The Dynamic Consequences of Incentive Schemes: Evidence from Salesforce Compensation

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Abstract

We develop, estimate, and use a dynamic structural model of agent behavior under a nonlinear period dependent output based contract, to measure the cost of dynamic moral hazard induced by the incentives of agents to time their allocation of effort. We assess policies by which firms may mitigate such perverse timing-related incentives. Our framework is flexible enough to handle the observed features of real-world incentives schemes. We utilize a rich dataset that involves complete details of sales, and compensation plans for a set of 90 sales-people for a period of 4 years at a large consumer-product company in the US. Our estimates from the data suggest significant incentives for strategic timing arising from the structure of the compensation scheme.

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1 Introduction

There is now a growing literature that has used firms’ internal databases to understand the role of incentives in motivating workers (see Lazear 2000a for a recent review). Much of this literature has focused on the role of output-based incentives in solving the agency problem for the principal: when effort cannot be monitored, output-based contracts can address the moral hazard problem induced by agents’ hidden actions.\(^1\) The optimal contract under this situation must balance risk and insurance, and is typically smooth, and nonlinear in output (e.g. Holmstrom 1979; Grossman and Hart 1983). These nonlinearities however, generate a countervailing cost for the principal, by generating a dynamic incentive for the agent to perversely manipulate the timing of effort. Such dynamic moral hazard, if significant, may well negate the positive effect of output based contracts to a firm. While the empirical literature on static moral hazard is rich, the literature on measuring such dynamic moral hazard is relatively sparse.

The goal of this paper is to develop, estimate, and use a dynamic structural model of agent behavior under a nonlinear period dependent output based contract, to measure the cost of such dynamic moral hazard, and to assess policies by which firms may mitigate these. Our framework is explicitly concerned with the dynamics induced by such contracts, and is flexible enough to handle the observed features of real-world incentives schemes. We consider the latter aspect especially important to a credible measurement of the cost of moral hazard. As Stiglitz (1991), and Ferrall and Shearer (1999) note, observed contracts in the real world, seldom follow the smooth, mathematically complex formulae prescribed as optimal by the theory. For instance, contracts for compensating sales-forces - the focus of our empirical application - are ubiquitously discrete and kinked, featuring quotas, bonuses and ceilings. In a survey of Fortune 500 firms, Joseph and Kalwani (1998) report that 95% of compensation schemes they survey had some combination of quotas and commissions, or both.\(^2\) These aspects complicate the analysis by generating dynamics in the actions of agents. A proper accounting of these dynamics then becomes critical to the evaluation and

\(^1\)An alternative motivation of output-based contracts is that it may help attract and retain the best sales-people (Lazear 1986; Godes 2003; Zenger and Lazarini 2004). This paper abstracts away from these issues since our data does not exhibit any significant turnover in the sales-force.

\(^2\)Quotas specify discontinuous, nonlinear compensation policies for agents when their output crosses pre-specified thresholds.
improvement of the incentive scheme.

Our empirical setting pertains to contracts for compensating sales-force agents at an American Fortune 500 company. Personal selling via such sales-forces is now an important part of the economy. In a review of sales-force practice, Albers and Mantrala (2008) note, “Dartnell’s 30th Sales Force Compensation Survey: 1998–1999 reports the average company spends 10% and some industries spend as much as 40% of their total sales revenues on sales force costs. In total, the US economy is estimated to spend $800 billion on sales forces, almost three times the amount spent on advertising in 2006 (Zoltners et al. 2008)”. To illustrate the nature of dynamic moral hazard in these contracts, consider the following example of a typical sales-force incentive scheme: agents are paid a guaranteed salary plus a lump-sum bonus if sales are higher than a pre-specified quota each quarter. Under this scheme sales-agents who achieve the level of sales required for the bonus in the current compensation cycle may have a perverse incentive to postpone additional effort. This enables the agent to use the sales generated from the postponed effort to attain the quota in the next compensation cycle.

Our assessment of the cost of such dynamic inefficiencies, is related to three particular strategies that the extant literature suggests could be used by the principal to address these. These comprise of linearity, ratcheting and periodicity in the incentive scheme. We discuss each in turn below. The contracts we observe at the focal firm in our empirical application features ratcheting and a quarterly periodicity, but has non-linear compensation plans.

Linearity implies a simple plan that specifies compensation as a linear function of output. In an important paper, Holmstrom and Milgrom (1987), showed that under the specific assumptions, and considering an agent who does not value the timing of payments, a linear scheme can achieve the second best outcomes for the firm. In their model, a linear scheme leads to the right choice of effort for the agent, because it applies the same incentive pressure on the agent no matter what his past performance has been (i.e. no dynamic moral hazard). We use our model to measure how much better off the firm may be under an unconstrained counterfactual linear contract, and under a constrained linear contract in which the firm pays out the same total

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3Sales or revenue targets are termed ‘quotas’ in the industry and in the marketing literature.
4Similar distortions can take place with a commission based plan (as in our application) when coupled with compensation plans that involve discontinuities based on quota ceilings and floors.
incentive compensation to its employees as its current contract. These metrics serve as our measure of the cost of dynamic moral hazard.

A second strategy for the firm to use subjective quotas that are updated according to the agent’s currently observed performance (Lazear 1986). Such a policy, sometimes referred to as “ratcheting”, is a common feature of several real world compensation schemes (e.g. Weitzman 1980; Leone, Misra and Zimmerman 2004), and is also a feature of the plan used by the firm in our empirical application. Ratcheting permits the firm to adjust the quota based on past performance and thereby permits learning and error correction. Further, the fact that quotas can be ratcheted up discourages agents from moving future sales to current periods. However, ratcheting has a disadvantage in that it may potentially accentuate dynamic inefficiencies if forward-looking agents manipulate current effort so as to obtain favorable quotas in the future. A priori therefore, the merits or demerits of a ratcheting as a policy to mitigate dynamic inefficiencies is an empirical question. Hence, a second question we consider is how much the firm would be better off from moving to a no-ratcheting policy. This metric assess the value (or cost) to the firm of a commitment to not changing the contract.

A third aspect we consider is the period-dependency of compensation plans. We assess whether longer or shorter incentive horizons can mitigate intertemporal shifting of effort. In particular, many compensation contracts specify quarterly, monthly or yearly periods during which marginal compensation can be earned on generated sales. Long periods reduce timing and deadline-related considerations, but may generate undesirable shirking in the early part of the compensation cycle. Short-periods reduce the shirking incentive, but may re-introduce deadline effects on a more frequent basis. Hence, our final assessment pertains to the extent to which the firm would be better off by changing the periodicity of its compensation plan.

Evaluating such policies requires a framework that incorporates the features of real world compensation schemes, and the dynamic incentives generated by them. The extant models in the literature are inadequate for analyzing the counterfactuals because actual compensation schemes contain several aspects - kinked profiles, discrete bonuses, finite horizons, heterogeneity across agents and significant demand uncertainty - that are hard to accommodate into a single all-encompassing theoretical framework, but nevertheless, significantly impact agent behavior. Developing randomized experiments in which alternative compensation policies are implemented
may not be feasible in several situations. Our approach is to develop a structural
model of agent behavior that incorporates the forward-looking behavior in which
agents allocate effort over time by maximizing a discounted stream of payoffs. We
outline methods to estimate the parameters indexing the model and show how the
model can be solved using numerical dynamic programming techniques.

Estimation of the model is complicated by the fact that effort is unobserved. We
introduce a methodology that exploits the richness of our data, an informative struc-
ture, and recent advances in estimation methods to facilitate the identification of
this latent construct. In particular, following the intuition in Copeland and Monnet
(2007), we describe how intertemporal linkages helps identify effort from sales data
in salesforce compensation settings. We model agents as maximizing intertemporal
utility, conditional on the current compensation scheme, and their expectations about
the process by which quotas would be updated based on their chosen actions. Our
empirical approach is to estimate, in a first stage, the structural parameters involving
the sales person’s utility function. We then simulate, in a second stage, her behav-
ior given a changed compensation profile. The estimator for the first stage of our
empirical strategy is based on the recent literature on 2-step estimation of dynamic
decisions (Hotz and Miller 1993; Bajari, Benkard and Levin 2007). Our approach is
to non-parametrically estimate agent-specific policy functions, and use these, along
with the conditions for the optimality of the observed actions, to estimate the struc-
tural parameters. We use our estimates to generate the empirical distribution of
agent preferences, which we use to simulate the behavior of the agent-pool under
counterfactual compensation profiles.

A practical concern with the use of two-step estimators has been the presence of
unobserved serially-correlated state variables which prevent consistent non-parametric
estimation of first-stage policy functions and transitions. In particular, this ruled out
models with unobserved heterogeneity (though see Arcidiacono & Miller 2008 for a
recent approach that handles discrete unobserved heterogeneity). We are able to ad-
dress this problem due to the availability of panel data of relatively long cross-section
and duration for each agent, which facilitates estimation agent-by-agent. This enables
a non-parametric accommodation of unobserved heterogeneity. Given the estimates
from the first stage, we evaluate agent behavior and sales under the counterfactual
by solving the agents’ dynamic programming problem numerically. We believe we
are the first in the empirical literature to model the intertemporal problem facing
sales-agents and to measure the dynamic effect of quotas and ratcheting in a real
world setting.

Our model-free analysis of the data reveals significant evidence for strategic timing
considerations by sales-agents. In particular, we find evidence that high effort levels
are extended as the agent strives to “make quota” within the quarter, but that effort
is adjusted downward once the agent is “in the money”. Our results indicate that the
current compensation scheme is highly leveraged - the equivalent, linear contract that
achieves the same quarterly sales on average as the current policy, requires 6.32%,
commission on sales. We also find that ratcheting does little to correct within period
effort shading while changes to the contract by incorporating linearity or moving to a
monthly interval are profit enhancing. Our specific recommendations to the firm were
put in place at the beginning of the year and initial feedback reveals that there have
been significant improvements in revenues, profits and employee satisfaction since the
implementation of these recommendations.

Our paper adds into a small empirical literature that has explored the dynamic ef-
ects of incentive schemes. Despite the preponderance of nonlinear incentive schemes
in practice, the empirical literature analyzing these, and the effect of quotas on sales-
force effort in particular, has remained sparse. Part of the reason for the paucity of
work has been the lack of availability of agent-level compensation and output data.
The limited empirical work has primarily sought to provide descriptive evidence of
the effect of compensation schemes on outcomes (e.g. Healy 1985, in the context of
executive compensation, Asch 1990, in the context of army-recruiters paid via non-
linear incentives; and Courty and Marschke 2004 ***). Oyer (1998) was the first to
empirically document the timing effects of quotas, by providing evidence of jumps in
firms’ revenues at the end of quota-cycles, that are unrelated to demand-side factors.

