Is the Volatility of the Market Price of Risk due to Intermittent Portfolio Re-balancing? by Chien, Cole, and Lustig

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February 2010

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- Why Sharpe ratio and equity premium are high in average (1 to 4).
- Why Sharpe ratio is countercyclical w/ intermittent rebalancing (1, 4 to 6).
- Other issues: leverage, highest Sharpe ratio, level of constraints.

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- Passive non Participants: 50%, can NOT hold equity, $\varpi^* = 0$,
  save in uncontingent bonds, subject to idiosyncratic uninsurable risk.

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- Then, we obtain a **representative agent** economy, regardless of size of Passive non Participants.
- Result follow because dividend and labor income are perfectly correlated.
- Adding Passive continuous rebalancers does not change result if $\varpi^* = \frac{1}{1+\psi}$.

Second Non-Reason for High Sharpe Ratio

- Assume that:
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- Then, interest rates may be lower, but the multiplicative equity premium is the same as in **Representative Agent Economy** .
- It is as if the economy has a different constant value of $\hat{\beta}$ and no aggregate shocks.
- Equivalence uses the multiplicative nature of η_t and CRRA: normalize consumption by Y_t.
- Since $\hat{\beta}$ is constant, time invariant exposure to aggregate shock.
- In this case for continuous rebalancers holding ϖ_t fixed is optimal.

High Average Sharpe Ratio and Equity Premium

- Leverage of Active Traders
  - passive non participants save uncontingent, due to precautionary reasons.
  - Hence, in equilibrium, active traders must borrow uncontingent.
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  - priced by agent with max MRS across states (Luttmer)
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  - see Table for case with complete traders.

Countercyclical Risk Prices

- Consider Intermittent Rebalacing Traders
- in non-rebalancing periods, they have no access to dividends.
- Active Traders absorb a disproportional share of the changes
- in an recession (expansion), the risk prices are high (low).
- describe mechanism in detail.

High Sharpe Ratio After a Bad Aggregate Shock

irb trader (vs active trader) in non-rebalancing period $t-1$:

- buy (sell) more equity with dividends.
- finance consumption by borrowing against equity in 'broker account' .

irb trader (vs active trader) in rebalancing period t:

- sell (buy) extra equity to repay loan. Reverse the positions.

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b_{t-1} + c_{t-1} &= \gamma Y_{t-1}\eta_{t-1} + b_{t-2}R_{t-2} \\
s_t V_t + b_t + c_t &= \gamma Y_t \eta_t + b_{t-1}R_{t-1} + s_{t-2}\left(1 + \frac{D_{t-1}}{V_{t-1}}\right)(V_t + D_t) \\
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- Random case: after a **low dividend** , active trader **sell less equity** : and thus he is more exposed to further shocks (more leveraged).

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- Is the evidence in the paper the right type?
  Most household don't trade in equities often, and lots of trade involve no net cash flow (Inv. Compay Inst.)