{1m}\) as the real number such that

\[
P(\hat{S}_{1m}, \hat{S}_{1m}) = p_m(\hat{S}_{1m}, \omega), \tag{55}
\]

where \(s_m(\cdot)\) in the r.h.s. of (55) is taken to be the identity function. Then, we conjecture \(s_m(\cdot)\) has the following form: for \(\hat{S}_1 \leq \hat{S}_{1m}, s_m(y) = y\) and for \(\hat{S}_1 > \hat{S}_{1}, s_m(y)\) is implicitly defined by

\[
p(S_i, s_m(S_i)) = (1 - q(S_i)) p(S_i) + q(S_i) \left[ E(X | S_i, S_j \leq s_m(S_i)) - \alpha \text{Var}(X | S_i, S_j \leq s_m(S_i)) \right]. \tag{56}
\]

By the same logic in the proof of Lemma 2, the conjectured disclosure rule is indeed optimal for the issuer given the specified beliefs. Finally, we have to impose the ex-ante indifference condition for the issuer:

\[-c + E(p(S_i, \omega)) = -2c + E \left[ \max \{p(\hat{S}_1, \hat{S}_2), p(\hat{S}_1, \omega)\} \right],
\]

or

\[c = E \left[ \max \{p(\hat{S}_1, \hat{S}_2), p(\hat{S}_1, \omega)\} \right] - E \left( p(S_i, \omega) \right). \tag{57}
\]

For a given cost \(c\), therefore, an equilibrium is given by a probability \(q\) and a function \(s_m(\cdot)\) such that (55), (56) and (57) hold. Existence of this equilibrium requires the following parameter restriction. It can be verified, from the construction of the equilibrium, that the r.h.s. of (57) is a strictly increasing function of \(q\), with

\[
\lim_{q \uparrow 1} \left\{ E \left[ \max \{p(\hat{S}_1, \hat{S}_2), p(\hat{S}_1, \omega)\} \right] - E \left( p(S_i, \omega) \right) \right\} = \infty
\]

\[
\lim_{q \downarrow 0} \left\{ E \left[ \max \{p(\hat{S}_1, \hat{S}_2), p(\hat{S}_1, \omega)\} \right] - E \left( p(S_i, \omega) \right) \right\} = \hat{c},
\]

with \(\hat{c} = E \left[ \max \left\{ p(\hat{S}_1), p(\hat{S}_1, \hat{S}_2) \right\} - p(S_i) \right]\). Therefore, for this equilibrium to exist, it has to be \(c > \hat{c}\). This equilibrium is supported by off-equilibrium beliefs that, upon observing no
published rating, the issuer deviated from the disclosing rule (that is, the issuer solicited one or two ratings but published none) as opposed to the soliciting rule (the issuer solicited no ratings). Soliciting no ratings is therefore never profitable.
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