A related literature also seeks to empirically describe the effect of incentives, more
broadly, on output (e.g. Chevalier and Ellison 1999; Lazear 2000a; Hubbard 2003;
Bandiera, Baransky and Rasul 2005; see Pendergast 1999 for a review). We comple-
ment this literature by detecting and measuring the dynamic inefficiencies associated
with compensation schemes. The descriptive evidence on quotas are mixed. Using
data from a different context, and a different compensation scheme, Steenburgh (2008)
reports descriptive evidence that agents facing quotas in a durable-goods company do
not tend to reduce effort in response to lump-sum bonuses. In contrast, Larkin (2006)
uses reduced form methods to document the distortionary effects of compensation
schemes on the timing and pricing of transactions in technology-markets. Our paper is also related to the work of Ferrall and Shearer (1999), and Paarsch and Shearer (2000), who estimate static, structural models of worker behavior, while modeling the optimal contract choice by the firm. Unlike our context, the compensation contracts in their papers are linear, and do not generate dynamic incentives for the agent. The closest paper to ours in spirit is Copeland and Monnet (2007) who estimate a dynamic model to analyze the effects of non-linear incentives on agents’ productivity in sorting checks. Our institutional context, personal selling by salesforce agents, adds several aspects that warrant a different model, analysis, and empirical strategy from Copeland and Monnet’s context. Unlike their industry, demand uncertainty plays a key role in our setting; this generates a role for risk aversion, and a trade-off between risk and insurance in our contracts. Further, ratcheting, an important dynamic affecting agent effort in our setting, is not a feature of their compensation scheme. Ratcheting generates a dynamic across compensation periods, in addition to dynamics induced within the period by the nonlinearity. Finally, this paper complements papers related to the analysis of managerial compensation (e.g. Jensen and Murphy, 1990; Margiotta and Miller, 2000).

The rest of this paper is structured as follows: We begin with a description of our data and some stylized facts. We then introduce our model followed by the estimation methodology. Finally we present counterfactuals and conclude with a discussion.

2 Patterns in the Data and Stylized Facts

In this section, we start by presenting some stylized facts of our empirical application, and also provide model-free evidence for the existence of strategic timing by sales-agent in our data. We use the reduced form evidence and the stylized facts presented here to motivate our subsequent model formulation and empirical strategy.

2.1 Data and Compensation Scheme

Our data come from the direct selling arm of the salesforce division of a large consumer-product manufacturer in the US with significant market-share in the focal category (we cannot reveal the name of the manufacturer, or the name of the category due to confidentiality reasons). The category of interest involves a non-pharmaceutical product available via prescriptions to consumers from certified physi-
cians. Importantly, industry observers and casual empiricism suggests that there is little or no seasonality in the underlying demand for the product. The manufacturer employs 87 sales-agents in the U.S. to advertise and sell its product directly to each physician (also referred to as a “client”), who is the source of demand origination. Agents are assigned their own, non-overlapping, geographic territories, and are paid according to a non-linear period-dependent compensation schedule. We note in passing that prices play an insignificant role since the salesperson has no control over the pricing decision and because price levels remained fairly stable during the period for which we have data.\(^5\) The compensation schedule involves a fixed salary that is paid irrespective of the sales achieved by the agent, as well as a commission on any sales generated above a quota, and below a ceiling. The salary is paid monthly, and the commission, if any, is paid out at the end of the quarter. The sales on which the output-based compensation is earned are reset every quarter. Additionally, the quota may be updated at end of every quarter depending on the agent’s performance (“ratcheting”). Our data includes the complete history of compensation profiles and payments for every sales-agent, and monthly sales at the client-level for each of these sales-agents for a period of about 3 years (38 months).

Quarterly, kinked compensation profiles of the sort in our data are typical of most real world compensation schemes, and have been justified in the theory literature as a trade-off between the optimal provision of incentives versus the cost of implementing more complicated schemes (Raju and Srinivasan 1996), or as optimal under specific assumptions on agent preferences and the distribution of demand (Oyer 2000). Consistent with the literature, our conversations with the management at the firm revealed that the primary motivation for quotas and commissions is to provide “high-powered” incentives to the salesforce for exerting effort in the absence of perfect monitoring. We also learned that the motivation for maintaining a “ceiling” on the compensation scheme stemmed from a desire to hedge against large “windfall” payouts to agents, which may result from large changes in demand due to reasons unrelated to agent effort. The latter observation suggests that unanticipated shocks to demand are likely important in driving sales.

The firm in question has over 15,000 SKU-s (Stock Keeping Units) of the product.

\(^5\)In other industries, agents may have control over prices (e.g. Bharadwaj 2002). In such situations, the compensation scheme may also provide incentives to agents to distort prices to “make quota”. See Larkin (2006), for empirical evidence from the enterprise resource software category.
The product portfolio reflects the large diversity in patient profiles, needs and usage characteristics for the product. The product portfolio of the firm is also characterized by significant new product introduction and line extensions reflecting the significant investments in R&D and testing in the industry. New product introductions and line extensions reflect both new innovations as well as new usage regimens for patients uncovered by fresh trials and testing. The role of the sales-agent is primarily informative, by providing the doctor with updated information about new products available in the product-line, and by suggesting SKU-s that would best match the needs of the patient profiles currently faced by the doctor. While agents’ frequency of visiting doctors is monitored by the firm, the extent to which he “sells” the product once inside the doctor’s office cannot be monitored or contracted upon. In addition, while visits can be tracked, whether a face-to-face interaction with a doctor occurs during a visit is within the agent’s control (e.g., an unmotivated agent may simply “punch in” with the receptionist, which counts as a visit, but is low on effort). In our application, we do not separately model these dimensions of sales-calls, and interpret all factors by which an agent shifts a doctor’s sales as effort.

2.2 The Timing of Effort

Our primary interest is in the extent to which nonlinearities in the compensation schemes provide incentives to salespeople to manipulate the timing of transactions. We start by looking in the data to see whether there exists patterns consistent with such strategic timing. As Oyer (1998) pointed out, when incentives exist for agents to manipulate timing, output (i.e. sales) should look lumpy over the course of the sales-cycle. In particular, we expect to see spikes in output when agents are close to the end of the quarter (and most likely to be close to “making quota”). Figure 1 plots the sales achieved in each month by a set of sales-agents. Figure 1 reveals significant spikes at the end of quarters suggesting that agents tend to increase effort as they reach closer to quota. In Figure 2, we present analogous plots that suggest that agents also tend to reduce effort within the quarter. We plot patterns in sales (normalized by total sales across all months in the data) for four agents. The shaded regions in Figure 2 highlights quarters in which sales fell in the last month of the quarter, perhaps because the agent realized a very large negative shock to demand early in the quarter.

6The firm does not believe that sales-visits are the right measure of effort. Even though sales-calls are observed, the firm specifies compensation based on sales, not calls.
and reduced effort, or because he “made quota” early enough, and hence decided to postpone effort to the next sales-cycle. We now explore how these sales-patterns are related to how far the agent is from his quarterly quota. Figure 3 shows nonparametric estimates of the relationship between sales ($y$-axis) and the distance to quota ($x$-axis), computed across all the sales-people for the first two months of each quarter in the data. We define the distance to quota as $(\text{Cumulative Sales at beginning of month-quota}) / \text{quota}$. From Figure 3, we see that the distance to quota has a significant influence on the sales profile. Sales (proportional to effort) tend to increase as agents get closer to quota, suggesting increasing effort allocation, but fall once the agent reaches about 40% of the quota in the first 2 months, suggesting the agent anticipates he would “make the quota” by the end of the quarter. The decline in sales as the agent approaches quota is also consistent with the ratcheting incentive, whereby the agent reduces effort anticipating his quota may be increased in the next cycle, if he exceeds the ceiling this quarter. To further explore the effect of quotas, we present in Figure 4.

\footnote{One alternative explanation for these patterns is that the spikes reflect promotions or price changes offered by the firm. Our extensive interactions with the management at the firm revealed that prices were held fixed during the time-period of the data (in fact, prices are rarely changed), and no additional promotions were offered during this period.}
Patterns suggesting agent ‘gave up’

Figure 2: Agents reduce effort within quarters

Figure 3: Sales vs distance to quota
non-parametric plots of the % quota attained by the end of month $T - 1$ versus the % quota attained by the end of month $T (= 2, 3)$, across all agents and quarters. Figure 4 suggests patterns that are consistent with intertemporal effort allocation due to quotas. In particular, when far away from quota in month $T - 1$ ($x \in 0.2, 0.4$), the profile is convex, suggesting a ramping up of effort. When the agent is close to quota in month $T - 1$ ($x \in 0.5, 0.8$), the profile is concave suggesting a reduction in the rate of effort allocation. Finally, figure 4 also shows that most agents do not achieve sales more than $1.4 \times$ quota, which is consistent with the effect of the ceiling (which was set to be $1.33 \times$ quota by the firm during the time-period of the data).

Figure 5 presents the analogous relationship, with plots for each agent in the data. Figure 5 shows that the concavity that we uncover is robust, and is not driven by pooling across agents.

2.3 Alternative Explanations: Seasonality, Buyer-side Stockpiling

In the results above, we have documented that kinked sales patterns related to quota-deadlines exist, and that these are correlated with the agent’s distance to quota. We now consider whether these patterns are due to alternative phenomena unrelated to
the effects of compensation schemes. The two leading explanations are a) demand side seasonality; and b) buyer side stockpiling. In the remainder of this section, we discuss how the institutional features of our setting, as well as the availability of some additional data enable us to rule out these explanations.

A priori, seasonality is not a compelling consideration due to the fact that the disease condition that our product treats is non-seasonal. Patient demand for the product tends to be flat over the year. Our extensive discussions with sales-agents as well as the management at the firm suggest that stockpiling by clients (i.e. doctors) is also not a relevant consideration in this category. First, as noted before, there are a large number of SKU-s available from the firm (about 15,000). The doctor is concerned about patient satisfaction and health, both of which are strongly linked to finding an exact match between the patient’s needs and the right SKU from this large product set. Ex ante, the distribution of patient profiles, needs and usage characteristics arriving at his office for the coming month is uncertain. These considerations precludes stockpiling of SKU-s at the doctors office. The firm solves this supply-chain problem by shipping the product directly to the consumer from its warehouse, upon receipt of an online order from the doctor made at his office during a patient
appointment.

We now show how these aspects are borne out in our data. To explore these effects, we start by augmenting our sales and compensation-data with some limited information available to us on the number of sales-calls made by each agent at each client every month. This “call” data is available for a subset of the time-period of the data (the last 18 months). The sales-call is not a decision variable for the agent; specifically, neither the number of calls nor the allocation of calls across clients is under the control of the agent. Using technology that tracks location, management is able to monitor the number and incidence of calls made by each agent (but not the quality/content of each call), and pre-specifies the number of calls every agent has to make each month. Each agent adheres closely to this top-down management specification. Further, management also specifies that the proportion of calls made to each client type faced by agents be the same. An extensive analysis of the sales-calls data, available on request, finds evidence corroborating these aspects of management policy.

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Notes: 1Nobs = 210,615. 2Dependent variable is Sales (in $) per week achieved by an agent in a given month at a given client. 3A financial quarter has 13 weeks. The firm codes months within the quarter such that months 1 & 2 have 4 weeks each, while month 3 has 5 weeks. The effect of weeks is controlled for by looking at sales/week. 4Robust standard errors, clustered at the client level reported. 5Client fixed effects are equivalent to accommodating sales-agent fixed effects as clients do not overlap across agents. Each agent has a separate territory.

Figure 6: Fixed Effects Regressions of Sales per week across Agent-Months

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8 The sales territories that agents are assigned to are charted such that agents may serve a different number of clients but were required to make the same number of total calls. This is accomplished by a categorization of clients into multiple tiers with the number of calls required varying across tiers.
We use the call data for the 18 months in conjunction with the sales information to explore the role of seasonality and stockpiling. Figure (6) reports on fixed effects regressions of the sales per week achieved each month by agents, controlling for client fixed effects (client fixed effects are equivalent to agent-fixed effects as agents have exclusive territories). Column [1] of Figure (6) shows that at the client level, on average, agents generate about $781 of prescriptions per client in the first month of the quarter. Column [1] also shows that output in month 3 is higher: incremental prescriptions of about $70 ($14.1 per week × 5 weeks), are written in the third month of every quarter, potentially responding to higher sales-effort. This effect however is the sum total of both true agent effort, as well as any other factors affecting sales. To measure the effect of effort, column [2] adds sales-calls into the regression; we see that sales-calls (an indirect proxy for effort), is significant in explaining sales. Column [3] now asks whether the productivity of these sales-calls is different across months of the quarter. Strikingly, we see that the productivity of sales-calls is significantly larger in the third month of the quarter compared to the beginning of the quarter (22.6 in month 3 vs. 0). This is not a scaling issue as the total number of calls per month is fixed by the firm, and is a constant. These results suggest that more effort is put into sales-calls later in quarter than early in the quarter, and suggest potential shirking in month 1.

To test for seasonality, we now exploit the behavior of physicians when they are not exposed to any sales-calls. We test whether, all else equal, the sales in months 1-3 of physicians when they are not exposed to any calls are statistically significantly different from one another. If in the absence of being called on (i.e. no sales force effort), say month 3 sales are higher, we would ascribe that to potential seasonality in underlying demand for the product. Column [3] reports on a test of whether $I(month2) = I(month3) = 0$. We find that the null is not rejected, implying no seasonality, consistent with the anecdotal evidence for the category.

We now test for stockpiling, or intertemporal substitution more generally, at the client level. We present two sets of results. First, we first add lagged sales into the model in column [4]. We find that lagged sales is not significant suggesting little stockpiling. We base our second test on the following idea: if stockpiling is important, we would find that sales will be lower than average in quarters which follow quarters in which sales were higher than average.

Figure (7) reports on results from fixed effects regressions in which we regress the
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Figure 7: Testing Buyer Stockpiling

sales in the 1st month of quarter \( \tau \) on the sales on the last month of quarter \((\tau - 1)\). Column [3] of Figure (7) shows that the lagged sales variable is not significant. We repeat the regressions by using the sales in the 2nd and the 3rd month of quarter \( \tau \) as the dependent variables (columns 2 and 1). The variables are not significant. Although not reported, these results hold if we use the total sales in quarter \( \tau \) and/or \((\tau + 1)\) as dependent variables.

Taken together, these results suggest that seasonality and buyer intertemporal substitution are not significant considerations for these data. These features arise from the specifics of our empirical setting. We anticipate that both these aspects are likely to be important in other situations, for instance, those involving durable-good selling, where intertemporal substitution is well known to be significant. Our conversations with the firm indicates then that much of the effect of agent-effort in our context is to generate brand-switching in the prescriptions written by the doctor. Several substitutes are available in the market for the products of our focal firm, and substitution effects are strong.\(^9\) Taken together, the above model-free evidence also point to the existence of significant effects of the compensation scheme on agent’s intertemporal effort allocations in these data, and motivates the dynamics incorporated into the model of agent effort.

\(^9\)We cannot fully rule out competitive effects arising from the co-movement in compensation policies of competing firms. It could be that high effort regimes for the focal firm coincides with low-effort regimes for competitors. Then, the sales effect in the 3rd month is a function of both the high effort induced by the focal firms’ compensation, as well the low effort of competing agents induced by their compensation policies. Our informal evidence suggest this is not the case (the closest competitor pays its agents on a monthly plan). Nevertheless, int he absence of competitive data, we cannot cast more light on this issue.
Discussion  Our above discussion highlights three facts regarding salesperson effort: (i) Salespeople are forward looking in that they allocate current effort in anticipation of future rewards; (ii) they act in response to their current quarter compensation environment by increasing and reducing effort relative to their quarter goals; and, (iii) salespeople take into account the impact of their current actions on subsequent changes in future firm compensation policies. These facts will play key roles in the development of our formal model of dynamic effort allocation. We discuss this next.

3  A Model of Dynamic Effort Allocation

We consider the intertemporal effort allocation of an agent facing a period-dependent, non-linear compensation scheme. The compensation scheme involves a salary, \( \alpha_t \), paid in month \( t \), as well as a commission on sales, \( \beta_t \). The compensation scheme is period-dependent in the sense that it specifies that sales on which the commission is accrued is reset every \( N \) months. The compensation scheme is non-linear in the sense that the commission \( \beta_t \) may depend discontinuously on the extent to which his total sales over the sales-cycle, \( Q_t \), exceeds a quota, \( a_t \), or falls below a ceiling \( b_t \). The extent to which the ceiling is higher than the quota determine the range of sales over which the agent is paid the marginal compensation. While our framework is general enough to accommodate compensation schemes where \( \{ \alpha_t, \beta_t, a_t, b_t \} \) change over time, our empirical application has the feature that the salary, \( \alpha \) and the commission-rate, \( \beta \) are time-invariant, and that the ceiling \( b_t \) is a known deterministic function of the quota \( a_t \). We develop the model in the context of this simpler compensation plan. The choice of the structure of the incentive scheme by the firm is determined by reasons outside of our model. Our approach will be to solve for the agent’s effort policy taking the firm’s compensation policy as given, and to use the model to simulate agent-effort for counterfactual compensation profiles. Let \( I_t \) denote the months since the beginning of the sales-cycle, and let \( q_t \) denote the agent’s sales in month \( t \). Further, let \( \chi_t \) be an indicator for whether the agent stays with the firm. \( \chi_t = 0 \) indicates the agent has left the focal company and is pursuing his outside option.\(^{10}\) The total sales, \( Q_t \), the current quota, \( a_t \), the months since the beginning of the cycle \( I_t \), and his employment

\(^{10}\)We assume that once the agent leaves the firm, he cannot be hired back (i.e. \( \chi_t = 0 \) is an absorbing state)
status $\chi_t$ are the state variables for the agent’s problem. We collect these in a vector $s_t = \{Q_t, a_t, I_t, \chi_t\}$, and collect the observed parameters of his compensation scheme in a vector $\Psi = \{\alpha, \beta\}$.

### 3.1 Actions

At the beginning of each period, the agent observes his state, and chooses to exert effort $e_t$. Based on his effort, sales $q_t$ are realized at the end of the period. We assume that the sales production function satisfies three conditions.

1. Current sales is a strictly increasing function of current effort.
2. Current sales are affected by the state variables only through their effect on the agent’s effort.
3. Unobservable (to the agent) shocks to sales are additively separable from the effect of effort.

Condition 1 is a fairly innocuous restriction that more effort result in more sales. Monotonicity of the sales function in effort enables inversion of the effort policy function from observed sales data. Condition 2 implies that the quota, cumulative sales or months of the quarter do not have a direct effect on sales, over and above their effect on the agent’s effort. As is discussed in more detail below, this “exclusion” restriction is facilitates nonparametric identification of effort from sales data. Condition 2 rules out reputation effects for the agent (the fact that an agent has achieved high sales in the quarter does not make him more likely to achieve higher sales today); and also rules out direct end-of-the-quarter effects on sales (we find support for these restrictions in our data). Condition 3 is a standard econometric assumption. Based on the above, we consider sales-functions of the form,

$$ q_t = g(e_t; z, \mu) + \varepsilon_t $$ (1)

where, $g(.)$ is the sales production function, such that $\frac{\partial g(e_t)}{\partial e_t} > 0$, $\mu$ is a vector of parameters indexing $g(.)$; $z$ is a vector of observed factors (such as the number and type of clients in an agent’s sales-territory) that affects his demand; and $\varepsilon_t$ is a mean-zero agent and month specific shock to demand that is realized at the end of the period, which is unobserved by the agent at the time of making his effort decision.
We assume that \( \varepsilon_t \) is distributed I.I.D. over agents and time-periods with distribution \( G_\varepsilon(.) \), to the estimated from the data. \( \varepsilon_t \) serves as the econometric error term in our empirical model (we present our econometric assumptions in detail in §4.1). In our empirical work, we will consider specifications in which the production function \( g(.) \) is heterogeneous across agents. For now, we suppress the subscript “i” for agent for expositional clarity.

3.2 Per-period utility

The agents’ utility is derived from his compensation, which is determined by the incentive scheme. We write the agent’s monthly wealth from the firm as, \( W_t = W(s_t, e_t, \varepsilon_t, \mu, \Psi) \). We model his utility each month as derived from the wealth from the firm minus the cost of exerting effort. We denote the cost function as \( C(e_t; d) \), where \( d \) is a parameter to be estimated. We assume that agents are risk-averse, and that conditional on \( \chi_t = 1 \), their per-period utility function is,

\[
\begin{align*}
    u_t &= u(Q_t, a_t, I_t, \chi_t = 1) = E\{W_t\} - r \text{ var}\{W_t\} - C(e_t; d) \\
\end{align*}
\]

Here, \( r \) is the agent’s coefficient of constant absolute risk aversion, and the expectation and variance of wealth is taken with respect to the demand shocks, \( \varepsilon_t \). The specification in equation (2) is attractive since it can be regarded as a second order approximation to an arbitrary utility function.\(^{11}\) We now discuss the transition of the state variables that generate the dynamics in the agent’s effort allocation problem. The payoff from leaving the focal firm and pursuing the outside option is normalized to zero,

\[
\begin{align*}
    u_t &= u(Q_t, a_t, I_t, \chi_t = 0) = 0 \\
\end{align*}
\]

3.3 State Transitions

There are two sources of dynamics in the model. The non-linearity in the compensation scheme generates a dynamic into the agent’s problem because reducing current effort increases the chance to cross, say, the quota threshold tomorrow. A second dynamic is introduced since the agent’s current effort also affects the probability that his compensation structure is updated in the future. Hence, in allocating his effort

\(^{11}\)In case of the standard linear compensation plan, exponential CARA utilities and normal errors this specification corresponds to an exact representation of the agent’s certainty equivalent utility..
each period, the agent also needs to take into account how current actions affect his expected future compensation structure. These aspects are embedded in the transitions of the state variables in the model. In the remainder of this section, we discuss these transitions. Subsequently, we present the value functions that encapsulate the optimal intertemporal decisions of the agent.

The first state variable, total sales, is augmented by the realized sales each month, except at the end of the quarter, when the agent begins with a fresh sales schedule, i.e.,
\[
Q_{t+1} = \begin{cases} 
Q_t + q_t & \text{if } I_t < N \\
0 & \text{if } I_t = N
\end{cases}
\]  

We assume that the agent has rational expectations about the transition of his quota, \( a_t \). We use the observed empirical data on the evolution the agent’s quotas to obtain the transition density of quotas over time. We estimate the following transition function that relates the updated quota to the current quota, as well as the performance of the agent relative to that quota in the current quarter,
\[
a_{t+1} = \sum_{k=1}^{K} \theta_k \Gamma (a_t, Q_t + q_t) + v_{t+1} \quad \text{if } I_t < N
\]
\[
= 0 \quad \text{if } I_t = N
\]

In equation (5) above, we allow the new quota to depend flexibly on \( a_t \) and \( Q_t + q_t \), via a \( K \)-order polynomial basis indexed by parameters, \( \theta_k \). The term \( v_{t+1} \) is an I.I.D. random variate which is unobserved by the agent in month \( t \). The distribution of \( v_{t+1} \) is denoted \( G(v, .) \), and will be estimated from the data. Allowing for \( v_{t+1} \) in the transition equation enables us to introduce uncertainty into the agent’s problem. In our empirical work, we extensively test different specifications for the ratcheting policy, and provide evidence that the associated errors \( v_{t+1} \) are not serially correlated in the specifications we use. Lack of persistence in \( v_{t+1} \) imply that all sources of time-dependence in the agent’s quota updating have been captured, and that the remaining variation is white noise.

The transition of the third state variable, months since the beginning of the quarter, is deterministic,
\[
I_{t+1} = \begin{cases} 
I_t + 1 & \text{if } I_t < N \\
1 & \text{if } I_t = N
\end{cases}
\]  

Finally, the agent’s employment status in \( (t + 1) \), depends on whether he decides to

\[\text{We use this flexible polynomial to capture in a reduced-form way, the manager’s policy for updating agents’ quotas.}\]
leave the firm in period $t$. The employment state tomorrow is thus a control variable for the agent today, and is described below.

### 3.4 Optimal Actions

Given the above state-transitions, we can write the agent’s problem as choosing effort to maximize the present-discounted value of utility each period, where future utilities are discounted by the factor, $\rho$. We collect all the parameters describing the agent’s preferences and transitions in a vector $\Omega = \{\mu, d, r, G_{\varepsilon}(.), G_v(.) , \theta_{k,k=1,...,K}\}$. In month $I_t < N$, the agent’s present-discounted utility under the optimal effort policy can be represented by a value function that satisfies the following Bellman equation,

$$V(Q_t, a_t, I_t, \chi_t; \Omega, \Psi) =$$

$$\max_{\chi_{t+1} \in (0,1), e > 0} \left\{ \begin{array}{l}
  u(Q_t, a_t, I_t, \chi_t, e; \Omega, \Psi) \\
  + \rho \int_{\varepsilon} V(Q_{t+1} = Q_t, q(\varepsilon_t, e), a_{t+1} = a_t, I_t + 1, \chi_{t+1}; \Omega, \Psi) f(\varepsilon_t) d\varepsilon_t 
\end{array} \right\}$$

The value in period $I_t + 1$ is stochastic from period $I_t$’s perspective because the effort in period $I_t$ is decided prior to the realization of $\varepsilon_t$, which introduces uncertainty into the cumulative sales attainable next period. Hence, the Bellman equation involves an expectation of the $(I_t + 1)$–period value function against the distribution of $\varepsilon_t$, evaluated at the states tomorrow. Similarly, the Bellman equation determining effort in the last period of the sales-cycle is,

$$V(Q_t, a_t, N, \chi_t; \Omega, \Psi) =$$

$$\max_{\chi_{t+1} \in (0,1), e > 0} \left\{ \begin{array}{l}
  u(Q_t, a_t, N, \chi_t, e; \Omega, \Psi) \\
  + \rho \int_{\varepsilon} \int_{v_t} V(Q_{t+1} = 0, a_{t+1} = a_t, \chi_{t+1}; \Omega, \Psi) f(\varepsilon_t) f(v_{t+1}) d\varepsilon_t dv_{t+1} 
\end{array} \right\}$$

At the end of the sales-cycle, the cumulative sales is reset and the quota is updated. The value in the beginning of the next cycle is again stochastic from the current perspective on account of the uncertainty introduced into the ratcheted future quota by the demand shock, $\varepsilon_t$, and the quota-shock, $v_{t+1}$. Hence, the Bellman equation in (8) involves an expectation of the $1^{st}$ period value function against the distribution of both $\varepsilon_t$ and $v_{t+1}$.
Conditional on staying with the firm, the optimal effort in period \( t \), \( e_t = e(s_t; \Omega, \Psi) \) maximizes the value function,

\[
e(s_t; \Omega, \Psi) = \arg \max_{e > 0} \{ V(s_t; \Omega, \Psi) \}
\]  

(9)

The agent stays with the firm if the value from employment is positive, i.e.,

\[
\chi_{t+1} = 1 \quad \text{if} \quad \max_{e > 0} \{ V(s_t; \Omega, \Psi) \} > 0
\]

Given the structure of the agent’s payoffs and transitions, it is not possible to solve for the value function analytically. We solve for the optimal effort policy numerically via modified policy iteration. The state-space for the problem is discrete-continuous, of dimension \( \mathbb{R}^2 \times (N + 1) \). The two continuous dimensions (\( Q_t \) and \( a_t \)) are discretized, and the value function is approximated over this grid for each discrete value of \( N \) and employment status. One iteration of the solution took 120 seconds on a standard Pentium PC. Further computational details of the algorithm are provided in Appendix A. We now present the technique for the estimation of the model parameters.

4 Empirical Strategy and Estimation

Our empirical strategy is motivated by the intended use of the model, which is to obtain a relative evaluation of the outcomes for the firm under a changed compensation scheme. This requires a method to simulate the outcomes for the firm under new compensation schemes. Consider a new compensation plan \( \varphi(q(e); \Psi) \), where \( \Psi \) indexes the parameters governing the features of the new plan (e.g. a revised salary, bonus, commission rate, quota etc.).\(^{13}\) The firm’s present discounted payoffs under \( \varphi(q(e); \Psi) \) are,

\[
\Pi_\varphi = \int \int \sum_{\tau=0}^{\infty} \beta^\tau [q(e_\varphi) - \varphi(q(e_\varphi))] \, d\mathcal{F}(\mu, r, d) \, d\mathcal{G}_\varepsilon (\varepsilon_\tau)
\]  

(10)

where \( (e_\varphi) \) is the effort policy expended by the agent when faced with compensation policy \( \varphi(q(e)) \),

\[
e_\varphi = \arg \max_{e > 0} V(s; e) \{ \mu, r, d \}, \varphi(.)
\]

\(^{13}\)Implicity, \( \Psi \) can be a function of the agent’s characteristics, \( \Psi = \Psi(\mu, r, d, \mathcal{G}_\varepsilon (\cdot)) \). For example, a counterfactual contract we consider is a linear contract characterized by a fixed salary and commission specific to each agent. In this contract, the optimal salary and commission rate are a function of the agent’s preferences. We suppress the dependence of \( \Psi \) on these features for notational simplicity.
In equation (10), $F(\mu, r, d)$ is the joint CDF across agents in the firm of demand parameters, risk aversion and the cost of effort. Our approach will be to use the model to simulate effort and sales under the counterfactual plans conditioning on estimates of $F(\mu, r, d)$ and $G(\varepsilon)$. A comparison of current policy quantities $\{\Pi^*, q^*, e^*\}$ to the counterfactual then facilitates a relative evaluation of the current plan to other potentially, better possibilities. The key object of econometric inference is thus the joint distribution of preferences, $F(\mu, r, d)$ and of demand uncertainty, $G(\varepsilon)$. In the section below, we discuss a methodology that delivers estimates of these distributions.

Our discussion below comprises two steps. In step 1, we discuss how we use the observed data on sales and compensation plans across agents to estimate the parameters of the agents’ preferences, as well as the functions linking sales to effort. In step 2, we discuss how we use these parameters, along with our dynamic programming (henceforth DP) solution to simulate the agent’s actions under counterfactual compensation profiles. In the remainder of this section, we first discuss our econometric assumptions, and then present details on the specific compensation scheme in our data. Subsequently, we describe the procedure for estimation of the parameters of the model.

4.1 Econometric Assumptions

The econometric assumptions on the model are motivated by the nature of the data, as well as the intended procedure for estimation. The observed variation to be explained by the model is the observed correlation of sales across agents and months with the distance to their quotas, as well as the variation of sales across agents, which are a function of the agents’ effort. The computational challenge in estimation derives from the fact that the model implies that each agent’s effort, and consequently, their sales, are solutions to a dynamic problem that cannot be solved analytically.

One approach to estimation would be to nest the numerical solution of the associated DP into the estimation procedure. This would be significantly numerically

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14 Implicitly, in equation (10), we assume that the distribution of demand shocks, $G(\varepsilon)$ stays the same under the counterfactual. In equation (10), we do not intergate against the ratcheting shocks $G_v(\varepsilon)$, because all the counterfactual contracts we consider involve no ratcheting. Consideration of counterfactual contracts that involve ratcheting would require a model for agents’ belief formation about quota updating under the new compensation profile, which is outside of the scope of the current analysis. Future research could consider solving for the optimal quota updating policy, under the assumption that agents’ beliefs regarding ratcheting are formed rationally. See Nair (2007) for one possible approach to solving for beliefs in this fashion applied to durable good pricing.
intensive since the DP has to be repeatedly solved for each guess of the parameter vector. Instead, our estimation method builds on recently developed methods for two-stage estimation of dynamic models (e.g. Hotz and Miller 1993; Bajari, Benkard and Levin 2007, henceforth BBL), which obviates the need to solve the DP repeatedly. Under this approach, agents’ policy functions - i.e., his optimal actions expressed as a function of his state - as well as the transition densities of the state variables are estimated non-parametrically in a first-stage; and subsequently, the parameters of the underlying model are estimated from the conditions for optimality of the chosen actions in the data. We face two difficulties in adapting this approach to our context. First, the relevant action - effort - is unobserved to the econometrician, and has to be inferred from the observed sales. This implies that we need a way to translate the sales policy function to an “effort policy function”. Second, unobserved agent heterogeneity is likely to be significant in this context, since we expect agents to vary significantly in their productivity. The dependence of sales on quotas induced by the compensation scheme generates a form of state dependence in sales over time, which in the absence of adequate controls for agent heterogeneity generates well-known biases in the estimates of the effort policy. However, handling unobserved heterogeneity in the context of 2-step Hotz-Miller type estimators has been difficult to date (there has been recent progress on this topic; please see Arcidiacono and Miller 2008).

We address both issues in our proposed method. To handle the first issue, we make a parametric assumption about the sales-production function. We discuss below why a non-parametric solution is not possible. We are able to handle the second issue due to the availability of sales-information at the agent-level of unusually large cross-section and duration, which enables us to estimate agent-specific policy functions, and to accommodate non-parametrically the heterogeneity across agents. We discuss the specific assumptions in more detail below.

4.1.1 Preliminaries

The model of agent optimization presented in §3 implies that the optimal effort each period is a function of only the current state $s_t$. To implement a two-step method, we thus need to estimate non-parametrically in a first-stage, the effort policy function, $e_t = \hat{e}(s_t)$. The effort policy function is obtained parametrically from the sales-policy function. To see the need for a parametric assumption, recall from §3 that we consider
sales-production functions of the form,

\[ q_t = g(e_t(s_t), z) + \varepsilon_t \]

For clarity, we suppress the variable \( z \), as the argument below holds for each value of \( z \). Let \( f(s_t) \equiv g(e_t(s_t)) \).

**Remark 1** If at least two observations on \( q \) are available for a given value of \( s \), the density of \( f(s) \) and \( \varepsilon \) are separately non-parametrically identified (Li and Vuong 1998).

**Remark 2** Given the density of \( f(s) \), only either \( g(s) \) or \( e(s) \) can be estimated non-parametrically.

Remark 2 underscores the need for a parametric assumption on the relationship between sales and effort. One option to relax this would be to obtain direct observations on agent’s effort, via say, survey data, or monitoring. This of course, changes the character of the principal-agent problem between the agent and the firm. Unobservability of agent effort is the crux of the moral hazard problem in designing compensation schemes. Hence, we view this parameterization as unavoidable in empirical models of salesforce compensation.

We now discuss how we use this assumption, along with the sales data to estimate the sale-production function. For each agent in the data, we observe sales at each of \( J \) clients, for a period of \( T \) months. In our empirical application \( T \) is 38 (i.e., about 3 years), and \( J \) is of the order of 60-300 for each agent. The client data adds cross-sectional variation to agent-level sales which aids estimation. To reflect this aspect of the data, we add the subscript \( j \) for client from this point onward. In light of remark 2 we assume that the production function at each client \( j \) is linear in effort,

\[ q_{jt} = h_j + e_t + \varepsilon_{jt} \]

\[ = h_j(z_j) + e(s_t) + \varepsilon_{jt} \]

The linear specification is easy to interpret: \( h_j \) can be interpreted as the agent’s time-invariant intrinsic “ability” to sell to client \( j \), which is shifted by client characteristics \( z_j \). We now let \( h_j \equiv \mu'z_j \), and let \( e(s_t) = \lambda'\theta(s_t) \), where \( \gamma \) is a \( R \times 1 \) vector of parameters indexing a flexible polynomial basis approximation to the monthly effort policy function. Then, the effort policy function satisfies,

\[ q_{jt} = \mu'z_j + \lambda'\theta(s_t) + \varepsilon_{jt} \]
We assume that $\varepsilon_{jt}$ is distributed I.I.D. across clients. We can then obtain the demand parameters and the effort policy function parameters from the following minimization routine,

$$\min_{\mu, \lambda} \| q_{jt} - (\mu' z_j + \lambda' \theta (s_t)) \|$$

As a by product, we also obtain the effort policy function for the month $t$ as,

$$\hat{\varepsilon}_t = \hat{\lambda}' \theta (s_t)$$

and the time-specific error distribution,

$$\hat{\varepsilon}_t = \sum_j \left( q_{jt} - (\hat{\mu}' z_j + \hat{\lambda}' \theta (s_t)) \right)$$

which is then used to estimate the empirical distribution of $\varepsilon_t$ for each agent.\(^{15}\) This distribution is an input to solving the dynamic programming problem associated with solution of the model for each agent. We sample with replacement from the estimated empirical distribution for this purpose.

Finally, at the end of this step, we can recover the predicted overall sales for the agent which determines the agent’s overall compensation. Summing equation (13) across clients, the overall sales in month $t$ is,

$$q_t = \sum_j q_{jt} = h + J e_t + \varepsilon_t$$

where, $h = \sum_{j=1}^{J} \hat{\mu}' z_j$, and $\varepsilon_t = \sum_{j=1}^{J} \hat{\varepsilon}_{jt}$.

**Intuition for estimation of effort:** Intuitively, we can think of identification of the effort policy by casting the estimator in equation (13) in two steps,

- **Step 1:** Estimate time-period fixed effects $\varpi_t$ as, $q_{jt} = \mu' z_j + \varpi_t + \varepsilon_{jt}$
- **Step 2:** Project $\varpi_t$ on a flexible function of the state variables as $\varpi_t = \lambda' \theta (s_t)$

The client-level data facilitates the estimation of time-period specific fixed effects in Step 1. Equation (13) combines steps 1 & 2 into one procedure. We discuss the identification of the model in further detail below.

We now discuss the specifics of the compensation scheme in our dataset, and derive the expression for the monthly expected wealth for the agent given the above econometric assumptions.

\(^{15}\)Alternatively, one could assume a parametric density for $\varepsilon$ and use maximum likelihood methods. The advantage of our nonparametric approach is that we avoid the possibility of extreme draws inherent in parametric densities and the pitfalls that go along with such draws.
4.2 Compensation scheme

The incentive scheme in our empirical application has two noteworthy features. First, the agent’s payout is determined based on his quarter-specific performance. Thus, $N = 3$, and cumulative sales, which affect the payout, are reset at the end of each quarter. Second, the commission scheme is non-linear, involving a salary, a quota and a ceiling. The monthly salary $\alpha$ is paid out to the agent irrespective of his sales. If his current cumulative sales are above quota, the agent receives a percentage of a fixed amount $\beta$ as commission. The percentage is determined as the proportion of sales above $a_t$, and below a maximum ceiling of $b_t$, that the agent achieves in the quarter. Beyond $b_t$, the agent receives no commission. For the firm in our empirical application, $\beta = \$20,000$, and the ceiling was always set 33% above the quota, i.e., $b_t = \frac{4}{3}a_t$. Figure 8 depicts the compensation scheme. We can write the agent’s wealth, $W(s_t,e_t,\varepsilon_t;\mu,\Psi)$ in equation (2) as,

$$W(s_t,e_t,\varepsilon_t;\mu,\Psi) = \alpha + \beta \left[ \left( \frac{Q_t+q_t-a_t}{b_t-a_t} \right) \mathbb{I}(a_t \leq Q_t + q_t \leq b_t) + \mathbb{I}(Q_t + q_t > b_t) \right] \mathbb{I}(I_t = N)$$

$$= \alpha + \beta \left[ 3 \left( \frac{Q_t+q_t-a_t}{a_t} \right) \mathbb{I}(a_t \leq Q_t + q_t \leq b_t) + \mathbb{I}(Q_t + q_t > b_t) \right] \mathbb{I}(I_t = N)$$

$$= \alpha + \beta \min \left\{ \frac{3(Q_t+q_t-a_t)}{a_t}, 1 \right\} \mathbb{I}(Q_t + q_t > a_t) \mathbb{I}(I_t = N) \tag{18}$$

Thus, at the end of each sales-cycle, the agent receives the salary $\alpha$, as well as a incentive component, $\beta \times \min \left\{ \frac{3(Q_t+q_t-a_t)}{a_t}, 1 \right\}$, on any sales in excess of quota. If it is not the end of the quarter, $\mathbb{I}(I_t = N) = 0$, and only the salary is received. Finally, assume that the cost function in (2), $C(e)$, is quadratic in effort, i.e. $C(e_t) = \frac{de_t^2}{2}$, where $d$ is a parameter to be estimated.

4.3 Estimation procedure

We now present the steps for estimation of the model parameters. The estimation consists of two steps, the first for a set of “auxiliary” parameters, and the second for a set of “dynamic parameters.” We discuss these in sequence below.
4.3.1 Step 1: Nonparametric estimation of policy function and state transitions

The goals of the first step are two-fold. First, we estimate the demand parameters $\mu$, as well as the distribution of demand shocks $G(v)\varepsilon_\tau$ for each agent. Second, we estimate an effort policy function, as well as transitions of the state variables for each agent. We use both set of objects to estimate $F(\mu, r, d)$ in step 2.

The effort policy function is related to observed sales via equation (13). The demand parameters and the demand shock distribution are obtained as by-products of estimating equation (13). We estimate the effort policy agent-by-agent. For each agent, data are pooled across the agent’s clients, and equation (13) estimated via least squares. An advantage of this approach is that we are able to handle heterogeneity across agents non-parametrically.

The next step is to estimate the parameters $(\theta_k, G(v, \cdot))$ describing the transition of the agent’s quotas in equation (5). This is a series estimator which we estimate via non-linear least squares. Since quotas vary only at the quarter-level, we do not estimate the quota transitions agent-by-agent. Instead, we pool the data across agents to
estimate the quota transition function allowing for agent fixed-effects. The distribution of ratcheting shocks, $G_v(.)$, are estimated non-parametrically from the residuals from this regression.

The law of motion of the other state variables (month of the quarter) does not have to be estimated since it does not involve any unknown parameters. This concludes step 1. Since we have estimated $\mu$ agent by agent, we can construct its marginal CDF ($F(\mu)$) using a simple estimator,

$$F(\mu) = \frac{1}{N} \sum_{i=1}^{N} I(\mu_i \leq \mu).$$ \hspace{1cm} (19)

The only remaining object to be estimated is the conditional distribution of the risk aversion, $r$, and the cost parameter, $d$, $F(r,d|\mu)$. Step 2 below delivers estimates of $F(r,d|\mu)$.

4.3.2 Step 2: Estimation of $F(r,d|\mu)$

We estimate the “dynamic” parameters $r$ and $d$ using the methods proposed in BBL for the case of continuous controls. The BBL estimator is a minimum distance estimator that finds parameters which minimize a set of moment inequality conditions. We propose to estimate the parameters by imposing two moment conditions that arise naturally in the class of principal-agent problems. In particular, let $s_0$ be an initial state for an agent, $(r^*, d^*)$ be the true parameters, and $e^*$ the optimal effort policy at the true parameters. Then, $(r^*, d^*)$ must satisfy,

1. Individual Rationality (IR): $V(s_0; e^*, r^*, d^*) \geq 0$

2. Incentive Compatibility (IC): $V(s_0; e^*, r^*, d^*) \geq V(s_0; e', r^*, d^*)$

where $V(s_0; e^*, r^*, d^*)$ is the value function corresponding to the optimal policy $e^*$, and $V(s_0; e', \theta^*)$ is the present discounted utility corresponding to any other feasible policy, $e' \neq e^*$. The IR constraint says that the agent should at least be as better off working with the firm, as leaving the firm and pursuing his outside option. The IC constraint says that the agent should obtain higher utility in present discounted terms under the optimal effort policy, compared to any other feasible effort policy. Following BBL, we propose to estimate $r^*, d^*$ by finding the set of parameters that minimize violations of these conditions over a random sample of the state space. In
what follows, we assume that the optimal policy function \( e^* = e^* (s_0) \) has already been estimated in step 1, and is available to the econometrician. Begin by defining the following quantities,

\[
Z(s_0; e^*) = \begin{bmatrix} \mathbb{E}(W) & \nabla(W) & \mathbb{C}(e) \end{bmatrix}
\]

\[
\theta = \begin{bmatrix} 1 & r & d \end{bmatrix}
\]

(20)

(21)

where \( \theta \) are parameters of interest, \( s_0 \) is an initial state, \( e^* \) is the estimated optimal effort policy function and \( Z(s_0; e^*) \) has components,

\[
\mathbb{E}(W) = E_{e^*|s_0} \sum_{t=0}^{\infty} \beta^t E_{e^*} [W(s, e^* (s))]
\]

\[
\nabla(W) = E_{e^*|s_0} \sum_{t=0}^{\infty} \beta^t E_{e^*} [W(s, e^* (s))^2 - E_{e^*} [W(s, e^* (s))]^2]
\]

\[
\mathbb{C}(e) = E_{e^*|s_0} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t e^* (s)^2
\]

(22)

(23)

The value function based on the optimal effort policy can then be expressed as,

\[
V(s_0; e^*, \theta) = Z(s_0; e^*)' \theta
\]

(24)

Similarly, for any alternative policy function \((e' \neq e^*)\), the perturbed value function is,

\[
V(s_0; e', \theta) = Z(s_0; e')' \theta
\]

(25)

Define the following two moment conditions,

\[
g_1(s_0; \theta) = \min (V(s_0; e^*, \theta), 0)
\]

\[
g_2(s_0, e'; \theta) = \min (V(s_0; e^*, \theta) - V(s_0; e', \theta), 0)
\]

(26)

and let \( g(s_0, e'; \theta) = [g_1(s_0; \theta) \ g_2(s_0; \theta)]' \).

Let \( H(.) \) be a sampling distribution over states \( s_0 \) and alternative feasible policies \( e' \). Define an objective function,

\[
Q(\theta) = \int [g(s_0, e'; \theta)]' \Lambda [g(s_0, e'; \theta)] dH(s_0, e')
\]

(27)

where, \( \Lambda \) is a \( 2 \times 2 \) weighting matrix. Clearly, the true parameter vector \((\theta = \theta^*)\) must satisfy,

\[
Q(\theta^*) = \min_{\theta} (Q(\theta)) = 0
\]

(28)
Following BBL, we estimate $\theta^*$ by minimizing the sample analog of $Q(\theta)$. The function is $Q(\theta)$ is obtained by averaging its evaluations over $NR$ I.I.D. draws of $s_0$ from a uniform distribution over the observed support of states for the agent. At each $s_0$, we generate alternative feasible policies by adding a normal error term to the estimated optimal effort policy. Using these, we forward simulate the terms in equation (22) to evaluate the moments at each guess of the parameter vector. The linearity of the value functions in $\theta$ imply that we can pre-compute $Z(s_0; e^*)$ and $Z(s_0; e')$ prior to parameter search, reducing computational time. In principle, an “optimal” $\Lambda$ that weights each of the moment conditions based on their informativeness about $\theta$ would give the most efficient estimates. However, the econometric theory for the optimal $\Lambda$ for inequality estimators of this sort are still to be developed. Hence, in practice, we set $\Lambda$ equal to the identity matrix. This yields consistent but potentially inefficient estimates. Further computational details of our estimation procedure are presented in Appendix (A).

We perform estimation agent by agent. The main computational burden arises from forward-simulating value functions and implementing the non-linear search separately for each agent (i.e. we solve 87 separate minimization problems). For each, we obtain point estimates of $r, d|\mu$. We use these to construct a non-parametric estimate of the CDF across agents, $F(r, d|\mu)$ as in the earlier section.

In general, the approach above yields point estimates of the parameters. Point estimation implies that the optimizer finds no other value of $\theta$ other than $\theta^*$ for which $Q(\theta) = 0$. A critical determinant to the point identification of the parameters is $H(.)$. In particular, $H(.)$ has to have large enough support over the states and alternative feasible effort policies to yield identification. This in turn requires that we a) pick the alternative feasible policies “intelligently”, such that they are informative of $\theta$; and b) more importantly, the econometrician has access to sufficient data (i.e. state points), on which non-parametric estimates of the optimal policy are available, and from which $s_0$-$s$ can be sampled. In application, we found that perturbations of the effort policy that were too far away from the effort policy were uninformative of the parameter vector. We use “small” perturbations (see Appendix (A) for precise details), which combined with the richness of our data, yield point identification of the parameters in our context for all the agents in the data.
4.4 Discussion: Identification

We now provide a more detailed discussion of identification in our model. In particular, we discuss how intertemporal linkages in observed sales identifies an agent’s unobserved effort allocation over time. The first concern is that effort has to be inferred from sales. In particular, looking at equation (11), we see that sales is explained by two unobservables, the first, effort, and the second, client-specific demand shocks. How can the data sort between the effects of either? The key identifying assumptions are,

1. Effort is a deterministic function of only the state variables.  
2. Effort is not client specific - i.e., the agent allocates the same effort to each client in a given month.

We believe the first assumption is valid since we believe we have captured the key relevant state variables generating the intertemporal variation in agent effort. Further, after including a rich-enough polynomial in the state variables in equation (11), we can reject serial correlation in the residuals, εjt (i.e. the remaining variation in sales is only white noise). Assumption 1 is also consistent with our dynamic programming model which generates a deterministic policy by construction. We believe the second assumption is reasonable. In separate analysis (not reported), we use limited data on the number of sales calls made by agents to each of the clients to check the validity of this assumption. Our analysis of these data finds that the allocation of calls across clients is not significantly related to the quotas and past performance, suggesting that effort more broadly, is not being tailored to each individual client.

Given these two assumptions, effort is identified by the joint distribution over time of the agent’s current sales, and the extent to which cumulative sales are below or above the quota and the ceiling. To see this, recall that the optimal policy implies that the agent expends high effort when he is close to the quota, irrespective of month. The agent expends low effort when he has either crossed the ceiling in a given quarter, or when he is very far away from the quota in an early month. Under the former situation, the marginal benefit of an additional unit of effort is higher when expended in the next quarter; the same is true under the latter, since he has very little chance of reaching the quota in the current quarter. The model assumes
that sales are strictly increasing in effort. Hence, if we see an agent achieve high sales across clients when he is close to the quota we conclude that effort is high. If we see low sales early on in the quarter, and when the quarter’s sales have crossed the ceiling, we conclude that effort is low. Our identification argument is based essentially on the fact that variation in effort over time is related to variation in the distance to quota over time, and is similar to the identification of productivity shocks in the production economics literature (see e.g. Olley and Pakes 1996; Ackerberg, Caves and Frazer 2006). In addition to the dynamic patterns observed in sales the identification of effort is also aided by the manner in which it enters the sales function. Note that, in our model, effort is implemented as the ‘effectiveness’ of detailing. Since the marginal effect of detailing (given adequate data) can be non-parameterically identified across time and individuals, a flexible projection of these effects on state variables will give us a non-parametric (and consistent) estimate of the effort policy function.

A related concern is how the effect of ratcheting is identified separately from the intertemporal substitution induced by the quota structure. The data are able to sort between these two separate dynamics in the following way. The extent of decline in the agent’s observed sales after he crossed the ceiling in any quarter informs the model about the extent of intertemporal effort allocation induced by the quota structure. However, note that in the absence of ratcheting, effort, and hence, sales, should be strictly increasing between the quota and the ceiling. Hence, the extent of decline in the agent’s observed sales after he crosses the quota, and before he attains the ceiling informs the model about the extent to which ratcheting plays a role. Figure 9 depicts the identification argument pictorially. The two other key parameters that are estimated in step 3 above are the cost $(d)$ and risk aversion parameter $(r)$. The cost of effort parameter is identified from the fact that sales are above the intercept in the first two months of the quarter. That is, if effort were costless, it would be optimal to exert no effort in the first two months and meet any target in the third month alone. The fact that effort is costly induces a capacity constraint on how much sales can be generated in any given month. This, along with the structure of the sales response function, acts as the primary identification mechanism for the cost of effort parameter. Finally, the risk aversion parameter is identified by the degree to which effort (sales) changes due to changes in the variance of wealth. This variation in wealth is generated by within-agent factors that shift demand over time that are unrelated to the agent’s distance to quota.
Figure 9: Identification of intertemporal inefficiencies from sales profile
5 Data and Estimation Results

Table 10 presents summary statistics from our data. The salesforce has 87 salespeople who are about 43 years of age on average, and have been with the firm for approximately 9 years. The firm did not significantly hire, nor have significant employee turnover in this sales-department during the time-period of the data.\footnote{So as to avoid concerns about learning-on-the job, and its interactions with quotas, 5 sales-agents, who had been with the firm for less than 2 years were dropped from the data.} The average salesperson in the salesforce earns \$67,632 per annum via a fixed salary component. The annual salary ranges across the salesforce from around \$50,000 to about \$90,000. The firm’s output-based compensation is calibrated such that, on a net basis, it pays out a maximum of \$20,000 per agent per quarter, if the agent achieves 133\% of their quarterly quota. On an average this implies that the agent has a 77\%-23\% split between fixed salary and incentive components if they achieve all targets. This is roughly what is achieved in the data. Across agents-quarters in the data, the average proportion of quarterly payout due to incentives is 16.8\% (std. dev. 20.9\%). Agents differ in terms of the number of clients they have, but are balanced in terms of the type of clients and the total calls they are required to make. For example, a particular salesperson may have a small number of clients but may be required to call on them more frequently, while another may have many more clients but may be asked by the firm to call on each of them relatively infrequently. The firm attempts to ensure that the total number of calls is balanced across agents.\footnote{For a more involved discussion of calls and the role it plays, along with empirical support of agents’ adherence to management policy regarding calls, see Appendix A.}

The mean quota for the salesforce is about \$357,116 per quarter. The mean attained sales stands at \$381,210, suggesting that the firm does a fairly good job of calibrating quotas and effort levels. This is further evidenced by the fact that the range and dispersion parameters of the cumulative sales at the end of the quarter and the quota levels are also fairly close. Figure (10) plots a histogram across agent-months of the extent to which agents beat the quota. We see that agents beat the quota frequently (mean=0.07). Conditional on beating the quota, many of the agent-months do not feature crossing the ceiling. However, the range is large, and ceiling is also crossed a few times, perhaps due to unanticipated demand shocks.

From Table (10), it appears on average that the firm adopts an asymmetric ratcheting approach to quota setting. When salespeople beat quotas the average increase
in subsequent quarter quotas is about 10%, but on the flip side, falling short of
quotas only reduces the next quarter quota by about 5.5%. This is consistent with
some other earlier studies (e.g. Leone, Misra and Zimmerman 2005) that document
such behavior at other firms, and is also consistent with our conversations with the
firm management. Finally, the table documents that monthly sales average about
$138,149, a fairly significant sum.

5.1 Results from estimation

We now report the results from estimation. We first discuss the results from the first
stage, which includes estimation of the effort policy function, and the quota transition
process. Subsequently, we discuss the results from the estimation of the cost function
and risk aversion parameters.

5.1.1 Effort Policy

The effort policy function was estimated separately for each agent using a flexible
Chebychev polynomial basis approximation. We approximate the effort policy using
the tensor product of basis functions of dimension 2 in each of the two continuous
state variables (cumulative sales and quota), allowing month specific intercepts, and
allowing the first two basis functions to be month specific. We find that this specifi-
cation fits the data very well. On average, we are able to explain about 79% of the variation in observed sales. Figure (11) plots a histogram of the $R^2$ values from the estimation across agents.

![Histogram of $R^2$ values from First-stage Effort Policy Estimation](image)

Figure 11: Histogram of $R^2$ values from First-stage Effort Policy Estimation

Rather then present estimates of the parameters of the basis functions approximating the effort policy, we present the estimates in graphical form. Figure 12 and Figure 13 represent the average estimated policy across salespeople using a perspective and its contour plot. Looking at Figure 12 and Figure 13, we see that the data shows a clear pattern whereby effort tends to increase in quotas, which supports the “effort inducement” motivation for quotas noted by the theory. The variation of effort with cumulative sales is also intuitive. When cumulative sales are less than quota (areas to the left of the diagonal), the agent tends to increase effort. When cumulative sales are much greater than quota (areas to the right of the diagonal line), there is little incentive for the agent to exert further effort, and sales decline.

We now present contours of the effort policy estimated for the individual agent. Figure 14 shows the contours for nine of the salespeople. We find that there is considerable heterogeneity across the salespeople, which is consistent with wide variation in agent productivity. At the same time, we find that the basic pattern described above remain true. Similar to the average contour plot discussed below, we see sales increase with quota but fall after cumulative sales have exceeded quota.
Figure 12: Estimated Effort Policy Function

Figure 13: Contours of the Estimated Effort Policy
Figure 14: Examples of estimated effort policy functions across salespeople.
5.1.2 Ratcheting Policy

We now discuss the estimated transition process for ratcheting. Figure (15) presents results from regressions in which we project the quota in quarter $\tau$ on flexible functions of agent’s sales and quotas in quarter $(\tau - 1)$. Due to the fact that quotas vary only at the quarter-level, we estimate a pooled specification with agent fixed-effects. We are able to explain about 78% of the variation in quotas over time. Figure (15) also reports Breusch-Godfrey statistics for tests of 1st and 2nd order serial correlation in the ratcheting errors. Lack of serial correlation will imply that our flexible specification has captured all sources of persistence in the manager’s quota updating policy. We see that with sufficient terms in the polynomial approximation, we are able to reject serial correlation in the ratcheting residuals.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Param</th>
<th>t-stat</th>
<th>Param</th>
<th>t-stat</th>
<th>Param</th>
<th>t-stat</th>
<th>Param*</th>
<th>t-stat*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.18</td>
<td>12.48</td>
<td>2.74</td>
<td>11.43</td>
<td>2.05</td>
<td>4.73</td>
<td>-0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td>$a(t-1)$</td>
<td>0.42</td>
<td>8.42</td>
<td>0.24</td>
<td>4.52</td>
<td>1.29</td>
<td>5.83</td>
<td>0.69</td>
<td>1.40</td>
</tr>
<tr>
<td>$Q(t-1)$</td>
<td>0.32</td>
<td>6.09</td>
<td>0.17</td>
<td>2.92</td>
<td>-0.59</td>
<td>-2.57</td>
<td>1.42</td>
<td>7.76</td>
</tr>
<tr>
<td>$a(t-1)^2$</td>
<td></td>
<td>-0.12</td>
<td>-4.89</td>
<td>-0.08</td>
<td>-0.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q(t-1)^2$</td>
<td></td>
<td>0.09</td>
<td>3.22</td>
<td>-0.33</td>
<td>-13.62</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a(t-1)^3$</td>
<td></td>
<td></td>
<td>0.00</td>
<td>0.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q(t-1)^3$</td>
<td></td>
<td>0.04</td>
<td>30.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Agent fixed effects included? | N | Y | Y | Y

$R^2$ | 0.481 | 0.565 | 0.576 | 0.785

Breusch-Godfrey (1) p-value* | 0.000 | 0.000 | 0.003 | 0.136

Breusch-Godfrey (2) p-value** | 0.000 | 0.000 | 0.006 | 0.236

*Preferred specification. Nobs = 1,044. Quotas and sales have been normalized to 100,000-s of $s$. *Tests against the null of zero 1st order serial correlation in the presence of a lagged dependent variable. **Tests against the null of zero 2nd order serial correlation in the presence of a lagged dependent variable.

Figure 15: Estimation of the Ratcheting Policy

5.1.3 Second Stage Parameter Estimates

The remaining elements needed for the evaluation of counterfactuals is an estimate for the joint distribution of the cost of effort ($d$) and risk aversion parameters ($r$). In this section we present estimates conditioned on the point estimates of $\mu_i$. Figure (16) presents the estimated joint PDF of the two parameters.

We find that there is a large amount of heterogeneity on both parameters. This is especially the case for the risk aversion parameter that varies quite widely from about 0.0018 to approximately 0.33. The large values are primarily on account of the fact...
that for some salespeople the variance of earnings across the sample period is low, resulting in risk aversion parameters that are correspondingly large. A more appropriate construct to examine is the monthly average risk premium \( \frac{1}{T} V(W) \), which has a mean of around $341.22 (median = $281.36).

The density of the cost of effort parameter is much tighter with a mean of 0.0508 (median = 0.0471). There is still substantial heterogeneity in \( d \) as well with values ranging from 0.02 through 0.16. The parameters values translate approximately to a mean (across the salesforce) cost of effort \( \frac{1}{T} C(e) \) of around $4015.6 per month. While not reported here, standard errors were computed using a bootstrap approach and are available from the authors upon request. The cost of effort parameter \( (d) \) was significant for all agents at the 0.05 level while the risk aversion parameter \( (r) \) was significantly different from zero for 81 out of the 87 salespeople at the 0.05 level.

The characterization of \( F(d, r|\mu) \) completes the discussion of the estimation of various parameters needed for policy experiments. In what follows, we discuss the solution of the dynamic programming problem faced by the salesperson and follow that up with a discussion of counterfactuals.
6 Results from the Dynamic Model

We now discuss the results from simulations of the optimal policy. The optimal policy is an output from the DP, taking the parameter estimates as given. We discuss the main qualitative features of the solution, and present simulations comparing output under the observed compensation scheme to the first-best, as well as to the linear plan suggested by Holmstrom and Milgrom (1987).

6.1 Optimal Policy

The optimal policy evaluated for a representative agent is presented in Figure 17. The optimal policy predicts the agent exerts positive effort when cumulative sales are close to quota. When the agent is past quota, the incentive to put in additional effort is zero, and effort is set to zero. When the cumulative sales are far less than quota, the chance of making quota in the current quarter is low, and the agent again sets effort to zero, preferring to postpone effort to the next quarter.

The value function for the agent is plotted in Figure 18. Consistent with the pattern in the policy function, we see that the agent generates value from expending effort only in a range around quota. At the current level of effort, the agent’s values

---

18 The simulation below use a risk aversion of 0, and set the cost function parameter $d = 5000$. 

are all positive; the agent is thus making positive utility on a present discounted basis, which we interpret as some indication that the agent’s participation constraint is not binding.

We now compare the predictions from the dynamic model of effort to the sales patterns in the data. Recall that the model takes the demand-side parameters as given, and produces an effort policy for these estimates. Since we intend to use our model to generate counterfactual predictions of agent effort, we wish to verify that the output from our fairly complex dynamic model, computed for the observed compensation policy, is able to reproduce reasonably the patterns we see in the data (for similar arguments, see for example, Dube, Hitsch and Chintagunta 2005; Nair 2007). In figure 19, we plot the observed sales in the data versus the predicted sales from the model for the 3 months of the quarter. We simulate the predicted sales by a three steps procedure. In step 1, we make 1000 draws of a 3$\times$1 vector of demand shocks ($\varepsilon_t$), from the empirical distribution of the demand shocks estimated from the data (see §4.1.1). In step 2, we used the predicted effort policy from the model to simulate agent effort, as well as the associated realization of sales, for the three months of the quarter, for each vector of the demand shocks. For now, we use the
average quota in the data as the value of the quota state variable. Finally, in step 3 we average across the 1000 predicted sales histories to generate a monthly sales prediction from the model. In Figure 19, the prediction is plotted as the solid line, and the observed sales (averaged across all agents and quarters) is plotted as the dotted line. We see that the policy does a remarkably good job of replicating the patterns, as well as the level of sales, in the data. The predicted policy underpredicts sales slightly in the initial months of the quarter, and overpredicts in the last month; nevertheless, the basic pattern - that effort is ramped up in the last month of the quarter - is captured well by the model.

7 Counterfactuals

In what follows we compare the current compensation policy at the focal firm to a series of alternatives with the aim of uncovering the causes and remedies of distortions that it may be causing. These counterfactuals are based on the evaluation of the average quota in the data as the value of the quota state variable. Finally, in step 3 we average across the 1000 predicted sales histories to generate a monthly sales prediction from the model. In Figure 19, the prediction is plotted as the solid line, and the observed sales (averaged across all agents and quarters) is plotted as the dotted line. We see that the policy does a remarkably good job of replicating the patterns, as well as the level of sales, in the data. The predicted policy underpredicts sales slightly in the initial months of the quarter, and overpredicts in the last month; nevertheless, the basic pattern - that effort is ramped up in the last month of the quarter - is captured well by the model.

We plan to update this by running the simulation agent-by-agent, and also by incorporating ratcheting.
firms expected profits as defined in equation (10) under each alternate compensation policy. The empirical analog of (10) is constructed as follows:

\[ \hat{\Pi}_\varphi = \frac{1}{T \times NS} \sum_{s=1}^{NS} \sum_{t=0}^{T} \beta^t [q(e_\varphi ; \Psi^s) - \varphi(q(e_\varphi) ; \Psi^s)] \]  

(29)

where \( \varphi(q(e) ; \Psi^s) \) is the compensation policy evaluated at a given draw \( \Psi^s \) from \( G_\varepsilon(\varepsilon_\tau) \times F(\mu, r, d) \) and \( (e_\varphi) \) is the effort policy expended by the agent when faced with compensation policy \( \varphi(q(e_\varphi) ; \Psi^s) \)

\[ e_\varphi = \arg \max_{e>0} V(s; e | \Psi^s, \varphi(.)) \]

For our counterfactuals we use \( T = 25 \) and \( NS = 500 \). We average out expected profits over \( T \) to convey results at the monthly level.

### 7.1 Quota Ratcheting

To assess the role of ratcheting we solved each agent’s dynamic program for the cases where ratcheting was present and when it was not. We solved to model conditioning on the current state-space at the focal firm. Our results indicate that moving to a no ratcheting policy would have no significant impact on the profitability of the firm. Repeated solutions based on different draws from the distributions of errors and parameters fell on both sides of zero with the average being about 0.251%. In dollar terms this turns out to be a difference of about $325 per agent per month, which is rather small considering that average revenues at the same level are over $135,000. This result suggests that the ratcheting policy in place at the firm has reached a steady state and there are no particular advantages (or disadvantages) from having it in place, apart from the costs of having to administer the scheme.

We note here that while the adoption of a ratcheting policy in our general DP solution induces slightly higher effort levels it does not change the overall nature of the effort policy function. That is, the low effort induced by the distance to quota or by cumulative sales being larger than the ceiling remains. Figure 20 presents the median agent’s effort policy function under the observed ratcheting policy. We find that the patterns in effort (and consequently in sales) are driven by within period

\[ \text{To guard against the influence of outliers we integrate the profit function only over the interquartile range and renormalize the results.} \]
considerations and are not affected to a significant degree by changes in the ratcheting policy.

To investigate the role of ratcheting further we also resolved the dynamic program for a number of different values of the initial state-space. We find that eliminating quota ratcheting results, on average, in a drop of about 2.83% in overall profits suggesting that ratcheting has a positive role (Lazear 1986) to play in the management of effort dynamics. A further examination of these counterfactuals reveals that the benefits of ratcheting accrue from the ability of the firm to rectify errors in the quota setting process. On the one hand, ratcheting permits the firm to raise quotas when an agent’s past performance suggests that quota attainment is easy. In the absence of such ratcheting the firm remains committed to the initial low quota which results in suboptimal incentive payments to the agent. On the flip side, if the quota is set too high, we see a ‘giving-up’ phenomena on the part of the agent which is again sub-optimal form the firms point of view. A ratcheting policy, by allowing the firm to adjust quotas dynamically, allows for such errors to be rectified.
7.2 Incentive Periodicity

The structure of information revelation (monthly) at the focal firm coupled with the periodicity of the incentive payments induces a distortive effect of the effort allocated by the agents. Put simply, it pays to play the “wait-and-see” game where agents allocate low levels of effort in early periods of the incentive horizon, gather demand shocks and then allocate higher effort in later periods. As long as the quota thresholds are not prohibitively high this strategy is optimal for the agent. It is, therefore, quite natural to inquire about the impact that the periodicity of the plan has on effort. To answer this we solved the agent’s problem by reducing the incentive horizon to one month. Under this counterfactual the agent no longer has the room to shirk since information about performance is only revealed at the end of the month.

Our counterfactuals suggest that reducing the periodicity of the compensation plan to be monthly would increase expected profits to the firm by 6.46%. For completeness, we also solved the agents problem by extending the periodicity to 6 and 12 months and found that profits were significantly lower. We note here, that the fact that expected profits are increasing in the frequency of incentive payouts is not necessarily a general result. In our application the selling cycles are quite short which facilitates the short incentive periods. In other industries where the selling cycle is longer (say for durable office products) it may indeed be optimal to lengthen the incentive horizon. Also note that we have assumed the increase in the frequency of incentive payouts is costless to the firm. Discussions with management at our focal firm revealed that the costs of moving from three months to one month were not a significant factor.

7.3 Linearity

The implementation of a linear (in output) compensation plan eliminates dynamics from the picture and, in the absence of discounting, permits aggregation for any arbitrary incentive horizon (see Holmstrom and Milgrom 1987). With reasonable levels ofdiscounting, linear plans still have the ability to smooth out effort (and consequently sales) within the incentive period. In what follows we implement three counterfactuals where we compare the firm’s existing compensation scheme to monthly, linear contracts which contain a fixed salary and a commission on sales. Each counterfactual places different assumptions on the nature of the linear plan and offers insights
into the impact of linearity on dynamic effort.

To begin, we solve for the linear compensation scheme which would obtain the equivalent sales as the current compensation policy used by the firm. We find that a linear commission rate of around 6.32% would give roughly the same level of dollar sales to the firm. Under this plan, incentives would increase which would also increase the risk premium needed to be paid out to the agent. Even with the firm lowering salaries (to match the IC constraint) overall profits would be slightly lower than the current plan. Note that a commission rate of 6.32% implies that the median salesperson would expect to earn over $22,000 in quarterly incentives compared to the current levels of $5000 if they attain the quota ceiling. In other words, the current plan does a good job of inducing strong incentives by using the non-linearity effectively.

As a next exercise, we computed the optimal linear compensation scheme for the firm. In this scheme both the salary and the incentive rate were allowed to vary across salespeople. In other words, each salesperson received a tailored plan. We found that under this compensation policy the optimal commission rate would range between 2.1% and to 10.61% with an average of about 7.61%. On average, under this new compensation policy, sales would improve by about 14.9% while profits would increase by approximately 11.1%.

Our final counterfactual, involves the simulation of a feasible plan as proposed by the firm. In this counterfactuals salaries are fixed at their current levels and we solve for a common commission rate for the salesforce as a whole. We find that the optimal commission under this scenario is about 6.81%. While the compensation costs to the firm are significantly higher (on account of the fixed salaries) the plan is still incrementally profitable compared to the current plan at the firm. Sales, under this feasible plan, would increase by about 8.6% with profits increasing by 5.2%.

A key feature of each of these plans is the inherent ability to remove dynamic distortions in effort. An examination of effort and sales patterns reveals that the scalloped pattern seen in the data (see Figure 1) has been eliminated.

\footnote{For this counterfactual we fixed each agent’s parameters rather that drawing from the joint distribution.}
8 Epilogue

Based on our analysis and counterfactuals we recommended that the firm alter its compensation policy. In particular we advocated the adoption of the feasible compensation scheme discussed earlier. As mentioned earlier, this plan predicts that sales would increase by around 8.6%. While we are not permitted reveal the exact details of the new compensation scheme, we can say that the firm adopted most of our suggestions. The features of the new scheme included,

1. No quotas or ratcheting
2. A monthly incentive periodicity
3. A linear commission rate on output.

The new compensation policy was put in place nationally for the new year beginning January 1, 2009. We have received data about the performance of individual agents in the first 5 months since the change. On average, sales are up by over 13% compared to a year ago in each month. In addition, sales are fairly flat with some wiggles on account of demand errors. Our calculations reveal that firm profits must be up by at least 11% over the same period assuming that there are no significant incremental costs involved with the adoption of the new compensation plan. Finally, the firm also reports that employee satisfaction with the new plan is high. This is important, since it reveals that the changes made were not simply simply reallocating surplus away from the agents but rather are increasing the total size.

9 Conclusions

This paper considers the dynamics induced by non-linear output based incentive schemes for compensating sales-force agents. Nonlinearities and kinks in the sales-force contract generates dynamic moral hazard that creates incentives for agents to postpone effort when they are “in the money” or have already “made quota”, or when agents shade effort to influence expected revisions in their quotas. Such intertemporal reallocation of effort can have adverse consequences for the firm. This paper seeks to provide evidence for, and measures of, the extent of such dynamic inefficiencies. The model free evidence suggest that quotas have a significant effect on the allocation of
Estimates from the dynamic model suggest that the extent of inefficiency is large, and suggest that re-optimizing the current compensation scheme would have large payoffs to the firm. Initial evidence from the focal firm reveal significant gains from adopting changes to the compensation policy based on the counterfactuals done.

10 References


Table 1: Descriptive Statistics of Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
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</thead>
<tbody>
<tr>
<td><strong>Agent Demographics</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Salary</td>
<td>$67,632.28</td>
<td>$8,585.13</td>
<td>$51,001.14</td>
<td>$88,149.78</td>
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<tr>
<td>Incentive Proportion at Ceiling</td>
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<td>0.02</td>
<td>0.8</td>
<td>0.28</td>
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<tr>
<td>Age</td>
<td>43.23</td>
<td>10.03</td>
<td>27</td>
<td>64</td>
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<tr>
<td>Tenure</td>
<td>9.08</td>
<td>8.42</td>
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<td>29</td>
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<td>Number of Clients</td>
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<td>19.09</td>
<td>63</td>
<td>314</td>
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<tr>
<td><strong>Quarter Level Variables</strong></td>
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<tr>
<td>Quota</td>
<td>$357,116</td>
<td>$95,680.74</td>
<td>$197,898.81</td>
<td>$801,966.82</td>
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<tr>
<td>Cumulative Sales (end of quarter)</td>
<td>$381,210</td>
<td>$89,947.66</td>
<td>$171,009.11</td>
<td>$767,040.98</td>
</tr>
<tr>
<td>Percent Change in Quota (when positive)</td>
<td>10.01%</td>
<td>12.48%</td>
<td>0.00%</td>
<td>92.51%</td>
</tr>
<tr>
<td>Percent Change in Quota (when negative)</td>
<td>-5.53%</td>
<td>10.15%</td>
<td>-53.81%</td>
<td>-0.00%</td>
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<tr>
<td><strong>Monthly Level Variables</strong></td>
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<td></td>
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<tr>
<td>Monthly Sales</td>
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<td>$383,19.34</td>
<td>$45,581.85</td>
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<td>Cumulative Sales (beginning of month)</td>
<td>$114,344</td>
<td>$985,94.65</td>
<td>$0</td>
<td>$65,247.25</td>
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<tr>
<td>Distance to Quota (beginning of month)</td>
<td>$278,858</td>
<td>$121,594.2</td>
<td>$20,245.52</td>
<td>$83,5361.10</td>
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<tr>
<td>Number of Salespeople</td>
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</table>
Appendix A: Computational Details

This appendix provides computational details of solving for the optimal policy function in equation (9) and for implementing the BBL estimator in equation (28).

Solution of Optimal Policy Function  The optimal effort policy was solved using modified policy iteration (see, for e.g., Rust 1996 for a discussion of the algorithm). The policy was approximated over the two continuous states using 10 points in each state dimension, and separately computed for each of the discrete states. The expectation over the distribution of the demand shocks $\varepsilon_t$ and the ratcheting shocks $v_{t+1}$ were implemented using Monte Carlo integration using 1000 draws from the empirical distribution of these variates for the agent. The maximization involved in computing the optimal policy was implemented using the highly efficient SNOPT solver, using a policy tolerance of 1E-5.

Estimation of Agent Parameters  We discuss numerical details of implementing the BBL estimator in equation (28). The estimation was implemented separately for each of the 87 agents. The main details relate to the sampling of the initial states, the generation of alternative feasible policies, and details related to forward simulation. For each, we sampled a set of 1002 initial state points uniformly between the minimum and maximum quota and cumulative sales observed for each agent, and across months of the quarter. At each of the sampled state points, we generated 500 alternative feasible policies by adding a normal variate with standard deviation of 0.35 to the estimated optimal effort policy from the first stage (effort is measured in 100,000-s of dollars). Alternative feasible policies generated by adding random variates with large variances (e.g. 5), or by adding noise terms to effort policies at only a small subset of state points, were found to be uninformative of the parameter vector. At each sampled state point, we simulated value functions for each of the 500 alternative feasible policies by forwards-simulating the model 36 periods ahead. The sample analog of the moment conditions are then obtained by averaging over the sampled states and alternative policies.